Mathematical and phenomenological elements of the twin-tori model of physics and cosmology.

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Abstract:

In this, a follow up to a previous paper 'A Short Article On A Newly Proposed Model Of Cosmology' (viXra:0909.0005), some of the basic mathematical structures to be used in the formulation of the model are shown, and several advantages are discussed. The paper then takes a more phenomenological approach and several simple (1+1) dimensional models are explored.

Introduction:

Over the course of the last 3-4 hundred years one thing stands out about the advancement of physical models of reality and the advancement of the science of cosmology. These two things are deeply intertwined, and a revolution in physical thinking such as that brought about by the replacement of Newtonian physics (which implied a universe that is infinite in space and yet static and described very accurately the motions of objects and the movements of the planets), by relativistic physics (which implies a finite and dynamic universe, obeying strict laws derived from physical principles and explained things such as the precession of the perihelion of mercury), almost uniformly comes with a revolution in cosmological thinking. The same can be said of the partial replacement of relativistic physics with semi-classical physics, in such cases of the hawking model of black hole entropy and the inflationary big bang model.

However, only over the last 100-150 years has another factor has become prominent. This factor is the development of mathematics and mathematical methods. Newtonian physics can almost completely be formulated in terms of simple algebra, geometry, and differential and integral equations (the Lagrangian & Hamiltonian Formulations are excluded here). Special Relativity requires algebra and differential equations also, but in addition it requires a different form of mathematics for its clearest expression, Group Theory & Affine Spaces (Minkowski space), this was totally new to most physicists at the time and was not totally understood until a long time after its publication in this form.

Quantum Mechanics again requires new and more abstract mathematics in addition to the traditional mathematics of algebra, ODE's and PDE's. Group theory, vector spaces of infinite dimension (Hilbert Space), Functional Analysis, a deep understanding of matrix theory and operator theory etc.

Quantum Field theory and General relativity also require new mathematics for their best formulation. The purpose of this paper is to give a short development of the mathematical structures and techniques used in the Twin-Tori model, in the form in which they will be used in the model itself. The development of the mathematics itself will be totally rigorous and in line with the same kind of development as it would be done in pure mathematics, but the language in which its role in the model will be described may be a little more heuristic. Also included are some phenomenological aspects of the model and some simple (1+1)-dimensional models that emerge from the physics of the new model, in reference to quantum gravity and the detailed modelling of the behaviour of dark matter and dark energy.

A Heuristic Overview Of The Twin Tori Model:

The Twin-Tori model has some of its roots in the Darkfield Theory of Dan Visser [1] [2]. It takes as its central axiom that the universe is in the shape of a finite, dynamical torus. Basic phenomenology is then used in an attempt to explain the dynamics of the observable universe that are currently 'explained' by the presence of dark energy. This leads, through theoretical considerations to an integral equation with the 'dark energy force' as its subject:

$$\int (\alpha F_{de}^2 + \gamma)\, dx = k \qquad \text{where } k \in \mathbb{R}$$

Dimensional analysis and evaluation shows that: $\alpha = G$ ($\approx 6.67 \times 10^{-27}$ N m^2 kg^{-2}) and $\gamma = -\frac{1}{4}c^4\hbar^2 G M^6$ ($\approx 10^{-61} M^6$) where M is the mass equivalent of the amount of dark energy producing the force.

This equation implies that: $\alpha F_{de}^2 + \gamma = 0 \Rightarrow F_{de} = \pm\frac{1}{2} (L_{PLANCK})^2 c^5 M^3 G^{-1}$.

In the Darkfield Theory of Dan Visser this equation is derived from a thought experiment, but it is constrained so that only the solution with the negative sign is produced. To interpret this equation is not difficult, if the dark energy force is negative then in this case it produces a repulsive force (whilst Newtonian gravity also produces a negative force and this force is attractive it is just a matter of reversing the direction in which the force is defined to act), and so in the Darkfield Theory model the location of dark energy is defined as a 'dark-energy globe' in the centre of the torus-shaped universe. The Dark energy is contained 'in the hole of the doughnut' so to speak. From a physical perspective this produces problems, in that this dark-energy globe is outside the bounds of the universe and is thus an untestable element theory.

As the starting point for the twin-tori model the quadratic integral for the dark energy force is derived as the imaginary part of a two-part complex integral, derived from a Quantum-Gravity framework to be published soon. Due to theoretical considerations and the desire to produce a consistent and potentially testable theory we take the positive sign in the equation, and thus consider dark energy to produce an *attractive* force. This is a break from the convention in physics that dark energy is a repulsive form of energy. However we retain consistency with data and current thinking by reinterpreting the placement of dark energy in the grander scale of cosmology.

In the twin-tori model, as can be surmised from the name, there exist two interacting tori, although actually there exists only 1 material torus and the other is contained inside an empty 'tube' inside. The torus is, for a want of a better description, made of dark energy. In the centre part of the ring of the torus there exists a 'tube' of radius
$\approx 10^{90}$ m. This 'tube' is not empty space, even in the quantum sense, but is our universe. This universe is both much larger than that predicted by the big bang model and yet much more dense. The boundary of our universe is essentially the inner boundary of the hollow dark energy torus, which extends some 10^{500} km from its inner boundary to its outer edge. This does not produce a Multiverse theory. This does not mean that after the 10^{500}km the ere is nothing either, these are just the boundaries at which the predictability of the twin-tori models breaks down.

Additional predictions within the model include the existence of population 3 stars, which the big bang model does not refer to, and also theoretical boundaries of the number of brown dwarf stars within a given envelope radius.

Fundamental structures within the twin-tori model:

In this section several important mathematical structures of direct relevance to the twin-tori models are either developed using standard axioms, or are defined as new. At least 2 of the proofs included are relatively trivial but to increase the readability of this paper to as many interested people as possible they are included for completeness.

Theorem 1: The set of rational number $\mathbb{Q}$ forms a field.
To prove this assertion is simple, and requires only the axioms of a group, and those of a field.

Define a set G, with binary operations + and $\times$: $\{G,+,\times\}$
Axioms of a Field:

I. The set G forms an abelian group under the binary operation +, with the identity element called 0.
II. Define G'=G-{0}, then the system {G',$\times$} is an abelian group with an identity element called e.
III. $\forall$a,b,c $\in$ G then a $\times$ (b+c) = (a$\times$b) + (a$\times$c).

To see that these axioms are satisfied by the rational numbers is trivial. The field of rational numbers is now taken to be the field of scalars over which a vector space is defined. However this vector space is not a space of algebraic or geometrical vectors, but the 'vectors' in this case are defined to be functions of the spacetime co-ordinates. These functions have a physical interpretation within the domain of the twin-tori models. They describe the motion of the particles assigned to them, in the direction of the spacetime co-ordinate of the function. An example may make this clear:

An electron is instantaneously at the origin of the 4-dimensional Cartesian co-ordinate system defined by it. It has motion in each of the 4 spacetime directions. Mathematically this is described within the twin-tori models by assigning to the electron a 4-vector of functions of the spacetime co-ordinates:

$(ct, x, y, z) \rightarrow (\sigma(ct), \tau(x), \varphi(y), \psi(z))$

These functions are taken to form a vector space, in accordance with quantum mechanics. What is meant by this is that wave-particle duality still holds and, as such, so does the superposition principle, and so if there were 2 particles described by: $X=(a(ct), b(x), c(y), d(z))$ and $Y=(e(ct), f(x), g(y), h(z))$ then the linear combination $AX+BY$ where A and B are rational numbers in this case (elements of the field of scalars) describes a new particle. It is in this way that the functions form a vector space according to the usual axioms. All functions are, in this case, assumed to be continuous and differentiable at least twice. Within the framework of the model the equations, and relevant physics, occasionally give rise to functions where this is not obvious and so this must be proven in those cases. This is not the case in this paper and is mentioned only for information purposes.

<u>Theorem 2: The set of all continuous and twice differentiable functions of a single variable forms a linear vector space over the field of rational numbers.</u>

A major advantage of this framework, as it has been presented, is that in simple situations the partial differential equations describing motion and time evolution of particles or systems can be mostly solved by the method of separation of the variables, greatly simplifying the analysis. In more complex situations however, and hence in most but not all realistic situations, this is not the case. However it also allows for motion in only one of the spacetime directions to be considered at any one time, and of course this can always be accomplished by a rotation of the co-ordinate system, as well as other simplifying tools such as Lorentz transformations to allow for analysis in the rest frame of the particle etc.

The fundamental symmetries of the theory are not all fully known yet, and hence a full discussion here is impossible. However it is possible to say at this point that rotations, including those which linearly mix time and spatial co-ordinates, in any number $m \leq n$ dimensions are possible. Where n is the number of spatial dimensions in which the theory is set.

Definition: The algebra formed by the spacetime functions:

The spacetime functions of single variable form groups with respect to their own parameters: $G_1(x)$, $G_2(y)$, $G_3(z)$, $G_0(ct)$. This imposes conditions on the forms the functions can take, however it is not appropriate to go into this here. However to avoid physical inconsistencies such as boson→fermion transformations, and
Baryonic matter→Dark matter transformations, which cannot be allowed, we must define commutation relations for the functions of the spacetime co-ordinates.

Let $f(x) \in G_1(x)$, $F(y) \in G_2(y)$ then
$[f(x), F(y)] \stackrel{def}{=} 2F(y)f(x) \Rightarrow \{f(x), F(y)\} = 0 \ \forall f(x) \in G_1(x), \forall F(y) \in G_2(y)$
Where $[A,B] = AB-BA$ is the commutator, and $\{C,D\}=CD+DC$ is the anti-commutator.

Also: $\{f(x),g(z)\}=\{F(y),H(z)\}=0$
$\Rightarrow [f(x),g(z)] \stackrel{def}{=} 2f(x)g(z) \quad \forall f(x) \in G_1(x), \forall g(z) \in G_3(z)$
$\Rightarrow [F(y),H(z)] \stackrel{def}{=} 2F(y)H(z) \ \ \forall F(y) \in G_2(y), \forall H(z) \in G_3(z)$
And now the only element left is the time-space element and the self-commutation rules.

$[f(ct),g(x)] \stackrel{def}{=} [f(ct),h(y)] \stackrel{def}{=} [f(ct),j(z)] \stackrel{def}{=} 0 \ \forall f(ct) \in G_0(ct)$ etc
$[f(x),g(x)] \stackrel{def}{=} [h(y),i(y)] \stackrel{def}{=} [j(z),k(z)] \stackrel{def}{=} [m(ct),n(ct)] \stackrel{def}{=} 0$
$\Rightarrow G_0(ct),G_1(x),G_2(y),G_3(z)$ are all isomorphic to each other and all abelian.

A paper on the physical relevance and application of this mathematics, and more details of it, will be published by the author as soon as it is completed.

Application of twin-tori quantum gravity to simple (1+1)-dimensional Cosmology:

In this section a simple application of some of the physics of the twin-tori model is given, purely as an example of the style of work.

Starting with the equation for the expansion of the universe in (1+1) dimensions, an ODE, we derive its linearly independent solutions and simulate these for different values of the structure constants:

The equation for length expansion as demonstrated in the twin-tori model is given by:

$$D^2x + t^{-1}Dx=0$$

To obtain the general solution for this equation is a very difficult task and is only possible in practice in numerical form. However it can be shown that a linearly independent solution to this equation is given by $x=A\ln(t)$, Where $A\in\mathbb{R}$, however using the restrictions imposed earlier in this paper it is obvious that $A\in\mathbb{Q}$. If we use this to model the expansion of the (1+1) dimensional universe whilst taking $A=1$ for simplicity we obtain a table of the form given in appendix 1.

Conclusions:

The twin-tori model, when interpreted as a physical theory of the universe, enables us to understand the most fundamental facts of the intersection of the standard fields of astronomy, cosmology, particle physics and string phenomenology from a basic understanding of dark energy and dark matter as what can be reduced to a quantum field theory which includes gravity. Whilst in its current form it is insufficiently developed to allow a full understanding of any of these things, there are hints and idea's appearing that allow us to speculate that one day a full understanding will be possible. In many ways the twin-tori model is in very much the same situation as string theory in that whist it is making progress rapidly and understanding is always increasing it has not yet reached the point where it can be fully accepted. We can only hope that this happens soon.

Appendix 1:

Below is a table of the relative expansion dynamics in a (1+1) dimensional twin-tori model, where t is in units of seconds divided by the speed of light and x is in light-seconds. (t=1,x=0) may be interpreted as the illusive big bang, and a failure of the simplified twin-tori equations.

t	x
1	0
10	2.302585
100	4.60517
1000	6.907755
10000	9.21034
100000	11.51293
1000000	13.81551
10000000	16.1181
1E+08	18.42068
1E+09	20.72327
1E+10	23.02585
1E+11	25.32844
1E+12	27.63102
1E+13	29.93361
1E+14	32.23619
1E+15	34.53878
1E+16	36.84136
1E+17	39.14395
1E+18	41.44653
1E+19	43.74912
1E+20	46.0517
1E+21	48.35429
1E+22	50.65687
1E+23	52.95946
1E+24	55.26204
1E+25	57.56463
1E+26	59.86721
1E+27	62.1698
1E+28	64.47238

References:

[1] www.darkfieldnavigator.com, Dan Visser

[2] 'Time Torus', Dan Visser

[3] A Short Essay On A Newly Proposed Model Of Cosmology, Dan Visser, Chris Forbes, Keith Lees: viXra:0909.0005