On a Four Dimensional Unified Field Theory
of the
Gravitational, Electromagnetic, Weak and the Strong Force.

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Abstract. The Gravitational, Electromagnetic, Weak & the Strong force are here brought together under a single roof via an extension of Reimann geometry to a new geometry (coined Reimann-Hilbert Space); that unlike Reimann geometry, preserves both the length and the angle of a vector under parallel transport. The affine connection of this new geometry – the Reimann-Hilbert Space, is a tensor and this leads us to a geodesic law that truly upholds the Principle of Relativity. The geodesic law emerging from the General Theory of Relativity (GTR) is well known to be in contempt of the Principle of Relativity which is a principle upon which the GTR is founded. The geodesic law for particles in the GTR must be formulated in special (or privileged) coordinate systems i.e. gaussian coordinate systems whereas the Principle of Relativity clearly forbids the existence of special (or privileged) coordinate systems in manner redolent of the way the Special Theory of Relativity forbids the existence of an absolute (or privileged) frame of reference. In the low energy regime and low spacetime curvature the unified field equations derived herein are seen to reduce to the well known Maxwell-Procca equation, the none-abelian nuclear force field equations, the Lorentz equation of motion for charged particles and the Dirac Equation. Further, to the already existing four known forces, the theory predicts the existence of yet another force. We have coined this the super-force and this force obeys $SU(4,4)$ gauge invariance. Furthermore, unlike in the GTR, gravitation is here represented by a single scaler potential, and electromagnetic field and the nuclear forces are described by the electromagnetic vector potential (A_μ) which describes the metric tensor i.e. $g_{\mu\nu} = A_\mu A_\nu$. From this $(g_{\mu\nu} = A_\mu A_\nu)$, it is seen that gravity waves may not exist in the sense envisaged by the GTR.

Keywords: Principle *of* Equivalence, Law *of* Congruency, Frame *of* Reference, System *of* Coordinates, Coordinate Transformation, Frame Transformation.

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"I am not interested in the spectrum of this and that atom ...
I want to know whether God had a choice in building the Universe."

– **Albert Einstein** (1879-1955)

I. INTRODUCTION

FROM a philosophical level, unification of all the forces of Nature imply beauty, simplicity and a purpose of design. The dream of unification of all the forces of nature in its present pursuit probably began in 1849 in the Royal Academy of Sciences in London with Michael Faraday (1791 – 1867) soon after his great works in electrodynamics when he tried to experimentally find a relationship between the electromagnetic and gravitational force – for obvious reasons he failed (Thomas 1991). In his book – *GRAVITY*, George Gawmow wrote: we open-quote; In the laboratory of Michael Faraday, who made many important contributions to the knowledge of electricity and magnetism, there is an interesting entry in 1849. It reads:

> *"Gravity. Surely this force must be capable of an experimental relation to electricity, magnetism, and other forces, so as to build it up with them in reciprocal action and equivalent effect. Consider for a moment how to go about touching this matter by facts and trial."*

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But the numerous experiments this famous British physicist undertook to discover such a relation were fruitless, and he concluded this section of his diary with these words:

> *"Here end my trials for the present. The results are negative. They do not shake my strong feeling of the existence of a relation between gravity and electricity, though they give no proof that such a relation exists."*

We close-quote. As evidenced from the last entry in his laboratory notebook, despite the failure to find this intimate relationship between gravitation and electromagnetism, Michael Faraday unshakably believed that all the forces of nature were but manifestations of a single **universal force** and ought therefore to be inter-convertible into one another in much the same manner as electricity and magnetism. Inspired by Albert Einstein's success to bring to altar and marry the Principle *of* Relativity and gravitation, the pursuit to achieve this seemingly elusive dream of unification of the forces of Nature remains much alive to the present day and is the theme of the present reading. If what is presented herein is viable and or anything to go-by; or a correct description of physical and natural reality – as we would like to believe; then this reading may well be a significant contribution toward the attainment of this dream.

Regarding the forces of Nature as described above, a Unified Field Theory (UFT) in the physics literature is a theory that proposes to bring any of the four interactions or forces into one coherent and consistent theoretical framework that conforms with experience. A Grand Unified Theory (GUT) is a theory that proposes to bring all the forces with the exception of the gravitational force, into one coherent and consistent theoretical framework and a Theory *Of* Everything (TOE) is a theory that proposes to bring all the four forces into one "giant", coherent and consistent theoretical framework which is consistent with basic facts and natural reality. The present attempt is the ambitious attempt on the so-called TOE. The title of the reading clearly suggests that this reading is about a UFT and not about a TOE. We have chosen this modest title for philosophical reasons that are not necessary to clarify here. We thus persuade the reader to accept this rather modest title.

Since the renaissance of the dream of a UFT was set-forth in 1925 by Albert Einstein (1879 – 1955) after the emergence of his General Theory of Relativity (GTR) and this being a result of Herman Weyl's beautiful, elegant but failed attempt, which was the first such on a unification of electromagnetism and gravitation (Weyl 1918), great progress has been made in the effort to achieving a better understanding of the natural World on this footing. Herman Weyl embarked on his grandiose work in 1918 after inspiration from Einstein's great works in GTR. The GTR is an elegant and beautiful but incomplete unification theory of spacetime and matter. Weyl achieved his theory by pure mathematical reasoning and his effort brought-forth and into being the powerful gauge concept without which the current efforts of unification could not be. To this day the two forces (gravity and electromagnetism) theoretically stand side-by-side independent of each other and the attempts to bring them together has since been abandoned if not forgotten as a historical footnote.

The GTR is one of the pillars of modern physics and it has not only revolutionalized our way of viewing space, time and matter but has also greatly advanced our knowledge insofar as unity of Nature is concerned. The search for a unified theory of all the forces of nature has largely continued on a theoretical front and as already mentioned, beginning with Herman Weyl (1918, 1927*a*, 1927*b*) and thereafter followed by Theodore Kaluza (1921), Albert Einstein (1919, 1920, 1921*a*, 1921*b*, 1923, 1945), Oscar Klein (1926), Erwin Schrödinger (1948), Sir Arthur S. Eddington (1921) and many others. These authors sought a unified theory of the gravitational and electromagnetic force because gravitation and electromagnetism – then, were the only forces known to humankind. Latter, with the discovery of the nuclear and sub-nuclear forces – as already mentioned, the attempts to unify gravitation with electromagnetism were abandoned by the mainstream physicists with the simple remark that this was a fruitless adventure for the reason that the subatomic forces needed to be taken into account.

The emergence of, or the discovery of the existence of subatomic forces marked a new era in the history of physics bringing forth another pillar of modern physics – Quantum Field Theory (QFT). The effort of unification now largely depended on both observations and theoretical insight because the quantum phenomena must be taken into account and this requires counter-intuitive pondering & delicate observations of the quantum phenomena since it is alien to our everyday experience in that it defies common sense. Despite the fact that we don't understand the deeper meaning of the quantum phenomena well over 80 years after the emergence of Quantum Theory (QT); unremitting and unwavering attempts on the unification of all the known forces of Nature has proceeded undaunted and unabated. Further, this is despite the fact that most if not all efforts to apply the rules applicable to the quantum phenomena to the gravitational phenomena that apply well to the other forces, has brought nothing frustration to the pine-ing physicist.

In the effort of unification, it is believed or supposed that the two key pillars of modern physics – QT and the GTR – behold the secrets to the "final unification program" and these must fuse into one consistent theory but much to the chagrin of the esoteric and curious practitioners in this field, these two bodies of knowledge appear to be fragmently disjoint in that they seem little adapted to fusion into one harmonious, coherent and consistent unified theoretical system. They do not directly contradict – though they have taken physics to the terrains of philosophy and religion because of their adamant refusal to come to the alter and marry. Their marriage is thought to be absolutely essential because it is generally agreed that a complete, unified, and deeper understanding of the Natural World lies in bringing the two theoretical systems together into one coherent and consistent unified structure since each describe a different world – for there to be unity, it is logical that there must be one world. It is thus the **dream** of most if not all practicing theoretical physicist to find such a system if it is exist to begin with. The belief and faith is that such a system ought to exist in order to preserve beauty, simplicity, an independent reality and harmony in the Natural World.

The first ever successful UFT was that by the Scottish physicist James Clerk Maxwell (1831 – 1879). He brought the electric and magnetic forces into one theoretical framework (Maxwell 1973). Amongst others, Maxwell's theory showed that light is part and parcel of electricity and magnetism. Maxwell's theory was however not consistent with Newtonian mechanics – a very successful theory at that time. The inconsistency between Maxwellian and Newtonian world views lead Einstein to ponder deeper into the intimate relationship between space and time, and by so doing he [Einstein] arrived at a new theory now known as the Special Theory *of* Relativity (STR) (Einstein 1905). Preserving the Maxwellian world view, the STR asymptotically overturned the Newtonian doctrine of absolute space and time by proposing that time and space were not absolute as Newton had wanted or postulated, but relative – different observers measure different time lapses and length depending on their relative states of motion. We will elaborate further in §(II) on this. The STR applies to inertial observers and Einstein did not stop there but proceeded to generalize the STR to include non-inertial observers thus arriving at the simple, elegant and all-time beautiful GTR which as presently understood is essentially is a theory of the gravitational phenomena.

Naturally, after the achievement of the GTR, the next task is to bring the other forces within the framework of the GTR or the GTR into the framework of the other forces, which is to bring the GTR into QT or QT into the GTR. To achieving this, the main thrust amongst the majority of the present day physicist is to seek a GUT, where upon it is thought that ideas to finding a TOE will dawn and shade light on the way forward (see e.g. Salam 1981). Currently, the only successful unification of forces in the micro-world is the 1967 – 68 theory by Sheldon Lee Glashow, Steven Weinberg & Abdus Salam. They succeeded in showing that the Weak & Electromagnetic force can be brought together into one theoretical framework. Since then, no satisfactory attempts (that is, experience and theory are in harmony) have come forth. The promising Standard Model *of* Particle Physics is also a good unification of the Weak, the Strong and the Electromagnetic force but many questions, largely theoretical ones, remain unanswered.

According to the popular science media, the most promising theoretical attempts made to date that bring the sub-nuclear forces together including the gravitational force are the theories that embrace the notion of extra dimensions beyond the known four of space and time such as String Theory. It is said by string theory's foremost proponents that this theory offers the best yet clues about a unified theory that en-campuses all the forces of nature and at the sametime it is not understood (e.g. Witten 2005).

It is our view and the view shared by many that the draw-back of theories that employ extra-dimensions is that they do not submit themselves to experience hence there is little room if any at all, to know whether these theories conform with natural reality. We are of the opinion that no matter how beautiful, elegant and appealing or seductive a theory may be or may appear to be, it ought only to be accepted as a truly physical theory if and only if it successfully submits itself to experience, otherwise it remains but an elegant piece of mathematics probably best left to be admired by mathematicians and mathematically minded poets and philosophers.

From a purely physical stand-point, there is not much one can say if anything at all about ideas based on the notion of higher dimensions since they do not naturally submit themselves to experience and the reason given is that "our collective technology as a human-race has not reached that level where we can submit these theories to experience" or that "the conditions of experience to test these ideas are only found at the unique moment of birth of space and time." As someone that wishes to fathom the mysteries of the natural world, we so much would love that string theory be the right theory given its exquisite beauty, elegance and far reaching imagination but at the sametime, we find it hard to forever keep our heard stuck in the sands thereof knowing that there is no way to verifying the theory.

Adding further to highlight the discontentness and or frustration with string theory, Smolin (2006) a leading theoretical physicist, who is a founding member and researcher at the Perimeter Institute for Theoretical Physics is of the opinion that string theory is at a dead end and openly encourages young physicists to investigate new alternatives because there is not much chance that string theory will be verified in the foreseeable future. In fact, he and others argue convincingly that string theory is not even a fully formed theory in the true sense and spirit of a scientific theory but is just but a conjecture because the theory has not been able to prove any of the exotic ideas posited by it.

The discovery of darkenergy and darkmatter he [Smolin] says is not even explained by string theory and is proving troublesome for the theory's foremost advocates. Further, Smolin (2006) writes in his book "*The Trouble with Physics*", that he believes that physicists are making the mistake of searching for a theory that is "beautiful" and "elegant" like string theory but instead they should seek falsifiable theories that can be backed up by experiments. Seeking beauty and elegance in a theory is a philosophy developed by Paul Dirac (see e.g. Kragh 1990) – this is a philosophy which we follow with the important difference that we believe that all ideas that purport to describe the true physical World, no matter how elegant and beautiful they may appear, they must naturally submit themself to experience well within the premises on which these ideas are founded.

In the spirit of or on the advice of Smolin (2006), we seek a new avenue of thought. We demanded of all the Laws *of* Physics to **absolutely** remain invariant and or covariant under both the change of the System *of* Coordinates and Frame *of* Reference and more importantly that the physics under a change of the System *of* Coordinates remains **absolutely** invariant. In this way, we seek to realize fully the Principle *of*

Equivalence by extending it to include the physical description of events in any given System *of* Coordinates and Frame *of* Reference. Before leaving this section, it is important to mention here that this reading is directed to a speicalized audience of "proffesionals" in the field of unification. We assume the reader has a good access to the STR, the GTR, Quantum Electrodynamics (QED), Quantum Flavour-dynamics (QFD) and Quantum Chromodynamics (QCD).

II. SPECIAL & GENERAL RELATIVITY

The STR was developed by Einstein in an effort to iron-out the inconsistencies between Newtonian mechanics and Maxwellian electrodynamics. The problem at hand was as follows;

(1) After a careful study of the great works of Galileo Galilee (1542 – 1642), Isaac Newton (1642 – 1727) founded a body of knowledge that beheld that in moving from one inertial Frame *of* Reference to another time preserved its nature absolutely. That is to say, given the three space dimensions and also that of time – suppose we have two inertial observers (the primed and unprimed) whose space-time coordinates are (x, y, z, t) and (x', y', z', t') respectively, with one moving along the x – *axis* relative to the other at a speed v, then, the two observers' coordinates intervals are related:

$$\begin{pmatrix} \Delta x' \\ \Delta y' \\ \Delta z' \\ c\Delta t' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & v/c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ c\Delta t \end{pmatrix} \quad (1)$$

declared Galileo in his great-works; c here and after denotes the speed of light. Essentially this is the entire conceptual constitution of Newtonian spacetime and the above transformation laws are known as the Galilean Transformation Law (GTL). What the law implies is that (assuming that the law is fundamentally true) all objects in the Universe move relative to one another – there is no such thing as absolute motion. On the other hand, the GTL predicts that, like time $(t' = t)$, acceleration $(a = d^2x/dt^2 : a' = a)$ is an absolute quantity. This means that motion is both absolute and relative. This apparent contradiction bothered Newton and lead to many philosophical debates between him and some of his contemporaries – How can motion be relative while acceleration is absolute, is acceleration not some kind of motion or is it a special kind of motion? they pondered in wonderment. Newton proposed that accelerations be measured relative to the immovable absolute space which he identified with the background of the "fixed" stars. We shall not go into this difficult philosophical subject.

(2) Maxwell's theory however predicted that light was a wave and its speed – in fragment contradiction with the Newtonian doctrine; was a universal constant. While this clashed with the Newtonian doctrine, it solved another problem, that of the existence of absolute space (or Frame *of* Reference). That is, if the speed of light were absolute; it [light] ought [in accordance with the Galilean Principle *of* Relativity] to move relative to some universal frame of reference that is at absolute rest. Also light being a wave means it ought to move through some medium – this medium would then naturally explain Newton's doctrine of absolute space and time, so it was thought. This hypothetical medium was then postulated to exist and it was coined the *Aether*. Attempts to detect this aether by measuring the speed of the Earth through its passage suggested that there is no such thing as an aether. With the aether having escaped detection by one the finest and most beautiful experiment ever carried out by humankind – the Michelson-Mosley Experiment (MM-Experiment) (Michelson 1881, 1887), theoretical attempts to save the aether paradigm were championed by notable figures such as the great Dutch physicist Hendrick Lorentz (1853 – 1928) amongst others. Lorentz's theory (Lorentz 1895) preserved the aether hypothesis by proposing that the lengths of objects underwent physical length contraction relative to the stationery aether (Lorentz-Fitzgerald contraction) and a change in the temporal rate (time dilation). At that time, this appeared to reconcile electrodynamics and Newtonian physics by replacing the GTL with a new set of transformation laws which came to be known as the Lorentz Transformation Law (LTL). If $\triangle t$, $\triangle x$, $\triangle y$, $\triangle z$ are the time and space separation relative to the aether and $\triangle t'$, $\triangle x'$, $\triangle y'$, $\triangle z'$ the time and space separations in the moving frame (speed v), then:

$$\begin{pmatrix} \Delta x' \\ \Delta y' \\ \Delta z' \\ c\Delta t' \end{pmatrix} = \begin{pmatrix} \Gamma & 0 & 0 & v\Gamma/c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ v\Gamma/c & 0 & 0 & \Gamma \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ c\Delta t \end{pmatrix} \quad (2)$$

where: $\Gamma = \left(1 - v^2/c^2\right)^{-1/2}$. The above are the LTL. Indirectly, after much careful pondering on the negative result of the MM-Experiment: by considering the apparent contradictions between Newtonian and Maxwellian electrodynamics; with a leap of *faith* and **boldness**, Einstein cut the Gordian knot and then untied it thereafter by the following reasoning; *If* – he asked; we accept the Laws *of* Electromagnetism as fundamental and also Newtonian Laws of motion as fundamental, then there ought not to be a contradiction when Newtonian Laws of motion are applied to inertia Frames *of* Reference in which the Electrodynamic Laws hold good and when these laws are transformed to an equivalent reference frame within the framework of Newtonian mechanics. Either of the two must be at fault or both. Newtonian mechanics, then had stood the test of time – for nearly 250 years it passed all the experimental tests to which it was submitted and was almost taken for granted as a self evident truth, an axiom of science and a tutology, so much that the celebrated physicist and philosopher Lord Kelvin – was amongst other prominent and highly esteemed thinkers of his time; so confident of Newtonian mechanics that he proclaimed before the turn of the past century that *"There is nothing new to be discovered in physics now. All that remains is more and more precise measurement."* – we know now, he was not right, Einstein was to soon show this.

On the other hand electrodynamics was a new field where more elaborate experiments to confirm it where yet to be carried out. It is here that Einstein boldly & faithfully cut sharply though the thick dark clouds hovering over the horizon of science, chopping and un-tieing the Gordian knot by upholding electrodynamics as more fundamental than Newtonian mechanics and thus went on to replace it with a new mechanics by putting forward the following two postulates:

1. The Laws *of* Physics are the same for all inertial frames of reference in uniform relative motion.

2. The speed of light in free space is the same for all inertial observers.

The first postulate, known also as the **Principle *of* Relativity**, dispels the notion that there is such a thing as a preferred or absolute Frame *of* Reference. The Laws *of* Physics must be the same in equivalent Frames *of* Reference. Inertial Frames *of* Reference have the same status of motion in that Newton's first Law holds good in them. If the first postulate were true and Maxwells theory were a fundamental theory of nature, then the second postulate follows immediately since Maxwell's theory predicts explicitly that the speed of light has a definite numerical value. The constancy of the speed of light predicted here lead us via Einstein's great insight to rethink our view of space and time. Time for different frames of reference runs at different rates and lengths are not absolute but depend on the observers state of motion. The LTL follow immediately from these two postulates but with the important difference that the aether hypothesis is not any longer necessary.

This is the entire conceptual content of the STR. Einstein was not satisfied with the STR because it only dealt with observers in uniform relative motion and he wanted to know how the Laws *of* Nature manifest themselves in the case of non-inertial observers and the quest for an answer to this question culminated in the GTR (Einstein 1915). The problem with non-inertial observers is that gravitation becomes a problem since it is an all pervading "non-vanishing force". By analyzing the motion of a body in free-fall in a gravitational field, Einstein was able to overcome the problem of gravitation by noting that if gravitational mass (m_g) and inertia mass (m_i) were equal or equivalent, then gravitation and acceleration are equivalent too (Einstein 1907). Because of the importance of this, it came to be known as the **Principle *of* Equivalence**. This meant that the effect(s) of acceleration and gravitation are the same – one can introduce or get rid of the gravitational field by introducing acceleration into the system. The deep rooted meaning of the Principle *of* Equivalence is that Physical Laws should remain the same in a local Frame *of* Reference in the presence of a gravitational field as they do in an inertial Frame *of* Reference in the absence of gravitation. In Einstein's own words:

Principle *of* Equivalence: "We shall therefore assume the complete physical equivalence of a gravitational field and the corresponding acceleration of the reference frame. This assumption extends the Principle *of* Relativity to the case of uniformly accelerated motion of the reference frame."

A consequence of this is that no mechanical or optical experiment can locally distinguish between a uniform gravitational field and uniform acceleration. It is here that we would like to point out that the Principle *of* Equivalence as used in the formulation of the GTR does not demand that the physics must remain invariant. By "***the physics***" we mean that the description of a physical event ought to remain invariant unlike for example in black-hole physics – depending on the System *of* Coordinates employed (and not the Frame *of* Reference – **this is important**), a particle can be seen to pass or not pass through the Schwarzschild sphere for the same observer supposedly under the same conditions of experience. Also the chronological ordering of events is violated – that is, the Law *of* Causality is not upheld. For example, in a rotating Universe as first pointed-out by the great mathematician and philosopher, Kant Gödel (1949); it is possible to travel back in time meaning to say it is possible in principle to violate the Second Law *of* Thermodynamics. Though the idea of time travel is very fascinating and appealing to the mind, it is difficult to visualize by means of binary logical reasoning how it can work in the Physical World as we know it. From intuition, the Laws *of* Nature must somehow have it deeply engrained and embedded in them the non-permissibility of time travel.

Therefore, we must demand that the physics, that to say, the physical state and chronological ordering of events, must remain invariant – that is, extend the Principle *of* Equivalence to include the physical state or physical description of events and the Law *of* Causality. Because this must be universal and important, let us call the extended Principle *of* Equivalence to what we shall coin the Law *of* Congruency:

Law *of* Congruency: Physical Laws have the same form in all equivalent Frames *of* Reference independently of the System *of* Coordinates used to express them and the complete physical state or physical description of an event(s) emerging from these laws in the respective Frames *of* Reference must remain absolutely and independently unaltered i.e. invariant and congruent; by the transition to a new System *of* Coordinates.

This forms the basic guiding principle of the present theory. The deeper meaning of the Law *of* Congruency is that it should not be permissable to transform a singularity by employing a different set of coordinates as is common place in the study of the Schwarzchild metric of spacetime. If the singularity exists, it exists independently of the System *of* Coordinates and Frame *of* Reference used – it is intrinsc and permanent. Therefore if we are to have no singularities, the theory itself must be free of these. If a particle is seen not to pass through the event horizon, it will not be seen to pass the event horizon no matter the System *of* Coordinates employed and the Frame *of* Reference to which the current situation is transformed into.

Back to the main vein; the Principle *of* Equivalence is in the context of Riemann geometry, mathematically embodied in the mathematical expression:

$$g_{\mu\nu;\sigma} = g_{\mu\nu,\sigma} + \Gamma^{\lambda}_{\sigma\mu}g_{\lambda\nu} + \Gamma^{\lambda}_{\sigma\nu}g_{\mu\lambda} = 0, \tag{3}$$

where $g_{\mu\nu}$ is the metric tensor describing the geometry of space-time and:

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}g^{\lambda\alpha}\left\{g_{\alpha\mu,\nu} + g_{\nu\alpha,\mu} - g_{\mu\nu,\alpha}\right\}, \tag{4}$$

are the affine connections or the Christoffel symbols (first defined in the reading Christoffel 1869). The affine connections play an important role in that they relate tensors between different Frames *of* Reference and Systems *of* Coordinates. Its draw back insofar as Physical Laws are concerned is that it is not a tensor. It transforms as:

$$\Gamma^{\lambda'}_{\mu'\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\nu}}{\partial x^{\nu'}}\frac{\partial x^{\lambda'}}{\partial x^{\lambda}}\Gamma^{\lambda}_{\mu\nu} + \frac{\partial x^{\lambda'}}{\partial x^{\lambda}}\frac{\partial^2 x^{\lambda}}{\partial x^{\mu'}\partial x^{\nu'}}. \tag{5}$$

The extra term on the right makes it a non-tensor. Most of the problems facing the GTR can be traced back to the non-tensorial nature of the affine connections – some of the problems will be highlighted in the succeeding section.

Both the invariance and covariance of Physical Laws under a change of the System *of* Coordinates and or Frame *of* Reference is, in Riemann geometry encoded and expressed through the invariance of the line element:

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}. \tag{6}$$

The line element is a measure of the distance between points in spacetime and remains invariant under any kind of transformation of the Frame *of* Reference and or the System *of* Coordinates. This is the essence of the GTR. From this Einstein was able to deduce that gravitation is and or can be described by the metric tensor – $g_{\mu\nu}$, thus, according to the Einstein doctrine of gravitation, it [gravitation] manifests itself as the curvature of space-time. Through his [Einstein] own intuition & imagination, he was able to deduce that the curvature of space-time ought to be proportional to the amount of matter-energy present – a fact that has been verified by numerous experiments. The resulting law emerging from Einstein's thesis is:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu} + \Lambda g_{\mu\nu}, \tag{7}$$

which is the well known Einstein's field Equation of Gravitation where:

$$R_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu,\lambda} - \Gamma^{\lambda}_{\mu\lambda,\nu} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\lambda\nu} - \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\lambda\mu}, \tag{8}$$

is the contracted Riemann curvature tensor and $T_{\mu\nu} = \varrho v_{\mu}v_{\nu} + pg_{\mu\nu}$ is the stress and energy tensor where ϱ is the density of matter, p is the pressure and v_{μ} the four velocity, $\kappa = 8\pi G/c^4$ is the Einstein constant of gravitation with G being Newton's universal constant of gravitation, c the speed of light and Λ is the controversial and so-called cosmological constant term added by Einstein so as to stop the Universe from expanding (Einstein 1917). Einstein was motivated to include the cosmological constant because of the strong influence from the astronomical wisdom of his day that the Universe appeared to be static and thus was assumed to be so. Besides this, the cosmological constant fullfiled Mach's Principle (Mach 1893), a principle that had inspired Einstein to search for the GTR and the thus thought that the GTR will have this naturally embedded in it – to his disatisfaction, the GTR did not exactly fullfil this in the manner Einstein had envisaged. Mach's principle forbids the existence of a truly empty space and at the sametime supposes that the inertia of an object is due to the induction effect(s) of the totality of all-matter in the Universe.

III. PROBLEM & QUEST

In our view, the major problem that the GTR faces is that it is based on pure Riemann geometry – a geometry that is well known to violate the Principle *of* Equivalence at the affine level because the affine connections are not tensors. If pure Riemannian geometry is to be the true geometry to describe the Natural World, then, no Laws *of* Physics should exist at the affine level of Riemann geometry. However, this is not so, since the Geodesic Law:

$$\frac{d^2x^{\lambda}}{ds^2} + \Gamma^{\lambda}_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds} = 0, \tag{9}$$

that describes the path and motion of particles in spacetime emerges at the affine level. Thus accepting Riemann geometry as a true geometry of nature means we must accept contrary to the Principle *of* Relativity that there exists in Nature preferred Frames *of* Reference and Systems *of* Coordinates because the above Geodesic Law leads us to formulating the equation of motion in prefaced Frames *of* Reference and System *of* Coordinates, namely, geodesic System *of* Coordinates also know as gaussian System *of* Coordinates. Gaussian System *of* Coordinates are those coordinate systems such that $g_{\mu\nu,\sigma} = 0$. It can be shown for example that given a flat space-time in which say the rectangular System *of* Coordinates (where $g_{\mu\nu,\sigma} = 0$ holds) are used to begin with; where [in the rectangular System *of* Coordinates] the affine vanish identically in this system and changing the System *of* Coordinates to spherical, the affine do not vanish. Further, the scaler $v_\lambda \dot{v}^\lambda$ is not a scaler in the GTR. The dot over the four velocity, i.e. $\dot{v}^\lambda$ represents the time derivative hence $\dot{v}^\lambda$ is the four acceleration. One can verify that $v_\lambda \dot{v}^\lambda$ is not a scaler by talking the term involving the affine in (9) to the otherside of the equality sign and then multiplying bothsides by v_α and thereafter contracting the indices ($\lambda = \alpha$). After the said operations, we will have on the left-hanside of the equation a scaler and on the right a pseudo-scaler – how can this be? This is a serious desideratum, akin to the Newton-Maxwell conundrum prior to Einstein's STR – a conundrum of how to reconcile or comprehend the apparent contradiction of the prediction of Maxwell's theory's that demanded that the speed of light be a universal and absolute speed and the gallilean philosophy of relativity that there is no such thing as a universal and absolute speed in the Universe.

Given for example, that the affinities represent forces as is the case in the GTR, this means a particle could be made to pass from existence into non-existence (or vise-versa) by simply changing the System *of* Coordinates. This on its own violates the Laws *of* Logic and the need for Nature to preserve an independent reality devoid of magic. For this reason, there is a need to ask:

"What exactly do we mean by a System *of* Coordinates and Frame *of* Reference and what relationship should these have to Physical Laws so that the Law *of* Congruency is upheld?"

This shall constitute the subject of the next section. Clearly, the only way out of this conundrum is to seek – as Einstein, Schrödinger etc have done; a theory in which the affinities have a tensor form hence in the present approach, the first and most important guide is to seek tensorial affinities. Einstein, Schrödinger etc have made attempts along these lines only to fail. The reason for their failure may perhaps stem from the fact that theirs was a mathematical exercise to try to find a set of tensorial affinities from within the framework of the classical spacetime of Riemannian geometry.

IV. NATURE OF TIME

"Absolute, true, and mathematical time, of itself, and from its own nature, flows equable without relation to anything external ..."

– **Sir Isaac Newton** (1642 – 1727)

We already know from the STR that time does not transform absolutely when dealing with different Frames *of* Reference and this was Einstein's radical new idea that changed forever our view of time. We should say, we are not about to change this but simple "*clip the wings*" of our use of this idea when dealing with *coordinate systems* (not *reference frames*). We ask the question whether or not the time coordinate is invariant under a change of the System *of* Coordinates? The answer to this questions shall provide an answer to the question paused in the preceeding section namely, whether or not it is right that the change of a coordinate system should lead to a change in the physics as happens in blackhole physics. In conclusion, we shall establish that time – *viz* when transforming between different Systems *of* Coordinates – is a scaler quantity and this manifests itself as a self-evident-truth beyond any doubt whatsoever. In order that we accomplish our mission in this part of this reading, it is necessary that we begin by defining succinctly what we mean by Frame of Reference and System of coordinates – these two are used interchangeably in most textbooks of physics.

For example, starting with the Schwarzschild metric; Stephani (Stephani 2004) in his effort of trying to describe events near and at the event horizon of in blackhole, goes on to say "We seek coordinate systems which are better adapted to the description of physical processes ...". This is nothing more than an admission that physics in different coordinate systems will be different – there exist systems of coordinates that are unsuitable for the description of physical events. Why should this be so? Physics and or physical processes should never be dependent on the choice of coordinates – at the very least, this is in-contempt of the sacrosanct Principle *of* Relativity. Let us devote some little time to understanding what is a coordinate system and a reference frame/system and thereafter look deeper into the meaning of what these really are.

System *of* Coordinates: When thinking about space, it is extremely useful to think of it as constituting of points, each labeled so that one can distinguish one point from another – each point is and must be unique. These labels are called coordinates. One must choose these labels in such a way that it is easy to manipulate. In practice, numbers are used because we understand and can manipulate them. To manipulate these labels, a universal and well defined rule must be set out so as to label and manipulate the labels and this is what is called the *System of Coordinates*. One ought to be free to choose any coordinate system of their choice provided the labeling scheme makes each point to be unique because any space exists independent of the system of coordinates used. Examples of System *of* Coordinates are the spherical coordinates (r, θ, ϕ), rectangular (x, y, z), cylindrical (r, θ, z) and curvilinear (x_1, x_2, x_3) to mention but a few. The coordinate itself has no physical significance but only its relative distance from other coordinates is what is of physical significance. Due to Minkowski's brilliant insight, we must add a forth dimension (t) in order to label the arena where physical events take place, i.e. for spacetime where spherical coordinates are used to label space, we have (r, θ, ϕ, t), and likewise for rectangular spacetime coordinates we have (x, y, z, t) etc. The question is, for example when we have to make a transition from say rectangular spacetime coordinates to say spherical spacetime coordinates (r, θ, ϕ, t), do we the right to alter the forth dimension? We shall provide an answer to this in a shortwhile.

Frame *of* Reference: After having chosen a system of coordinates of our liking, suppose we station an observer at every-point of space. For any given System *of* Coordinates (rectangular, spherical, curvilinear etc) there exists a point that one can call the point of origin, this point can be any-point, there ought not to be a preferred point. In the usual three dimensions of space, this point is the point (0, 0, 0) – this choice gives the easiest way to manipulate the coordinates. Once the observer has set the $(0, 0, 0)$ point, they will set up about this point $(0, 0, 0)$, their axis and the set of axis then constitutes the *Frame of Reference*. The observer that has declared their point of origin and has set their frame of reference "sees" every other point relative to the (0, 0, 0) point thus this point is their point of reference which together with the set of axis is in the usual language of STR is the Frame of Reference. The Frame *of* Reference thus provides one with a reference point $(0, 0, 0)$ and a set of axes relative to which the observer can measure the position and motion of all other points in spacetime as seen in other Frames *of* Reference.

The above defines a Frame *of* Reference and we hope the reader is able to make a clear distinction between the two – that is a System *of* Coordinates and Frame *of* Reference. It follows that the STR is concerned with nature of Physical Laws under a change of the Frame *of* Reference, that is, from one-point of spacetime to another depending on these points's state of motion while the GTR is concerned with nature of Physical Laws under both a change of the System *of* Coordinates and Frame *of* Reference. The STR posits that the Laws *of* Physics remain the same for observers in uniform relative motion with the GTR positing through the Principle *of* Equivalence that even for observers in uniform relative acceleration the Laws *of* Physics remain the same and these are the same as for those observers in uniform relative motion. The GTR goes further and extends this to encamps different System *of* Coordinates by maintaining that the Laws *of* Physics remain invariant under a change of System *of* Coordinates. We will point out here a logical flew in the GTR in its endeavors. This is deeply rooted in its treatment of time under a change of the System *of* Coordinates. The logical flew lies in the equal-footing treatment of the space and time coordinates applicable to the STR or transformation between different but equivalent Frames *of* Reference being unconsciously extended to describe natural processes under a change of the System *of* Coordinates. Let us look closely at the coordinate transformation law:

$$\Delta x^{\mu'} = \left(\frac{\partial x^{\mu'}}{\partial x^{\mu}}\right) \Delta x^{\mu}. \tag{10}$$

Lets pluck out the time coordinate, that is $\mu' = \mu = 0$. It follows that a time difference of $\triangle t'$ in the primed System *of* Coordinates is related to the time lapse $\triangle t$ in the unprimed System *of* Coordinates by:

$$\triangle t' = \left(\frac{\partial x^{0'}}{\partial x^{0}}\right) \triangle t. \tag{11}$$

Clearly, if $\partial x^{0'}/\partial x^0 \neq 1$ (identically not equal to unity) or is a function of position or anything for that matter that has a numerical value other than unity, then this means that for different System *of* Coordinates time moves at different rates. We here have *time dilation* intimately associated with the way in which we label point in spacetime?! Herein lies the problem:

This means a photon can be blue or red-shifted by just changing the system of coordinates!

Red or blue shifting is a physical process but changing of the system of coordinates is not a physical process at all! Here we have it - this is the source of our problems in our endeavors to completely understand nature from the current GTR viewpoint especially when it comes to blackholes, we alter time and again the *time*-coordinate to read ourself of singularities but in so doing we are making a physical alteration and not a an alteration of the way we label spacetime. Clearly, the only way in which a photon's physical state will remain invariant is if time preserved its nature under a change of the System *of* Coordinates. This could mean time is not a vector but a scaler when it comes to coordinate transformations. If time behaved as predicted by equation (11) with $\partial x^{0'}/\partial x^0 \neq 1$, it could mean all physical events in spacetime are affected by a change of the System *of* Coordinates and as already stated it means the way in which we label points does has a realisable physical significance?! This on its own makes no physical or logical sense at all and constitute a serious desideratum – it allows for *magic*, that is, one would choose at will a System *of* Coordinates of their liking and they would give a different description from that of another observer that employs a different set of coordinates of the same physical phenomena or event in spacetime. A priori to this analysis and also a posteriori justified, is that, it is absolutely necessary that we put forward the following **Protection Postulate** so as to uphold the Law *of* Congruency:

Postulate I: In order to preserve the physical state and the chronological evolution of a physical system in the transition from one System *of* Coordinates to another, of itself and from its own nature time must flow equable without relation to anything external – it must remain invariant under any kind of transformation of the System *of* Coordinates.

It is not difficult to show that if a particular or all spatial coordinates where to transform in a non-linear manner with respect to the corresponding coordinate, events and or points in spacetime will cease to be unique and also the physics is altered just by changing the System *of* Coordinates! In order to strictly preserve the physics and second to preserve the uniqueness of events when a transition to a new System *of* Coordinates is made, it is necessary to put forward another protection postulate:

Postulate II: In order to preserve the physics when a transition to a new coordinate system is made and for this same transition to preserve the uniqueness of physical events in spacetime, the points in the new coordinate system for a non-periodic coordinate system, must be linear and have a one-to-one relation with the old one and in the case of a periodic coordinate system the periodicity must be ignored.

Linearity has a two-fold meaning here: (1) suppose in a transformation of the coordinate system from A to B a point in the coordinate system A has more than one corresponding coordinate for a non periodic coordinate system like spherical coordinate system (this periodicity can be ignored because it does not physically place the point to another point in the same space), then in such a coordinate transformation, events cease to be unique and this must be guarded against – hence the second postulate.

Mathematically speaking, the first postulate means that when it comes to coordinate transformations, time is a scaler quantity, that is, **for** a coordinate transformation and **not** a tranformation of the reference frame:

$$\left(\frac{\partial x^{0'}}{\partial x^{0}}\right) \equiv 1. \tag{12}$$

We thus have established here that time must behave as a scaler when transforming from one system of spacetime coordinates to another and this is not so when transforming from one frame of reference to another. Because of this, let us adopt the terminology **coordinate scaler** or **coordinate vector** to mean a quantity behaves as a scaler under a coordinate transformation and likewise we will have a **frame scaler** and **frame vector** to mean a quantity that transforms as a scaler or vector when transforming from one frame of reference to the other.

V. THEORY

We shall seek a geometry that gives tensorial affinities in such a way that one can obtain both the respective geometries on which quantum and classical physics are founded. Quantum physics is defined on a Hilbert space or Hilbert geometry while classical physics is founded on the classical spacetime of Riemannian geometry. The main idea and thrust is to find a geometry that fuses these two geometries into one super geometry whose resultant affines are tensors. Let us begin by defining these two geometries and fuse them in such a manner as described above – that is, the resulting affine must be tensors.

Hilbert Space: Given the object $\psi = \sum_{j=0}^{\infty}\psi_j$, then, every inner product $\langle,\rangle$ on a real or complex vector space $\mathcal{H}$ gives rise to a norm:

$$ds_H^2 = \langle\psi,\psi\rangle = \psi^\dagger\psi = \sum_{j=0}^{\infty}\psi_j^\dagger\psi_j, \tag{13}$$

and the space $\mathcal{H}$ is said to be a Hilbert space if it is complete with respect to this norm. Completeness in this context means that any cauchy sequence of elements of the space converges to an element in the space, in the sense that the norm of differences approaches zero. On the other hand we define a Riemannian space:

Riemann Spacetime: A space is said to be Riemannian if the norm is invariant under a coordinate transformation such that the metric of the space satisfies the fundamental theorem of Reimann geometry, that is the covariant derivative equation (3) resulting in the definition of the affine connection as given by equation (4).

From these spaces as defined above, one can by a closer inspection of the Riemann geometry imagine a union of both the Riemann and Hilbert space. Let us coin this space the Riemann-Hilbert Space (RHS). This space is some-kind of a Reimann Space in its formulation with it embedded the Hilbert objects that gives the space the necessary machinery to overcome the criticism leveled earlier against pure Riemann geometry that of the affinities being non-tensorial.

Riemann-Hilbert Spacetime: If the metric tensor is defined $g_{\mu\nu} = \hat{\mathbf{e}}_\mu \cdot \hat{\mathbf{e}}_\nu$ then, for the ordinary flat spacetime geometry of Minkowski where $g_{\mu\nu} = \eta_{\mu\nu}$, the unit vectors that would give this metric are the four objects:

$$\begin{array}{c} \hat{\mathbf{e}}_0 = i\begin{pmatrix}-i\\0\\0\\0\end{pmatrix}, \ \hat{\mathbf{e}}_1 = i\begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \\ \hat{\mathbf{e}}_2 = i\begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \ \hat{\mathbf{e}}_3 = i\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \end{array}. \tag{14}$$

Notice that the components or length of the axis unit vectors are all constants – why is this so? Is it really necessary that they become constants and at the sametime is it really necessary that the significant component of these unit vectors be equal? Just for a minute, suppose we set up a $3D$ system of coordinates in the usual space that we inhabit with three orthogonal axis. Let each of these axes have an observer, say X monitors the $x - axis$ and Y monitors the $y - axis$ and like wise Z monitors the $z - axis$. Along each of these axis the observer can define a unit length and it need not be equal to that of the others. Having defined their unit length to compare it with that of the others, they will have to measure the resultant vector which is the magnitude of the vector sum of the three "unit" vectors along their respective axis. This setting does not affect anything in the physical world for as long as one commits to mind that the unit vectors along each of the axis are different and they have in mind the length of the resultant unit vector. This little picture tells us we can have variable unit vectors along each of the axis that is:

$$\hat{\mathbf{e}}_0 = i\begin{pmatrix}\psi_0\\0\\0\\0\end{pmatrix},\quad \hat{\mathbf{e}}_1 = i\begin{pmatrix}0\\\psi_1\\0\\0\end{pmatrix},\quad \hat{\mathbf{e}}_2 = i\begin{pmatrix}0\\0\\\psi_2\\0\end{pmatrix},\quad \hat{\mathbf{e}}_3 = i\begin{pmatrix}0\\0\\0\\\psi_3\end{pmatrix}, \tag{15}$$

where $\psi_j = \psi_j(x)$ for $j = 0, 1, 2, 3$ real variable functions. If as usual the position vector in this space is given by $\mathbf{X} = x^\mu \hat{\mathbf{e}}_\mu$ where x^μ is the usual spacetime coordinate in Riemann geometry, then, it is not difficult for one to see that the resulting metric from the above set of unit vectors will be diagonal, the meaning of which is that all the off-diagonal terms will equal zero. We must in general be able to obtain a metric with non-zero components and not only diagonal as is the case if the unit vectors are as given in equation (15). For this to be so, that is, obtain a metric with non-zero components, we will need to have:

$$\hat{\mathbf{e}}_\mu^{(a)} = \frac{1}{2} i\phi A_\mu \gamma_\mu^{(a)} \begin{pmatrix}\psi_0\\\psi_1\\\psi_2\\\psi_3\end{pmatrix} = \frac{1}{2} i\phi A_\mu \gamma_\mu^{(a)} \psi, \tag{16}$$

where $\gamma_\mu^{(a)}$ is a set of three (hence the index $a = 1, 2, 3$) 4×4 matrices with $\gamma_\mu^{(1)} = \gamma_\mu$ being the usual 4×4 Dirac matrices and:

$$\gamma_0^{(2)} = \begin{pmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\mathbf{I}\end{pmatrix},\quad \gamma_i^{(2)} = \tfrac{1}{2}\begin{pmatrix}2\mathbf{I} & i\sqrt{2}\boldsymbol{\sigma}_i\\ -i\sqrt{2}\boldsymbol{\sigma}_i & -2\mathbf{I}\end{pmatrix}, \tag{17}$$

and:

$$\gamma_0^{(3)} = \pm\begin{pmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\mathbf{I}\end{pmatrix},\quad \gamma_i^{(3)} = \mp\tfrac{1}{2}\begin{pmatrix}2\mathbf{I} & i\sqrt{2}\boldsymbol{\sigma}_i\\ -i\sqrt{2}\boldsymbol{\sigma}_i & -2\mathbf{I}\end{pmatrix}, \tag{18}$$

and ϕ is a 4×4 scaler and A_μ is a 4×4 vector, $\mathbf{I}$ is the 2×2 identity matrix, and $\boldsymbol{\sigma}_i$ are the 2×2 Pauli matrices and these are given in (A.2). We set these objects A_μ and ϕ to be 4×4 matrices because the theory requires this to be so and the reader is not going to be reminded of this. Now, from the above, we have: $\left(ds^{(a)}\right)^2 = \hat{\mathbf{e}}_\mu^{(a)\dagger}\hat{\mathbf{e}}_\nu^{(a)} dx^\mu dx^\nu$, it follows that: $\left(ds^{(a)}\right)^2 = \rho\varphi g_{\mu\nu}^{(a)} dx^\mu dx^\nu$ where:

$$g_{\mu\nu}^{(a)} = \frac{1}{\rho\varphi}\left\{\hat{\mathbf{e}}_\mu^{(a)\dagger}, \hat{\mathbf{e}}_\nu^{(a)}\right\}, \tag{19}$$

and $\{,\}$ is the usual anti-commutation bracket and this anti-commutation is in the indices, $\rho = \psi^\dagger\psi$ and $\varphi = \phi^\dagger\phi$. From all the above, it follows that the metric tensor is given by:

$$g_{\mu\nu}^{(a)} = A_\mu^{(a)\dagger} A_\nu^{(a)}, \tag{20}$$

and if this metric tensor is to be symmetric as it must, then $A_\mu^{(a)} = A_\mu^{(a)\dagger}$, hence $A_\mu^{(a)}$ must be a real function. Written in full the three metrics are:

$$\left[g_{\mu\nu}^{(1)}\right] = \begin{pmatrix} A_0^{(1)}A_0^{(1)} & 0 & 0 & 0\\ 0 & -A_1^{(1)}A_1^{(1)} & 0 & 0\\ 0 & 0 & -A_2^{(1)}A_2^{(1)} & 0\\ 0 & 0 & 0 & -A_3^{(1)}A_3^{(1)} \end{pmatrix}, \tag{21}$$

and:

$$\left[g_{\mu\nu}^{(2)}\right] = \begin{pmatrix} A_0^{(2)}A_0^{(2)} & A_0^{(2)}A_1^{(2)} & A_0^{(2)}A_2^{(2)} & A_0^{(2)}A_3^{(2)}\\ A_1^{(2)}A_0^{(2)} & -A_1^{(2)}A_1^{(2)} & A_1^{(2)}A_2^{(2)} & A_1^{(2)}A_3^{(2)}\\ A_2^{(2)}A_0^{(2)} & A_2^{(2)}A_1^{(2)} & -A_2^{(2)}A_2^{(2)} & A_2^{(2)}A_3^{(2)}\\ A_3^{(2)}A_0^{(2)} & A_3^{(2)}A_1^{(2)} & A_3^{(2)}A_2^{(2)} & -A_3^{(2)}A_3^{(2)} \end{pmatrix}, \tag{22}$$

and:

$$\left[g_{\mu\nu}^{(3)}\right] = \begin{pmatrix} A_0^{(3)}A_0^{(3)} & -A_0^{(3)}A_1^{(3)} & -A_0^{(3)}A_2^{(3)} & -A_0^{(3)}A_3^{(3)}\\ -A_1^{(3)}A_0^{(3)} & -A_1^{(3)}A_1^{(3)} & -A_1^{(3)}A_2^{(3)} & -A_1^{(3)}A_3^{(3)}\\ -A_2^{(3)}A_0^{(3)} & -A_2^{(3)}A_1^{(3)} & -A_2^{(3)}A_2^{(3)} & -A_2^{(3)}A_3^{(3)}\\ -A_3^{(3)}A_0^{(3)} & -A_3^{(3)}A_1^{(3)} & -A_3^{(3)}A_2^{(3)} & -A_3^{(3)}A_3^{(3)} \end{pmatrix}, \tag{23}$$

it is seen that the metric $g_{\mu\nu}^{(3)}$ is simple the metric $g_{\mu\nu}^{(2)}$ under the transformation $A_k \longmapsto -A_k$. Also, we note that the metric $g_{\mu\nu}^{(a)}$ is invariant under $A_\mu \longmapsto -A_\mu$.

The line element equation, $\left(ds^{(a)}\right)^2 = \rho\varphi g_{\mu\nu}^{(a)} dx^\mu dx^\nu$; is similar in form to that for the scalar-tensor theories of gravity in which ρ is a pure scalar quantity (Brans 1961). Scaler-Tensor theories are an alternative theory to Einstein's GTR whose endeavor is similar to the present, that is, incorporate or unify quantum phenomena with the gravitational phenomena.

Unlike scaler-tensor theories, the object ρ shall here be chosen such that it is not a scaler as in Brans-Dicke Theory. This choice of ρ affords us the opportunity and the economy to unchain ourself from the bondage of non-tensorial affinities as will be seen shortly because we can forcefully choose this object in such a way that the resultant affine connections are tensors. Comparing this with Reimann geometry and demanding that in the limiting case, that is $\rho = 1$, the RHS reduces to the well known Reimann space – would; require that we make the substitution $g_{\mu\nu} \longmapsto \tilde{g}_{\mu\nu}^{(a)} = \rho\varphi g_{\mu\nu}^{(a)}$ into equation (3), that is:

$$g^{(a)}_{\mu\nu;\sigma} = \rho\varphi\left(g^{(a)}_{\mu\nu,\sigma} + \bar{\Gamma}^{\lambda}_{\sigma\mu}g^{(a)}_{\lambda\nu} + \bar{\Gamma}^{\lambda}_{\sigma\nu}g^{(a)}_{\mu\lambda} + Q_{\sigma}g^{(a)}_{\mu\nu} + G_{\sigma}g^{(a)}_{\mu\nu}\right) = 0, \tag{24}$$

where $Q_{\sigma} = \partial_{\sigma}\ln\rho$ and $G_{\sigma} = \partial_{\sigma}\ln\varphi$ and $\bar{\Gamma}^{\lambda}_{\sigma\nu}$ is the new affine connection. From this equation, one can deduce that:

$$\bar{\Gamma}^{\lambda}_{\sigma\nu} = \Gamma^{\lambda}_{\sigma\nu} + \mathrm{Q}^{\lambda}_{\sigma\nu} + \mathrm{G}^{\lambda}_{\sigma\nu}, \tag{25}$$

where $\Gamma^{\lambda}_{\sigma\nu}$ is the usual Christophel affine connection and $\mathrm{G}^{\lambda}_{\sigma\nu}$ is a new tensorial connection given by:

$$\mathrm{G}^{\lambda}_{\sigma\nu} = \frac{1}{2}g^{(\lambda\alpha}_{(a)}\left\{g^{(a)}_{\alpha\sigma}G_{\nu} + g^{(a)}_{\nu\alpha}G_{\sigma} - g^{(a)}_{\sigma\nu}G_{\alpha}\right\}, \tag{26}$$

while $\mathrm{Q}^{\lambda}_{\sigma\nu}$ is also a new but non-tensorial affine connection given by:

$$\mathrm{Q}^{\lambda}_{\sigma\nu} = \frac{1}{2}g^{\lambda\alpha}_{(a)}\left\{g^{(a)}_{\alpha\sigma}Q_{\nu} + g^{(a)}_{\nu\alpha}Q_{\sigma} - g^{(a)}_{\sigma\nu}Q_{\alpha}\right\}. \tag{27}$$

Now, taking advantage of the fact that the liberty is wholly ours to make a proper choice of ψ, let us seize the moment and demand (here and now) as set out in §(III) that the affine connection ($\bar{\Gamma}^{\lambda}_{\mu\nu}$) be a tensor. We will achieve this by making a suitable choice of ρ. We shall also require that our choice be such that the object ψ be defined on the Hilbert space – the subtle aim being to identify this object with the quantum mechanical spinor wavefunction. First things first, it is clear that if we envisage the material field to be defined by the Dirac wavefunction, then ρ can not be a scaler. If it is a scaler, this reduces the theory to a theory much akin to Weyl's un-successful unified theory (Weyl 1918, 1927*a*, 1927*b*) and at the same time, the inclusion of the scaler field φ will be rendered void.

We note that if Q_{μ} is chosen such that it transformations:

$$Q_{\mu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}}Q_{\mu} - 2\frac{\partial^2 x^{\lambda}}{\partial x_{\lambda}\partial x^{\mu'}} \tag{28}$$

this would lead to $Q^{\lambda}_{\mu\nu}$ to transform as:

$$Q^{\lambda'}_{\mu'\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\nu}}{\partial x^{\nu'}}\frac{\partial x^{\lambda'}}{\partial x^{\lambda}}Q^{\lambda}_{\mu\nu} - \frac{\partial x^{\lambda'}}{\partial x^{\lambda}}\frac{\partial^2 x^{\lambda}}{\partial x^{\mu'}\partial x^{\nu'}}. \tag{29}$$

The above transformation law clearly and immediately verifies the fact that the affine connection, $\bar{\Gamma}^{\lambda}_{\mu\nu}$, is indeed a tensor. At this point, we have achieved with relative ease to obtain tensorial affinities and thus the task now is to obtain physically meaningful field equations that conform with natural reality. Before leaving this section, we must find the transformation properties of the object ρ and this will have to be done from (29). From this we see that if $\psi' = S\psi$ where S' is some 4×4 transformation matrix; then, from this transformation equation ($\psi' = S\psi$) and (29) we will have to have: $\rho' = \Phi\rho$, where:

$$\Phi = \exp\left(\int\left[\frac{\partial x^{\lambda'}}{\partial x^{\lambda}}\frac{\partial^2 x^{\lambda}}{\partial x^{\mu'}\partial x^{\nu'}}\right]dx^{\mu'}\right). \tag{30}$$

For $\rho' = \Phi\rho$ to hold, this would require that: $S^{\dagger}S = \Phi\mathcal{I}$, where here and after $\mathcal{I}$ is the 4×4 identity matrix. This means a constraint is placed on how the object ψ can transform.

A. Electromagnetic and non-Abelien Force Fields

Given the metric: $g^{(a)}_{\mu\nu} = A^{(a)}_{\mu}A^{(a)}_{\nu}$, and planking this into the Christoffel symbol as defined in (5), one obtains:

$$\Gamma^{(a)\lambda}_{\mu\nu} = \frac{1}{2}\left[\partial_{\nu}\left(A^{(a)\lambda}A^{(a)}_{\nu}\right) + \partial_{\mu}\left(A^{(a)}_{\nu}A^{(a)\lambda}\right) - \partial^{\lambda}\left(A^{(a)}_{\mu}A^{(a)}_{\nu}\right)\right], \tag{31}$$

and differentiating the products in the brackets and rearranging, one obtains:

$$\Gamma^{(a)\lambda}_{\mu\nu} = \frac{1}{2}A^{(a)}_{\mu}F^{(a\mu)\lambda}{}_{\nu} + \frac{1}{2}A^{(a)}_{\nu}F^{(a\nu)}{}_{\mu}{}^{\lambda}, \tag{32}$$

where:

$$F^{(a\lambda)}_{\mu\nu} = \left[\partial_{\mu}, A^{(a)}_{\nu}\right] + \overbrace{A^{(a)}_{\nu}\partial_{\mu}\ln A^{(a)}_{\lambda}}^{\textbf{non-linear term}}, \tag{33}$$

which has the form of a non-abelian field. The λ is – just like the a in the bracket in the superscript; not an active index but a label informing us that, in the partial derivative ∂_{μ} appearing in the non-linear term, the vector being differentiated is the one with this index-λ, i.e. $A^{(a)}_{\lambda}$.

Given (33) and that nuclear forces are described by non-abelien fields of the form as in (33), the temptation to identify the vector field $A^{(a)}_{\mu}$ with the vector field describing the nuclear forces is very much tempting and irresistible. Succumbing to this temptation will only be worthwhile if and only if one can show that the resulting field equations do describe the nuclear forces as we know them. This task of showing that the resulting field equation do describe the known forces, is done in §(VI), (VII), (VIII); and in (IX) we predict the existence of a new and an as yet undiscovered force. We note that because of the term $\partial_{\mu}\ln A^{(a)}_{\lambda}$ in (33); this field $F^{(a\lambda)}_{\mu\nu}$, does not really have the exact form of the non-abelien fields that we know from QED, QFD and QCD. Further we note that if we impose the condition $\partial_\mu \ln A^{(a)}_\lambda = g^{(a\lambda)}_{*\mu} A^{(a)}_\mu$; then we will obtain a field that is similar to what we know from QED, QFD and QCD.

The object $g^{(a\lambda)}_{*\mu} = g^{(a\lambda)}_{*\mu}(x)$ is a four vector and not a tensor as one may blindly deduce from the way it is written, i.e. with the two indices μ and λ; further, it has the dimensions of inverse length and the asterisk has been inserted to differentiate this object from the metric tensor. For reasons to become clear latter, the object $g^{(a\lambda)}_{*\mu}$ is better called a vector-charge. Each of the vector-fields $A^{(a)}_\mu$ carries this vector-charge $g^{(a\lambda)}_{*\mu}$, that is:

$$\begin{aligned} A^{(a)}_0 &= \exp\left(\int g^{(a0)}_{*\mu} dx^\mu\right), \\ A^{(a)}_1 &= \exp\left(\int g^{(a1)}_{*\mu} dx^\mu\right), \\ A^{(a)}_2 &= \exp\left(\int g^{(a2)}_{*\mu} dx^\mu\right), \\ A^{(a)}_3 &= \exp\left(\int g^{(a3)}_{*\mu} dx^\mu\right), \end{aligned} \tag{34}$$

and from this we see that the vector-charge $g^{(a0)}_{*\mu}$ is associated with the field $A^{(a)}_0$; and $g^{(a1)}_{*\mu}$ with the field $A^{(a)}_1$ etc. Now, substituting $\partial_\mu \ln A^{(a)}_\lambda = g^{(a\lambda)}_{*\mu} A^{(a)}_\mu$ into (33), we are lead to:

$$F^{(a\lambda)}_{\mu\nu} = \left[\partial_\mu, A^{(a)}_\nu\right] + \overbrace{g^{(a\lambda)}_{*\mu} A^{(a)}_\mu A^{(a)}_\nu}^{\textbf{non-abelien term}} , \tag{35}$$

which has the same form as the usual non-abelien force field. At this point, we have managed to obtain the field tensor normally associated with the nuclear forces, the task now is to find the field equations that correspond to reality – that is, equations that describe what we know about the forces of nature. Besides the tedious task of tracking the many indices that will come along, as will be seen, this task [of finding the field equations] is not much of a task as compared to the task of arriving at the idea of how to arrive at the non-Reimann geometry that is capable of yeilding this very result of a metric whose components describe the four potentials $A^{(a)}_\mu$. Given that from the GTR, the 10 metric components all describe the gravitational field, to leap from this and convience of this very metric being described by just four objects and this objects describing not the gravitational field but the electromagnetic field other nucler forces is to us not an easy leap.

B. Field Equations

Source Coupled Field Equations: Reimann geometry is built on the idea of parallel transport of vectors along a given path. A good intuitive description of parallel transport is perhaps that by Baez (2009). Say one starts at the north pole holding a javelin that points horizontally in some direction, and they carry the javelin to the equator, always keeping the javelin pointing 'in as same a direction as possible', subject to the constraint that it point horizontally, that is, tangent to the earth and in so doing we the idea is that we're taking 'space' to be the 2-dimensional surface of the earth, and the javelin is the 'little arrow' or 'tangent vector', which must remain tangent to 'space'. After marching down to the equator, march 90 degrees around the equator, and then march back up to the north pole, always keeping the javelin pointing horizontally and 'in as same a direction as possible'. Obviously, because the surface of the earth is curved, by the time one gets back to the north pole, the javelin will be pointing in a different direction.

Parallel transport is an operation that takes a tangent vector and moves it along a path in space without turning it (relative to the space) or changing its length akin to the a person that carries a javelin as described above. In flat space we can say that the transported vector is parallel to the original vector at every point along the path. In curved space as described above, the original and final vector after the parallel transport operation are not concident and the change in this can be computer as will be done below.

If say we have a vector v^λ and we parallel transport this vector along a closed circuit $ABCD$ in the order $A \longrightarrow B$ then $B \longrightarrow C$ then $C \longrightarrow D$ and then finally $D \longrightarrow A$. The changes of this vector along these paths are:

$$\begin{aligned} dv^\lambda_{AB} &= -\Gamma^\lambda_{\mu\nu}(x) v^\nu(x) da^\mu \\ dv^\lambda_{BC} &= -\Gamma^\lambda_{\mu\nu}(x+da) v^\nu(x+da) da^\mu \\ dv^\lambda_{CD} &= +\Gamma^\lambda_{\mu\nu}(x+db) v^\nu(x+da) da^\mu \\ dv^\lambda_{DA} &= +\Gamma^\lambda_{\mu\nu}(x) v^\nu(x) db^\mu \end{aligned} \tag{36}$$

where $\Gamma^\lambda_{\mu\nu}$ and v^μ are evaluated at the location indicated in the parenthesis and the vector da^μ is the vector along $\vec{AB}$ and likewise the vector db^μ is the vector along $\vec{BC}$. Collectining these terms (i.e. $dv^\lambda_{AB} + dv^\lambda_{BC} + dv^\lambda_{CD} + dv^\lambda_{DA}$) yields the overall change (dv^λ) suffered by v^λ, i.e.:

$$dv^\lambda = \frac{\partial(\Gamma^\lambda_{\mu\nu} v^\nu)}{\partial x^\alpha} db^\alpha da^\mu - \frac{\partial(\Gamma^\lambda_{\mu\nu} v^\nu)}{\partial x^\beta} db^\beta da^\mu, \tag{37}$$

and this further reduces to:

$$dv^\lambda = \left(\Gamma^\lambda_{\mu\nu,\alpha} v^\nu - \Gamma^\lambda_{\mu\nu}\Gamma^\lambda_{\sigma\alpha} v^\sigma\right) db^\alpha da^\mu - \left(\Gamma^\lambda_{\mu\nu,\beta} v^\nu - \Gamma^\lambda_{\mu\delta}\Gamma^\delta_{\sigma\beta} v^\sigma\right) db^\beta da^\mu, \tag{38}$$

and using the identities $da^\mu \Gamma^\lambda_{\mu\nu,\sigma} = da^\alpha \Gamma^\lambda_{\alpha\nu,\sigma}$ one arrives at:

$$dv^\lambda = \left(\Gamma^\lambda_{\mu\nu,\alpha} - \Gamma^\lambda_{\mu\alpha,\nu} + \Gamma^\lambda_{\delta\alpha}\Gamma^\delta_{\mu\nu} - \Gamma^\lambda_{\delta\nu}\Gamma^\delta_{\mu\alpha}\right) v^\mu db^\alpha da^\nu, \tag{39}$$

and this can be written compactly as:

$$dv^{\lambda} = R^{\lambda}_{\mu\sigma\nu} v^{\mu} da^{\sigma} db^{\nu}, \tag{40}$$

where $R^{\lambda}_{\mu\sigma\nu}$ is the curvature tensor (see e.g. Kenyon 1990; or any good book on GTR). The above result is the important reason why we have gone through all the above calculation, namely to find (via this exposition) the mathematical relationship that informs us of the change that occurs for a any given vector after parallel transport. In Reimann geometry, the affines are not tensors and this leads to a vector altering its direction if it is transported in a closed circuit as above.

For a moment, let us shy-away from the abstract world of mathematics and pause a perdurable question to the reader. Suppose one is in a freely falling laboratory and this laboratory moves in a gravitational field in a closed circuit such that the laboratory leaves a given point and latter it returns to the same-point and throughout its path at all points it is in free-fall. The best scenario is a laboratory orbiting a central massive body. If in this laboratory we have a stationery object – do we (or does one) expect that after a complete orbit this object will have its motion altered? Or, does one expect that an object (inside the laboratory) that – say, has a specific momentum (relative to the laboratory) will after a complete circuit alter its momentum without any external force being applied to the free-falling system?

If this did happen, then Newton's First Law of motion that defines inertia systems of reference is violated and it would mean that there is no such thing as an inertial system of reference; actually this renders the Principle *of* Equivalence obsolete. Surely, something must be wrong because the sacrosanct Principle *of* Equivalence can not be found in this wanting-state.

We say this renders the Principle *of* Equivalence obsolete because for a system in free-fall like the laboratory above, according to the Principle *of* Equivalence; it is an inertial system throughout its journey thus we do not expect an object in an inertial system to alter its momentum without a force being applied to it. The none preservation of angles during parallel transport in Reimann geometry is in violation of the Principle *of* Equivalence if it is understood that parallel transport takes place in a geodesic system of reference i.e. inertial systems of reference.

Naturally, we expect that for an observer inside the laboratory, they should observe a zero net change in the momentum. This in the context of parallel transport of vectors means that such a spacetime will transport vectors (in free-falling frames) in a manner such that after a complete circuit the transported vector and the original will still have the same magnitude and direction i.e. $dv^{\lambda} = 0$. Actually, this means that throughout its transport, the magnitude and direction of the vector must be preserved. Reimann geometry does not preserve the angles but only the length of the vector. The only way to have both the angles and the length preserved is if the affinites are tensors and the curvature tensor of such a spacetime will be identically equal to zero. We have already discovered a geometry whose affines are tensors. All we need to do now is to make the transformation: $\Gamma^{\lambda}_{\mu\nu} \longmapsto \bar{\Gamma}^{\lambda}_{\mu\nu}$ so that:

$$dv^{\lambda} = \bar{R}^{\lambda}_{\mu\sigma\nu} v^{\mu} da^{\sigma} db^{\nu}, \tag{41}$$

where:

$$\bar{R}^{\lambda}_{\mu\sigma\nu} = \overbrace{\bar{\Gamma}^{\sigma}_{\mu\nu,\lambda} - \bar{\Gamma}^{\lambda}_{\mu\sigma,\nu}}^{\textbf{Linear terms}} + \overbrace{\bar{\Gamma}^{\lambda}_{\mu\alpha}\bar{\Gamma}^{\alpha}_{\sigma\nu} - \bar{\Gamma}^{\lambda}_{\nu\alpha}\bar{\Gamma}^{\alpha}_{\sigma\mu}}^{\textbf{non-Linear terms}}. \tag{42}$$

and the fact that $dv^{\lambda} = 0$ implies $\bar{R}^{\lambda}_{\mu\sigma\nu} = 0$, because $(v^{\mu}, da^{\sigma}, db^{\nu}) \neq 0$, hence thus it follows that:

$$\bar{R}^{\lambda}_{\mu\sigma\nu} = 0, \tag{43}$$

is the field equation that we seek and this field equation as we will see shortly – describes both the field $A^{(a)}_{\mu}$ and its sources (ϕ, ψ). Contracting the λ and σ indices to get the equivalent of the Reimann tensor, we obtain:

$$\bar{R}_{\mu\nu} = 0, \tag{44}$$

and further raising the μ index and then contracting it with ν to get the equivalent of the Ricci scaler, we obtain:

$$\bar{R} = 0. \tag{45}$$

Equations (43), (44) and (45) are the source coupled field equations. For the present purpose, we only consider the linear terms of the curvature tensor (43). The justification for this is that we believe that the Laws *of* Physics currently known, have been discovered in the low energy and low curvature regime and in this regime, the none linear terms in the curvature tensor (42) are (should be) small enough for us to neglect. By making this approximation, we are making a check on the present new ideas presented herein to see if these ideas reduce to what we already know. As will be seen very shortly, we are able to recover equations already familiar to us.

As a starting point, equation (45), leads to the equation: $\Box \ln \rho\varphi = 0$, where $\Box = \partial^{\mu}\partial_{\mu}$. This equation is in actual fact a conservation of the current $Q_{\mu} + G_{\mu}$; it is a different form of writing:

$$\partial^{\mu}(Q_{\mu} + G_{\mu}) = 0, \tag{46}$$

which written in this form one clearly and easy sees that it is a conservation law of the current $Q_{\mu} + G_{\mu}$.

Now, equation (45) written in terms of the fields: $F^{(a\lambda)}_{\mu\nu}$, Q_{μ} and G_{μ} is:

$$\frac{1}{2}A^{(a)}_{\mu}D^{(a\lambda)}_{\lambda}F^{(a\lambda)\lambda}{}_{\nu} + \frac{1}{2}A^{(a)}_{\nu}D^{(a\lambda)}_{\lambda}F^{(a\lambda)}{}_{\mu}{}^{\lambda} + \partial_{\mu}G_{\nu} + \partial_{\mu}Q_{\nu} + \left(G^{\lambda}\partial_{\lambda} + Q^{\lambda}\partial_{\lambda}\right)g^{(a)}_{\mu\nu} = 0, \tag{47}$$

and multiplying this by $A^{(a)\mu}$ and remembering that $A^{(a)\mu}A^{(a)}_{\nu} = g^{\ \mu}_{(a)\nu}$, the resultant equation is: $D^{(a\lambda)}_{\mu}F^{(a\lambda)\mu}{}_{\nu} - J^{(a)}_{\nu} - VA^{(a)}_{\nu} = 0$ and this can be written more neatly as:

$$D^{\mu}_{(a\lambda)}F^{(a\lambda)}_{\mu\nu} - J^{(a)}_{\nu} - VA^{(a)}_{\nu} = 0, \tag{48}$$

where $D^{(a\lambda)}_{\mu} = \partial_{\mu} + g^{(a\lambda)}_{*\mu}A^{(a)}_{\mu}$ and $J^{(a)}_{\nu} = J^{(a)}_{(1)\nu} + J^{(a)}_{(2)\nu}$ where $J^{(a)}_{(1)\nu} = -A^{(a)\mu}\partial_{\mu}\partial_{\nu}\ln\varphi$ is a vector current, $J^{(a)}_{(2)\nu} = -A^{(a)\mu}\partial_{\mu}\partial_{\nu}\ln\rho$ is a pseudo-vector current and $V = -\partial^{\mu}(\ln\rho\varphi)\partial_{\mu}$. Other than the appearance of the pseudo-vector current, clearly equation (48) is the Maxwell-Procca equation!

Source-Free Field Equations: For the source-free field equations, we know that the curvature tensor (45) satisfies the identity:

$$\bar{R}^{\sigma}_{\alpha\mu\nu;\lambda} + \bar{R}^{\sigma}_{\alpha\lambda\mu;\nu} + \bar{R}^{\sigma}_{\alpha\nu\lambda;\mu} = 0 \tag{49}$$

and contracting the indices σ and α of this equation, it is not difficult to see that one arrives at:

$$D^{(a\lambda)}_{\sigma}F^{(a\lambda)}_{\mu\nu} + D^{(a\lambda)}_{\nu}F^{(a\lambda)}_{\sigma\mu} + D^{(a\lambda)}_{\mu}F^{(a\lambda)}_{\nu\sigma} = 0, \tag{50}$$

which is the source free field equation. We still can obtain another source free field equation and this is by differentiating (48) with respect to ∂^{ν}; so doing we obtain:

$$D^{\mu}_{(a\lambda)}D^{\nu}_{(a\lambda)}F^{(a\lambda)}_{\mu\nu} - V\partial^{\nu}A^{(a)}_{\nu} = 0, \tag{51}$$

and if we assume the Lorentz gauge $\partial^{\nu}A^{(a)}_{\nu} = 0$, then this equation reduces to:

$$D^{\mu}_{(a\lambda)}D^{\nu}_{(a\lambda)}F^{(a\lambda)}_{\mu\nu} = 0. \tag{52}$$

With equations (48) and (50), we have arrived at the desired field equations.

We note that, if we consider $\left(\phi, \psi, A^{(a)}_{\mu}\right)$ as giving the complete description of a fundamental particle, then, this fundamental particle will carry four fields, that is: $F^{(a0)}_{\mu\nu}$, $F^{(a1)}_{\mu\nu}$, $F^{(a2)}_{\mu\nu}$ and $F^{(a3)}_{\mu\nu}$. These fields are not independent entities, but an integral part of the system of the particle $\left(\phi, \psi, A^{(a)}_{\mu}\right)$; they can not be separated from the particle $\left(\phi, \psi, A^{(a)}_{\mu}\right)$. As will be seen in §(VII) and (VIII), we can have an arrangement where one of these fields is an abelien field while the other three are non-abelien fields and also a setting where two of these fields are abelien fields while the other two are non-abelien fields. Given this and that if we are to think of these fields ($F^{(a\lambda)}_{\mu\nu}$) as quarks, then perhaps, one will be able to explain why quarks are inseparable and also why these quarks come in threes and pairs.

Furthermore, we note that for each of these fields ($F^{(a\lambda)}_{\mu\nu}$) there will be three types of them and these three types are labeled by $a = 1, 2, 3$ – we know that each quark-type comes in three "colors", or more clearly there are three types of each quark. Would the label-a be a label of the colors? We will not try to investigate this in this reading; all we want is to float the idea that these fields can be thought of as quarks – and additionally; so as to keep this reading as a reading whose aim is to set for further exploration, the mathematical foundations of a UFT. We will not try to cement this idea (the the field $F^{(a\lambda)}_{\mu\nu}$ may represent quarks) in the present reading but in further readings.

In the succeeding sections we will proceed to show that the components of the metric have the capability to explain the known natural forces, that is, the Electromagnetic force, the Weak and the Strong and this depends on the values that $g^{(a\lambda)}_{*\mu}$ takes and as will be seen, the already rich library of the worked out mathematics of the these forces makes the task of showing this a relatively easy task.

C. Dirac Equation

We show here that under certain conditions, the present theory yeilds the Dirac Equation. If say $\mathbf{e}_{\mu}$ is any general unit vector, then $\partial^{\mu}\mathbf{e}_{\mu} = \cos\theta$ where θ is the angle between that unit vector and the tangent surface at the point where this unit vector is located. Given this definition and that of the unit vector of the RHS: $\hat{\mathbf{e}}^{(a)}_{\mu}$; then $\partial^{\mu}\hat{\mathbf{e}}^{(a)}_{\mu} = \cos\theta$. For this spacetime (RHS), clearly the angle θ must be a 4×1 scaler object (i.e. rank one scaler), that is:

$$\theta = \begin{pmatrix}\theta_0\\ \theta_1\\ \theta_2\\ \theta_3\end{pmatrix} \text{ and } \cos\theta = \begin{pmatrix}\cos\theta_0\\ \cos\theta_1\\ \cos\theta_2\\ \cos\theta_3\end{pmatrix}. \tag{53}$$

With this, the equation $\partial^{\mu}\hat{\mathbf{e}}^{(ab)}_{\mu} = \cos\theta$ implies:

$$iA_\mu\tilde{\gamma}^\ell\gamma_\mu^{(a)}\partial^\mu\psi + \left(\frac{1}{\rho\varphi}\frac{\partial\left(\tilde{\gamma}^\ell\gamma_\mu^{(a)}iA_\mu\rho\varphi\right)}{\partial x_\mu}\right)\psi = \frac{\cos\theta}{\rho\varphi}. \tag{54}$$

Now, making the (posteriori justified) setting:

$$\begin{array}{rcll} J^\mu\left(\tilde{\gamma}^\ell\gamma_\mu^{(a)}A_\mu\right) & = & 0 & \ldots\textbf{(a)} \\ G^\mu\left(\tilde{\gamma}^\ell\gamma_\mu^{(a)}A_\mu\right) & = & 0 & \ldots\textbf{(b)} \\ m_0c\mathcal{I} & = & -i\hbar\partial^\mu\left(\tilde{\gamma}^\ell\gamma_\mu^{(a)}A_\mu\right) & \ldots\textbf{(c)} \end{array} \tag{55}$$

as a gauge condition (or constraint) where m_0 is the rest-mass of the particle (which is real), $\hbar$ is Planck's constant and then multiplying the resultant by $\tilde{\gamma}^\ell$ (remembering that $\tilde{\gamma}^\ell\tilde{\gamma}^\ell = \mathcal{I}$), we are lead to:

$$iA_\mu\gamma_\mu^{(a)}\partial^\mu\psi - \left(\frac{m_0c}{\hbar}\right)\tilde{\gamma}^\ell\psi = \tilde{\gamma}^\ell\left(\frac{\cos\theta}{\rho\varphi}\right). \tag{56}$$

As one would naturally expect, that the unit vector at a point be perpendicular to the tangent surface at that point, in which case $\cos\theta = 0$, this means the above reduces to:

$$iA_\mu\gamma_\mu^{(a)}\partial^\mu\psi = \left(\frac{m_0c}{\hbar}\right)\tilde{\gamma}^\ell\psi; \tag{57}$$

this equation is the curved spacetime Dirac Equation proposed in the reading Nyambuya (2007). We have droped the superscript a in A_μ because this is present in $\gamma_\mu^{(a)}$.

Actually, if (56) is to be Lorentz invariant as we would expect it to, we **must** have $\cos\theta = 0$ hence the imposition of the condition $\cos\theta = 0$ has a sound justification for its existence.

In the reading Nyambuya (2007*b*) – out of the need for simplicity, we advanced that an interchange of a particle's electromagnetic field $A_\mu \longmapsto -A_\mu$ requires us to simultaneously make the transformation $m_0 \longmapsto -m_0$. From the gauge condition (55 c), this transformation ($A_\mu \longmapsto -A_\mu$ and $m_0 \longmapsto -m_0$) finds justification.

Now moving on to the real issue of why we decided to include this section on the Dirac Equation; other than the fact that showing that this equation does under certain conditions arise from the present theory (giving it [off cause!] some ground to stand); we want to discuss an extension of the curved spacetime Dirac Equation – namely; Nyambuya (2009).

In Nyambuya (2009), we did show that the Dirac Equation can be generalised to describe both bosons and fermions. According to our present understanding *viz*, from the accepted literature, bosons are described by a zero-rank scaler function while fermions are described by the four component Dirac function ψ. As will be seen in §(VI), (VII), (VIII) and (IX) we are not only going to have to describe bosons using the Dirac four component function ψ but use this same equation to describe them [bosons]. The fact that we will use the four componet function and the same equation describing fermions to describe bosons does not mean the bosons we are describing are not bosons. So we want to clear this here and now. We direct the reader to the reading Nyambuya (2009) for this. In its bare formulation, the Dirac Equation describes only spin-$1/2$ particles but in Nyambuya (2009), we did show that it [Dirac Equation] can be written in a more general form to describe in general any spin particle (i.e $s = 1/2, 1, 3/2, 2, \ldots, n/2\ldots: n = 1, 2, 3, \ldots$ etc) and this generalisation extends to (57) as-well.

D. Gauge Invariance

The metric $\tilde{g}_{\mu\nu}^{(a)} = \rho\varphi A_\mu^{(a)\dagger}A_\nu^{(a)}$ is $U(1,4)\times SU(2,4)$, $U(1,4)\times SU(2,4)\times SU(2,4)$, $U(1,4)\times SU(3,4)$ and $U(1,4)\times SU(4,4)$ gauge invariant. Typically in the study of nuclear forces, that is in QFD and QCD for example, one often talks of $SU(2)$ and $SU(3)$ gauge invariance respectively. This can be generalised to $SU(N)$ gauge invariance where $N = 2, 3, 4, \ldots$. In general an $SU(N)$ field is defined on an N dimensional space. The dimensionality is determined by the components of the spinor for the force field, that is for $SU(2)$ gauge invariance of the Weak forces has two fields (Ψ_{QFD}) and for $SU(3)$ gauge invariance of the Strong forces has three fields (Ψ_{QFD}), i.e.:

$$\Psi_{QFD} = \begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} \quad \Psi_{QCD} = \begin{pmatrix}\Psi_1\\ \Psi_2\\ \Psi_3\end{pmatrix}. \tag{58}$$

We can define an $SU(N,D)$ gauge group where $N \leq D$ and $N = 2, 3, 4, \ldots$. and this is the usual $SU(N)$ group written on a D-dimensional space. For example, an $SU(2,4)$ gauge group is a four component field that is — like the $SU(2)$; span by three generators and an $SU(3,4)$ gauge group is a four component field that is — like the $SU(3)$; span by eight generators. Further, an $U(1,4)\times SU(2,4)$ gauge group is four component field that is span by three generators of the $SU(2)$ group and one generator of the $U(1)$ group.

To show this – that is; $\tilde{g}_{\mu\nu}^{(a)} = \rho\varphi A_\mu^{(a)\dagger}A_\nu^{(a)}$ is $U(1,4)\times SU(2,4)$, $U(1,4)\times SU(2,4)\times SU(2,4)$, $U(1,4)\times SU(3,4)$ and $U(1,4)\times SU(4,4)$ gauge invariant, we subject the object $\left(\phi, \psi, A_\mu^{(a)}\right)$ to the decomposition:

$$\begin{array}{rcll} \phi & = & \sum_j \mathcal{T}_j\phi_j & \ldots\textbf{(a)} \\ \psi & = & \sum_j \mathcal{T}_j\psi_j & \ldots\textbf{(b)} \\ A_\mu^{(a)} & = & \sum_j \mathcal{T}_j A_{j\mu}^{(a)} & \ldots\textbf{(c)} \end{array} \tag{59}$$

where $\left(\phi_j, \psi_j, A_{j\mu}^{(a)}\right)$ are generators of the $U(1,4)\times SU(2,4)$, $U(1,4)\times SU(2,4)\times SU(2,4)$, $U(1,4)\times SU(3,4)$ and $U(1,4)\times$

$SU(4,4)$ group and the $\mathcal{T}$-matrices are 4×4 orthogonal matrices that obey the Cliford algebra: $\left[\mathcal{T}_i,\mathcal{T}_j\right] = if_{ijl}\mathcal{T}_l$, and f_{ijl} are the suitable structural constants for that particular gauge group. The $\mathcal{T}$-matrices are listed are listed in Appendix (A) and these will be discussed in §(VI), (VII), (VIII) and (IX) where we deal with these symmetries individually. From this – i.e. (59), the metric can now be decomposed as:

$$\tilde{g}^{(a)}_{\mu\nu} = \rho\varphi g^{(a)}_{\mu\nu} = \sum_j \rho_j\varphi_j A^{(a)\dagger}_{j\mu}A^{(a)}_{j\nu} \tag{60}$$

where $\rho_j = \psi^\dagger_j\psi_j$ and $\varphi_j = \phi^\dagger_j\phi_j$; and this metric written in this decomposed form is clearly invariant under the transformation:

$$\begin{array}{lll} \phi_j & \longmapsto \mathcal{U}_j\phi_j + \mathcal{N}_j\chi_j & \ldots(\mathbf{a}) \\ \psi_j & \longmapsto \mathcal{U}_j\psi_j + \mathcal{N}_j\Psi_j & \ldots(\mathbf{b}) \\ A^{(a)}_{j\mu} & \longmapsto \mathcal{U}_jA^{(a)}_{j\mu} + \mathcal{N}_j\partial_\mu\theta^{(a)}_j & \ldots(\mathbf{c}) \end{array}, \tag{61}$$

where χ_j and $\theta^{(a)}_j = \theta^{(a)}_j(x)$ are arbitrary scaler fields, Ψ_j is an arbitrary spinor field and the $\mathcal{U}_j$ and $\mathcal{N}_j$ is a set of 4×4 matrices such that:

$$\begin{array}{ll} \mathcal{U}^\dagger_j\mathcal{U}_j = \mathcal{I} & \ldots(\mathbf{a}) \\ \mathcal{N}^\dagger_i\mathcal{N}_j = 0 & \ldots(\mathbf{b}) \\ \mathcal{U}^\dagger_i\mathcal{N}_j + \mathcal{N}^\dagger_i\mathcal{U}_j = 0 & \ldots(\mathbf{c}) \end{array} \tag{62}$$

where $\mathcal{N}_j = a\left(\mathcal{U}_j - \mathcal{U}^\dagger_j\right) + ib\left(\mathcal{U}_j + \mathcal{U}^\dagger_j\right)$ where $(a,\ b)$ are arbitrary constants. The matrices $\mathcal{U}_j$ are the set of matrices belonging to the $SU(N,D)$ group group and these are listed in Appendix (A).

Just as $\left(\phi,\psi,A^{(a)}_\mu\right)$ is a complete system making up a particle, $\left(\phi_j,\psi_j,A^{(a)}_{j\mu}\right)$ is also a system making up a particle and on the same footing $\left(\chi_j,\Psi_j,\mathcal{A}^{(a)}_{j\mu}\right)$ must be viewed as a complete system making up a particle. While $\left(\phi_j,\psi_j,A^{(a)}_{j\mu}\right)$ maybe a complete system, this system of particles makes the system $\left(\phi,\psi,A^{(a)}_\mu\right)$ it is most logical to suppose that the particles $\left(\phi_j,\psi_j,A^{(a)}_{j\mu}\right)$ must exist within the system $\left(\phi,\psi,A^{(a)}_\mu\right)$. On the question of the arbitrary particle $\left(\chi_j,\Psi_j,\mathcal{A}^{(a)}_{j\mu}\right)$, we shall not try to address the question of what kind of a particle $\left(\chi_j,\Psi_j,\mathcal{A}^{(a)}_{j\mu}\right)$ is, but certainly it is an arbitrary particle. We will discuss in §(VI), (VII), (VIII) and (IX) the symmetries $U(1,4)\times SU(2,4)$, $SU(2,4)\times SU(2,4)$, $U(1,4)\times SU(3,4)$ and $U(1,4)\times SU(4,4)$.

Now, the fact that the metric can be decomposed (as in 60) directly leads to another fact that the connections can also be decomposed in the same manner, that is:

$$\bar{\Gamma}^\lambda_{\mu\nu} = \sum_j \bar{\Gamma}^\lambda_{j\mu\nu}, \tag{63}$$

and further this means the curvature tensor is also decomposable, that is:

$$\bar{R}^\lambda_{\mu\sigma\nu} = \sum_j \bar{R}^\lambda_{j\mu\sigma\nu}, \tag{64}$$

and given that $\bar{R}^\lambda_{\mu\sigma\nu} = 0$, this means $\sum_j \bar{R}^\lambda_{j\mu\sigma\nu} = 0$, and this implies:

$$\bar{R}^\lambda_{j\mu\sigma\nu} = 0, \tag{65}$$

hence thus the gauge fields $A^{(a)}_{j\mu}$ obey the same field equations as $A^{(a)}_\mu$.

VI. ELECTROMAGNETIC FORCE

We will show that equations (48) and (50) can explain the three known forces of nature acting on the atomic nucleus. We shall start by showing that these equations do reproduce the first and second group of Maxwell's equations exactly. All we need to do is to show that the force field $F^{(a\lambda)}_{\mu\nu}$ exhibits $U(1,4)$ symmetry. However as shall be seen, it does not exclussively exhibit $U(1,4)$-symmetry. In all other cases, it exhibits $U(1,4)$-symmetry in-conjunction with $SU(2,4)$, $SU(3,4)$ and $SU(4,4)$-symmetries, i.e. $U(1,4)\times SU(2,4)$, $U(1,4)\times SU(3,4)$, $U(1,4)\times SU(3,4)$-symmetry. Clearly, the symmetries $U(1,4)\times SU(2,4)$ and $U(1,4)\times SU(3,4)$ *must* represent the Electroweak and Electrostrong forces respectively.

Now, we proceed to the task; if just one of the $g^{(a\lambda)}_{*\mu}$ is zero, one of the four fields $F^{(a\lambda)}_{\mu\nu}$ is $U(1,4)$ invariant, that is:

$$\begin{array}{llll} \text{If } g^{(a0)}_{*\mu} = 0, & \text{then } F^{(a0)}_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu & : & \text{Case } \lambda = 0, \\ \text{If } g^{(a1)}_{*\mu} = 0, & \text{then } F^{(a1)}_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu & : & \text{Case } \lambda = 1, \\ \text{If } g^{(a2)}_{*\mu} = 0, & \text{then } F^{(a2)}_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu & : & \text{Case } \lambda = 2, \\ \text{If } g^{(a3)}_{*\mu} = 0, & \text{then } F^{(a3)}_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu & : & \text{Case } \lambda = 3. \end{array} \tag{66}$$

Under the transformations $A^{(a)}_\mu \longmapsto \mathcal{U}A^{(a)}_\mu + \mathcal{N}\partial_\mu\theta(x)$ and $\partial_\mu \longmapsto \mathcal{U}^\dagger\partial_\mu$, where $\mathcal{U}(x) = \exp[i\theta(x)]$ is some unitary 4×4 matrix; the above fields $(F^{(a\lambda)}_{\mu\nu})$ are invariant. From this we see that the field that corresponds to the case $g^{(a\lambda)}_{*\mu} = 0$ must be the electromagnetic field and the fields corresponding to the cases $g^{(a\lambda)}_{*\mu} \neq 0$ are non-abelien gauge fields. We will be demonstrate this in the succeeding sections.

VII. ELECTROSTRONG FORCE

We proceed further to identify the equations describing $SU(3,4)$ gauge fields which in accordance with experience must be the Strong force written in four dimensions. Now, as stated above if just one of the $g_{*\mu}^{(a\lambda)}$ is zero the corresponding field $(F_{\mu\nu}^{(a\lambda)})$ represents the electromagnetic field. In this case where one of the $g_{*\mu}^{(a\lambda)}$'s, there will be three non-zero, i.e.: $(0, g_\mu^{*1}, g_\mu^{*2}, g_\mu^{*3})$, $(g_\mu^{*0}, 0, g_\mu^{*2}, g_\mu^{*3})$ and $(g_\mu^{*0}, g_\mu^{*1}, g_\mu^{*2}, 0)$. As an example, in the case $(0, g_\mu^{*1}, g_\mu^{*2}, g_\mu^{*3})$, the four fields $F_{\mu\nu}^{(a\lambda)}$ are:

$$\begin{aligned} F_{\mu\nu}^{(a0)} &= \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} & \ldots \textbf{(a)}\\ F_{\mu\nu}^{(a1)} &= \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} + g_\mu^{*1} A_\mu^{(a)} A_\nu^{(a)} & \ldots \textbf{(b)}\\ F_{\mu\nu}^{(a2)} &= \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} + g_\mu^{*2} A_\mu^{(a)} A_\nu^{(a)} & \ldots \textbf{(c)}\\ F_{\mu\nu}^{(a3)} &= \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} + g_\mu^{*3} A_\mu^{(a)} A_\nu^{(a)} & \ldots \textbf{(d)} \end{aligned} \tag{67}$$

where clearly $(F_{\mu\nu}^{(a0)})$ is an abelien gauge field and the rest are not. All the fields $(F_{\mu\nu}^{(a\lambda)})$ are $U(1,4)$ invariant.

Now making the decomposition:

$$A_\mu^{(a)} = \sum_{s=1}^{8} \mathcal{T}_{ks} A_{ks\mu}^{(a)} \tag{68}$$

where $s = 1, 2, 3, ..., 8$ labels the eight different generators of the $SU(3,4)$ group and $k = 1, 2, 3, 4$ labels the four different configurations of this group and $\mathcal{T}_{ks}$ are 4×4 matrices that satisfy the Clifford algebra: $\left[\mathcal{T}_{ki}, \mathcal{T}_{kj}\right] = i f_{ijl}^S \mathcal{T}_{kl}$ where the f_{ijl}^S are the structural constants suitable for the $SU(3,4)$, these matrices are listed in (A 5).

$$A_{1s\mu}^{(a)} = \begin{pmatrix} 0 \\ A_{1s1}^{(a)} \\ A_{1s2}^{(a)} \\ A_{1s3}^{(a)} \end{pmatrix} \; A_{2s\mu}^{(a)} = \begin{pmatrix} A_{2s0}^{(a)} \\ 0 \\ A_{2s2}^{(a)} \\ A_{2s3}^{(a)} \end{pmatrix} \; A_{3s\mu}^{(a)} = \begin{pmatrix} A_{3s0}^{(a)} \\ A_{3s1}^{(a)} \\ 0 \\ A_{3s3}^{(a)} \end{pmatrix} \; A_{4s\mu}^{(a)} = \begin{pmatrix} A_{4s0}^{(a)} \\ A_{4s1}^{(a)} \\ A_{4s2}^{(a)} \\ 0 \end{pmatrix}. \tag{69}$$

From the above, we see that one of the fields $A_\mu^{(a)}$ is zero and the other three are non-zero. This also applies to the components of ψ. After the decomposition (68), the fields $F_{\mu\nu}^{(a\lambda)}$ also decompose into:

$$\begin{aligned} F_{\mu\nu}^{(a0)} &= \textstyle\sum_{s=1}^{8} \mathcal{T}_{ks} F_{ks\mu\nu}^{(a0)} & \ldots \textbf{(a)}\\ F_{k\mu\nu}^{(a1)} &= \textstyle\sum_{s=1}^{8} \mathcal{T}_{ks} F_{ks\mu\nu}^{(a1)} & \ldots \textbf{(b)}\\ F_{k\mu\nu}^{(a2)} &= \textstyle\sum_{s=1}^{8} \mathcal{T}_{ks} F_{ks\mu\nu}^{(a2)} & \ldots \textbf{(c)}\\ F_{k\mu\nu}^{(a3)} &= \textstyle\sum_{s=1}^{8} \mathcal{T}_{ks} F_{ks\mu\nu}^{(a3)} & \ldots \textbf{(d)} \end{aligned} \tag{70}$$

and the fields $F_{ks\mu\nu}^{(a\lambda)}$ are given by:

$$\begin{aligned} F_{ks\mu\nu}^{(a0)} &= \partial_\mu A_{ks\nu}^{(a)} - \partial_\nu A_{ks\mu}^{(a)} & \ldots \textbf{(a)}\\ F_{ks\mu\nu}^{(a1)} &= \partial_\mu A_{ks\nu}^{(a)} - \partial_\nu A_{ks\mu}^{(a)} + g_{*\mu}^{(a1)} \mathcal{T}_{ks} A_{ks\mu}^{(a)} A_{ks\nu}^{(a)} & \ldots \textbf{(b)}\\ F_{ks\mu\nu}^{(a2)} &= \partial_\mu A_{ks\nu}^{(a)} - \partial_\nu A_{ks\mu}^{(a)} + g_{*\mu}^{(a2)} \mathcal{T}_{ks} A_{ks\mu}^{(a)} A_{ks\nu}^{(a)} & \ldots \textbf{(c)}\\ F_{ks\mu\nu}^{(a3)} &= \partial_\mu A_{ks\nu}^{(a)} - \partial_\nu A_{ks\mu}^{(a)} + g_{*\mu}^{(a3)} \mathcal{T}_{ks} A_{ks\mu}^{(a)} A_{ks\nu}^{(a)} & \ldots \textbf{(d)} \end{aligned} \tag{71}$$

and now making the setting:

$$g_\mu^{*\lambda} \mathcal{T}_{ks} A_{ks\mu}^{(a)} A_{ks\nu}^{(a)} \equiv \sum_m \sum_l -i g_{es}^\lambda f_{sml} A_{km\mu}^{(a)} A_{kl\nu}^{(a)}, \tag{72}$$

for the fields in (78) (b), (c) and (d) yeilds:

$$\begin{aligned} F_{ks\mu\nu}^{(a0)} &= \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} & \ldots \textbf{(a)}\\ F_{ks\mu\nu}^{(a1)} &= \partial_\mu A_{ks\nu}^{(a)} - \partial_\nu A_{ks\mu}^{(a)} - i g_{es}^1 f_{kslm} A_{kl\mu}^{(a)} A_{km\nu}^{(a)} & \ldots \textbf{(b)}\\ F_{ks\mu\nu}^{(a2)} &= \partial_\mu A_{ks\nu}^{(a)} - \partial_\nu A_{ks\mu}^{(a)} - i g_{es}^2 f_{kslm} A_{kl\mu}^{(a)} A_{km\nu}^{(a)} & \ldots \textbf{(c)}\\ F_{ks\mu\nu}^{(a3)} &= \partial_\mu A_{ks\nu}^{(a)} - \partial_\nu A_{ks\mu}^{(a)} - i g_{es}^3 f_{kslm} A_{kl\mu}^{(a)} A_{km\nu}^{(a)} & \ldots \textbf{(d)} \end{aligned}, \tag{73}$$

where it must be understood that the indices m, l in (73) are being summed just as in (72). What is the meaning and motivation of the condition (72)? We have simple written these fields in exactly the same way they are written in QFD and QCD. On the other hand, we note that this condition transforms the self interaction of the 8-gauge-field $A_{ks\mu}^{(a)}$ with there of the 8-gauge-fields, that is: the left handside of (72) which is : $g_\mu^{*\lambda} \mathcal{T}_{ks} A_{ks\mu}^{(a)} A_{ks\nu}^{(a)}$ contains on one gauge-field $A_{ks\nu}^{(a)}$ whereas the right handside contains a mix (this is seen in m, n in the indices of the gauge fields) of all the 8-gauge field; $-i g_{es}^\lambda f_{sml} A_{km\mu}^{(a)} A_{kl\nu}^{(a)}$. The object g_{es}^λ is a constant that gives the strength of the force and the subscript (es) is just a dummy-label for **e**lectro**s**trong force.

Now, if we subject the metric $\tilde{g}_{k\mu\nu}^{(a\lambda)} = \sum_{s=1}^{8} \rho_{ks} \varphi_{ks} g_{ks\mu\nu}^{(a\lambda)}$ where $\rho_{ks} = \psi_{ks}^\dagger \psi_{ks}$, $\varphi_{ks} = \phi_{ks}^\dagger \phi_{ks}$ and $g_{ks\mu\nu}^{(a\lambda)} = A_{ks\mu}^{(a)\dagger} A_{ks\nu}^{(a)}$, to the transformation:

$$\begin{array}{llll} \phi_{ks} & \longmapsto & \mathcal{U}_{ks}\phi_s + \mathcal{N}_{ks}\chi_{ks} & \ldots(\mathbf{a}) \\ \psi_{ks} & \longmapsto & \mathcal{U}_{ks}\psi_s + \mathcal{N}_{ks}\Psi_{ks} & \ldots(\mathbf{b}) \\ A^{(a)}_{ks\mu} & \longmapsto & \mathcal{U}_{ks}A^{(a)}_{s\mu} + \mathcal{N}_{ks}\partial_\mu\theta^{(a)}_s & \ldots(\mathbf{c}) \\ A^{(a)}_{\mu} & \longmapsto & \mathcal{U}A^{(a)}_{\mu} + \mathcal{N}\partial_\mu\theta^{(a)} & \ldots(\mathbf{d}) \end{array}, \quad (74)$$

where: $\mathcal{U}_{ks} = \exp\left[i\int\mathcal{T}_{ks}\theta_{ks}(x)\right]$ is an $SU(3,4)$ unitary matrix, $\mathcal{U}$ is a $U(1,4)$ unitary matrix; this metric – $\tilde{g}^{(a\lambda)}_{k\mu\nu}$; remains invariant – the meaning of which is that the equations thereof, remain invariant, hence thus the above field setup is $U(1,4)\times SU(3,4)$ invariant.

VIII. ELECTROWEAK FORCE

Following the above procedure, if just two of the $g^{*\lambda}_\mu$ are equal to zero and the rest are none-zero, then, just as in the case of the Electrostrong force, we will result force field will ve $U(1,4)\times SU(2,4)$ invariant and this describe the electroweak force. There is going to be six combinations with $g^{*\lambda}_\mu$ are equal to zero and the rest are none-zero, i.e.: $(0,0,g^{*2}_\mu,g^{*3}_\mu)$, $(0,g^{*1}_\mu,0,g^{*3}_\mu)$, $(0,g^{*1}_\mu,g^{*2}_\mu,0)$, $(g^{*0}_\mu,0,0,g^{*3}_\mu)$, $(g^{*0}_\mu,0,g^{*2}_\mu,0)$ and $(g^{*0}_\mu,g^{*1}_\mu,0,0)$. We submit $A^{(a)}_{k\mu}$ to the decomposition:

$$A^{(a)}_{k\mu} = \sum_{w=1}^{3}\mathcal{T}_{kw}A^{(a)}_{kw\mu}, \quad (75)$$

where the $A^{(a)}_{kw\mu}$ are the generators of the $SU(2,4)$ group and T_{iw} are 4×4 Pauli matrices $w=1,2,3$ and $k=1,2,..6$ and these matrices are shown in (A 2) and these likewise satisfy the Clifford Algebra: $\left[\mathcal{T}_{ki},\mathcal{T}_{kj}\right] = if^W_{ijl}\mathcal{T}_{kl}$, where the f^W_{ijl} are the structural constants suitable for the $SU(2,4)$.

$$A^{(a)}_{1w\mu} = \begin{pmatrix} 0 \\ 0 \\ A^{(a)}_{1w2} \\ A^{(a)}_{1w3} \end{pmatrix} A^{(a)}_{2w\mu} = \begin{pmatrix} 0 \\ A^{(a)}_{2w1} \\ 0 \\ A^{(a)}_{2w3} \end{pmatrix} A^{(a)}_{3w\mu} = \begin{pmatrix} 0 \\ A^{(a)}_{3w1} \\ A^{(a)}_{2w2} \\ 0 \end{pmatrix} A^{(a)}_{4w\mu} = \begin{pmatrix} A^{(a)}_{4w0} \\ 0 \\ 0 \\ A^{(a)}_{2w3} \end{pmatrix} A^{(a)}_{5w\mu} = \begin{pmatrix} A^{(a)}_{5w0} \\ 0 \\ A^{(a)}_{5w2} \\ 0 \end{pmatrix} A^{(a)}_{6w\mu} = \begin{pmatrix} A^{(a)}_{6w0} \\ A^{(a)}_{6w1} \\ 0 \\ 0 \end{pmatrix} \quad (76)$$

The gauge fields $A^{(a)}_{kw\mu}$ represent the mediating gauge Bosons of the Electrostrong force.

$$\begin{array}{lll} F^{(a0)}_{k\mu\nu} = \sum_{w=1}^{3} F^{(a0)}_{\mu\nu} & \ldots(\mathbf{a}) \\ F^{(a1)}_{k\mu\nu} = \sum_{w=1}^{3} F^{(a1)}_{ks\mu\nu}\mathcal{T}_{ks} & \ldots(\mathbf{b}) \\ F^{(a2)}_{k\mu\nu} = \sum_{w=1}^{3} F^{(a2)}_{ks\mu\nu}\mathcal{T}_{ks} & \ldots(\mathbf{c}) \\ F^{(a3)}_{k\mu\nu} = \sum_{w=1}^{3} F^{(a3)}_{ks\mu\nu}\mathcal{T}_{ks} & \ldots(\mathbf{d}) \end{array} \quad (77)$$

where:

$$\begin{array}{ll} F^{(a0)}_{k\mu\nu} = \partial_\mu A^{(a)}_{kw\nu} - \partial_\nu A^{(a)}_{kw\mu} & \ldots(\mathbf{a}) \\ F^{(a1)}_{k\mu\nu} = \partial_\mu A^{(a)}_{kw\nu} - \partial_\nu A^{(a)}_{kw\mu} + g^{*1}_\mu\mathcal{T}_{ks}A^{(a)}_{kw\mu}A^{(a)}_{kw\nu} & \ldots(\mathbf{b}) \\ F^{(a2)}_{k\mu\nu} = \partial_\mu A^{(a)}_{kw\nu} - \partial_\nu A^{(a)}_{kw\mu} + g^{*2}_\mu\mathcal{T}_{ks}A^{(a)}_{kw\mu}A^{(a)}_{kw\nu} & \ldots(\mathbf{c}) \\ F^{(a3)}_{k\mu\nu} = \partial_\mu A^{(a)}_{kw\nu} - \partial_\nu A^{(a)}_{kw\mu} + g^{*3}_\mu\mathcal{T}_{ks}A^{(a)}_{kw\mu}A^{(a)}_{kw\nu} & \ldots(\mathbf{d}) \end{array} \quad (78)$$

and now making the setting:

$$g^{*\lambda}_\mu\mathcal{T}_{kw}A^{(a)}_{kw\mu}A^{(a)}_{kw\nu} \equiv \sum_m\sum_l -ig^{\lambda}_{ew}f^W_{sml}A^{(a)}_{km\mu}A^{(a)}_{kl\nu}, \quad (79)$$

for the fields in (78) (b), (c) and (d) yeilds:

$$\begin{array}{ll} F^{(a0)}_{kw\mu\nu} = \partial_\mu A^{(a)}_{kw\nu} - \partial_\nu A^{(a)}_{kw\mu} & \ldots(\mathbf{a}) \\ F^{(a1)}_{kw\mu\nu} = \partial_\mu A^{(a)}_{kw\nu} - \partial_\nu A^{(a)}_{kw\mu} - ig^{1}_{ew}f^W_{kslm}A^{(a)}_{kl\mu}A^{(a)}_{km\nu} & \ldots(\mathbf{b}) \\ F^{(a2)}_{kw\mu\nu} = \partial_\mu A^{(a)}_{kw\nu} - \partial_\nu A^{(a)}_{kw\mu} - ig^{2}_{ew}f^W_{kslm}A^{(a)}_{kl\mu}A^{(a)}_{km\nu} & \ldots(\mathbf{c}) \\ F^{(a3)}_{kw\mu\nu} = \partial_\mu A^{(a)}_{kw\nu} - \partial_\nu A^{(a)}_{kw\mu} - ig^{3}_{ew}f^W_{kslm}A^{(a)}_{kl\mu}A^{(a)}_{km\nu} & \ldots(\mathbf{d}) \end{array}, \quad (80)$$

where it must be understood that the indices m,l in (73) are being summed just as in (72). The meaning and motivation of the condition (79) is the same as (79)? The object g^{λ}_{es} is a constant that gives the strength of the Electroweak force and the subscript (ew) is just a dummy-label for **e**lectro**w**eak force.

Now, if we subject the metric $\tilde{g}^{(a\lambda)}_{k\mu\nu} = \sum_{w=1}^{3}\rho_{kw}\varphi_{kw}g^{(a\lambda)}_{kw\mu\nu}$ where $\rho_{kw} = \psi^\dagger_{kw}\psi_{kw}$, $\varphi_{kw} = \phi^\dagger_{kw}\phi_{kw}$ and $g^{(a\lambda)}_{kw\mu\nu} = A^{(a)\dagger}_{kw\mu}A^{(a)}_{kw\nu}$, to the tranformation:

$$\begin{array}{llll} \phi_{kw} & \longmapsto & \mathcal{U}_{kw}\phi_s + \mathcal{N}_{kw}\chi_{kw} & \ldots(\mathbf{a}) \\ \psi_{kw} & \longmapsto & \mathcal{U}_{kw}\psi_s + \mathcal{N}_{ks}\Psi_{kw} & \ldots(\mathbf{b}) \\ A^{(a)}_{kw\mu} & \longmapsto & \mathcal{U}_{kw}A^{(a)}_{w\mu} + \mathcal{N}_{kw}\partial_\mu\theta^{(a)}_w & \ldots(\mathbf{c}) \\ A^{(a)}_{\mu} & \longmapsto & \mathcal{U}A^{(a)}_{\mu} + \mathcal{N}\partial_\mu\theta^{(a)} & \ldots(\mathbf{d}) \end{array}, \quad (81)$$

where: $\mathcal{U}_{kw} = \exp\left[i\int\mathcal{T}_{kw}\theta_{kw}(x)\right]$ is an $SU(2,4)$ unitary matrix, $\mathcal{U}$ is a $U(1,4)$ unitary matrix; this metric – $\tilde{g}^{(a\lambda)}_{k\mu\nu}$; remains invariant – the meaning of which is that the equations thereof, remain invariant, hence thus the above field setup is $U(1,4)\times SU(2,4)$ invariant.

IX. SUPER FORCE

In the event that $g_{*\mu}^{(a\lambda)} \neq 0$ for all $\mu = 0,1,2,3$; then the resulting force fields is a new and an as yet undiscovered $SU(4,4)$ force field.

$$\begin{array}{ll} F_{\mu\nu}^{(a0)} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} + g_{*\mu}^{(a0)} A_\mu^{(a)} A_\nu^{(a)} & \ldots \textbf{(a)} \\ F_{\mu\nu}^{(a1)} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} + g_{*\mu}^{(a1)} A_\mu^{(a)} A_\nu^{(a)} & \ldots \textbf{(b)} \\ F_{\mu\nu}^{(a2)} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} + g_{*\mu}^{(a2)} A_\mu^{(a)} A_\nu^{(a)} & \ldots \textbf{(c)} \\ F_{\mu\nu}^{(a3)} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} + g_{*\mu}^{(a3)} A_\mu^{(a)} A_\nu^{(a)} & \ldots \textbf{(d)} \end{array} \quad (82)$$

and making the decomposition:

$$A_\mu^{(a)} = \sum_{l=1}^{15} \mathcal{T}_l A_{l\mu}^{(a)}, \quad (83)$$

where the $A_{l\mu}^{(a)}$ are the generators of the $SU(4,4)$ group and T_l are 4×4 $SU(4,4)$ matrices listed in (A 6), and $l = 1,2,3,...,15$. Having slipt the gauge field $A_\mu^{(a)}$ into 16 gauge fields $A_{l\mu}^{(a)}$, as before, the fields $F_{\mu\nu}^{(a\lambda)}$ slipt as-well, that is:

$$\begin{array}{ll} F_{l\mu\nu}^{(a0)} = \partial_\mu A_{l\nu}^{(a)} - \partial_\nu A_{l\mu}^{(a)} + g_{*\mu}^{(a0)} A_{l\mu}^{(a)} A_{l\nu}^{(a)} & \ldots \textbf{(a)} \\ F_{l\mu\nu}^{(a1)} = \partial_\mu A_{l\nu}^{(a)} - \partial_\nu A_{l\mu}^{(a)} + g_{*\mu}^{(a1)} A_{l\mu}^{(a)} A_{l\nu}^{(a)} & \ldots \textbf{(b)} \\ F_{l\mu\nu}^{(a2)} = \partial_\mu A_{l\nu}^{(a)} - \partial_\nu A_{l\mu}^{(a)} + g_{*\mu}^{a2} A_{l\mu}^{(a)} A_{l\nu}^{(a)} & \ldots \textbf{(c)} \\ F_{l\mu\nu}^{(a3)} = \partial_\mu A_{l\nu}^{(a)} - \partial_\nu A_{l\mu}^{(a)} + g_{*\mu}^{(a3)} A_{l\mu}^{(a)} A_{l\nu}^{(a)} & \ldots \textbf{(d)} \end{array} \quad (84)$$

and as before we introduce the gauge constraint:

$$g_\mu^{*\lambda} \mathcal{T}_{ks} A_{ks\mu}^{(a)} A_{ks\nu}^{(a)} \equiv \sum_m \sum_l -ig_s^\lambda f_{sml} A_{km\mu}^{(a)} A_{kl\nu}^{(a)}, \quad (85)$$

for the fields in (78) (b), (c) and (d) yeilds:

$$\begin{array}{ll} F_{\mu\nu}^{(a0)} = \partial_\mu A_{l\nu}^{(a)} - \partial_\nu A_{l\mu}^{(a)} - ig_s^1 f_{kslm} A_{l\mu}^{(a)} A_{km\nu}^{(a)} & \ldots \textbf{(a)} \\ F_{l\mu\nu}^{(a1)} = \partial_\mu A_{l\nu}^{(a)} - \partial_\nu A_{l\mu}^{(a)} - ig_s^1 f_{kslm} A_{l\mu}^{(a)} A_{km\nu}^{(a)} & \ldots \textbf{(b)} \\ F_{l\mu\nu}^{(a2)} = \partial_\mu A_{l\nu}^{(a)} - \partial_\nu A_{l\mu}^{(a)} - ig_s^2 f_{kslm} A_{l\mu}^{(a)} A_{km\nu}^{(a)} & \ldots \textbf{(c)} \\ F_{l\mu\nu}^{(a3)} = \partial_\mu A_{l\nu}^{(a)} - \partial_\nu A_{l\mu}^{(a)} - ig_s^3 f_{kslm} A_{l\mu}^{(a)} A_{km\nu}^{(a)} & \ldots \textbf{(d)} \end{array}, \quad (86)$$

All these equations are invariant under the tranformation:

$$\begin{array}{lll} \phi_l & \longmapsto \mathcal{U}_l \phi_l + \mathcal{N}_l \chi_{ks} & \ldots \textbf{(a)} \\ \psi_l & \longmapsto \mathcal{U}_l \psi_l + \mathcal{N}_l \Psi_{ks} & \ldots \textbf{(b)} \\ A_{l\mu}^{(a)} & \longmapsto \mathcal{U}_l A_{l\mu}^{(a)} + \mathcal{N}_l \partial_\mu \theta_s^{(a)} & \ldots \textbf{(c)} \end{array}, \quad (87)$$

where: $\mathcal{U}_l = \exp[-i\mathcal{T}_l\theta_l(x)]$ is an $SU(4,4)$ unitary matrix hence thus these fields are $SU(4,4)$ invariant.

X. NEW GEODESIC LAW

Lastly before entering into a general discussion of the entire body of work presented herein, let us address the problem raised in §(III) of the geodesic law namely that it is neither invariant nor covariant under a change of the system of coordinates and/or change in the frame of reference. In order to derive the equation of motion in the GTR, one needs to formulate this equation first from a gaussian coordinate system and thereafter make a transformation to a coordinate system of their choice. As, already said in that section, this is in contempt of the very principle upon which the GTR is founded – *the Principle of Relativity* – which requires that one should be free to formulate the geodesic equation of motion in a coordinate system of their choice without having to start from a gaussian coordinate system. The geodesic law equation (9) is derived (upon making proper algebraic operations) from the Lagrangian:

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu} \frac{dx^\mu}{ds}\frac{dx^\nu}{ds}, \quad (88)$$

by inserting this into the Lagrangian equation of motion, namely:

$$\frac{d}{ds}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}\right) - \frac{\partial \mathcal{L}}{\partial x^\mu} = 0. \quad (89)$$

In the present, our geometry's metric has been replaced by $\rho\varphi g_{\mu\nu}$, thus we will have to effect this into the Langragian by $\mathcal{L} \longmapsto \rho\varphi\mathcal{L}$, that is:

$$\mathcal{L} = \frac{1}{2}\rho\varphi g_{\mu\nu}^{(a)} \frac{dx^\mu}{ds}\frac{dx^\nu}{ds},. \quad (90)$$

Using this Langragian in (89), one arrives at the geodesic equation:

$$\frac{d^2x^\lambda}{ds^2} + \bar{\Gamma}_{\mu\nu}^{(a)\lambda} \frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0, \quad (91)$$

and this geodesic equation unlike (9), does not require us to formulate the geodesic equation of motion in the prefared gaussian coordinate system, but in any coordinate system one chooses or desires and this is because $\bar{\Gamma}_{\alpha\nu}^{\mu}$ is a tensor!

We show, as a way of justification for the importance of this equation, that; in the low energy and curvature regime, this equation (91); for the case $\lambda = i$, reduces to the Lorentz equation. To show this, first let us make the setting $\beta^\mu = dx^\mu/ds$ and second, we make the approximation for low energies that: $\beta^0 \sim 1$ and $\beta^k \ll 1$ and this means we will have to drop the terms $\beta^i\beta^j$ because these terms will be small. Hence thus for first order approx., we will have $\bar{\Gamma}^i_{0\nu}\beta^0\beta^\nu$ and so doing we find that $\bar{\Gamma}^i_{0\nu}\beta^0\beta^\nu \simeq A_0^{(a)}\beta^\nu F^i_\nu + (G^i + Q^i)$, and it follows that:

$$\frac{d^2\vec{\mathbf{x}}}{d\tau^2} + cA_0^{(a)}v^\nu F_\nu^{(a)j}\hat{\mathbf{e}}_j + \vec{\nabla}Q = -\vec{\nabla}\Phi, \tag{92}$$

where $j = 1, 2, 3$, $Q = c^2 \ln\rho$, $\Phi = c^2 \ln\varphi$ and $ds = cd\tau$. This is the Lorentz equation for a particle traveling inside an electromagnetic field under the forces ∇Q and $-\nabla\Phi$.

Now we would like to justify the inclussion of the scaler field φ. We introduced this field so that emergent force from it – that is: $-\nabla\Phi$; will be act as the known gravitational field on astronomical scales while the emergent force from ρ – that is: ∇Q; be a force that will act on the nuclear scale. If we did not include φ, we where going to have other than the forces, $cA_0^{(a)}v^\nu F_\nu^{(a)j}\hat{\mathbf{e}}_j$; just one force to identify with the gravitational force, that is: ∇Q, and it would have been difficult to justity this as the gravitation force given that this force is not a vector.

Now, to make the force ∇Q a long range force and $-\nabla\Phi$ a long range force we have to set the currents $(G^\mu,\ Q^\mu)$ such that:

$$\partial^\mu G_\mu = (1/\mathcal{R}_g)^2 \ln\varphi + \kappa_D\varrho_D \quad \text{and} \quad \partial^\mu Q_\mu = (1/\mathcal{R}_p)^2 \ln\rho, \tag{93}$$

where κ_D is a constant and ϱ_D is the distibution of darkenergy, $\mathcal{R}_g$ and $\mathcal{R}_p$ are the range of the forces $-\nabla\Phi$ and ∇Q respectively. We have explained in the reading Nyambuya (2009*b*) our proposal for the distribution of darkmatter and how to modify the law of gravitational to include this darkmatter. We will not go into trying to explain this here but direct the reader to this reading (Nyambuya 2009*b*). From (93) and (46), it follows that:

$$\rho = \varphi^{(\mathcal{R}_p/\mathcal{R}_g)^2} e^{\kappa_D\rho_D\mathcal{R}_p^2}. \tag{94}$$

Now equation (93) can be written in full, i.e. where we make use of the fact that $G_\mu = \partial_\mu \ln\varphi$ and $J_\mu = \partial_\mu \ln\rho$, hence:

$$\Box(\ln\varphi) = \left(\frac{1}{\mathcal{R}_g}\right)^2 (\ln\varphi) + \kappa\varrho_D \quad \text{and} \quad \Box(\ln\rho) = \left(\frac{1}{\mathcal{R}_p}\right)^2 (\ln\rho), \tag{95}$$

and taking in the simplest case of radially dependent solutions, that is: $\varphi = \varphi(r)$ and $\rho = \rho(r)$, then we will have $\ln\varphi = k_a e^{-r/\mathcal{R}_g}/r$ and $\ln\rho = k_q e^{-r/\mathcal{R}_p}/r$ where (k_a, k_q) are constants. From what we already known, these constants are $(k_a = G\mathcal{M}, k_q = g_Y^2)$, hence thus:

$$\begin{aligned} \Phi &= -\left(\tfrac{G\mathcal{M}}{r}\right)\left[\exp\left(-\tfrac{r}{\mathcal{R}_g}\right) + \left(\tfrac{\alpha_D}{G\mathcal{M}}\right)\exp\left(-\tfrac{r}{\mathcal{R}_D}\right)\right] \quad \ldots (\mathbf{a}),\\ Q &= -\left(\tfrac{g_Y^2}{r}\right)\exp\left(-\tfrac{r}{\mathcal{R}_p}\right) \quad \ldots (\mathbf{b}) \end{aligned} \tag{96}$$

and these type of potentials are called the Yukawa potentials after Japanese's (first Nobel Prize winner – 1949) theoretical physicist Hideki Yukawa (1907 – 1981) who showed in the 1935 that such a potential arises from the exchange of a massive scalar field such as the field of the pion. In particle physics, a pion is any of three subatomic particles: $\pi,^-, \pi^+, \pi^0$. Pions are the lightest mesons and play an important role in explaining low-energy properties of the strong nuclear force whose mass is . Since the field mediator is massive the corresponding force has a certain range due to its decay, which range is inversely proportional to the mass. If the mass is zero, then the Yukawa potential becomes equivalent to a Coulomb potential, and the range is said to be infinite. The potential is negative, denoting that the force is attractive. The constant g_Y is a real number; it is equal to the coupling constant. The coupling constant, is a number that determines the strength of an interaction.

If $\mathcal{R}_g$ is the size the typical size of galaxies and $\mathcal{R}_p$ is the typical size of the atom, then the force $\vec{\mathbf{F}}_g = -\nabla\Phi$ will act or will be stronger on, solar and galactic scales while $\vec{\mathbf{F}}_Q = \nabla V_Q$ will act on the quantum scale. Clearly, $\vec{\mathbf{F}}_g$ must be the gravitational force and $\vec{\mathbf{F}}_Q$ its equivalent on the quantum scale. To a larger extent, this means the force that causes apples to fall has little significance on the quantum scale and this force is replaced by the quantum gravitational force $\vec{\mathbf{F}}_Q$ on this scale. We would like to highlight to the reader that this section requires a full separate reading and we have just browsed here through the ideas. All we wanted to do is to show that the inclusion of the the scaler field ϕ to be part and parcel of unit vector as ψ is neccessary.

XI. DISCUSSION & CONCLUSIONS

We have shown in this reading that it is possible – in principle; to describe all the known forces of Nature using a 4D geometric theory that needs not the addition of extra dimensions as is the case with string and string related theories. In the succeeding paragraph and those that follow thereafter, we shall discuss the the results of our investigation in point form. Our discussion will be limited to what we have discovered here and we shall not try to make comparisons of the ideas here with the many proposals of UFTs (e.g. Garrett 2007) and the reason for this is to avoid a much as is possible any confusion.

1. In Reimann geometry, the metric tensor is described by 10 different potentials and in turn these potentials describe the gravitational force. In the RHS, the metric tensor is described by just 4 different and not 10 different potentials and these potentials describe not the gravitational but

the nuclear forces. At the very least, this is a paradigm shift! The RHS on which the present theory is founded, is different from that of employed in Reimann spacetime in that the unit vectors of the RHS are variable at all points on this continuum.

2. An important out-come which lead to the ideas laid down here is the revision carried out of what is a frame of reference and a system of coordinates. This revision has lead us to the idea that it is erroneous to treat time much the same as we do when dealing with frames of reference. It has been concluded that the way in which we have treated time and space when it comes to coordinate transformation since Minkowski's 1908 pronouncement in his now famous lecture that:

   > *"The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."*

   is partly at fault because we have treated transformations between reference frames and systems of coordinates in a manner that makes no physical distinction between the two. If this is the case, that space and time be treated on an equal footing irrespective of whether we are dealing with space and time coordinates or frames of reference, it could mean that the labeling of points in spacetime has a dynamic physical meaning – this as has been argued is clearly not right and this is what leads us coordinates systems that give different physical of the same physical phenomena like with the Schwarzchild metric which has a singularity and this singularity is transformed away by the Eddington-Finkelstein coordinates. The physics from these two coordinates systems is very different yet they both describe the same physical phenomena.

3. We note that while the gravitational force has been brought under the same roof with the nuclear forces, it is not unified with these forces in a manner Michael Faraday had hoped, that is; it be interconvertible to other forces just as the electric and magnetic forces are interconvertable into one another. We see here that the gravitational field that acts on the macroscale is less significant on the atomic scale where the Yukawa force is much stronger than the classcial gravitational field.

$$\bar{\Gamma}^{\lambda}_{\mu\nu} = \underbrace{\overbrace{\Gamma^{\lambda}_{\mu\nu}\left(A^{(a)}_{\mu}\right)}^{\textbf{Nuclear Forces}} + \overbrace{\mathrm{Q}^{\lambda}_{\mu\nu}(\psi)}^{\textbf{Q. Gravitation}} + \overbrace{\mathrm{G}^{\lambda}_{\mu\nu}(\phi)}^{\textbf{C. Gravitation}}}_{\textbf{Unified Forces}} \tag{97}$$

4. We have been able achieve one thing that Einstein sought (not that Einstein's opinion is a fact, but merely that him begin the insipiration to many in this field his thoughts on the subject are important) in a unified theory – that is, the material field ψ must be part and parcel of the fabric of spacetime. Einstein is quoted as having said the left handside of his equation is like marble and the right handside is like wood and that he found wood so ugly that his dream was to turn wood into marble. These feelings of Einstein against his own GTR are better summed up in his own words in a letter to Georges Lemaître (1894–1966) the Belgian Roman Catholic priest on September 26 1947:

   > *"I have found it very ugly that the field equation should be composed of two logically independent terms which are connected by addition. About justification of such feelings concerning logical simplicity is to difficult to argue. I can not help to feel and I am unable to believe that such an ugly thing should be realized in Nature."*

   Einstein hoped that the final theory must be such that the ponderable material function (ψ) must emerge from the geometry of the theory – this off course has been achieved. The wavefunction is part and parcel of the fabric of the RHS – it is part of the metric that defines this spacetime. However, this field, is distinct, the meaning of which is that is as fundamental as the other field ϕ and $A^{(a)}_{\mu}$, these fields are not derivable from the other, but stand as distinct and fundamental fields.

5. The equations discovered here – and more importantly the geodesic equation of motion; are completely gauge invariant and covariant under a change of the system of coordinates as well as under a change of the reference frame. This gauge invariance and covariance holds even in the non-linear regime. The geodesic equation of motion reproduces the Lorentz equation of motion. However, we note that this equation needs a deeper inspection *viz* its meaning to the relation between inertia and gravitational mass.

6. The field equation that we explored in this reading are linear approximations and this is suppose to hold in the regime of lowe energy and low curvature.

7. The gravitational field of GTR is described by the metric and in the low energy regime, Einstein's equation predict the existence of gravitational radiation. Given that in the present theory, the metric no longer describes the gravitational force, but the electromagnetic potential, it follows that Einstein's gravitational waves do not exist! There are currently at least four major experiments running the effort of which is to detect Gravitational waves. These experiment are Laser Interferometer Gravitational Wave Observatory (LIGO) ([46]) which is a joint project between scientists at MIT and Caltech in the USA; The Virgo detector which is an Italian project; Geo 600 is a Gravitational wave detector located in Hannover, Germany; AIGO which is an Australian project.

The gravitational waves which these experiments are searching for are those understood from the GTR. This curvature is caused by the presence of mass - the more massive the object is, the greater the curvature it causes, and hence the more intense the gravity. When these massive objects move around in spacetime, the curvature will change in accordance to the motion of the object thus causing ripples in spacetime which then spread outward at the speed of light like ripples on the surface of a pond. These ripples are what is then called gravitational waves in the GTR. To date no direct evidence of their existence has yet come forth.

In the present theory, mass will produce radiation which is indistinguishable from Electromagnetic radiation. The simplest example of a strong source of gravitational waves as understood from the GTR is a spinning neutron star with a small mountain on its surface. The mountain's mass will cause curvature of the spacetime. Its movement will "stir up" spacetime, much like a paddle stirring up water. The waves will spread out through the Universe never stopping or slowing down. Gravitational wave according to the present are what we already know as electromagnetic waves because the four vector potential comprises the metric. Since Einstein's prediction of gravitational waves, they have never been observed to this very day. Should the present prove viable or correct, it would render these efforts as fruitless.

8. The equations explored here have been in the linear range of low energy and low curvature regime. In this range, we have had to drop the non-linear terms in the new curvature tensor in equation (42). From this we have been able to derive about all the equations we know in electromagnetism, the weak and the strong force. From this it follows that when we use these equations (in electromagnetism, the weak and the strong force), they work because we are in the low energy and low curvature regime. Further, it follows that in the high energy and high curvature regime, the non-linear part in the curvature tensor (42) will have to manifests itself. Thus, the high energy and high curvature regime will provide the real testing grounds for the ideas presented herein.

9. Though we have not really rigorously verified this, the equations derived here seem to be suggesting that a particle will be composed of three or two internal non-abelien fields and these fields are inseparable from the particle. Given that quarks are observed as internal non-abelien fields that appear inseparable, this suggests that the present ideas may have in them the answer as to what really quarks, why are they no inseparable, why to they come in pairs and threes, and peharps why do they exhibit fractional electronic charges. Thus predicted supper force have four internal non-abelien fields, and from the forgoing, it means that if this $SU(4,4)$ particle is discovered, it is suppose to comprise of four quarks.

10. Lastly, it is expected of a UFT to say something about dark-matter and darkenergy (see e.g. Rubin & Ford 1971; Rubin *et al.* 1985; Zwisky 1933, 1937). The present reading is silent on the matter. This does not mean it does not have anything to do with this subject. Work on the inclusion of darkmatter and darkenergy began enerstly with the reading Nyambuya (2009) in which the dark field (which explains the darkmatter and darkenergy) have been introduced as a four cosmological vector field Λ_μ. In this reading (Nyambuya 2009), we introduced this field by making the transformation $\partial_\mu \longmapsto \partial_\mu + \Lambda_\mu$. We introduced this four cosmological vector field to explain the apparent asymmetry between matter and antimatter. We note that in the present theory, this vector can be introduced by the addition of a darkpotential we $\phi_D = \exp\left(\int \Lambda_\mu dx^\mu\right)$ to the unit vector (16), that is: $\hat{\mathbf{e}}_\mu^{(a)} = \frac{1}{2} i\phi\phi_D A_\mu \gamma_\mu^{(a)}\psi$ where $\Lambda_\mu = \Lambda_\mu(x^k, t)$. This leads to a dark-affine connection:

$$\mathrm{D}^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha}\left\{g_{\alpha\mu}\Lambda_\nu + g_{\nu\alpha}\Lambda_\mu - g_{\mu\nu}\Lambda_\alpha\right\}, \tag{98}$$

hence thus the resultant affine connection would be:

$$\bar{\Gamma}^\lambda_{\mu\nu} = \underbrace{\overbrace{\Gamma^\lambda_{\mu\nu}\left(A^{(a)}_\mu\right)}^{\textbf{Nuclear Forces}} + \overbrace{\mathrm{Q}^\lambda_{\mu\nu}(\psi)}^{\textbf{Quantum Gravitation}} + \overbrace{\mathrm{G}^\lambda_{\mu\nu}(\phi)}^{\textbf{Classical Gravitation}} + \overbrace{\mathrm{D}^\lambda_{\mu\nu}\left(\Lambda_\mu\right)}^{\textbf{Dark Forces}}}_{\textbf{TOE}} \tag{99}$$

The new definition of of the unit vector, that is $\hat{\mathbf{e}}_\mu^{(a)} = \frac{1}{2} i\phi\phi_D A_\mu \gamma_\mu^{(a)}\psi$, leads to:

$$iA_\mu\gamma^{(a)}_\mu \partial^\mu\psi + i\Lambda^\mu\gamma^{(a)}_\mu\psi = \left(\frac{m_0 c}{\hbar}\right)\psi, \tag{100}$$

just as in Nyambuya (2008). From this, the resulting geodesic equation of motion is:

$$\frac{d^2\vec{\mathbf{x}}}{d\tau^2} - qv^\nu F_\nu{}^j \hat{\mathbf{e}}_j + \vec{\nabla} Q = -\vec{\nabla}\Phi - \mathbf{\Lambda} c^2 \tag{101}$$

where $\mathbf{\Lambda} = \Lambda^j \hat{\mathbf{e}}_j$ where $j = 1, 2, 3$.

In closing I would like to say that, while further work needs to be done, if it turns out that this theory is a true description of natural reality or anything to go by as I believe it to be, then, it is without a doubt that the train and ground for a grander understanding of the natural world from a unified perspective has been set forth. It seems to me, this theory is something worthwhile to spend my time on.

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APPENDIX A: U(1,4) AND SU(N,D) MATRICES

We list the $U(1,4)$ and $SU(N,D)$ matrices, specifically the $U(1,4)$, $SU(2,2)$, $SU(2,4)$, $SU(3,3)$, $SU(3,4)$ and the $SU(4,4)$. In general, an $SU(N,D)$ group of matrices is a set of (N^2-1) matrices and these matrices are $D\times D$ matrices where $N\leq D$ and $N=2,3,4,....$ and these satisfy the Cliford algebra: $\left[\mathcal{T}_i,\mathcal{T}_j\right]=if_{ijk}\mathcal{T}_k$ where f_{ijk} are the suitable structural constants for that group. In simpler or usual terms, this is the usual $SU(N)$ group written on a D-dimensional space. We begin by listing the $U(1,4)$ matrices. These are four dimensional unitary matrices such that $\mathcal{U}_j^\dagger\mathcal{U}_j=\mathcal{I}$ and there are sixteen of these matrices. If γ^μ are the usual Dirac-gamma matrices, then the sixteen unitary matrices are $(\gamma^\mu,\mathcal{I},\gamma^5,\sigma^{\mu\nu},\gamma^\mu\gamma^5)$ where $\sigma^{\mu\nu}=\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu$ and $\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3$ – these matrices are the 16 $\tilde{\gamma}$-matrices. Written in full, these matrices are given:

$$\begin{array}{ccccc}
\begin{pmatrix}\mathbf{I}&\mathbf{0}\\\mathbf{0}&-\mathbf{I}\end{pmatrix} & \begin{pmatrix}\mathbf{0}&\boldsymbol{\sigma}_1\\-\boldsymbol{\sigma}_1&\mathbf{0}\end{pmatrix} & i\begin{pmatrix}\mathbf{0}&\boldsymbol{\sigma}_2\\-\boldsymbol{\sigma}_2&\mathbf{0}\end{pmatrix} & \begin{pmatrix}\mathbf{0}&\boldsymbol{\sigma}_3\\-\boldsymbol{\sigma}_3&\mathbf{0}\end{pmatrix} & \Big|\,\gamma^\mu\\
\begin{pmatrix}\mathbf{I}&\mathbf{0}\\\mathbf{0}&\mathbf{I}\end{pmatrix} & \begin{pmatrix}\mathbf{0}&\boldsymbol{\sigma}_1\\\boldsymbol{\sigma}_1&\mathbf{0}\end{pmatrix} & i\begin{pmatrix}\mathbf{0}&\boldsymbol{\sigma}_2\\\boldsymbol{\sigma}_2&\mathbf{0}\end{pmatrix} & \begin{pmatrix}\mathbf{0}&\boldsymbol{\sigma}_3\\\boldsymbol{\sigma}_3&\mathbf{0}\end{pmatrix} & \Big|\,\gamma^0\gamma^\mu\\
\begin{pmatrix}\mathbf{0}&\mathbf{I}\\-\mathbf{I}&\mathbf{0}\end{pmatrix} & \begin{pmatrix}\boldsymbol{\sigma}_1&\mathbf{0}\\\mathbf{0}&-\boldsymbol{\sigma}_1\end{pmatrix} & i\begin{pmatrix}\boldsymbol{\sigma}_2&\mathbf{0}\\\mathbf{0}&-\boldsymbol{\sigma}_2\end{pmatrix} & \begin{pmatrix}\boldsymbol{\sigma}_3&\mathbf{0}\\\mathbf{0}&-\boldsymbol{\sigma}_3\end{pmatrix} & \Big|\,\gamma^\mu\gamma^5\\
\begin{pmatrix}\mathbf{0}&\mathbf{I}\\\mathbf{I}&\mathbf{0}\end{pmatrix} & \begin{pmatrix}\boldsymbol{\sigma}_1&\mathbf{0}\\\mathbf{0}&\boldsymbol{\sigma}_1\end{pmatrix} & i\begin{pmatrix}\boldsymbol{\sigma}_2&\mathbf{0}\\\mathbf{0}&\boldsymbol{\sigma}_2\end{pmatrix} & \begin{pmatrix}\boldsymbol{\sigma}_3&\mathbf{0}\\\mathbf{0}&\boldsymbol{\sigma}_3\end{pmatrix} & \Big|\,\gamma^0\gamma^\mu\gamma^5
\end{array} \tag{A.1}$$

The fifth column gives the compact form of this particular row. The $\mathbf{I}$ are the 2×2 identity matrices and $\boldsymbol{\sigma}_j$ are the 2×2 Pauli matrices given in (A.2). These 16 matrices do not form a group Lie Group because they do not satisfy the Cliford algebra.

1. SU(2,2) Matrices

The $SU(2,2)$ are the Pauli matrices and these are given by:

$$\boldsymbol{\sigma}_1=\frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix},\ \boldsymbol{\sigma}_2=\frac{1}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix},\boldsymbol{\sigma}_3=\frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}, \tag{A.2}$$

and these are listed from left to right in the order $\mathcal{T}_1,\mathcal{T}_2,\mathcal{T}_3$ respectively. From these we derive the $SU(2,4)$ matrices.

2. SU(2,4) Matrices

For the $SU(2,4)$ set of matrices, there are 6 sets and in relation to the electroweak force, these are related to the settings $(0,0,g_\mu^{*2},g_\mu^{*3})$, $(0,g_\mu^{*1},0,g_\mu^{*})$, $(0,g_\mu^{*1},g_\mu^{*2},0)$, $(g_\mu^{*0},0,0,g_\mu^{*3})$, $(g_\mu^{*0},0,g_\mu^{*2},0)$ and $(g_\mu^{*0},g_\mu^{*1},0,0)$.

$\mathcal{T}_{kw}$

(**k=1:**) For the configuration $(0,0,g_\mu^{*2},g_\mu^{*3})$, we have:

$$\frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix},\ \frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&-i\\0&0&i&0\end{pmatrix},\ \frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}, \tag{A.3}$$

These matrices are listed from left to right in the order $\mathcal{T}_{k1},\mathcal{T}_{k2},\mathcal{T}_{k3}$ where in this case $k=1$. Thise order will be maintained when listing the ther matrices hereafter.

(**k=2:**) For the configuration $(0,g_\mu^{*1},0,g_\mu^{*3})$, we have:

$$\frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&1&0&0\end{pmatrix},\ \frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&-i\\0&0&0&0\\0&i&0&0\end{pmatrix},\ \frac{1}{2}\begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&-1\end{pmatrix}, \tag{A.4}$$

(**k=3:**) For the configuration $(0,g_\mu^{*1},g_\mu^{*2},0)$, we have:

$$\frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&0\end{pmatrix},\ \frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&-i&0\\0&i&0&0\\0&0&0&0\end{pmatrix},\ \frac{1}{2}\begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&0\end{pmatrix}. \tag{A.5}$$

(**k=4:**) For the configuration $(g_\mu^{*0},0,0,g_\mu^{*3})$ we have:

$$\frac{1}{2}\begin{pmatrix} 0&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{pmatrix}, \frac{1}{2}\begin{pmatrix} 0&0&0&-i\\0&0&0&0\\0&0&0&0\\i&0&0&0\end{pmatrix}, \frac{1}{2}\begin{pmatrix} 1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&-1\end{pmatrix}. \tag{A.6}$$

(**k=5:**) For the configuration $(g_\mu^{*0}, 0, g_\mu^{*}, 0)$, we have:

$$\frac{1}{2}\begin{pmatrix} 0&0&1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{pmatrix}, \frac{1}{2}\begin{pmatrix} 0&0&-i&0\\0&0&0&0\\i&0&0&0\\0&0&0&0\end{pmatrix}, \frac{1}{2}\begin{pmatrix} 1&0&0&0\\0&0&0&0\\0&0&-1&0\\0&0&0&0\end{pmatrix}. \tag{A.7}$$

(**k=6:**) For the configuration $(g_\mu^{*0}, g_\mu^{*1}, 0, 0)$, we have:

$$\frac{1}{2}\begin{pmatrix} 0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}, \frac{1}{2}\begin{pmatrix} 0&-i&0&0\\i&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}, \frac{1}{2}\begin{pmatrix} 1&0&0&0\\0&-1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}. \tag{A.8}$$

To have 3×6 $SU(2,4)$ matrices implies the existence of $3\times 6 = 18$ gauge bosons for the $SU(2,4)$-force. Taking into account that these gauge bosons $(A_{kw\mu}^{(a)})$ come in three types $a = 1, 2, 3$, this leads to a total of $3\times 6\times 3 = 54$ and if we add the photon that comes along with these force, we will have a total of $54 + 1 = 55$ gauge bosons for the $SU(2,4)$-force.

3. SU(2,4) × SU(2,4) Matrices

There are two configurations of the $SU(2,4) \times SU(2,4)$ group hence there are two set of matrices leading to $SU(2,4)\times SU(2,4)$ gauge invariance of $\left(\phi, \psi, A_\mu^{(a)}\right)$: $(\boldsymbol{\mathcal{T}_{kl}^{(SS)}}$ **:**)

(**k=1:**) For the first configuration is given by:

$$\begin{pmatrix} \boldsymbol{\sigma}_1 & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\sigma}_1\end{pmatrix}, \begin{pmatrix} \boldsymbol{\sigma}_2 & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\sigma}_2\end{pmatrix}, \begin{pmatrix} \boldsymbol{\sigma}_3 & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\sigma}_3\end{pmatrix} \tag{A.9}$$

(**k=2:**) For the second configuration is given:

$$\begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma}_1\\ \boldsymbol{\sigma}_1 & \mathbf{0}\end{pmatrix}, \begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma}_2\\ \boldsymbol{\sigma}_2 & \mathbf{0}\end{pmatrix}, \begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma}_3\\ \boldsymbol{\sigma}_3 & \mathbf{0}\end{pmatrix} \tag{A.10}$$

(**k=3:**) For the third configuration is given:

$$\begin{pmatrix} \boldsymbol{\sigma}_1 & \boldsymbol{\sigma}_1\\ \boldsymbol{\sigma}_1 & \boldsymbol{\sigma}_1\end{pmatrix}, \begin{pmatrix} \boldsymbol{\sigma}_2 & \boldsymbol{\sigma}_2\\ \boldsymbol{\sigma}_2 & \boldsymbol{\sigma}_2\end{pmatrix}, \begin{pmatrix} \boldsymbol{\sigma}_3 & \boldsymbol{\sigma}_3\\ \boldsymbol{\sigma}_3 & \boldsymbol{\sigma}_3\end{pmatrix} \tag{A.11}$$

where $\boldsymbol{\sigma}_k$ and $\mathbf{0}$ are the 2×2 Pauli matrices (given in A.2) and the null matrix respectively.

4. SU(3,3) Matrices

The 3×3 Gell-Mann matrices are given:

$$\tfrac{1}{2}\begin{pmatrix} 0&1&1\\1&0&0\\0&0&0\end{pmatrix} \tfrac{1}{2}\begin{pmatrix} 0&-i&0\\i&0&0\\0&0&0\end{pmatrix} \tfrac{1}{2}\begin{pmatrix} 1&0&0\\0&-1&0\\0&0&0\end{pmatrix} \tfrac{1}{2}\begin{pmatrix} 0&0&1\\0&0&0\\1&0&0\end{pmatrix} \tfrac{1}{2}\begin{pmatrix} 0&0&-i\\0&0&0\\i&0&0\end{pmatrix} \tfrac{1}{2}\begin{pmatrix} 0&0&0\\0&0&1\\0&1&0\end{pmatrix} \tfrac{1}{2}\begin{pmatrix} 0&0&0\\0&0&-i\\0&i&0\end{pmatrix} \tfrac{1}{2\sqrt{3}}\begin{pmatrix} 1&0&0\\0&1&0\\0&0&-2\end{pmatrix}, \tag{A.12}$$

and these are listed from left to right in the order $\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3, \mathcal{T}_4, \mathcal{T}_5, \mathcal{T}_6, \mathcal{T}_7, \mathcal{T}_8$, respectively. From these we derive the $SU(3,4)$ matrices.

5. SU(3,4) Matrices

For the $SU(3,4)$ set of matrices, there are 4 sets and in relation to the electrostrong force, these are related to the settings $(0, g_\mu^{*1}, g_\mu^{*2}, g_\mu^{*3})$, $(g_\mu^{*0}, 0, g_\mu^{*2}, g_\mu^{*3})$, $(g_\mu^{*0}, g_\mu^{*1}, 0, g_\mu^{*3})$, and $(g_\mu^{*0}, g_\mu^{*1}, g_\mu^{*2}, 0)$: $(\boldsymbol{\mathcal{T}_{ks}^{(s)}}$ **:**)

(**k=1:**) For the configuration $(0, g_\mu^{*1}, g_\mu^{*2}, g_\mu^{*3})$, we have:

$$\begin{array}{cccc} \frac{1}{2}\begin{pmatrix} 0&0&0&0\\ 0&0&1&1\\ 0&1&0&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&0&0\\ 0&0&-i&0\\ 0&i&0&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&0&0\\ 0&1&0&0\\ 0&0&-1&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&0&0\\ 0&0&0&1\\ 0&0&0&0\\ 0&1&0&0 \end{pmatrix} \\ \frac{1}{2}\begin{pmatrix} 0&0&0&0\\ 0&0&0&-i\\ 0&0&0&0\\ 0&i&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&-i\\ 0&0&i&0 \end{pmatrix} & \frac{1}{2\sqrt{3}}\begin{pmatrix} 0&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&-2 \end{pmatrix} \end{array}. \tag{A.13}$$

(**k=2:**) For the configuration $(g_\mu^{*0}, 0, g_\mu^{*2}, g_\mu^{*3})$, we have:

$$\begin{array}{cccc} \frac{1}{2}\begin{pmatrix} 0&0&1&1\\ 0&0&0&0\\ 1&0&0&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&-i&0\\ 0&0&0&0\\ i&0&0&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&0&-1&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&0&1\\ 0&0&0&0\\ 0&0&0&0\\ 1&0&0&0 \end{pmatrix} \\ \frac{1}{2}\begin{pmatrix} 0&0&0&-i\\ 0&0&0&0\\ 0&0&0&0\\ i&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&-i\\ 0&0&i&0 \end{pmatrix} & \frac{1}{2\sqrt{3}}\begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&0&1&0\\ 0&0&0&-2 \end{pmatrix} \end{array}. \tag{A.14}$$

(**k=3:**) For the configuration $(g_\mu^{*0}, g_\mu^{*1}, 0, g_\mu^{*3})$, we have:

$$\begin{array}{cccc} \frac{1}{2}\begin{pmatrix} 0&1&0&1\\ 1&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&-i&0&0\\ i&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 1&0&0&0\\ 0&-1&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&0&1\\ 0&0&0&0\\ 0&0&0&0\\ 1&0&0&0 \end{pmatrix} \\ \frac{1}{2}\begin{pmatrix} 0&0&0&-i\\ 0&0&0&0\\ 0&0&0&0\\ i&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&0&0\\ 0&0&0&1\\ 0&0&0&0\\ 0&1&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&0&0\\ 0&0&0&-i\\ 0&0&0&0\\ 0&i&0&0 \end{pmatrix} & \frac{1}{2\sqrt{3}}\begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&0&0\\ 0&0&0&-2 \end{pmatrix} \end{array}. \tag{A.15}$$

(**k=4:**) For the configuration $(g_\mu^{*0}, g_\mu^{*1}, g_\mu^{*2}, 0)$, we have:

$$\begin{array}{cccc} \frac{1}{2}\begin{pmatrix} 0&1&1&0\\ 1&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&-i&0&0\\ i&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 1&0&0&0\\ 0&-1&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&1&0\\ 0&0&0&0\\ 1&0&0&0\\ 0&0&0&0 \end{pmatrix} \\ \frac{1}{2}\begin{pmatrix} 0&0&-i&0\\ 0&0&0&0\\ i&0&0&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&0&0\\ 0&0&1&0\\ 0&1&0&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0&0&0&0\\ 0&0&-i&0\\ 0&i&0&0\\ 0&0&0&0 \end{pmatrix} & \frac{1}{2\sqrt{3}}\begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&-2&0\\ 0&0&0&0 \end{pmatrix} \end{array}. \tag{A.16}$$

To have 8×4 $SU(3,4)$ matrices implies the existence of $8 \times 4 = 32$ gauge bosons for the $SU(3,4)$-force. Taking into account that these gauge bosons $(A_{kw\mu}^{(a)})$ come in three types $a = 1, 2, 3$, this leads to a total of $8 \times 4 \times 3 = 66$ and if we add the photon that comes along with this force, we will have a total of $66 + 1 = 67$ gauge bosons for the $SU(3,4)$-force.

6. SU(4,4) Matrices

There is only one set of $SU(4,4)$ and this set is:
($\mathcal{T}_l^{(S)}$ **:**)

$$\begin{array}{ccccc}
\frac{1}{2}\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} &
\frac{1}{2}\begin{pmatrix}0&-i&0&0\\i&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} &
\frac{1}{2}\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} &
\frac{1}{2}\begin{pmatrix}0&0&1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{pmatrix} &
\frac{1}{2}\begin{pmatrix}0&0&-i&0\\0&0&0&0\\i&0&0&0\\0&0&0&0\end{pmatrix} \\
\frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&0\end{pmatrix} &
\frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&i&0\\0&-i&0&0\\0&0&0&0\end{pmatrix} &
\frac{\sqrt{3}}{6}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-2&0\\0&0&0&0\end{pmatrix} &
\frac{1}{2}\begin{pmatrix}0&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{pmatrix} &
\frac{1}{2}\begin{pmatrix}0&0&0&-i\\0&0&0&0\\0&0&0&0\\i&0&0&0\end{pmatrix} \\
\frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&1&0&0\end{pmatrix} &
\frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&i\\0&0&0&0\\0&-i&0&0\end{pmatrix} &
\frac{1}{2}\begin{pmatrix}0&0&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix} &
\frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&i\\0&0&-i&0\end{pmatrix} &
\frac{1}{2\sqrt{6}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-3\end{pmatrix}
\end{array} \tag{A.17}$$

(Adapted from Lee & Chen-Tsai 1965)

To have 16 $SU(4,4)$ matrices implies the existence of 16 gauge bosons for the $SU(4,4)$-force. Taking into account that these gauge bosons ($A^{(a)}_{k\mu}$) come in three types $a = 1, 2, 3$, this leads to a total of $16 \times 3 = 48$ and if we add the photon that comes along with these force, we will have a total of $48 + 1 = 49$ gauge bosons for the $SU(2,4)$-force. In total, counting the $SU(2,4)$, $SU(3,4)$ and the $SU(4,4)$-force, we have $55 + 67 + 49 = 171$ intermidiating gauge bosons.

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