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Complete Exposition of Non-primes Generated from a Geometric Revolving Approach by 8×8 Sets of Related Series, and thereby *ad negativo* Exposition of a Systematic Pattern for the Totality of Prime Numbers

Abstract

We present a certain *geometrical* interpretation of the natural numbers, where these numbers appear as *joint products of 5- and 3-multiples* located at specified *positions* in a *revolving chamber*. Numbers without factors 2, 3 or 5 appear at *eight* such positions, and any prime number larger than 7 manifests at one of these eight positions after a specified amount of rotations of the chamber. Our approach determines the sets of rotations constituting primes at the respective eight positions, as the *complements* of the sets of rotations constituting *non-primes* at the respective eight positions. These sets of rotations constituting non-primes are exhibited from a *basic 8×8-matrix* of the *mutual products* of the eight prime numbers located at the eight positions in the *original* chamber. This 8×8-matrix is proven to generate *all* non-primes located at the eight positions in *strict rotation regularities* of the chamber. These regularities are expressed in relation to the multiple 11^2 as an anchoring *reference point* and by means of convenient *translations* between certain classes of multiples. We find the expressions of rotations generating *all* non-primes located at *same* position in the chamber as a set of *eight related series*. The *total* set of non-primes located at the eight positions is exposed as *eight* such sets of eight series, and with each of the series *completely* characterized by *four simple variables* when compared to a reference series anchored in 11^2. This represents a *complete* exposition of non-primes generated by a quite simple mathematical structure. *Ad negativo* this also represents a *complete* exposition of all *prime numbers* as the union of the eight complement sets for these eight non-prime sets of eight series.

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We start out with a recapitulation of an excerpt (with some linguistic adjustments for the sake of clarity) from the article by Johansen (2006: 127-9) presented as lecture at *18. Workshop in Hadronic Mechanics* at University of Karlstad, Sweden, June 2005, regarding the *first* of two methods announced by the article to disclose the generative pattern of prime numbers:

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We write the natural numbers as combined multiples of the numbers 5 and 3: [1]

(1) $N = m5 + n3; \quad m>0, n>0.$

Obviously, this split code 5:3 can be performed to cover any sequence of whole numbers by simply lowering the bottom values of m and n.

From (1) we construct the following matrix:

[1] The profound significance of the split code 5:3 in "Nature's code" is acknowledged and argued in the pioneering monumental work of Peter Rowlands (2007), and also with some stated connection (Rowlands 2007: 530, 550) to the contribution in Johansen (2006).

Fig. 1 ***The Revolving Chamber***

(1) N = m5 + n3; m>0, n>0.

5

3

n \ m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18 .. m
1	8	13	18	23	28	33	38	43	48	53	58	63	68	73	78	83	88	93
2	11	16	21	26	31	36	41	46	51	56	61	66	71	76	81	86	91	96
3	14	19	24	29	34	39	44	49	54	59	64	69	74	79	84	89	94	99
4	17	22	27	32	37	42	47	52	57	62	67	72	77	82	87	92	97	102
5	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105
6	23	28	33	38	43	48	53	58	63	68	73	78	83	88	93	98	103	
7	26	31	36	41	46	51												
8	29	34	39	44	49	54												
9	32	37	42	47	52	57												
10	35	40	45	50	55	60												
11	38	43	48															
12	41	46	51															
.																		
.																		
n																		

There exist three possibilities to make a cut in the matrix in such a way that every number shows up only once. We may denote these three bands of numbers by means of colour terms:

1) *The Blue Band*, corresponding to the five upper rows.
2) *The Red Band*, corresponding to the three left columns.
3) *The Violet Band*, corresponding to a double diagonal field unfolding from first six columns of The Blue Band, or from first ten rows of The Red Band.

There can not be any prime numbers in the row for n=5, nor in the columns that are multiples of m=3. Ignoring these rows and columns (illustrated by the black grid in fig. 1), prime candidates can only appear in the remaining "chambers" of the bands. Further, prime candidates can only appear at spots in the chambers where *odd* numbers are located (illustrated with the colours blue, red and violet, respectively). We notice that these spots are distributed in a zigzag pattern inside each chamber, and that this pattern alternates with its mirror pattern when progressing horizontally or vertically along a band. In the present context we will only study The Blue Band.

To easily get a picture of the underlying prime number generator, we first imagine *all* remaining odd (blue) numbers in The Blue Band as being prime numbers. This is the case for the first two chambers of The Blue Band. However, in the third chamber, which can be imagined as constituted from the first (whole) *rotation* of the left, first chamber, the number of 49, i.e. 7×7, shows up as the first anomaly not being any prime number. Analogous anomalies will be the case for all powers of 7, as well as for all "clean multiples" of 7 (meaning those having a factor in a preceding chamber) located in chambers further to the right on The Blue Band. 7 is the only lower number *outside* and *before* our matrix, which acts as a "bullet" and "shoots out" odd numbers in The Blue Band, removing their prime number candidature. For example, the number of 77 is shot out from the prime number universe in chamber no. 5 after two rotations of chamber no. 1, being a multiple of the bullets 7 and 11. Prime numbers from the first chamber will deliver the same "ammunition" when exposed for sufficient rotations to manifest multiples made up as internal *products* of these prime numbers. Such multiples occur at corresponding "arrival spots" in upcoming chambers after further rotations. For example, the number of 143 is shot out from the prime number universe in chamber no. 10 after four rotations of chamber no.2 , being a multiple of the factor "bullets" 11 and 13. Quite obviously, *all* multiples of primes will expose the same pattern of shooting out corresponding prime number candidates occurring in proceeding chambers, without regard to the number of rotations of chamber no. 1 or no. 2 manifesting the prime factor bullets of the multiple. Hence, the over-all process of shooting out prime candidates can be imagined as successive out-shooting during consecutive rotation of chambers no. 1 and 2, due to more and more multiples from prime bullets, located in preceding chambers, becoming manifest along with further chamber rotations. This elimination process of prime candidates is obviously *exhaustive*. *All* prime candidates which is *not* shot out from the multiples of prime bullets occurring at preceding chambers *have* to be primes. Therefore, a complete mathematical description of this successive out-shooting of prime candidates will automatically *ad negativo* implicate also a complete, successive description of the generation of prime numbers. Here the prime numbers appear as the numbers *remaining* in chambers of The Blue Band *after* the shoot-out procedure has passed through the chamber where the prime candidate is located.

I have not been able to find a mathematical formula to grasp this successive out-shooting of prime numbers from the chamber rotation constituting The Blue Band. However, this simple model gives a clear understanding of the fact that there ***is*** *an order in the prime number generation, as well as a good intuitive understanding of* ***what*** *kind of order it is. For example, it is easy to understand that this generative order includes as a tendential law more and more prime number candidates to be shot out with increasing amount of rotations. To work out the mathematics of this may be difficult, but there is no mystery around concerning the underlying, inner workings of the prime number generator.*
(Johansen 2006: 129)

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We will now develop the sought mathematical formula(s) referred to in the quote.

We apply the following notation of the blue (odd) numbers' positions inside a chamber, using their positions inside the first two chambers as illustration:

Left chamber:
a_1: Position of 13
a_2: Position of 11
a_3: Position of 19
a_4: Position of 17

Right chamber:
b_1: Position of 23
b_2: Position of 31
b_3: Position of 29
b_4: Position of 37

Then, all blue numbers in The Blue Band can be written as one of these positions combined with a specific number of rotations. As an example, 71 can be written as $(2,a_2)$, meaning that 71 emerges at the position a_2 after 2 rotations of the original (left) chamber. Accordingly, 67 will be written as $(1,b_4)$, etc.

Fig. 2 *The basic 8×8-matrix of the non-primes generator in the revolving chamber*

	Number position							
	Left chamber				Right chamber			
	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4
Start:	13	11	19	17	23	31	29	37
Step								
1			13- 5(13) 17- 9(17) 23-17(23) 37-45(37)			11- 3(11) 19-11(19) 29-27(29) 31-31(31)		
2		13- 7(17)		19-14(23) 37-50(41)	11- 4(13) 17-10(19)		29-29(31)	23-21(29) 31-37(37)
3		19-18(29) 31-42(41)			23-23(31) 29-35(37)	17-12(23) 37-52(43)		11- 5(17) 13- 7(19)
4	17-16(29) 31-44(43)	23-28(37)	19-19(31) 29-39(41)				11- 6(19) 13- 9(23) 37-57(47)	
5	11- 8(23) 19-23(37) 23-31(48) 37-60(49)			13-12(29) 17-17(31) 29-41(43) 31-48(47)				
6	13-13(31) 29-45(47)	37-65(53)	11-10(29) 31-50(49)				17-20(37) 19-25(41) 23-32(43)	
7		11-11(31) 29-47(49)			31-53(53) 37-72(59)	13-15(37) 23-35(47)		17-22(41) 19-26(43)
8		17-24(43)		11-13(37) 23-37(49)	13-17(41) 19-27(47)		31-60(59)	29-50(53) 37-74(61)

Fig. 2 describes the basic distributive structure of positions (illustrated as columns) in the chambers, manifesting from the specific numbers of rotations (illustrated in red) of the eight initial position numbers (illustrated in bold black) of the original chamber (i.e. chamber no.1, the left, and chamber no.2, the right, taken together), where these rotations correspond to stepwise multiplications of the respective original position numbers with progressively larger multiplicators (illustrated in blue). The succession of multiplications goes as follows, taking as example 11 as multiplicand:

(2)

Multiplicator		**Product - position - rotations**
1. row: the multiplicand number itself.		11×11 at b_2 after 3 rotations
2. row: the closest blue number larger than itself.		11×13 at b_1 after 4 rotations
3. row: the 2. closest number larger than itself.		11×17 at b_4 after 5 rotations
4. row: the 3.	"	11×19 at b_3 after 6 rotations
5. row: the 4.	"	11×23 at a_1 after 8 rotations
6. row: the 5.	"	11×27 at a_3 after 10 rotations
7. row: the 6.	"	11×31 at a_2 after 11 rotations
8. row: the 7.	"	11×37 at a_4 after 13 rotations

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9. row: the 8.	"	11×(11+30) at b_2 after (3+11) rotations
10.row: the 9.	"	11×(13+30) at b_1 after (4+11) rotations
11.row: the 10.	"	11×(17+30) at b_4 after (5+11) rotations
…………....	….	…………………………
…………….	….	…………………………

As an example we can look at the number in the box $(8,b_3)$ that manifests at position b_3, i.e. the same position as 29 in the original chamber, after the original number 31 is multiplied with the multiplicator 59 which is situated at the 8. row, i.e. 7 steps after the number 31 itself acts as multiplicator on itself. This box is reached after 60 rotations of the original chamber.

For each of the eight different position numbers in the original chamber, the position in the 9. row (i.e. after 8 steps of the succession) is identical with the original position. We introduce the following notations: n_0 for the initial position number in the original chamber, *p* for the position, *s* for the number of steps. Then we always have with regard to *position* as function of n_0 and s, that p for the product of an multiplicand n_0 and a multiplicator 8 steps after the multiplicator n_0, is identical to p for n_0^2. The same obviously is the case with respect to p for the product of an multiplicand n_0 and any multiplicator 8m steps after n_0, when m is a natural number. (From now on we apply the symbol *m* with this meaning instead of the one in context (1).)

Since 8 steps for any n_0 represent an addition of $n_0 30$ to n_0^2 in the product, the statements above implies, respectively, for the function p:

(3) $p\,[n_0\,(n_0+30)] = p\,(n_0^2)$

(4) $p\,[n_0\,(n_0+m30)] = p\,(n_0^2)$

Quite obviously, we have corresponding formulas for the position in the 2. row vs. the 10., 18., 26.,… row; for the position in the 3. row vs. the 11., 19., 27.,… row; etc.

This means that with respect to *position*, the 8 sequence of positions characteristic for the products progressing in steps from n_0^2 , just repeats in 8 steps cycles along with increasing m, with each sequence of products progressing in the equally positioned 8 steps from $n_0^2 + mn_0 30$. And the same cyclic repetition of positioning, just slided, must also be the case for products progressing in steps from $n_0(n_0+d) + mn_0 30$, when *d* denotes the addition corresponding to the actual row at the n_0-path. (As an example, for the 11.row in (2) we have d=17-11=6; m=1; and the position cycle for $n_0=11$ is slided two steps.) Including d and such sliding, we have the following formulas extending, respectively, (3) and (4):

(5) $p\,[n_0(n_0+d+30)] = p\,[n_0(n_0+d)]$

(6) $p\,[n_0\,(n_0+d+m30)] = p\,[n_0(n_0+d)]$

With regard to the number of *rotations*, we always have that after 8 steps the number of rotations manifesting the product of row 9 is n_0 larger than for row 1, for example 14-3 = 11 rotations larger in the case of 11 being the multiplicand of the product.

In rows larger than 9, the addition of rotations to the amount of rotations for row 1, will be larger than n_0, with rows 10 and 11 of (2) indicating the pattern of additions. We will later establish the general formulas.

Formula (4) exposes the generation of the whole set of rhythmic positioning of all blue (in the sense of fig. 1) number multiples located at the 8 paths departing from the 8 n_0 multiplicands of the original chamber. However, formula (4) does *not* expose the positions of blue number multiples located at paths departing from multiplicands *larger* than the 8 ones of the original chamber.

Nevertheless, the exhaustive paths of positions originating from such larger multiplicands can easily be reduced to the *same* 8 paths of positions originating from the *initial* n_0 multiplicands.

Contemplating, as an example, the 9. row of (2), 11(11+30) is equivalent to (11+30)11, i.e. 41×11. (3) implies that the position of 41×41 is identical to the position of 11×41, and hence by transitivity identical also to the position of 11×11. Quite obviously, the same homology must be the case for additions d to the multiplicator, implying, for example, that the position of 41(41+6) is identical to the position of 11(11+6), i.e. to the position at the third step-row for the path departing from the 11-multiplicand. Also quite obviously, (4) implies the same homology for *larger* multiplicands when made-up of *plural* additions of 30 to the initial multiplicand, for example for multiplicands 71,101,131… when compared to the initial multiplicand 11. We apply *n* to generally denote the amount of such 30-additives to any of the 8 initial multiplicands. (From now on we apply the symbol *n* with this meaning instead of the one in context (1)). Then we can state the following formulas in analogy to formulas (3) and (4), respectively:

(7) $p\ [(n_0+30)n_0] = p\ (n_0^2)$

(8) $p\ [(n_0+n30)n_0] = p\ (n_0^2)$

Further, in analogy to formulas (5) and (6), respectively, we have:

(9) $p\ [(n_0+d+30)n_0] = p\ [n_0(n_0+d)]$

(10) $p\ [(n_0+d+n30)n_0] = p\ [n_0(n_0+d)]$

We will soon develop the exact specific formulas, consistent with (6) and (10), for the 8 paths generating multiples departing from the 8 initial multiplicand n_0's, as well as the according exact specific formulas consistent with direct *combination* of (6) and (10). Before proceeding to the specific formulas, we will present some general reflection on the significance of what is already implied in the relations exposed in fig. 2.

Fig. 2 shows that for all initial multiplicands n_0, during the stepwise path through 8 rows, all columns (and step-rows) are filled once and only once. For example, the path of multiplicand 11 fills each original position (column) in the chamber once and only once. Formulas (6) and (10), taken together, cover *all* possible multiplicands of blue numbers. This implies that every thinkable multiplicand path merely duplicates one of the 8 paths exposed in fig. 2, characteristic for the 8 basic n_0 multiplicands constituting the blue numbers in the original chamber. Hence, this must hold whatever the magnitude of a multiplicand. This is an interesting peculiarity. It means that:
(a) there only exist 8 different paths for all blue numbers when acting as factors to generate larger multiples composed by blue numbers;
(b) each of these paths is uniquely given by which multiplicand n_0-path a product belongs to from formulas (6) and (10);
(c) each of the 8 paths visits all 8 original n_0-positions (columns) in the chamber once in a sequence of 8 steps;
(d) the *whole* possibility space of products of blue numbers is basically identical to the structure of fig. 2 with its 8 different paths, and represents merely an extension generated from fig. 2 and nothing else.

This means that all thinkable non-primes are represented and generated as the boxes of fig. 2 filled with the specified elements constituted by initial n_0-numbers progressing as specified amounts and intervals of rotations. Hence, the basic structure generating the whole set of non-primes (and hence *ad negativo* of primes) is exposed with mathematically precise qualification already from fig. 2.

We notice that in the 8×8 matrix of fig. 2, *empty boxes* are located in *all* rows and columns, with a total of 34 empty boxes out of the 64 boxes in the matrix. More specifically, 34 boxes contain 0 elements; 6 boxes contain 1 element; 18 boxes contain 2 elements; 2 boxes contain 3 elements; and 4 boxes contain 4 elements. Also, we notice a pair wise structuring of the boxes: Step-row 2 and step-row 8 have their boxes located in identical columns, and two such boxes in the same column contain identical amounts of elements when comparing the two corresponding step-rows. The same is the case for the pair of step-row 3 vs. step-row 7, as well as for the pair of step-row 4 vs. step-row 6. The last pair, of step-row 1 vs. step-row 5, have their boxes located in *different* columns, but still exhibit equal *amounts* of boxes (2 boxes) as well as the same *distribution of amounts of elements* between their boxes (4 elements in each of the 2 boxes).

How are these *empty* boxes to be interpreted? To take column a_2 as an example, the empty boxes of step-rows 1 and 5, tell that no path from any of the initial 8 n_0's will ever hit position a_2 in the revolving chamber after 1 and 5 steps from n_0. Extending the consideration by including increasing m or n, it also tells, consistent with (6) and (10), that no such path will hit position a_2 after 1,9,17,25,33… steps or after 5,13,21,29,37… steps from *any* of the 8 n_0's.

This means that in all these cases the paths will hit *another* position than a_2 in the chamber. Therefore, in all these cases we *know* that the number in the chamber at the a_2 position is a *prime*. However, this is with respect to the reference frame constituted by the position of the 8 initial numbers n_0 and their number of *steps*. A certain number of steps departing from a certain multiplicand n_0 implies a certain amount of rotations. For *different* n_0's the same number of steps will imply *different* amounts of rotations. Knowing that position a_2 is not a multiple from any n_0 taking, say, 9 steps, does not guarantee that position a_2 is not a multiple from a number of steps *different* from 9. The number of rotations corresponding to 9 steps after *one* n_0, can be the same as the number of rotations corresponding to *other* than 9 steps from *another* n_0, and the last path may deliver a multiple at position a_2 after this amount of rotations. Therefore, the unique determination of primes in all empty boxes, following from the reference frame of positions and *steps*, does not directly yield a corresponding unique determination of primes from the reference frame of positions and *rotations*.

How are the *filled* boxes to be interpreted with respect to their elements vs. not-elements? If we take box $(2,a_2)$ as an example, it tells that a multiple on the path departing from $n_0=13$ will occur at position a_2 after 2,10,18,26… steps of the path. It also tells that multiples generated from the 7 *other* n_0's, different from 13, *never* will occur at position a_2 after 2,10,18,26….steps of *their* paths. This means that with respect to the reference frame of positions and *steps*, the number in the chamber at the a_2 position after said series of steps, is a *prime* if – and only if – n_0 is different from 13. However, for the same reason as stated above with respect to *empty* boxes, this unique determination of primes vs. non-primes in *filled* boxes, contemplated from the reference frame of positions and *steps*, does not directly yield a corresponding unique determination of primes from the reference frame of positions and *rotations*.

Therefore, both with regard to the empty and to the filled boxes of fig. 2, it poses technical challenges of mathematical *translation* to step from a unique determination of primes vs. non-primes in the reference frame of positions and *steps*, to a corresponding unique determination in the reference frame of positions and *rotations*. Such translation is required to reach unique results to be easily interpreted in agreement with our conventional notion of natural numbers, primes and non-primes, a task we will pursue during the rest of our article.

However, one could imagine the matrix of fig. 2 as a *basic* geometric reference frame to distinguish between primes and non-primes, *without* considering its constitution and interpretation from fig. 1 indicating the link between fig. 2 and our conventional conception of natural numbers. Then one could interpret the 8 n_0's as 8 basic distinctions or initial ordinal numbers, and any element in a filled box of the matrix as uniquely characterized from two primary variables: initial distinction n_0 and s, and two secondary variables: m and n (interpreted as path repetitions in the n_0/s-matrix). From these four variables we could *define* primes as the complete set of potential positions in the matrix not becoming actualized as positions on the eight paths (i.e. the set of all positions in empty boxes and all non-actualized positions in filled boxes). In such a basic *non*-conventional geometric reference frame the unique determination of primes vs. non-primes then could be considered *already* solved in virtue of our above treatment connected to fig. 2. It may be useful to keep in mind that the choice of reference frame is a question about choice of *convention* in mathematics. Therefore, there is a certain element of *relativism* required to uphold that the prime number generator is *not* completely exposed already from the considerations above.

The reference frame of positions and *steps* seems enlightening also with regard to the *quantitative progression* in the distribution of primes vs. non-primes. Contemplating the matrix of fig. 2 as a "board" filled with steps from the 8 n_0's as "chess pieces" moving somewhat pawn-like, 34 fields are never touched by any piece, and no field are touched by more than 4 out of 8 pieces. Repeating the moves by setting m=1 for all 8 pieces, just repeats the same exact distribution of touches among the fields, and the same for every new cycle from increasing m. *Each* successive increase in m by 1 must correspond to a *constant* absolute increase in *rotations* (of the original chamber) for each piece when touching a certain field, compared to the number of rotations touching the same field at the board in the preceding cycle. From only considering increase in m (i.e. not increase in n), the number of new non-primes emerging from every new cycle/board will be a *constant*, and with the same distribution on the board as in the preceding cycles. However, in spite of the correspondence of this with *constant* absolute increase in *rotations* for each piece/field, these constancies are accompanied by the non-primes of one board becoming more *scattered* among natural numbers with increasing m. For m=0, the non-primes span from 11×11 to 37×61; for m=1, the span increases to the difference 37×91 – 11×41; for m=2, the span increases further to the difference 37×121 – 11×71, and so on. Hence, an identical amount of emerging new non-primes, appearing with identical positions in the reference frame of the board matrix, will manifest *skewed* when interpreted as a distribution with a certain density in the progress of the revolving chamber or on the conventional progressing "line" of natural numbers. However, there is hardly any mystery connected to this skewness, since its systematic pattern and exact mathematical underpinnings are exposed from the relations in fig. 2.

Quite obviously, when considering the effects from increasing n (instead of m) an analogous systematic skewness is revealed, still without much mathematical mystery connected to its regularity. And the same must be the case when contemplating the *total* result from increase in m and n, with respect to the quantitative progression in the density distributions of primes vs. non-primes interpreted along the number line. It is outside the scope of this article to develop adequate formulas for such distributions, reckoning such developments a secondary issue compared to our aim of presenting the sets of series generating with completeness the *exact* non-primes (and hence, *ad negativo*, also the primes themselves). However, the remarks above may be suited to improve the general understanding of establishing the matrix of positions and steps in fig. 2 as a fruitful reference frame for translation into prime number treatments.

One might object that the non-primes generated from the prime of 7 are not included in fig. 2, and hence fig. 2 does not suffice to locate *all* non-primes. Basically, this objection shows not to represent much of a disturbance, though it leads to a slight modification. The path from the multiplicand 7 displays this structure:

(11)

Row	**Product**	**Position**	**Rotations**
1	7×7	a_3	1
2	7×11	a_4	2
3	7×13	b_2	2
4	7×17	b_3	3
5	7×19	a_1	4
6	7×23	a_2	5
7	7×29	b_1	6
8	7×31	b_4	6
9	7×37	a_3	8 (=1+7)
10	7×41	a_4	9 (=2+7)
11	7×43	b_2	9 (=2+7)

We see that with regard to *positions* the path from 7 is *identical* to the path from the multiplicand 37, which is consistent with formulas (3) and (4). Therefore, the path from multiplicand 7 does not add anything new with regard to the succession of positions illustrated in fig. 2. However, it adds – of course – something new with regard to *rotations*, shooting out more numbers as non-primes than the path from 37. The only modification necessary to also include the path from 7, is to establish the rotations corresponding to *this* path as the basic ones, and then add the rotations from the paths of 37, 67, 97 etc. from this basis with the correct formulas, as we shall do later on. In principle, the path from 7 does not represent any additional difficulty.

A basic strategy in our approach is to map and group the total distribution of non-primes from the multiples manifesting at the paths progressing from the 8 initial position numbers (at top of fig. 2), each starting with its own square:

(a) Departing from 11^2, all (blue number) larger multiples of the multiplicand 11 are exposed in a sequence of 8 steps, the path from n_0=11, where the sequence merely repeats itself in succession with increase of m.
(b) Repeating the same path (with respect to its stepping structure, i.e. positioning in the matrix of fig. 2, but now connected to other amounts of rotations), in order to cover all paths that depart from $(11+n30)^2$, where the structural path of (a) repeats itself in succession with increase of n.
(c) Moving to 13^2, and generate all multiples of 13 as exposed in a cyclic sequence of 8 steps, the path from n_0=13, in analogy with (a).
(d) Repeating (b) with n_0=13, in order to cover all paths that depart from $(13+n30)^2$ in succession with increase of n.
(e) Repeating (c) and (d) for the six remaining initial n_0's (however, using 7 instead of 37 for the eight and last one).

This strategy will not all the time generate *new* non-primes, but such overlap does not matter for our purpose which is to exhibit the generator for non-primes in order to *ad negativo* locate the total set of *non*-primes.

The first step-row of fig. 2 reveals that *all squares* of blue numbers are positioned *either* at a_3 *or* at b_2, i.e at only *two* of the 8 possible positions. The squares are distributed to a_3 vs. b_2 in a 50/50-relation (when presupposing filled rotated chambers before looking at the squares of their numbers). More specifically, numbers situated at the positions a_1, a_4, b_1 and b_4 in the state of the revolving chamber where they manifest, always have their squares at a_3 (the position of initial number 19), and the numbers situated at the remaining four positions in the state of the revolving chamber where they manifest, always have their squares at b_2 (the position of initial number 31).

Let us investigate the pattern of non-primes generated from the 8 n_0-paths when m and n increases. We use the occupied box *positions* (corresponding to the 8 n_0's at the top of fig. 2) as our observation post and look for the pattern of rotations, following different paths, all reaching this position as its destination.

We may start with investigating the 8 boxes in the b_4 column (the position of 37 in the original chamber). First we look at the second step-row of this column. We see from fig. 2 that this box is reached as a multiple (element in the box) located either at the path from n_0=23 or at the path from n_0=31.

For the 23-path the box is reached with 29 as the multiplicator, due to 29 being the number giving the product of 23 for the *second* row of the 23-multiplicand (the multiplicator giving the product for the *first* row, 23×23, is 23 itself). And we see from fig. 2 that this box is reached after 21 rotations of the original chamber for the initial 37-position, i.e.:
(12) $23\times29 = 37 + 21\times30$.

After a cycle of 8 further steps for the 23-multiplicand, the same box is reached for the 10. row of the 23-multiplicand, with the multiplicator 59 (=29+30) giving the product 23×59. This multiple is reached after 23 more rotations of the chamber, i.e. from an addition of 23×30 to the first entry into this box at the 23-path. This is just an example of the general regularity that the same box in the matrix, containing a multiplicand n_0, in this case 23, always is re-entered after n_0 more rotations – and therefore also after mn_0 more rotations. Hence, after the 23-multiplicand has left the box and circumscribed one cycle of 8 more steps with successive increase of its (blue number) multiplicator, the same box is re-entered by the multiple:

(13) 23(29+30)

which, consistent with (13), also can be written:

(13a) 37 + 30(21+23)

All such entries into the box from the 23-multiplicand due to increase in m, are obviously covered by the set of multiples:

(14) 23(29+m30)

which can also be written:

(14a) 37 + 30(21+m23); m=0,1,2…belonging to N.

We now define the *total* 23-path as the *set* of paths from the 23-multiplicand (given by the multiples covered by the (14) expression), from the (23+30)-multiplicand, from the (23+ 2×30)-multiplicand etc., i.e. as the set of paths from the (23+30n)-multiplicands. Then the expression (14) only covers the multiples generated from such multiplicands determined by n=0. We also have to cover the multiples from multiplicands 53, 83, 113 etc. that reaches the box $(2,b_4)$ as well. For the 53-multiplicand the multiples entering the box are given by the set

(15) 53(59+m30)

alternatively expressed as

(15a) 23×29 + 30[23+29+30 + m(23+30)]

or:

(15b) 37 + 30[21+23+29+30 + m(23+30)]

Generalized, for the (23+n30)-multiplicand the numbers entering the box are given by the set:

(16) (23+n30) [(29+n30) +m30]

alternatively expressed as:

(16a) 23× 29 + 30[n(23+29) + m23 + n30(n+m)]

or:

(16b) 37 + 30[21+ n(23+29) + m23 + n30(n+m)]

Just as a simple example, the number 4717=53×89 are the expressions (16), (16a) and (16b) with n=1 and m=1.

The expression inside the brackets in (16b) gives *all* the numbers of *rotations*, at the 37-position in the original chamber, that determines all the non-primes located in the box $(2,b_4)$ from successive increases in m and n, in so far as these multiples are located at the total 23-path. (We have not yet considered multiples in the box located at the total *31-path*, the *other* element in the box $(2,b_4)$.)

When we increase n in (16), we get all the possible *multiplicands* that is anchored in 23 and generate non-primes in this box. When increasing m, we get all the non-primes generated from all the *cycles* of every one of these multiplicands.

Noted in passing, with regard to the uneven distribution of primes over the field of natural numbers, expressions of type (16b) offers the key to a precise understanding of the relations generating the tendency to overall falling occurrence of prime numbers. Fig. 2 implies the existence of 64 – and only 64 – such expressions.

This was from the non-primes entering the box from the total *23-path*. As already stated, *additional* non-primes enter the box from the total 31-path. By analogy to (16) these are given by the set:

(17) (31+n30) [(37+n30) +m30]

alternatively expressed as:

(17b) $37 + 30[37+ n(31+37) +m31 + n30(n+m)]$

The union of (16) and (17) gives *all* the non-primes located at the box $(2,b_4)$, i.e. all non-primes entering position b_4 of the chamber after 2, 10, 18 etc. steps of a multiplicand.

All this was for the box $(2,b_4)$ in the matrix. However, the *total* amount of non-primes located in the column b_4, corresponding to the *complete* set of rotations resulting in entries at the 37-position of the original chamber, is not given before also adding such non-primes occurring in the matrix for the b_4-column in the filled boxes of *rows 3, 7 and 8*. Let us now establish the corresponding formulas for these three remaining boxes, in analogy with the reasoning above:

Box $(3,b_4)$:
Non-primes entering this box from the total *11-path*, are given by the set:
(18) $(11+n30) [(17+n30) +m30]$
alternatively expressed as:
(18b) $37 + 30[5+ n(11+17) +m11 +n30(n+m)]$

Non-primes entering this box from the total *13-path*, are given by the set:
(19) $(13+n30) [(19+n30) +m30]$
alternatively expressed as:
(19b) $37 + 30[7+ n(13+19) +m13 +n30(n+m)]$

Box $(7,b_4)$:
Non-primes entering this box from the total *17-path*, are given by the set:
(20) $(17+n30) [(41+n30) +m30]$
alternatively expressed as:
(20b) $37 + 30[22+ n(17+41) +m17 + n30(n+m)]$

Non-primes entering this box from the total *19-path*, are given by the set:
(21) $(19+n30) [(43+n30) +m30]$
alternatively expressed as:
(21b) $37 + 30[26+ n(19+43) +m19 + n30(n+m)]$

Box $(8,b_4)$:
Non-primes entering this box from the total *29-path*, are given by the set:
(22) $(29+n30) [(53+n30) +m30]$
alternatively expressed as:
(22b) $37 + 30[50+ n(29+53) +m29 + n30(n+m)]$

Non-primes entering this box from the total *37-path*, are given by the set:
(23) $(37+n30) [(61+n30) +m30]$
alternatively expressed as:
(23b) $37 + 30[74+ n(37+61) +m37 +n30(n+m)]$

Finally, we also have non-primes entering this box from the total *7-path*, given by the set:
(24) $(7+n30) [(31+n30) +m30]$
alternatively expressed as:
(24b) $37 + 30[6+ n(7+31) +m7 +n30(n+m)]$

Exchanging n with (n+1) in expressions (24) and (24b) makes these expressions identical with, respectively, expressions (23) and (23b). This means that inclusion of the 7-path adds nothing new to the 37-path, except for the value of n=0 in expressions (24) and (24b). This is included most conveniently simply by defining the definition domain of n in expressions (23) and (23b) as starting not from n=0 as in the expressions from (16) to (23), but from n=-1. By this slight extension, all the novelty of the 7-path becomes already entailed in the 37-path. Then we no longer need the expressions

(24) and (24b), nor do we anymore need to consider any autonomous role of the prime 7 in our proceeding considerations about primes and non-primes. The role of the prime 7 in the revolving approach is completely represented by n=-1 in expressions (23) and (23b). This fact appears obvious already from looking at the structure of the original chamber, where 7 would hold the position b_4 of the initial number 37 if the chamber was rotated one turn to the left in fig. 2, with a corresponding left rotation of the box $(8,b_4)$ to become located in the column for 7 as initial b_4.

Obviously, the same will be the case when observing the patterns of non-prime entries for other boxes than $(8,b_4)$ and for other columns than b_4. In all such cases the significance of number 7 is restricted to the value of n=-1 in expressions analogous to expressions (23) and (23b). Also, in all such cases this significance of number 7 is restricted to the boxes in fig. 2 where 37 occurs, since the 7-path only occurs along the 37-path. For all other boxes we still can ignore the value of n=-1 and let n start with n=0.

The *union* of the numbers covered by the expressions from (16) to (23) gives the complete set of non-primes located in column b_4, i.e. *all* thinkable non-primes generated at position b_4, occupied by 37 in the original chamber, after rotations of the original chamber, *whatever* the starting position n_0 in the original chamber these non-primes are generated at the path from. The *union* of the sub-expressions inside the brackets of the expressions from (16b) to (23b) gives the complete set of *what* numbers of rotations these non-primes at position b_4 are situated at, and by this the exact magnitudes of these non-primes. From this it follows automatically that all numbers situated at position b_4 after *other* numbers of rotations *have to be primes*. Therefore, by this procedure we can determine *ad negativo* the whole sequence of primes situated at destination position b_4 in the rotating chamber.

Obviously, we can repeat the same procedure also for the seven *remaining* columns, different from b4, each of these seven with a corresponding set of eight expressions analogous to the expressions from (16) to (23), and each of these seven with a corresponding set of eight expressions analogous to the expressions from (16b) to (23b), with the numbers of rotations resulting in non-primes given by the sub-expressions inside the brackets. Therefore, by this procedure we can determine *ad negativo* the whole sequence of primes situated also at the seven *other* destination positions in the rotating chamber. By thereafter taking the *union* of all the eight sets of non-primes corresponding to the eight different destination positions (each set with eight sub-sets), we have generated all thinkable non-primes, and can conclude *ad negativo* that *all the rest* of blue numbers generated by the rotating chamber *have to be primes*.

It is obvious that applying the same procedure for the seven remaining columns, i.e. destination positions, as the procedure generating the expressions from (16) to (23) and from (16b) to (23b), does not introduce any additional difficulty compared to the execution of the procedure applied for column b_4. All the resulting expressions manifest from repeated applications of the same procedure. We will name these two complete sets as, respectively, *the primary 64-set of non-prime expressions* and *the primary 64-b set of non-prime expressions*. We do not bother to list the expressions from performing these trivial operations. Instead we will juxtapose all the according sub-expressions, deduced from these operations and exhibiting the respective amounts of rotations, homologous to those inside the brackets of (16b) to (23b), in a *joint figure*, to better overview the total pattern.

It seems suitable to use the expression for the *lowest* product in fig. 2, i.e. 11×11, as an anchoring point of reference for the other sub-expressions. In analogy to the sub-expression inside the brackets of (16b) we write the sub-expression, describing the number of rotations, for this product number as:
(25) $3 + 11n + 11(m+n) + n30(m+n)$

Fig. 3 ***The 8×8 universal matrix of (11,11)-related additives of rotations for complete generation of non-primes***

	11	**13**	**17**	**19**	**23**	**29**	**31**	**37**
11	0n 3-31							
13	2n 4-23	2n +2(m+n) 5-19						
17	6n 5-37	6n+2(m+n) 7-11	6n+6(m+n) 9-19					
19	8n 6-29	8n+2(m+n) 7-37	8n+6(m+n) 10-23	8n+8(m+n) 11-31				
23	12n 8-13	12n+2(m+n) 9-29	12n+6(m+n) 12-31	12n+8(m+n) 14-17	12n+12(m+n) 17-19			
29	18n 10-19	18n+2(m+n) 12-17	18n+6(m+n) 16-13	18n+8(m+n) 18-11	18n+12(m+n) 21-37	18n+18(m+n) 27-31		
31	20n 11-11	20n+2(m+n) 13-13	20n+6(m+n) 17-17	20n+8(m+n) 19-19	20n+12(m+n) 23-23	20n+18(m+n) 29-29	20n+20(m+n) 31-31	
37	26n 13-17	26n+2(m+n) 15-31	26n+6(m+n) 20-29	26n+8(m+n) 23-13	26n+12(m+n) 28-11	26n+18(m+n) 35-23	26n+20(m+n) 37-37	26n+26(m+n) 45-19
41		30n+2(m+n) 17-23	30n+6(m+n) 22-37	30n+8(m+n) 25-29	30n+12(m+n) 31-13	30n+18(m+n) 39-19	30n+20(m+n) 42-11	30n+26(m+n) 50-17
43			32n+6(m+n) 24-11	32n+8(m+n) 26-37	32n+12(m+n) 32-29	32n+18(m+n) 41-17	32n+20(m+n) 44-13	32n+26(m+n) 52-31
47				36n+8(m+n) 27-23	36n+12(m+n) 35-31	36n+18(m+n) 45-13	36n+20(m+n) 48-17	36n+26(m+n) 57-29
49					38n+12(m+n) 37-17	38n+18(m+n) 47-11	38n+20(m+n) 50-19	38n+26(m+n) 60-13
53						42n+18(m+n) 50-37	42n+20(m+n) 53-23	42n+26(m+n) 65-11
59							48n+20(m+n) 60-29	48n+26(m+n) 72-23
61								50n+26(m+n) 74-37

Rotations for the platform for the additives, the reference box (11,11): 3 + 11n + 11(m+n) + n30(m+n)

In fig. 3 we represent the 64 sub-expressions, i.e. the numbers of rotations generating non-primes, as certain *additions* to the "platform" or "ground zero" of expression (25). For a convenient overview we downplay the numbers situated in the far left joint of the sub-expressions (as the number 3 in the (25) expression), from now on coined *first-numbers*, in the main (upper) lines in the boxes of the fig. 3 matrix. Instead we list these first-numbers in the lines below the main parts of said additions. These 64 first-numbers are marked in red in fig. 2 as well as in fig. 3, and are merely transported from fig. 2 to fig. 3. In fig. 3 we also list, immediately to the right of the first-numbers in red, the respective *columns* of fig. 2 for the multiples in the boxes in fig. 3, indicated by the *initial* position numbers in the original chamber for the respective columns of fig. 2.

Horizontally, at the top of fig. 3, we list in succession the factors in the original chamber, acting as multiplicands in the 64 basic products represented in fig. 2. Vertically, to the left of fig. 3, we list in succession the numbers acting as multiplicators in the 64 basic products represented in fig. 2. Hence, all the 64 basic products, and all the clusters of non-primes generated from each of them, are also represented in fig. 3.

The general formula to find the respective additions for the basic products is established as follows:

We denote the multiplicator factors in the far left column of fig. 3 with the symbol *v* and the multiplicand factors at the top of fig. 3 with the symbol *w*. When ignoring the first-numbers, the remaining sub-expression inside the brackets, as for example in the expression (16b), has the general form:
(26) $(w+v)n + vm + n30(n+m)$

We rewrite this to:
(27) $[(11 + w-11) + (11 + v-11)]n + (11 + v-11)m + n30(n+m)$

Ignoring the first-number 3 in expression (25), and then subtracting (25) from (27) gives:
(28) $(v-11)n + (w-11)(n+m)$

Hence, for any expression of the additives in the matrix of fig. 3 the coefficients of n and (n+m) are simply found by subtracting 11 from, respectively, v and w.

The interpretation of the expression for a box in fig. 3 is as follows, taking (29,23) as an example: The expression of this box lists the numbers of rotations generating all possible non-primes when increasing m and/or n from this box, identical with the upper line of box $(2,b_4)$ in fig. 2, which is the product 23×29. This product represents the *second* step for the multiplicand 23-factor which has as its *first* step the product 23×23 (the element represented by the third line in box $(1,a_3)$ in fig. 2). The expression for box (29,23) in fig. 3, consistent with the upper line of the box $(2,b_4)$ in fig. 2, gives *all* the non-primes generated from possible sets of (n,m) departing from the *lowest* among products of the 23-multiplicand when these products are located at the *same position* in the chamber as the initial position number 37 (i.e. in column b_4 of fig. 2). This lowest product, 23×29, emerges after 21 rotations of the original chamber, 21 being the first-number of this box. Departing from this basic product with successive increases in m and n, the expression for box (29,23) in fig. 3 gives the amounts of additional rotations manifesting *all upcoming* products arriving at the same position of the chamber from the paths of said departure. This expression, when added to expression (25), corresponds to the sub-expression inside the brackets of expression (16b). However, the correct expression for the completed addition requires that also the first-numbers are included in the calculation. Since the first-number for the (11,11)-box, as stated in (25), is 3, the complete addition from the expression of box (11,11) to the expression of box (29,23) is:
(29) $(21-3) + 18n + 12(m+n)$

As an illustration we may inspect the non-prime generated from n=3 and m=2 in the box (29,23). Putting these values of n and m into (25), gives 541 rotations from the first number (11x11) in the (11,11)-box, a box which has all its numbers situated at position number 31 of the chamber. The non-prime located in this box for these values of n and m is then given by 541×30 + 31, which is 16261. The factor given from n=3 is 11+3×30=101, the factor given from m=2 is 11+(3+2)30=161, and the product is 101×161=16261. Putting m=2 and n=3 into (29) gives another non-prime after 132 additional rotations, situated at position number 37 in the (29,23)-box. This results in the non-prime 37 + 30(541+132), which is 20227. For confirmation: The factor given from n=3 is 23+3×30=113, the factor given from m=2 is 29+(3+2)30=179, and the product is 113×179=20227.

Interpreted in this way we realize that fig. 3 presents a *complete* compilation and juxtaposition of the primary 64-b set of non-prime expressions. Therefore, fig. 3 represents *all the non-primes there are*, as well as *precise generation* of them in an *exact* and quite *simple* pattern. *Ad negativo*, *all* numbers in the total set of revolving blue numbers *not* represented in fig. 3, *have to be primes*. Hence, a *generative order*, in the sense of David Bohm (1987), for generation of non-primes vs. primes is thereby exposed, and the challenge reduces to unfolding the implications of this generative order in further simplifications and details with regard to different aspects of prime number mathematics.

Fig. 3 contains the same 64 basic products as fig. 2, but applies other criteria for grouping and ordering these products into different boxes in an overall scheme. In fig. 2 the criteria are the 8 positions in the original chamber (columns, corresponding to the eight initial position numbers) and the 8 steps (rows) from each of these initial positions, constituting the 8 basic multiplicand-paths which generates the complete set of 8 total multiplicand-paths through additional rotations from stepwise recycling of the 8-step sequences when increasing m and n. In fig. 3 the criteria are the 8 successive position numbers (columns) acting as multiplicand factors in the 64 basic products, and their successive multiplicators (rows), to make up the 64 basic products. Different from fig. 2, fig. 3 also exposes directly the *formulas* for *all* numbers of rotations generating all *further* non-primes from the 64 basic products, by grouping these further non-primes into the same 64 anchoring boxes (hence, also with the same chamber positions) as the respective basic products. Also in fig. 3 the different *steps* in the 8-cycle, at which such further products are located (always identical to the step number of their anchoring basic product), are easily visible, as the steps in the *non-empty* boxes of the respective columns, for example (29,23) as the second non-empty box, and hence the second step, reckoned from the top, in the column for the 23-multiplicand. In fig. 3 the 8 step numbers for the multiplicands (represented by the columns) appear as the 8 successive bands of box *diagonals* in the figure. Fig. 2 not only *exhibits* the respective step numbers for the initial multiplicands, but primarily presents a matrix classification *based* on the step numbers (i.e. the rows) and the chamber position (i.e. the columns), which makes easy a direct read-out of the step numbers in the filled boxes of the columns for the respective chamber positions. If we go to fig. 3 and, as an example, inspect the step numbers for the basic products occurring in the revolving chamber at position number 37 (i.e. column b_4 of fig. 2), such products occurring in the *same* row 2 of fig. 2, i.e. 23×29 and 31×37, occur in fig. 3 in *different columns* (no. 5 and no. 7 from the left) as well as in *different rows* (no. 6 and no. 8 from the top). However, their manifestation in the *same box* of fig. 2, still *reappears* in fig. 3, namely as the combination of their common location in *band diagonal* no. 2 from the top (hence: step number 2) and their common number 37 to the *right in the second line* of their box expressions (hence: chamber position as of initial number 37).

We notice that the basic products in fig. 3 are distributed in such a manner that the same position number (as 37) occurs once and only once in each of the eight columns of fig. 3. This is a reappearance of the fact that all the 8 multiplicands in fig. 2 occur once and only once in each column. Hence, the complete set of non-primes in fig. 3 occurring at position number 37 is generated from one and only one basic box, as well as arriving into the same box, in each of the eight columns. The same is of course the case for the complete set of non-primes occurring at the seven *other* position numbers. We also notice that basic products occurring with the *same position number* in fig. 3 always occur at *different rows*. (In many cases there occur *gaps* between such rows. This is due to the fact that boxes for each position number only occur 8 times in fig. 3, while fig. 3 encompasses 15 rows. If we lifted the 7 lowest rows 8 rows upwards in fig. 3, corresponding to subtracting 30 from involved multiplicators, the gaps would disappear.)

In fig. 2 we can also read out step numbers for the respective *multiplicators* in the 64 basic products. Each multiplicator occurs once and only once in each step-row (and in each column) of fig. 2, but with the modification that the lowest multiplicators disappear successively downwards in the scheme in tandem with becoming substituted by multiplicators from the chamber rotated once, i.e. the number of the substitute constituted by the replaced number added with 30. Also in fig. 3 the pattern of this substitution is easily seen, by noticing that the multiplicators in the lower half of the figure, successively replace the multiplicators at the top,

In fig. 3 only the boxes in the 11-column have expressions without (m+n). This is merely due to the choice of (11,11) as the ground zero reference frame which enables presentation of the expressions of the other boxes by simply reducing these to corresponding additions to expression (25) for the reference box. If instead we had chosen, for example, (23,23) as such a reference frame, the boxes in the *23*-column would have been the only ones having additive expressions without (m+n).

For any value of (n,m), fig. 3 gives 64 related non-primes. Hence, we can regard the whole field of blue non-primes as generated from stepwise increase of m and n, each step generating a new "*board*" of 64 non-primes in an exposed rhythmic pattern (however, with some inclusion of overlap between numbers on the new board and non-primes having already occured at earlier boards).

For further understanding of this pattern, it seems fruitful to introduce:
(30) $m' = m+n$

Moving back to expressions (16), (16a) and (16b), these can then be rewritten as:
(31) $(23+n30)\ (29+m'30)$
(31a) $23\times 29 + 30(29n+23m'+30m'n)$
(31b) $37 + 30(21+29n+23m'+30m'n)$

Trivially, also the primary 64 and 64-b *sets* of non-prime expressions can be rewritten in analogy with this, in order to provide some simplification of the overall pattern. We could have achieved these simplified expressions directly, if we at the introduction of (16) had chosen to define the set of m as constrained by m having to be equal to or larger than n. (Without such a constraint it would not be possible to execute our strategy described earlier, i.e. the succession from (a) to (e), since multiplicands for many values of (n,m) then would be *larger* than multiplicators of same products.)

To further explore fig. 3 we rewrite (25) as:
(32) $3 + 11(n+m') + 30m'n$

We will now examine the pattern of rotations generated by variations of m' and n in the (32)-expression for rotations making up the (11,11)-box.

For convenience we study the pattern of variations by a structuring that defines the variable among two, which displays the *largest* value, as consistently the *same* variable. (Below we have chosen m' to denote this largest of the two variables.) This is convenient because it gives a simpler structuring, while at the same time such a choice does not effect any pair of values to be missed in the exposition, due to both variables playing identical and interchangeable roles in expression (32).

Fig. 4 ***The set of rotations for non-prime box (11,11) at position number 31 in the revolving chamber***

n+m'	m'	m'n	**C= 11(n+m') + 30m'n**	Horizontal increase in C
0	0	0	0	0
1	1	0	11	11
2	2 1	0 1	22 52	22 30
3	3 2	0 2	33 93	33 60
4	4 3 2	0 3 4	44 134 164	44 90 30
5	5 4 3	0 4 6	55 175 235	55 120 60
6	6 5 4 3	0 5 8 9	66 216 306 336	66 150 90 30
7	7 6 5 4	0 6 10 12	77 257 377 437	77 180 120 60
8	8 7 6 5 4	0 7 12 15 16	88 298 448 538 568	88 210 150 90 30
9	9 8 7 6 5	0 8 14 18 20	99 339 519 639 699	99 240 180 120 60
10	10 9 8 7 6 5	0 9 16 21 24 25	110 380 590 740 830 860	110 270 210 150 90 30
11	11 10 9 8 7 6	0 10 18 24 28 30	121 421 661 841 961 1021	121 300 240 180 120 60
12				132 330 270 210 150 90 30
13				143 360 300 240 180 120 60
14				154 390 330 270 210 150 90 30
…	…	…	…	….
…	…	…	…	….
			0 52 164 336 568 860	
			11 41 71 101 131 161	
			11 41 71 101 131 161	
			11 41 71 101 131 161	
			… … … … … …	
			Vertical increase in C	

The first column from left displays increasing (n+m') which we use as a convenient starting reference frame in examining the variations. The second (matrix) column shows the corresponding different possibilities for figures of m'. The third (matrix) column shows the corresponding different possibilities for figures of m'n for the respective values of m'. The fourth (matrix) column shows the corresponding different figures for the overwhelming part C of the (11,11)-box expression (only ignoring the trivial first-number of 3 rotations for the basic 11×11 product). The fifth (matrix) column shows the corresponding different figures for horizontal increase in C departing from the first sub-column of the fourth column. The (matrix) rows at the bottom of the figure show the corresponding different figures for vertical increase in C from the top number of each sub-column of the fourth column.

Just as an example, n=4 and m=3 gives (m',n)=(7,4) and n+m'=11. The corresponding number in the matrix C is 961.Adding the first-number 3 of the (11,11)-box to 961 and multiplying with 30, gives 28920. Adding to this the position number for the (11,11)-box, 31, gives 28951 as the non-prime represented by the number 961 in matrix C. For confirmation: (n,m)=(4,3) for the box (11,11) gives from the value of n the factor 11 + 4×30 = 131, and from the value of m the factor (11+4×30) + 3×30 = 221, i.e. 131×221 which is 28951.

Adding 3 to the set C exposed in fourth column, we achieve the exhaustive set of rotations arriving in the (11,11)-box. (We also achieve the values of m', n and m'n for a certain number of rotations if looking to the *first three* columns; however, some rotations have more than one such set of values and will transpire more than once in the fourth column.) This is the *total* set of non-primes located in box (11,11), *one* of the eight boxes of fig. 3 having its multiples located at position number 31. In addition there will be analogous sets of non-primes located in the seven *other* boxes of multiples arriving at position number 31. Subtracting the *union* of these eight sets of non-primes, we know that *all remaining numbers of rotations result in primes* at position number 31. For a pin-downed understanding the challenge is reduced to merely working out the most simple and convenient *expressions* of these sets of non-primes and primes.

In fig. 4 we notice plural possibilities for working out such convenient expressions, when inspecting both horizontal and vertical increase in C, and also when inspecting diagonal increase in C (as well as when inspecting the main underlying structure for such increases in C, namely m'n).

To simplify the overall pattern even more, we present fig. 5 which follows from a simple read-out of fig. 4 (matrix) columns:

Fig. 5 ***Make-up of the set of rotations for non-prime box (11,11) at position number 31 in the revolving chamber***

	3+	0+	
	3+	1+	
	3+	2+	1
	3+	3+	2
	3+	4+	3 + 1
	3+	5+	4 + 2
	3+	6+	5 + 3 + 1
	3+	7+	6 + 4 + 2
	3+	8+	7 + 5 + 3 + 1
	3+	9+	8 + 6 + **4** + 2
	3+	10+	9 + 7 + 5 + 3 + 1
	3+	11+	10+ 8 + 6 + 4 + 2
	…	…	……………………..
	….	….	………………………
n		0	1 2 3 4 5

Colour coding:
30's
11's
1's

In fig. 5 rows represent n+m', with n+m'=0 for the top row; and diagonals represent n increasing from left to right, with n=0 at the far left diagonal (in blue). The first-number of the box is represented by the column to the left (in green).

Fig. 5 represents the overall make-up of C from its composition of 11's (in blue) and 30's (in black). The black and blue numbers, when given a certain interpretation, represent the complete set of C. This interpretation is as follows: A *blue* number is to be interpreted as the product of 11 and the blue number of the row. A *black* number in a row, is to be interpreted as this product added with the product of 30 and the *sum* of the black number and *all preceding* black numbers to the left of it in the same row. When merely adding to these C elements, represented by the respective blue and black number interpretations, the initial 3 rotations (in green) constituting the first-number of box (11,11), fig. 5 in its infinite extension provides a compressed and *exhaustive* representation of the *total* set of rotations for which the box (11,11) is filled.

The *set* of non-factorized numbers included in the box are then given directly by multiplying the respective amounts of rotations with 30, and to this product add 31 (the position number in the original chamber generating the first product, 11×11, in the box).

Just as an example:

The amount of rotations represented by the black **4** at the row with blue 9 (i.e. n+m'=9) in the figure:
(33) $3+ 11\times9 + 30(8+6+\mathbf{4}) = 642$
The natural number corresponding to this place in the revolving chamber after this amount of rotations:
(34) $642\times30 + 31 = 19291$
Hence, this black 4 in fig. 5, when interpreted in this manner, is just another way of writing the number 19291. Since this number is included in fig. 5, it is positioned in box (11,11) and with necessity a non-prime. Just for confirmation: This black 4 is located in fig. 5 at the position for n+m'=9 and m'=6, which gives n=3 and m=3. This gives the factor $(11+3\times30)$ from the value of n, and from the value of m the other factor $[(11+3\times30) + 3\times30]$, i.e. the product 101×191 which is 19291.

All rotations *not* represented by blue and black numbers in fig. 5, represent primes if – and only if – they also are not represented in corresponding figures/formulas for any *other* of the eight boxes corresponding to position number 31 in the revolving chamber.

With regard to convenient expressions for C, it may also be useful to contemplate the structure of *vertical* increase illustrated in fig. 4. Moving back to (25), we subtract the first-number 3 and rewrite the remaining part C of the expression as:
(35) $(30n^2+22n) + m(30n+11)$

Quite obviously, the *first* joint of (35) expresses the *top row* of the matrix for vertical increase in C at the bottom of fig. 4. The value of n indicates the number of steps to the right to generate the top number for the respective (sub-)columns in this matrix.

The *second* joint of (35) expresses the *additives* to the respective top numbers, constituting the respective (sub-)columns in the matrix when added to the top numbers. The value of m indicates the number of steps downwards from the top to generate these (sub-)columns.

Taken together, the two joints of (35) in this way express the whole matrix of vertically increasing C, with the number of rotations for any position being indicated by the values of n (for the column position) and m (for the row position). Just as an example, we may look at the position with its column 3 steps to the right and its row 2 steps downwards. According to fig. 4 the number of rotations for this position is to be read out of the figure as 336+101+101 which is 538. For confirmation, putting n=3 in the first joint of (35) gives 336, and putting m=2 in the second joint gives 202, and the addition of these two values for the joints give 538.

This clarifies how expression (25) is manifested and embodied in fig. 4, and how the matrixes of fig. 4 visualize increases in m and n. Quite obviously, we can easily also relate the *additives* of fig. 3 to the matrix of vertically increase in C of fig. 4, by decomposing such an additive into two sub-additives to the respective two joints of (35). As an example, when the additive for box (19,19) is written 16n+8m, the first sub-additive, 16n, gives $(30n^2+38n)$ when added to the first joint of (35), and the second sub-additive, 8m, gives m(30n+19) when added to the second joint of (35). Applying the same procedure for all 64 additives of fig. 3, we can represent the whole picture of non-prime generation by such decomposed 8×8 *additives* to the matrix of vertical increase in C of fig. 4.

The different positions in the matrix of vertical increase in C from fig. 4 hold corresponding positions in fig. 5. The top row of this matrix appear as the numbers represented by the column with 1's to the right in fig. 5, with the top black 1 representing the number 52 for (n,m)=(1,0), the next black 1 representing the number 164 for (n,m)=(2,0), etc. The left column of the matrix appears as the numbers represented by the left diagonal of natural numbers (illustrated in blue) in fig. 5, the next left column of the matrix appears as the numbers represented by the next left diagonal of natural numbers (illustrated in black) in fig. 5, etc. Just as an example, the number 639 at position (n,m)=(3,3) in the vertical matrix of fig. 4, appears as the corresponding position indicated by the number **4** in black bold in fig. 5, which is interpreted as 639 added with the first-number 3. For each black number in fig. 5, *n* is represented by the number of steps downwards from the same blue number to arrive at the black number; and *m* is represented by the number of steps diagonally from a diagonal's initial number 1, required to arrive at the black number. In this way, fig. 5 provides a *direct read-out of n and m*. Therefore, the respective rotation numbers for each of the positions in fig. 5 also can be directly calculated from expression (35), as constituted from the two joints of the expression.

*

After this clarification of crucial attributes of the box (11,11), applied as the anchoring reference box consistent with fig. 3, we now will broaden the treatment and use, as earlier in this article, the eight boxes for rotations filling the position number 37 as illustration. In accordance with fig. 3, these boxes are given by the following respective additions to box (11,11):

(36)

(17,11) 6n
(19,13) 8n + 2m’
(29,23) 18n +12m’
(37,31) 26n +20m’
(41,17) 30n + 6m’
(43,19) 32n + 8m’
(53,29) 42n +18m’
(61,37) 50n +26m’

To illustrate the general procedure we use the box (19,13) as an exemplar, and insert the addition of 8n+2m’ into the expression for C in fig. 4 for the respective corresponding values of n and m’. This superposition gives the following pattern:

Fig. 6 ***The set of rotations for non-prime box (19,13) at position number 37 in the revolving chamber, related to the set for (11,11)***

n+m’	m’	m’n	**C= 11(n+m’) + 30m’n + (8n+2m’)**
0	0	0	0+0
1	1	0	11+2
2	2 1	0 1	22+4 52+10
3	3 2	0 2	33+6 93+12
4	4 3 2	0 3 4	44+8 134+14 164+20
5	5 4 3	0 4 6	55+10 175+16 235+22
6	6 5 4 3	0 5 8 9	66+12 216+18 306+24 336+30
7	7 6 5 4	0 6 10 12	77+14 257+20 377+26 437+32
8	8 7 6 5 4	0 7 12 15 16	88+16 298+22 448+28 538+34 568+40
9	9 8 7 6 5	0 8 14 18 20	99+18 339+24 519+30 639+36 699+42
10	10 9 8 7 6 5	0 9 16 21 24 25	110+20 380+26 590+32 740+38 830+44 860+50
11	11 10 9 8 7 6	0 10 18 24 28 30	121+22 421+28 661+34 841+40 961+46 1021+52
…	……………….	………………	………………………………………………………
…	……………….	………………	………………………………………………………

In analogy to fig. 4 for box (11,11) we notice strict regularities with regard to horizontal, vertical and diagonal increase also for box (19,13), only with some slight modification of the simplicities. For the total picture we also must include the difference between the first-numbers of the boxes (11,11) and

(19,13), which are 3 and 7, respectively. For complete comparison between the rotations in the two boxes, all the blue numbers in fig. 6 must be added with this difference of 7-3=4.

Just as an example, n=4 and m=3 gives (m’,n)=(7,4) and n+m’=11. The corresponding number in the matrix C is 961+46=1007. Adding to this the first-number 7 of the (19,13)-box, gives 1014 which multiplied with 30 gives 30420. Adding to this the position number for the (19,13)-box, 37, gives 30457 as the number represented by the number 961(+46) in matrix C. For confirmation, (n,m)=(4,3) in the box (19,13) gives from the value of n the factor 13 + 4×30=133, from the value of m the factor (19 + 4×30) + 3×30 = 229, and from these factors the product 133×229 which is 30457.

In order to further simplify the overview of this, we illustrate the total pattern for box (19,13) in analogy with fig. 5 for box (11,11):

Fig. 7 ***Make-up of the set of rotations for non-prime box (19,13) at position number 37 in the revolving chamber***

3+4+ 0+0
3+4+ 1+1
3+4+ 2+2 + **1**+3
3+4+ 3+3 + **2**+3
3+4+ 4+4 + **3**+3 + **1**+3
3+4+ 5+5 + **4**+3 + **2**+3
3+4+ 6+6 + **5**+3 + **3**+3 + **1**+3
3+4+ 7+7 + **6**+3 + **4**+3 + **2**+3
3+4+ 8+8 + **7**+3 + **5**+3 + **3**+3 + **1**+3
3+4+ 9+9 + **8**+3 + **6**+3 + **4**+3 + **2**+3
3+4+ 10+10 + **9**+3 + **7**+3 + **5**+3 + **3**+3 + **1**+3
3+4+ 11+11 + **10**+3 + **8**+3 + **6**+3 + **4**+3 + **2**+3
…… ……………………………………………………..
…… ……………………………………………………...
n 0 1 2 3 4 5

Colour coding:
30’s
11’s
2’s (i=1 for b=2)
2’s (a=3 for b=2)
1’s f(19,13) = 3+ 4 = f(11,11) + [f(19,13)-f(11,11)]

The respective additions to fig. 5 and box (11,11), implied in the extension to box (19,13) and representing 2(4n+m’) as stated in (36), are illustrated in *red*, and are to be interpreted in analogy to the reading of the black numbers in fig. 5, with the sole difference that the inserting supplements of reds represent a successive, uniform addition of 3(·2) rotations to each black number in fig. 5. Also, in fig. 7 there occurs an initial addition to all blue numbers (hence also with implications to the interpretations of all black numbers) with a *violet* number equal to the respective blue ones for each row. Finally, we have to apply the first-number for box (19,13) which is 3+4=7, with the additive 4 illustrated in *light green*.

We recognize the number corresponding to the earlier position (m’,n)=(7,4), as the segment with black boldface **4** in the lowest row in fig. 7. The calculation of this non-prime from a read-out interpretation of this black number in fig. 7 goes as follows:
3+4 + (11×11 + 11×2) + 30(10+8+6+**4**) + 4×3×2 = 7 + 143 + 840 + 24 = 1014

From the pattern illustrated in fig. 7 we realize that the non-primes filling the box (19,13) can be written as nice series in analogy to and only slightly less simple than the non-primes filling the box (11,11) patterned in fig. 5. We also realize that the *union* of the non-primes filling the two boxes does not represent much further complication of the pattern for each of the boxes, and is adequately described by using the box (11,11) as reference frame or template. In fig. 7 the pattern and series of box (11,11) are still easily visible by simply ignoring the non-black numbers (and the dark green 3's). The pattern and series of the union of the non-primes filling the two boxes manifest visibly clear by simply first observing fig. 7 ignoring the reds, violets and light greens; then add these into the observation; and afterwards combine these two points of view into a superposition.

Said union is to be understood as the union of the set of non-primes filling one of the eight boxes at position number 31 and the set of non-primes filling one of the eight boxes at the *different* position number 37, namely the boxes (11,11) and (19,13), respectively. However, it is of special interest to look at unions of box sets at the *same* position number to reveal the total set of primes vs. non-primes for each of the eight position numbers. Also in cases with *equal* position numbers, the procedure for finding the pattern for and unifying the sets, will be analogous to the unifying procedure above.

On this background we will now examine the sets for the eight boxes of position number 37, stated in (36), also for the seven *remaining* sets arriving from the boxes different from (11,11), in analogy to fig. 7 for box (19,13).

We introduce the following concepts and symbols:
Base number (*b*); meaning a joint factor for the coefficients *k(n)* of n and *k(m')* of m' (fig. 7 from fig. 6 implies b=2).
Initial number (*i*); meaning the coefficient of m' divided by the base number, thus giving the multiple of 11's when already including the base number as a factor, for violet numbers of a box expression (fig. 7 from fig. 6 implies i=1).
Adding number (*a*); meaning the additives of uniform numbers appearing from the difference between coefficients of m' and n', displayed from fig. 7 on as uniform reds tied to the blacks.
We also state the related concept already established:
First-number (f); displayed in green colour(s) from fig. 5 on.

Simple contemplation of fig. 6 gives these equations:
(37) $i = k(m')/b$
(38) $a = [k(n) - k(m')]/b$

Then we have for the eight expressions in (36), when choosing the largest possible values of b:

(39)

Box	Expression	b (base number)	i (initial number) (multiple of violets)	a (adding number) (uniform reds)	f (first number)
(17,11)	6n	6's	0	1	5
(19,13)	8n + 2m'	2's	1	3	7
(29,23)	18n +12m'	6's	2	1	21
(37,31)	26n +20m'	2's	10	3	37
(41,17)	30n + 6m'	6's	1	4	22
(43,19)	32n + 8m'	8's	1	3	26
(53,29)	42n +18m'	6's	3	4	50
(61,37)	50n +26m'	2's	13	12	74

When inserting these values of b, i, a and (f-3) also for the seven boxes different from (19,13) into fig. 7, the overall figure that manifests from such superposition of all eight (b,i,a,f)-sets gives a quite simple overview of the basic pattern and series making up the *total* set of non-primes at position number 37 in the revolving chamber, and therefore – *ad negativo* – also of the total set of *primes* located at position number 37.

We may simplify this a bit further, by giving priority to seeking a *uniform* value of b for the eight expressions, and by presenting a simpler relation between the eight expressions by means of applying a suitable procedure for certain *translations.*

To make a complete representation of all non-primes the 8×8-board of fig. 3 was proved to be sufficient, so there was no need to consider *non-filled* boxes in fig. 3 (or located *beyond* the range of this figure). In spite of this we will now examine if a certain consideration of such may be useful in some respects. The structure of the pattern for the filled boxes in fig. 3 indicates that there is no problem to extend this pattern, ad infinitum, hence also to non-filled boxes if this is fruitful for some task. To quickly achieve general insight with regard to such extensions, we use the imagined filled box (41,41) as exemplar.

Extending the pattern for additive expressions of fig. 3 to box (41,41) gives this box the expression:
(40) 30n + 30m'

We discover the first-number for box (41,41) while inspecting fig. 3, by first locating the first-number for box (41,11), which is the first-number of box (11,11), i.e. 3, and to this add 11 (since the addition of 11, and its multiples, ensures that we return to box (11,11)), which gives 14. After this the first-number for box (41,41) is found by the first-number of box (41,11), i.e. 14, and to this add 41 (since the addition of 41, and its multiples, ensures that we return to box (41,11)), which gives 55.

In theory and stated in passing, we could also have discovered the first-number by alternatively first moving *horizontally* in fig. 3, to box (11,41), and then vertically to box (41,41).For both methods the first-number 55 of box (41,41) is found as the first-number 3 of box (11,11), added with one of the two position factors in this last box, 11, to represent *one* direction of rotations, and to this add the *other* position factor of the box, in this case also 11, to represent the supplementary orthogonal direction. However, this alternative route is not in agreement with our basic strategy which departs from the square of a multiplicand and forbids multiplicands (the top row of fig. 3) to be larger than multiplicators (the left column of fig. 3) in any box, which would be the case for the imagined box (11,41). There is no need to complicate the matter by leaving this strategy, and therefore we still will ignore this alternative route.

From the nature of this method moving from box (11,11) to box (41,41) we realize that this is just an example of a *general* procedure to find the first-number for the "higher" imagined box anchored in each of the connected filled boxes of fig. 3. If we denote a filled box of fig. 3 as box (v,w), we have, quite obviously, with respect to the first-numbers for the connected higher box:

(41) f(v+30,w+30) = f(v,w) + v + w + 30

Also when v is different from w, as for example for box (19,13), which is to be connected to box (49,43), the two alternative methods yield the same result, merely exchanging the position of v and w in the calculation of (41); in this case 7 + 13 + (19+30), vs. 7 + 19 + (13+30).

(41) is equivalent to:
(42) f(v+30,w+30) = f(v,w) + (v-11) + (w-11) + 52

i.e.:
(43) f(v+30,w+30) = f(v,w) + (v-11) + (w-11) + f(41,41)-f(11,11)

After this clarification with regard to the first-numbers, we apply the same procedure as when establishing fig. 6 for the (19,13)-box. Thus, we achieve the following figure for box (41,41):

Fig. 8 ***The set of rotations for non-prime box (41,41) at position number 31 in the revolving chamber, related to the set for box (11,11)***

n+m’	m’	m’n	**C=** **11(n+m’) + 30m’n + (30n+30m’)**						Difference between first-numbers
0	0	0	0+0						+(55-3)
1	1	0	11+30						+(55-3)
2	2 1	0 1	22+60	52+60					+(55-3)
3	3 2	0 2	33+90	93+90					+(55-3)
4	4 3 2	0 3 4	44+120	134+120	164+120				+(55-3)
5	5 4 3	0 4 6	55+150	175+150	235+150				+(55-3)
6	6 5 4 3	0 5 8 9	66+180	216+180	306+180	336+180			+(55-3)
7	7 6 5 4	0 6 10 12	77+210	257+210	377+210	437+210			+(55-3)
8	8 7 6 5 4	0 7 12 15 16	88+240	298+240	448+240	538+240	568+240		+(55-3)
9	9 8 7 6 5	0 8 14 18 20	99+270	339+270	519+270	639+270	699+270		+(55-3)
10	10 9 8 7 6 5	0 9 16 21 24 25	110+300	380+300	590+300	740+300	830+300	860+300	+(55-3)
11	11 10 9 8 7 6	0 10 18 24 28 30	121+330	421+330	661+330	841+330	961+330	1021+330	+(55-3)

For box (11,11) we have for a pair (m’,n):
(44) C(11,11) = 11(n+m’) + 30m’n

Added to the first-number for the box this gives the total amount of rotations *R* for the pair:
(45) R(11,11) = 3 + C(11,11)

For box (41,41) we have for a pair (m’,n):
(46) C(41,41) = C(11,11) + 30(n+m’)

(44) inserted into (46) gives:
(47) C(41,41) = 11(n+m’) + 30m’n + 30(n+m’)

equivalent to:
(48) C(41,41) = 41(n+m’) + 30(n+m’)

Added to the first-number for the box this gives the total amount of rotations for the pair:
(49) R(41,41) = 55 + C(41,41)

For box (41,41) we have for the pair (m’-1,n-1), as an implication of (48), when using the symbol *C‘* to denote the C-expression of this last pair:
(50) C’(41,41) = 41(n-1+m’-1) + 30(m’-1)(n-1)

equivalent to:
(51) C’(41,41) = 11(n+m’) + 30m’n – 52

Added to the first-number for the box this gives the total amount of rotations *R'* for this pair:
(52) R’(41,41) = 55 + C’(41,41)

(44) and (51) gives:
(53) C’(41,41) = C(11,11) – 52

(52) and (53) gives:
(54) R’(41,41) = (55-52) + C(11,11)

(54) and (45) gives:
(55) R’(41,41) = R(11,11)

This means that the total amount of rotations for (m',n) of box (11,11) is *always* identical with the total amount of rotations for (m'-1,n-1) of box (41,41), whatever the value of m' or n.

This simple connection between the rotations of box (11,11) and box (41,41) is quickly interpreted and confirmed when looking into fig. 8. Here we can move between the two alternatives (as easily seen by inspecting the m' column matrix), to express the identical amount of rotations by taking a *knight-like step* in the C matrix: one step to the right (meaning n increasing with 1) followed by two steps downwards (resulting in also m' increasing with 1), as well as by stepping in the opposite direction. We merely have to make sure afterwards to remember including the difference between the first-numbers of the two boxes.

As an example, if we look at (m',n)=(5,3) at the row n+m'=8 in the figure, we find the corresponding black number 538 in the (11,11)-box of the C-matrix in fig. 8. Adding the blue number tied to it, 240, which indicates a necessary addition to manifest the number in the (41,41)-box at the same position (m',n)=(5,3), we get 778. Adding to this the difference, i.e. 52, between the two first-numbers of these two boxes, we get 830, which fig. 3 exposes as identical to the amount of C-rotations for the pair with (m',n)= (6,4) in the (11,11)-box. Thus, we have performed a knight-like move between the two boxes. Adding to 830 the first-number 3 for the (11,11)-box, gives 833 rotations which multiplied with 30 gives 24990. Adding to this the position number 31 for the (11,11)-box gives 25021 which therefore is the natural number corresponding to (m',n)=(5,3) in box (41,41) and identical to the natural number corresponding to (m',n)=(6,4) in box (11,11).

For confirmation: (m',n)=(6,4) for box (11,11) is equivalent to (n,m)=(4,2) for the same box, which from the value of n gives the factor 11 + 4×30 = 131, from the value of m the factor (11 + 4×30) + 2× 30 = 191, and from these the product 131×191 which is 25021. Trivially, (m',n)=(5,3) for box (41,41) is equivalent to (n,m)=(3,2) for the same box, which from the value of n gives the factor 41 + 3×30 = 131, from the value of m the factor (41 + 3×30) + 2×30 = 191, and from these factors the same product 131×191 which is 25021.

Hence, fig. 8 provides *two alternative representations* of this same number, one framed in box (41,41) with (m',n)=(5,3), and another framed in box (11,11) with (m',n)=(6,4), and the example illustrates the details in the correct calculations when performing the knight-like step from the first representation to the second (as well as, indirectly, the mirrored opposite route).

This identity is no surprise, since the structure in establishing fig. 3 guarantees that all numbers in the top row of fig. 3 can be interpreted as multiplicands and all numbers in the column to the far left as multiplicators. Hence, the choice of reference box (which is (11,11) in fig. 3) and the accompanying 8×8 reference board for each pair of (m',n) anchored in the box is a question about convention and adequacy.

In the case of (41,41) the identity is related to the multiplicand of the box being identical with 11added with 30 (11 added with 1 rotation) and the multiplicator being identical with 11 added with 30 (11 added with 1 rotation). Thus, the box (41,41) is basically just another clothing of the box (11,11). Applying the box (41,41) as reference-box instead of (11,11), and the according modification of the (32)-expression, does not represent any basic novelty of information, and is just a question about convenience. The identity implies that we can move freely to and fro the two boxes.

However, the alternation between the two boxes (11,11) and (41,41) as illustrated in fig. 8 has the restriction that some numbers in box (11,11) can not be reached from the box (41,41) when supposing absence of negative values of n and m' in the definition domain for box (41,41). This makes it impossible to step forward from box (41,41) to arrive at numbers in the box (11,11) with n=0 or m'=0. This is the case for all numbers located in the left column in the C matrix of box (11,11), due to these numbers having n=0. This restriction is easily overcome by a modest extension of the definition domain for n and m' for box (41,41) to also include values of n=-1, as well as to include the value of m'=-1 in one special case, namely to make up the pair (m',n)=(0,0) in box (11,11) from a pair one step lower in box (41,41), i.e. the pair (-1,-1). The result of this extension is presented in the following figure:

Fig. 9 ***The extended set of rotations for non-prime box (41,41) at position number 31 in the revolving chamber to cover the complete set for box (11,11)***

n+m' m'	m'n	**C= 11(n+m') + 30m'n + (30n+30m')**	Difference between first-numbers
-2 -1	+1	8-60	+(55-3)
-1 0	0	-11-30	+(55-3)
0 1 0	-1 0	-30+0 0+0	+(55-3)
1 2 1	-2 0	-49+30 11+30	+(55-3)
2 3 2 1	-3 0 1	-68+60 22+60 52+60	+(55-3)
3 4 3 2	-4 0 2	-87+90 33+90 93+90	+(55-3)
4 5 4 3 2	-5 0 3 4	-106+120 44+120 134+120 164+120	+(55-3)
5 6 5 4 3	-6 0 4 6	-125+150 55+150 175+150 235+150	+(55-3)
6 7 6 5 4 3	-7 0 5 8 9	-144+180 66+180 216+180 306+180 336+180	+(55-3)
7 8 7 6 5 4	-8 0 6 10 12	-163+210 77+210 257+210 377+210 437+210	+(55-3)
8 9 8 7 6 5 4	-9 0 7 12 15 16	-182+240 88+240 298+240 448+240 538+240 568+240	+(55-3)
9 10 9 8 7 6 5	-10 0 8 14 18 20	-201+270 99+270 339+270 519+270 639+270 699+270	+(55-3)
10 11 10 9 8 7 6 5	-11 0 9 16 21 24 25	-220+300 110+300 380+300 590+300 740+300 830+300 860+300	+(55-3)
11 12 11 10 9 8 7 6	-12 0 10 18 24 28 30	-239+330 121+330 421+330 661+330 841+330 961+330 1021+330	+(55-3)

The necessary additions to fig. 8 to include the required *negative* values of n or m' are marked in violet. By this modest extension of the definition domain for (41,41), all numbers in box (11,11) are within reach from the corresponding numbers in box (41,41) exposed in the figure. Therefore, it is just a question about convenience if these numbers, and their underlying factor products, are represented by fig. 9 or by fig. 4, and we can apply and alternate between the two alternative representations as adequate.

In analogy to presenting fig. 5 from fig. 4, we now present the following fig. 10 from a simple read-out of fig. 9. Fig. 10 constitutes a translated, unambiguous and covering alternative to fig. 5, hence duplicating the relation between fig. 9 and fig. 4.

Fig. 10 ***Make-up of the extended set of rotations for non-prime box (41,41) at position number 31 in the revolving chamber to cover the complete set for box (11,11)***

```
      3+52+                          (-1)- [-2-2]
      3+52+                           0-  [-1-1]
      3+52+                         1-   0+0
      3+52+                       2-   1+1
      3+52+                     3-    2+2 + 1
      3+52+                   4-    3+3 + 2
      3+52+                 5-    4+4 +  3 + 1
      3+52+               6-   5+5 +   4 + 2
      3+52+             7-   6+6 +   5 + 3 + 1
      3+52+           8-   7+7 +   6 + 4 + 2
      3+52+         9-    8+8 +  7 + 5 + 3 + 1
      3+52+      10-    9+9 +  8 + 6 + 4 + 2
      3+52+    11-  10+10 + 9 + 7 + 5 + 3 + 1
      3+52+  12- 11+11 + 10 + 8 + 6 + 4 + 2
      ….  ……………………………………….
   n       -1      0       1    2    3    4    5
```

Colour coding:
30's
11's
30's
1's   f(41,41) = 3 + 52 = f(11,11) + [f(41,41)-f(11,11)]

Negative numbers symbolized by black numbers to the *left* of the blue numbers at the diagonal axis n=0, are to be read-out *from right to left*, departing from the number located at the zero axis for the row in question. Such reading is indicated by putting the negative sign at the *right* of black numbers located left of the zero axis. As an example, the number represented as **5-** in the figure, i.e. the number at row n+m'=6 with m'=5 and n=-1, is to be interpreted by the following calculation from the zero axis towards the left:
(4× 11 + 4×30) **-5×**30 = 164 – 150 = 14, which added to the first-number 55 gives 69 rotations at position number 31, i.e. the number 2101. For confirmation, the pair (m',n)=(5,-1) is equivalent with (n,m)=(-1,6) which is read as the product of the factors (41- 1×30) and [(41- 1×30) + 6×30], which is the product 11×19 identical to 2101. Hence, in the figure **5-** is to be interpreted as the unique representation of this multiple 2101.

Numbers inside brackets do not belong to the set, but are included in the figure because they expose the extension of the same pattern to negative values of (n+m') when the numbers in brackets are included in the *calculation* of the top numbers to the left of the diagonal axis n=0. As an example, the number at the top left, represented as **(-1)-** in the figure, i.e. the number with m'=-1 and n=-1, is to be interpreted by the following calculation from the zero axis towards the left:
(-2×11 -2×30) – (**-1×**30) = -82 + 30 = -52, which added to first-number 55 gives 3 rotations at position number 31, i.e. the number 121 which is 11×11.

Thus, the numbers in brackets can be said to play a hidden role in the make-up of the set.

*

The identity (55) is just an example of a *general* identity relation between R(v,w) and R'(v+30,w+30). To make this perfectly clear, we state the general equations in analogy to the performed procedure from (44) to (55):

Applying the general expression of (27) for box coefficients, and generalizing the approach from equation (44), we get the following general equation with respect to a box (v,w):
(56) C(v,w) = C(11,11) + (v-11)n + (w-11)m’

Addition of first-number to (56) gives:
(57) R(v,w) = f(v,w) + C(v,w)

For a box (v+30,n+30):
(58) C(v+30,w+30) = C(41,41) + (v-11)n + (w-11)m’

Adding the first-number for this box to give the total amount of rotations for the pair:
(59) R(v+30,w+30) = f(v+30,w+30) + C(v+30,w+30)

For box (v+30,w+30) we have for the pair (m’-1,n-1), as an implication of (58):
(60) C’(v+30,w+30) = C’(41,41) + (v-11)(n-1) + (w-11)(m’-1)

equivalent to:
(61) C’(v+30,w+30) = C’(41,41) + (v-11)n + (w-11)m’ – (v-11 + w-11)

Adding the first-number for this box to give the total amount of rotations for the pair:
(62) R’(v+30,w+30) = f(v+30,w+30) + C’(v+30,w+30)

We can rewrite (53) as:
(63) C’(41,41) = C(11,11) – [f(41,41) - f(11,11)]

(56) and (63) gives:
(64) C’(41,41) = C(v,w) - [(v-11)n + (w-11)m’] - [f(41,41) - f(11,11)]

(61) and (64) gives:
(65) C’(v+30,w+30) = C(v,w) - [f(41,41) - f(11,11)] – (v-11 + w-11)

(62) and (65) gives:
(66) R’(v+30,w+30) = C(v,w) - [f(41,41) - f(11,11)] – (v-11 + w-11) + f(v+30,w+30)

(66) and (43) gives:
(67) R’(v+30,w+30) = C(v,w) + f(v,w)

(67) and (57) gives:
(68) R’(v+30,w+30) = R(v,w)

Therefore, we can translate any box in fig. 3 with its expression to an equivalent higher box by adding 1 (or any natural number) rotation (i.e. the number of 30) to both the multiplicand and the multiplicator, hence adding (30n+30m’) (or multiples of that) to the expression of the first box. This modification can suitably be written as just an addition of 30n+30m’ to the additive expression for the corresponding box in fig. 3. For example, we can rewrite the expression for (19,13), which is 8n+2m’, as the imagined box (49,43) with the expression 38n+32m’, merely remembering that the last expression also must include the value -1 for n, as well as and -1 for m’ in the special case where n+m’=-2.

Now we perform such a rewriting for the six boxes in (39) different from boxes (29,23) and (37,31), while we also present the rewritten expressions uniformly with the same joint (highest) base number, b=2. This gives the following pattern (with numbers for pre-translated boxes in parentheses, and applying (41) to calculate first-numbers for the translated boxes):

(69)

Box with translation	Expression	Base number b	Initial number i	Adding number a	First-number f
(29,23)	2(9n+ 6m’)	2’s	6	3	21
(37,31)	2(13n+10m’)	2’s	10	3	37
(17,11)->(47,41)	2(18n+15m’)	2’s (6’s)	15(0)	3(1)	63(5)
(19,13)->(49,43)	2(19n+16m’)	2’s (2’s)	16(1)	3(3)	69(7)
(41,17)->(71,47)	2(30n+18m’)	2’s (6’s)	18(1)	12(4)	110(22)
(43,19)->(73,49)	2(31n+19m’)	2’s (8’s)	19(1)	12(3)	118(26)
(53,29)->(83,59)	2(36n +24m’)	2’s(6’s)	24(3)	12(4)	162(50)
(61,37)->(91,67)	2(40n +28m’)	2’s(2’s)	28(13)	12(12)	202(74)

By performing the adequate translations we have achieved: 1) uniform expression of the base numbers; 2) synchronous successive increases in n, m’ and f when moving top-down from box to box; and 3) uniform expression of adding numbers in two classes (3 for top four expressions; 3×4 for bottom four expressions).

For the five upper translations we must remember also to include values of n=-1 and the one case of m’=-1 in the definition domain for the expressions. For the last translation, namely the box of (61,37), a further modification is necessary, since in this case an extension to the value of -1 *already* has been shown to be necessary (cf. (24) and (24b)).

In (24b) the C-expression inside the brackets is:
(70) $38n + 7m + n30(n+m)$

Exchanging m with m’=m+n, this translates to:
(71) $31n + 7m' + 30m'n$

From the coefficients it is obvious that this can be interpreted as the C-expression for the imagined box (31,7) related to fig. 3. The only difference from the discussion above is that we in this case take one step down to a *lower* box than those filled in fig. 3. Hence, the box (31,7) relates to the box (61,37) in homology to the relation between a box (v,w) and its higher box (v+30,w+30), and is to be interpreted as a box in the set (v-30,w-30).

For a clarification of this further modification we illustrate the set of rotations related to box (61,37) in the following figure:

Fig. 11 ***The extended set of rotations for non-prime box (91,67) at position number 37 in the revolving chamber to cover the complete set for box (31,7) by means of box (61,37) and box (41,41)***

n+m’	m’	m’n	**C= 11(n+m’) + 30m’n + (50n+26m’) + (30n+30m’)**				
-4	-2	4	76-152-120				
-3	-1	2	27-126-90				
-2	0 -1	0 1	-22-100-60	8-76-60			
-1	1 0	-2 0	-71-74-30	-11-50-30			
0	2 1 0	-4 -1 0	-120-48+0	-30-24+0	0+0+0		
1	3 2 1	-6 -2 0	-169-22+30	-49+2+30	11+26+30		
2	4 3 2 1	-8 -3 0 1	-218+4+60	-68+28+60	22+52+60	52+76+60	
3	5 4 3 2	-10 -4 0 2	-267+30+90	-87+54+90	33+78+90	93+102+90	
4	6 5 4 3 2	-12 -5 0 3 4	-316+56+120	-106+80+120	44+104+120	134+128+120	164+152+120
…	…………	………………	………………………………………………………………..				
…	…………	………………	………………………………………………………………..				

The reference set of rotations for box (11,11) is illustrated in *black* (cf. fig. 4). The set for box (61,37), manifested on this background of box (11,11), is represented by the added suffixes in *blue* (analogous to establishing fig. 6). The set for box (91,67), manifested on this background of box (61,37), is represented by the supplementary added suffixes in *pink* (analogous to establishing fig. 8). The *complete* set for box (91,67) to cover the whole set for box (61,37) is achieved by adding the supplementary sections with *violet* prefixes (analogous to establishing fig. 9). Finally, the complete set for box (91,67) to cover the whole set for box (31,7) is achieved by also adding the supplementary sections with *green* prefixes.

If we want to express a number at (m',n) in box (61,37) by the corresponding number at (m'-1,n-1) in its higher box (91,67), the relation between the two numbers appears in fig. 11 when performing a knight's move: one column to the left in matrix C, and then two rows downwards in matrix C. Just as an example, we can look at (m',n) = (2,2) in box (61,37). This number, located at the bottom right of fig. 11, has C=164+152=316. (The pink section, 120, has to be ignored since box (61,37) does not contain any pink suffix numbers.) The corresponding number is located at (1,1) in box (91,67), which has C=52+76+60=188. The difference between these two C-values is 128, which is identical to the difference between the first-numbers for the two boxes, 202 for box (91,67) and 74 for box (61,37). Thus, the procedure represents a straight-forward combination of the earlier procedures described in connection to fig. 6 and fig. 8. As in the case for fig. 9 and for the reasons clarified there, in order to achieve a *complete* representation of the set of the lower box by means of the set the higher box, it is also necessary to include the supplementary sections illustrated in violet to account for required *negative* values of the higher box.

Let's now look at the possibility of expressing a number at (m'+1,n+1) in box (31,7) by the corresponding number at (m'-1,n-1) in the higher box *two* levels above itself, i.e. the box (91,67), with the box (61,37) acting as an intermediary. Then we have to perform *two* knight moves in sequence, moving from the number at (m'-1,n-1) in box (91,67), via (m',n) of box (61,37), into (m'+1,n+1) in box (31,7), for example from the number at (0,0) via (1,1) to (2,2). If convenient, we of course also can perform the double knight move the opposite way.

To cover the *complete* set of (31,7) we then must include some *further* negative values of n (as well as one for m'), just as we did to ensure that the complete set of (61,37) was covered by (91,67) in the figure. This is achieved by simply repeating the same procedure once more. These supplementary numbers are illustrated in green in the figure. For example, the number at the top of matrix C in fig. 11, the number for (m',n)=(-2,-2), is the representation in box (91,67) of the number at (0,0) in box (31,7). By this moderate extension *all* numbers in box (31,7) have a unique correspondent number in box (91,67) in the figure.

In fig. 11 the numbers in box (61,37) are illustrated as the black/violet/green numbers of box (11,11) added with a *blue* suffix. The numbers in box (91,37) are illustrated as these numbers in box (61,37) added with a *pink* suffix. The numbers in box (7,37) are not directly illustrated, but this restriction is passed straight-forward by a reading that just *switches the sign of the pink* suffixes (reflecting the underlying switching of the sign before (30n+30m') in the expression of C). Then we can move between box (61,37) and box (31,7) analogous to the described movement between box (91,67) and box (61,37).

Just as an example, we can look at (m',n) = (1,1) in box (61,37). This number has C=52+76=128. The corresponding number in box (31,7) is the number at (2,2) which has C=164+152-120=196 (switching the sign of the pink suffix). The difference between these two C-values is 68, which is identical to the difference between the first-numbers for the two boxes, 74 for box (61,37) and 6 for box (31,7).

By combining these two analogous procedures into one we can perform a double knight move directly from box (91,67) to box (31,7). Then we have to include the pink suffixes of the numbers *both* in the departure box and in the destination box, but switch the sign of the suffix, while we can ignore the intermediary box (61,37) (without any pink suffix).

Just as an example, we can look at the correspondence between the number at (-1,-1) in box (91,67) and the number at (1,1) in box (31,7). The first number has C=8-76-60=-128. Its corresponding number in the box two levels below, i.e. at (1,1) in box (31,7), has C=52+76-60=68 (after having switched the sign for the pink suffix +60 at (1,1)).The difference between these two C-values is 196, which is identical to the difference between the first-numbers for the two boxes, 202 for box (91,67) and 6 for box (31,7). (Inclusion of negative rotations in matrix C of fig. 11 does of course not imply any possibility for negative *overall* rotations when the relevant first-numbers are brought into the calculation.)

Thus, in this combined procedure we do not have to consider the intermediary number in box (61,37). Nor do we have to consider the first-number of this box, because it has opposite sign in the two sub-procedures making up the double knight move and therefore cancels out. Still, the box (61,37) functions as an underlying template to understand the double knight move, and its numbers are included as parts of the segments constituting the numbers of both the departure box and the destination box.

Obviously, the contemplation above is general in its nature and does not depend on the values of v and w in the intermediary box. Therefore, we can give this interrelationship a general formulation with respect to a box (v,w) as illustrated as in fig. 11, with values (m’,n) in definition domains as exhibited in fig. 11, and the sign of the pink suffix switched from positive in the figure for (m’+1,n+1) into negative when interpreting numbers for (m’-1,n-1). If we use R^+ to denote rotations in the upside box (v+30,w+30) and R^- to denote rotations in the downside box (v-30,w-30), we have the following general identity between the rotations constituting such corresponding non-primes between these two boxes:

(72) $R^-(m'+1,n+1) = R^+(m'-1,n-1) + [f(v+30,w+30) - f(v-30,w-30)]$

By this identity all numbers in a box can as an alternative be completely and uniquely represented by numbers in the corresponding box *two* levels above.

*

We will now seek for a representation of fig. 11 in a compressed illustration analogous to our earlier figures of make-ups of rotation sets. However, first it is adequate as an intermediary step to establish a certain *extension* of the procedure performed when establishing fig. 10 which enabled the rotations of box (11,11) to become completely covered by rotations in its upside box (41,41). This extension is necessary because of the introduction in fig. 11 of the *double* knight move, having to account for further negative values n and m' than those mapped in fig. 10. We perform this extension by extrapolating one step further the procedure which established fig. 10, in order to also achieve a complete covering between translated boxes *two* steps apart. From such extrapolation we achieve the following figure:

Fig. 12 ***Make-up of the extended set of rotations for non-prime box (41,41) at position number 31 in the revolving chamber to cover the complete set for its translated box two board-steps lower***

```
3+52                              -1-  [(-3)]- [-4-4]
3+52                            0-  [(-2)]-  [-3-3]
3+52                          1-  (-1)- [-2-2]
3+52                        2-   0-   [-1-1]
3+52                      3-   1-    0+0
3+52                    4-   2-    1+1
3+52                  5-   3-    2+2 +  1
3+52                6-   4-    3+3 +  2
3+52              7-   5-    4+4 +  3 + 1
3+52            8-   6-    5+5 +   4 + 2
3+52          9-   7-    6+6 +   5 + 3 + 1
3+52       10-  8-    7+7 +   6 + 4 + 2
3+52     11-   9-   8+8 +  7 + 5 + 3 + 1
3+52   12-   10-   9+9 +  8 + 6 + 4 + 2
3+52  13-  11-  10+10 + 9 + 7 + 5 + 3 + 1
3+52 14- 12- 11+11 + 10 + 8 + 6 + 4 + 2
    ------  --------  ---------  ------------------------
n          -2    -1      0      1     2    3    4    5
```

Colour coding:
30's
11's
30's
1's f(41,41) = 3 + 52 = f(11,11) + [f(41,41)-f(11,11)]

The interpretation of fig. 12 is analogous to the one of fig. 10.

As an example, the number represented as **7-** at the diagonal n=-2 in the figure, i.e. the number with m'=6 and n=-2, is to be interpreted as follows by calculation from the zero axis towards the left: (4×11 + 4×30) + 30(-5 **-7**) = 164-360 = -196 which added to first-number 55 gives -141 rotations at position number 31 of the chamber. This gives the number 31 – 141×30 = - 4199. For confirmation: (m',n)=(6,-2) for box (41,41) is equivalent to (n,m)=(-2,8) for the box, which from the value of n gives the factor 41 + (- 2)×30 = -19, from the value of m the factor -19 + 8×30 = 191, and from these factors the product -19×221 which is -4199.

Since such a negative number is *not* to be included in the complete set of non-primes exhibited in fig. 3, the extension of box (41,41) in fig. 12 does not establish a legitimate double knight move when interpreted *in isolation*. The double knight move in fig. 11 was between boxes (91,67) and (31,7), and in general it takes place between boxes (v+30,w+30) and (v-30,w-30). When interpreting box (41,41) in fig.12 in analogy to fig.11 it does not correspond to the *intermediary* box, which is box (v,w)=(61,37) in fig. 11, but to the *highest* box, box (v+30,w+30)=(91,67). This correspondence is in fig. 11 displayed by the fact that in expressions for each (m',n) it is the box (41,41) which occurs as a joint (black number) in the segment (black+blue+pink) for (91,67). The corresponding *intermediary* boxes are (11,11), in black, and (61,37), in black+blue, consistent with these being forming the pair of boxes located at the same 8×8 board of fig. 3. The corresponding *lower* boxes are (-19,-19), in black, and (31,7), in black+blue-pink(sign-inverted). This last correspondence therefore implies an asymmetry between negative vs. positive factors constituting each of the two boxes. Technically, the negative box (-19,-19) does not pose any complication. Its first-number is 11, calculated by (41) from f(11,11)=3. However, negative boxes and negative non-primes are beyond the horizon of our treatment. The only challenge is to understand how they are to be interpreted *inside* our horizon and *why* they appear here.

(It may be noted in passing that box (41,41) *could* have been brought to function as the intermediary box, between box (71,71) and box (11,11), if the *blue* suffixes in fig. 9 were added to the respective segments in order to function just as the *pink* suffixes do in fig. 11, with related *sign-switching* in the calculations of the double knight moves.)

Due to the *negative* anchoring of box (-19,-19), it is not any surprise that the *further* negative values (when compared to those of fig. 10) of n mapped in fig. 12 can yield negative and seemingly irrelevant rotations, as illustrated by the case of the non-prime -4199 for **7-**. On the other hand, in order to yield correct results by double knight move translation between two boxes in an *isolated* interpretation, box (41,41) in fig. 12 *must* be interpreted in relation to box (11-30,11-30) which is box (-19,-19). How is this paradox between the relevance and non-relevance of such negative boxes to be reconciled?

The purpose with the extension represented in fig. 12 is not any *isolated* contemplation, but the function of these extensions as *parts* of the segments to adequately express the numbers in box (31,7) in a complete covering translation from box (91,67), as well as the function of such extensions as *parts* of the segments for *other* boxes at position number 37 when performing analogous covering translations by double knight moves. (And the same also with regard to functions of such extensions as *parts* of segments in analogous covering translations for boxes at the seven *other* position numbers.) In these contexts, such added segments as of fig. 12 do not represent *autonomous* non-prime numbers, but merely *parts* of *segments* constituting such numbers. The relevance of the extension of fig. 12 is its significance for exposing the concise *pattern* in this make-up.

As a supplementary example to make the read-out of fig. 12 perfectly clear, we may look at the number at the top left, represented as **-1** in the figure, i.e. the number with m'=-2 and n=-2. This number is to be interpreted as follows by calculation of rotations from the zero axis towards the left: (-4×11 – 4×30) + 30[-(-3) –**(-1)**] = -164 + 120 = -44 which added to first-number 55 gives 11 rotations at position number 31 of the chamber. This gives the number 31 + 11×30 = 361. For confirmation: (m',n)=(-2,-2) for box (41,41) is equivalent to (n,m)=(-2,0) for the box, which from the value of n gives the factor 41 + (- 2)×30 = -19, from the value of m the factor -19 + 0×30 = -19, and from these factors the product (-19)×(-19) which is 361. (It may be of some interest to note that in this case, different from the case above of **7-**, a negative value of n, as well as of m', yields a *positive* non-prime.) This product is identical to the product for the corresponding two-steps lower box (-19,-19) when (m'n)=(0,0), which also emerges from the first-number of box (-19,-19), i.e. 11, as 31 + 11×30. This interrelationship between the two boxes is, of course, in obvious agreement with formula (72). In this way we see how the number represented as **-1** in fig. 12 functions as a "ground zero" reference point to anchor double knight moves between (91,67) and (31,7). In fig. 11 this **-1** of fig. 12 is represented as the combination of the green prefix, 76, and the pink suffix,-120, at the upper left segment of matrix C, so that the *lowest* non-prime of this matrix C, i.e. 31×7, manifests from the mere addition of the blue insert,-152, to this combination. (For confirmation: 91×67 + (-44-152)30 = 217 = 31×7.)

This indicates how the extension performed in fig. 12 is necessary and sufficient to allow box (91,67) to cover the *complete* set of non-primes contained in box (31,7) via a corresponding extension of the definition domains of m and n for box (41,41). Hence, box (91,67) covers completely the connected box (31,7) via double knight moves, *by means of* box (41,41) covering completely box (-19,-19) via double knight moves. In this way the set of non-primes contained in box (31,7) is *completely* covered also from a box translation *two* board-steps above itself, i.e. box (91,67), by means of a correspondent covering translation between the upper and the lower box connected to reference box (11,11). Thus, the make-up of fig. 12 serves as the intermediary reference make-up to accomplish complete covering of box (31,7) from the extended set of box (91,67) at position number 37

From the general nature of the procedure it is clear that homologous covering between translated boxes two board-steps apart, can be performed for *any* box in the fig. 3 8×8 board, by means of the covering translation between the upper and the lower box connected to reference box (11,11). Thus, also with respect to covering translations spanning *two* board-steps, covering translations between reference boxes still function as anchoring reference translations for performing covering translations between all other boxes.

After having presented the make-up of fig. 12 as an adequate intermediary, we can now move on to present the make-up of non-prime box (91,67) extended to cover the *complete* set of its translated box (31,7) two board-steps lower:

Fig. 13 ***Make-up of the extended set of rotations for non-prime box (91,67) at position number 37 in the revolving chamber to cover the complete set for its translated box (31,7) two board-steps lower***

6+196+ 12- (**-1**)**-** 12-[(**-3**)]**-** [**-4**-4]
6+196+ 12-**0-** 12-[(**-2**)]**-** [**-3**-3]
6+196+ 12-**1-** 12-(**-1**)**-** [**-2**-2]
6+196+ 12-**2-** 12-**0-** [**-1**-1]
6+196+ 12-**3-** 12-**1-** **0**+0
6+196+ 12-**4-** 12-**2-** **1**+1
6+196+ 12-**5-** 12-**3-** **2**+2 + **1**+12
6+196+ 12-**6-** 12-**4-** **3**+3 + **2**+12
6+196+ 12-**7-** 12-**5-** **4**+4 + **3**+12 + **1**+12
6+196+ 12-**8-** 12-**6-** **5**+5 + **4**+12 + **2**+12
6+196+ 12-**9-** 12-**7-** **6**+6 + **5**+12 + **3**+12 + **1**+12
6+196+ 12-**10-** 12-**8-** **7**+7 + **6**+12 + **4**+12 + **2**+12
6+196+ 12-**11-** 12-**9-** **8**+8 + **7**+12 + **5**+12 + **3**+12 + **1**+12
6+196+ 12-**12-** 12-**10-** **9**+9 + **8**+12 + **6**+12 + **4**+12 + **2**+12
6+196+ 12-**13-** 12-**11-** **10**+10 + **9**+12 + **7**+12 + **5**+12 + **3**+12 + **1**+12
6+196+ 12-**14-** 12-**12-** **11**+11 + **10**+12 + **8**+12 + **6**+12 + **4**+12 + **2**+12
…… ………………………………………………………………………. ……
n -2 -1 0 1 2 3 4 5

Colour coding:
30's
11's
56's (i=28 for b=2)
2's (a=12 for b=2)
1's f(91,67) = 6+196 = f(31,7) + [f(91,67)-f(31,7)]

The values of n are represented by the diagonals, with the values of n+m', constituting the succession of rows, represented at the diagonal for n=0, i.e. the zero axis.

Each number indicated by having a part (blue or black) in boldface, without brackets, represents a specific non-prime, and no additional non-primes occur in this set. Hence, the numbers in boldface, with the according extensions of their series, represent *all* the non-primes filling the box (31,7) at position number 37 in the revolving chamber. Numbers in boldface inside brackets do not belong to the set, but are exposed to show their hidden role in the generation of the over-all pattern of non-primes in the box.

The interpretation of the figure to read out non-primes at the *right* of the zero axis departs from the general first-number of this set, 202, and addresses from there firstly the blue(+violet) segment at the row in question and adds this joint number to the first-number. This constitutes the non-prime at the zero axis for the row. The black(+red) positive segments are then interpreted as this basic non-prime with the stepwise additions, from left to right, of all the black(+red) segments located to the right of the zero axis ending with including the addition of the black(+red) segment itself.

As an example, the non-prime represented by the segment with black boldface number **3** in the row n+m'=6 is calculated as follows:
$(6+196) + (6\times11 + 6\times56) + 30(5+\mathbf{3}) + 2\times12\times2 = 202 + 402 + 240 + 48 = 892$

Multiplied with 30 and added with the position number 37 this gives the non-prime 26797.
As exhibited by the position in the figure, this number has (m',n)=(4,2) which is equivalent to (n,m)=(2,2). For box (91,67) the value of n=2 gives the factor $67 + 2\times30 = 127$, and the value of m=2 gives the factor $(91 + 2\times30) + 2\times30 = 211$. These factors give the product 127×211 which is 26797, confirming this being the non-prime represented by said boldface number in black.

The interpretation of the figure to read out non-primes at the *left* of the zero axis also departs from the general first-number, 202, added with the blue(+violet) segment at the row in question. The black(+red) negative segments are then interpreted as this basic non-prime with the stepwise additions, from left to right, of all the black(+red) segments located to the left of the zero axis but ending with including the addition of the black(+red) segment itself.

As an example, the non-prime represented by the segment with black boldface number **7-** in the row n+m'=4 is calculated as follows:
$(6+196) + (4\times11 + 4\times56) + 30(-5\ \mathbf{-7}) - 2\times12\times2 = 202 + 268 - 360 - 48 = 62$
.
Multiplied with 30 and added with the position number 37 this gives the non-prime 1897.
As exhibited by the position in the figure, this number has (m',n)=(6,-2) which is equivalent to (n,m)=(-2,8). For box (91,67) the value of n=-2 gives the factor $67 + (-2)\times30 = 7$, and the value of m=8 gives the factor $[91 + (-2)\times30] + 8\times30 = 271$. These factors give the product 7×271 which is 1897, confirming this being the non-prime represented by said boldface number in black.

As a final example, the non-prime represented by the segment with black boldface number **(-1)-** in the row n+m'= -4, i.e. located at the top left in the figure, is calculated as follows:
$(6+196) + (-4\times11 - 4\times56) + 30[-(-3) - \mathbf{(-1)}] - 2\times12\times2 = 202 - 268 + 120 - 48 = 6.$

Multiplied with 30 and added with the position number 37 this gives the non-prime 217.
As exhibited by the position in the figure, this number has (m',n)=(-2,-2) which is equivalent to (n,m)=(-2,0). For box (91,67) the value of n=-2 gives the factor $67 + (-2)\times30 = 7$, and the value of m=0 gives the factor $[91 + (-2)\times30] + 0\times30 = 31$. These factors give the product 7×31 which is 217, confirming this being the non-prime represented by said boldface number in black. This non-prime is identical to the lowest non-prime in box (7,31), illustrating that fig. 13 represents a complete covering of the non-primes in box (7,31).

*

Fig. 13 represents the most extended pattern of the eight boxes at number position 37, due to being the only one having to perform *double* knight moves in the translation to the lower corresponding box in order to cover all non-primes in the lowest box. The make-up of the five boxes in (69) performing *single* knight moves in the translation are easily illustrated from applying fig. 13 as an exemplar, by just deleting the far left diagonal for n=-2 and the top row for m=-2, and simply exchange the respective values of variables b, i, a and f with the respective values presented in (69). The make-ups of the patterns corresponding to these other boxes, and even more the two boxes (29,23) and (37,31) without any knight move and translation, appear as trivial on the background of fig. 13. For this reason we do not bother to present them in analogous separate figures.

Thus, merely varying the values of the variables (b,i,a,f) as determined by (69), is sufficient to expose all the eight sets of series in patterns similar to fig. 13. It is a trivial task to write out the eight sets of mathematical series corresponding to these eight patterns. We do not bother to do this since the figurative illustrations with fig. 13 as exemplar provide a simpler comprehension of the precise structure of the series. The *totality* of non-primes located at position number 37 in the revolving chamber is then expressed as the *union* of these eight sets. Thus, the totality of *primes* located at position number 37 in the revolving chamber is determined *ad negativo* as the set of all numbers located at position number 37, which are *not* a member of this union of eight sets of series. When simply combining *all* these eight sets in one *superimposed* figure of the same type as fig. 13, we have achieved *the* figure of the generative pattern for the *totality* of non-primes located at position number 37 in the revolving chamber. The non-primes located at position number 37 is then completely determined as all numbers of rotations *not* represented in this superimposed figure.

There will occur some overlaps between numbers in the different series constituting the unified set of non-primes at position number 37, but this is of course without relevance for identifying the totality of non-primes at this position, and therefore also without relevance for identifying *ad negativo* the totality of *primes* at this position of the chamber.

From the general nature of our approach and its procedures it is obvious that the treatment of the *seven remaining unified position sets*, corresponding to the seven position numbers *different* from 37 in the revolving chamber, can be performed in homology to the above analysis.

Now we will present these seven remaining sets, each made up by eight boxes, with covering translations that seems adequate, and with corresponding key values of (b,i,a,f) for the respective boxes:

(Symbol -> denotes box translations in *one* board-step; symbol => denotes box translations in *two* board-steps; and boldfaced boxes are the eight boxes with multiplicand 37, the multiplicand-path applied as exemplar earlier in the article.)

(73a) ***Key values of (b,i,a,f) for the eight translated boxes at position number 11***

Box with translation	Expression	Base number b	Initial number i	Adding number a	First-number f
(17,13)	2(3n+ m’)	2	1	2	7
(29,19)	2(9n+ 4m’)	2	4	5	18
(37,23)	2(13n+ 6m’)	2	6	7	28
(41,31)	2(15n+10m’)	2	10	5	42
(53,37)	2(21n+13m’)	2	13	4×2	65
(31,11)->(61,41)	2(25n+15m’)	2	15	2×5	11+72
(43,17)->(73,47)	2(31n+18m’)	2	18	13	24+90
(49,29)->(79,59)	2(34n+24m’)	2	24	2×5	47+108

(73b) ***Key values of (b,i,a,f) for the eight translated boxes at position number 13***

Box with translation	Expression	Base number b	Initial number i	Adding number a	First-number f
(23,11)	2(6n+ 0m’)	2	0	2×3	8
(29,17)	2(9n+ 3m’)	2	3	2×3	16
(43,31)	2(16n+10m’)	2	10	2×3	44
(49,37)	2(19n+13m’)	2	13	2×3	60
(31,13)->(61,43)	2(25n+16m’)	2	16	3×3	13+74
(37,19)->(67,49)	2(28n+19m’)	2	19	3×3	23+86
(41,23)->(71,53)	2(30n+21m’)	2	21	3×3	31+94
(47,29)->(77,59)	2(33n+24m’)	2	24	3×3	45+106

(73c) ***Key values of (b,i,a,f) for the eight translated boxes at position number 17***

Box with translation	Expression	Base number b	Initial number i	Adding number a	First-number f
(29,13)	2(9n+ m’)	2	0	4×2	12
(31,17)	2(10n+ 3m’)	2	3	7	17
(43,29)	2(16n+ 9m’)	2	9	7	41
(47,31)	2(18n+10m’)	2	10	4×2	48
(23,19)->(53,49)	2(21n+19m’)	2	19	2	14+72
(41,37)->**(71,67)**	2(30n+28m’)	2	28	2	50+108
(37,11)=>(97,71)	2(43n+30m’)	2	30	13	13+216
(49,23)=>(109,83)	2(49n+36m’)	2	36	13	37+264

(73d) ***Key values of (b,i,a,f) for the eight translated boxes at position number 19***

Box with translation	Expression	Base number b	Initial number i	Adding number a	First-number f
(13,13)	2(n+ m’)	2	1	0	5
(17,17)	2(3n+ 3m’)	2	3	0	9
(23,23)	2(6n+ 6m’)	2	6	0	17
(37,37)	2(13n+13m’)	2	13	0	45
(29,11)->(59,41)	2(24n+15m’)	2	15	3×3	10+70
(31,19)->(61,49)	2(25n+19m’)	2	19	2×3	19+80
(41,29)->(71,59)	2(30n+24m’)	2	24	2×3	39+100
(49,31)->(79,61)	2(34n+25m’)	2	25	3×3	50+110

(73e) ***Key values of (b,i,a,f) for the eight translated boxes at position number 23***

Box with translation	Expression	Base number b	Initial number i	Adding number a	First-number f
(13,11)	2(n+ 0m’)	2	0	1	4
(19,17)	2(4n+ 3m’)	2	3	1	10
(31,23)	2(10n+ 6m’)	2	6	2×2	23
(37,29)	2(13n+ 9m’)	2	9	2×2	35
(41,13)->(71,43)	2(30n+16m’)	2	16	2×7	17+84
(47,19)->(77,49)	2(33n+19m’)	2	19	2×7	27+96
(53,31)->(83,61)	2(36n+25m’)	2	25	11	53+114
(59,37)->(89,67)	2(39n+28m’)	2	28	11	72+126

(73f) ***Key values of (b,i,a,f) for the eight translated boxes at position number 29***

Box with translation	Expression	Base number	Initial number	Adding number	First-number
		b	i	a	f
(19,11)	2(4n+ 0m')	2	0	2×2	6
(23,13)	2(6n+ 1m')	2	1	5	9
(37,17)	2(13n+ 3m')	2	3	2×5	20
(43,23)	2(16n+ 6m')	2	6	2×5	32
(47,37)	2(18n+13m')	2	13	5	57
(41,19)->(71,49)	2(30n+19m')	2	19	11	25+90
(59,31)->(89,61)	2(39n+25m')	2	25	2×7	60+120
(31,29)=>(91,89)	2(40n+39m')	2	39	1	29+240

(73g) ***Key values of (b,i,a,f) for the eight translated boxes at position number 31***

Box with translation	Expression	Base number	Initial number	Adding number	First-number
		b	i	a	f
(11,11)	2(0n+ 0m')	2	0	0	3
(19,19)	2(4n+ 4m')	2	4	0	11
(29,29)	2(9n+ 9m')	2	9	0	27
(31,31)	2(10n+10m')	2	10	0	31
(23,17)->(53,47)	2(21n+18m')	2	18	3	12+70
(43,37)->**(73,67)**	2(31n+28m')	2	28	3	52+110
(37,13)=>(97,73)	2(43n+31m')	2	31	4×3	15+220
(47,23)=>(107,83)	2(48n+36m')	2	36	4×3	35+260

By performing the adequate translations we have achieved:
1) uniform expression of the base numbers for all boxes in all unified sets;
2) synchronous successive increases in n, m' and f when moving top-down from box to box in each of the unified sets;
3) simplistic expression of adding numbers into few classes in each of the unified sets.

From the values of the variables (b,i,a,f) given by (73a)-(73g) also the seven remaining unified position sets, each with eight boxes, is exhibited as seven sets of eight series, in patterns analogous to fig. 13. The unified set from (73a) gives the rotations for all non-primes occurring at position number 11, and *ad negativo* all *primes* occurring at position number 11. (73b) gives all non-primes occurring at position number 13, and *ad negativo* all *primes* occurring at position number 13; etc. The *totality* of non-primes located at position numbers in the revolving chamber is then expressed as the *union* of these eight unified position sets; i.e. as *8×8 sets of related series*, each having a simple mathematical expression read out from (b,i,a,f) analogous to the one illustrated by fig. 13. Thus, the totality of *primes* is determined *ad negativo* as the set of all numbers (presupposing that they belong to the "blue" numbers in the sense of fig. 1) that are *not* a member of this union of eight unified sets of series. We can determine the totality of primes either as the complement of this *over-all* union of non-primes (in the universe of blue numbers), or, alternatively, as the union of the complements of *each* of the eight unified sets of non-primes (belonging to the respective eight universes of the eight position numbers.) Following the last approach we determine the primes in succession for each of the eight position numbers, and reach the complete set of primes in the end as the union of these eight complements. Presupposing the position number, the primes are given uniquely for each of the eight complements *solely* from the amount of rotations. Following the first approach, the primes are given uniquely only for the *combination* of position number and amount of rotations.

Some numbers of rotations of the original chamber will not manifest in *any* of these eight position number sets, expressing that all the eight numbers in the revolved chamber for such numbers of rotations are not filled, i.e. are primes. Some other numbers of rotations will manifest in *more* than one of the eight position number sets, expressing that there are more than one non-prime in the revolved chamber for such numbers of rotations.

When simply combining *all* the eight unified sets in one overall 8×8 *superimposed figure* of the same type as fig. 13, we have achieved *the* figure of the generative pattern for the *totality* of non-primes located at position numbers in the revolving chamber. The *primes* are then completely determined and exposed as all (blue) numbers in the revolving chamber *not* represented in this superimposed figure.

From these results it seems likely that the total set of non-primes can be further simplified from adequate mathematical contemplation of the over-all unified pattern. Whatever degree of such simplification*, the generative revolving code distinguishing primes from non-primes has been disclosed exhaustively and concisely by our exposition*.

The revolving generative pattern of non-primes exposes the accompanying generative pattern of primes as its background *gestalt*. Due to the simple serial constitution of the related sets making-up the generative pattern of non-primes, and the few and simple variables occurring in these series, it seems likely that our revolving approach also should be able to yield a *positive* formulation of the generative pattern of primes by performing a gestalt switch. The *existence* of such a disclosed positive formulation is already indicated by our second, positive method presented in Johansen (2006: 129-30), which exhibits the whole prime number set as an epiphenomenon from generative Fibonacci-series. This positive formulation implied a discovery of a new (to our knowledge) method to decide with possible certainty whether an arbitrary number is a prime or not, from merely inspecting the relevant sequences of the Fibonacci series and performing a few simple arithmetic operations. (At least it has been checked as correct for the first 1000 Fibonacci numbers.) However, we have not provided a *proof* for why this simple method works.

It seems likely that many crucial problems in prime number mathematics can find their solution from this revolving generative approach. When the generative pattern is exposed and understood, it should be easier to discover the adequate mathematical formulations to pose and solve the problem at hand.

As an example, our exhibition removes the basic mystery about twin primes. There are three and only three possibilities for twin primes: the pairs have to occur at position numbers 11&13, at 17&19 or at 29&31 in their according boxes. Both numbers in the pair to be a prime simply means that the two respective position numbers in the chamber never occur filled for the *same* amount of rotations. The challenge then reduces to express the set of rotations where this is the case for the respective pairs. To take 17&19 as an example, we will have twin primes for all rotations where *neither* the unified set of eight series (73c) *nor* the unified set of eight series (73d) is filled; i.e. for all rotations where *none* of the 16 boxes in these two sets are filled. In principle and in some sense, the solution of the twin prime problem might be said to have already been provided with our exhibition of the precise generative pattern for the six relevant unified sets, by simply grouping these six unified sets into the three pairs with each "twin set" manifesting as the superposition of its two sub-patterns. However, the challenge remains to formulate such twin prime occurrences in further simplified mathematical expressions.

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Our exposition has presented an exhaustive and unique completion of non-primes generated as 8×8 sets of related *series* with a simple mathematical structure, and thereby an exhaustive and unique determination of prime numbers as the remaining *complement* of these 8×8 sets of series in a specified universal set with eight sub-sets. Thus, non-primes vs. primes are completely, uniquely, autonomously and constructively determined *without* successive trial-and-error factorizing and multiplication, from a *limited* amount of series, and exposed in specified, peptized and strict *regularity*.

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