A Time-dependent Local Representation of Energy
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1. Abstract

Whereas globally energy propagates continuously as a wave, in this note we consider that energy can be represented locally by an exponential of time. Using such representation we are able to show that *Planck's Formula* for blackbody radiation is an *exact mathematical identity*. We are also able to explain the *photoelectric effect* without the *photon hypothesis* and derive an equation relating the photoelectric current to the intensity and frequency of radiation. This equation conforms well with graphical characteristics of the *actual* experimental data. Moreover, this representation of energy explains the *quantum hypothesis,* provides a simple intuitive explanation of the *double-slit experiment* and permits a definition of the *temperature of radiation*. It also explains the physical meaning of Planck's constant *h* and why it exists.

2. Introduction

We have proven elsewhere the following mathematical equivalence,

$$E(t) = E_0 e^{\gamma\, t} \text{ if and only if } E_0 = \frac{\eta\, \nu}{e^{\eta\, \nu / \kappa\, \mathcal{T}} - 1} \tag{1}$$

where *E(t)* is integrable, $\eta = \int_0^\tau E(u)\,du$, $\mathcal{T} = \left(\frac{1}{\kappa}\right)\frac{\eta}{\tau}$ and κ is a scalar constant.

We have also shown that for any integrable function *E(t)*,

$$\lim_{\tau \to 0} \frac{\eta\, \nu}{e^{\eta\, \nu / \kappa\, \mathcal{T}} - 1} = E_0 \text{ , or that } E_0 \approx \frac{\eta\, \nu}{e^{\eta\, \nu / \kappa\, \mathcal{T}} - 1} \tag{2}$$

Planck's Formula for blackbody radiation states that

$$E = \frac{h\nu}{e^{h\nu/kT} - 1} \text{ ,} \tag{3}$$

where E is energy, ν is frequency, T is temperature, h is Planck's constant and k is Boltzmann's constant. It is well known that this formula fits the experimental data remarkably well. Note that *Planck's Formula* **(3)** has the exact mathematical form to **(1)**. This suggests that energy can be represented by an exponential. Or, if we were to consider theoretical and practical limitations to experimental data, then we can assume that $E(t)$ is only an integrable function and *Planck's Formula* would then be the 'best fit possible'. So **(3)** can be compared to **(2)** in any event**.** But to avoid limits and approximations, however, and to have a mathematical formalism that deals with exact identities, we will consider that *Planck's Formula* is an exact relationship. Thus we argue, by **(1),** energy can be represented locally by an exponential of time. We have,

Time-dependent Local Representation of Energy: With radioactive decay we know the 'source' is 'cooling' down exponentially. Similarly with radiation, we assume the 'sensor' is 'heating' up exponentially. We adapt the view that while globally energy propagates continuously as a wave, locally energy interacts discretely in increments of ΔE . We have the following representations of energy.

1. *Globally, radiation energy from a 'source' propagates continuously as a wave.*
2. *Locally, radiation energy at the 'sensor' is given by an exponential function of time,*
   $E(t) = E_0 e^{\gamma\, t}$, *where E_0 is the intensity and ν is the frequency (rate) of radiation.*
3. *At local equilibrium the average energy locally at a point is equal to the average energy globally per degree of freedom.*
4. *When local equilibrium is attained, the function $E(t)$ 'collapses' to E_0 (see figure 1) and a discrete amount of energy ΔE is manifested (absorbed or emitted).*

Thus locally at a point of interaction, and *before* 'local equilibrium' is attained (when the 'average energy locally' at a point equals to the 'average energy globally' per degree of freedom) energy can be represented by,

$$E(t) = E_0 e^{v\,t} \qquad (4)$$

where E_0 is the intensity of radiation and v is the frequency of radiation. Equilibrium necessitates that a *minimal* amount of energy accumulate in order that energy be manifested *(this explains the existence of Planck's constant h)*. This minimal amount of locally accumulated energy before energy can be manifested we claim is h, Planck's constant (note that the units of h match this formulation). When 'local equilibrium' is attained, energy is manifested (absorbed or emitted) in increments of $\Delta E = E(t) - E(t_0)$. Thus, when 'local equilibrium' is attained (and so an amount of energy ΔE is manifested), the above exponential representation **(4)** 'collapses' and resets again to E_0, since $E(t) - \Delta E = E_0$ *(see figures 1 and 2)*.

3. Planck's Formula

For any integrable function *E(t)*, we have the following **mathematical identity.**

$$\eta = \int_0^{\eta / E_{av}} E(u)du, \ \text{ where } \eta = \int_0^{\tau} E(u)du \text{ and } E_{av} = \frac{\eta}{\tau} \qquad (5)$$

For a 'source' of radiation with temperature T, using **(4)** we can represent energy locally by $E(t) = E_0 e^{v\,t}$, where E_0 is the intensity of radiation and v is the frequency of radiation. Since the 'average energy of the source' per degree of freedom is kT, at 'local equilibrium' we have $E_{av} = kT$ and $\eta = h$ (the associated accumulation of energy) from above. Using these values in the mathematical identity **(5)**, we have

$$h = \int_0^{h/kT} E_0 e^{v\,u}du = \frac{E_0}{v}\left[e^{hv/kT} - 1 \right] \qquad (6)$$

and from **(6)**, we get *Planck's Formula*,

$$E_0 = \frac{hv}{e^{hv/kT} - 1} \qquad (7)$$

It is also mathematically true that

$$E(t) = E_0 e^{rt} \ \textit{ if and only if } \ E_0 = \frac{\Delta E}{e^{\Delta E / E_{av}} - 1} \qquad (8)$$

and that,

$$E_0 = \frac{\Delta E}{e^{\Delta E / E_{av}} - 1} \ \textit{ is independent of } \ \Delta E \text{ and } \Delta t \qquad (9)$$

Since **(5)** above is an *exact mathematical identity*, we have the following interesting conclusion:

***Planck's Formula* is an *exact mathematical identity* that describes the *interaction of energy (e.g. measurement)*. That is to say, from (8) above, the mathematical relationship between the (instantaneous) energy** E_0 **at the 'sensor', the energy absorbed by the 'sensor'** ΔE **, and the average energy at the 'sensor'** $\overline{E}$ **.**

Note further that since by **(9)** above $E_0 = \dfrac{\Delta E}{e^{\Delta E / E_{av}} - 1}$ is *independent* of ΔE, this formula can always be

reduced to *Planck's Formula*, with $\Delta E = h\nu$ and $\bar{E} = kT$. This would then explain why *Planck's Formula* is such a remarkably exact fit to the experimental data.

From the above results, we can also conclude that the time required for an accumulation of energy h to occur is given by,

$$\tau = \frac{h}{kT} \tag{10}$$

4. The Quantum Hypothesis
The *Quantum Hypothesis* simply states that energy is manifested (absorbed or emitted) in discrete units of size $h\nu$:

$$\Delta E = nh\nu \tag{11}$$

where n is an integer, h is Planck's constant and ν is the frequency of radiation.

The *Quantum Hypothesis* was used by Planck and Einstein to derive *Planck's Formula* and explain the *Photoelectric Effect*. Using the representation of energy described above in section **2**, we can mathematically explain the *Quantum Hypothesis*:

For any exponential function, we have the following **mathematical equivalence**:

$$E(t) = E_0 e^{\nu t} \quad \textbf{if and only if} \quad \Delta E = \eta \nu \tag{12}$$

$$\text{where } \Delta E = E(t) - E(t_0) \quad \text{and} \quad \eta = \int_{t_0}^{t} E(u)du$$

From the local representation of energy **(4)**, we have $E(t) = E_0 e^{\nu t}$, where E_0 is the intensity of radiation and ν is the frequency of radiation. For energy to be manifested (absorbed or emitted) we need to have 'local equilibrium' and therefore some minimal amount of accumulated energy. We take this minimal amount of accumulated energy to be Planck's constant h. From **(12)**, the manifested energy ΔE will then be,

$$\Delta E = h\nu \tag{13}$$

Since once this amount of energy is manifested the exponential 'collapses' (*see figures 1 and 2*), we get

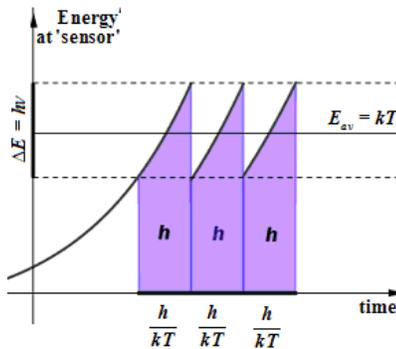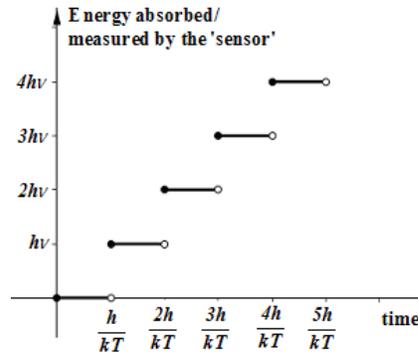$$\Delta E = nh\nu \tag{14}$$

figure 1

figure 2

*Note that the graph E vs. t in figure 2 is a **linear step function**.*

5. The Photoelectric Effect

Photoelectric emission is characterized by the following experimental facts:
1. For a given metal surface and frequency of incident radiation, the rate at which photoelectrons are emitted (the photoelectric current) is directly proportional to the intensity of the incident light.
2. The energy of the emitted photoelectron is independent of the intensity of the incident light but depends on the frequency of the incident light.
3. For a given metal, there exists a certain minimum frequency of incident radiation below which no photoelectrons are emitted. This frequency is called the threshold frequency. *(can be disputed)*
4. The time lag between the incidence of radiation and the emission of photoelectrons is very small, less than 10^{-9} second.

Explanation of the Photoelectric Effect without the Photon Hypothesis: Let v be the frequency (rate) of radiation of an incident light on a metal surface and let α be the frequency (rate) of absorption of this radiation by the metal surface. The combined rate locally at the surface will then be $v - \alpha$ and using the representation of energy in section **2**, the radiation energy at the surface can be represented by $E(t) = E_0 e^{(v - \alpha)t}$, where E_0 is the intensity of radiation of the incident light. If we let P be the 'accumulation of energy' locally at the surface over a 'time pulse' τ, then by **(12)** we'll have that,

$$\Delta E = P(v - \alpha) \tag{15}$$

If as before we let Planck's constant h be the 'accumulation of energy' for an electron, the number of electrons n_e over the 'pulse of time' τ will then be $n_e = \dfrac{P}{h}$ and the energy of an electron ΔE_e will be given by

$$\Delta E_e = \frac{\Delta E}{n_e} = h(v - \alpha) \tag{16}$$

Since $P = \displaystyle\int_0^\tau E_0 e^{(v-\alpha)u} du = E_0 \left[\dfrac{e^{(v-\alpha)\tau} - 1}{(v-\alpha)} \right]$, we can calculate the photoelectric current I to be

$$I = \frac{n_e}{\tau} = \frac{P}{h\tau} = E_0 \left[\frac{e^{(v-\alpha)\tau} - 1}{h(v-\alpha)\tau} \right] \tag{17}$$

The 'absorption rate' α is a characteristic of the metal surface, while the 'pulse of time' τ is assumed to be constant for fixed experimental conditions. The quantity $\left[\dfrac{e^{(v-\alpha)\tau} - 1}{h(v-\alpha)\tau} \right]$ would then be *constant*.

Combining these and using **(16)** and **(17)** we have *The Photoelectric Effect:*
1) For incident light of fixed frequency v and fixed metal surface, the photoelectric current I is proportional to the intensity E_0 of the incident light.
2) The energy ΔE_e of a photoelectron depends only on the frequency v and not on the intensity E_0 of the incident light. It is given by the equation $\Delta E_e = h(v - \alpha)$ where h is Planck's constant and the 'absorption rate' α is a property of the metal surface.
3) If ΔE_e is taken to be the kinetic energy of a photoelectron, then for incident light with frequency v less than the 'threshold frequency' α the kinetic energy of a photoelectron would be negative and so there will be no photoelectric current.
4) The photoelectric current is almost instantaneous ($< 10^{-9}$ sec.), since for a single photoelectron we have that $\Delta t = \dfrac{h}{kT} < 10^{-9}$ sec. by equation **(10)** above.

4

Preliminary experimental graphical analysis:
Many experiments since the classic 1916 experiments of Millikan have shown that there is photoelectric current even for frequencies below the threshold, contrary to the accepted explanation by Einstein. In fact, the original experimental data of Millikan show an asymptotic behavior of the (current) vs (energy) curves along the energy axis with no clear 'threshold frequency'. The photoelectric equations **(16)** and **(17)** we derived above agree with these experimental anomalies, however.

In an article Richard Keesing of York University, UK , states,

> *I noticed that a reverse photo-current existed ...   and try as I might I could not get rid of it.*
> *...*
> *My first disquieting observation with the new tube was that the ιν curves had high energy tails on them and always approached the voltage axis asymptotically. I had been brought up to believe that the current would show a well defined cut off, however my curves just refused to do so.*
> *...*
> *Several years later I was demonstrating in our first year lab here and found that the apparatus we had for measuring Planck's constant had similar problems.*
> *...*
> *After considerable soul searching it suddenly occurred on me that there was something wrong with the theory of the photoelectric effect ...*

In the same article, taking the original experimental data from the 1916 experiments by Millikan, Prof. Keesing plots the following graphs,
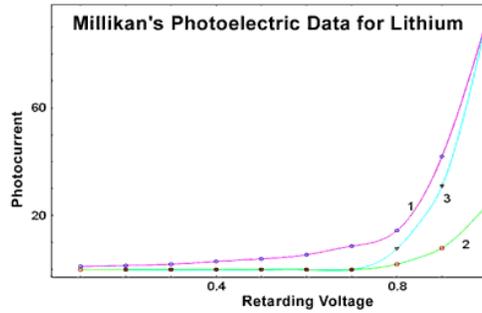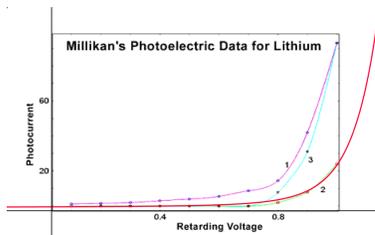

figure 3

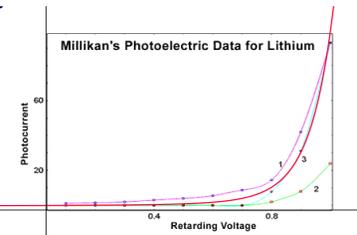We analyze the asymptotic behavior of equation **(17)** by using a function of the same form as **(17)**.
Note: we use d to account for slight shifts of experimental graphs made for clarity

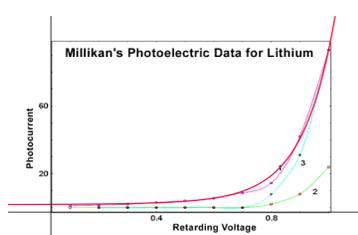$$f(x) = \frac{A\left(e^{b(x-c)} - 1\right)}{x - c} + d \tag{18}$$

The following graphs match the above experimental data to various graphs (in red) of equation **(18)**

A=0.13 b=1.98 c=5.95 d=0,07
figure 4

A=0.09 b=2.07 c=4.88 d=0.09
figure 5

A=0.05 b=1.41 c=3.04 d=0.18
figure 6

6. The *double-slit* Experiment:

The results derived in this paper using the representation of energy described in section **2** suggest that while the 'measurement' of energy is made in discrete units (*equal size sips*), the 'propagation' of energy can be continuous. This view explains the *wave-particle duality* and the *measurement problem* of Quantum Mechanics. This view also helps explain the *double-slit experiment* below:

The 'light burst' at the detection screen (the 'sensor') in a *double-slit experiment* may not signify the arrival of "the" electron emitted from the 'source' and going through one or the other of the slits as a particle then strikes the screen. Rather, what we have is an *'event'* of a light burst by some atom on the detection screen, having absorbed enough radiant energy to cause it to 'pop' (much like popcorn at seemingly random manner once a seed has absorbed enough heat energy). The parts of the detection screen that are 'illuminated' more frequently by radiation energy will of course show more 'popping'.

What is emitted when an electron is 'fired' is a burst of energy which propagates continuously as a wave and going through both slits 'illuminates' the detection screen in the typical interference pattern observed. For small bursts of energy, the 'illumination' of the screen in an interference pattern may not be manifested instantly, however. If locally the accumulation of energy has not reached the minimum threshold, energy will not be manifested in a discrete 'light burst' (in accordance with the above results). If the emitted bursts of energy are very small, the 'accumulation of energy' locally at various points of the detection screen will build up slowly, but more so in certain parts of the screen. Thus, the interference pattern will emerge only after a longer period of time, as more atoms absorb enough energy to cause them to 'pop' more frequently at those locations of the screen.

This explanation is a natural consequence of the representation of energy we have formulated in section **2**. It is also logically consistent and totally compatible with the 'probability distribution' interpretation of Quantum Mechanics.

7. The Temperature of Radiation

Temperature is typically defined as a thermodynamic property of large systems consisting of many particles. It is the average kinetic energy of the system. Such definition plainly does not apply to radiation, and so radiation is said not to 'have temperature'. The effects of radiation can be felt as temperature, but radiation does not have temperature. This is rather odd. We adopt the view instead that temperature is as much a 'physical property' as a theoretical concept. And though temperature cannot be defined for radiation in the sense of thermodynamics, nonetheless temperature exists for radiation. Following the same ideas we applied above regarding energy, we are able to define a quantity that behaves like the *'temperature of radiation'*.

Consider some incident radiation. Consider that locally at a point this energy is given by an integrable function of time $E(t)$. We can define the *temperature of radiation* as $\mathcal{T} = \left(\dfrac{1}{\kappa}\right)\dfrac{\eta}{\tau}$,

where $\eta = \displaystyle\int_{0}^{\tau} E(u)\,du$ and κ is an arbitrary constant used for purposes of scaling. Clearly, the faster the accumulation of energy builds up, the higher the temperature will be. Note further that the average energy at a point is given by $E_{av} = \kappa\,\mathcal{T}$. This conforms well with the average kinetic energy kT per degree of freedom for a large system. Note further that both such temperatures play the same role in **(1)** and in *Planck's Formula* **(3)**.

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