How the classical world got its localization:
An elementary account of how the age of the universe may be implicated in the quantum-classical transition

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Abstract
An expanding universe of finite duration appears to impose limits on the temporal and spatial extent of quantum waves. These limitations seem to be able to bring about localization for sufficiently large quantum objects that can resemble classical behavior. A threshold for a transition from quantum to classical behavior of a physical object is derived in terms of the magnitude of its moment of inertia.

Introduction

Why do macroscopic objects always seem to be well localized in space? The studies on which this paper is based suggest that sufficiently large quantum objects will become localized automatically simply by being present in a finite, expanding universe. The limitations imposed by such a universe on the temporal extent of quantum wave functions and on the uncertainties associated with semiclassical particle motion appear to lead to spatially localized behavior resembling classical behavior. The results of these studies also indicate that a major parameter that separates classical from quantum behavior on this basis is the moment of inertia of the physical object.

Dealing with quantum mechanics in the universe at large presents a tremendously challenging problem requiring input from general relativity as well as relativistic quantum mechanics and cosmology. However, despite the true difficulties and complexity of the problem, it would still seem to be worthwhile to try to address questions of interest using simple and straightforward approaches as available. At worst, dismal failure ensues; at best, we may be able to get some hints of answers to important questions. Accordingly, a simple approach to examining the behavior of quantum waves in a time-limited universe is presented.

The duration of the universe since its inception at the Big Bang limits the total amount of time that has been available for any physical process, and in particular would seem to set an obligatory maximum duration or extent in time for any quantum wave. Thus, instead of describing a quantum object using a wave function that is infinitely extended in time, it would appear that we should consider dealing with a quantum wave packet of finite duration in time, that is consequently also correspondingly limited in space. From this perspective, a quantum object in a temporally limited universe would be expected to exhibit a bandwidth both in energy and momentum, and thus to exhibit a maximum extent in space in conjunction with its maximum duration in time. This limitation in spatial extent could provide for localization of an otherwise delocalized quantum object.

In the present study, we are following up on earlier studies that have suggested that certain large-scale properties of the universe may have a role in affecting the transition between quantum behavior and classical behavior of objects. These studies appear to have shown that certain properties of the universe related to its expansion or temporal duration that can be expressed in terms of the Hubble constant or similar parameters can have a role in effecting a quantum-classical transition. Those studies have led to the conclusion that sufficiently large objects must behave classically, and the studies individually introduce threshold criteria for “sufficiently large”. Interestingly enough, these earlier papers that were variously based on different physical arguments have come up with roughly similar criteria for a threshold between quantum and classical behavior. Arguments based on the Heisenberg uncertainty principle in the presence of Hubble expansion; random motion in the context of the stochastic quantum theory; wave packet dispersion over time; and constraints on wave packet behavior due to the finite temporal extent of the universe have all led to closely related criteria for a critical threshold size separating quantum behavior from classical behavior.[1-5]

In the present paper, we introduce a slightly different approach and a simpler method of derivation, and we also show how such criteria can be expressed more directly and simply in terms of a threshold moment of inertia.

Quantum and classical descriptions of objects and our present observations of them

In the simplest case in classical mechanics, a free object is fully described by its instantaneous position and momentum, both of which can be precisely defined, so that the object is spatially localized. In quantum mechanics, a free object is generally described by a wave function that is characterized by a precisely defined momentum and a precisely defined energy, but which extends throughout all of time from the infinite past to the infinite future, and throughout all of space, so that such a free quantum object is being described as existing for all time from the infinitely removed past to the infinitely removed future, with equal probability for it to be found at any spatial location in the universe.

How do we move from a description in terms of a distributed wave to a description in terms of a localized object?

We live in a world that has existed for only a finite duration of time since it was created in the Big Bang. This limits the total amount of time available for any physical process. In particular, this finite lifetime of our universe would seem to set a maximum limit on the duration of any quantum wave, as observed at present.

To provide somewhat more familiar examples we may consider other types of waves. Other types of waves or wave packets begin at their initiation and for purposes of present observation can be regarded as lasting until now, although as time goes on they can extend into the future. A water wave created by a disturbance to a water surface at a time T ago will exhibit a limited extent in space and time, and when observed at the present it would be described by a duration T in time. Similarly, if we switched on a laser a time interval T ago, the description of the electromagnetic wave train or wave pulse at present would be based on a finite extension in space and time, with a duration in time equal to T.

Thus, instead of describing freely moving objects by using infinitely extended wave functions having pure monochromatic frequencies, it would seem that the wave functions that we use to characterize these objects now as observed at the present time would be better described by wave packets whose extent in time would necessarily be less than the lifetime of the universe. These quantum waves could still be nearly monochromatic with fairly sharply defined frequencies, since the lifetime of the universe is very long compared to most processes of physical interest. However, these quantum wave packets representing objects in our universe would necessarily exhibit bandwidths in frequency because of the limited duration in time of the wave functions that describe them. In fact, it would be expected that these quantum wave packets would exhibit frequency widths that would be roughly inversely proportional to the duration in time of the wave packets.

Wave packets and their widths

While idealized waves can be treated as extending throughout all of space and all of time, as we have noted, real waves occur as wave pulses or wave trains that are limited in their extent in time and space. It turns out that the finite duration and/or finite extent of such a wave packet implies that such a wave packet will have a finite spread in wavelength or wave number, essentially a bandwidth. These and other properties of wave packets can be analyzed and derived formally and understood in terms of their Fourier transforms.[6]

There are rather general properties of wave behavior (or periodic behavior in general) that create certain intrinsic relationships and limitations associated with the waves. It is a basic property of wave motion that the product of the duration in time of a wave train and the width (or spread, or bandwidth) of its frequency components Δf must be greater than or comparable to a number that is very roughly of the order of magnitude of 1:

$$\Delta t \cdot \Delta f \geq 1 \tag{1}$$

Similarly, there is an unavoidable constraint on the spatial extent and wave number content of a localized wave packet, that the product of the width of a wave packet in space Δx and the width of the spread of wave numbers Δk of the waves that compose the wave packet must be greater than or comparable to a number roughly of the order of magnitude one:

$$\Delta x \cdot \Delta k \geq 1 \tag{2}$$

These are fundamental limitations on physical systems.[6]

In quantum mechanics, the frequency and the wavelength of a quantum wave are related to the energy and momentum of the quantum object by the relationships $E = hf$ and $p =$

hk = h/λ respectively (where E is the energy, f is the frequency, p is the momentum, k is the wave number, λ is the wavelength, and h is Planck's constant). If we multiply the preceding equations (1) and (2) by Planck's constant, we find that they can be reexpressed as the following Eqns. (3) and (4):

$$\Delta t \cdot \Delta E \geq h \tag{3}$$

And:

$$\Delta x \cdot \Delta p \geq h \tag{4}$$

These can be recognized as statements of the Heisenberg uncertainty relations. Thus, the relationships between the widths or spreads in parameter values associated with general wave behavior are formalized in quantum mechanics in the Heisenberg uncertainty relations (and can be done so with somewhat more precision than that presented here, when framed in terms of defined widths).[7]

We will be primarily interested in exploring cases for which these parameters reach their limiting values, that is, the cases for which the products in Eqns. (1 - 4) are all at their lower limits, in which the products on the left hand side of the equations are approximately equal to (rather than greatly exceeding) the limiting values on the right hand side of these equations. We will be examining cases of the smallest intrinsic widths or uncertainties allowed, and will be disregarding extrinsic uncertainties originating from other sources.

As we have noted, the duration of the universe since its inception at the Big Bang limits the total amount of time available for any physical process, and in particular would seem to set an obligatory maximum temporal extent for any quantum wave. Thus, instead of having a quantum object described by a wave function infinitely extended in time, it would necessarily be described by a wave packet of finite duration in time, as observed at the present time. Such a wave packet would therefore also be correspondingly limited in space.[6] As a result, it would appear that a quantum object in a finite universe must exhibit a bandwidth both in energy and momentum, and exhibit a maximum extent in space in conjunction with its maximum duration in time.

If the duration in time of a quantum wave packet is limited by a maximum available time T_{max}, then we would describe the associated width of the wave packet in time as limited by a maximum value given approximately by:

$$\Delta t \approx T_{max} \tag{5}$$

We could then expect in general terms that the width in energy of the quantum wave packet would be limited approximately by the requirements of Eqn. (3) together with Eqn. (5) so that for a limited wave packet the minimum value of the energy width would be given approximately by:

$$\Delta E \approx h/\Delta t \approx h/T_{max} \tag{6}$$

But the energy of the quantum wave here is the kinetic energy associated with the quantum object, and it is related to the momentum by $E = p^2/2m$, or $p = (2mE)^{½}$ where m is the mass of the quantum object. The width in energy associated with the quantum wave will consequently be related to the width in momentum of the quantum wave. For sufficiently narrow widths in energy, a difference relationship based on the equation $p = (2mE)^{½}$ can be approximated by the differential relationship and can be expressed by the equation:

$$\Delta p \approx (m/2E)^{1/2}\Delta E \tag{7}$$

Combining Eqn. (7) with Eqn. (6), we find that the minimum width in energy of a limited quantum wave packet would be associated with a minimum width in momentum for the wave packet that is given by:

$$\Delta p \approx (m/2E)^{1/2}\Delta E \approx (m/2E)^{1/2}(h/T_{max}) \tag{8}$$

Combining Eqn. (8) with Eqn. (4), we can find the maximum spatial width of the limited quantum wave packet as:

$$\Delta x \approx h/\Delta p \approx h(m/2E)^{-1/2}(T_{max}/h) \approx T_{max}(2E/m)^{1/2} \tag{9}$$

Thus, we find that an object in a time-limited universe would have a quantum wave that would be restricted in its spatial extent to a maximum value that is given by the preceding equation, Eqn. (9). It would seem that the quantum wave packet associated with a quantum object in a time-limited universe could not extend further than this in space. So, a quantum object in a time-limited universe would seem to be necessarily intrinsically localized because of the finite duration of the universe that it exists in.

Eqn. (9) is telling us that objects with large masses will have small intrinsic wave packets. It is also telling us that objects with small kinetic energies will have small intrinsic wave packets.

In fact, Eqn. (9) would suggest that objects at rest would have vanishingly small wave packets. However, a time-limited universe would restrict the range of frequencies present, since waves with extremely low frequencies could not have exhibited periodicity during the time available. As a result, it would seem that there must be a lower limit on the frequency of quantum waves, and therefore there must be a lower limit on the kinetic energy of any quantum object in such a universe. The lower limit on frequency would be approximately:

$$f_{min} \approx 1/T_{max} \tag{10}$$

and, using the fact that energy and frequency are related by E = hf, the associated lower limit on kinetic energy will be approximately:

$$E_{min} \approx h/T_{max} \tag{11}$$

Thus, in such a time-restricted universe, it would seem that the objects that we regard as objects at rest or stationary objects could not have exactly zero kinetic energy, but rather would actually be characterized by having such extremely small kinetic energies.

If we want to evaluate the uncertainty in location or maximum wave packet width of objects that are moving extremely slowly, so slowly that we would regard them as stationary objects, we can combine Eqn. (9) and Eqn. (11) to obtain the result:

$$\Delta x \approx T_{max}(2E/m)^{1/2} \approx (2hT_{max}/m)^{½} \tag{12}$$

This result can be viewed as a kind of intrinsic core width of the quantum wave structure associated with any object in a time-limited universe.

We will use the Hubble time as an estimate of the lifetime of the universe since its inception at the Big Bang. Since the Hubble time is the inverse of the Hubble constant H_o, we can reexpress Eqn. (12) in terms of the Hubble constant as:

$$\Delta x \approx (2h/mH_o)^{½} \tag{13}$$

We note that the core width associated with the quantum wave structure will be smaller for higher mass objects, varying inversely with the square root of the mass.

A threshold criterion for classical behavior

A threshold criterion for classical behavior (already developed in earlier studies) is based on a requirement that the size of the core quantum wave structure associated with an object (or the quantum uncertainty in its location) should be smaller in extent than the physical size of the extended object (in contrast to having the quantum wave structure extend beyond the physical size of an extended object, in order for it to exhibit more typically quantum behavior).[1-5] On this basis, the requirement for classically behaved objects would be that the region of non-zero probability density associated with the location of the center of mass of the object would be confined to the interior of the object, so that the object would be localized and would not have its wave structure extend appreciably beyond its physical extent; while the requirement for objects to exhibit more typically quantum behavior would be that the quantum probability density would extend beyond the boundaries of the object, so that the object would be more delocalized.

Thus, if we designate the size of the object L, the criterion that we have derived here in a very rough approximate manner for the threshold of classical behavior would be:

$$L \approx \Delta x \approx (2h/mH_o)^{1/2} \tag{14}$$

This approximate equation provides a rough estimate of the threshold size; objects of larger sizes for a given mass would be expected to have their quantum wave structure localized within their boundaries and hence to behave in a largely classical manner. For a given mass, objects having sizes considerably larger than this critical value would be expected to exhibit classical behavior in their translational motion, whereas objects having sizes appreciably smaller than this critical value would be expected to be able to exhibit quantum behavior.

Various earlier papers based on different approaches have led to roughly similar criteria for a quantum-classical boundary.(1-5) In the simplest case, these requirements led to a definition of the critical size for an object. The critical size associated with an object of mass m was defined by the equation:(4)

$$L_{cr} = [h/(4\pi m H_o)]^{1/2} \tag{15}$$

These studies were based on establishing a separation between quantum and classical behavior with respect to translational motion, and discussed the significance of the critical threshold parameters in terms of size, mass, density and related parameters characterizing the object. In continuing this discussion, we will use this previously introduced threshold criterion, rather than the somewhat similar rough estimate in Eqn. (14).

The moment of inertia and its role

It turns out that the threshold separating classical from quantum behavior based on such requirements can also be expressed and examined even more easily in terms of the magnitude of the moment of inertia of the object in question.

The moment of inertia is a measure of an object's inertia with respect to rotation or its tendency to resist angular acceleration; somewhat more quantitatively, the moment of inertia is a measure of the resistance of an object to angular acceleration about a given axis that is equal to the sum of the products of each element of mass in the body and the square of the element's distance from the axis. Thus, the moment of inertia of a classical object is given as the sum over the discrete point elements of mass composing the body of the product (mr^2), where m is a discrete element of mass and r is the distance of the mass element from a fixed axis; or in the case of a continuous distribution of matter, the moment of inertia is defined as the corresponding integral.(8)

We are concerned primarily with the moment of inertia of an extended object with respect to its center of mass. This can be written in a general form dependent only on the mass and a linear measure of the size of the object. Based on dimensional analysis alone, the moment of inertia I of a non-point object must take the form: $I = k_I m R^2$, where m is the total mass, R is the radius of the object measured from the center of mass, and k_I is a dimensionless constant, the inertia constant, that depends on the object's distribution of mass.[8] (As an example, the moment of inertia of a solid sphere about its center of mass is given by $I = (2/5)mR^2$.)

Thus, we can see that, within an order of magnitude or so, the moment of inertia I of a classical object with respect to its center of mass can be estimated at least roughly by the quantity mL^2, where L is a length parameter describing the size of the object:

$$I \approx mL^2 \tag{16}$$

Recasting the threshold criterion in terms of moment of inertia

We can write an equation for the critical threshold size of an object that roughly separates quantum behavior from classical behavior in terms of the moment of inertia of the object rather than in terms of its mass and size. If we combine Eqn. (16) with Eqn. (15), we find that the value $I \approx h/4\pi H_o$ gives us an approximate expression for a value of the moment of inertia that would be expected to separate objects that would behave classically from those that would behave quantum mechanically according to these criteria. This provides us with an estimate of a threshold value for the moment of inertia that would be expected to separate quantum behavior from classical behavior, which we will designate the threshold moment of inertia:

$$I_{th} = h/4\pi H_o \tag{17}$$

This provides a very straightforward criterion for a boundary separating objects potentially exhibiting quantum behavior from those necessarily exhibiting classical behavior due to properties of the universe as a whole.

We can evaluate this threshold numerically; we will use $h = 6.63 \times 10^{-34}$ joule-seconds as the value for Planck's constant, and $H_o = 2.3 \times 10^{-18}$ sec^{-1} as the value for the Hubble constant. Inserting these values into Eqn. (17), we can calculate a numerical value for the parameter that we have called the threshold moment of inertia in mks or SI units as:

$$I_{th} = 2.3 \times 10^{-17} \text{ kg·m}^2 \tag{18}$$

This result tells us that, approximately speaking, any object with a moment of inertia larger than about 10^{-17} kg·m^2 would be expected to behave in a classical manner, while any object with a moment of inertia smaller than about 10^{-17} kg·m^2 may exhibit quantum behavior.

How does this result fit the real world? At a planetary scale, where classical behavior is manifestly present, the moment of inertia of the Earth is approximately 8×10^{37} kg·m^2, which is some 54 orders of magnitude larger than the critical threshold moment of inertia given in Eqn. (18). On a submicroscopic scale, at sizes for which quantum behavior is prevalent, there are nuclear moments of inertia of about 5×10^{-54} kg·m^2, which is some 37 orders of magnitude smaller than the critical threshold moment of inertia given in Eqn. (18). The value for the critical threshold moment of inertia itself would correspond very roughly to the moment of inertia of an object of size approximately 0.1 mm and having a density of 1 gm/cc or a mass of about a microgram. Everyday experience shows that in fact all objects larger than this do behave classically. Below this value for the moment of inertia there is an extensive range of smaller mesoscale objects that would be expected to behave quantum mechanically on the basis of this criterion alone, but in fact appear to behave classically under some circumstances, presumably due to the effects of decoherence and other phenomena that could bring quantum behavior into classicality.[5,9]

In conclusion

It would seem that the age of the universe may indeed be implicated in the quantum-classical transition. We have obtained fairly reasonable threshold criteria based on localization that appear to result from limitations imposed on quantum objects by the limited duration of the universe. The threshold criterion which we have expressed in terms of the magnitude of the moment of inertia of an object would seem to enable us to set at least a rough boundary above which generally classical behavior prevails.

In closing, a remark made by Howard Georgi and Sheldon Glashow in a quite different connection might be paraphrased:[10] We present a series of hypotheses and speculations leading to a conclusion…Our hypotheses may be wrong and our speculations idle, but the uniqueness and simplicity of the scheme would seem to be reasons enough that it be taken seriously.

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