Fractional dynamics and the Standard Model for particle physics

Ervin Goldfain

Photonics Center of Excellence, Welch Allyn inc., 4619 Jordan Road, P.O. Box 187

OptiSolve Consulting, 4422 Cleveland Road, Syracuse, N.Y. 13215

Abstract

Fractional dynamics is an attractive framework for understanding the complex phenomena that are likely to emerge beyond the energy range of the Standard Model for particle physics (SM). Using fractional dynamics and complex-scalar field theory as a baseline, our work explores how physics on the high-energy scale may help solve some of the open questions surrounding SM. Predictions are shown to be consistent with experimental results.

Key words: Renormalization Group flow; Fractional dynamics; Feigenbaum scaling; Standard Model

PACS numbers: 11.10.Hi; 11.10. Ef; 12.60. - i; 12.90. + b.

1. Introduction and motivation

As of 2006, predictions derived from the Standard Model of elementary particles (SM) — a body of knowledge discovered in the early 1970's — agrees with all the experiments that have been conducted to date. Nevertheless, the majority of particle theorists feel that SM is not a complete framework, but rather an "effective field theory" that needs to be extended by new physics at some higher energy scale reaching in the TeV region. The most cited reasons for this belief are: a) the recent discovery of neutrino oscillations and masses; b) SM does not include the contribution of gravity and gravitational corrections to both quantum field theory and renormalization group (RG) equations; c) SM does not fix the large number of free parameters that enter the theory (in particular the spectra of masses, gauge couplings and fermion mixing angles); d) SM has a gauge hierarchy problem, which requires fine-tuning; e) SM postulates that the origin of electroweak symmetry breaking is the Higgs mechanism, whose confirmation is sought in future accelerator experiments. The number and physical attributes of the Higgs boson are neither explained by SM nor fixed from first principles, f) SM does not clarify the origin of its underlying $SU(3)\times SU(2)\times U(1)$ gauge group and why quarks and leptons occur in certain representations of this group, g) SM does not explain why the weak interactions are chiral, that is, why only fermions with one handedness experience the force transmitted by the triplet of massive vector bosons W^{+}, W^{-}, Z^{0}.

Despite years of research on multiple fronts, there is currently no compelling and universally accepted resolution to the above-mentioned challenges. A large body of proposed extensions of SM exists, each of them attempting to resolve some unsatisfactory aspects of the theory while introducing new unknowns. Expanding on a series of recent contributions centered on RG, nonlinear dynamics, chaos and fractal geometry [3-4, 6-10, 12-14, 17-19], our work explores how the physics on the TeV regime may shed light onto some of the open questions surrounding SM.

The paper is organized as follows: section 2 surveys the motivation for fractional dynamics in the far ultraviolet region of field theory. The principle of local scale invariance is briefly introduced in section 4. Fractional dynamics of a "toy" model based on complex scalar fields is analyzed in section 5. Sections 6 to 8 discuss how critical behavior in continuous dimension acts as source of massive field theories and makes connection to SM data. Concluding remarks are presented in the last section. We emphasize from the outset the introductory nature of our work. As such, its content is not aimed to be either entirely rigorous or formally complete. Independent research efforts are required to confirm, develop or disprove these preliminary results.

2. Fractional dynamics and the far ultraviolet region of field theory

It is generally believed that quantum field theory breaks down near the so-called Cohen-Kaplan threshold of $\sim 100\ TeV$ as a result of exposure to large vacuum fluctuations and strong-gravitational effects. No convenient redefinition of observables is capable of turning off the dynamic contribution of these effects. For instance, it is known that the zero-point vacuum energy diverges quadratically in the presence of gravitation. Quantum field theory in Euclidean space-time discards the zero-point vacuum energy through the use of a normal time ordering procedure [5, 20]. Because vacuum energy is gravitating and couples to all other field energies present at the quantum level, cancellation of the zero-point term is no longer possible when gravitational effects are significant. Likewise, this strong coupling regime of the far ultraviolet region suggests that even asymptotically free theories such as QCD reverse their properties in response to arbitrarily large non-perturbative effects. In fact, complex dynamics of quark-gluon plasma is expected to arise near the so-called transition temperature [16].

The non-linearity of the underlying field theory combined with the far-from-equilibrium dynamics induced by highly unstable vacuum fluctuations are prone to lead to self-organized criticality [6]. Because dynamical instabilities can develop on long time-scales, the macroscopic description of phenomena in terms of conventional differential operators breaks down. This is, in essence, the main argument for using fractal operators in the far TeV region of field theory and for the passage from ordinary to fractional dynamics [14, 18-19, 21]. Since application of fractals in contemporary physics has become far ranging, the interest in fractional dynamics has grown at a steady pace in the last decade. There is now a broad range of applications of fractional dynamics in research areas where fractal attributes of underlying processes and the onset of long-range correlations demand the use of fractional calculus. These areas include, but are not limited to, wave propagation in complex and porous media, models of systems with chaotic and pseudo-chaotic dynamics, random walks with memory, colored noise and pattern formation, anomalous transport and Levy flights, studies of scaling phenomena and critical behavior, plasma physics, turbulence, quantum field theory, far-from-equilibrium statistical models, complex dynamics of traffic networks and so on (for a brief review of current applications, see [21-26]).

3. Conventions and assumptions

a) Einstein summation convention is applied throughout.

b) the Poincare index is denoted by $\mu = 0,1,2,3$.

c) we study a basic "toy" model containing a single pair of massive complex-scalar fields $\varphi(x), \varphi^{*}(x)$.

d) the analysis is carried out exclusively at the classical level. Suppression of quantum attributes and transition to classical behavior is the result of decoherence induced by steady exposure to large random fluctuations [27, 28].

e) following [22-26] we use in our work the left Caputo fractional derivative defined as

$$D^{\alpha}\varphi(x) \doteq \frac{1}{\Gamma(n-\alpha)}\int_{0}^{x}\frac{\varphi^{(n)}(s)}{(x-s)^{\alpha+1-n}}ds \qquad (1)$$

where $n-1<\alpha<n$ and $\varphi^{(n)}(s) \doteq {d^n\varphi(s)}/{ds^n}$. We note that, in addition to using (1), many studies based on fractional calculus often start from alternative operators such as Riemann-Liouville and Grunwald-Letnikov derivatives and integrals (see [30-33] for details).

f) space-time variables and fields are suitably normalized as dimensionless observables.

g) assuming that the field dynamics has low-level fractionality, we use the so-called ε-expansion to perform the transition from first order to Caputo derivatives of order $\alpha \doteq 1-\varepsilon$ according to the prescription [22]

$$
\begin{aligned}
D^{1-\varepsilon}\varphi(x) &= \partial\varphi(x) + \varepsilon D_1\varphi(x) \\
D_1\varphi(x) &\doteq \partial\varphi(0)\ln|x| + \gamma\partial\varphi(x) + \int_0^x \partial^2\varphi(s)\ln|x-s|\,ds
\end{aligned}
\qquad (2)
$$

4. Local scale invariance at the onset of fractional dynamics

The novel symmetry principle that underlies the onset of fractional dynamics in the TeV region is the *local scale invariance* of the theory [17]. There is a two-fold rationale for the onset of this symmetry, namely,

a) field dynamics is *scale-invariant*. This is equivalent to stating that, in dimensional regularization scheme, the outcome of the regularization procedure does not depend on the particular choice of $\varepsilon = 4-d$ [20].

b) by analogy with the definition of the Lipschitz-Hölder exponent and to ensure compliance with relativity [14, 17], we take the continuous dimension parameter ε to denote a *locally* defined function of space-time coordinates, $\varepsilon(x)$. In addition, we assume that $\varepsilon(x)$ may be expressed either as a *contravariant* $\varepsilon^i(x)$ or a *covariant* $\varepsilon_i(x)$ four-vector. This motivates us to formally extend (2) to

$$
\begin{aligned}
&D^{1-\varepsilon_\mu(x)}\varphi(x) = \partial^\mu\varphi(x) + \varepsilon^\mu(x)D_1^\mu\varphi(x) \\
&D_1^\mu\varphi(x) \doteq \partial^\mu\varphi(0)\ln|x| + \gamma\partial^\mu\varphi(x) + \int_0^x (\partial^2)^\mu\varphi(s)\ln|x-s|\,ds \\
&D_{1-\varepsilon_\mu(x)}\varphi(x) = \partial_\mu\varphi(x) + \varepsilon_\mu(x)D_{1,\mu}\varphi(x) \\
&D_{1,\mu}\varphi(x) \doteq \partial_\mu\varphi(0)\ln|x| + \gamma\partial_\mu\varphi(x) + \int_0^x (\partial^2)_\mu\varphi(s)\ln|x-s|\,ds
\end{aligned}
\tag{3}
$$

5. Fractional dynamics of the complex-scalar field

The goal of this section is to show that the principle of local scale invariance and the introduction of fractional dynamics lead to a mechanism of gauge boson-fermion unification that is fundamentally distinct from the mechanism advocated by supersymmetry.

The Lagrangian density of our model is

$$L \doteq \partial_\mu\varphi(x)\,\partial^\mu\varphi^*(x) - m^2\varphi^*(x)\varphi(x) \tag{4}$$

where m the mass of the field and x is a shorthand notation for (x^μ) or (x_μ). Using the framework of Caputo derivatives and ε-expansion [22], one obtains

$$L \doteq D_\mu^{1-\varepsilon(x)}\varphi(x)\, D^{\mu,1-\varepsilon(x)}\varphi^*(x) - m^2\varphi^*(x)\varphi(x) \tag{5}$$

where the locally defined infinitesimal dimension $\varepsilon(x)$ satisfies the condition $\varepsilon(x)\cdot x \ll 1$. Our aim is to show that, in contrast with conventional field theory embodied in (4), use of Caputo derivatives guarantees invariance under local gauge transformations *without* the explicit need for gauge fields and covariant operators. To this end, let us perform the local phase change

$$
\begin{aligned}
\varphi(x) &\to e^{i\alpha(x)}\varphi(x) \\
\varphi^*(x) &\to e^{-i\alpha(x)}\varphi^*(x)
\end{aligned}
\tag{6}
$$

Up to a first order approximation, Caputo derivatives transform as [22]

$$
\begin{aligned}
D_\mu^{1-\varepsilon(x)}\varphi(x) &\to D_\mu^{1-\varepsilon(x)}[e^{i\alpha(x)}\varphi(x)] = \partial_\mu[e^{i\alpha(x)}\varphi(x)] + \varepsilon(x)D_{1,\mu}^{1}[e^{i\alpha(x)}\varphi(x)] \\
D^{1-\varepsilon(x),\mu}\varphi^*(x) &\to D^{1-\varepsilon(x),\mu}[e^{-i\alpha(x)}\varphi^*(x)] = \partial^\mu[e^{-i\alpha(x)}\varphi^*(x)] + \varepsilon(x)D_1^{1,\mu}[e^{-i\alpha(x)}\varphi^*(x)]
\end{aligned}
\tag{7}
$$

where

$$
\begin{aligned}
D_{1,\mu}^{1}(x) &\doteq \ln|x|\partial_\mu[e^{i\alpha(0)}\varphi(0)] + \int_0^x \partial_\mu^2[e^{i\alpha(\lambda)}\varphi(\lambda)]\ln|x-\lambda|\,d\lambda + \gamma\partial_\mu[e^{i\alpha(x)}\varphi(x)] \\
D_1^{1,\mu}(x) &\doteq \ln|x|\partial^\mu[e^{-i\alpha(0)}\varphi^*(0)] + \int_0^x \partial^{2,\mu}[e^{-i\alpha(\lambda)}\varphi^*(\lambda)]\ln|x-\lambda|\,d\lambda + \gamma\partial^\mu[e^{-i\alpha(x)}\varphi^*(x)]
\end{aligned}
\tag{8}
$$

in which γ stands for the Euler constant and

$$
\partial_\mu[e^{i\alpha(x)}\varphi(x)] = e^{i\alpha(x)}[\partial_\mu + i\partial_\mu\alpha(x)]\varphi(x) \tag{9}
$$

Local gauge invariance of (5) is preserved if Caputo derivatives transform covariantly, that is

$$
\begin{aligned}
D_\mu^{1-\varepsilon(x)}[e^{i\alpha(x)}\varphi(x)] &= e^{i\alpha(x)}D_\mu^{1-\varepsilon(x)}\varphi(x) \\
D^{\mu,1-\varepsilon(x)}[e^{-i\alpha(x)}\varphi^*(x)] &= e^{-i\alpha(x)}D^{\mu,1-\varepsilon(x)}\varphi^*(x)
\end{aligned}
\tag{10}
$$

On account of (7) and (10), we arrive at the following set of conditions

$$
\boxed{ie^{i\alpha(x)}\varphi(x)\partial_\mu\alpha(x) = \varepsilon(x)\,\{D_{1\mu}^{1}[e^{i\alpha(x)}\varphi(x)] - e^{i\alpha(x)}D_{1\mu}^{1}\varphi(x)\}} \tag{11}
$$

$$\boxed{(-i)e^{-i\alpha(x)}\varphi^*(x)\,\partial_\mu\alpha(x) = \varepsilon(x)\,\{D_1^{1,\mu}[e^{-i\alpha(x)}\varphi^*(x)] - e^{-i\alpha(x)}D_1^{1,\mu}\varphi^*(x)\}}$$

The direct consequence of (11) is that gauge fields *are no longer required* in a field theory built on fractional dynamics. The compensating role of the vector bosons is played by the *continuous dimension parameter* $\varepsilon(x)$. This conclusion is consistent with previous studies and points to a novel unification mechanism of gauge boson and fermion fields, including classical gravitation. This mechanism is fundamentally different from the unification scheme postulated by supersymmetry and related quantum field models [20].

6. Emergence of massive field theories

It is known that, allowing elementary particles to have nonzero masses in quantum field theory violates local gauge and weak isospin symmetries imposed on the standard model lagrangian. The mechanism of so-called spontaneous symmetry breaking (SSB) posits that the vacuum itself acquires a non-zero charge distribution that leaves the Lagrangian invariant and generates both fermion and vector boson masses [5, 20]. In SM, massive fermions exist in both left-handed and right-handed states. The only Dirac field operators that yield a non-vanishing mass are bilinear products of fields having the form

$$m\overline{\psi}\psi = m(\overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R) \tag{12}$$

However, such mass terms mix right and left-handed spinors and are forbidden from the Lagrangian on account of violation of the weak isospin symmetry [5, 20]. Stated differently, since ψ_L represents a SU(2) doublet and ψ_R a SU(2) singlet, the product of the two is not a singlet, as it ought to be in order to preserve the weak isospin symmetry. An immediate question that arises from the previous section is whether or not SSB still exists in a field theory based on fractional dynamics. Stated differently, can mass terms be introduced in the Lagrangian without violating local and weak isospin symmetries?. To answer this question, we note that the first order Caputo operator may be defined either from the "left" or from the "right" and, in general, the effect produced by $D^{1-\varepsilon(x)}$ is not identical with the effect produced by $D^{1+\varepsilon(x)}$. It follows that the proper description of fractional differentiation requires a *doublet* of Caputo operators $\begin{pmatrix} D^{1-\varepsilon_L(x)} \\ D^{1+\varepsilon_R(x)} \end{pmatrix}$ and a *doublet* of scalars $\begin{pmatrix} \varepsilon_L(x) \\ \varepsilon_R(x) \end{pmatrix}$ with $\varepsilon_{L,R}(x)\cdot x \ll 1$. Therefore, mass terms that correspond to Dirac bilinears assume the form:

$$\boxed{\psi \to D^{-\varepsilon_L}\psi \ \text{ or } \ \psi \to D^{\varepsilon_R}\psi} \qquad \textbf{for singlets}$$

$$\boxed{\begin{pmatrix}\psi_1 & \psi_2\end{pmatrix} \to \begin{pmatrix} D^{-\varepsilon_L} \\ D^{+\varepsilon_R} \end{pmatrix}\begin{pmatrix}\psi_1 & \psi_2\end{pmatrix} \ \text{ and } \ \begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix} \to \begin{pmatrix} D^{-\varepsilon_L} & D^{+\varepsilon_R}\end{pmatrix}\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}} \qquad \textbf{for doublets} \tag{13}$$

It can be seen that these mass terms automatically preserve the weak isospin symmetry in a similar manner in which the Higgs scalar doublet works in the electroweak model [5, 20, 29].

7. Critical behavior in continuous dimension

The previous sections have shown that the concept of dimension takes on a key role in the far ultraviolet region of field theory. Here we elaborate on this conjecture by making connection to the philosophy of the Renormalization Group and critical behavior in continuous dimension [1, 2]. To streamline the derivation, we refer in what follows to the original Lagrangian (4). Let us start by adding a potential term to (4), that is

$$L \doteq \partial_\mu\varphi(x)\,\partial^\mu\varphi^*(x) - m^2\varphi^*(x)\varphi(x) + \lambda^2\,[\varphi^*(x)\varphi(x)]^2 \tag{14}$$

Here $g = \lambda^2$ represents the self-interaction strength of the field. According to the renormalization group, the so-called beta-function defines how g "flows" with the sliding energy scale μ, that is

$$\beta(g) \doteq \frac{d(g)}{dt} \tag{15}$$

where

$$t \doteq \ln\left(\frac{\mu}{\mu_0}\right) \tag{16}$$

for an arbitrary reference scale μ_0. In the basin of attraction of a critical point t_c the field correlation length scales as

$$\xi \approx \left|t - t_c\right|^{-\nu} \tag{17}$$

Here, critical exponent ν is given by [1, 2]

$$\nu^{-1} = -\left.\frac{\partial\beta}{\partial(g)}\right|_{g=g^*} \tag{18}$$

and g^* stands for a fixed point of the beta-function, $\beta(g^*) = 0$.

One can exploit the interchangeable roles played by the sliding scale t and the dimension parameter $d(x)$ as follows. Assume that d_c represents the critical dimension for which g flows into the fixed point g^*. The mass of the complex-scalar field is known to be inversely proportional to the divergent correlation length and vanishes identically at the fixed point [1, 2]

$$m[g^*(d_c), d_c] = 0 \tag{19}$$

In the basin of attraction of g^* the field develops mass according to the power law

$$m[g^*(d_c), d(x)-d_c] \approx |d(x)-d_c|^{\nu(d_c)} = |\varepsilon(x)|^{\nu(d_c)} \tag{20}$$

where

$$\nu^{-1}(d_c) = -\frac{\partial \beta}{\partial (g)}\bigg|_{g^*(d_c)} \tag{21}$$

We are led to conclude that, as the fixed point is asymptotically approached and the continuous space-time dimensionality collapses to $d(x) \to d_c = 4$, the complex-scalar field becomes massless, in agreement with conventional quantum field theory. Numerical analysis yields $\nu(d_c) = 0.5$ for $d_c = 4$ which is found to match well the value reported in the literature [1-2].

An important observation is now in order. Following the universal properties of the RG flow near the onset of chaos in low-dimensional maps, the dimensional control parameter $\varepsilon(x) = |d_c - d(x)|$ is expected to asymptotically approach the critical value $\varepsilon_\infty = 0$ according to the geometric progression [15]:

$$\varepsilon_n(x) - \varepsilon_\infty \approx a_n(x) \cdot \delta^{-n} \tag{22}$$

in which $n \gg 1$ is the index defining the number of iteration steps, δ stands for a scaling constant that is representative for the class of dynamical maps under consideration and $a_n(x)$ is a coefficient which becomes asymptotically independent of n and x, that is, $a_\infty = a$. Substituting (22) in (20) produces to the mass scaling series

$$\boxed{M_n[g^*(d_c), d_c - d_n(x)] \approx [a_n \delta^{-n}]^{1/2}} \tag{23}$$

We may go a step further and state that, given the generic link between the coupling flow and the corresponding flows of masses and fields in RG [5, 20, 29], similar scaling pattern develops for g_n and the underlying fields of the theory, η_n. We thus expect to obtain, for $n \gg 1$

$$\boxed{g_n - g^*(d_c) \approx \delta^{-\lambda(d_c)n}}$$
$$\boxed{\eta_n - \eta^*(d_c) \approx \delta^{-\zeta(d_c)n}} \quad (24)$$

where $\lambda(d_c)$ and $\zeta(d_c)$ represent two additional critical exponents dependent on d_c.

8. Universal scaling of fermion masses

Period-doubling bifurcations are defined by $n = 2^m$, with $m > 1$ [15]. Replacing in (23) yields the following mass series:

$$M_m(x) \approx \sqrt{a_{2^m}(x)} \cdot \delta^{-2^{m-1}} \quad (25)$$

where $\delta = 4.669...$ represents the Feigenbaum constant for the onset of chaos in unimodal maps. The ratio of two arbitrary masses is therefore

$$\frac{M_l(x)}{M_m(x)} \approx \sqrt{\frac{a_{2^l}(x)}{a_{2^m}(x)}} \cdot \frac{\delta^{-2^{l-1}}}{\delta^{-2^{m-1}}} \quad (26)$$

where $\lim \frac{a_l}{a_m} = 1$ as $l, m \to \infty$. Thus, for two sufficiently distant consecutive terms in the mass series, the dependence of $a_n(x)$ on the space-time variable may be suppressed and we obtain

$$\frac{M_l}{M_{l+1}} \approx \sqrt{\frac{a_{2^l}}{a_{2^{l+1}}}} \cdot \delta^{2^{l-1}} = \sqrt{a_{2^l,2^{l+1}}} \cdot \delta^{2^{l-1}} \quad (27)$$

It is important to emphasize that (27) provides only a *first-order approximation* considering that a) (27) is less accurate if the iteration index is not large enough, that is, if $l \approx O(1)$, b) there is a fair amount of uncertainty involved in determining the quark mass spectrum [11]. Numerical results derived from (27) are displayed in the table below. This table contains a side-by-side comparison of estimated versus actual mass ratios for charged leptons and quarks and a similar comparison of gauge coupling ratios. All masses and couplings are evaluated at the energy scale given by the top quark mass. Quark masses are averaged using the most recent reports issued by the Particle Data Group [11]. Specifically, m_u = 2.12 MeV; m_d = 4.22 MeV; m_s = 80.9 MeV; m_c = 630 MeV; m_b = 2847 MeV; m_t = 170,800 MeV.

Scaling ratio	2^{l-1}	Experimental value	Estimated value
m_u/m_c	4	3.365×10^{-3}	2.104×10^{-3}
m_c/m_t	4	3.689×10^{-3}	2.104×10^{-3}
m_d/m_s	2	0.052	0.046
m_s/m_b	2	0.028	0.046
m_e/m_μ	4	4.745×10^{-3}	2.104×10^{-3}
m_μ/m_τ	2	0.061	0.046
$(\alpha_{EM}/\alpha_W)^2$	2	0.045	0.046
$(\alpha_{EM}/\alpha_s)^2$	4	2.368×10^{-3}	2.104×10^{-3}

Table 1: Fermion masses and coupling ratios

The scaling sequence of charged leptons and quarks may be graphically summarized with the help of the following diagrams:

$$\overbrace{d \quad s}^{\delta^2} \qquad \overbrace{s \quad b}^{\delta^2}$$

$$\overbrace{u \quad c}^{\delta^4} \qquad \overbrace{c \quad t}^{\delta^4}$$

$$\overbrace{e \quad \mu}^{\delta^4} \qquad \overbrace{\mu \quad \tau}^{\delta^2}$$

$$\overbrace{\alpha_{EM}^2 \qquad \alpha_W^2}^{\delta^2} \qquad \overbrace{\alpha_W^2 \qquad \alpha_s^2}^{\delta^2}$$

Based on the above scheme, one may infer that neutrinos masses are arranged according to the possible pattern:

$$\overbrace{\nu_e \qquad \nu_\mu}^{\delta^{16}} \qquad \overbrace{\nu_\mu \qquad \nu_\tau}^{\delta^4}$$

It is also instructive to note that quarks and charged leptons follow a different period doubling path. To this end, let us organize the charged lepton and quark masses in a collection of triplets, that is

$$M_l \doteq \begin{bmatrix} m_e & m_\mu & m_\tau \end{bmatrix} \qquad M_q \doteq \begin{bmatrix} m_u & m_d \\ m_c & m_s \\ m_t & m_b \end{bmatrix} \tag{28}$$

It can be seen that the mass scaling for adjacent quarks stays constant within either one of the triplets (u,c,t) or (d,s,b), whereas the mass scaling for charged leptons varies as a geometric series in δ_2^2 within the triplet (e,μ,τ). This finding points out toward a symmetry breaking mechanism that segregates lepton and quark phases in the process of cooling from the far ultraviolet region of field theory to the low-energy region of SM.

9. Concluding remarks

We have argued that fractional dynamics represents an analytic framework suitable for the description of physical phenomena that are likely to arise in the TeV realm of particle physics. Unless conventional quantum field theory, fractional dynamics describes far-from-equilibrium statistical processes that give raise to manifest scale invariance, non-local correlations and extensive symmetry breaking. Using fractional dynamics and the benchmark example of complex-scalar field theory, we have explored the potential spectrum of phenomena that is likely to emerge beyond the energy range of SM. Based on this framework, we have shown that, near the ultraviolet boundary of field theory, a) gauge bosons and fermions become unified through a fundamentally different mechanism than the one advocated by supersymmtry; b) SSB and the emergence of massive field theories occur as a result of critical behavior in continuous dimension; c) particles develop a family structure that is tied to the universal transition to chaos in unimodal maps. First order predictions were found to match reasonably well current experimental data. However, as pointed out in section 1, our goal is not to formulate a comprehensive solution to the host of open challenges surrounding SM. Concurrent research efforts are needed to confirm or falsify these preliminary findings. In particular, the long-awaited operation of the Large Hadron Collider and similar high-energy accelerator sites should soon produce experimental evidence that backs or disproves our model.

10. References

[1] Ballhausen H., Berges J., Wetterich C. Critical Phenomena in Continuous Dimension. Available from http://arxiv:hep-th/0310213.

[2] Ballhausen H. Renormalization Group Flow Equations and Critical Phenomena in Continuous Dimension and at Finite Temperature, Doctoral Thesis, Faculty of Physics and Astronomy, Ruprecht-Karls-University Heidelberg.

[3] Morozov A., Niemi A. J. Can Renormalization Group Flow End in a Big Mess? Available from http://arxiv:hep-th/0304178.

[4] Damgaard P. H., Thorleifsson G. Chaotic renormalization-group trajectories. Phys. Rev. A 1991; 44: 2378-2741.

[5] See e.g. Ryder L.H. Quantum Field Theory, Cambridge University Press; 1996.

[6] El Naschie M. S. The VAK of vacuum fluctuation, spontaneous self-organization and complexity interpretation of high-energy particle physics and the mass spectrum. Chaos, Solitons and Fractals 2003;18: 401-420.

[7] El Naschie M. S. Elementary prerequisites for E-infinity. Chaos, Solitons and Fractals 2006;30: 579-605.

[8] El Naschie M.S. A review of E-infinity theory and the mass spectrum of high-energy particle physics. Chaos, Solitons and Fractals 2004, 19:209-36.

[9] El Naschie M.S. On the universality class of all universality classes and E-infinity space-time physics. Chaos, Solitons and Fractals (2006). doi:10.1016/j.chaos.2006.08.017

[10] El Naschie M. S. "Feigenbaum scenario for turbulence and Cantorian E-infinity theory of high-energy particle physics", Chaos, Solitons and Fractals (2006). Available from doi:10.1016/j.chaos.2006.08.014

[11] See e.g. Particle Data Group http://pdg.lbl.gov/2005/reviews/quarks_q000.pdf.

[12] Goldfain E. Derivation of lepton masses from the chaotic regime of the linear σ-model. Chaos, Solitons and Fractals 2002; 14: 1331-1340.

[13] Goldfain E. Feigenbaum scaling, Cantorian space-time and the hierarchical structure of standard model parameters. Chaos, Solitons and Fractals 2006; 30:324-331.

[14] Goldfain E. Fractional dynamics and the TeV regime of field theory. Available from doi:10.1016/j.cnsns.2006.06.001

[15] McCauley J. L. Chaos, Dynamics and Fractals; An Algorithmic Approach to Deterministic Chaos. Cambridge Univ. Press, 1993.

[16] Cheng M. *et al.* Transition temperature in QCD. Phys. Rev. D 2006; 74: 054507.

[17] Goldfain E. Local scale invariance, cantorian space-time and unified field theory. Chaos, Solitons and Fractals 2005; 23:701-710.

[18] Goldfain E. Complexity in quantum field theory and physics beyond the standard model. Chaos, Solitons and Fractals 2006; 28: 913-922.

[19] Goldfain E. Complex Dynamics and the High-energy Regime of Quantum Field Theory. Intl. Journal of Nonlinear Sciences and Numerical Simulation 2005; 6:3-223-234.

[20] Kaku M. Quantum Field Theory, a modern introduction. Oxford University Press; 1993.

[21] West B J., Bologna M., Grigolini P. Physics of Fractal Operators. Springer; 2003.

[22] Tarasov V. E., Zaslavsky G. M. Dynamics with Low-Level Fractionality. Available from http://arxiv:physics/0511138.

[23] Tarasov V. E. Fractional Generalization of Gradient and Hamiltonian Systems. Journal of Physics A. 2005; 38:26-5929-5943.

[24] Tarasov V. E., Zaslavsky G. M. Fractional dynamics of coupled oscillators with long-range interaction. Available from http://arxiv:nlin.PS/0512013.

[25] Laskin N., Zaslavsky G. M. Nonlinear Fractional Dynamics on a Lattice with Long Range Interactions. Available from http://arxiv:nlin.SI/0512010.

[26] Tarasov V.E., Zaslavsky G. M. Fractional Ginzburg-Landau equation for fractal media. Available from http://arxiv:physics/0511144.

[27] Joos, E *et al.* Decoherence and the Appearance of a Classical World in Quantum Theory, 2nd edition, Springer; 2003.

[28] Schlosshauer M. Decoherence, the Measurement Problem, and Interpretations of Quantum Mechanics. Rev. of Modern Phys. 2004; 76-1267-1305.

[29] Donoghue J. F. *et al.* Dynamics of the Standard Model, Cambridge University Press; 1994.

[30] Samko S. G. *et al* Fractional Integrals and Derivatives; Theory and Applications, Gordon and Breach, New York; 1993.

[31] Oldham K.B and Spannier J. Fractional Calculus, Academic Press, New York; 1974.

[32] Miller K.S. and Ross B. An Introduction to Fractional Calculus and Fractional Differential Equations, Wiley and Sons, New York; 1993.

[33] Podlubny I. Fractional Differential Equations, Academic Press, New York; 1999.