On the Speed of Rotation of the Isotropic Space: Insight into the Redshift Problem

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Abstract: This study applies the mathematical method of chronometric invariants, which are physically observable quantities in the four-dimensional space-time (Zelmanov A. L., *Soviet Physics Doklady*, 1956, vol. 1, 227–230). The isotropic region of the space-time is considered (it is known as the isotropic space). This is the home of massless light-like particles (e.g. photons). It is shown that the isotropic space rotates with a linear velocity equal to the velocity of light. The rotation slows in the presence of gravitation. Even under the simplified conditions of Special Relativity, the isotropic space still rotates with the velocity of light. A manifestation of this effect is the observed Hubble redshift explained as energy loss of photons with distance, for work against the non-holonomity (rotation) field of the isotropic space wherein they travel (Rabounski D. *The Abraham Zelmanov Journal*, 2009, vol. 2, 11–28). It is shown that the light-speed rotation of the isotropic space has a purely geometrical origin due to the space-time metric, where time is presented as the fourth coordinate, expressed through the velocity of light.

Contents:

This presentation is dedicated to Hermann Minkowski (1864–1909), on the 100th anniversary of his publication of "Raum und Zeit".

§1. Foreword. When I presented *Hubble Redshift due to the Global Non-Holonomity of Space** [1], the scientific community asked: Why do you believe that the isotropic space (the home of photons) rotates with

*The presentation was also delivered, in two parts, at Meetings of the American Physical Society, held in Spring, 2009 [2, 3]. A brief account of the study was preliminary publushed in [4].

the velocity of light, and what are its foundations in the basic space-time geometry?

Naturally, this question is not trivial, and cannot be answered in brief. I therefore decided to provide the answer, in detail, in this special presentation.

This problem will be considered in the framework of both General Relativity and Special Relativity. In both cases, it will be observed that the sign-alternating structure of the space-time metric, where time is presented as the fourth coordinate $x^0 = ct$, expressed through the velocity of light, is solely responsible reason for the ligh-speed rotation of the isotropic space. Now, I have to offer all the explanations to the attention of readers.

§2. A short explanation of the isotropic space. First of all we need to give a short explanation of the isotropic space and of its origin in the geometric structure of space-time.

The basic space-time of the General Theory of Relativity is a four-dimensional pseudo-Riemannian space, with the signature $(-{+}{+}{+})$ or $({+}{-}{-}{-})$. This is one member of the family of Riemannian spaces, the metric spaces where the square of distance between any two infinitely close points is set up by the square form $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$. This form is invariant along all the space (that also is specific to Riemannian spaces). Due to invariance of the metric, the length of any n-dimensional vector Q^α, being transferred in parallel to itself in a Riemannian space of n-dimensions, remains unchanged: $Q_\alpha Q^\alpha = g_{\alpha\beta}\, Q^\alpha Q^\beta = const$. This is known as *Levi-Civita parallel transfer*, due to Tullio Levi-Civita, and is specific to Riemannian spaces. The kind metric $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta = inv$ is referred to as a *Riemannian metric*, in memory of Bernhard Riemann who introduced it in the 1850's. The prefix "pseudo" means a class of Riemannian spaces, where the metric is sign-alternating. In this case, algebraically, the diagonal components $g_{\alpha\alpha}$ of the fundamental metric tensor $g_{\alpha\beta}$ do not bear the same sign. Geometrically, this means that two types of coordinate axes are present in the space: the axes of real coordinates (the "plus" sign in the diagonal components) and the axes where coordinates are imaginary (the "minus" sign). Pseudo-Riemannian spaces were introduced in 1908 by Hermann Minkowski, who first considered a particular case of these, having four dimensions, wherein one axis is imaginary and three other axes are real, or, alternatively, one axis is real while the other three are imaginary*. So, the

*In a general case, pseudo-Riemannian spaces can have any number of dimensions, with any combitations of positive and negative signs in the signature.

signature of the space metric is $(-+++)$ or $(+---)$, respectively. Thus, Minkowski emphasized time as a coordinate axis $x^0 = ct$, which is segregate from three axes of the spatial spread. Historically, he studied a highly simplified case, where the space metric can be reduced, by transformation of the coordinates, to a simplest diagonal form, where $g_{\alpha\alpha} = \{-1, +1, +1, +1\}$ or $\{+1, -1, -1, -1\}$, and the non-diagonal components of $g_{\alpha\beta}$ are zero. This is the basic space-time of the Special Theory of Relativity. In this case, the Riemann-Christoffel curvature tensor is zero, so the space is non-curved as can be illustrated by a pile of flat spatial sections (three-dimensional spaces) threaded up onto the time axis. This simplified case of the four-dimensional pseudo-Riemannian space is known as *Minkowski's space*. This, however, differs from a four-dimensional pseudo-Euclidean space, which also is non-curved, but all spatial coordinates are homogeneous therein (the unit coordinate marks are uniformly distributed along the coordinate axes). In contrast, the spatial coordinates can be inhomogenous in Minkowski's space, producing some forces therein.

The four-dimensional pseudo-Riemannian space is not a "monolite" single spread as a sign-definite metric space. Due to its sign-alternating metric, it is presented with two segregate spreads:

a) The non-isotropic space (space-time), where the time interval and the spatial interval always differ from each other. As such, $ds^2 \neq 0$ and any world-vector's length is $Q_\alpha Q^\alpha = const \neq 0$ in the space. Thus, this is the home for mass-bearing particles (such a particle, being characterized with the world-vector $P^\alpha = m_0 \frac{dx^\alpha}{ds}$, has a non-zero rest mass $P_\alpha P^\alpha = m_0 = const \neq 0$).

b) The isotropic space (space-time), where the time interval and the spatial interval have the same length. As such, the space-time interval is always zero ($ds^2 = 0$). Any world-vector of the isotropic space has zero length ($Q_\alpha Q^\alpha = const = 0$). The isotropic space particles are characterized with the world-vector $P^\alpha = \frac{m}{c} \frac{dx^\alpha}{d\tau}$, expressed through the relativistic mass m and the observable time interval $d\tau$. They have zero rest mass ($P_\alpha P^\alpha = const = 0$), but non-zero relativistic mass and energy according to $E = mc^2$. All isotropic space particles move at the velocity of light. Thus, these are massless light-like particles, e.g. photons.

This terminology, "non-isotropic" and "isotropic", does not seem to be very successful when being applied to the space-time regions. This is because, for a physicist, the terms mean something different than in the geometry of pseudo-Riemannian spaces. A physicist, when hearing that something (space or medium) is non-isotropic, thinks about the presence of a preferred direction in it. Conversely, the absence of a preferred direction is regularly understood as isotropy. Relativists and mathematicians refer to a space region as isotropic if $ds^2 = 0$ therein, so the length of any world-vector is zero: the vector is equally targeting all four-dimensional directions. On the other hand, spatial vectors of the isotropic space, having one dimension lesser than the four-dimensional space itself, are not world-vectors therein. The vectors have surely non-zero lengths, and target their specific spatial direction. That is, the term "isotropic" is attributed to the four-dimensional space (space-time) of photons, but is unrelated to the three-dimensional space where photons travel (it can be isotropic or anisotropic, depending on the particular physical conditions in it).

I, and the relativists in general, adhere to this terminology, because it is well accepted in the scientific literature on the space-time geometry and the theory of relativity.

§3. The light-speed rotation. We are going to consider the isotropic space from the viewpoint of a regular observer, whose home is the non-isotropic space filled with mass-bearing particles. Thus, his reference body is a rigid physical body over which a real (deformed) coordinate net is spanned, and real clocks are located on its surface. To find physical quantities, registered by the observer, we should project the four-dimensional quantities onto the time line and coordinate net of his body of reference. This problem was resolved, in 1944, by Abraham Zelmanov. His mathematical apparatus of chronometric invariants [5–7] targets physically observable quantities for a regular observer at rest with respect to his body of reference.

In particular, the theory introduces the chronometrically invariant (physically observable) intervals of time and the spatial coordinates as the projections of the interval of the four-dimensional coordinates dx^α onto the observer's time line and the spatial section. The observable spatial coordinates meet the regular three-dimensional coordinates, while the physically observable time interval

$$d\tau = \sqrt{g_{00}}\, dt + \frac{g_{0i}}{c\sqrt{g_{00}}}\, dx^i = \sqrt{g_{00}}\, dt - \frac{1}{c^2}\, v_i\, dx^i \tag{3.1}$$

depends on the gravitational potential $\mathrm{w} = c^2 (1 - \sqrt{g_{00}})$ and the linear velocity of rotation of the observer's three-dimensional space

$$v_i = -\, c\, \frac{g_{0i}}{\sqrt{g_{00}}}\,. \tag{3.2}$$

The chronometrically invariant metric tensor

$$h_{ik} = -\,g_{ik} + \frac{g_{0i}\,g_{0k}}{g_{00}} = -\,g_{ik} + \frac{1}{c^2}\,v_i\,v_k\,, \tag{3.3}$$

obtained as the spatial projection of the fundamental metric tensor $g_{\alpha\beta}$, gives the chronometrically invariant (observable) spatial interval

$$d\sigma^2 = h_{ik}\,dx^i dx^k. \tag{3.4}$$

Due to these formulae, the space-time interval $ds^2 = g_{\alpha\beta}\,dx^\alpha dx^\beta$ is expressed through the observable time interval and the observable spatial interval as

$$ds^2 = c^2 d\tau^2 - d\sigma^2, \tag{3.5}$$

that is true in the space-time of General Relativity, because the observable quantities, $d\tau$ and $d\sigma$, take all components of the fundamental metric tensor $g_{\alpha\beta}$ into account. This is in contrast to the analogous formula of Special Relativity, $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$, which assumes that only the diagonal terms of $g_{\alpha\beta}$ are non-zero, and are units.

Now, I show how rotation of the isotropic space can be easyly found with use of the mathematical method of physically observable quanities (chronometric invariants).

Two physical conditions specific to the isotropic space,

$$ds^2 = 0\,, \qquad c^2 d\tau^2 = d\sigma^2 \neq 0\,, \tag{3.6}$$

were highlighted in §2*. These conditions set that the time spead and the spatial spread meet each other everywhere in the isotropic space.

Time and regular three-dimensional space can meet each other in terms of the linear velocity of rotation of the space, according to the definition of the velocity (3.2).

This can be visualized by introducing a locally geodesic frame of reference in the point of observation (where the observer is located). The main advantage of such a reference frame is that it is the same, within infinitesimal vicinities of the point of observation, for all other regions of the space (space-time)†.

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*The second condition, $c^2 d\tau^2 = d\sigma^2 \neq 0$, is stronger. This is because the first, $ds^2 = c^2 d\tau^2 - d\sigma^2 = 0$, includes also the fully degenerate case $c^2 d\tau^2 = d\sigma^2 = 0$, which means something out of the isotropic space due to the full degeneration of the observable time durations and the observable lengths.

†Locally geodesic coordinates and reference frames rise from Riemann's pioneering studies, and are much explained in the scientific literature. For instance, see §7 of Petrov's *Einstein Spaces* [8].

Within infinitesimal vicinities of any point of such a reference frame the fundamental metric tensor is

$$\tilde{g}_{\alpha\beta} = g_{\alpha\beta} + \frac{1}{2}\left(\frac{\partial^2 \tilde{g}_{\alpha\beta}}{\partial\tilde{x}^\mu \partial\tilde{x}^\nu}\right)(\tilde{x}^\mu - x^\mu)(\tilde{x}^\nu - x^\nu) + \ldots\,, \tag{3.7}$$

i.e. its components $\tilde{g}_{\alpha\beta}$ at a point, located in the vicinities, are different from $g_{\alpha\beta}$ at the point of observation to within only the higher order terms, which can be neglected. Therefore, at any point of a locally geodesic reference frame the fundamental metric tensor can be considered constant, so its first derivatives (the Christoffel symbols) and the second derivatives (the space curvature) are zero.

As a matter of fact, within infinitesimal vicinities of any point located in a Riemannian space, a locally geodesic reference frame can be set up. At the same time, at any point of this locally geodesic reference frame, a tangential flat Euclidean space can be set up so that this reference frame, being locally geodesic for the Riemannian space, is the global geodesic for that tangential flat space.

The fundamental metric tensor of a flat Euclidean space is constant, so the values of the tensor $\tilde{g}_{\alpha\beta}$, taken in the vicinities of a point of the Riemannian space, converge to the values of the tensor $g_{\alpha\beta}$ in the flat space tangential at this point. Therefore, we can build a system of basis vectors $\vec{e}_{(\alpha)}$, which are located along the coordinate axes in this flat space, and tangential to curved coordinate lines of the Riemannian space in the point of observation.

It should be noted that, in a general case, real coordinate lines in Riemannian spaces are curved, inhomogeneous, and are not orthogonal to each other. So the lengths of the basis vectors may sometimes be very different from unity.

We denote a four-dimensional vector of infinitesimal displacement by $d\vec{r} = \{dx^0, dx^1, dx^2, dx^3\}$. So $d\vec{r} = \vec{e}_{(\alpha)} dx^\alpha$, where components of the basis vectors $\vec{e}_{(\alpha)}$ tangential to the coordinate lines are $\vec{e}_{(0)} = \{e^0_{(0)}, 0, 0, 0\}$, $\vec{e}_{(1)} = \{0, e^1_{(1)}, 0, 0\}$, $\vec{e}_{(2)} = \{0, 0, e^2_{(2)}, 0\}$, $\vec{e}_{(3)} = \{0, 0, 0, e^3_{(3)}\}$. The scalar product of the vector $d\vec{r}$ with itself is $d\vec{r}d\vec{r} = ds^2$. On the other hand, $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$. As a result we arrive at the formula

$$g_{\alpha\beta} = \vec{e}_{(\alpha)}\vec{e}_{(\beta)} = e_{(\alpha)}e_{(\beta)} \cos\left(x^\alpha; x^\beta\right), \tag{3.8}$$

which shows how components of the fundamental metric tensor of the observer's space depend on the lengths of the basis vectors (tangential to his real coordinate axes, inhomogeneous and curved), and on the angle between them.

In particular, formula (3.8) gives

$$g_{00} = e^2_{(0)}\,, \tag{3.9}$$

$$g_{0i} = e_{(0)}e_{(i)}\cos\left(x^0; x^i\right), \tag{3.10}$$

$$g_{ik} = e_{(i)}e_{(k)}\cos\left(x^i; x^k\right). \tag{3.11}$$

Finally, applying these to the definitions of v_i (3.2) and h_{ik} (3.3), we derive how these depend on the lengths of the basis vectors $\vec{e}_{(0)}$ and $\vec{e}_{(i)}$ (tangential to the real coordinate axes, inhomogeneous, and curved), and on the angle between them. That is

$$v_i = -\,c\,e_{(i)}\cos\left(x^0; x^i\right), \tag{3.12}$$

$$h_{ik} = e_{(i)}e_{(k)}\left[\cos\left(x^0; x^i\right)\cos\left(x^0; x^k\right) - \cos\left(x^i; x^k\right)\right]. \tag{3.13}$$

Consider these equations under the isotropic space condition, $c^2d\tau^2 = d\sigma^2 \neq 0$. According to this condition, time and regular three-dimensional space meet each other. Geometrically, this means that the time basis vector $\vec{e}_{(0)}$ meets all three spatial basis vectors $\vec{e}_{(i)}$ (this fact does not mean, however, that the spatial basis vectors coincide, because the time basis vector is the same for all the spatial frame). In other words, $\cos\left(x^0; x^k\right) = \pm 1$ everywhere in the isotropic space. Also, in observing a photon, only its direction of motion (direction of travelling light) is counted, and $e_{(0)} = e_{(i)}$ along it (according to the isotropic space condition). This can be exressed through the gravitational potential $\mathrm{w} = c^2\left(1-\sqrt{g_{00}}\right)$, because $e_{(0)} = \sqrt{g_{00}}$ in a general case (3.9). Finally, in the isotropic space, we have

$$\cos\left(x^0; x^k\right) = \pm 1\,, \qquad e_{(i)} = e_{(0)} = \sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2}\,, \tag{3.14}$$

and, hence,

$$v_i = \mp\sqrt{g_{00}}\;c_i = \mp\left(1-\frac{\mathrm{w}}{c^2}\right)c_i\,, \tag{3.15}$$

$$h_{ik} = \left(1-\frac{\mathrm{w}}{c^2}\right)^2\left[1-\cos\left(x^i; x^k\right)\right], \tag{3.16}$$

where c^i is the chronometrically invariant three-dimensional vector of the physically observable velocity of light, $c_ic^i = h_{ik}\,c^ic^k = c^2$.

According to the formula derived (3.15), we immediately come to the following conclusion:

> The isotropic space rotates, at each its point and in each direction where a photon travels, with a linear velocity equal to the velocity of light. This fundamental rotation can slow down relative to the light speed in the presence of gravitation.

§4. Consequences of the light-speed rotation. Now, we investigate two sequels of the light-speed rotation of the isotropic space.

First consequence. Consider physically observable time $d\tau$ (3.1), which is dependent on the linear velocity of rotation of space. This is proper time, registered by the observer. It is always positive ($d\tau > 0$) due to his recognition of the past and the future. Therefore, the coordinate time function of an object, the function $\frac{dt}{d\tau}$, manifests how this object travels along the time axis with respect to the observer.

When expressing the coordinate time function from the definition of $d\tau$ (3.1), we obtain

$$\frac{dt}{d\tau} = \frac{1}{\sqrt{g_{00}}}\left(1 + \frac{1}{c^2}\, v_i\, \mathrm{v}^i\right), \qquad \mathrm{v}^i = \frac{dx^i}{d\tau}\,, \tag{4.1}$$

where v^i is the chronometrically invariant (physically observable) velocity of the object we observe.

Substituting the observable velocity of photons $\mathrm{v}^i = c^i$ and the linear velocity of the light-speed rotation (3.15), specific to the isotropic space as we obtained above, we consider a case where the time basis vector is directed oppositely to the spatial basis vectors, so $\cos(x^0; x^k) = -1$ and, hence, $v_i = -c_i$. (The second case, $\cos(x^0; x^k) = +1$, leads to nonsense in the coordinate time function.) We obtain

$$\frac{dt}{d\tau} = \frac{1}{\sqrt{g_{00}}}\left(1 - \sqrt{g_{00}}\right). \tag{4.2}$$

It is evident that the photon coordinate time stops, $\frac{dt}{d\tau} = 0$, when $\sqrt{g_{00}} = 1$ and, hence, the gravitational potential $\mathrm{w} = c^2(1 - \sqrt{g_{00}})$ becomes $\mathrm{w} = 0$, implying the absence of gravitational fields. In the presence of gravitation we have $\sqrt{g_{00}} < 1$, so the photon coordinate time function increases with the value of the gravitational potential, and the isotropic space rotation is slowing down from the light speed.

The stopping of the photon coordinate time function reflects that they, the particles of the isotropic space, move at the velocity of light. Light signals are mediators in synchronization of clocks (Einstein's method of synchronization). In this process, a light signal transfers zero-point of the time coordinate from one clock to another. Thus, from the point of view of a regular observer, the isotropic space particles are “resting-in-time”: their coordinate time is stopped with respect to his coordinate time, while physically observable time is not at rest due to their visible motion. In other words, photons rest in time, while we are moving along the time axis with respect to them. Therefore, the photon coordinate time function is always zero in the absence of gravitational fields. According to the formula (4.2), only gravitation is able to enforce the coordinate time of a photon to be flowing with respect to that of the observer.

Second consequence. It is interesting to ponder whether the light-speed rotation of the isotropic space has any influence on the space curvature. It is doubtful that this rotation can be attributed only to the curved space-time of General Relativity. To illustrate, consider the Riemann-Christoffel curvature tensor $R_{\alpha\beta\mu\nu}$. It is built on the second derivatives of the fundamental metric tensor $g_{\alpha\beta}$, and on its first derivatives, according to its definition

$$R_{\mu\nu\sigma\cdot}^{\cdots\alpha} = \frac{\partial\Gamma^{\alpha}_{\sigma\mu}}{\partial x^{\nu}} - \frac{\partial\Gamma^{\alpha}_{\mu\nu}}{\partial x^{\sigma}} + \Gamma^{\beta}_{\mu\sigma}\Gamma^{\alpha}_{\nu\beta} - \Gamma^{\beta}_{\mu\nu}\Gamma^{\alpha}_{\sigma\beta}\,, \tag{4.3}$$

where $\Gamma^{\alpha}_{\mu\nu} = g^{\alpha\sigma}\,\Gamma_{\mu\nu,\sigma} = \frac{1}{2}\,g^{\alpha\sigma}\left(\frac{\partial g_{\mu\sigma}}{\partial x^{\nu}} + \frac{\partial g_{\nu\sigma}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}}\right)$.

In this formula, according to the definition of v_i (3.2), we should use $g_{0i} = -\frac{1}{c}\,v_i\sqrt{g_{00}}$. Hence, even if $\sqrt{g_{00}} = 1$ (no gravitational fields), v_i should have an influence on the Riemann-Christoffel tensor. But this is true, only if $g_{0i} \neq const$. In the isotropic space, in the absence of gravitation, as shown above, $v_i = -c_i$ and, hence, $g_{0i} = \frac{1}{c}\,c_i$. If rotation of the isotropic space is stationary and vorticeless, $v_i = -c_i$ is independent from the spatial coordinates and time, so its first and second derivatives are zero. In other words, there is not a goal of this rotation into the curvature tensor. Thus, with the diagonal spatial metric* where $g_{kk} = \{-1,-1,-1\}$ or $\{+1,+1,+1\}$, we arrive at the condition of Special Relativity, which is $R_{\mu\nu\sigma\cdot}^{\cdots\alpha} = 0$.

Therefore, even in the framework of the simplified conditions of Special Relativity, the isotropic space still rotates with the velocity of light.

*We know, according to the theorem introduced by Émile Cotton [9], that any three-dimensional square form can be reduced to the diagonal unit form. This means, in particular, that, if a four-dimensional space (space-time) is free of gravitation ($g_{00} = 1$) and its three-dimensional metric g_{ik} is stationary, the space-time metric is reducible to the diagonal unit form $g_{\alpha\alpha} = \{+1,-1,-1,-1\}$ or $\{-1,+1,+1,+1\}$ (see §46 of Petrov's *Einstein Spaces* [8]). This is the case considered by the Special Theory of Relativity, and is known as Minkowski's space (space-time).

§5. The origin of the fundamental rotation. Why does the isotropic space rotate at the velocity of light? In other words, wherefrom does the rotation originate? To answer this question, we should turn to the geometric structure of space-time.

The basic space-time of the General Theory of Relativity is the four-dimensional pseudo-Riemannian space, which metric is sign-alternating so that the time axis is emphasized as $x^0 = ct$. The space-time signature, $(+---)$ or $(-+++)$, was first pointed out by Hermann Minkowski in his famous *Raum und Zeit* [14] as the origin of the relativistic transformations of the spatial coordinates and time, which distinguishes relativistic physics from classical physics.

If a sign-definite signature, $(++++)$ or $(----)$, the world would have four spatial coordinates where time is a spatial parameter as found in classical physics. In this case, no difference from the laws of classical physics would be observed, but simply four spatial coordinates instead of three ones. Accordingly, $ds^2 = 0$ that is the isotropic space condition which differs an isotropic region from a non-isotropic one, would mean that the space has been shrunk into a point. So, $ds^2 \neq 0$ is true everywhere in the space. No splitting into isotropic and non-isotropic regions is possible. All the space is a single non-isotropic spread.

In contrast, in a space of the sign-alternating signature as above, the isotropic space condition $ds^2 = 0$ is expanded as to contain non-zero time and spatial spreads, equal to each other in the length. As a result, the isotropic region ($ds^2 = 0$) and the non-sotropic region ($ds^2 \neq 0$) co-exist in the space.

Therefore the isotropic space (the home of photons), i.e. the region determined by the condition $ds^2 = 0$, is due only to the sign-alternating space metric which emphasizes time as a segregate axis of the space.

The next step in understanding the light-speed rotation of the isotropic space is visualized by consideration of the formula (3.12). This formula, $v_i = -c\, e_{(i)} \cos(x^0; x^i)$, shows how the linear velocity of the rotation of the observer's space depends on the lengths of the spatial basis vectors $\vec{e}_{(i)}$ (tangential to his real coordinate axes, inhomogeneous and curved), and on the angle between them and the time basis vector $\vec{e}_{(0)}$. The velocity of light appears in the formula, as well as in the other formulae of relativistic physics, due to the fact that time is presented here as the fourth coordinate axis, $x^0 = ct$, where the velocity plays a rôle of numerical coefficient.

If one assumes another numerical coefficient of the same dimension, say u cm/sec, so the time coordinate axis is presented as $x^0 = ut$, the formula has to be changed as*

$$v_i = -\,u\,e_{(i)}\cos\,(x^0;\,x^i)\,. \tag{5.1}$$

Once the isotropic space condition $ds^2 = 0$ applied to the space-time metric $ds^2 = g_{\alpha\beta}\,dx^\alpha dx^\beta = e_{(\alpha)}\,e_{(\beta)}\cos\,(x^\alpha;\,x^\beta)$ as we did in §2 and §3, we obtain $\cos\,(x^0;\,x^i) = -1$ and, hence,

$$v_i = -\,u_i \tag{5.2}$$

in the space. In other word, when assuming $x^0 = ut$ instead of $x^0 = ct$, we immediately arrive at a result that the isotropic space rotates with a linear velocity equal to u.

As a result of what has been said above, we arrive at the conclusion that the isotropic space rotates with the velocity of light due to two purely geometric conditions:

a) The space-time metric is sign-alternating. The signature, $(+---)$ or $(-+++)$, emphasizes time as the fourth coordinate $x^0 = ct$ containing the velocity of light as a numerical coefficient.

b) The isotropic space condition $ds^2 = 0$. This is a sequel of the first condition. Namely, because the signature emphasizes the time axis $x^0 = ct$, there is in the space-time a region where the space-time spread is zero ($ds^2 = 0$), while the time spead and the spatial spread are non-zero, and are equal to each other.

The conditions are true in the framework of both General Relativity and Special Relativity, because the same signature condition exists, independent of the presence of the space curvature or the other factors which alter the basic geometries of the theories.

So, the light-speed rotation of the isotropic space has a purely geometrical origin due to the sign-alternatic structure of the space-time metric, where time is presented as the fourth coordinate axis $x^0 = ct$.

§6. A topological interpretation of the result. How can we imagine that the isotropic space rotates with the velocity of light? In searching for a native illustration of this result, we turn our attention to the concepts of topology as the best way of understanding something in many-dimensional space geometry.

According to the concepts of topology [10], a finite symmetric system can be considered as a topological spread mapped into a spherical space. Can we apply these views to our Universe?

*In this case, the respective changes appear in Lorentz' transformations and in all other formulae of relativistic physics.

Observational astronomy manifest the precense of the event horizon in the cosmos, and also the homogeneity and symmetry of the Metagalaxy to within a first order approximation. Therefore the Universe is homogeneous and isotropic on the average.

Also, as was proved by Zelmanov in the 1950's, in the framework of the General Theory of Relativity, spatial infinitude of the homogeneous isotropic cosmological models depends on the frame of reference from which we observe the universe [11,12]. In other words, if a homogeneous isotropic universe, being observed in one reference frame, is infinite, it may be finite in another reference frame. Zelmanov enunciated this result as the *Infinite Relativity Principle.* Thus, being located in a universe of infinite spread, we can always move to a specific frame of reference wherein the universe seems finite.

So, we can consider the Universe as a finite spread, which is homogeneous and isotropic on the average. Therefore, we can apply the aforementioned topological views to the Universe as a whole.

In addition, we should take into account that only one geodesic line can be drawn through a given point in a given direction, and the unique geodesic line can be either non-isotropic or isotropic (see §6 of Petrov's *Einstein Spaces* [8] or §101 of Raschewski's *Riemannsche Geometrie und Tensoranalysis* [13] for detail). That is, the isotropic and non-isotropic regions of space-time have no common points.*

Therefore, we do consider the Universe as two segregate speads (isotropic and non-isotropic), each mapped into a respective spherical space of the same radius of curvature. These two spherical spaces are equivalent to the surfaces of two concentric hyperspheres, which have the same radius, but are not coincident with each other. The surface of the isotropic hypersphere is the home of isotropic trajectories, while the non-isotropic hypershere's surface is the home of non-isotropic trajectories.

We are going to consider an observer who is located in the hypersphere's surface.

Any spherical formation of n dimensions (created by a spherical space of $n-1$ dimensions) is directed in its "parental" space of $n+1$ and higher dimensions. This can be easily understood, because in any

*This result can be illustrated in Minkowski's diagram, which is the plane paper (two-dimensional) representation of the four-dimensional pseudo-Riemannian space (space-time). Once a moving mass-bearing particle increases its velocity so much that it approaches the velocity of light, its non-isotropic trajectory in the diagram tries to reach the light cone (isotropic region) but never meets it as the particle never reaches the velocity of light. Even if the particle is moving infinitesimally close to the velocity of light, its trajectory is close to the light cone but never meets it. So, the isotropic and non-isotropic regions have no common points.

circle, the two-dimensional spherical formation created by a respective circumference (one-dimensional spherical space), is obviously directed in the three-dimensional space. If a hypothetical one-dimensional observer, located in a circumference, sees that every other (one-dimensional) object of the circumference moves with respect to him with a constant velocity, this is equivalent to the circumference rotating as a whole with respect to his position in it, with the same velocity along the direction in which he looks. Due to this rotation, an inertial force acts on all one-dimensional objects of the circumference, according to the angular velocity and the radius of it. This force has the same numerical value at all the objects, and is directed othogonally to the circumference. As a result, all objects moving relative to the observer along the circumference are carried out, by the force, in the directions opposite to their motion. This is manifested as an additional acceleration braking the objects.

Analogously, a three-dimensional observer located in the isotropic hypersphere's surface (isotropic spherical space) sees that any other object of the surface moves with respect to him with the velocity of light along his direction of observation. This is equivalent to stating that the surface rotates as a whole with the velocity of light in the direction of his observation. Because the polar axis of the rotation is directed in the "parental" space of the hypersphere, the inertial force produced due to the rotation is directed orthogonally to the hypersphere's surface in each of its points, and is equally applied to all objects of the surface. Trying to carry the moving objects to the direction orthogonal to the surface, along which they travel, the force produces a braking acceleration on the objects. Because all objects of the isotropic space (massless particles, e.g. photons) move with the velocity of light, the additional braking acceleration cannot slow down their motion, but only change their energy (frequency). As was shown in my previous study [1], this "braking effect" is observed as Hubble redshift which is explained as energy loss of photons with distance, for work against the non-holonomity (rotation) field of the isotropic space wherein they travel.

§7. Conclusions. We have considered how the isotropic space (the home of photons) appears to a hypothetical "light-like" observer located in it. Such an observer cannot accompany his reference body to which he compares all his measurements (a real physical body, e.g. a cosmic rigid body, located in the non-isotropic space). Therefore the result of his observation differs from that obtained by a regular observer who always accompanies his reference body in the non-isotropic space. Meanwhile, this approach gives an advantage to see the real physical propertites of the isotropic space. Here I would like to emphasize the most important of the obtained results:

1. In the four-dimensional pseudo-Riemannian space, which is the basic space-time of the General Theory of Relativity, the isotropic region (isotropic space, the home of photons) rotates, basically, with a linear velocity equal to the velocity of light. The fundamental rotation is slowed down in the precence of gravitation.
2. Even in a very simplified case of Minkowski's space — the basic space-time of the Special Theory of Relativity — the isotropic space of photons still rotates with the velocity of light. This means that a Galilean frame of reference (completely free of gravitation and rotation) is not possible in the isotropic space, since the latter always has an associated rotation.*
3. The fundamental rotation was found, in the frameworks of both General Relativity and Special Relativity, proceeding from only two obviously geometric conditions: a) the space-time metric is sign-alternating, $(+---)$ or $(-+++)$, where the time axis is emphasized as $x^0 = ct$, and b) $ds^2 = 0$ everywhere in the isotropic space. This means that the rotation has a purely geometrical origin due to the sign-alternating structure of the space-time metric.
4. In the framework of topology, the Universe can be presented as two segregate spreads (isotropic and non-isotropic) mapped onto two concentric hyperspheres, which have the same radius, but are not coinciding with each other. The fact that any object of the isotropic space moves relative to the observer with the velocity of light is eqivalent to an isotropic hypersphere which rotates with the velocity in its "parental" space of higher dimensions.

Thus, the isotropic space (the home of photons) rotates, at each of its points, with a linear velocity which is, basically, equal to the velocity of light. This fact was unfortunately overlooked during one hundred years commencing in Hermann Minkowski's 1908 famous presentation published posthumously†, in 1909, as *Raum und Zeit* [14]. Minkowski was the first person who pointed out that the Special Theory of Relativity,

*This is in contrast to the non-isotropic region (non-isotropic space) inhabited with mass-bearing particles. In Minkowski's space, as proven in the framework of the Special Theory of Relativity, we can reduce any motion to rectilinear and uniform form by transformations of the spatial coordinates and time. Therefore, a regular observer can find a Galilean frame of reference everywhere in the positions allowed for him in Minkowski's space.

†This presentation was delivered by Minkowski, a few months before the publication, at the *80th Assembly of German Natural Scientists and Physicians*, held

introduced by Albert Einstein three years before, in 1905, is explained in a four-dimensional space (space-time), where time is the fourth coordinate axis $x^0 = ct$, while the three-dimensional space of the observer moves with the velocity of light along it. Now, we clearly understand that this picture is not complete. It should be added to the light-speed rotation of the isotropic space (the home of photons). In other words, Minkowski's formula $x^0 = ct$ means not only the light-speed motion of the observer's space along the time axis, upstairs in Minkowski's diagram, but also the light-speed rotation of the surface of the isotropic cone which illustrates the isotropic space therein. It is significant that this understanding arrives on the anniversary of his *Raum und Zeit*, which was published exactly one hundred years ago. Therefore, I dedicate this paper to the memory of Hermann Minkowski (1864–1909), the pathfinder of space-time geometry.

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