extra dimensions? Are they necessary ?

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Plan of the talk

What is known about string theory, LQG, and also other models

Deceleration parameter, q(Z) and its impact upon DM/ DE and Cosmological expansion rates

Entropy balance as to sum total of entropy of galactic black holes, As compared to the total entropy of the universe

Based on**:**

-A.W. Beckwith, due to the following .,

- . ***viXra:0909.0023*** ***[pdf]*** *submitted on 8 Sep 2009*

- **viXra:0909.0018 [pdf]** ***submitted on 6 Sep 2009***
- **viXra:0909.0017 [pdf]** ***submitted on 5 Sep 2009***
- **viXra:0909.0016 [pdf]** ***submitted on 5 Sep 2009***

Motivations

Calculation of q(Z) deceleration parameter may have links to DM/DE

Crucial to provide accurate theoretical predictions
for the statistics of anisotropies and structure formation
Here is the problem. The Galaxy hierarchy formation is
Wrong. Need a replacement for it.

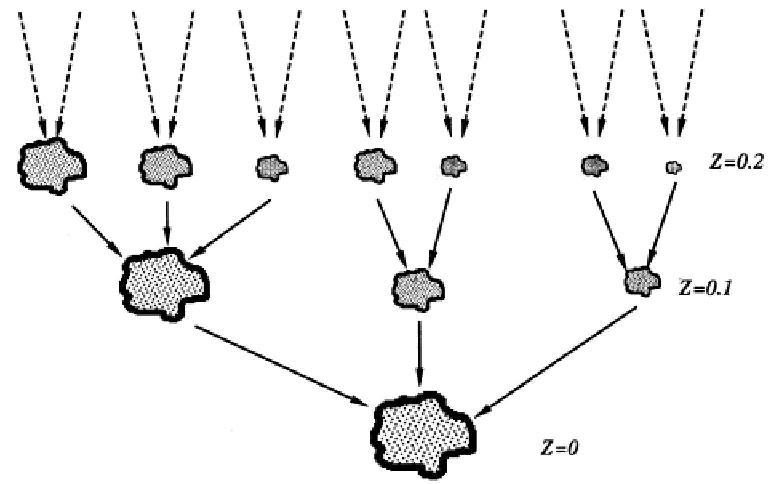


Figure 1. A schematic representation of a halo merging history 'tree'.

Large-scale deceleration and Acceleration re commencing?, sign of q(Z)

If gravitons have a slight mass, can we obtain DE expansion ? YES we can. And the 5 dimensional q(Z) deceleration is the same as the 4 dimesional q(Z) deceleration result!

To examine this, consider the following graph. It has DE /DM connectivity, due to KK graviton Linkage. Z ~ X, red shift. I.e. Z<.55 to Z ~0 is when acceleration increases

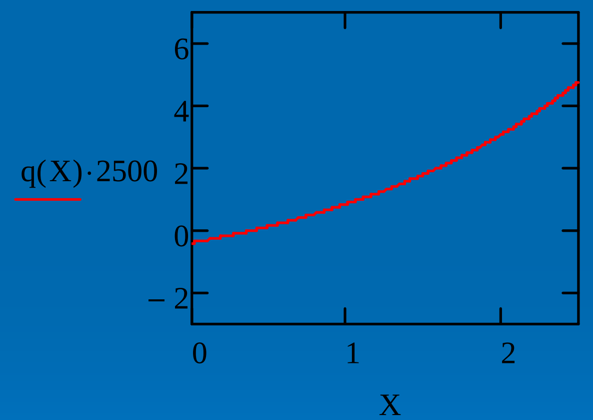


Figure 4 b: re duplication of basic results of Marcio E. S. Alves, Oswaldo D. Miranda, Jose C. N. de Araujo, 2009, using their parameter values, with an additional term of C for'Dark flow' added, corresponding to one KK additional dimensions. Results show asymptotic 'collapse' of de celebration parameter as one comes away from the red shift Z =1100 of the CMBR 'turn on ' regime for de coupling of photons from 'matter' , in end of 'dark ages' Figures **4a,** and **4b** suggest that additional dimensions are permissible. They do not state that the initial states of GW/ initial vaccum states have to form explicitly due to either quatum or

If addition of higher dimensions is not the problem then what IS the problem ? In a word, it is the introduction of compression of initial states, in the origins of gravitons, and other sources of entropy.

I.e. if gravitons, and other initial processes create relic entropy, then what is the difference between LQG, and string theory & KK theory?

String theory leads to more entropy produced by Super massive Black holes in the center of Galaxies, than would LQG.
The relevant formula to consider is the following.

Total amount of entropy = ? Sum of entropy in Super massive black holes at the center of galaxies

Up to 1 million SM black holes!

Now for the relevant S (entropy) formula

$$S_{Black-Hole}^{\surd} \sim 10^{90} \cdot \left[\frac{M}{10^{6} \cdot M_{Solar-Mass}} \right]^{2}$$

√

$$S_{Total} \sim 10^{88}$$

String theory, with large additional dimensions leads to one million super massive Black holes with the first entropy result which does NOT equal the 2nd result

We are now identifying the problem, I.e. not Extra dimensions, but HUGE extra dimensions

- Large extra dimensions also leads to extremely low relic GW predicted frequencies. I.e. as low as one HERTZ. Huge extra dimensions are tied in with highly compressed, NON classical squeezed states at the onset of inflation. Way to no where,fast.

- Giovannini (1995) used STRING theory with compact, TINY higher **dimensions to predict relic GW up to 10 to the 10 power Hz. I.e. Calabi Yau manifold. Arkani Hamid, et al. want to have not tiny higher dimensions, but huge higher dimensions! Huge higher dimensions imply entropy imbalance between Number of SM black holes and entropy of Universe.**

Post-inflationary nonlinear gravitational dynamics is common to all scenarios

Sachs-Wolfe effect: a compact formula. Inputs From this affected by entropy generation

$$\frac{\Delta T}{T} = \frac{1}{3}\phi_{\varepsilon}^{(1)} + \frac{1}{18}\left(\phi_{\varepsilon}^{(1)}\right)^2 - \frac{K}{10} - \frac{5}{9}\left(a_{NL} - 1\right)\left(\phi_{\varepsilon}^{(1)}\right)^2$$

Post-inflation non-linear evolution of gravity: order unity NG

$$K = 10\nabla^{-4}\partial^i\partial_j\left(\partial^i\phi_{\varepsilon}^{(1)}\partial_j\phi_{\varepsilon}^{(1)}\right) - \frac{10}{3}\nabla^{-2}\left(\partial^i\phi_{\varepsilon}^{(1)}\partial_i\phi_{\varepsilon}^{(1)}\right)$$

Initial conditions set during or after inflation

$$\zeta^{(2)} = 2a_{NL}\left(\zeta^{(1)}\right)^2 \quad \begin{cases} \text{standard scenario} \\ a_{NL} = 1 + O(\varepsilon, \eta) \\ \\ \text{curvaton scenario} \\ a_{NL} = \dfrac{3}{4r} - \dfrac{r}{2} \end{cases}$$

N.B., Matarrese, Riotto, Phys. Rev. Lett. (2004)

Extracting the non-linearity parameter $f_{\rm NL}$

$$\frac{\Delta T}{T} = \frac{1}{3}\left(\phi_{(1)} + f_{\rm NL} * \phi_{(1)}^2\right)$$

$$\boxed{f_{\rm NL}(\mathbf{k}_1,\mathbf{k}_2) = \frac{5}{3}(a_{NL} - 1) - \frac{1}{6} + \mathrm{K}(\mathbf{k}_1,\mathbf{k}_2)}$$

Connection between theory and observations

This is the proper quantity measurable by CMB experiments, via the phenomenological analysis by *Komatsu and Spergel* (2001)

$$\mathrm{K}(\mathbf{k}_1,\mathbf{k}_2) = 3\frac{(\mathbf{k}_1 \cdot \mathbf{k})(\mathbf{k}_2 \cdot \mathbf{k})}{\mathrm{k}^4} - \frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)}{\mathrm{k}^2} \qquad \mathrm{k} = |\,\mathbf{k}_1 + \mathbf{k}_2\,|$$

Now for a surprising fact about LQG, which does not need additional dimensions

- BOJOWALD (2008) came up with a model of a bounce effect from a contracting universe, which leaves the question of if or not squeezed states (non classical in behavior) are necessary. ‘Quantum’ bounce means shift from contraction to super inflation.

Non classical initial states would affect generation of entropy , implying entropy of one million SM black holes at center of one million galaxies = entropy of Universe ~ 10 to the 90^{th} power magnitude.

Squeezing of initial states may be either tiny, or moderate or very large. I.e. it is an OPEN Problem, BOJOWALD (2008)

Conclusion. I prefer LQG

Angular decomposition

$$a_{lm} = \int d^2\mathbf{n}\ \frac{\Delta T}{T}(\mathbf{n})\, Y^*_{lm}(\mathbf{n})$$

The linear and non-linear parts of the temperature fluctuations correspond to a linear Gaussian part and a non-Gaussian contribution

$$a_{lm} = a^{L}_{lm} + a^{NL}_{lm}$$

At linear order

$$a^{\mathrm{L}}_{\ell m} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} \phi_{1i}(\mathbf{k}) \Delta^{(1)}_{\ell}(k) Y^*_{\ell m}(\hat{\mathbf{k}})$$

Initial fluctuations

Linear radiation transfer function

Ex: Linear Sachs-Wolfe $\Delta^{(1)}_l(k) = j_l[k(\eta_0 - \eta_*)]/3$

Linear ISW $\Delta^{(1)}_l(k) \propto \int d\eta\, g'(\eta)\ j_l[k(\eta_0 - \eta)]$

2nd-order radiation transfer function on large-scales

Express the observed CMB anisotropies in terms of the quadratic curvature perturbations

$$a_{\ell m}^{\rm NL} = 4\pi(-i)^\ell \int \frac{d^3k}{(2\pi)^3}\left[K_0(\mathbf{k})\Delta_\ell^{0(2)}(k) + K_1(\mathbf{k})\Delta_\ell^{1(2)}(k) + K_2(\mathbf{k})\Delta_\ell^{2(2)}(k)\right] Y_{\ell m}^*(\hat{\mathbf{k}})$$
$$+ (4\pi)^2 \sum_{L_1 M_1, L_2 M_2} (-i)^{L_1+L_2} \mathcal{G}_{\ell L_1 L_2}^{m M_1 M_2} \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3} \phi_*^{(1)}(\mathbf{k}_1)\phi_*^{(1)}(\mathbf{k}_2)\, \Delta_{L_1 L_2}(k_1,k_2)\, Y_{L_1 M_1}(\hat{\mathbf{k}}_1) Y_{L_2 M_2}(\hat{\mathbf{k}}_2)$$

with $\phi_*^{(1)}$ the gravitational potential at last scattering, and K_n are convolutions

$$K_n(\mathbf{k}) = \frac{1}{(2\pi)^3}\int d^3k_1 d^3k_2 \delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}) f_n(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k})\phi_*^{(1)}(\mathbf{k}_1)\phi_*^{(1)}(\mathbf{k}_2)$$

with kernels

$$f_0(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}) = -\frac{5}{3}(a_{\rm nl}-1)-1\,, \qquad f_1(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}) = 1$$
$$f_2(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}) = 3\frac{(\mathbf{k}_1\cdot\mathbf{k})(\mathbf{k}_2\cdot\mathbf{k})}{k^4} - \frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k^2}$$

$\Delta_l^{n(2)}(k)$ and $\Delta_{L_1 L_2}(k_1,k_2)$ generalize the radiation transfer function at second-order

2nd-order transfer functions for the Sachs-Wolfe

$$\Delta_\ell^{0(2)}(k) = \frac{1}{3} j_\ell(k(\eta_0 - \eta_*))$$

$$\Delta_\ell^{1(2)}(k) = \frac{7}{18} j_\ell(k(\eta_0 - \eta_*))$$

$$\Delta_\ell^{2(2)}(k) = -\frac{1}{3} j_\ell(k(\eta_0 - \eta_*))$$

$$\Delta_{L_1 L_2}(k_1, k_2) \equiv 0 .$$

$$\longleftrightarrow \quad \boxed{-\frac{5}{3}(a_{NL} - 1)}$$ Primordial NG

Non-linear evolution of the gravitational potentials ***after*** Inflation and
additional 2nd-order corrections to temperature anisotropies

2nd-order transfer functions for the late ISW

$g(\eta) = \phi^{(1)}(\mathbf{x},\eta)/\varphi_0$ growth suppression factor

$$\Delta_\ell^{0(2)}(k) = 2\int_{\eta_m}^{\eta_0} d\eta \frac{g'(\eta)}{g_m} j_\ell[k(\eta_0-\eta)]$$

$$\Delta_\ell^{1(2)}(k) = \int_{\eta_m}^{\eta_0} d\eta \frac{B_1'(\eta)}{g_m^2} j_\ell[k(\eta_0-\eta)]$$

$$\Delta_\ell^{2(2)}(k) = -\int_{\eta_m}^{\eta_0} d\eta \bar{B}(\eta) j_\ell[k(\eta_0-\eta)]$$

$-\frac{5}{3}(a_{NL}-1)$ Primordial NG

Non-linear evolution of the gravitational potentials ***after*** inflation

$$\Delta_{L_1L_2}(k_1,k_2) = -4\int_{\eta_m}^{\eta_0} d\eta \frac{g''(\eta)}{g_m} j_{L_1}[k_1(\eta_0-\eta)]\int_{\eta_0}^{\eta} d\tilde{\eta}\frac{g(\tilde{\eta})}{g_m} j_{L_2}[k_2(\eta_0-\tilde{\eta}]$$
$$+ 2\int_{\eta_m}^{\eta_0} d\eta \frac{g'(\eta)}{g_m} j_{L_1}[k_1(\eta_0-\eta)]\left[2\int_{\eta_m}^{\eta_0} d\eta \frac{g'(\eta)}{g_m} j_{L_2}[k_2(\eta_0-\eta)] + \frac{1}{3} j_{L_2}[k_2(\eta_0-\eta_*)]\right]$$

Additional second-order corrections to temperature anisotropies $(\text{ISW})^2$

Expression for 2nd-order Early ISW, vector and tensor modes available as well in B. N., Matarrese S., Riotto A., 2005, JCAP 0605

On large scales:
NG = NG from gravity (universal)+
NG primordial

Gravity itself is non-linear
Non-linear (second-order) GR perturbations in the standard cosmological model introduce some order unity NG:

√ we would be in trouble if NG turned out to be very close to zero

√ such non-linearities have a non-trivial form:
their computation ≡ core of the (large-scale) radiation transfer function at second-order

WHAT ABOUT SMALLER SCALES ?

Aim: - have a full radiation transfer function at second-order
for *all scales*

- in particular:
compute the CMB anisotropies generated by the non-linear dynamics of the photon-baryon fluid for subhorizon modes at recombination (acoustic oscillations at second-order)

Remember: crucial to extract information from the bispectrum are the scales of acoustic peaks according to the phenomenological analysis of *Komatsu and Spergel (2001)*

2nd-order CMB Anisotropies on all scales

Apart from gravity account also for: a) Compton scattering of photons off electrons

b) baryon velocity terms **v**

Boltzmann equation for photons

$$\frac{df}{d\eta} = a\,C[f]$$

Collision term

$$\underbrace{\frac{df}{d\eta} = \frac{\partial f}{\partial \eta} + \frac{\partial f}{\partial x^i}\frac{dx^i}{d\eta} + \frac{\partial f}{\partial p}\frac{dp}{d\eta} + \frac{\partial f}{\partial n^i}\frac{dn^i}{d\eta}}_{\text{Gravity effects}}$$

+ Boltzmann equations for baryons and CDM+ Einstein equations

Metric perturbations

Poisson gauge

$$ds^2 = a^2(\eta)\left[-e^{2\Phi}d\eta^2 + 2\omega_i dx^i d\eta + (e^{-2\Psi}\delta_{ij} + \chi_{ij})dx^i dx^j\right]$$

ω_i and χ_{ij} second-order vector and tensor modes.

Examples: using the geodesic equation for the photons

$$\frac{1}{p}\frac{dp}{d\eta} = -\mathcal{H} + \Psi' - \Phi_{,i}\, n^i e^{\Phi+\Psi} - \omega_i'\, n^i - \frac{1}{2}\chi_{ij}' n^i n^j \longrightarrow$$

Redshift of the photon
(***Sachs-Wolfe and ISW effects***)

$$\frac{dn^i}{d\eta} = (\Phi_{,k} + \Psi_{,k})\, n^k n^i - \Phi^{,i} - \Psi^{,i} \longrightarrow$$

Direction of the photons changes
due to gravitational potentials
Lensing effect (it arises at second-order)

PS: Here the photon momentum is $\mathbf{p} = pn^i$; $\quad p^2 = g_{ij}P^iP^j$
$(P^\mu = dx^\mu(\lambda)/d\lambda$ quadri-momentum vector)

Photon Boltzmann equation

Expand the distribution function in a linear and second-order parts around the zero-order Bose-Einstein value

$$f(x^i,p,n^i,\eta)=f^{(0)}(p,\eta)+f^{(1)}(x^i,p,n^i,\eta)+\frac{1}{2}f^{(2)}(x^i,p,n^i,\eta)$$

$$f^{(0)}(p,\eta)=2\,\frac{1}{\exp\left\{\frac{p}{T(\eta)}\right\}-1}$$

Left-hand side $df/d\eta$

$$\begin{aligned}\frac{df}{d\eta} &= \frac{df^{(1)}}{d\eta}+\frac{1}{2}\frac{df^{(2)}}{d\eta}-p\frac{\partial f^{(0)}}{\partial p}\frac{d}{d\eta}\left(\Phi^{(1)}+\frac{1}{2}\Phi^{(2)}\right)+p\frac{\partial f^{(0)}}{\partial p}\frac{\partial}{\partial\eta}\left(\Phi^{(1)}+\Psi^{(1)}+\frac{1}{2}\Phi^{(2)}+\frac{1}{2}\Psi^{(2)}\right)\\ &- p\frac{\partial f^{(0)}}{\partial p}\frac{\partial\omega_i}{\partial\eta}n^i-\frac{1}{2}p\frac{\partial f^{(0)}}{\partial p}\frac{\partial\chi_{ij}}{\partial\eta}n^in^j\,,\end{aligned}$$

The collision term C[f]

√ Up to ***recombination*** photons are tightly coupled to electrons via Compton scatterings e(**q**) γ(**p**) $\leftrightarrow$ e(**q'**) γ(**p'**). The collision term governs ***small scale anisotropies and spectral distorsions***

√ Important also for ***secondary scatterings***: reionization, kinetic and thermal Sunyaev-Zeldovich and Ostriker-Vishniac effects

√ **Crucial points to compute the second-order collision term:**

1) Little energy $\delta\varepsilon/T$ is transferred $\longrightarrow$ expand in the perturbations ***and*** in $\delta\varepsilon/T \approx q/m_e$. At linear order the Boltzmann equations depend only on n^i, **at second-order there is energy exchange (p dependence) and thus spectral distorsions**.

2) Take into account **second-order baryon velocity $v^{(2)}$**

3) Take into account **vortical components** of (first-order × first-order)

The 2nd-order brightness equation

$$\Delta^{(2)\prime} + n^i \frac{\partial \Delta^{(2)}}{\partial x^i} - \tau' \Delta^{(2)} = S$$

with $\tau' = -\bar{n}_e \sigma_T a$ optical depth

Source term $S = S^{(2)} + S^{(I\times I)}$

Second-order baryon velocity

Sachs-Wolfe effects

$$S^{(2)} = 4\Psi^{(2)\prime} - 4\Phi^{(2)}_{,i} n^i - 8\omega_i' - 4\chi_{ij}' n^i n^j - \tau' \left[\Delta^{(2)}_{00} - \Delta^{(2)} + 4\mathbf{v}^{(2)} \cdot \mathbf{n} - \frac{1}{2} \sum_{m=-2}^{2} \frac{\sqrt{4\pi}}{5^{3/2}} \Delta^{(2)}_{2m} Y_{2m}(\mathbf{n}) \right]$$

$$\begin{aligned} S^{(I\times I)} = &-8\Delta^{(1)}\left(\Psi^{(1)\prime} - \Phi^{(1)}_{,i} n^i\right) + 2n^i(\Phi^{(1)} + \Psi^{(1)})\partial_i(\Delta^{(1)} + 4\Phi^{(1)}) \\ &+ \left[(\Phi^{(1)}_{,j} + \Psi^{(1)}_{,j}) n^i n^j - (\Phi^{(1),i} + \Psi^{(1),i})\right] \frac{\partial \Delta^{(1)}}{\partial n^i} \\ &- \tau' \Big[2(\delta_e^{(1)} + \Phi^{(1)})\left(4\mathbf{v}\cdot\mathbf{n} + \Delta_0^{(1)} - \Delta^{(1)} + \frac{1}{2}\Delta_2^{(1)} P_2(\hat{\mathbf{v}}\cdot\mathbf{n})\right) \\ &+ 2(\mathbf{v}\cdot\mathbf{n})\left[\Delta^{(1)} + 3\Delta_0^{(1)} - \Delta_2^{(1)}\left(1 - \frac{5}{2}P_2(\hat{\mathbf{v}}\cdot\mathbf{n})\right)\right] - v\Delta_1^{(1)}(4 + 2P_2(\hat{\mathbf{v}}\cdot\mathbf{n})) + 14(\mathbf{v}\cdot\mathbf{n})^2 - 2v^2 \Big] \end{aligned}$$

Gravitational lensing

Quadratic-Doppler effect

Coupling velocity and linear photon anisotropies

Hierarchy equations for multipole moments

$$\Delta^{(2)\prime} + ik\mu\Delta^{(2)} - \tau'\Delta^{(2)} = S(\mathbf{k}, \mathbf{n}, \eta)$$

$\mathbf{e}_3 = \hat{\mathbf{k}}$ and $\mathbf{k} \bullet \mathbf{n} = \mu$

Expand the temperature anisotropies in multipole moments

$$\Delta^{(i)}_{\ell m} = (-i)^{-\ell}\sqrt{\frac{2\ell+1}{4\pi}}\int d\Omega\,\Delta^{(i)} Y^*_{\ell m}(\mathbf{n})$$

⇨ System of coupled differential equations

$$\Delta^{(2)\prime}_{\ell m}(\mathbf{k}, \eta) = k\left[\frac{\kappa_{\ell m}}{2\ell-1}\Delta^{(2)}_{\ell-1,m} - \frac{\kappa_{\ell+1,m}}{2\ell+3}\Delta^{(2)}_{\ell+1,m}\right] + \tau'\Delta^{(2)}_{\ell m} + S_{\ell m}$$

Free-streeming of photons:
From $n \bullet \nabla$. It gives a projection effect
of inhomogeinities onto anisotropries
At linear order responsible of the hierarchy

Suppression of anisotropies

Residual scattering effects and gravity

Integral Solution

$$\Delta^{(2)\prime} + ik\mu\Delta^{(2)} - \tau'\Delta^{(2)} = S(\mathbf{k}, \mathbf{n}, \eta)$$

$\mathbf{e}_3 = \hat{\mathbf{k}}$ and $\mathbf{k} \bullet \mathbf{n} = \mu$

One can derive an integral solution in term of the Source

$$\Delta^{(2)}(\mathbf{k}, \mathbf{n}, \eta_0) = \int_0^{\eta_0} d\eta S(\mathbf{k}, \mathbf{n}, \eta) e^{ik\mu(\eta - \eta_0) - \tau}$$

Important:

√ The main information is contained in the **Source, which contains peculiar effects from the non-linearity of the perturbations**

√ The integral solution is a formal solution (the source contains second-order moments up to l=2), but still more convinient than solving the whole hierarchy

CMB anisotropies generated at recombination

$$\Delta^{(2)}(\mathbf{k}, \mathbf{n}, \eta_0) = \int_0^{\eta_0} d\eta S(\mathbf{k}, \mathbf{n}, \eta) e^{ik\mu(\eta-\eta_0)-\tau}$$

KEY POINT: Extract all the effects generated at recombination (i.e. Isolate from the Source all those terms $\propto$ optical depth τ')

$$S = -\tau' S_* + S'$$

$$\Delta^{(2)}(\mathbf{k},\mathbf{n},\eta_0) = \int_0^{\eta_0} d\eta(-e^{-\tau}\tau')\, S_*(\mathbf{k},\mathbf{n},\eta)e^{ik\,\mu(\eta-\eta_0)} \simeq S_*(\mathbf{k},\mathbf{n},\eta_*)e^{ik\,\mu(\eta_*-\eta_0)}$$

Visibility function sharply peaks at recombination epoch η_*

$$S = -\tau' S_* + S'$$

Yields anisotropies generated at recombination due to the ***non-linear dynamics of the photon-baryon fluid***

$$\begin{aligned}
S_* &= \Bigg[\underline{\Delta_{00}^{(2)} + 4\Phi^{(2)} + 4\mathbf{v}^{(2)} \cdot \mathbf{n}} - \frac{1}{2} \sum_{m=-2}^{2} \frac{\sqrt{4\pi}}{5^{3/2}} \Delta_{2m}^{(2)} Y_{2m}(\mathbf{n}) + 2(\delta_e^{(1)} + \Phi^{(1)}) \left(\Delta_0^{(1)} - \Delta^{(1)} + 4\mathbf{v} \cdot \mathbf{n} + \frac{1}{2} \Delta_2^{(1)} P_2(\hat{\mathbf{v}} \cdot \mathbf{n})\right) \\
&+ 2(\mathbf{v} \cdot \mathbf{n}) \left[\Delta^{(1)} + 3\Delta_0^{(1)} - \Delta_2^{(1)} \left(1 - \frac{5}{2} P_2(\hat{\mathbf{v}} \cdot \mathbf{n})\right)\right] - v\Delta_1^{(1)} (4 + 2P_2(\hat{\mathbf{v}} \cdot \mathbf{n})) + 14(\mathbf{v} \cdot \mathbf{n})^2 - 2v^2 + 8\Delta^{(1)}\Phi^{(1)} \\
&+ 16\Phi^{(1)2} + 4\Psi^{(1)} \left[\Delta^{(1)} + 3\Delta_0^{(1)} - \Delta_2^{(1)} \left(1 - \frac{5}{2} P_2(\hat{\mathbf{v}} \cdot \mathbf{n})\right)\right]\Bigg]
\end{aligned}$$

Need to know the evolution of the photon energy density $\Delta^{(2)}_{00}$, velocity and gravitational potentials around recombination

Boltzmann equations for massive particles

The Source term requires to know the evolution of ***baryons and CDM***

Left-hand side

$$\frac{dg}{d\eta} = \frac{\partial g}{\partial \eta} + \frac{\partial g}{\partial x^i}\frac{dx^i}{d\eta} + \frac{\partial g}{\partial q^i}\frac{dq^i}{d\eta}$$

just extend to a massive particle with mass m and energy $E=(m^2+q^2)^{1/2}$

example:

$$\begin{aligned}\frac{dq^i}{d\eta} &= -(\mathcal{H}-\Psi')q^i + \Psi_{,k}\frac{q^i q^k}{E}e^{\Phi+\Psi} - \Phi^{,i}Ee^{\Phi+\Psi} + \Psi_{,i}\frac{q^2}{E}e^{\Phi+\Psi} - E(\omega^{i\prime} + \mathcal{H}\omega^i) - (\chi^{i\prime}{}_k + \omega^i{}_{,k} - \omega_k^{,i})E \\ &+ \left[\mathcal{H}\omega^i\delta_{jk} - (\chi^i{}_{j,k} + \chi^i{}_{k,j} + \chi_{jk}^{,i})\right]\frac{q^j q^k}{E}.\end{aligned}$$

Collision terms: electrons are coupled to protons via Coulomb scatt.
driving $\delta_e = \delta_p = \delta_b$ and $v_e = v_p \equiv v$ (“***baryons***”);
+ Compton scatterings $e\gamma$

$$\begin{aligned}\frac{dg_e}{d\eta}(\mathbf{x},\mathbf{q},\eta) &= \langle c_{ep}\rangle_{QQ'q'} + \langle c_{e\gamma}\rangle_{pp'q'} \\ \frac{dg_p}{d\eta}(\mathbf{x},\mathbf{Q},\eta) &= \langle c_{ep}\rangle_{qq'Q'}\end{aligned}$$

Momentum continuity equation

$$\frac{\partial(\rho_b v^i)}{\partial\eta} + 4(\mathcal{H} - \Psi')\rho_b v^i + \Phi^{,i} e^{\Phi+\Psi}\rho_b + e^{\Phi+\Psi}\left(\rho_b \frac{T_b}{m_p}\right)^{,i} + e^{\Phi+\Psi}\frac{\partial}{\partial x^j}(\rho_b v^j v^i) + \frac{\partial\omega^i}{\partial\eta}\rho_b + \mathcal{H}\omega^i\rho_b$$
$$= -n_e\sigma_T a\bar{\rho}_\gamma\left[\frac{4}{3}(v^{(1)i} - v_\gamma^{(1)i}) + \frac{4}{3}\left(\frac{v^{(2)i}}{2} - \frac{v_\gamma^{(2)i}}{2}\right) + \frac{4}{3}\delta_b^{(1)}(v^{(1)i} - v_\gamma^{(1)i}) + v_j^{(1)}\Pi_\gamma^{ji}\right]$$

Photon velocity

$$(\rho_\gamma + p_\gamma)v_\gamma^i = \int \frac{d^3p}{(2\pi)^3} f p^i$$

2nd-order velocity

$$\frac{4}{3}\frac{v_\gamma^{(2)i}}{2} = \frac{1}{2}\int \frac{d\Omega}{4\pi}\Delta^{(2)}\, n^i - \frac{4}{3}\delta_\gamma^{(1)} v_\gamma^{(1)i}$$

Quadrupole moments of photon distribution

$$\Pi_\gamma^{ij} = \int \frac{d\Omega}{4\pi}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right)\left(\Delta^{(1)} + \frac{\Delta^{(2)}}{2}\right)$$

Acoustic oscillations at second-order

In the tight coupling limit the energy and momentum continuity equations for photons and baryons reduce to

$$\left(\Delta_{00}^{(2)\prime\prime} - 4\Psi^{(2)\prime\prime}\right) + H\frac{R}{1+R}\left(\Delta_{00}^{(2)\prime} - 4\Psi^{(2)\prime}\right) - c_s^2\nabla^2\left(\Delta_{00}^{(2)} - 4\Psi^{(2)}\right) =$$

$$\frac{4}{3}\nabla^2\left(\Phi^{(2)} + \frac{\Psi^{(2)}}{1+R}\right) + S_\Delta' + H\frac{R}{1+R}S_\Delta' - \frac{4}{3}\partial_i S_V^i$$

$$\Delta_{00}^{(2)} = \delta_\gamma^{(2)}$$

$$c_s = \frac{1}{\sqrt{3(1+R)}}, \qquad R = \frac{3}{4}\frac{\rho_b}{\rho\gamma}$$

The quadrupole moment at recombination

$$S_\Delta(R=0) = \left(\Delta_{00}^{(1)2}\right)' - \frac{16}{3}\Psi^{(1)}\partial_i \mathrm{v}_\gamma^{(1)i} + \frac{16}{3}\left(\mathrm{v}_\gamma^2\right)' + \frac{8}{3}(\eta-\eta_i)\partial^i\Psi^{(1)}\partial_i\Delta_{00}^{(1)}$$

$$S_V^i(R=0) = \frac{8}{3}\mathrm{v}_\gamma^{(1)\mathrm{i}}\,\partial_j\,\mathrm{v}_\gamma^{(1)\mathrm{j}} + \frac{1}{4}\partial^i\Delta_{00}^{(1)2} - 2\partial^i\left(\Psi^{(1)}\right)^2 - \Psi^{(1)}\partial^i\Delta_{00}^{(1)} + \frac{8}{3}(\eta-\eta_i)\partial^j\Psi^{(1)}\partial_j\,\mathrm{v}_\gamma^{(1)\mathrm{i}}$$

$$-2\omega^i - \frac{3}{4}\partial_j\Pi^{(2)ij}$$

Two important differences w .r. s to the linear case:

1) At second-order vector perturbaztions are inevitably generated

2) At second-order the quadrupole of the photons is no longer suppressed in the tight coupling limit

$$\boxed{\Pi^{(2)ij} \simeq \frac{8}{3}\left(\mathrm{v}^\mathrm{i}\mathrm{v}^\mathrm{j} - \frac{1}{3}\delta^{ij}\,\mathrm{v}^2\right)}$$

Similar term analyzed by W. Hu in ApJ 529 (2000) in the context of reionazation

Non-linear dynamics at recombination

Modes entering the horizon during the matter epoch ($\eta^{-1}_{*} < k < \eta^{-1}_{EQ}$)

$$\Delta^{(2)}_{00}(k,\eta) = \int \frac{d^3k_1 d^3k_2}{(2\pi)^3} \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) f(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k},\eta) \Psi^{(1)}(0,\mathbf{k}_1)\Psi^{(1)}(0,\mathbf{k}_2)$$

Acoustic oscillations of primordial non-Gaussianity

$$f(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k},\eta) \approx \frac{54}{5}(\mathrm{a}_{\mathrm{NL}}-1) - \frac{2}{5}(9\,\mathrm{a}_{\mathrm{NL}}-19)\cos(kc_s\eta)$$

$$-\frac{2}{7}\left(\frac{9}{10}\right)^2 G(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k})\eta^2 \longrightarrow$$

Non-linear evolution of gravity

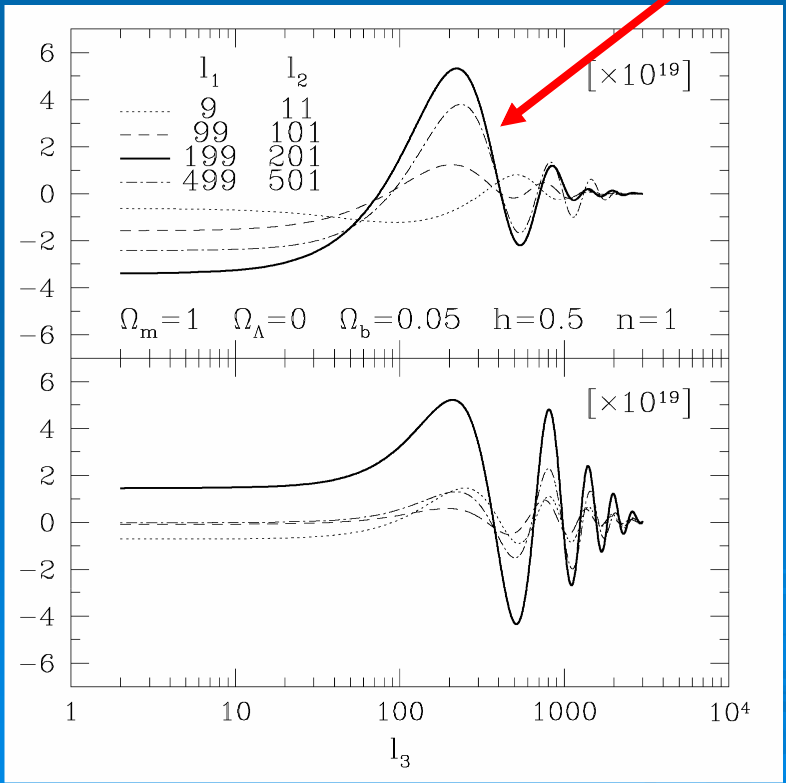
$$\Delta^{(1)}_{00} = \frac{6}{5}\Psi^{(1)}(0,\mathbf{k})\cos(kc_s\eta) - \frac{18}{5}\Psi^{(1)}(0,\mathbf{k})$$

$$G(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}) = \mathbf{k}_1 \bullet \mathbf{k}_2 - \frac{10}{3}\frac{(\mathbf{k}\bullet\mathbf{k}_2)(\mathbf{k}\bullet\mathbf{k}_1)}{k^2}$$

Linear vs. *full* radiation transfer function

$$f\left(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k},\eta\right)\approx\frac{54}{5}(\mathrm{a}_{\mathrm{NL}}-1)-\frac{2}{5}(9\,\mathrm{a}_{\mathrm{NL}}-19)\cos(kc_s\eta)-\frac{2}{7}\left(\frac{9}{10}\right)^2 G(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k})\eta^2$$

primordial non-Gaussianity is transferred linearly: Radiation Transfer function at first order



Non-linear evolution of gravity: the core of the 2nd order transfer function

(how these contributions mask the primordial signal?
how do they fit into the analysis of the bispectrum?)

Numerical analysis in progress (N.B., Komatsu, Matarrese, Nitta, Riotto)

(from Komatsu & Spergel 2001)

Modes entering the horizon during radiation epoch

($k > \eta^{-1}_{EQ}$)

$$\Delta_{00}^{(2)}(k,\eta)=\int\frac{d^3k_1 d^3k_2}{(2\pi)^3}\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})f(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k},\eta)\Psi^{(1)}(0,\mathbf{k}_1)\Psi^{(1)}(0,\mathbf{k}_2)$$

$$\begin{aligned} f(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k},\eta) &\approx -18(\mathrm{a}_{\mathrm{NL}}-1)\cos(kc_s\eta) \\ &\quad + f_1(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k})\ \left(\mathbf{cos}(k_1c_s\eta)\right)\mathbf{cos}(k_2c_s\eta)-\mathbf{cos}(k_3c_s\eta)\Big) \\ &\quad + f_2(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k})\ \mathbf{sin}(k_1c_s\eta)\,\mathbf{sin}(k_2c_s\eta) \end{aligned}$$

In this case the driving force is the quadrupole $\Pi \propto \mathbf{v}_\gamma^{\mathbf{2}}$

$$\Delta_{00}^{(1)}=6\Psi^{(1)}(0,\mathbf{k})\cos(kc_s\eta) \qquad \mathbf{v}_\gamma^{(1)i}=-i\frac{k^i}{k}\frac{9}{2}\Psi^{(1)}(0,\mathbf{k})\sin(kc_s\eta)c_s$$

Modes entering the horizon during radiation epoch (II): The Meszaros effect

Around the equality epoch η_{EQ} Dark Matter starts to dominate

⇨ Consider the ***DM perturbations on subhorizon scales during the radiation epoch***

Meszaros effect:

$$\delta_d^{(1)}(k\eta >> 1) \approx A\Psi^{(1)}(0)\ln[Bk\eta] \qquad A \approx -9.6,\, B \approx 0.44$$

⇨ **This allows to fix the gravitational potential at $\eta > \eta_{EQ}$ through the Poisson equation and to have a more realistic and accurate analytical solutions for the acoustic oscillations from the equality onwards**

Meszaros effect at second-order

Combining the energy and velocity continuity equations of DM

$$\delta_d^{(2)''} - 3\Psi^{(2)''} - s_1' + H(\delta_d^{(2)'} - 3\Psi^{(2)'} - s_1) = s_2$$

$$s_1 = 4\delta_d^{(1)'}\Psi^{(1)} - 6\left(\Psi^{(1)2}\right)' + \left(\delta_d^{(1)2}\right)' - 2\mathbf{v}_d^{(1)}\delta_{d,i}^{(1)} + 2\Psi_{,k}^{(1)}\mathbf{v}_d^{(1)k}$$

$$s_2 = \nabla^2\Phi^{(2)} - 2\partial_i(\Psi^{(1)'}\mathbf{v}_d^{(1)i}) + \nabla^2\mathbf{v}_d^{(1)2} + 2\nabla^2\Phi^{(1)2}$$

for a R.D. epoch

$$\Phi^{(2)} \sim \frac{oscill.}{\eta^2}$$

Solution

$$\delta_d^{(2)} - 3\Psi^{(2)} - \int_0^\eta d\eta' s_1(\eta') = C_1 + C_2\ln(\eta) - \int_0^\eta d\eta' s_2(\eta')\ \eta'\ [\ln(k\eta') - \ln(k\eta)]$$

Initial conditions:

$$C_1 = \delta_d^{(2)}(0) - 3\Psi^{(2)}(0) = \left[\frac{27}{2}(a_{NL} - 1) + \frac{9}{4}\right]\Psi^{(1)}(0,\mathbf{k}_1)\Psi^{(1)}(0,\mathbf{k}_2)$$

$$C_2 = 0$$

Meszaros effect at second-order

Dark Matter density contrast on subhorizon scales for $\eta < \eta_{EQ}$

$$\delta_d^{(2)}(k\eta >> 1) \approx \Big[-3A(a_{NL}-1)\ln(Bk\eta) + A^2 \ln(Bk_1\eta) A \ln(Bk_2\eta) + 6.6A \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1 c_s^2} \left(\frac{1.2}{2} [\ln(k_1 c_s \eta)]^2 - \ln(Bk_1\eta)\ln(k_1 c_s \eta) \right) + \frac{9}{2c_s^4} k^2 \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1^2 k_2^2} [\ln(k\eta)]^2 \Big] \Psi^{(1)}(0,\mathbf{k}_1)\Psi^{(1)}(0,\mathbf{k}_2)$$

Can be used for two pourposes:

√ fix the initial conditions for the evolution of the gravitational potential and photons perturbations at $\eta > \eta_{EQ}$ for subhorizon modes

√ Interesting for NG and Large Scale Structure studies: determine the full second-order transfer function for matter perturbations (primordial non-Gaussianity parametrized by a_{NL} is transferred linearly, but the core of the transfer function is given by the remaining terms).
See N.B, Matarrese & Riotto, JCAP2222, for the case of modes entering the horizon after equality

Second-order transfer function

First step: calculation of the full 2-nd order radiation transfer function on large scales (low-l), which includes:

- ✓ NG initial conditions
- ✓ non-linear evolution of gravitational potentials on large scales
- ✓ second-order SW effect (and second-order temperature fluctuations on last-scattering surface)
- ✓ second-order ISW effect, both early and late
- ✓ ISW from second-order tensor modes (unavoidably arising from non-linear evolution of scalar modes), also accounting for second-order tensor modes produced during inflation

Second step: solve Boltzmann equation at 2-nd for the photon, baryon and CDM fluids,which allows to follow CMB anisotropies at 2-nd order at all scales;

this includes both scattering and gravitational secondaries, like:

- ✓ Thermal and Kinetic Sunyaev-Zel'dovich effect
- ✓ Ostriker-Vishniac effect
- ✓ Inhomogeneous reionization
- ✓ Further gravitational terms, including gravitational lensing (both by scalar and tensor modes), Rees-Sciama effect, Shapiro time-delay, effects from second-order vector (i.e. rotational) modes, etc. …

In particular we have computed the non-linearities at recombination

(Bartolo, Matarrese & A.R. 2005+)

Conclusions

√ Up to now a lot of attention focused on the bispectrum of the curvature Perturbation ζ. ***However this is not the physical quantity which is observed is (the CMB anisotropy)***

√ Need ***to provide an accurate theoretical prediction of the CMB NG in terms of the primordial NG seeds*** $\Rightarrow$ ***full second-order radiation transfer function at all scales***

√ Future techniques (predicted angular dependence of f_{NL}, extensive use of simulated NG CMB maps, measurements of polarization and use of alternative statistical estimators) might help NG detection down to $f_{NL}\sim 1$: need to compute exactly the ***predicted amplitude and shape of CMB NG from the post-inflationary evolution of perturbations***.