Is entropy related to the synchronization of the input/output power of a system of oscillators?

S. Halayka*

December 21, 2009

Abstract

The objective of this paper is to identify a way to relate entropy with the synchronization of the input/output power of a system of oscillators. This view is ultimately reconciled through an examination of the geometric differences that exist between 2D shell and 3D lattice oscillator arrangements. Keywords: generalizations of lattices, black holes, measures of information, entropy. MSC codes: 06B75, 83C57, 94A17.

1 Introduction

This paper's objective is to argue that entropy is related to the synchronization of energy flux (e.g., synchronization of interaction via streams of exchange quanta) involving a system of oscillators. Maximal synchronization is taken to occur when each and every oscillator both sends and receives its relative maximum amount of power in such a way that the power is equidistributed along all of the oscillator's respective degrees of freedom.

The system's n oscillators each have n degrees of freedom: one inherent degree, plus $(n-1)$ non-inherent degrees that associate it with the remaining $(n-1)$ oscillators. In total, the system has n^2 degrees. Each oscillator can receive or send (e.g., "input", "output") energy from either or both of the two opposing directions (e.g., "left", "right") that exist along each degree (e.g., in total, there are 4 ways to interact per degree).

In terms of quantum gravity, the Bekenstein entropy bound (e.g., via the Hawking area theorem and the generalized second law of thermodynamics) infers that the entropy S of a Schwarzschild black hole is proportional to the area of its event horizon (e.g., a 2D shell of radius R_s)

$$R_s = \frac{2GE}{c^4}, \tag{1}$$

$$A = 4\pi R_s^2 = \frac{16\pi G^2 E^2}{c^8}, \tag{2}$$

*shalayka@gmail.com

$$S \propto A, \tag{3}$$

and that the black hole's interior represents the smallest spherical region that this amount of entropy can be fit into.

With regard to the black hole's macroscopic state as observed from some distance far from the black hole's event horizon where gravitational time dilation is negligible (e.g., $r \gg R_s$, $d\tau/dt \approx 1$), the black hole appears to emit electromagnetic energy (e.g., Hawking radiation) that is characteristic of the Planck black body spectrum associated with the temperature

$$T = \frac{\hbar c^5}{k 8\pi G E}. \tag{4}$$

An intuitive plausibility argument for this phenomenon involving oscillator (e.g., particle-antiparticle) creation and annihilation is given in [1].

The first law of thermodynamics (e.g., the fundamental thermodynamic relation) defines the change in statistical thermodynamic entropy, the change in energy, and temperature to be

$$dS = \frac{dE}{T}, \tag{5}$$

$$\frac{dS}{dE} = \frac{1}{T}. \tag{6}$$

Given that $S \propto A$, the change in event horizon area with respect to the change in rest energy dA/dE can be used as a template to determine the black hole's statistical thermodynamic entropy

$$\frac{dA}{dE} = \frac{32\pi G^2 E}{c^8}, \tag{7}$$

$$\underbrace{A = \frac{dA}{dE}\frac{E}{2}}_{\text{template}} \implies \underbrace{S = \frac{dS}{dE}\frac{E}{2}}_{\text{solution}}, \tag{8}$$

$$S = \frac{1}{T}\frac{E}{2} = \frac{k 4\pi G E^2}{\hbar c^5} = \frac{kA}{4\ell_p^2}. \tag{9}$$

Dropping Boltzmann's constant k converts statistical thermodynamic entropy into Shannon entropy (e.g., information), which is given in terms of natural digits (e.g., "nats", base e number system)

$$S = \frac{A}{4\ell_p^2} = \pi \frac{R_s^2}{\ell_p^2}. \tag{10}$$

Here ℓ_p represents the Planck length

$$\ell_p = \sqrt{\frac{\hbar G}{c^3}} \approx 1.6 \times 10^{-35} \text{ metres}. \tag{11}$$

The holographic principle further expands on this Planck length quantization of the black hole. See [2–5].

2 Method

Rather than quantizing the black hole in terms of the Planck length, the remainder of this argument quantizes the black hole in terms of the Planck energy E_p instead. Consider a 2D shell consisting of n uniformly distributed Planck oscillators that are taken to possess maximal entropy

$$E_p = \sqrt{\frac{\hbar c^5}{G}} \approx 1.9 \times 10^9 \text{ Joules}, \tag{12}$$

$$n = \frac{E}{E_p}, \tag{13}$$

$$\frac{n}{A} = \frac{c^8}{E E_p 16\pi G^2}, \tag{14}$$

$$S = 4n^2\pi, \tag{15}$$

where equations (10) and (15) are equivalent as a matter of course. It is taken that the factor of 4 represents the previously mentioned number of ways that interaction can occur along each of the individual n^2 degrees (e.g., input/output, left/right). For lack of a simpler physically-related explanation, the factor of π is taken to represent rotation (e.g., radians, spin).

The Planck oscillators are related to the frequency

$$f_p = \frac{E_p}{h} \approx 2.9 \times 10^{42} \text{ Hertz}. \tag{16}$$

Each Planck oscillator is taken to oscillate along all of its n degrees equally over time, outputting the maximum amount of power possible to both the left and right of each degree

$$P' = \frac{E_p f_p}{n} = \frac{E_p^2}{nh} \approx \frac{5.7 \times 10^{51}}{n} \text{ Joules per second}. \tag{17}$$

The total output power of the entire set of Planck oscillators is $2n^2P'$, where one half is sent to the black hole's interior and one half is sent to its exterior. Barring Hawking radiation, the static solitary black hole is taken to neither gain nor lose rest energy over time, and so the same amount of output power sent to the interior/exterior must be simultaneously returned back to the Planck oscillators as input power (e.g., also $2n^2P'$). As a result, all 4 ways of interacting are simultaneously maximal for all n^2 degrees.

The following is a detailed breakdown of the input/output power *per oscillator*, where n is an *odd* number (e.g. see Fig. 1):

1. Each of the $(n-1)$ non-inherent degrees outputs P' to the interior (e.g., $(n-1)P'$ total), which is eventually reabsorbed by the other oscillators as input.

2. Each of the $(n-1)$ non-inherent degrees also outputs P' to the exterior (e.g., also $(n-1)P'$ total), which is taken to be lost.

3. The oscillator's inherent degree is taken to point toward/away from the centre of the black hole (e.g., not toward/away from one of the other oscillators), and so the power that it outputs to both the interior and exterior (e.g., $2P'$ total) is also taken to be lost (e.g., is not eventually reabsorbed by some other oscillator).

Where the required total input power per oscillator is $2nP'$, it appears that $(n-1)P'$ comes from the black hole itself (e.g., via the internal halves of the non-inherent degrees), and so $(n+1)P'$ must come from the vacuum.

Oppositely, where n is an *even* number (e.g. see Fig. 2), the oscillator's inherent degree points not only toward/away from the centre of the black hole, but also toward/away from some other oscillator (e.g. the two oscillators are antipodal). The power that the oscillator outputs to the exterior along its inherent degree (e.g., P' total) is still taken to be lost, but the power that it outputs to the interior along its inherent degree (e.g., P' total) is now taken to be eventually reabsorbed by some other oscillator. Where the required total input power per oscillator is $2nP'$, it appears that nP' comes from the black hole itself (e.g., via the internal halves of the inherent and non-inherent degrees), and so nP' must come from the vacuum.

It is taken that an oscillator can only oscillate along one degree at any given time (e.g., where a change of degree occurs f_p times per second), and so each of the oscillator's degrees must possess a unique positive integer phase number from within the closed interval $[1,\, n]$. In terms of the linear power distribution model given here, the black hole's exchange quanta are of energy-momentum $E_p = p_p c$ (e.g., inherent wavelength $c/f_p = \lambda_p$), and are spaced apart along the degrees by steplengths of $n\lambda_p = \lambda' = n 2\pi \ell_p$. Where i and j refer to the ith oscillator's jth degree (e.g., coincidentally, this jth degree relates the ith oscillator to the jth oscillator), a solution for phase is:

$$\theta(i,j) = \begin{cases} j - (n - i + 1) & \text{if} \quad j > n - i + 1, \\ j + i - 1 & \text{if} \quad j \leq n - i + 1. \end{cases} \tag{18}$$

By using this solution to plot $n \times n$ matrices[1] (e.g., related to Fig. 1 for $n = 5$, and Fig. 2 for $n = 4$)

$$\mathbf{M}(5) = \begin{bmatrix} \mathbf{1} & 2 & 3 & 4 & 5 \\ 2 & \mathbf{3} & 4 & 5 & 1 \\ 3 & 4 & \mathbf{5} & 1 & 2 \\ 4 & 5 & 1 & \mathbf{2} & 3 \\ 5 & 1 & 2 & 3 & \mathbf{4} \end{bmatrix}, \quad \mathbf{M}(4) = \begin{bmatrix} \mathbf{1} & 2 & 3 & 4 \\ 2 & \mathbf{3} & 4 & 1 \\ 3 & 4 & \mathbf{1} & 2 \\ 4 & 1 & 2 & \mathbf{3} \end{bmatrix}, \tag{19}$$

the following properties of the solution are illustrated:

1. As required, no two degrees (e.g., elements) of any one oscillator (e.g., row) share the same phase number.

[1]. . . of the Hankel variety, where the skew diagonals are constant. . .

2. With respect to non-inherent degrees (e.g., those *not* along the main diagonal $\mathbf{M}_{ii}$), counterbalancing output streams share the same phase number (e.g., the matrix is symmetric $\mathbf{M}_{ij} = \mathbf{M}_{ji}$).

3. With respect to inherent degrees (e.g., those along the main diagonal $\mathbf{M}_{ii}$), all phase numbers are unique where n is an odd number. Where n is an even number, only one half of the inherent degree phase numbers are unique because the oscillators are in antipodal pairs.

Perhaps this solution is loosely related to total colouring in graph theory [6, 7], where chromatic number is synonymous with phase number.

It is taken that the left and right of each degree are also slightly out of phase with respect to each other. To simplify, where left is synonymous with interior and right is synonymous with exterior:

$$\theta(i,j)_{\text{Interior}} = \theta(i,j), \tag{20}$$

$$\theta(i,j)_{\text{Exterior}} = \theta(i,j) - 1/2. \tag{21}$$

All said, the black hole would neither gain nor lose rest energy over time even though there would be astronomical amounts of energy continually being carried in and out of each and every one of its constituent oscillators (e.g., $4nP'$ Joules per second, per Planck oscillator). From this alone it seems that gravitation is doubly symmetric (e.g., spin-2) with respect to the ways of interacting per degree (e.g., simultaneous left-input, right-input, left-output, and right-output by default), and so its mechanism is the cycling (e.g., counterbalanced bidirectional emission) of energy (e.g., $4n^2P'$ Joules per second, per Schwarzschild black hole).[2] On the other hand, electromagnetism would be only singly symmetric (e.g., spin-1, only simultaneous left-output and right-output by default, non-counterbalanced bidirectional emission of energy).[3]

3 Results

To summarize the properties of the black hole:

1. Power output is matched by power input by default. Barring Hawking radiation, no net change in the system's rest energy occurs over time even though some of its exchange quanta (e.g., gravitons) are lost to the exterior. Gravitation is doubly symmetric.

2. In terms of Bachmann-Landau notation, the complexity of energy exchange for each degree is $\Theta(1)$.

[2] In classical terms, the gravitational time dilation that a body undergoes at a distance of $r > R_s$ is related to the black hole's rest energy by $d\tau/dt = \sqrt{1 - 2GE/(rc^4)}$. The factor of 2 represents the doubly symmetric nature of the gravitational field.

[3] As such, a field that is half symmetric (e.g., only left-output or right-output by default, non-counterbalanced unidirectional emission of energy) is spin-1/2, and a field that is involved with no such power input or output is spin-0 (e.g., a scalar field, non-directional).

3. The complexity of energy exchange for each Planck oscillator is $\Theta(n)$.

4. The complexity of energy exchange for each black hole is $\Theta(n^2)$. Entropy is proportional to the system's rest energy squared.

5. The n oscillators lie uniformly distributed along a 2D shell (e.g., event horizon). Measure the combined distance from any one oscillator to all of the other $(n-1)$ oscillators. This combined value is roughly the same for all oscillators. As well, none of the n oscillators block any of the other $(n-1)$ oscillators from directly exchanging energy with any of the remaining $(n-2)$ oscillators. That is, entropy can be maximal only where blocking does not occur.

On the other hand, consider the low entropy of a cube of weakly self-gravitating sodium chloride (e.g., Na^+ and Cl^- ions, table salt). As temperature reduces toward absolute zero and input/output power reduces toward the minimum (e.g., see: zero-point energy), the ions align to form a 3D lattice of uniformly spaced points:

1. The primary form of energy exchange in this case is light, where power output is not matched by power input by default. Where $T > 0$ Kelvin always, a net negative change in the system's rest energy occurs over time as some of its exchange quanta (e.g., photons) are lost to the exterior. Electromagnetism is singly symmetric.

2. The complexity of energy exchange for each degree is $\Theta(1)$.

3. The complexity of energy exchange for each ion is $\Theta(1)$. Ignoring inherent degrees altogether, each ion directly exchanges energy *primarily* with up to only 6 nearest neighbours along the lattice, regardless of the total number of ions within the entire system.

4. The complexity of energy exchange for each system is $\Theta(n)$. Entropy is proportional to the system's rest energy.

5. The n ions lie uniformly distributed along a 3D lattice. Measure the combined distance from any one ion to all of the other $(n-1)$ ions. This combined value varies from ion to ion. As well, some of the n ions block some of the other $(n-1)$ ions from directly exchanging energy with some of the remaining $(n-2)$ ions. That is, entropy cannot be maximal where blocking occurs.

4 Discussion

To generalize, where i and j refer to the ith oscillator's jth degree, the entropy of a system of n oscillators is taken to be

$$P'(i) = \frac{E(i)^2}{nh}, \tag{22}$$

$$\underbrace{f(u,v) = \frac{\min(u,v)}{\max(u,v)}}_{\text{normalized ratio}}, \tag{23}$$

$$\begin{aligned}\gamma(i,j) = {} & f(P^{\text{In}}_{\text{Left}}(i,j), P'(i)) + f(P^{\text{In}}_{\text{Right}}(i,j), P'(i)) \\ & + f(P^{\text{Out}}_{\text{Left}}(i,j), P'(i)) + f(P^{\text{Out}}_{\text{Right}}(i,j), P'(i)),\end{aligned} \tag{24}$$

$$S = \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma(i,j)\pi. \tag{25}$$

This entropy calculation accounts for various types of systems:

1. $E(i) \neq E_p$. The oscillators need not be Planck oscillators.
2. $P^{\text{Both}}_{\text{Both}}(i,j) \neq P'(i)$. With regard to equation (23), entropy contribution reduces as $P^{\text{Both}}_{\text{Both}}(i,j)$ become increasingly smaller *or* larger than $P'(i)$ (e.g., $P'(i)$ is an equilibrium point).
3. $E(i) \neq$ *constant*, as is with chemical compounds (e.g., sodium chloride).

It is important to note that even if this linear power distribution model is incorrect (e.g., $P'(i) \neq E(i)^2/(nh)$), the remaining principles represented by equations (23 - 25) (e.g., power synchronization/equilibrium, 4-way interaction, and spin) do not automatically become incorrect as well. That is, the importance of the role that $P'(i)$ plays is irrespective of $P'(i)$'s specific algebraic constitution.

It is also important to note that systemic phase synchronization, like that which is described by equation (18), can occur even where $P^{\text{Both}}_{\text{Both}}(i,j) = \textit{constant}$, $P^{\text{Both}}_{\text{Both}}(i,j) \ll P'(i)$. If phase synchronization were the source of entropy (contrary to the argument presented here), this would mean that a system of oscillators of arbitrarily large individual rest energy could still possess maximal entropy even where power input/output is arbitrarily close to zero. It seems counterintuitive to ascribe maximal entropy to a relatively non-interacting system such as this (e.g., relatively non-interacting in comparison to the possibility represented by $P^{\text{Both}}_{\text{Both}}(i,j) = P'(i)$ anyway), and so power synchronization is taken to be the source of entropy in this argument, not phase synchronization.

It seems that the argument presented here is in dispute with entropy models that take maximal entropy to be $S = A/\ell_p^2$, not $S = A/(4\ell_p^2)$ (e.g., see page 294 in [4]). Applying the concepts argued for here, this means that $S = 16n^2\pi$ and so the maximum entropy contribution per degree would be $16\pi = 8 \times 2\pi$, not $4\pi = 2 \times 2\pi$. This is undesirable, given that this implies that gravitation is a spin-8 interaction, even though it is widely considered to be spin-2. To be fair however, this discrepancy could also simply mean that the spin-related interpretation of the factor of π argued for here is incorrect.

Thanks to T. Nagoshi and V. Rolem for their inspiring work on shells.

References

[1] Schutz BF. A First Course in General Relativity. (1985) ISBN: 9780521277037

[2] 't Hooft G. Dimensional Reduction in Quantum Gravity. (1993, 2009) arXiv:gr-qc/9310026

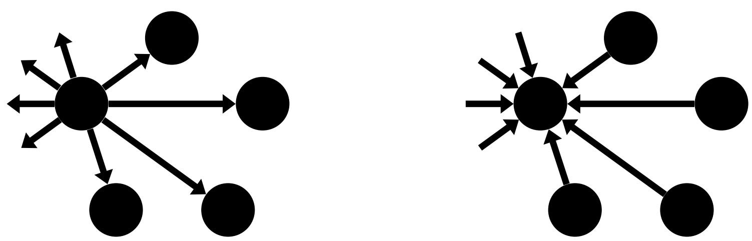
[3] 't Hooft G. The Holographic Principle. (2000) arXiv:hep-th/0003004v2

[4] Susskind L. The Black Hole War: My Battle with Stephen Hawking to Make the World Safe for Quantum Mechanics. (2009) ISBN: 9780316016407

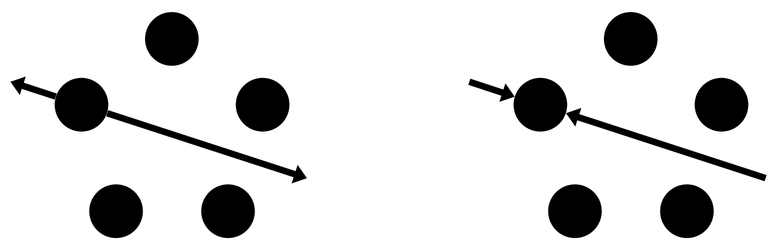
[5] Sidharth BG. The Planck Oscillators and the Black Hole Thermodynamics Correspondence. (2009) Int J Theor Phys 48: 2427-24331

[6] Behzad M. Graphs and their chromatic numbers. (1965) Ph.D. Thesis, Michigan State University

[7] Vizing VG. Some unsolved problems in graph theory. (1968) Russian Mathematical Surveys 23(6):125

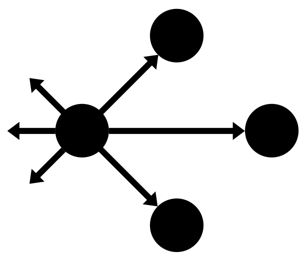


(a) Output along non-inherent degrees. (b) Input along non-inherent degrees.

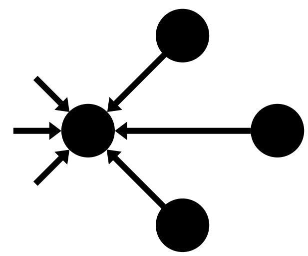


(c) Output along inherent degree. (d) Input along inherent degree.

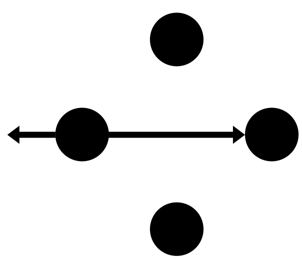
Figure 1: Diagram of a Schwarzschild black hole that illustrates input/output *per oscillator*, where $n = 5$ (e.g., an odd number). For ease of visualization, a 1D shell arrangement (e.g., a circle) has been used in place of a 2D shell arrangement. Regarding subfigures 1(b) and 1(d), the total number of inputs per oscillator is $2n = 10$, where $(n - 1) = 4$ come from the black hole and $(n+1) = 6$ come from the vacuum. It is interesting to note that the black hole's total number of synchronized inputs/outputs (e.g., $S/\pi = 4n^2 = 100$) remains the same for both 1D and 2D shell arrangements. Entropy is independent of the dimension of the shell. Lattice arrangements do not exhibit this property, in that there are 4 nearest neighbours in 2D, 6 in 3D, etc. Entropy is proportional to the dimension of the lattice.



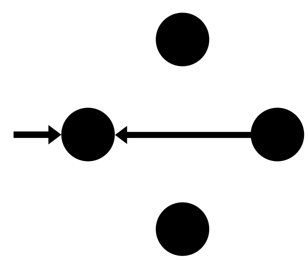
(a) Output along non-inherent degrees.



(b) Input along non-inherent degrees.



(c) Output along inherent degree.



(d) Input along inherent degree.

Figure 2: Diagram of a Schwarzschild black hole that illustrates input/output *per oscillator*, where $n = 4$ (e.g., an even number). It is important to note that even though the horizontal arrows from subfigures 2(a) and 2(b) are duplicated by subfigures 2(c) and 2(d), these two identical sets of arrows represent two different degrees (e.g., one non-inherent, one inherent). Regarding subfigures 2(b) and 2(d), the total number of inputs per oscillator is $2n = 8$, where $n = 4$ come from the black hole and $n = 4$ come from the vacuum.