

Why does the electron and the positron possesses the same rest mass but different charges of equal modulus and opposite signs??.And why both annihilates??

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Abstract

We demonstrate how Rest Masses and Electric Charges are generated by the $5D$ Extra Dimension of a Universe possessing a Higher Dimensional Nature using the Hamilton-Jacobi equation in agreement with the point of view of Ponce De Leon explaining in the generation process how and why antiparticles have the same rest mass m_0 but charges of equal modulus and opposite signs when compared to particles and we also explains why both annihilates.

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1 Introduction

In this work we will analyze how the $5D$ generates the rest-masses and the electric charges seen in $4D$ using the Hamilton-Jacobi Equation according to the formalism of Ponce De Leon. Masses and Charges are Geometrical Effects of a Hidden Fifth Dimension^{1,2}. We will avoid Confinement and Compactification Mechanisms³ and we will adopt the Ponce de Leon point of view⁴ of Space-Time-Matter theory where matter in $4D$ is purely geometric in nature and a Large $5D$ Extra Dimension is needed to get a consistent description of the properties of matter observed in $4D$. According to Ponce De Leon the mathematical support for Space-Time-Matter theory is given by the theorem of Campbell-Magaard.⁵ All the matter fields seen in $4D$ are generated by a geometrical effect due to the presence of the $5D$.⁶ The variation of the rest masses and electric charges of the particles seen in $4D$ is an indirect observation of the existence of the $5D$ and also according to Ponce De Leon these variations of rest masses and electric charges can be regarded as new physical phenomena unambiguously associated with the experimental existence of Extra Dimensions and according to Ponce De Leon this can provide a wealth of new physics.⁷ We adopt here the $5D$ General Relativity Ansatz given by Ponce De Leon according to the following equation (eq 12 and 14 pg 4 in [3], eq 5 pg 5 in [5] without Conformal Factors $\Omega(y) = 1$)

$$dS^2 = g_{\alpha\beta}(x^\rho, y) dx^\alpha dx^\beta - \Phi^2(x^\rho, y) dy^2 \quad (1)$$

Note that this Ansatz is exactly the same presented by Basini-Capozziello and can be made equal to the one presented by Shiromizu-Maeda-Sasaki making the term dS^2 equal in all of them. The variation of the rest-mass due to the presence of the $5D$ Extra Dimension is given by ([3] eq 20, eq 13 pg 6 in [5] without Conformal Factors $\Omega(y) = 1$, [7] eq 1 and eq 21 pg 5 in [4]):

$$m_0 = \frac{M_5}{\sqrt{1 - \Phi^2 \left(\frac{dy}{ds}\right)^2}} \quad (2)$$

providing the following form for eq 20 pg 5 in [4]⁸

$$u^4 = \frac{dx^4}{ds} = -\Phi \left(\frac{dy}{ds} \right). \quad (3)$$

The variation of the electric charge due to the presence of the $5D$ Extra Dimension is given by (eq 19 pg 5 in [4])⁹:

¹see top of pg 2 in [3]

²note that Ponce De Leon also points out the fact that $11D$ Supergravity and $10D$ Superstrings also evolved from the Klein Compactification Mechanism

³see pg 2 in [4]. Ponce De Leon argues that the Cylindrical Condition is not needed and also argues that we may live in a Universe of Large Extra Dimensions, so the Compactification Mechanism is not needed too. see also pg 2 in [5]. Large Extra Dimensions are introduced in BraneWorld and *STM* theories with different motivations. we keep the point of view of *STM*

⁴see pg 2 in [3]

⁵see pg 2 in [3]

⁶see pg 2 in [4] and pg 2 in [5]

⁷see again pg 2 in [3], pg 2 in [4]

⁸without Electromagnetic Potential and a Spacelike Metric see bottom of pg 4 before section 2.2 in [4]

⁹the reader would ask why the + sign in an equation that originally have the - sign?. see eqs 55, 58 and 60 in [3] and eq 18 in [4]

$$q = \frac{M_{(5)}\Phi u^4}{\sqrt{1-(u^4)^2}} = \pm \frac{M_5\Phi^2 \frac{dy}{ds}}{\sqrt{1-\Phi^2(\frac{dy}{ds})^2}} \quad (4)$$

The equation of Hamilton-Jacobi for the Action S defined by $S = S(x^\mu, y)$ is given by the following expression (eq 11 pg 6 in [5] without Conformal Factors $\Omega(y) = 1$)¹⁰:

$$g^{\mu\nu} \left(\frac{\partial S}{\partial x^\mu} \right) \left(\frac{\partial S}{\partial x^\nu} \right) - \frac{1}{\Phi^2} \left(\frac{\partial S}{\partial y} \right)^2 = M_{(5)}^2 \rightarrow g^{\mu\mu} \left(\frac{\partial S}{\partial x^\mu} \right) \left(\frac{\partial S}{\partial x^\mu} \right) - \frac{1}{\Phi^2} \left(\frac{\partial S}{\partial y} \right)^2 = M_{(5)}^2 \quad (5)$$

The rest-mass m_0 seen in $4D$ is given by (eq 12 pg 6 in [5]):

$$g^{\mu\nu} \left(\frac{\partial S}{\partial x^\mu} \right) \left(\frac{\partial S}{\partial x^\nu} \right) = m_0^2 \rightarrow g^{\mu\mu} \left(\frac{\partial S}{\partial x^\mu} \right) \left(\frac{\partial S}{\partial x^\mu} \right) = m_0^2 \quad (6)$$

Note that we can write the Hamilton-Jacobi equation as follows:

$$m_0^2 - \frac{1}{\Phi^2} \left(\frac{\partial S}{\partial y} \right)^2 = M_{(5)}^2. \quad (7)$$

$$m_0^2 = M_{(5)}^2 + \frac{1}{\Phi^2} \left(\frac{\partial S}{\partial y} \right)^2. \quad (8)$$

$$m_0 = \sqrt{M_{(5)}^2 + \frac{1}{\Phi^2} \left(\frac{\partial S}{\partial y} \right)^2}. \quad (9)$$

Look now to this form of the Hamilton-Jacobi equation (eq 17 pg 5 in [4])¹¹:

$$m_0^2 - \frac{q^2}{\Phi^2} = M_{(5)}^2, \quad (10)$$

$$m_0^2 = M_{(5)}^2 + \frac{q^2}{\Phi^2}, \quad (11)$$

$$m_0 = \sqrt{M_{(5)}^2 + \frac{q^2}{\Phi^2}}, \quad (12)$$

From the equations above it can be seen that the rest-mass m_0 in $4D$ is obtained from partial derivatives of the $5D$ Action $S = S(x^\mu, y)$ with respect to the $4D$ Spacetime Coordinates while the electric charge q is obtained from the same Action but with partial derivatives related to the Extra Coordinate (see eq 18 pg 5 in [4]) (see also eqs 55, 58 and 60 in [3]). This is exactly the purpose of the Hamilton-Jacobi Equation: to extract masses and charges from the $5D$ Extra Dimensional Formalism

$$q = \pm \frac{\partial S}{\partial y} \rightarrow m_0 = \sqrt{g^{\mu\mu} \frac{\partial S}{\partial x^\mu}} \quad (13)$$

The equations of the rest mass m_0 and electric charge q written in function of the $5D$ Extra Dimension shows how masses and charges are generated by the Higher Dimensional Nature of the Universe.

¹⁰Spacelike Metric and diagonalized metrics for the right terms below

¹¹Spacelike Metric also

$$m_0 = \frac{M_5}{\sqrt{1 - \Phi^2(\frac{dy}{ds})^2}} \quad (14)$$

$$q = \pm \frac{M_5 \Phi^2 \frac{dy}{ds}}{\sqrt{1 - \Phi^2(\frac{dy}{ds})^2}} = \pm m_0 \Phi^2 \frac{dy}{ds} \quad (15)$$

Examining the Table of Elementary Particles given below¹²

| Particle | spin (\hbar) | B | L | T | T ₃ | S | C | B* | charge (e) | m_0 (MeV) | antipart. |
|----------------|------------------|-----|---|-----|----------------|----|---|----|------------|-------------|------------------|
| u | 1/2 | 1/3 | 0 | 1/2 | 1/2 | 0 | 0 | 0 | +2/3 | 5 | \bar{u} |
| d | 1/2 | 1/3 | 0 | 1/2 | -1/2 | 0 | 0 | 0 | -1/3 | 9 | \bar{d} |
| s | 1/2 | 1/3 | 0 | 0 | 0 | -1 | 0 | 0 | -1/3 | 175 | \bar{s} |
| c | 1/2 | 1/3 | 0 | 0 | 0 | 0 | 1 | 0 | +2/3 | 1350 | \bar{c} |
| b | 1/2 | 1/3 | 0 | 0 | 0 | 0 | 0 | -1 | -1/3 | 4500 | \bar{b} |
| t | 1/2 | 1/3 | 0 | 0 | 0 | 0 | 0 | 0 | +2/3 | 173000 | \bar{t} |
| e^- | 1/2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | -1 | 0.511 | e^+ |
| μ^- | 1/2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | -1 | 105.658 | μ^+ |
| τ^- | 1/2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | -1 | 1777.1 | τ^+ |
| ν_e | 1/2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0(?) | $\bar{\nu}_e$ |
| ν_μ | 1/2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0(?) | $\bar{\nu}_\mu$ |
| ν_τ | 1/2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0(?) | $\bar{\nu}_\tau$ |
| γ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | γ |
| gluon | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | gluon |
| W ⁺ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | +1 | 80220 | W ⁻ |
| Z | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 91187 | Z |
| graviton | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | graviton |

Examine first the group of the Quarks $udscbt$. All these particles possess a defined rest-mass m_0 seen in 4D and a defined electric charge q . Suppose that in 5D all these Quarks are the same Quark with the same 5D rest-mass M_5 and the Dimensional Reduction from 5D to 4D or the Hamilton-Jacobi equation "projects" these "different" rest-masses m_0 seen in 4D as "images" of the same 5D rest-mass M_5 being the differences between each Quark generated by the respective Spacetime Coupling term $\sqrt{1 - \Phi^2(\frac{dy}{ds})^2}$ assigned for each Quark. Hence for example the Quark u and the Quark t have the same 5D rest-mass $M_5 = 1$ but different Geometries from the 5D to 4D Dimensional Reduction generates two different Spacetime Couplings for each Quark $\sqrt{1 - \Phi_u^2(\frac{dy[u]}{ds})^2}$ and $\sqrt{1 - \Phi_t^2(\frac{dy[t]}{ds})^2}$ "projecting" in 4D two different rest-masses $m_0 = 5$ for the Quark u and $m_0 = 173000$ for the Quark t as "images" of the same 5D Quark mass $M_5 = 1$. This point of view could perhaps lead to a major revolution in Particle Physics¹³

Examining now the electric charges q or better the relation $\frac{q}{m_0}$

$$\frac{q}{m_0} = \pm \frac{1}{\sqrt{g^{\mu\mu}}} \frac{\partial x^\mu}{\partial y} = \pm \Phi^2 \frac{dy}{ds} \quad (16)$$

Electric charges q are functions of the 4D rest-masses m_0 . Note in the given Table of Elementary Particles that all the particles that possess charges q also possess masses m_0 . There are no particles with electric

¹²extracted from the Formulary Of Physics by J.C.A. Wevers available on Internet

¹³see pg 60 in [6] and bottom of pg 2 and pg 3 in [7]

charge q and rest mass $m_0 = 0$. This is one of the most important consequences of the Hamilton-Jacobi equation in the Ponce De Leon Formalism. For our Quarks u and t different Mass-to-Charge Couplings $\pm\Phi^2 \frac{dy}{ds}$ one for the Quark $u \pm\Phi_u^2 \frac{dy[u]}{ds}$ and another for the Quark $t \pm\Phi_t^2 \frac{dy[t]}{ds}$ associated to the rest masses $m_0(u) = 5$ and $m_0(t) = 173000$ will generate the same electric charge $q(u) = +\frac{2}{3}$ and $q(t) = +\frac{2}{3}$ for both Quarks. Their respective antiparticles \bar{u} and \bar{t} possesses the same rest-masses but electric charges of different signs $q(\bar{u}) = -\frac{2}{3}$ and $q(\bar{t}) = -\frac{2}{3}$. The explanation why antiparticles have the same rest-masses of particles but different signs for electric charges will be given in Section 2 but we can say right now that the difference is being generated by the Mass-to-Charge Couplings $\pm\Phi^2 \frac{dy}{ds}$.

2 Rest-Masses and Electric Charges seen in a 4D Spacetime but being generated by a 5D Spacetime due to the Geometrical Nature of the Hamilton-Jacobi Equation. The approach of Ponce de Leon

The 5D Geodesics Equations tells nothing about the masses and the charges of the particles seen in 4D (See pg 2 and 3 in [4], See pg 2,3 and 4 in [5]). The masses and charges of the particles seen in 4D are also generated by the 5D Spacetime in a very attractive way. We can have a small group of particles in the 5D Spacetime each one having the same rest mass M_5 but due to different Spacetime Couplings between the 5D and 4D Spacetimes two particles having the same rest-mass in 5D will appear with different rest-masses m_0 in the 4D Spacetime. The Spacetime Coupling projects for each 5D particle a different image in 4D. The same is also true for electric charges (See pg 3 in [4]). This is a very interesting point of view: for example we have 6 Quark each one having a different rest mass m_0 seen in 4D but it might be possible that all the Quarks in 5D have the same rest mass M_5 and due to different Spacetime Couplings the same 5D Quark appears in the 4D Spacetime with different images each one being a different projection of the 5D Spacetime into the 4D Spacetime one. The masses and charges generated in the 4D Spacetime as a geometrical projection from the 5D Spacetime are explained by the Hamilton-Jacobi equation (See pg 4 in [5], See pg 3 in [4]). In this Section we follow the procedures and the approach of Ponce De Leon.

According to Ponce de Leon there are three possibilities for the projection of a 5D Spacetime into a 4D Spacetime giving three possible values for the 5D rest mass M_5 (See eqs 3 to 5 pg 3 in [6]¹⁴):

- Timelike 5D Geodesics:

$$dS^2 > 0 \curvearrowright dS^2 = ds^2 - \Phi^2 dy^2 \curvearrowright ds^2 > \Phi^2 dy^2 \curvearrowright 1 > \Phi^2 (dy/ds)^2 \curvearrowright M_5 > 0 \quad (17)$$

- Null-like 5D Geodesics:

$$dS^2 = 0 \curvearrowright dS^2 = ds^2 - \Phi^2 dy^2 \curvearrowright ds^2 = \Phi^2 dy^2 \curvearrowright 1 = \Phi^2 (dy/ds)^2 \curvearrowright M_5 = 0 \quad (18)$$

- Spacelike 5D Geodesics:

$$dS^2 < 0 \curvearrowright dS^2 = ds^2 - \Phi^2 dy^2 \curvearrowright ds^2 < \Phi^2 dy^2 \curvearrowright 1 < \Phi^2 (dy/ds)^2 \curvearrowright M_5 < 0 \quad (19)$$

¹⁴without conformal factors

- Case 1)- particles in a Timelike 5D Spacetime Ansatz $dS^2 > 0$ with a 5D rest-mass $M_5 > 0$ giving a 4D rest-mass $m_0 > 0$

All the relations between M_5, m_0, dS^2 and ds^2 are given by the following equation(See eq 22 pg 5 in [3])¹⁵:

$$\frac{dS}{M_5} = \frac{ds}{m_0} \quad (20)$$

Now we will introduce the mathematical demonstration of the Hamilton-Jacobi Equation: Starting with the contravariant component of the 5D Momentum P^Q defined in function of M_5 as being $P^Q = M_5 U^Q$ (See eq 15 pg 5 in [3], See eq 6 pg 5 in [5]) where $U^Q = (dx^q/dS, dy/dS)$ and $U^Q U_Q = 1$ because $U^Q = g^{QQ} U_Q$ and $U_Q = g_{QQ} U^Q$ giving $U^Q U_Q = g^{QQ} U_Q g_{QQ} U^Q = g^{QQ} g_{QQ} U_Q U^Q$ but we know that $g^{QQ} g_{QQ} = 1$ then $U^Q U_Q = 1$

Defining the contravariant and the covariant components of the Momentum in 5D and the product between the components we have(See eq 16 pg 5 in [3], See eq 7 pg 5 in [5]):

$$P^Q = M_5 U^Q \quad (21)$$

$$P_Q = M_5 U_Q \quad (22)$$

$$P(5) = P^Q P_Q = M_5 U^Q M_5 U_Q = M_5^2 U^Q U_Q = M_5^2 \quad (23)$$

The product between components of the 5D Momentum is given by:

$$P(5) = P^Q P_Q = M_5^2 \quad (24)$$

But we know that $Q = 0, 1, 2, 3, 4$ being the script 4 the Fifth Dimension

Also we know that dS^2 the 5D Spacetime Metric is not entirely seen by a 4D observer. The 4D observer can only access the 4D part of the trajectory(See abstract and pg 2 in [5], See pg 4 before eq 11 in [3]). Hence the 4D observer can only measure the 4D Momentum defined by its contravariant and covariant components as follows(See eq 17 pg 5 in [3], See eq 8 pg 5 in [5]):

$$p^q = m_0 U^q \quad (25)$$

$$p_q = m_0 U_q \quad (26)$$

with $p = P$ and being $q = 0, 1, 2, 3$ but also with:

$U^q = g^{qq} U_q$ and $U_q = g_{qq} U^q$ giving $U^q U_q = g^{qq} U_q g_{qq} U^q = g^{qq} g_{qq} U_q U^q$ and since $g^{qq} g_{qq} = 1$ then $U_q U^q = 1$ just like its 5D counterpart

then we should expect for:

$$p(4) = p^q p_q = m_0 U^q m_0 U_q = m_0^2 U^q U_q = m_0^2 \quad (27)$$

Hence the product between components of the 4D Momentum is given by:

$$p(4) = p^q p_q = m_0^2 \quad (28)$$

¹⁵we do not consider here conformal factors

Then the product between components of the 5D Momentum can be written as:

$$p(5) = P^Q P_Q = p^a p_a + P^4 P_4 = M_5^2 \quad (29)$$

with:

$$p(5) = p(4) + P^4 P_4 = M_5^2 \quad (30)$$

but we know that

$$p(4) = p^a p_a = m_0^2 \quad (31)$$

Then we should expect the following expression given below for the product between components of the 5D Momentum(See eq 19 pg 5 in [3],See eq 9 pg 5 in [5]):

$$p(5) = p^a p_a + P^4 P_4 = M_5^2 \curvearrowright p(5) = m_0^2 + P^4 P_4 = M_5^2 \quad (32)$$

Considering now the following 5D Spacetime Ansatz defined below as(See eq 14 pg 4 in [3],See eq 5 pg 5 in [5])¹⁶:

$$dS^2 = g_{qr}(x^w, y) dx^q dx^r - \Phi^2(x^w, y) dy^2 \quad (33)$$

$$dS^2 = ds^2 - \Phi^2(x^w, y) dy^2 \quad (34)$$

$$ds^2 = g_{qr}(x^w, y) dx^q dx^r \quad (35)$$

Where w is the affine parameter and S is the 5D Action defined by(See pg 6 between eqs 10 and 11 in [5]):

$$S = S(x^w, y) \quad (36)$$

We can define the covariant components of the 5D or 4D Momentum in function of the 5D Action given above as follows:

$$p_q = P_q = -\frac{\partial S}{\partial x^q} \quad (37)$$

$$p_r = P_r = -\frac{\partial S}{\partial x^r} \quad (38)$$

$$p_4 = P_4 = -\frac{\partial S}{\partial y} \quad (39)$$

but we know that:

$$p^q = g^{qr} p_r \quad (40)$$

Rewriting the product between components of the 5D Momentum in function of the 5D Action we should expect for:

¹⁶without conformal factors

$$p(5) = p^a p_a + P^4 P_4 = M_5^2 = g^{qr} p_r p_q + P^4 P_4 = M_5^2 \quad (41)$$

Then we finally arrive at the Hamilton-Jacobi equation as defined by Ponce De Leon given below:

$$g^{qr} \frac{\partial S}{\partial x^q} \frac{\partial S}{\partial x^r} + P^4 P_4 = M_5^2 \quad (42)$$

but we also know that

$$P^4 = g^{44} P_4 \quad (43)$$

Hence the Hamilton-Jacobi equation now becomes:

$$g^{qr} \frac{\partial S}{\partial x^q} \frac{\partial S}{\partial x^r} + g^{44} P_4^2 = M_5^2 \quad (44)$$

but g_{44} from the 5D Spacetime Ansatz is given by $g_{44} = -\Phi^2$. Hence and since $g^{44} = -1/(\Phi^2)$ the Hamilton-Jacobi Equation as defined by Ponce De Leon is now (See eq 11 pg 6 in [5])¹⁷¹⁸:

$$g^{qr} \frac{\partial S}{\partial x^q} \frac{\partial S}{\partial x^r} - \frac{1}{\Phi^2} P_4^2 = M_5^2 \quad (45)$$

$$g^{qr} \frac{\partial S}{\partial x^q} \frac{\partial S}{\partial x^r} - \frac{1}{\Phi^2} \left(\frac{\partial S}{\partial y} \right)^2 = M_5^2 \quad (46)$$

being the 4D rest mass m_0 given by (See eq 12 pg 6 in [5]):

$$m_0^2 = g^{qr} \frac{\partial S}{\partial x^q} \frac{\partial S}{\partial x^r} \quad (47)$$

Then the Hamilton-Jacobi equation as defined by Ponce De Leon can now be written as (See eqs 17 and 18 pg 5 in [4])¹⁹²⁰²¹²²:

$$m_0^2 - \frac{1}{\Phi^2} \left(\frac{\partial S}{\partial y} \right)^2 = M_5^2 \quad (48)$$

We already know that $\frac{dS}{M_5} = \frac{ds}{m_0}$. Hence we should expect for:

$$\frac{dS}{ds} = \frac{M_5}{m_0} \quad (49)$$

But $dS^2 = ds^2 - \Phi^2 dy^2$ giving $(dS/ds)^2 = 1 - \Phi^2 (dy/ds)^2 = (M_5/m_0)^2$

From the expressions above we can write the expressions given below:

$$M_5^2 = m_0^2 \left[1 - \Phi^2 \left(\frac{dy}{ds} \right)^2 \right] \quad (50)$$

$$m_0^2 = \frac{M_5^2}{1 - \Phi^2 \left(\frac{dy}{ds} \right)^2} \quad (51)$$

¹⁷QED:Quod Erad Demonstratum

¹⁸without conformal factors and spacelike signature for the extra dimension

¹⁹QED:Quod Erad Demonstratum

²⁰spacelike signature for the extra dimension

²¹note that this equation do not have conformal factors

²²note also that the electric charge is defined as the extra component of the 5D Momentum. this agrees with pg 3 in [4]

And finally we arrive at the relation between the rest mass m_0 seen in a $4D$ Spacetime and the rest mass M_5 from the $5D$ Spacetime according to Ponce De Leon(See eq 20 pg 5 in [3],See eq 21 pg 5 in [4],See eq 13 pg 6 in [5])²³²⁴²⁵

$$m_0 = \frac{M_5}{\sqrt{1 - \Phi^2 \left(\frac{dy}{ds}\right)^2}} \quad (52)$$

From the equation above it is now easy to see why two particles with the same rest mass M_5 in a $5D$ Spacetime (or two specimens of the same $5D$ particle) can appear in the $4D$ Spacetime with different rest masses m_0 looking apparently as different particles however the particles seen in $4D$ are different projections or different images of two identical $5D$ particles because each $5D$ particle and each $4D$ image moves with a different $5D$ Spacetime Ansatz dS^2 generating in the $4D$ Spacetime different terms of the form $\sqrt{1 - \Phi^2 \left(\frac{dy}{ds}\right)^2}$ each term for each particle. The term $\sqrt{1 - \Phi^2 \left(\frac{dy}{ds}\right)^2}$ is the Spacetime Coupling between the $5D$ rest mass M_5 and the $4D$ rest mass m_0 .

Now its time to turn back to the Hamilton-Jacobi equation

$$g^{qr} \frac{\partial S}{\partial x^q} \frac{\partial S}{\partial x^r} - \frac{1}{\Phi^2} \left(\frac{\partial S}{\partial y}\right)^2 = M_5^2 \quad (53)$$

$$m_0^2 - \frac{1}{\Phi^2} \left(\frac{\partial S}{\partial y}\right)^2 = M_5^2 \quad (54)$$

where the electric charge q is defined as(See eq 18 pg 5 in [4])

$$q = P_4 = -\frac{\partial S}{\partial y} \quad (55)$$

From the equation above we can see that the electric charge seen in a $4D$ Spacetime is obtained purely by the derivative of the Hamilton-Jacobi Action S with respect to the extra dimension. In this case the $4D$ Spacetime electric charge q according to Ponce De Leon is generated by a pure geometric effect originated in the $5D$ Spacetime.

Rewriting the Hamilton-Jacobi equation according to Ponce De Leon as follows(See again eq 17 pg 5 in [4])²⁶:

$$g^{qr} \frac{\partial S}{\partial x^q} \frac{\partial S}{\partial x^r} - \frac{q^2}{\Phi^2} = M_5^2 \quad (56)$$

$$m_0^2 - \frac{q^2}{\Phi^2} = M_5^2 \quad (57)$$

we can have a clear perspective about how the $5D$ Spacetime Action $S = S(x^w, y)$ generates in a $4D$ Spacetime the masses and charges of all the Elementary Particles observed.(See pg 6 between eqs 12 and 13 in [5]).

Combining together the Hamilton-Jacobi equation and the relation between the $5D$ rest mass M_5 and the $4D$ rest mass m_0 both as defined by Ponce De Leon we will find the following interesting result:

²³QED:Quod Erad Demonstratum

²⁴spacelike signature for the extra dimension

²⁵without conformal factors

²⁶QED:Quod Erad Demonstratum

$$m_0^2 - \frac{q^2}{\Phi^2} = M_5^2 \quad (58)$$

$$M_5^2 = m_0^2[1 - \Phi^2(\frac{dy}{ds})^2] \quad (59)$$

$$m_0^2 - \frac{q^2}{\Phi^2} = m_0^2[1 - \Phi^2(\frac{dy}{ds})^2] \quad (60)$$

dividing the expression above by m_0^2 we should expect for:

$$1 - \frac{q^2}{m_0^2\Phi^2} = 1 - \Phi^2(\frac{dy}{ds})^2 \quad (61)$$

$$\frac{q^2}{m_0^2\Phi^2} = \Phi^2(\frac{dy}{ds})^2 \quad (62)$$

$$q^2 = m_0^2\Phi^4(\frac{dy}{ds})^2 \quad (63)$$

$$q = \pm m_0\Phi^2\frac{dy}{ds} \quad (64)$$

but we know that

$$m_0 = \frac{M_5}{\sqrt{1 - \Phi^2(\frac{dy}{ds})^2}} \quad (65)$$

Then we have the Ponce De Leon final expression for the electric charge seen in $4D$ Spacetime in function of the $5D$ rest mass M_5 (See eq 19 pg 5 in [4])²⁷²⁸

$$q = \pm \frac{M_5\Phi^2\frac{dy}{ds}}{\sqrt{1 - \Phi^2(\frac{dy}{ds})^2}} \quad (66)$$

This is another very interesting feature of the formalism developed by Ponce De Leon. Two identical particles in a given $5D$ Spacetime with the same rest-mass M_5 will appear not only with different rest masses m_0 in the $4D$ Spacetime looking apparently as different $4D$ particles or different $4D$ "images" of the same $5D$ particle with each "image" being defined by the each different $4D$ rest-mass m_0 and the differences between the $4D$ "images" are due to the different Spacetime Couplings for each $5D$ particle moving each particle in a different $5D$ Spacetime Ansatz dS^2 but also the electric charge q seen in $4D$ is a function of the $5D$ Spacetime. This means to say that two $5D$ Spacetime identical particles each one with the same rest mass M_5 will appear in the $4D$ Spacetime with different rest masses m_0 as a different $4D$ "images" of the same $5D$ particle but each "image" defined by the $4D$ rest mass m_0 possesses also an electric charge of positive or negative sign generated in $4D$ by the term $\Phi^2(dy/ds)$. Two $4D$ particles with the same $4D$ rest mass m_0 can have two possible values for the $4D$ electric charge:

²⁷QED:Quod Erad Demonstratum

²⁸we will explain why our result have the \pm sign

- $+m_0\Phi^2\frac{dy}{ds}$
- $-m_0\Phi^2\frac{dy}{ds}$

The result above explains why an Elementary Particle seen in $4D$ with a rest-mass m_0 have an electric charge q of a given sign (+ in the case of the quarks u and c and $-$ in the case of the quarks s and b) and for every charged Elementary Particle in $4D$ there exists (also in $4D$) a corresponding Elementary Anti-Particle of equal rest mass m_0 and an electric charge q equal in modulus to the charge q of the corresponding Elementary Particle but opposite signs ($-$ in the case of the anti-quarks u^- and c^- and $+$ in the case of the anti-quarks s^- and b^-)

This leads ourselves to the following combinations:

- positive matter corresponds to negative anti-matter

$$Matter(+) = q(+) = +m_0\Phi^2\frac{dy}{ds} \quad (67)$$

$$AntiMatter(-) = q(-) = -m_0\Phi^2\frac{dy}{ds} \quad (68)$$

- negative matter corresponds to positive anti-matter

$$Matter(-) = q(-) = -m_0\Phi^2\frac{dy}{ds} \quad (69)$$

$$AntiMatter(+) = q(+) = +m_0\Phi^2\frac{dy}{ds} \quad (70)$$

The term $\pm\Phi^2\frac{dy}{ds}$ is known as the Mass to Charge Coupling. It plays between the $4D$ rest mass m_0 and the $4D$ electric charge q a role almost similar to the role played between the $5D$ rest mass M_5 and the $4D$ rest mass m_0 by the Spacetime Coupling.

The scenario described above between matter and anti-matter is by far well-known but however one fundamental question remains:

- What generates this scenario in the first place????

This scenario can be entirely demonstrated mathematically by the formalism developed by Ponce De Leon.

- Here we go:

According to Ponce De Leon our $4D$ Universe lies in the intersection point between two different $5D$ BraneWorld Universes and the intersection point is the $5D$ Extra Dimension y when $y = 0$. One of these $5D$ BraneWorld Universes is the responsible for the Matter seen in our $4D$ Universe and the other $5D$ BraneWorld Universe is the responsible for the Antimatter seen in our $4D$ Universe. Below are the $5D$ Spacetime Ansatz of two different BraneWorld Universes defined in function of the Extra Dimension y and an affine parameter w as follows (See eq 55 pg 10 in [3]):

$$dS^2 = g_{qr}(x^w, +y)dx^q dx^r - \Phi^2(x^w, +y)dy^2 \curvearrowright y(+) \geq 0 \curvearrowright 5D\text{BraneWorldMatterUniverse} \quad (71)$$

$$dS^2 = g_{qr}(x^w, -y)dx^q dx^r - \Phi^2(x^w, -y)dy^2 \curvearrowright y(-) \leq 0 \curvearrowright 5D\text{BraneWorldAntiMatterUniverse} \quad (72)$$

Each one of these 5D BraneWorld Universes possesses particles of 5D rest-mass M_5 and perhaps these 5D particles are similar in both Universes. However according to the Ponce De Leon relations between the 5D rest-mass M_5 and the 4D rest-mass m_0 and the 4D electric charge q we have an interesting feature: the 4D "image" of one of these 5D Universes correspond to the 4D matter particles seen in our Universe while the 4D "image" of the other 5D Universe correspond to the 4D anti-matter particles also seen in our Universe and what is more remarkable: all this agrees with the Hamilton-Jacobi equation.

Our Visible 4D Universe lies exactly in the point $y = 0$

Each one of these 5D BraneWorld Universe defines an Action for the Hamilton-Jacobi equation as shown below (See eq 58 pg 10 in [3]):²⁹

$$S(+) = S(x^r, +y) \curvearrowright S(+) = S(x^q, +y) \quad (73)$$

Above is written the Action for the 5D BraneWorld Matter Universe

$$S(-) = S(x^r, -y) \curvearrowright S(-) = S(x^q, -y) \quad (74)$$

Above is written the Action for the 5D BraneWorld Anti Matter Universe

Using separation of variables for both Actions we have:

$$S(+) = S(x^r, +y) = A(x^r) + B(+y) \curvearrowright S(+) = S(x^q, +y) = A(x^q) + B(+y) \quad (75)$$

Above is the Action for the 5D BraneWorld Matter Universe with the 5D and 4D components separated.

$$S(-) = S(x^r, -y) = A(x^r) + B(-y) \curvearrowright S(-) = S(x^q, -y) = A(x^q) + B(-y) \quad (76)$$

Above is the Action for the 5D BraneWorld Anti-Matter Universe with the 5D and 4D components separated.

From above we can see that the 4D part of both Actions $A(x^q)$ or $A(x^r)$ are equal for both BraneWorld Universes. The difference lies in the 5D part of both Actions $B(+y)$ and $B(-y)$ responsible for the electric charge (remember that $q = -\frac{\partial S}{\partial y}$).

Considering for example the parts of the Action responsible for the 4D rest mass m_0 inside the Hamilton-Jacobi equation for the two 5D BraneWorld Universes defined below involving two particles: an electron and a positron lying the electron in the 5D BraneWorld Matter Universe $y(+) \geq 0$ and the positron lying in the 5D BraneWorld Antimatter Universe $y(-) \leq 0$ we have (See eq 55 pg 10 in [3]):

²⁹the Action of the Hamilton-Jacobi equation is described as a sum. see for example eq 66 pg 11 in [3], or eq 32 pg 11 in [5]

$$dS^2 = g_{qr}(x^w, +y)dx^q dx^r - \Phi^2(x^w, +y)dy^2 \curvearrowright y(+) \geq 0 \curvearrowright \text{Electron} \curvearrowright q(+) < 0 \quad (77)$$

$$dS^2 = g_{qr}(x^w, -y)dx^q dx^r - \Phi^2(x^w, -y)dy^2 \curvearrowright y(-) \leq 0 \curvearrowright \text{Positron} \curvearrowright q(-) > 0 \quad (78)$$

$$P(+)_q = -1 \times \frac{\partial S(+)}{\partial x^q} = -\frac{\partial A(x^q)}{\partial x^q} \quad (79)$$

$$P(+)_r = -1 \times \frac{\partial S(+)}{\partial x^r} = -\frac{\partial A(x^r)}{\partial x^r} \quad (80)$$

$$P(-)_q = -1 \times \frac{\partial S(-)}{\partial x^q} = -\frac{\partial A(x^q)}{\partial x^q} \quad (81)$$

$$P(-)_r = -1 \times \frac{\partial S(-)}{\partial x^r} = -\frac{\partial A(x^r)}{\partial x^r} \quad (82)$$

$$m_0^2 = g^{qr} \frac{\partial S(+)}{\partial x^q} \frac{\partial S(+)}{\partial x^r} = g^{qr} \frac{\partial A(x^q)}{\partial x^q} \frac{\partial A(x^r)}{\partial x^r} \quad (83)$$

$$m_0^2 = g^{qr} \frac{\partial S(-)}{\partial x^q} \frac{\partial S(-)}{\partial x^r} = g^{qr} \frac{\partial A(x^q)}{\partial x^q} \frac{\partial A(x^r)}{\partial x^r} \quad (84)$$

The result above is very important: it shows that the 4D part of the Hamilton-Jacobi equation in both BraneWorld Universes is equal generating equal rest masses m_0 . This explains for example why electron and positron have the same 4D rest-mass m_0

Looking now to the 5D part of the Actions responsible for the electric charge $q = -\frac{\partial S}{\partial y}$

$$P_4(+)= -\frac{\partial S(+)}{\partial y} = -\frac{\partial B(+y)}{\partial y} \curvearrowright y(+) \geq 0 \quad (85)$$

$$P_4(-)= -\frac{\partial S(-)}{\partial y} = -\frac{\partial B(-y)}{\partial y} \curvearrowright y(-) \leq 0 \quad (86)$$

We can clearly see that the part of the Action responsible for the charge in the 5D BraneWorld Matter Universe is equal in modulus but have an opposite sign when compared to the part of the Action responsible for the charge in the 5D BraneWorld Anti Matter Universe

Or in other words:

$$B(+y) = -B(-y) \curvearrowright B(-y) = -B(+y) \quad (87)$$

And this implies in

$$q_4(+)= -\frac{\partial B(+y)}{\partial y} = -\frac{\partial(-B(-y))}{\partial y} = \frac{\partial B(-y)}{\partial y} = -q_4(-) \quad (88)$$

$$q_4(-)= -\frac{\partial B(-y)}{\partial y} = -\frac{\partial(-B(+y))}{\partial y} = \frac{\partial B(+y)}{\partial y} = -q_4(+) \quad (89)$$

From above we can see that if $q(+)$ is the charge of the electron resulting in a $q(+)<0$ then $q(-)$ will have the same modulus but opposite signs resulting in a $q(-)>0$ for the positron

Again using the example of the electron

$q_4(+)$ < 0 then according to the Ponce De Leon 5D to 4D mass-to-charge relation:

$$q_4(+)= -\frac{M_5\Phi^2\frac{dy}{ds}}{\sqrt{1-\Phi^2(\frac{dy}{ds})^2}} \tag{90}$$

$$q_4(+)= -m_0\Phi^2\frac{dy}{ds} \tag{91}$$

We know that both the electron and the positron have the same 5D rest mass M_5 and the same 4D rest mass m_0

But Φ is the Scalar Field and looking back to the definition of the dS^2 in both 5D BraneWorld Universes we have:

$$\Phi^2(x^w, +y) \curvearrowright y(+)>=0 \tag{92}$$

Above is the square of Scalar Field for the 5D BraneWorld Matter Universe

$$\Phi^2(x^w, -y) \curvearrowright y(-)<=0 \tag{93}$$

Above is the square of the Scalar Field for the 5D BraneWorld Anti Matter Universe

Again using separation of variables we have:³⁰

$$\Phi(x^w, +y) = U(x^w)V(+y) \tag{94}$$

$$\Phi(x^w, -y) = U(x^w)V(-y) \tag{95}$$

From above and in a similar situation compared to the Action S for the Hamilton-Jacobi equation, the 4D part of each Scalar Field is equal in both 5D BraneWorld Universes and the difference between Scalar Fields in each BraneWorld Universe lies exclusively in the 5D part of each Scalar Field. Hence we can clearly see that

$$V(-y) = -V(+y) \curvearrowright V(+y) = -V(-y) \tag{96}$$

then we have:

$$\Phi^2(x^w, +y) = U^2(x^w)V^2(+y) \tag{97}$$

$$\Phi^2(x^w, -y) = U^2(x^w)V^2(-y) \tag{98}$$

implying directly in:

³⁰while the Action of the Hamilton-Jacobi equation in separation of variables is a sum the Scalar Field in separation of variables is a product. see for example eq 132 pg 19 in [6]

$$V^2(-y) = (-V(+y))^2 \curvearrowright V^2(+y) = (-V(-y))^2 \quad (99)$$

$$V^2(-y) = V^2(+y) \quad (100)$$

$$\Phi^2(x^w, +y) = \Phi^2(x^w, -y) \quad (101)$$

The square of the Scalar Field for the electron and the positron are exactly equal in both 5D BraneWorld Universes. Examining again the 4D equation of the electron charge:

$$q_4(+)= -m_0\Phi^2\frac{dy}{ds} \quad (102)$$

From above Φ^2 and m_0 are the same for the electron and the positron. Then the difference that generates two different charges of equal modulus and opposite signs in the 4D Universe according to

$$q = \pm m_0\Phi^2\frac{dy}{ds} \quad (103)$$

or even better for our example³¹:

$$q_4(\pm) = \mp m_0\Phi^2\frac{dy}{ds} \quad (104)$$

must reside in the term $\frac{dy}{ds}$

Note that from the equation above we can extract the equations of the electric charges $\mp q$ of both the electron and the positron as shown below:

- electron:

$$q_4(+)= -m_0\Phi^2\frac{dy(+)}{ds} \quad (105)$$

- positron:

$$q_4(-)= m_0\Phi^2\frac{dy(-)}{ds} \quad (106)$$

The electron is located in a 5D Spacetime where $y(+)\geq 0$ and then $\frac{dy(+)}{ds}\geq 0$ ³² while the positron is located in a 5D Spacetime where $y(-)\leq 0$ and then $\frac{dy(-)}{ds}\leq 0$ ³³.

Note that the term $\frac{dy}{ds}$ for the positron is exactly the same for the electron multiplied by -1 and vice versa. Then:

³¹note the difference between \pm and \mp . we defined the electron lying in the 5D Matter BraneWorld Universe $y(+)\geq 0$ with a $q(+)< 0$ and the positron lying in the 5D BraneWorld AntiMatter Universe $y(-)\leq 0$ with a $q(-)> 0$

³²assuming linear displacement in y

³³assuming again linear displacement in y

$$\frac{dy(+)}{ds} = -\frac{dy(-)}{ds} \quad (107)$$

$$\frac{dy(-)}{ds} = -\frac{dy(+)}{ds} \quad (108)$$

Again using the equation of the electron:

$$q_4(+)= -m_0\Phi^2\frac{dy(+)}{ds} \quad (109)$$

$$q_4(+)= -m_0\Phi^2\left(-\frac{dy(-)}{ds}\right) \quad (110)$$

$$q_4(+)= m_0\Phi^2\frac{dy(-)}{ds} \quad (111)$$

Note that we inserted in the equation of the electron the term $\frac{dy(-)}{ds}$ of the positron and since $m_0\Phi^2$ are the same for both particles the equation above no longer represents the electron because the motion now occurs in the 5D Universe $y(-) \leq 0$. Then:

$$q_4(-)= m_0\Phi^2\frac{dy(-)}{ds} \quad (112)$$

And this agrees with our previous equation for the positive charge of the positron.

- Case 2)- particles in a Null-Like 5D Spacetime Ansatz $dS^2 = 0$ with a 5D rest-mass $M_5 = 0$ giving a 4D rest-mass $m_0 = 0$

We have seen so far the case of particles in a Timelike 5D Spacetime Ansatz $dS^2 > 0$ with a 5D rest-mass $M_5 > 0$ giving a 4D rest-mass $m_0 > 0$. But what happens if the 5D Spacetime Ansatz dS^2 becomes Null-Like which means to say $dS^2 = 0$???

The first thing to take in mind is the fact that a Timelike 5D Spacetime Ansatz $dS^2 > 0$ always require a 5D rest-mass M_5 different than 0 otherwise dS/M_5 with $dS > 0$ and $M_5 = 0$ would produce an invalid result.

Then we cannot have 5D particles in a Timelike 5D Spacetime Ansatz $dS^2 > 0$ with a null 5D rest-mass $M_5 = 0$.

On the other hand if the 5D Spacetime Ansatz dS^2 becomes Null-Like $dS^2 = 0$ then the Ponce De Leon relation between the 5D rest-mass M_5 and the 4D rest-mass m_0 will require a zero 5D rest-mass M_5 otherwise since $dS^2 = 0$ then $ds^2 = \Phi^2 dy^2$ and $1 = \Phi^2(dy/ds)^2$. This will generate a zero Spacetime Coupling $\sqrt{1 - \Phi^2(dy/ds)^2} = 0$ and since according to Ponce De Leon $m_0 = M_5/\sqrt{1 - \Phi^2(dy/ds)^2}$ if $M_5 > 0$ and $\sqrt{1 - \Phi^2(dy/ds)^2} = 0$ we would get an invalid result for m_0

Then a Null-Like 5D Spacetime Ansatz $dS^2 = 0$ always require a 5D rest-mass $M_5 = 0$

Rewriting the Hamilton-Jacobi equation according to Ponce de Leon for the case of a zero 5D rest mass $M_5 = 0$ as follows:

$$g^{qr} \frac{\partial S}{\partial x^q} \frac{\partial S}{\partial x^r} - \frac{q^2}{\Phi^2} = 0 \quad (113)$$

$$m_0^2 - \frac{q^2}{\Phi^2} = 0 \quad (114)$$

We will obtain the following result(See eq 24 pg 6 in [4]):

$$m_0^2 = \frac{q^2}{\Phi^2} \quad (115)$$

$$m_0 = \pm \frac{q}{\Phi} \quad (116)$$

$$q = \pm m_0 \Phi \quad (117)$$

The two signs for the electric charge above are being generated by the term $1 = \Phi^2(dy/ds)^2$ or $1 = \pm\Phi(dy/ds)$

Or even better(See pg 6 after eq 24 in [4]):

$$\Phi(dy/ds) = \pm 1 \quad (118)$$

$$\frac{dy}{ds} = \pm \frac{1}{\Phi} \quad (119)$$

Note that like in the previous case the expression above encompasses the 5D BraneWorld Matter Universe for the electron with $y(+)$ ≥ 0 and the 5D BraneWorld Antimatter Universe for the positron with $y(-)$ ≤ 0 .

But we know that according to Ponce De Leon $q = -\frac{\partial S}{\partial y}$. Then we can write the following expression for the 4D rest mass m_0 generated from a Null Like 5D Ansatz $dS^2 = 0$ as follows(See eq 27 pg 6 in [3]):

$$m_0 = \pm \frac{1}{\Phi} \frac{\partial S}{\partial y} \quad (120)$$

From above we have the following expressions for the 4D rest-mass m_0 in a Null-Like 5D Spacetime Ansatz $dS^2 = 0$ ³⁴

$$m_0 = \frac{1}{\Phi} \frac{\partial S}{\partial y} \curvearrowright y(+)$$
 ≥ 0 (121)

$$m_0 = -\frac{1}{\Phi} \frac{\partial S}{\partial y} \curvearrowright y(-)$$
 ≤ 0 (122)

And both provides always a positive m_0 which means to say that in a Null-Like 5D Spacetime Ansatz $dS^2 = 0$ the rest-mass m_0 seen in 4D is obtained purely by the derivative of the Hsmilton-Jacobi Action with respect to the extra dimension as a pure geometrical effect originated in the 5D Spacetime and the 4D electric charge q is also generated by the same geometric effect originated in the 5D

³⁴note that the minus sign in the $y(-) \leq 0$ cancels with the minus sign giving a positive m_0 in this case.

Again back to the Hamilton-Jacobi equation according to Ponce De Leon as follows:

$$g^{qr} \frac{\partial S}{\partial x^q} \frac{\partial S}{\partial x^r} - \frac{1}{\Phi^2} \left(\frac{\partial S}{\partial y} \right)^2 = 0 \quad (123)$$

$$g^{qr} \frac{\partial S}{\partial x^q} \frac{\partial S}{\partial x^r} = \frac{1}{\Phi^2} \left(\frac{\partial S}{\partial y} \right)^2 \quad (124)$$

We will obtain this interesting result:³⁵

$$\sqrt{g^{qr} \frac{\partial S}{\partial x^q} \frac{\partial S}{\partial x^r}} = \pm \frac{1}{\Phi} \left(\frac{\partial S}{\partial y} \right) \quad (125)$$

For diagonalized metrics we have:³⁶

$$g^{rr} \frac{\partial S}{\partial x^r} \frac{\partial S}{\partial x^r} - \frac{1}{\Phi^2} \left(\frac{\partial S}{\partial y} \right)^2 = 0 \quad (126)$$

$$g^{rr} \left(\frac{\partial S}{\partial x^r} \right)^2 - \frac{1}{\Phi^2} \left(\frac{\partial S}{\partial y} \right)^2 = 0 \quad (127)$$

$$g^{rr} \left(\frac{\partial S}{\partial x^r} \right)^2 = \frac{1}{\Phi^2} \left(\frac{\partial S}{\partial y} \right)^2 \quad (128)$$

$$\sqrt{g^{rr}} \left(\frac{\partial S}{\partial x^r} \right) = \pm \frac{1}{\Phi} \left(\frac{\partial S}{\partial y} \right) \quad (129)$$

$$\sqrt{g^{rr}} \left(\frac{\partial y}{\partial x^r} \right) = \pm \frac{1}{\Phi} \quad (130)$$

$$\Phi = \pm \sqrt{g_{rr}} \left(\frac{\partial x^r}{\partial y} \right) \quad (131)$$

And at least we got for the Null-Like 5D Spacetime Ansatz $dS^2 = 0$ in a diagonalized metric a set of valid expressions for the Scalar Field Φ . One of these expressions corresponds to the 5D BraneWorld Matter Universe:

$$\Phi = \sqrt{g_{rr}} \left(\frac{\partial x^r}{\partial y} \right) \curvearrowright y(+) \geq 0 \quad (132)$$

While the other corresponds to the 5D BraneWorld Antimatter Universe:³⁷

$$\Phi = -\sqrt{g_{rr}} \left(\frac{\partial x^r}{\partial y} \right) \curvearrowright y(-) \leq 0 \quad (133)$$

In order to terminate this second case: we are now left with two different expressions for the Hamilton-Jacobi equation according to Ponce De Leon

³⁵note the signs \pm

³⁶the signs \pm appears again

³⁷again we assume a linear displacement with respect to y in order to use the minus sign of y to cancel the minus sign in the beginning of the expression giving a positive Scalar Field. The square of both expressions must match as we have seen before

$$m_0^2 - \frac{q^2}{\Phi^2} = M_5^2 \quad (134)$$

$$m_0^2 - \frac{q^2}{\Phi^2} = 0 \quad (135)$$

In the end of this section we provide a Table of Elementary Particles. Note that all the particles possessing an electric charge always possesses a rest mass. We can have particles of zero 4D rest mass $m_0 = 0$ (eg photons) but these particles will always have a null electric charge $q = 0$. We cannot have a particle with zero 4D rest-mass and a non-null electric charge. This is one of the most important consequences of the Hamilton-Jacobi equation according to Ponce De Leon formalism.

- Case 3)- particles in a Spacelike 5D Spacetime Ansatz $dS^2 < 0$ with a 5D rest-mass $M_5 < 0$ giving a 4D rest-mass $m_0 > 0$

We already know that $dS/M_5 = ds/m_0$ then since the 4D rest-mass m_0 is always positive³⁸ we must "always" have a negative 5D rest mass $M_5 < 0$ since $dS < 0$ in order to make the term dS/M_5 "always" positive. Also note that the 4D Spacetime Ansatz ds^2 is "always" Timelike or Null-Like.

- Lastly we would like to discuss a fundamental question: Why does the electron annihilates with the positron? Why each particle annihilates with its own antiparticle counterpart??
- Considering the case 1)- particles in a Timelike 5D Spacetime Ansatz $dS^2 > 0$ with a 5D rest-mass $M_5 > 0$ giving a 4D rest-mass $m_0 > 0$:

We already know that the equations relating the 4D rest-mass m_0 and the 4D electric charge q according to Ponce de Leon for a particle and its antiparticle counterpart are given by:

$$q_4(+)= -m_0\Phi^2\frac{dy(+)}{ds} \curvearrowright q_4(+)<0 \quad (136)$$

$$q_4(-)= m_0\Phi^2\frac{dy(-)}{ds} \curvearrowright q_4(-)>0 \quad (137)$$

Imagine that one particle and its antiparticle counterpart collides: Both are travelling in two different Timelike 5D Spacetime Ansatz $dS^2 > 0$ each one for each particle. Although we defined the antiparticles moving in the 5D BraneWorld Universe $y(-) \leq 0$ remember that $y(-)^2 \geq 0$ and consequently $dy^2 \geq 0$ and the square of the Scalar Fields is the same. Both 5D Spacetime Ansatz seems to be equal however "they" are not. Both particles and antiparticles share the same 4D Spacetime Universe and the same 4D rest-mass m_0 because according to the nature of the Hamilton-Jacobi equation and the formalism of Ponce De Leon two equal 5D rest masses M_5 but however located in two different 5D BraneWorld Universes are being projected into the same 4D Spacetime.

Suppose that our electron collides with our positron: we have now the following situations:

³⁸we do not consider here exotic matter

- Sum of the charges: Both particles possess charges of equal modulus but opposite signs. In the collision both charges enter in contact with each other. Consequently one charge will cancel the other. Then we should expect for:

$$q_4(+) + q_4(-) = 0 \quad (138)$$

- Sum of the masses: using the equation above we have:

$$q_4(+) + q_4(-) = -m_0\Phi^2 \frac{dy(+)}{ds} + m_0\Phi^2 \frac{dy(-)}{ds} = 0 \quad (139)$$

But we know that both particles share the same term $m_0\Phi^2$. Hence it seems to be legitimate to write:

$$q_4(+) + q_4(-) = m_0\Phi^2 \left(-\frac{dy(+)}{ds} + \frac{dy(-)}{ds} \right) = 0 \quad (140)$$

But we also know that:

$$\frac{dy(+)}{ds} = -\frac{dy(-)}{ds} \quad (141)$$

$$\frac{dy(-)}{ds} = -\frac{dy(+)}{ds} \quad (142)$$

Then we would have two situations:

$$q_4(+) + q_4(-) = m_0\Phi^2 \left(\frac{dy(-)}{ds} + \frac{dy(-)}{ds} \right) = 0 \quad (143)$$

$$q_4(+) + q_4(-) = m_0\Phi^2 \times 2 \times \left(\frac{dy(-)}{ds} \right) = 0 \quad (144)$$

$$q_4(+) + q_4(-) = m_0\Phi^2 \left(-\frac{dy(+)}{ds} - \frac{dy(+)}{ds} \right) = 0 \quad (145)$$

$$q_4(+) + q_4(-) = m_0\Phi^2 \times 2 \times \left(-\frac{dy(+)}{ds} \right) = 0 \quad (146)$$

Note that in order to produce a total charge of the system electron-positron equal to zero the term $m_0\Phi^2 \times 2$ ³⁹ must also become equal to zero. Hence the total mass of the system electron-positron according to the Hamilton-Jacobi equation will be zero. This leads us to an important conclusion:

- A zero $4D$ rest-mass requires a Null-Like $5D$ Spacetime Ansatz $dS^2 = 0$. The total rest-mass of the electron-positron $m_0 = 0$ seen in $4D$ is the mass of the observed photon that will appear in the collision. Then in the collision the electron-positron system changes the geometry from two different and independent Timelike $5D$ Spacetime Ansatz $dS^2 > 0$ to a single one and unified Null-Like $5D$ Spacetime Ansatz $dS^2 = 0$

³⁹the sum of two equal $4D$ rest-masses m_0 one for the electron and the other for the positron

We will terminate this Section with two fundamental questions(and possible answers):

- 1)-Why we have in our 4D Universe two "kinds" of "matter" for non-zero rest-mass particles:(Matter and Antimatter) and not a third one???
- 2)-Why Matter prevails over Antimatter and not the inverse????

This picture of two different 5D BraneWorld Universes one for Matter and the another for Antimatter suggest us that perhaps the Big Bang was a "shock-wave",a collision between two different "plane waves" in 5D that generated 13,7 billions of years ago what we know as the 4D Big Bang.(See abstract of [1] and abstract of and page 2 of [2].Note that in the last one it is mentioned explicitly the 4D Brane as the "plane" of the collision between two different 5D "plane waves" propagating in opposite directions along the Extra Dimension.)

A collision between two different 5D BraneWorld Universes is pictured below:(See eqs 23 and 24 pg 5 in [2])

$$dS^2 = n^2(t + \lambda y)dt^2 - a^2(t + \lambda y) \left[\frac{dr^2}{(1 - kr^2)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] - \Phi^2(t + \lambda y)dy^2, \quad (147)$$

$$dS^2 = n^2(t - \lambda y)dt^2 - a^2(t - \lambda y) \left[\frac{dr^2}{(1 - kr^2)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] - \Phi^2(t - \lambda y)dy^2, \quad (148)$$

The 5D BraneWorld Universe $y(+)$ > 0 represents the Matter in our 4D Universe and the 5D BraneWorld Universe $y(-)$ < 0 represents the Antimatter in our 4D Universe.According to Ponce de Leon they can be interpreted as plane-waves propagating in "opposite" directions along the fifth dimension, and colliding at $y = 0$.($k = -1, 0, +1$).

- If the Big Bang was a collision between two different 5D BraneWorlds 13,7 billions of years ago then we can easily figure out that:
- Although both these 5D Universes possessed the same kind of 5D rest-mass M_5 , one of these 5D Universes was more massive than the other.In this case the 5D Matter Universe M_5 that generates the 4D rest-masses m_0 for the electron and not for the positron.This can be the reason why Matter prevailed over Antimatter
- The reason why we have two "kinds" of matter seen in our 4D Universe is due to the fact that it was a collision between two 5D Universes of the same kind of 5D rest-mass and not a collision between three of four 5D Universes with different kinds of 5D rest-mass

Below there is presented a Chart of Elementary Particles. Note that all the Elementary Particles known always possesses a positive $4D$ rest mass m_0 : Examining carefully the Chart using the Ponce De Leon equations of mass and charge:

$$m_0 = \frac{M_5}{\sqrt{1 - \Phi^2 \left(\frac{dy}{ds}\right)^2}} \quad (149)$$

$$q = \pm \frac{M_5 \Phi^2 \frac{dy}{ds}}{\sqrt{1 - \Phi^2 \left(\frac{dy}{ds}\right)^2}} \quad (150)$$

| Particle | spin (\hbar) | B | L | T | T ₃ | S | C | B* | charge (e) | m_0 (MeV) | antipart. |
|------------|------------------|-----|---|-----|----------------|----|---|----|----------------|-------------|------------------|
| u | 1/2 | 1/3 | 0 | 1/2 | 1/2 | 0 | 0 | 0 | +2/3 | 5 | \bar{u} |
| d | 1/2 | 1/3 | 0 | 1/2 | -1/2 | 0 | 0 | 0 | -1/3 | 9 | \bar{d} |
| s | 1/2 | 1/3 | 0 | 0 | 0 | -1 | 0 | 0 | -1/3 | 175 | \bar{s} |
| c | 1/2 | 1/3 | 0 | 0 | 0 | 0 | 1 | 0 | +2/3 | 1350 | \bar{c} |
| b | 1/2 | 1/3 | 0 | 0 | 0 | 0 | 0 | -1 | -1/3 | 4500 | \bar{b} |
| t | 1/2 | 1/3 | 0 | 0 | 0 | 0 | 0 | 0 | +2/3 | 173000 | \bar{t} |
| e^- | 1/2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | -1 | 0.511 | e^+ |
| μ^- | 1/2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | -1 | 105.658 | μ^+ |
| τ^- | 1/2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | -1 | 1777.1 | τ^+ |
| ν_e | 1/2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0(?) | $\bar{\nu}_e$ |
| ν_μ | 1/2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0(?) | $\bar{\nu}_\mu$ |
| ν_τ | 1/2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0(?) | $\bar{\nu}_\tau$ |
| γ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | γ |
| gluon | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | gluon |
| W^+ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | +1 | 80220 | W^- |
| Z | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 91187 | Z |
| graviton | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | graviton |

We can easily see that:

- 1)-We can have a set of $5D$ Quarks all of them with the same given rest-mass M_5 in a given $5D$ Spacetime generating as $4D$ Spacetime "images" all the six $4D$ Quarks each one with its own $4D$ rest-mass m_0 because the same $5D$ rest-mass M_5 each one for each $5D$ Quark is being divided by different Spacetime Couplings each one for each $4D$ Quark
- 2)-The group of Leptons in $5D$ corresponds to two $5D$ set of particles. One for the Electron-Muon Group and the other for the Neutrino Group in a situation similar to the one described for Quarks. Both moves in Timelike $5D$ Spacetime Ansatz $dS^2 > 0$ however in the Ansatz for the Neutrino Group the derivative of the Hamilton-Jacobi Action with respect to the extra coordinate is zero .
- 3)-As pointed before all the charged particles possesses mass
- 4)-particle Z like the Neutrino Group is stationary in the $5D$ Spacetime

3 Conclusion

In Section 2 we analyzed the Hamilton-Jacobi equation using the Ponce De Leon formalism. We demonstrated how the $5D$ Spacetime generates as a projection in the $4D$ Spacetime the masses and electrical charges of all elementary particles and antiparticles and we explained why antiparticles have the same rest-mass of particles but electrical charges of opposite signs also using the Hamilton-Jacobi equation.

4 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke⁴⁰
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein^{41,42}

⁴⁰special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

⁴¹"Ideas And Opinions" Einstein compilation, ISBN 0 – 517 – 88440 – 2, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

⁴²appears also in the Eric Baird book Relativity in Curved Spacetime ISBN 978 – 0 – 9557068 – 0 – 6

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