# Value of High-Frequency Relic Gravitational Wave (HFRGW) Detection to Astrophysics and Fabrication and Utilization of the Li-Baker HFRGW Detector

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#### <u>Abstract</u>

A number of applications of High-Frequency Relic Gravitational Wave (HFRGW) detection to astrophysics are identified and a means for detecting them is presented that is sensitive enough to provide useful data. Observation of relic gravitational waves will provide vital information about the birth of the Universe and its early dynamical evolution as well as enable significant direct inferences to be drawn about the value of the Hubble parameter of the early universe and the cosmological scale factor. Other astrophysical applications involve the entropy growth of the early Universe, an ability to rule out alternatives to inflation, to pinpoint the energy scale at which inflation took place and to provide clues about the symmetries underlying new physics at the highest energies. Several alternative HFRGW detectors are described and the proposed Li-Baker HFRGW detector, which is theoretically sensitive to GW amplitudes, *A*, as small as 10<sup>-32</sup>, is discussed in detail. It is recommended that plans and specifications for the Li-Baker HFRGW detector be prepared in order to expedite its fabrication.

#### Introduction

"A detection of the special pattern produced by gravitational waves would be not only an unprecedented discovery, but also a direct probe of physics at the earliest observable instants of our Universe." -- from "The Origin of the Universe as Revealed Through the Polarization of the Cosmic Microwave Background" <u>http://arxiv.org/abs/0902.3796</u>. Grishchuk (2004) found the energy density of relic gravitational waves as given by  $\Omega_{gw}(v) = (\pi^2/3)h^2(v)(v/v_H)^2$  as shown in Fig. 1, and the sensitivity required,  $h_{rms}$ , as a function of HFRGW frequency, shown in Fig. 2. The parameter *n* in Figs. 1 and 2 is the slope of the primordial GW spectrum (Figs. 5 and 6 of Grishchuk, 2008) whose exact value is currently unknown but which is thought to be between 1.0 and 1.2. Measuring the HFRGW spectrum would allow its value to be evaluated much more accurately. Grishchuk (2007) showed that observation of relic gravitational waves to measure this

parameter will enable significant direct inferences to be drawn about the value of the Hubble parameter of the early universe and the cosmological scale factor, vital information about the birth of the Universe and its early dynamical evolution.



Figure 1: Predicted Relic Gravitational Wave Energy Density as a Function of Frequency (Grishchuk, 2008)

#### Value to Astrophysics

Grishchuk (2007) showed that the energy density of the relic HFGW is greatest at around **10GHz** and the amplitude sensitivity required at that frequency is about  $A \sim 10^{-30}$ . The problem is that HFGWs at such a high frequency cannot be detected by the existing low-frequency gravitational wave detectors such as LIGO, GEO600 and Virgo or the planned Advanced LIGO, Big Bang Observer and LISA detectors, which are limited to a maximum frequency of a few kHz. For example, the advertised frequency range for maximum sensitivity of the Laser Interferometer Gravitational Observatory (LIGO) is 40Hz to 2000Hz (Shawhan, 2004; Shoemaker, 2008). The problem with higher frequencies is that the interference pattern between the LIGO legs, and which is caused by the passage of a gravitational wave, must be observed. However, "at higher frequencies, the quantum nature of the laser beam (made of discrete photons, albeit a large number of them) limits the precision of the measurement. Increased laser power

would reduce the problem of quantum noise, but ultimately the LIGO (and other) interferometers (such as the Advanced LIGO and the proposed Laser Interferometer Space Antenna or LISA) are not suited to measuring gravitational waves that stretch or shrink the arms much more rapidly than the time a photon typically remains in the optical cavity, which is roughly a millisecond for these interferometers (thus about a one kilocycle frequency upper limit)." (Shawhan, 2004)



Figure 2: Spectrum of Relic Gravitational Wave Amplitudes as a Function of Frequency (Grishchuk, 2008)

With regards to entropy, as a generalized astrophysical application, it <u>can be thought of</u> as due to 'ignored' degrees of freedom, classically, and is generalized in general relativity by appealing to to extremising entropy for all the null surfaces of space time. This last quote is from Thanu Padamanadan's address on this topic in the 25th IAGRG meeting in the *Saha Institute* in Calcutta, India, as brought up in Beckwith (2009a, 2009b and 2009c) <del>p</del>Also, we should note that the entropy creation due to Dark Matter (DM) is different from accelerated entropy growth created by 'relic gravitons' which is discussed in <del>arXIV 0809.1454</del>. Beckwith (200?) and DM entropy is meant as an initial entropy background prior to the Cosmic Microwave Background or CMBR barrier, i.e. graviton entropy generation is due to relic conditions, and should be thought of in line with a refinement of Jack Ng's work on DM entropy as referenced by Beckwith's work on DM propulsion. Furthermore, as mentioned in Beckwith (200?) there is a short hand way to reference comparing graviton production via relic conditions, and neutrino physics, of the sort which can be measured in ICE CUBE data sets. As mentioned by Dr. Steinhardt of Princeton, to Beckwith (2008), for relic gravitons, there are  $10^5$  times more relic neutrinos one can expect to observe in data sets as opposed to an individual relic graviton.

The Li-Baker detector (Baker et al., 2008 and Li et al. 2008), especially due to its HFGW fractal membrane would be a way to help establish a linkage to HFGW, and neutrino physics. The importance of such a linkage cannot be over stated. And we have a chance with application of the Li-Baker detector to remove challenges to the theory of cosmological inflation.

Cosmic microwave background polarization offers an extraordinary opportunity to gain a first glimpse into the physics that shaped our Universe. Experimentalists have demonstrated that a coordinated attack on this problem over the coming decade will likely detect primordial gravity waves – thereby providing extensive information about new physics at ultra-high energy scales.

Finally, the theory of how to obtain such gravitational wave/ graviton 'signatures' in the CMBR itself has been worked out in minute detail. Dr. Fangyu Li of *Chongqing University* (Li and Nan ,2009) has the following abstract quote we cite due to its cogency as to presenting how to measure relic HFGW signatures:

"The displaying condition of strength, phase and polarization states of the high-frequency relic gravitational waves (HFRGWs) in the electromagnetic (EM) detecting systems is studied. It is shown that the displaying condition depends not only on sensitivity of the EM detecting systems and the amplitudes of the HFRGWs, but also on the phase, the polarization states of the HFRGWs and their matching to the EM detecting systems. In order to display simultaneously the strength, phase and polarization states of the resonant "monochromatic component" of the HFRGWs, an important necessary condition would be utilization of two or multiple different EM detectors."

Given that E and B field modes are used already to mimic GW in the first place, what <del>Dr.</del> Li and <del>Dr.</del> Yang Nan (Li and Nan, 2009) are offering is an analytical blue print as to making precise detection for RELIC GW by noting that "Since the frequencies  $(\sim 10^9 - 10^{10} \text{ Hz})$  of HFRGW in the microwave band are much higher than that of the usual celestial GWs, and their dimensionless amplitudes may be only  $h_{rms} \sim 10^{-30} - 10^{-34} / \sqrt{Hz}$ , thus suitable detecting scheme to the HFRGW would be special EM resonance systems and not usual GW detectors such as LIGO, Virgo et al".

# Alternative HFRGW Detectors

Let us consider alternative HFRGW detectors. One of the first suggested means for the detection of HFRGWs concerns electromagnetic detectors (Braginsky, et al. 1974 and Braginsky and Rudenko, V, 1978). Then Pegoraro, et al. (1978) suggested the use of

tuned resonant chamber HFGW detectors. Rudenko and Sazhin in 1980 proposed a Laser interferometer as a gravitational wave detector (somewhat similar to the current Japanese approach). In 1995 Tobar characterized multi-mode resonant-mass HFGW detectors and three years later in 1998 Ottaway, et al. proposed a compact injection-locked Nd:YAG laser for HFGW detection. And in 1999 Tobar suggested, microwave parametric transducers for the next generation of resonant-mass gravitational wave HFRGW detectors.

In the past few years, HFRGW detectors have been fabricated at *Birmingham University*, England, *INFN Genoa*, Italy and in Japan. These types of detectors may be promising for the detection of the HFRGWs in the GHz band (MHz band for the Japanese) in the future, but currently, their sensitivities are orders of magnitude less than what is required for the detection of HFRGWs from the big bang.

The Birmingham HFRGW detector shown Fig. 3 measures changes in the polarization state of a microwave beam (indicating the presence of a GW) moving in a waveguide about one meter across as shown in Fig. 3. Please see Cruise (2000); Ingley and Cruise (2001) and Ingley (2005). It is expected to be sensitive to HFRGWs having spacetime strains of  $A \sim 2 \times 10^{-13}$  /√Hz, where Hz is the GW frequency, and as usual A is a measure of the strain or fractional deformation in the spacetime continuum (dimensionless m/m).



Figure 3. Birmingham University HFRGW Detector.

The *INFN Genoa* HFRGW resonant antenna consists of two coupled, superconducting, spherical, harmonic oscillators a few centimeters in diameter. Please see Fig.4. The oscillators are designed to have (when uncoupled) almost equal resonant frequencies. In theory the system is expected to have a sensitivity to HFRGWs with size (fractional deformations) of about ~  $2x10^{-17}$  / $\sqrt{Hz}$  with an expectation to reach a sensitivity of ~  $2x10^{-20}$  / $\sqrt{Hz}$ . (Bernard, Gemme, Parodi, and Picasso (2001); Chincarini and Gemme (2003)). As of this date, however, there is no further development of the *INFN Genoa* HFRGW detector.



Figure 4. INFN Genoa HFRGW Detector.

The Kawamura 100 MHz HFRGW detector has been built by the Astronomical Observatory of Japan. It consists of two synchronous interferometers exhibiting an arms length of 75 cm. Please see Fig. 4. Its sensitivity is now about  $10^{-16}/\sqrt{\text{Hz}}$  (Nishizawa et al., 2008). According to Cruise (2008) of Birmingham University its frequency is limited to 100 MHz and at higher frequencies its sensitivity diminishes.



Figure 5. The National Astronomical Observatory of Japan 100MHz Detector. Cruise (2008).

# The Li-Baker HFRGW Detector

The Li-Baker HFGW detector was invented by R. M L Baker, Jr. of Transportation Sciences Corporation, California and GravWave®LLC and patented (<u>http://www.gravwave.com/docs/Chinese%20Detector%20Patent%2020081027.pdf</u>. Based upon the theory of Li, Tang and Zhao (1992) termed the **Li-effect**, the detector was proposed by Baker during the period 1999-2000, a patent for it was filed in P. R. China in 2001, subsequently granted in 2007, and preliminary details were published later by Baker, Stephenson and Li (2008).

The Li-Effect, the theoretical basis for the Li-Baker detector, was first published in 1992 and subsequently, some ten peer-reviewed papers have been published concerning it (Li and Tang (1997), Li *et al.* (2000), Li and Yang (2004), Baker and Li (2005), Baker, Li and Li (2006), Baker, Woods and Li (2006), Li and Baker (2007), Li, Baker and Fang (2007), Baker, Stephenson and Li (2008), and Li *et al.* (2008)). The capstone paper (Li, et al., 2008) presents all of the technical details and is included as **APPENDIX B.** The Li-Effect is *very different* from the classical *(inverse)* Gertsenshtein Effect. With the Li-Effect, a gravitational wave transfers energy to a separately generated electromagnetic (EM) wave in the presence of a static magnetic field. That EM wave has the same frequency as the GW (ripple in the spacetime continuum) and moves in the same direction. This is the "synchro-resonance condition," in which the EM and GW waves are synchronized (move in the same direction and have the same frequency).

The result of the intersection of the parallel and superimposed EM and GW beams, according to the Li-Effect, is *new EM photons moving off in a direction perpendicular to the beams and the magnetic field direction*. Thus, these new photons occupy a separate region of space (see Fig.6) that can be made essentially noise-free and the synchroresonance EM beam itself (in this case a Gaussian beam) is not sensed there, so it does not interfere with detection of the photons.



Figure 6. Detection Photons Sent to Locations that are Less Affected by Noise

The synchro-resonance solution of Einstein's field equations (Li, Baker, Fang, Stephenson and Chen, 2008 pp. 411 to 413) is radically different from the Gertsenshtein (1962) effect. The newer Li-Effect solution also uses coupling between EM and gravitational waves (Li, Tang and Zhao, 1992) that arises according to the theory of relativity. And a strong static magnetic field in the y-direction, B, is superimposed upon a GW propagating in the z-direction, as in the inverse Gertsenshtein effect. However, with the Li-Effect, there is an *additional* focused microwave beam ("Gaussian beam") at the expected frequency, phase and bandwidth of the HFGWs in the same direction (z) as the GW (as shown in Fig. 6).

Unlike the Gertsenshtein effect, a first-order perturbative photon flux (PPF), comprising the detection photons, will be generated in the *x*-direction. Since there is a 90 degree shift in direction, there is little crosstalk between the PPF and the superimposed EM wave (Gaussian beam), so the PPF signal can be isolated and distinguished from the effects of the Gaussian beam, enabling detection of the GW.

Here's how it works:

1. The perturbative photon flux (PPF), which signals the detection of a passing gravitational wave (GW), is generated when the two waves (EM and GW) have the same frequency, direction and phase. This situation is termed "synchroresonance." These PPF detection photons a are generated as the EM wave propagates along its *z*-axis path, which is also the path of the GWs, as shown in Fig. 3..

- 2. The magnetic field is in the y-direction. According to the Li-Effect, the PPF detection photon flux (also called the "Poynting Vector") moves out along the x-axis in both directions.
- 3. The signal (the PPF) and the noise, or background photon flux (BPF) from the Gaussian beam have very different physical behaviors. The BPF (background noise photons) are from the synchro-resonant EM Gaussian beam and move in the z-direction, whereas the PPF (signal photons) move out in the x-direction along the x-axis.
- 4. The PPF signal can be intercepted by electromagnetic-interference-shielded microwave receivers located on the *x*-axis (isolated from the synchro-resonance Gaussian EM field, which is along the *z*-axis). In addition, isolation is further improved by cooling the microwave receiver apparatus to reduce thermal noise background.

The resultant efficiency of detection of HFRGWs is very much greater than from the inverse Gertsenshtein effect, which has been exploited in some previously proposed HFGW detectors. The proposed novel Li-Baker detection system is shown in Fig. 4. The detector is sensitive to HFRGWs directed along the +z-axis, and the precise geometrical arrangement of the major components around this axis is the key to its operation.



Figure 7. Schematic of Ultra-Sensitive Li-Baker HFRGW Detector

The detector, shown in Fig. 7, has five major components.:

1. A Gaussian (focused, with minimal side lobes) microwave beam (GB) is aimed along the +z-axis at the same frequency as the intended HFGW signal to be detected (Yariv, 1975), typically in the GHz band, and also aligned in the same direction as the HFGW to be detected. The microwave transmitter's horn antenna is not shown, but would be located on the -z axis.

2. A static magnetic field **B**, generated by two powerful magnets (typically using powerful superconductor magnets such as those found in a conventional MRI medical body scanner), is directed along the y-axis.

3. Two paraboloid-shaped reflectors, which are formed from "fractal membranes" (Wen *et al.*, 2002; Zhou *et al.*, 2003; Hou *et al.*, 2005), are located in the y-z plane at the origin of the coordinate system to aim and focus the detection photons at diffraction-limited spot antennas connected to two microwave receivers. These reflectorsare segmented (similar to a Fresnel lens) and located back-to-back in the y-z plane. They are thin enough (less than a centimeter thick in the x-direction) to not block the z-directed Gaussian beam. These microwave reflectors reflect the x-directed detection photons (PPF) and reject the z-directed Gaussian-beam photons, which move parallel to the surface of the reflectors in the y-z plane.

4. High-sensitivity shielded microwave receivers are located at each end of the x-axis.

5. Interior noise from thermal photon generation is eliminated by cooling the Li-Baker detection apparatus to below ~48 mK (0.048 Kelvin). There are effectively no thermal photons at 10 GHz. Noise from the interior background photon flux (BPF) from the EM Gaussian beam is reduced to a negligible level by moving the receivers out to the side about a meter away from the EM beam and by a series of superconductor or microwave absorbent baffles to "shade" the receivers. Stray EM resulting from scattering of particulate matter near the apparatus and possible dielectric dissipation can be effectively suppressed by evacuating the apparatus to about 7.5x10<sup>-7</sup> Torr (a rather high vacuum).External noise is eliminated by the use of a steel and titanium cryogenic containment vessel surrounding the low-temperature Li-Baker detection apparatus.

## **Future work and prospects**

Please see papers as presented recently in the Vixra.org server by Beckwith, as to possible astro physical applications of this system, which may enable investigations as to the foundations of the physics, semi classical, and otherwise of the genesis of GW from big bang, and other similar astro physical processes. The authors, in particular owe a great deal to Gary Stephenson, and to Clive Woods, of LSU for their understanding of electronics which may enable full utilization o and development of sensing applications

of HFGW to not only astro physics, but communications, and other endeavors. Of course, the dept this project owes to Dr. Fangyu Li of Chongquing university cannot be over stated, as to his through development of theoretical sensing requirements for this hard ware system. Further developments of this document await input from other members of the Gravwave team.

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