DE-CELERATION PARAMETER Q(Z) AND INFLATON $\phi(t)$. I.E. HOW TO LINK EARLY UNIVERSE INFLATION WITH RE ACCELERATION? LINKS TO RADI OF THE UNIVERSE FORMALISM?

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The case for a four dimensional graviton mass (non zero) influencing reacceleration of the universe in five dimensions is stated; with emphasis upon if five dimensional geometries as given below give us new physical insight as to cosmological evolution. A calculated inflaton $\phi(t)$ may partly re-emerge after fading out in the aftermath of inflation. The inflaton may be the source of re acceleration of the universe, especially if the effects of a re emergent inflaton are in tandem with the appearance of macro effects of a small graviton mass, leading to a speed up of the rate of expansion of the universe at red shit value of Z ~ .423. A final statement as to how and why the radius of the universe question may be affected by these deliberations is presented , in terms of if the graviton is either purely a field theoretic , or semi classical object, as via t'Hoofts deterministic QM is presented in the end, as an open question.

1 Introduction: What can be said about DM and DE?

We will start with a first-principle introduction to detection of gravitational wave density using the definition given by Maggiore [1]

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f) \Rightarrow h_0^2 \Omega_{gw}(f) \cong 3.6 \cdot \left[\frac{n_f}{10^{37}}\right] \cdot \left(\frac{f}{1kHz}\right)^4 \qquad (1)$$

Where n_f is the frequency-based numerical count of gravitons per unit phase space. The author suggests that n_f may depend upon the interaction of gravitons with neutrinos in plasma during early-universe nucleation, as modeled by M. Marklund *et al* [2], which is a supposition the author[3] is investigating for a modification of a joint KK tower of 5 dimensional gravitons, as given by Maartens[4] for DM. Assume the stretching of early relic neutrinos that would lead to the KK tower of gravitons--for when $\alpha < 0$, is[3],

$$m_n(Graviton) = \frac{n}{L} + 10^{-65} \text{ grams} \qquad (2)$$

Appendix I summarizes what can be stated about this formulation, in terms of its origins. . Also Eq. (3) will be the starting point used for a KK tower version of Eq. (4) below. So from Maarten's [5] 2005 paper,

$$\dot{a}^2 = \left[\left(\frac{\tilde{\kappa}^2}{3}\left[\rho + \frac{\rho^2}{2\lambda}\right]\right)a^2 + \frac{\Lambda \cdot a^2}{3} + \frac{m}{a^2} - K\right] \quad (3)$$

Maartens [4]also writes $\dot{H}^2 = \left[-\left(\frac{\tilde{\kappa}^2}{2}\cdot[p+\rho]\cdot\left[1+\frac{\rho^2}{\lambda}\right]\right) + \frac{\Lambda \cdot a^2}{3} - 2\frac{m}{a^4} + \frac{K}{a^2}\right]$.

Also, if $\rho \cong -P$, for red shift values z between zero to 1.0-1.5 with equality, $\rho = -P$, for z between zero to **.5.** $a \equiv [a_0 = 1]/(1+z)$. As given by Beckwith[3]

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \equiv -1 - \frac{\dot{H}}{H^2} = -1 + \frac{2}{1+\tilde{\kappa}^2[\rho/m]\cdot(1+z)^4\cdot(1+\rho\ \ /2\lambda)} \approx -1 + \frac{2}{2+\delta(z)} \quad (4)$$

Eq. (4) assumes $\Lambda = 0 = K$, and the net effect is to obtain, a substitute for DE, by presenting how gravitons with a small mass done with $\Lambda \neq 0$, even if curvature **K** =0

2 Consequences of small graviton mass for reacceleration of the universe

In a revision of Alves *et. al,* [6] Beckwith[3] used a higher-dimensional model of the brane world and Marsden[6] KK graviton towers. The density ρ of the brane world in the Friedman equation as used by Alves *et. al*[7] is use by Beckwith[3] for a non-zero graviton

$$\rho \equiv \rho_0 \cdot (1+z)^3 - \left[\frac{m_g \cdot (c=1)^6}{8\pi G(\hbar = 1)^2}\right]\cdot\left(\frac{1}{14\cdot(1+z)^3} + \frac{2}{5\cdot(1+z)^2} - \frac{1}{2}\right) \quad (5)$$

I.e. Eq. (3) above is making a joint DM and DE model, with all of Eq. (4) being for KK gravitons and DM, and 10^{-65} grams being a 4 dimensional DE. Eq. (4) is part of a KK graviton presentation of DM/ DE dynamics. Beckwith[8] found at $z \sim$ **. 4**, a billion years ago, that acceleration of the universe increased, as shown in Fig. 1.

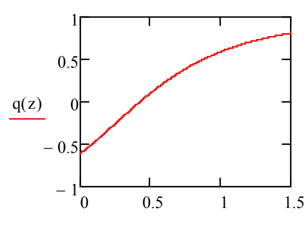

Fig. 1: Reacceleration of the universe based on Beckwith [3] **(note that q < 0 if z <.423)**

3. What if an inflaton re-emerges in space-time? At z ~ . 423?

Padmanabhan[7] has written up how the 2nd Friedman equation as of Eq. (5), which for **z ~. 423** may be simplified to read as $\dot{H}^2 \cong \left[-2\frac{m}{a^4}\right]$ would lead to an inflaton value of, when put in, for scale factor behavior as given by $a(t) \propto t^{\lambda}, \lambda = (1/2) - \varepsilon^{+}, 0 \le \varepsilon^{+} << 1$, of, for the inflaton[7] and inflation of

$$\phi(t) = \int dt \cdot \sqrt{-\frac{\dot{H}}{4\pi G}} \sim \sqrt{\frac{2m}{4\pi G}} \cdot \left[2\varepsilon^{+}\right] \cdot t^{2 \cdot \varepsilon^{+}} \qquad (6)$$

Which is assuming a decline of $a(t) \propto t^{\lambda}, \lambda = (1/2) - \varepsilon^{+}, 0 \le \varepsilon^{+} << 1$. As the scale factor of $a(t) \propto t^{\lambda}, \lambda = (1/2) - \varepsilon^{+}, 0 \le \varepsilon^{+} << 1$ had time of the value of roughly $a(t) \propto t^{\lambda}, \lambda = (1/2) - \varepsilon^{+}, 0 \le \varepsilon^{+} << 1$ have a power law relationship drop below $a(t) \propto t^{1/2}$, the inflaton took Eq. (7) 's value which may affect the increase in the rate of acceleration. We relate an energy state to the inflaton if $a(t) = a_0 t^{\lambda}$, then there is a potential of[7]

$$V(\phi) = V_0 \cdot \exp\left[-\sqrt{\frac{16\pi G}{\lambda}} \cdot \phi(t)\right] \qquad (7)$$

A situation where both $\lambda = (1/2) - \varepsilon^{+}$ grows smaller, and, temporarily, $\phi(t)$ takes on Eq. (7)'s value, even if the time value gets large, then there is infusion of energy by an amount dV. The entropy dS ≃dV/T, will lead, if there is an increase in V, as given by Eq. (6) a situation where there is an increase in entropy. If $S \approx N =$ number of graviton states[3,8] then we have an argument that the re emergence of an inflaton, with a reduction of Eq. (7) in magnitude may be part of gravitons playing a role in the re acceleration of the universe. Finally, Eq. (6) to Eq. (7) as combined with $S \approx N$ as referenced on pages 2 and 3 as a way to link graviton count with entropy may make inter connections between the inflaton picture of entropy generation and entropy connected/ generated with a numerical count of gravitons. What is needed is experimental verification of Eq. (6)

6. What can be said about entropy fluctuations and their role in graviton nucleation?

We offer for perusal, making use of Mukhanov's[8] book linking energy fluctuation and entropy. The bridge between early and later universe conditions will be raised, as far as making sense out of how quintessence arose as a factor initially, and also how its partial re appearance makes the Fig 1 graphics not so inexplicable.

To begin with the general expression as to fluctuations of entropy and entropy is, given by a de composition in Fourier space Mukhanov [8]writes as

$$\delta\varepsilon_k = -(\sigma \cdot k^2 \cdot \delta S_K)/(k^2 C_S^2 - 4\pi G\varepsilon_0) \quad (8)$$

The speed of sound, $C_S = 0$ in the present matter dominated era, and clearly is zero up through a billion years ago, which corresponds to red shift Z~ .423. As given by Lashkari and Brandenberger[9] in 2008, the non zero values of C_S, i.e. $C_S = 1/3$ in a radiation dominated era , according to string cosmology and string gas thermodynamics, with varying degrees of how C_S could approach 1, i.e. the speed of light as Z grew well above 1100. For the purpose of our demonstration of a bridge between entropy and gravitons, we will look at first what happens with $C_S = 0$, and then later comment upon the early universe era. To look at the situation a billion years ago, the following energy density formula will be utilized, utilizing in part Maarten's version of the Friedman equations

$$\varepsilon_0 = -\frac{3}{4\pi G} \cdot \left[-\kappa^2 \cdot \left[\frac{\rho}{6} + \frac{\rho^2}{3\lambda} + \frac{3P\rho}{2\lambda} \right] - \frac{m}{a^4} + \frac{\Lambda}{3} \right] = -\frac{3}{4\pi G} \cdot \left[\dot{H} + H^2 \right] \quad (9)$$

In the situation in which $P = -\rho$, the above simplifies to become for Z~.423, to Z ~0

$$\varepsilon_0\Big|_{Z=.423} = -\frac{3}{4\pi G} \cdot \left[-\frac{\kappa^2}{6}\rho \cdot \left[1 - \frac{\rho}{\lambda} \right] - \frac{m}{a^4} + \frac{\Lambda}{3} \right] = -\frac{3}{4\pi G} \cdot \left[\dot{H} + H^2 \right]. \quad (10)$$

Throw in the assumption made that the density, as given by Eq. (5) has a small graviton mass put in, and remove the cosmological constant, and then one has

$$\varepsilon_0\Big|_{Z=.423} = -\frac{3}{4\pi G} \cdot \left[-\frac{\kappa^2}{6}\rho \cdot \left[1 - \frac{\rho}{\lambda} \right] - \frac{m}{a^4} \right] \quad (11)$$

One then has, especially with $C_S \cong 0$, that when Z ~ .423 or smaller

$$\delta\varepsilon_k\Big|_{Z=.423} = (\sigma \cdot k^2 \cdot \delta S_K)/(3 \cdot \left[\frac{\kappa^2}{6}\rho \cdot \left[1 - \frac{\rho}{\lambda} \right] \right] + \frac{3m}{a^4}) \quad (12)$$

To first order, we assume, that $\frac{\rho}{\lambda} \approx .01$, and that so, if $k \approx a/L$ where a is the scale factor, and L is a physical "length" , that if L is very large, that of course, $k \approx a/L$ is not a major contributor, and that to a partial degree, one is seeing $\delta\varepsilon_k \propto \delta S_k$ in a positive sign contribution, as opposed to what happens in early universe cosmology, where $k \approx a/[L = l_{Planck}]$ where $l_{Planck} \propto 10^{-33}$ centimeters, so $k \approx a/\left[L = l_{Planck}\right]$ is enormous, so the following comes up, for large Z, say Z > 1100

$$\delta\varepsilon_k\Big|_{Z>1100} = -(\sigma \cdot \delta S_K)/(1 \geq C_S^2 > 1/9) \tag{13}$$

7. Quintessence $\tilde{\tilde{Q}}$, its relationship to expansion Q(a) and w(a) $\leftrightarrow V(\tilde{\tilde{Q}}(a))$

The issue of how quintessence $\tilde{\tilde{Q}}$ can be related to the inflaton $\phi(t)$ is not clear from most writing on the subject. Needless to say, we will present a first order link between the two, and how to reconstruct quintessence potentials and fields. Its relevance to inflaton physics, both in the beginning of inflation, and also to the problem of if inflaton re emergence is necessary for graviton contributions to re acceleration of the universe, a billion years ago. Caldwell and Kimonkowski [10] offer the following energy density value based upon a reconstructive value for w(a) which may be useful for explaining how gravitons contribute to re acceleration, with ρ_c a critical density value, $\Omega_{\tilde{\tilde{Q}}} \cong 1 - \Omega_m$, where $\Omega_m \leq 0.3 \Leftrightarrow w \leq 0.5$ in many cases, as given by Caldwell and Kimonkowski, i.e. looking at

$$\rho_{\tilde{\tilde{Q}}}(a) = \Omega_{\tilde{\tilde{Q}}}\rho_c \exp\left[3\int_a^{a)}[1 + w(a)] \cdot d\ln a\right] \tag{14}$$

This has, as noted by Caldwell and Kimonkowski [10] , some links with models of deceleration parameters of the form

$$q_0 = \frac{3\Omega_m}{1 + \Omega_m} - 1 \tag{15}$$

For what it is worth, the above presages that in the present era, that we have, to first order, $\rho_{\tilde{\tilde{Q}}}(a) \cong \Omega_{\tilde{\tilde{Q}}}\rho_c$, but our entire argument is with regards to having an effective mass of the inflaton is, in its own way, similar to a very small, non zero graviton mass. Ie. That of an effective mass $m_{\tilde{\tilde{Q}}} = \sqrt{\partial^2 V / \partial\tilde{\tilde{Q}}^2} << H$ of the inflaton. Here, in the range of very low varying, nearly constant $w(a) \neq -1,$, one can write

$$\partial^2 V / \partial\tilde{\tilde{Q}}^2 = -(3/2)(1 - w) \cdot \left[\dot{H} - (3/2) \cdot (1 + w)H^2\right] \tag{16}$$

For idenfication, the author will assume that

$$m \cong m_{\tilde{\tilde{Q}}} = \sqrt{\partial^2 V / \partial \tilde{\tilde{Q}}^2} << H \tag{17}$$

Whereas in the flat space solutions, FRW, one has

$$\dot{\rho}^{\bullet} + 4H\rho^{\bullet} = 0, \quad \rho^{\bullet} = \rho_0^{\bullet} \cdot [(a_0 = 1)/a]^4 \tag{18}$$

Leading to

$$m = \frac{\kappa^2}{3} \cdot \rho_0^* \cdot [a_0 \equiv 1]^4 \tag{19}$$

If there is a one to one situation where $w(a) \neq -1$, but is close to -1, one may be recovering a relationship between $\phi(t) \sim \sqrt{\frac{2m}{4\pi G}} \cdot [2\varepsilon^+] \cdot t^{2 \cdot \varepsilon^+}$ which slowly increases if Z ~ .423 and $\tilde{\tilde{Q}} \sim 3H\dot{\tilde{\tilde{Q}}} \Big/ \sqrt{\partial^2 V / \partial \tilde{\tilde{Q}}^2}$, i.e. if for large z one is setting the Hubble parameter $H = \left[\left(\frac{\tilde{\kappa}^2}{3} \left[\rho + \frac{\rho^2}{2\lambda} \right] \right) a^2 + \frac{m}{a^2} \right]^{1/2} \sim H_0$, and $a = 1/1 + z$, and a linear spatial fluctuation of the inflaton field governed by

$$\dot{\tilde{\tilde{Q}}} = (1/\dot{\delta}_m) \cdot \left[\delta\ddot{\tilde{\tilde{Q}}} + 3H\delta\dot{\tilde{\tilde{Q}}} + \left[V_{,\tilde{\tilde{Q}}\tilde{\tilde{Q}}} - \frac{1}{a^2}\nabla^2 . \right] \delta\tilde{\tilde{Q}} \right] \tag{17}$$

For what it is worth, we are assuming that when Z < .423, that $\delta\ddot{\tilde{\tilde{Q}}} \to 0$ and that to first order we are looking at $\dot{\tilde{\tilde{Q}}} \sim (1/\dot{\delta}_m) \cdot \left[\left[V_{,\tilde{\tilde{Q}}\tilde{\tilde{Q}}} \right] \delta\tilde{\tilde{Q}} \right]$, with $\delta\tilde{\tilde{Q}}$ a yet to be determined scalar fluctuation. Perhaps with a variant of a cosmic axion, or pseudo Nambu Goldstone Boson, as given by Caldwell and Kimonkowski [10] with a potential looking like

$$V = \mu^4 \cdot (1 - \cos\left[\delta\tilde{\tilde{Q}} \Big/ f \right] \ ) \tag{18}$$

. Also if we note that what is known as Axion monodromy , as given by a modification of a potential given by Bauman and McAllister [11] , may be used to present

$V = \mu^4 \cdot (1 - \cos\left[\delta\tilde{\tilde{Q}} / f\right])$ in terms of $\delta\tilde{\tilde{Q}} \equiv \varphi = \breve{a} f$, with $\breve{a}$ an axion , and with μ a dynamically driven scale, and $f > M_{Planck}$. The details of the axion monodromy are presented by McAllister, Silverstein, and Westphal [12] , and the remaining issue to resolve and look at would be to connect, as was brought up by Baumgart, Cheung, . Ruderman, Wang, and Yavin [13] constraints upon the evolution of axions and other DM models, so as to figure an inter relationship between an axion as an inflaton, as an example, and what Beckwith brought up in Eq. (2) above for DM. Couplings between the inflaton, as presented above, and other degrees of freedom, as related to by Bauman and McAllister [11] would be important to the problem of if there is a decay process, which in some sense reverses itself to a degree later, allowing for re acceleration of the universe.

8. What if the inflaton, and quintessence are manifestations of a complex field?

As brought up by Yurov[14], the following field is alleged to take on both inflaton and quintessence phenomenology.

$$\Phi(t) = \phi(t) \exp(\, i\,\theta(t)) / \sqrt{2} \tag{20}$$

In Yurov's [14]model, the above dual use, complex scalar field is part of a relatively simple chaotic potential he writes as, assuming cyclic behavior with $M = \Phi^2 \cdot \dot{\theta} =$ a constant value, that

$$V = \ddot{m}^2 \Phi^* \Phi \tag{21}$$

Making an equivalence between what Yurov is doing, and what was done, as borrowed from Beckwith would be in making a 1-1 identification between Eq. (3) above, and $\dot{H}^2 = \left[-\left(\frac{\tilde{\kappa}^2}{2} \cdot [p + \rho] \cdot \left[1 + \frac{\rho^2}{\lambda} \right] \right) + \frac{\Lambda \cdot a^2}{3} - 2\frac{m}{a^4} + \frac{K}{a^2} \right]$ with what Yurov postulated for Eq. (12) of Yurov's [24] manuscript for re acceleration of the universe one billion years ago, first starting with his so called Ricatti equation (after Eq. 2 of his manuscript)

$$\dot{H} + 3H^2 = V \tag{22}$$

As well as his Eqn. (1) values of

$$H^2 + \frac{\widehat{k}}{a^2} = \frac{1}{6} \cdot \left[\dot{\phi}^2 + \ddot{m}^2 \phi^2 + \frac{M^2}{\phi^2} \right] \tag{23}$$

The second inflation resulting in re acceleration which Yurov [14] postulates is with a scalar field, where $\phi_{0,+}$ is the re emergent scalar field a billion years ago which he claims fits

$$\phi_{+} = \left[\phi_{0,+}^{3} - \sqrt{3/2} \cdot \frac{3M^{2}t}{\vec{\tilde{m}}} \right]^{1/3} \tag{24}$$

Here is the author's tentative identification of how to link Eq. 22 and Eq. 23, with what was done by the author as far as Fig 1 above:

$$H^{2} = \frac{1}{6} \cdot \left[\dot{\phi}^{2} + \vec{\tilde{m}}^{2}\phi^{2} + \frac{M^{2}}{\phi^{2}} \right] \leftrightarrow \left(\frac{\tilde{\kappa}^{2}}{3} \left[\rho + \frac{\rho^{2}}{2\lambda} \right] \right) + \frac{m}{a^{4}} \tag{25}$$

Also, the 2nd identification, namely

$$\dot{H}^{2} \cong \left[-2\frac{m}{a^{4}} \right] \leftrightarrow \dot{H} = V - 3H^{2} \tag{26}$$

If Eq. (24) and Eq. (25) can be reconciled, and if the conditions as given by Eq. (14) can be used , as well, with $t_{Before-2nd-Exit}$ being time in which the 2nd scalar field emerged, i.e. some time of the order of a billion years ago, and t time after the big bang, i.e. ogf the order of 12 to thirteen billion years, with $0 \leq \varepsilon^{+} << 1$, i.e. infinitesimally small, then

$$\phi \sim \sqrt{\frac{2m}{4\pi G}} \cdot \left[2\varepsilon^{+} \right] \cdot t^{2\cdot\varepsilon^{+}} \sim \left[\phi_{0,+}^{3} - \sqrt{3/2} \cdot \frac{3M^{2}t_{Before-2nd-Exit}}{\vec{\tilde{m}}} \right]^{1/3} \tag{27}$$

With $\phi_{0,+}$ being a yet to be determined vacuum nucleation value for the inflaton, emergent field, obeying Eq. (24) and Eq. (25) for H, and also, as given by Yurov

$$H = \sqrt{U(\phi_{0,+})/3} = [1/3]^{1/2} \cdot \sqrt{\vec{\tilde{m}}^{2}\phi_{0,+}^{2} + \left[M^{2}/\phi_{0,+}^{2} \right]} \tag{28}$$

9. Examining information exchange between different universes?

As given by Yurov [14], again, there is formalism for the alleged first inflation which he gives as

$$\phi(t) = \phi_{0,-} - \sqrt{2/3} \cdot \vec{\vec{m}} \cdot t \tag{29}$$

Note, that $\phi_{0,-}$ is related to an inflaton (?) mass, m, via the formula as given by Yurov[14]

$$\phi_{0,-} = \sqrt{2/3} \cdot \vec{\vec{m}} \cdot \left[t_{1st-EXIT} \sim 10^{-35}\sec \right] \tag{30}$$

Whereas $t_{Before-2nd-Exit}$ the time after one billion years ago, when 2nd inflation started, and $t_{1st-EXIT} \sim 10^{-35}\sec$ is when first inflation ended. The linkage between the two is in commonality in the m parameter as chosen, in Eq. (30) above. Having said this, what is left unsaid is what would be numerical inputs as to constituting $\phi_{0,-}$. Here,

$$\vec{\vec{m}} \approx \sqrt{\frac{3}{8}} \cdot \left[\sqrt{\frac{3H^2}{4\pi G}}\Bigg|_{time \sim 10^{-35}\sec} + \sqrt{\frac{3H^2}{4\pi G}}\Bigg|_{time \sim 10^{-44}\sec} \right] \tag{31}$$

The term given by $\frac{3H^2}{4\pi G} >> V(t)\Big|_{time \sim 10^{-44}\sec}$ and so then the term for $\vec{\vec{m}}$ is largely determined the Friedman equation at the onset of the big bang, and at the end of the big bang. How this is linked to initial conditions, will be brought up via considering [6]

$$\left|\vec{\vec{m}}\right| \le \left[\frac{l^2}{4}\right] \tag{32}$$

The term l as done by Eq. (32) is for a line element usually reserved for five dimensions as can be seen in[6]

$$dS^2\Big|_{5-\dim} = \frac{l^2}{z^2} \cdot \left[\eta_{uv} dx^{\mu} dx^{v} + dz^2 \right] \tag{33}$$

Note the bound of Eq. (32) and its link to Eq. (33) in Brane world treatments of values of the 2nd inflaton, as given by Eq. (28). Furthermore, the assumption being made here is that the 5th dimensional 'length' $l \sim \breve{\lambda}$ in formula (34) below, which is pertinent to information packing in the transfer of information from prior to present universe, takes into consideration pertinent treatment of the tension values of the branes, in Brane world cosmology, according to tension [6] $\lambda = 3M_P^2 / 4\pi l^2$. Having a small $l \sim \breve{\lambda}$ value would be consistent with the approximation used above of $\rho/2\lambda \approx .01$ as mentioned above.

Beckwith[3] has concluded that the only way to give an advantage to higher dimensions as far as cosmology would be to look at if a fifth dimension may present a way of actual information exchange to give the following parameter input from a prior to a present universe, i.e. the fine structure constant, as given by [3]

$$\widetilde{\alpha} \equiv e^2 / \hbar \cdot c \equiv \frac{e^2}{d} \times \frac{\breve{\lambda}}{hc} \tag{34}$$

The wave length as may be chosen to do such an information exchange would be part of a graviton as being part of an information counting algorithm as can be put below, namely:

Argue that when taking the log, that the 1/N term drops out. As used by Ng [15]

$$Z_N \sim (1/N!) \cdot \left(V / \breve{\lambda}^3\right)^N \tag{35}$$

This, according to Ng,[15] leads to entropy of the limiting value of, if $S = (\log[Z_N])$ will be modified by having the following done, namely after his use of quantum infinite statistics, as commented upon by Beckwith[3]

$$S \approx N \cdot \left(\log\left[V/\lambda^3\right] + 5/2\right) \approx N \tag{36}$$

Eventually, the author hopes to put on a sound foundation what 'tHooft[11] is doing with respect to t'Hooft [11] deterministic quantum mechanics and equivalence classes embedding quantum particle structures. If one uses the wave functional

$$\Psi_{i,f}\ [\phi(\mathbf{x})]\Big|_{\phi \equiv \phi_{ci,cf}} = c_{i,f} \cdot \exp\left\{ -\int d\mathbf{x}\ \alpha \left[\phi_{Ci,f}(\mathbf{x}) - \phi_0(\mathbf{x}) \right]^2 \right\}, \tag{37}$$

With $\phi_0(x)$ being equivalence classes to fit in a kink anti kink structure with t'Hooft's work[16] and tied it in with equivalence classes, and mixed it in with a kink anti kink structure given by the following figures from Beckwith's dissertation [17]. The first one is involving the use of instantons and what is known as domain wall approximations. Fig 2a. below represents how a Cooper pair charge can be used to ascertain an instanton- anti instanton structure would be organized as of CDW, for quasi one dimensions. The second, Fig 2b is how an equivalence class structure could be put in, and what the consequences would be. I.e.

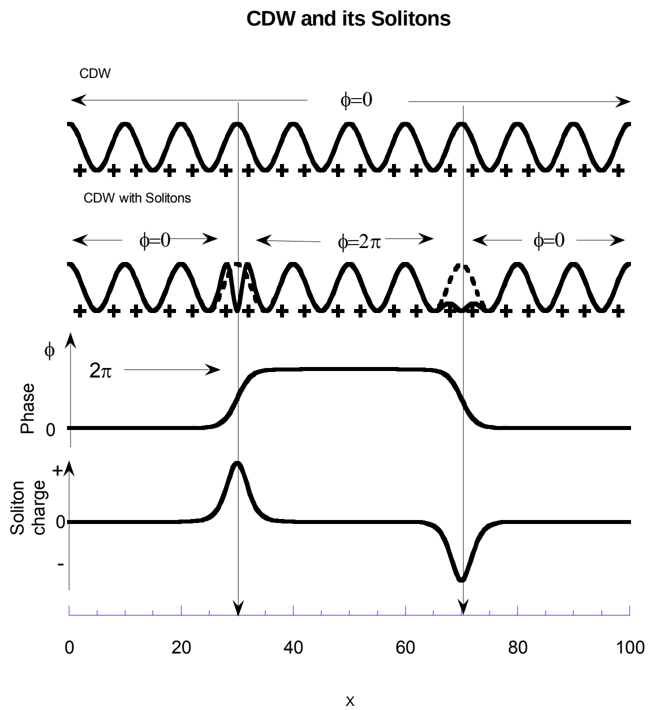

Fig. 2a: The pop up effects of an intanton-anti-instanton in Euclidian space[3, 17]

Doing so will answer the questions Kay[18] raised about particle creation, and the limitations of the particle concept in curved and flat space, i.e. the global hyperbolic space time which is flat everywhere expect in a localized "bump" of curvature. Furthermore, making a count of gravitons with $S \approx N \sim 10^{20}$ gravitons[3], with use of the formula from Lloyd [19], of $I = S_{total} / k_B \ln 2 = [\#operations]^{3/4}$ ~ 10^{20} as implying at least one operation per unit graviton, with gravitons being one unit of information, per produced graviton[3]. What the author, Beckwith, sees is that since instanton- anti instanton pairs does not have to travel slowly[20], as has been proved by authors in the 1980s that gravitons if nucleated in a fashion as indicated by Fig. 2b will be in tandem and not be influenced as indicated by Isbanez and *Verdaguer* [20] . The instanton – anti instanton structure allows for rapid travel.

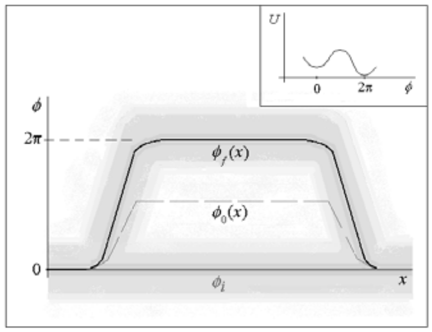

Fig. 2b: The pop up effects of an intanton-anti-instanton in Euclidian space[3, 17]

Also, an instanton - anti instanton structure may allow us to be able to answer the following. The stretch-out of a graviton wave, greater than the size of the solar system, gives, an upper limit of a graviton mass due to wave length $\lambda_{graviton} > 300 \cdot h_0 kpc$ $\Leftrightarrow m_{graviton} < 2 \times 10^{-29} h_0^{-1} eV$ [3]. I. e. stretched graviton wave, at ultra-low frequency, may lead to a low mass limit. However, more careful limits due to experimental searches, as presented by Buonanno [21] have narrowed the upper limit to $10^{-20} h_0^{-1} eV$. An instanton – anti instanton structure to the graviton, if confirmed, plus experimental confirmation of mass, plus perhaps $n \sim 10^{20}$ gravitons $\approx 10^{20}$ entropy counts, Eq. (23) implies up to $\approx 10^{27}$ operations. If so, there is a one-to-one relationship between an operation and a bit of information, so a graviton has at least one bit of information. And that may be enough to determine the conditions needed to determine if parameter inputs into Eq. (8) gives information and structure from a prior universe to our present cosmos. Finally, the datum referred to in Eq. (6) to Eq. (7) as combined with $S \approx N$ as a way to relate the graviton count with entropy may be a way to make inter connection between the inflaton picture of entropy generation and entropy connected/ generated with a numerical count of gravitons. This datum needs experimental confirmation and may be important to astro physics linkage of DE with DM, in the future. Eq. (6) and Eq. (7) if confirmed for Z ~ . 423 may prove, in part, that higher dimensions are necessary for cosmology. Also**,** Sahni and Habib [22] as of 1998 make a linkage between energy density of emergent particles, and the energy density of created particles behaving like an effective cosmological constant, leading generically to Ωm <1 in clustered matter. The author contends that the above formalism for a graviton as an emergent particle, with a slight mass in four dimensions is consistent with what Sahni and Habib[22] worked with, in 1998. Experimental verification of this would be important for determining if or not theories purporting to show increasing or decreasing values of the gravitational constant were valid, e.g. of the sort given by Singh [23] are based upon firm experimental foundations.

10. Conclusion : Radius of the universe problem ? Its connections to GW frequency ?

A final consideration, not to be minimized would be to get definite The problem of reconciling the existence of a graviton mass with quantum mechanics , in spin two particles usually having zero mass appears to be resolvable, and may imply a linkage between DE and DM in ways richer than suggested by Chapygin gas models [23,24] By a problem to be solved, the author is referring to the correspondence principle of quantum mechanics which usually dictated that spin 2 particles would have to , in four dimensions, have no mass. Note that the construction of a kink-anti-kink model as a bound for graviton mass creates conditions for modeling gravitons/ GW initially as a low-frequency phenomenon**.** Furthermore, the radius of the universe problem, as presented by Roos [24] will yield rich applications of the Friedmann equations used in this document, once there are falsifiable experimental criteria for determining both the Hubble Parameter $H = \frac{\dot{a}}{a}$ on the basis of choices of Friedman equations, and $\Omega \equiv \rho(t)/\rho_{critical}$, using variables chosen and described in this present paper. Both are pertinent to the problem of the radius of the universe[24] parameter set in Eq (38)

$$r_U \equiv \frac{1}{H \cdot \sqrt{|\Omega - 1|}} \tag{38}$$

Combining experimental confirmation of Eq. (1.57) with observations and appropriate use of different choices for $H = \frac{\dot{a}}{a}$ and $\Omega \equiv \rho(t)/\rho_{critical}$ may yield important research dividends, once appropriate measurement protocols are worked out for GW astronomy. Specifically, the author is convinced that analyzing Eq. (38) will be tied in, with appropriate analysis of the following diagram, given in Fig 3 below.. Note that this diagram explicitly also uses The relation between Ω_g and the spectrum $h(\nu_g, \tau)$ is often expressed as written by Grishchuk [25] , as

$$\Omega_g \approx \frac{\pi^2}{3}\left(\frac{\nu}{\nu_H}\right)^2 h^2(\nu, \tau), \tag{39}$$

The author looks to an interplay between Eq. (38) and Eq. (39) as a way to resolve questions as to if the universe is open and /or closed and also to shed some light as to the existence, or lack of relic GW which if detected may allow for a final choice between either purely 4 dimensional cosmology models, and what has been postulated to exist for higher dimensions in this manuscript and else where

Finding falsifiable criteria to evaluate an inter relationship between Eq. (38) and Eq(39) would also be akin to making sense of a known disjoint between mass, and massless versions of graviton equations, as given by **Appendix B**. Beckwith[3] claims that dealing with Eq. (38) properly would be akin to, if done with care investigating if semi classical criteria should be used to give credence to a t'Hooft[3,16] style deterministic quantum mechanical analysis of gravitons, and Gravitational wave generation.

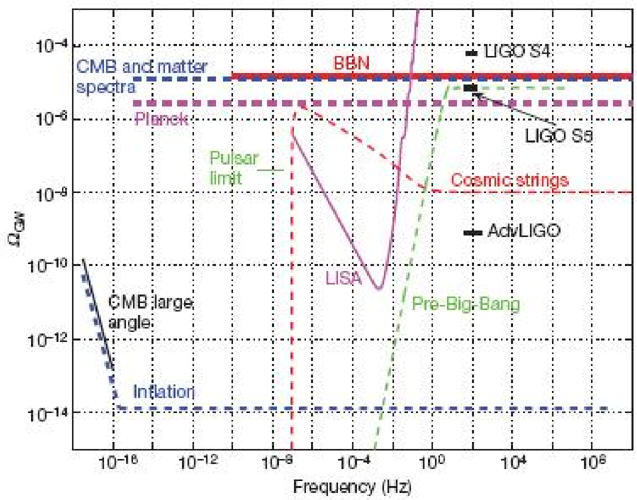

Figure 3. This figure from.B. P. Abbott et al[26]. **[50]** (2009) shows the relation between

Ω_g and frequency.

Appendix I. the origins of Eq. (2) of the main document

Note that Rubakov [27] writes KK graviton representation as, after using the following normalization $\int \frac{dz}{a(z)} \cdot [h_m(z) \cdot h_{\tilde{m}}(z)] \equiv \delta(m - \tilde{m})$ where J_1, J_2, N_1, N_2 are different forms of Bessel functions, to obtain the KK graviton/ DM candidate representation along RS dS brane world

$$h_m(z) = \sqrt{m/k} \cdot \frac{J_1(m/k) \cdot N_2([m/k] \cdot \exp(k \cdot z)) - N_1(m/k) \cdot J_2([m/k] \cdot \exp(k \cdot z))}{\sqrt{[J_1(m/k)]^2 + [N_1(m/k)]^2}} \quad \text{(A.1)}$$

This Eq. (A.1) is for KK gravitons having a TeV magnitude mass $M_Z \sim k$ (i.e. for mass values at .5 TeV to above a TeV in value) on a negative tension RS brane. What would be useful would be managing to relate this KK graviton, which is moving with a speed proportional to H^{-1} with regards to the negative tension brane with $h \equiv h_m(z \to 0) = const \cdot \sqrt{\frac{m}{k}}$ as an initial starting value for the KK graviton mass, before the KK graviton, as a 'massive' graviton moves with velocity H^{-1} along the RS dS brane. If so, and if $h \equiv h_m(z \to 0) = const \cdot \sqrt{\frac{m}{k}}$ represents an initial state, then one may relate the mass of the KK graviton, moving at high speed, with the initial rest mass of the graviton, which in four space in a rest mass configuration would have a mass lower in value, i.e. of $m_{graviton}(4 - Dim \quad GR) \sim 10^{-48} eV$, as opposed to $M_X \sim$

$M_{KK-Graviton} \sim .5\times 10^{9}\, eV$. Whatever the range of the graviton mass, it may be a way to make sense of what was presented by Dubovsky et.al. [28] who argue for graviton mass using CMBR measurements, of $M_{KK-Graviton} \sim 10^{-20} eV$ Dubosky et. al. [28] results can be conflated with Alves et. al. [5.] arguing that non zero graviton mass may lead to an acceleration of our present universe, in a manner usually conflated with DE, i.e. their graviton mass would be about $m_{graviton}(4-Dim \quad GR) \sim 10^{-48}\times 10^{-5} eV \sim 10^{65}$ grams.

Appendix B. The disjoint break down in Graviton mass , and massless version of field theory equations

Beckwith[3] claims that the deceleration parameter q (z) incorporating Eq. (3), Eq. (4) and Eq. (5) of the main text should give much the same behavior as Fig. 1 above. If so, then if one is differentiating between four and five dimensions by what is gained, in cosmology, one needs having it done via other criteria. The following is a real problem. As given by Maggiore [1] **,** the massless equation of the graviton evolution equation takes the form

$$\partial_{\mu}\partial^{\varpi} h_{\mu\nu} = \sqrt{32\pi G}\cdot\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T^{\mu}_{\mu}\right) \qquad \text{(B.1)}$$

When $m_{graviton} \neq 0$, the above becomes

$$\left(\partial_{\mu}\partial^{\varpi} - m_{graviton}\right)\cdot h_{\mu\nu} = \left[\sqrt{32\pi G} + \delta^{+}\right]\cdot\left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}T^{\mu}_{\mu} + \frac{\partial_{\mu}\partial_{\nu}T^{\mu}_{\mu}}{3m_{graviton}}\right) \qquad \text{(B.2)}$$

The mismatch between these two equations, when $m_{graviton} \to 0$**,** is due to $m_{graviton}h^{\mu}_{\mu} \neq 0$ as $m_{graviton} \to 0$ **,** which is due to setting a value of $m_{graviton}\cdot h^{\mu}_{\mu} = -\left[\sqrt{32\pi G} + \delta^{+}\right]\cdot T^{\mu}_{\mu}$

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