The Temperature of a Black Hole in a De-Sitter Space-Time

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Abstract:

A relation for the black-hole temperature in a De-Sitter type universe is determined in the first step of this paper. As a result of that, the upper and the lower temperature limits of the black hole are calculated, and then the limits of the radius of the universe containing the black hole. All these calculations are based upon the present values of the cosmological constant Λ. Further relations for the dependance of this temperature on Hubble's constant and the gravitationsal energy of the hardons was also derived.

Key Words: De-Sitter metric, cosmological constant, Hubble Constant, hardons quantum vaccum.

1 Introduction:

The space-time metric of a De Sitter universe containing a black hole is given by:

$$ds^2 = c^2\left[1-\frac{2GM}{rc^2}-\frac{\Lambda r^2}{3}\right]dt^2 - \left[1-\frac{2GM}{rc^2}-\frac{\Lambda r^2}{3}\right]^{-1} dt^2 - r^2 d\Omega^2 \quad (1)$$

where $d\Omega^2 = r^2\left[d\theta^{\,2} + \sin^2\theta \;\mathrm{d}\phi^{\,2}\right]$

In this case the metric posseses a horizon r_H at the roots of the equation:

$$r^3 - Ar + B = 0 \quad (2)$$

where: A = -3/Λ and B = 2GM / $c^2\Lambda$, and where the symbols have their usual meannings. Solving we obtain three roots , but only one is real. Keeping the real root, we have that:

$$r_H = \left(\frac{2}{3}\right)^{1/3}\frac{A}{\left[-9B+\left(27B^2-4A^3\right)^{1/3}\sqrt{3}\right]^{1/3}} + \frac{\left[-9B+\left(27B^2-4A^3\right)^{1/2}\sqrt{3}\right]^{1/3}}{\sqrt[3]{18}} \quad (3)$$

Because the temperature of a black hole is given by:

$$T = \frac{\hbar c}{4\pi\, \mathrm{k_B} r_H} \quad (4)$$

equation (4) becomes:

$$T=\frac{\hbar c}{4\pi\, k_B}\left[\begin{array}{l}-\dfrac{3}{\Lambda}\left(\dfrac{2}{3}\right)^{1/3}\left\{\dfrac{18GM}{\Lambda c^2}+18\left(\dfrac{G^2M^2}{\Lambda^2c^4}+\dfrac{1}{\Lambda^3}\right)^{1/2}\right\}^{-1/3}\\ \\ +18^{-1/3}\left\{-\dfrac{18GM}{\Lambda^2c^4}+18\left(\dfrac{G^2M^2}{\Lambda^2c^4}+\dfrac{1}{\Lambda^3}\right)^{1/2}\right\}^{1/3}\end{array}\right]^{-1} \qquad (5)$$

Looking at expression (5) we realize that regardless of what the mass of the black hole is the only significant contribution, comes, from the second term and so (6) becomes:

$$T=\frac{\hbar c}{4k_B\pi}\left[18^{-1/3}\left\{-\frac{18GM}{\Lambda c^2}+18\left(\frac{G^2M^2}{\Lambda^2c^4}+\frac{1}{\Lambda^3}\right)^{1/2}\right\}^{1/3}\right]^{-1} \qquad (6)$$

When we introduce in (6) the mass of quantum black holes M = 10^{15} g, or solar type black holes, M = 10^{33} g, into (6) the dominant term becomes Λ^{-3} since for the purpose of this calculations Λ has been chosen to be equal to 10^{-54} cm^{-2}. Finally the temperature of such a black hole will become:

$$T=\frac{\hbar c}{4\pi\, \mathrm{k_B}}\left[18^{-2/3}\frac{1}{\sqrt{\Lambda}}\right]^{-1}=0.550\left[\frac{\hbar c}{k_B}\right]\sqrt{\Lambda} \qquad (7)$$

Thus we see that the temperature of such a black hole is independent of it's mass and depends on the square root of the cosmological constant Λ, as well as $\hbar$, c and $\mathrm{k_B}$.

2 Temperature Calculations

To calculate the maximum possible temperature of such a black hole we refer to the work of Sivaram and Sabbata [1]. The authors give an expression for the maximum possible value of the cosmological constant in the early universe as follows:

$$\Lambda=\frac{c^3}{G\hbar}=10^{66}\,cm^{-2} \qquad (8)$$

Substituting in (7) $\Lambda=10^{66}\ and\ 10^{-54}\,cm^{-2}$ into (7) [1], [2] we obtain the maximum and minimum temperatures of our black hole in De-Sitter space. So we find:

$$T_{Max}=1.25\times10^{32}\ K$$

(9)

$$T_{Min}=1.25\times10^{-28}\,K$$

It is easy to recognize that the first temperature is the so called Planck's temperature and the second the lowest possible temperature of a black body in the universe.

3 Radius of the Universe Calculation

Assuming thermodynamic equilibrium between the black hole and a radiation-dominated universe, where the temperature changes rapidly according to the law

$$T(t) = \frac{A_0}{R(t)} \tag{10}$$

and using the initial condition that t = t(Planck) the radius of the universe is R(universe) = L(Planck) we can obtain the following relation:

$$R = \frac{0.296k_B}{\hbar c\sqrt{\Lambda}} \tag{11}$$

This, of course, may still constitute only a crude approximation. Substituting for the the two different values of the cosmological constant we obtain the following values for the radius of the universe:

$$R = 1.29 \times 10^{-33}\, cm$$

$$(12)$$

$$R = 1.29 \times 10^{27}\, cm$$

From (12) we can see that for such a universe the minimum achivable value of its radius is just the Planck length at early times when Λ is extremely large, and the maximum achivable radius is almost the known radius of the universe today. ($R_U = 10^{28}$ cm)

4 Black Hole Temperature Connection to Cosmological Parameters

The first cosmological constant connection with the black hole temperature can be obtained if we use the following definition for the cosmological constant:[2]

$$|\Lambda| = 21\left[\frac{H_0}{c}\right]^2 \tag{13}$$

This makes the black hole temperature in (7) equal to:

$$T_{bh} = 2.52\left[\frac{\hbar H_0}{k_B}\right] \tag{14}$$

And one now sees how the original black hole temperature can be related to Hubble's constant H_o. But the gravitational energy of a typical elementary particle (hadron) was shown to be [4]:

$$E_{grav} = \frac{Gm^3}{\hbar}c = \hbar H_0 \tag{15}$$

and with the help of (15) (14), this becomes:

$$T_{bh} = 2.52\left[\frac{E_{grav}}{k_B}\right] = 2.52T_{grav}(hardon) \tag{16}$$

This may suggest that the temperature of such a black hole is related to the contribution of all the hadrons that the black hole might contain.

5 Black Hole Temperature and the Zeldovich Cosmological Constant

Another way of associating the black hole temperature to Hubble's constant can be obtained if we consider the Zeldovich definition of the cosmological constant which can be derived from the energy tensor of the quantum field theory of polarized vaccuum. Recall that Zeldovich has found that the value of the cosmological constant Λ is given by:[5]

$$\Lambda_Z = \frac{G^2 m^6}{\hbar^4} \tag{17}$$

where all the symbols have their usual meanings, and m is the mass of the elementary particle. Substituting in (7) we obtain:

$$T = 0.55\left[\frac{Gc}{\hbar k_B}\right]m^3 \tag{18}$$

If now use Weinberg's relation to replace the mass of the elementary particle m:[6]

$$m_\pi = \left[\frac{\hbar^2 H_0}{Gc}\right]^{1/3} \tag{19}$$

we obtain:

$$T_{bh} = 0.55\left[\frac{\hbar H_0}{k_B}\right] = 0.55 T_{grav}(hardon) \tag{20}$$

This again may suggest that when Weinberg's relation is used for the mass of the elementary particle, the black hole temperature obtained is still related to the contribution of all the hadrons that the black hole might contain but this time to a less degree than that suggested by (16)

6 Conclusions:

In this kind of De-Sitter universe the temperature a black hole was found to independent of its mass but proportional to the square root of the cosmological constant. Depending on the value of the cosmological constant the cosmological De-Sitter black hole can have two limiting temperature values. For the maximum value of the cosmological constant at early times when quantum effects dominate in the history of the universe the temperature of the black hole reaches the Planck temperature, and for the value of the cosmological constant today (present era) the temperature coincides with that of the lowest value of the black body radiation in the universe. Next the maximum and minimum values of the radius of such a universe were calculated based on the max and minimum values of the cosmological constant. Finally using Weinberg's definition for the mass of an elementary particle and secondly the Zeldovich's definition of the cosmological constant a connection of the temperature to Hubble's constant, and finally to the gravitational energy of hardons and of their temperature was found.

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REFERENCES:

[1] Sivaram, C., and De Sabbata V., Astrophysics and Space Science, 176, 145-148, 1991.
[2] Islam, J., N., Introduction to Mathematical Cosmology, Cambridge University Pree, 1992, p:74.
[3] As in ref [2], p:74.
[4] Sivaram, C., Astrophysics and Space Science, 219, 135-141, 1994.
[5] Zeldovich Ya., B., Soviet Physics Uspeki, vol # 11, 381-393, 1968.
[6] Wienberg, S., Gravitation and Cosmology, Wiley, New York,1972, p:619.