Independence of capacitance from dielectric

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Abstract

It is shown that there exists a uniqueness theorem, stating that the charges given to a constant configuration of conductors take a unique distribution, which contrary to what is believed does not have any relation to the uniqueness theorem of electrostatic potential. Using this theorem we obtain coefficients of potential analytically. We show that a simple carelessness has caused the famous formula for the electrostatic potential to be written as $U = 1/2 \int \mathbf{D} \cdot \mathbf{E} dv$ while its correct form is $U = 1/2 \int \mathbf{D} \cdot \mathbf{E}_\rho dv$ in which $\mathbf{E}_\rho$ is the electrostatic field arising only from the external charges not also from the polarization charges.

Considering the above-mentioned material it is shown that, contrary to the current belief, capacitance of a capacitor does not at all depend on the dielectric used in it and depends only on the configuration of its conductors. We proceed to correct some current mistakes resulted from the above-mentioned mistakes, eg electrostatic potential energy of and the inward force exerted on a dielectric block entering into a parallel-plate capacitor are obtained and compared with the wrong current ones.

It is shown that existence of dielectric in the capacitor of a circuit causes attraction of more charges onto the capacitor because of the polarization of the dielectric. Then, in electric circuits we should consider the capacitor's dielectric as a source of potential not think wrongly that existence of dielectric changes the capacitor's capacitance. Difference between these two understandings are verified completely during some examples, and some experiments are proposed for testing the theory. For example it is shown that contrary to what the current theory predicts, resonance frequency of a circuit of RLC will increase by inserting dielectric into the capacitor (without any change of the geometry of its conductors). It is also shown that what is calculated as K (dielectric constant) is in fact $2 - (1/K)$.

1 Introduction

In the current electrostatic discussions it is stated that a solution of Laplace's equation which fits a set of boundary conditions is unique, and while this matter has not been proven in the case that these boundary conditions are the charges on the boundaries, the known charges on the boundaries are taken as boundary conditions. In the first section of this paper this problem is solved and then the coefficients of potential are obtained. In the current electromagnetic textbooks these coefficients are obtained through the above-mentioned unproven generalization of boundary conditions. Incorrectness of this way is also shown.

The relation $U = 1/2\int_V \mathbf{D}\cdot\mathbf{E}dv$ as the electrostatic potential energy of a system is an equation quite familiar to every physicist, but a careful scrutiny indicates an undoubted mistake existent in it. This mistake is simply arising from this fact that in the process of obtaining this equation, while accepting $\nabla\cdot\mathbf{D} = \rho$ where ρ is the external electric charge density, it is forgotten that in the primary equation of the electrostatic potential energy of the system the potential arising only from this ρ, ϕ_ρ, not also from the polarization charges be taken into account resulting in considering $\mathbf{E}$ (obtained from $-\nabla\phi$) instead of $\mathbf{E}_\rho$ (obtained from $-\nabla\phi_\rho$) which is the electrostatic field arising only from ρ not also from the polarization. This careful scrutiny is presented in the third section of this article. A great part of this section proceeds to some consequences of this same mistake including this current belief that the capacitance of a capacitor depends on its dielectric, while we shall prove that this is not at all the case and it depends only on the form of the configuration of the conductors of the capacitor.

To another much simple and obvious current mistake has been paid in another paper (Two kinds of potential difference for a capacitor): We connect a battery, which the potential difference between its poles is $\Delta\phi$, to the two plates of an uncharged capacitor until it will be charged. Then, what is the electrostatic potential difference between the plates of the charged capacitor? All the current literature on the subject answer that this electrostatic potential difference is the same potential difference between the poles of the battery, $\Delta\phi$, while this is not the case and is equal to $2\Delta\phi$.

As it is seen, the above current mistakes some of which being fundamental are totally in bases of the subject of Electromagnetism, and cannot be ignored, because not only are much widespread and taught in all the universities but also some of them are basis for some subsequent deductions in other branches of physics. This matter shows that in the progress of physics the attention should not be only to its rapidity but also to its profundity, otherwise, as in the case of this article, sometimes some of the obvious mistakes remain hidden from the physicists' view yielding probably very other wrong consequences.

2 Another uniqueness theorem in Electrostatics

2.1 Uniqueness theorem of charge distribution in conductors

In solving electrostatic problems there is a uniqueness theorem that distinctly states that when the electrostatic potential or the normal component of its gradient is given in each point of the bounding surfaces then if the potential is given in at least one point, the solution of Laplace's equation is unique, and otherwise we may add any constant to a solution of this equation. Unfortunately, sometimes negligence is seen in careful applying of the quite clear stated above boundary conditions. For instance without any reason the charges of bounding surfaces are taken as boundary conditions in terms of which the above theorem is applied in obtaining coefficients of potential of a system of conductors. The reasoning being used is this (see Foundations of Electromagnetic Theory by Reitz, Milford and Christy, Addison-Wesley, 1979): "Suppose there are N conductors in fixed geometry. Let all the conductors be uncharged except conductor j, which bears the charge Q_{j0}. The appropriate solution to Laplace's equation in the space exterior to the conductors will be given the symbol $\phi^j(x, y, z)$and the potential of each of the conductors will be indicated by $\phi_1^j, \phi_2^j, \cdots, \phi_j^j, \cdots, \phi_N^j$. Now let us change the charge of the jth conductor to λQ_{j0}. The function $\lambda\phi^j(x, y, z)$ satisfies Laplace's equation, since λ is a constant; that the new boundary conditions are satisfied by this function may be seen from the following argument. The potential at all points in space is multiplied by λ; thus all derivatives (and in particular the gradient) of the potential are multiplied by λ. Because $\sigma = \epsilon_0 E_n$, it follows that all charge densities are multiplied by λ. Thus the charge of the jth conductor is λQ_{j0} and all other conductors remain uncharged. A solution of Laplace's equation which fits a particular set of boundary conditions is unique; therefore we have found the correct solution, $\lambda\phi^j(x, y, z)$ to our modified problem. The conclusion we draw from this discussion is that the potential of each conductor is proportional to the charge Q_j of conductor j, that is $\phi_i^j = p_{ij}Q_j, (i = 1, 2, \cdots, N)$ where p_{ij} is a constant which depends only on the geometry."

The fault may be found in this reasoning is arising from the same incorrect distinction of boundary conditions. This fault is that a solution to Laplace's equation other than $\lambda\phi^j$ can be found such that it can make the charge of the jth conductor λfold retaining all other conductors uncharged. This solution can be $\lambda\phi^j(x, y, z) + c$ for a non-zero constant c. It is obvious that its gradient and therefore $\sigma = \epsilon_0 E_n$ arising from it compared with before are λ fold and then the charge of the jth conductor will be λ fold while all other conductors remain uncharged. But this solution is no longer proportional to the charge of the jth conductor, Q_j, ie we won't have $\phi^j(x, y, z) = p_{ij}Q_j$.

In order to clear obviously that the uniqueness theorem of potential does not include boundary conditions on charges, suppose that there is an initially uncharged conductor. We then give it some charge. We want to see when the given charge is definite whether potential function outside the conductor will or won't be determined uniquely by this theorem. We say that the given charge distributes itself onto the surface of the conductor and remain fixed causing that the potential of the equipotential surface of the conductor to become specified. With specifying of the conductor potential, potential function outside the conductor is determined uniquely according to the theorem. But important for us is knowing that whether form of the charge distribution onto the conductor surface is uniquely determined or not. One can say that maybe the charge can take another form of distribution on the surface causing another potential for the equipotential surface of the conductor and according to the theorem we shall have another unique function for the potential outside the conductor. In a geometric illustration there is not anything to prevent the above problem for a sharp conductor being solved with equipotential surfaces concentrated near either the sharp end or the other end; the charge is concentrated at the sharp end in the first and at the other end in the second case. Which occurs really is a matter that must be determined by another uniqueness theorem, uniqueness theorem of charge distribution, which has no relation to the uniqueness theorem of potential.

Analytical proof of this theorem is a problem that must be solved. That this theorem is valid can be understood by some thinking and visualizing. Separate from inner parts of the conductors consider external surfaces of the conductors as some conducting thin shells. Obviously if some charge is to distribute itself in these shells, the components of the charge, as a result of the repulsive forces, will take the most distant possible distances from one another, and even when for instance uncharged conducting shells are set in the vicinity of charged conducting shells, their conducting (or valence) charges will be separated in order that like charges take the most distant and unlike charges take the most neighboring possible distances from one another. What is clear is that these "most"s indicate to some unique situation. Therefore we can say that form of the surface charge distribution is a function of geometrical form of the conductors and then will be specified uniquely for a definite configuration of conductors.

2.2 Proportion of charge density to net charge

Now suppose that for a particular configuration of and definite amount of charge given to some conductors we can find two distributions of charge in the conductors in each of which the resultant electrostatic force on each infinitesimal partial charge due to other infinitesimal partial charges is outward normal to the conductor surface and there exists no tangential component for this force. (Of course these outward normal forces are balanced by surface stress in the material of the conductors.) Because there is not any tangential component for the mentioned forces, existence of these two charge distributions is possible. But because of the same configuration for the both, the uniqueness theorem of charge distribution necessitates that the both distribution be the same. We shall benefit form this matter soon.

We prove that in a constant configuration of some conductors from which only one has net charge, Q, change of this net charge form Q to λQ causes that the surface density in each point of the conductors' surfaces becomes λ fold: Visualize the constant situation existent before that Q becomes λQ . The charges in the conductors have a unique distribution according to the uniqueness theorem of charge distribution. In this distribution there exists a resultant electrostatic force exerted on each infinitesimal partial surface charge σda due to other partial charges which is outward normal to the conductor surface. Suppose that this distribution becomes nailed up in some manner, ie each partial charge becomes fixed in its position and no longer has the state of a conducting free charge (in order that won't probably change its position as a result of change of the charge). Now suppose each partial charge becomes λ fold in its position, ie we have for the new partial charge $\sigma' da = \lambda \sigma da$. Since the partial charges are nailed up, they are not free to redistribute themselves on the conductors' surfaces probably. It is obvious that resultant electrostatic force exerted on a partial charge $\sigma' da$ will be still outward normal to the conductor surface, since firstly this partial charge is λfold of previous σda and secondly each of other partial charges isλ fold of previous partial charges and then the only change in the resultant force on σda will be in its magnitude which becomes λ^2 fold, while its direction will remain unchanged. Therefore, by changing each σda to $\lambda \sigma da$ we have found a nailed up distribution for the charges which exerts a resultant force on each partial surface charge outward normal to the conductor surface, and furthermore, the only change in the net charges of the conductors is in the conductor bearing net charge Q previously which now bears λQ , and then it is obvious that if the partial charges get free from the nailed state will retain this distribution. Therefore, this distribution is a possible one, and according to what said previously based on the uniqueness theorem, is the same distribution that really occurs on the conductors' surfaces when the net charge of the mentioned conductor changes from Q to λQ .

2.3 Generalization of the uniqueness theorem and of the charge density proportion to net charge

In fact, the uniqueness theorem of charge distribution on the conductors is true in case of a particular configuration of conductors and a constant (nailed up) charge distribution and a constant set of linear dielectrics in the space exterior to the conductors, ie in such a case a charge given to the conductors causes a unique charge distribution on their surfaces. The truth of this theorem can be found out with some indications similar to previous ones.

Now consider a constant configuration of conductors and a constant set of linear dielectrics outside the conductors. There is no charge outside the conductors. We give a net charge to only one of the conductors. Certainly, according to the above theorem we shall have a unique charge distribution in the conductors. Suppose that the given charge of that conductor becomes λfold. We want to prove that the surface free charge densities on all of the conductors and also the dielectrics' polarizations will become λfold consequently.

Visualize the situation existent before that the given charge becomes λfold. An outward resultant force normal to the conductor surface is exerted on each partial surface charge σda due to other nonpolarization and polarization partial charges. Now suppose that all the nonpolarization (or free) partial charges be nailed up in their positions and then all the nonpolarization and polarization partial charges (ie the previous free charges and dielectrics' polarizations) become λfold. Obviously, in this case the resultant electrostatic force on each partial surface charge is outward normal to the conductor surface too (and only its magnitude has become λ^2fold). Furthermore, it is obvious that in each point of each dielectric the electrostatic field has only become λfold (without any change in its direction) and then we see that this field is proportional to the polarization at that point as must be so expectedly. Thus, if the charges get free from the nailed state, they will remain on their positions, and furthermore, the only change in net charges is in the above mentioned conductor, net charge of which has now become λfold. Therefore, this is a possible distribution and according to the above mentioned uniqueness theorem of charge distribution is unique and then is the same distribution that really occurs.

2.4 Superposition principle for the charge densities

We must also notice another point. We understood that in a configuration of some conductors that only one of them has net charge, charge distribution is unique. Suppose that we have N conductors and only conductor i has net charge (Q_i). The unique distribution that charges get, prescribes charge surface density $\sigma_i(\mathbf{r})$ (and polarization $\mathbf{P}_i(\mathbf{r})$) for each point of each conductor (and each point outside the conductors).

Now consider this same configuration of these conductors from which only conductor j (such that j is not equal to i) has net charge (Q_j). The unique distribution that charges get, prescribes charge surface density $\sigma_j(\mathbf{r})$ (and polarization $\mathbf{P}_j(\mathbf{r})$) for each point of each conductor (and each point outside the conductors).

It is clear intuitively that if we have this same configuration of the conductors from which only two conductors have net charges, the ith conductor has the same relevant net charge (Q_i) and the jth conductor has the same relevant net charge (Q_j), then the unique distribution that charges get, prescribes charge surface density $\sigma_i(\mathbf{r}) + \sigma_j(\mathbf{r})$ (and polarization $\mathbf{P}_i(\mathbf{r}) + \mathbf{P}_j(\mathbf{r})$) for each point of each conductor (and each point outside the conductors). This fact has generality for when each conductor has a specified net charge or when there is a fixed distribution of external charge outside the conductors (ie we can add contribution of this distribution towards forming charge surface density on the conductors (and forming polarization) to other contributions). We can even, when there are linear dielectrics, obtain surface charge distribution on the conductors by adding the charge surface density in each point on the conductors related to charge distribution in the absence of dielectrics to the charge surface density in the same point produced only by the polarizations of the dielectrics assuming that there exists no net charge in any conductor but only the polarizations exist.

Therefore, considering the theorems we have proven so far, we can conceive that in a system of some charged conductors and some fixed external charge distribution and some linear dielectrics if the net charge of a conductor becomes λfold, free partial charge surface density arising from that conductor, assuming that other conductors are uncharged and there are not any dielectrics or other external charges, will become λfold in each point on the conductors. It is evident that, considering the integral definition of electrostatic potential and assuming that the potential is zero at infinity, the partial potential arising from that conductor (ie in fact from its effect on forming the free charges) will become λfold in each point, too, and then the partial potential arising from that conductor will become λfold in each conductor which is an equipotential region for this partial potential. In other words, the free net charge of one of the conductors is proportional to the partial potential arising from the (effect of the free net) charge of that conductor (assuming that there are not any dielectrics or other external charges and that other conductors are uncharged) in each of the conductors: $(i = 1, 2, 3, \cdots, N)$ $\phi_i^j = p_{ij} Q_j$. Furthermore, this fact that each conductor is an equipotential region for this partial potential proves that p_{ij} depends only on the geometry of the configuration of the conductors and even does not depend on the dielectrics and their positions (or other external charge distributions outside the conductors), because, as we mentioned, this constant coefficient of the proportion, p_{ij}, is related to when we suppose that there are not at all any dielectrics (or other external charges) and infer that the charge surface densities will become λfold if the net charge of a conductor (the jth one) becomes λfold (assuming that other conductors are uncharged).

Now since the potential of each conductor is the sum of its partial potentials plus a constant, we have $\phi_i = \sum_{j=1}^{N} p_{ij} Q_j + c$. (Adding of c removes the worry arising from generalization of the necessity of the above reasoning that the partial potentials must be zero at infinity.)

3 Static potential energy and current mistakes

3.1 Static potential energy

We know that if a closed surface S contains external electric charge Q and polarization electric charge Q_P, then we shall have $\oint_S \mathbf{E}\cdot\hat{n}da = (Q+Q_P)/\epsilon_0$. In this relation $\mathbf{E}$ is the partial electrostatic field arising from both an elective distribution of external charge, the part of which inside the closed surface being equal to Q, and an elective distribution of polarization charge, the part of which inside the closed surface being equal to Q_P. (The word "elective" implies that the entire existent charge distribution is not necessarily taken into consideration, and similarly the word "partial" implies that maybe only a part of the existent field is intended. Notice the superposition principle of field and the linearity of potential.)

On the other hand we have $Q_P = \int_{S'} \mathbf{P}\cdot\hat{n}da + \int_V (-\nabla\cdot\mathbf{P})dv$ in which V is the volume of the dielectric enclosed by S, and S' is the surface of the conductors inside the closed surface S. In this relation $\mathbf{P}\cdot\hat{n}$ and $-\nabla\cdot\mathbf{P}$ are the the polarization charge densities of the elective distribution of polarization charge, and then we can say that in this relation $\mathbf{P}$ is an elective (ie not necessarily entire) distribution of electrostatic polarization. If using the divergence theorem we change the volume integral into the surface integral, we finally shall obtain $Q_P = -\oint_S \mathbf{P}\cdot\hat{n}da$. The comparison of this relation with the first relation of this section shows that $\oint_S(\epsilon_0\mathbf{E}+\mathbf{P})\cdot\hat{n}da = Q$ in which $\mathbf{P}$ is an elective distribution of polarization, and Q is the total charge of that part of the elective distribution of external charge which is inside the closed surface S, and $\mathbf{E}$ is the partial field arising from both the totality of the elective distribution of external charge and the totality of the elective distribution of polarization. On definition, the electric displacement vector is $\mathbf{D} = \epsilon_0\mathbf{E}+\mathbf{P}$. Then $\oint_S \mathbf{D}\cdot\hat{n}da = \int_V \rho dv$. This relation says that if $\mathbf{E}$ is arising from both ρ, which is an elective distribution of external electric charge, and $\mathbf{P}$, which is an elective distribution of electrostatic polarization, then the surface integration of $\mathbf{D} = \epsilon_0\mathbf{E}+\mathbf{P}$ on the closed surface S is equal to the totality of only that part of our elective external charge which is inside the closed surface. If we use the divergence theorem in the recent relation, we shall conclude $\nabla\cdot\mathbf{D} = \rho$.

Considering the above discussions the following deduction may be interesting. (In this deduction the expression "the $\mathbf{E}$ arising from both ρ and $\mathbf{P}$" is shown as "$\mathbf{E}(\rho, P)$".)

$$\left.\begin{array}{l}\mathbf{D_1} = \epsilon_0\mathbf{E}(\rho, P_1)+\mathbf{P_1} \Rightarrow \nabla\cdot\mathbf{D_1} = \rho \\ \mathbf{D_2} = \epsilon_0\mathbf{E}(\rho, P_2)+\mathbf{P_2} \Rightarrow \nabla\cdot\mathbf{D_2} = \rho\end{array}\right\} \Rightarrow \nabla\cdot\mathbf{D_1} = \nabla\cdot\mathbf{D_2}$$

or $\nabla\cdot(\epsilon_0\mathbf{E}(\rho, P_1)+\mathbf{P_1}) = \nabla\cdot(\epsilon_0\mathbf{E}(\rho, P_2)+\mathbf{P_2})$

The electrostatic potential energy of a bounded system of electric charges (which can exist in various forms of external charge, polarization charge, etc, eg in the form of cancelled charges, from the macroscopic viewpoint, in a molecule) having the density ρ, which is in fact the spent energy for assembling all the fractions of the charge differentially from infinity, is

$$U = \frac{1}{2}\int_{V_h} \rho(\mathbf{r})\phi(\mathbf{r})dv \tag{1}$$

in which V_h is the whole space and ϕ is the partial electrostatic potential due to the distribution of ρ. The way of obtaining the relation (1) can be seen in many of the electromagnetic texts.

As it is so actually in the tridimensional world of matter, we disburden ourselves from the dualizing the charge density as the surface and volume ones and say we have only the volume density of the electrostatic charge that, for instance, can have an excessive absolute amount on the surface of a charged electric conductor. Now we take into consideration an elective distribution of the volume density of the external (ie nonpolarization) electric charge, ρ. We want to obtain the electrostatic potential energy of this distribution. We know that $\nabla \cdot \mathbf{D} = \rho$ so that $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$ in which $\mathbf{P}$ is the elective distribution of the electrostatic polarization and $\mathbf{E}$ is the resultant field arising from both the elective distribution of the external electric charge density (ρ) and the polarization charge densities due to the elective distribution of the electrostatic polarization ($\mathbf{P}$). Since the electrostatic potential energy of this elective distribution of the external electric charge is $U = 1/2\int_{V_h} \rho\phi dv$, in which ($V_h$ is the whole space and) ϕ is only arising from ρ (not from both ρ and $\mathbf{P}$), we shall have $U = 1/2\int_{V_h} \phi\nabla\cdot\mathbf{D}dv$, and since $\int_{V_h} \phi\nabla\cdot\mathbf{D}dv = \int_{V_h} \nabla\cdot(\phi\mathbf{D})dv - \int_{V_h} \mathbf{D}\cdot\nabla\phi dv = \int_{S_h} \phi\mathbf{D}\cdot\hat{n'}da - \int_{V_h} \mathbf{D}\cdot\nabla\phi dv = 0 - \int_{V_h} \mathbf{D}\cdot(-\mathbf{E}_\rho)dv = \int_{V_h} \mathbf{D}\cdot\mathbf{E}_\rho dv$ (V_h and S_h being in turn the whole space and the total surfaces of the problem (which of course there is not any surface)), we shall have

$$U = \frac{1}{2}\int_{V_h} \mathbf{D}\cdot\mathbf{E}_\rho dv \tag{2}$$

in which as we said" U is the electrostatic potential energy of an elective distribution of the external electric charge with the density ρ, and we have $\nabla\cdot\mathbf{D} = \rho$ in which $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$ in which $\mathbf{P}$ is an elective distribution of electrostatic polarization and $\mathbf{E}$ is arising from both $\mathbf{P}$ and ρ, while $\mathbf{E}_\rho$ is the field arising only from ρ." It is obvious that this electrostatic potential energy has been distributed in the space with the volume density $u = 1/2\mathbf{D}\cdot\mathbf{E}_\rho$. (We saw previously that $\nabla\cdot\mathbf{D_1} = \nabla\cdot\mathbf{D_2}$. Uniqueness of the electrostatic potential energy of a definite distribution of external electric charges with the density ρ necessitates having $1/2\int_{V_h} \mathbf{D_1}\cdot\mathbf{E}_\rho dv = 1/2\int_{V_h} \mathbf{D_2}\cdot\mathbf{E}_\rho dv$; but although these total energies are equal to each other this won't necessarily mean that the energy densities are also the same, ie we cannot infer $\mathbf{D_1}\cdot\mathbf{E}_\rho = \mathbf{D_2}\cdot\mathbf{E}_\rho$ or $\mathbf{D_1} = \mathbf{D_2}$ (although their divergences are equal).)

It is very opportune to compare the above accurate definition of the electrostatic potential energy with what is set forth for discussion under this very title in the present electromagnetic books, and to pay attention to the existent inaccuracy in the definitions of the involved terms caused by the omission of the subscript ρ from the term $\mathbf{E}_\rho$. This is a sample of the existent inaccuracies in the present current electromagnetic theory specially in not correct distinguishing between different electric fields. This mistake has caused that, considering relation $\mathbf{D} = \epsilon\mathbf{E}$ for linear dielectrics, wrong relations like $u = 1/2\epsilon E^2 = 1/2D^2/\epsilon$ to be current in present electromagnetic textbooks. We shall pay to some other mistakes soon.

3.2 Independence of capacitance from dielectric

Consider a system consisting of some fixed perfect conductors and some linear dielectrics in the space exterior to the conductors and some fixed distribution of external charge density in this space. We want to obtain electrostatic potential energy arising from all the free net charges on these conductors, ie the electrostatic potential energy of that part of the charge distribution in all of the conductors which comes into existence as a result of these free net charges (which of course does not include electrostatic potential energy of the polarization and distribution of external charges and that (other) part of the charge distribution in all of the conductors which comes into existence as a result of these polarization and external charges). Since each conductor is an equipotential region for the potential arising from these free net charges, for this electrostatic potential energy we have $U = \frac{1}{2}\sum_{j=1}^{N} Q_j\phi_j$ from the relation (1), in which Q_j is the net charge of the conductor j and ϕ_j is the electrostatic potential on the conductor j arising from all free net charges of the conductors of the system (ie one related to free net charges themselves and their effect on the conductors, not also related to dielectric polarization and other external charges and their effect on the conductors). What is necessary to be emphasized again (and is important in the coming discussion) is that the ϕ_j's are arising only from net charges of the conductors not also from the polarization charges.

Using the coefficients of potential for this system we can also write $\phi_i = \sum_{j=1}^{N} p_{ij}Q_j$ in which Q_j is the net charge of the conductor j, and ϕ_i is the electrostatic potential on the conductor i arising from all (Q_j's ie all) net charges of the conductors of the system (ie one related to free net charges themselves and their effect in the conductors, not also related to dielectric polarization and other external charges and their effect on the conductors). Combining the two recent relations yields $U = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} p_{ij}Q_iQ_j$ for the electrostatic potential energy arising from free net charges of the conductors of a system consisting of some perfect conductors and probably some linear dielectrics and external charge distribution outside the conductors.

A capacitor is defined as two conductors (denoted by 1 and 2), from among a definite configuration of some conductors, that one of them bears net charge Q (Q being greater than or equal to zero) and the other one bears $-Q$. (Existence of net charges on other conductors in the configuration or of linear dielectrics or external charges outside the conductors and the effect which each has on these two conductors (ie 1 and 2) are not important at all. We shall find out this soon.)

By using the relation $\phi_i = \sum_{j=1}^{N} p_{ij} Q_j$ for the above capacitor we have:

$$\left.\begin{array}{l}\phi_1 = p_{11}Q + p_{12}(-Q) + 0\\ \phi_2 = p_{21}Q + p_{22}(-Q) + 0\end{array}\right\} \Rightarrow \Delta\phi = \phi_1 - \phi_2 = (p_{11} + p_{22} - 2p_{12})Q = Q/C$$

(We know that $p_{12} = p_{21}$ proof of which can be seen in many of the electromagnetic books.) We have attention that in the relation $\Delta\phi = Q/C, \Delta\phi$ is the potential difference between the potential arising from net charges of the conductors 1 and 2 (related to themselves and their effect in other conductors) on the conductor 1 and the potential arising from these charges (related to themselves and their effect in other conductors) on the conductor 2. Therefore, since the potential of other charges is not considered and considering linearity of potentials and that C, which is called as the capacitance of the capacitor, depends only on the form of the configuration of all (and not only two) of the conductors, it is obvious that existence of net charges on the conductors other than the conductors 1 and 2 and existence of any linear dielectrics or external charges in the space exterior to the conductors, so far as the configuration of the conductors is constant, are unimportant (and there is no need that one of the conductors 1 and 2 be shielded by the other, the way presented in some electromagnetic books for the potential difference independence of whether other conductors are charged). We specially emphasize again that so, we have proven that the capacitance (C) of a capacitor does not depend on whether there exist any dielectrics at all and only depends on the configuration of the conductors introduced in the definition of the capacitor.

Using the relation $U = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} p_{ij} Q_i Q_j$ we obtain

$$U = \frac{1}{2}Q^2/C = \frac{1}{2}Q\Delta\phi = \frac{1}{2}C(\Delta\phi)^2 \qquad (3)$$

for the electrostatic potential energy of the charges Q and $-Q$ (themselves and of their effect). We should emphasize again that in the recent relation, $\Delta\phi$ is the potential difference arising from the free charges Q and $-Q$ (and not also from eg polarization charges), and C depends only on the configuration of the conductors (and not also on eg existence or nonexistence of linear dielectrics).

At the end of this section let's obtain the capacitance of a capacitor consisting of two parallel plates in which the plates separation d is very small compared with the dimensions of the plates:

$$C = \frac{Q}{(\Delta\phi)_Q} = \frac{Q}{E_Q d} = \frac{Q}{\sigma d/\epsilon_0} = \frac{Q}{(Q/A)d/\epsilon_0} = \frac{\epsilon_0 A}{d},$$

in which $(\Delta\phi)_Q$ and E_Q are the potential difference and the electrostatic field arising from Q and $-Q$ (and not also from the polarization charges) respectively. Therefore, the capacitance of this capacitor is $\epsilon_0 A/d$ regardless of whether there exist any linear dielectrics between the parallel plates or not.

And now see the present books of Electricity and Magnetism in which without attention to this fact that $\Delta\phi$ must be arising only from the capacitor charge, the relation $\Delta\phi = Ed$, in which E is arising from not only the capacitor charge but also the linear dielectrics polarization charges, is used and consequently wrong expression $\epsilon A/d$ is obtained for the capacitance.

3.3 Dielectric as source of potential

We saw that the mathematical discussions presented so far proved independence of the capacitance of a capacitor from its dielectric. But this is doubtlessly surprising for the physicists and engineers, because they know well that dielectric has a substantial part in accumulation of charge in the capacitor. This section is intended for obviating this surprise.

It is made use often of electroscope to show the effect of dielectrics in capacitors. If the two conductors of a charged capacitor are connected to an electroscope, leaves of the electroscope will get away from each other. Now, if, without any change in the configuration of the capacitor's conductors, a dielectric is inserted between the two conductors of the capacitor, the leaves of the electroscope will come close to each other. Current justification of this phenomenon is as follows (eg see University Physics by Sears, Zemansky and Young, Addison-Wesley 1987): "The equation $C = Q/\Delta\phi$ shows the relation among the capacitor's capacitance, capacitor's charge, and the potential difference between the two conductors of the capacitor. When a dielectric is inserted into the capacitor, due to the orientation of the electric dipoles of the dielectric in the field inside the capacitor some polarization charge opposite to the charge of each conductor of the capacitor is induced on that surface of the dielectric which is adjacent to this conductor, and then the electrostatic field in the dielectric, and thereby the potential difference (between the two conductors), arising from both the capacitor's charge and this induced polarization charge is decreased. Then, the denominator of $C = Q/\Delta\phi$ decreases which results in increasing of the capacitance (C) considering that Q remains uncharged, ie the capacitor's capacitance increases by inserting a dielectric between the capacitor's conductors. That the leaves of the electroscope come closer to each other by inserting the dielectric is because of this same decreasing of the potential difference, $\Delta\phi$."

It is clear that considering the discussion presented in this article, the above justification is quite wrong, because $\Delta\phi$ is the potential difference arising only from the capacitor's charge not also from the polarization charge formed in the dielectric. But why do the leaves of the electroscope come closer to each other when a dielectric is inserted into the capacitor? Its reason is quite obvious. Metal housing and the leaves connected to the metal knob of the electroscope, themselves, are in fact a capacitor, which when are connected separately to the two conductors of the capacitor under measurement, a new (equivalent) capacitor will be formed consisting of two conductors: the first being one of the conductors of the capacitor under measurement and the electroscope's metal housing which is connected to it, and the second being the other conductor of the capacitor under measurement and the set of the knob and the leaves of the electroscope which is connected to this conductor. It is obvious that if the capacitor under measurement is charged at first, its charge now, after its connecting to the electroscope, will be distributed throughout the new formed capacitor and then a part of the charge of the primary capacitor now will go to the electroscope because of which the leaves of the electroscope will get away from each other (because the opposite charges induced in the electroscope will attract each other causing drawing of the leaves toward the electroscope's housing which itself means more separation of the leaves from each other).

By inserting the dielectric into the capacitor we cause creation of polarization charges in the dielectric which this, in turn, causes more charges of the new formed capacitor to be drawn towards the dielectric. Thus, the distribution of the charge will be changed in such a manner that a part of the charge distribution in the electroscope will go to the primary capacitor (or the one under measurement) to be placed as close as possible to the dielectric; this means decrease of the electroscope's charge which will cause its leaves to come closer to each other. Therefore, the act of the dielectric is change of the charge distribution in the new capacitor formed from the primary capacitor and the electroscope, not change of the capacitance of the primary capacitor.

Now, let's connect the two plates of a parallel-plate capacitor by a wire in the space exterior to the space between the plates. What will happen if a slice of a dielectric having a permanent electric polarization is inserted between the two plates of the capacitor? The polarized dielectric will cause induction of charge on the two plates; the positive surface of the slice will induce negative charge on the plate adjacent to it, and the negative surface will induce positive charge on the (other) plate adjacent to it. Induction of charge on the two plates, while they had no charge beforehand, means that while inserting the dielectric between the plates an electric current has been flowing in the wire from one plate to the other. In other words the dielectric acts like a power supply producing electric current or charging the capacitor. Then, we can attribute electric potential difference to it (like the potential difference between the two poles of a battery).

Now, how will the situation be if the inserted dielectric is not to have previous polarization but it is to be polarized because of the charge (or in fact the electric field produced by the charge) of the capacitor? Answer is that the situation will be similar to the same state of permanent polarization, and again the dielectric acts as a source of potential. Its physical and direct reason can be seen easily in the discussion we presented about the electroscope. There, we saw that inserting the dielectric, charge distribution was changed in such a manner that some more charges were accumulated on the conductors of the (primary) capacitor. It is clear that more accumulation of charge on the capacitor necessitates flowing of electric current in the circuit. Cause of this current and of the more accumulation of charge on the capacitor is the source of potential difference which we must attribute to the dielectric.

In this manner, the purpose of this section has been fulfilled practically; in electric circuits wherever a dielectric is to exist between the conductors of a capacitor, a proper source of voltage must be considered in the circuit in the same place of the dielectric. Such a voltage source causes accumulation of charges on the conductors of the capacitor more than when there exists no dielectric in the capacitor. One can say whether this act is not equivalent to defining, in principle, the capacitance of a capacitor equal to the charge accumulated on the capacitor (due to both the configuration of the capacitor's conductors and the electric induction in the conductors caused by the polarization of the dielectric) divided by the potential difference between the two conductors of the capacitor (which is the method that current instruments measuring capacitor's capacitances work based on it) and no longer considering the dielectric as a source of potential. Following example shows that consequences of such a definition in practice are not equivalent to the practical consequences of the main definition of capacitance of capacitor (although can be close to it under suitable conditions). We then shall investigate another example which will show, well, considerable differences that can come into existence if role of the dielectric as a power supply in the circuit is not taken into consideration, according to which a quite practical criterion for testing the theory presented in this section in comparison with the current theory will be presented.

3.4 Some examples as test

Let's connect the two plates of a dielectricless parallel-plate capacitor to the two poles of a battery. At the end of the section 3.2 we saw that the capacitance of such a capacitor is $\epsilon_0 A/d$ in which A is the capacitor's area and d is the distance between its plates. Then, according to the relation $C = Q/\Delta\phi$ for the capacitor's capacitance, we have $\epsilon_0 A/d = \sigma A/V$ in which σ is the surface density of the charge accumulated on the capacitor and V is the potential difference given to the two plates of the capacitor by the battery. In this manner we have:

$$\sigma d = \epsilon_0 V. \tag{4}$$

Now we fill the space between the two plates with a linear dielectric with the permittivity ϵ. We indicate the magnitude of the formed electric polarization in the dielectric by P. P is in fact equal to the surface density of the polarization charge in the dielectric. Suppose that a charge exactly equal to the polarization charge is induced on the plates of the capacitor. (Indeed, in the state of induction of charge in the capacitor due to the polarized dielectric between the capacitor's plates we should suppose that the two plates of the capacitor are connected to each other by a wire in the space exterior to the space between the plates; in other words in this state the battery existent in the circuit does not play any role except as a short circuit.) Then the charge induced on the capacitor due to the polarization of the dielectric is equal to PA. This charge, as we said, has been stored in the capacitor because of a source of potential difference, equal to V', which we must attribute to the dielectric; ie because of the potential difference V' exerted to the two plates of the capacitor the charge PA has been accumulated in the capacitor, and then the ratio PA/V' is equal to the capacitor's capacitance $\epsilon_0 A/d = PA/V'$. Considering that $P = (\epsilon - \epsilon_0)E = (\epsilon - \epsilon_0)\sigma/\epsilon$ in which E is the electrostatic field arising from both the external and polarization charges we infer from this relation that $V' = (\epsilon - \epsilon_0)\sigma d/(\epsilon\epsilon_0)$ which considering Eq. (4) results in

$$V' = (1 - \frac{\epsilon_0}{\epsilon})V = (1 - \frac{1}{K})V \tag{5}$$

Let's calculate sum of the charges (Q) accumulated on this capacitor (due to both the configuration of the capacitor's conductors and the induction arising from the (polarization of the) dielectric). For this act we must add the potential difference arising from the dielectric to the potential difference given by the battery and after that multiply the sum by the (real) capacitance of the capacitor $C = \epsilon_0 A/d$:

$$Q = (V + (1 - \frac{\epsilon_0}{\epsilon})V)\frac{\epsilon_0 A}{d} = (2 - \frac{\epsilon_0}{\epsilon})\frac{\epsilon_0 A}{d}V = (2 - \frac{1}{K})CV \tag{6}$$

Can we present another definition of capacitance of capacitor, for convenience in practice, equal to sum of the charges accumulated on the capacitor (consisting of the charges arising from both the configuration of the capacitor's conductors and the induction due to the dielectric) divided by the potential difference between the two capacitor's conductors, given to the capacitor only by the battery (or the circuit)? Considering Eq. (6) such a definition gives the following (newly defined) capacitance of our capacitor equal to

$$\frac{Q}{V} = (2 - \frac{\epsilon_0}{\epsilon})\frac{\epsilon_0 A}{d}. \tag{7}$$

Is this definition useful in practice, and does it yield real consequences? The answer is negative. It is sufficient only instead of a single capacitor to consider n capacitors connected in series such that the space between the plates of only one of them is filled with dielectric and to try to calculate the accumulated charges on the equivalent capacitor.

If all of these n capacitors were dielectricless, because of the identity between the capacitors the (shared) potential difference between the two plates of each of these capacitors would be V/n. When only one of these capacitors is filled with a linear dielectric with the permittivity ϵ, the potential difference related to this dielectric (as a source of potential), similar to Eq. (5) will be $(1-\epsilon_0/\epsilon)V/n$. Since these n capacitors are identical and the capacitance of each of them is $\epsilon_0 A/d$, the equivalent capacitance of these n capacitors which are connected in series will be obtained by solving the equation $1/C_1 = n/(\epsilon_0 A/d)$ for C_1 equal to $\epsilon_0 A/(nd)$. Therefore, the charge accumulated on each capacitor is equal to

$$(V + (1 - \frac{\epsilon_0}{\epsilon})\frac{V}{n})\frac{\epsilon_0 A}{nd} = (1 + \frac{\epsilon - \epsilon_0}{n\epsilon})\frac{\epsilon_0 A}{nd}V. \tag{8}$$

But now let's see if the capacitance of the capacitor having dielectric is to be equal to (7) while the capacitance of each of the other capacitors is equal to $\epsilon_0 A/d$, whether or not the charge accumulated on each capacitor will be obtained still equal to (8) when no longer the source of potential difference related to the dielectric is considered in lieu of considering (7) for the capacitance of the capacitor having dielectric. Equivalent capacitance of the capacitors which are in series will be obtained by solving the equation

$$\frac{1}{C_2} = \frac{n-1}{\epsilon_0 A/d} + \frac{1}{(2 - \frac{\epsilon_0}{\epsilon})\epsilon_0 A/d}$$

for C_2, and charge of each capacitor should be considered equal to $C_2 V$:

$$C_2 V = \frac{1}{n - 1 + \frac{\epsilon}{2\epsilon - \epsilon_0}} \cdot \frac{\epsilon_0 A}{d} V. \tag{9}$$

Obviously the coefficient of $\epsilon_0 AV/d$ in Eq. (8) is not equal to the coefficient of $\epsilon_0 AV/d$ in Eq. (9) except when $\epsilon = \epsilon_0$ or $n = 1$. Thus, we see that the new definition we tried to present for capacitance of capacitor is not so useful in practice (at least in this example does not give the real charge accumulated on the capacitors). But the ratio of these two coefficients is not so far from one. To see this fact let's indicate ϵ/ϵ_0 by K and obtain the ratio of the coefficient of $\epsilon_0 AV/d$ in Eq. (9) to the coefficient of $\epsilon_0 AV/d$ in Eq. (8):

$$\frac{(n - 1 + \epsilon/(2\epsilon - \epsilon_0))^{-1}}{(1/n) + ((\epsilon - \epsilon_0)/n^2\epsilon)} = 1/(1 + \frac{(K-1)^2(n-1)}{(2K-1)Kn^2})$$

It is seen that the degree of the term $(K-1)^2(n-1)/((2K-1)Kn^2)$ with respect to K is zero and with respect to n is -1; thus this term is close to zero practically, or in other words the ratio of the above-mentioned coefficients is close to one practically. This matter is itself a good reason that why the definition of capacitance in the form of capacitor's charge divided by the potential difference exerted on the capacitor's conductors (Eq. (7)) has been able to endure practically and the difficulties due to such a definition has remained hidden in practice. But, important for a physicist should be mathematical much exactness and discovery of what actually occurs or exists. In order to find out that such an exactness can be important even in practice (and then won't be negligible even for engineers) notice the following example.

Consider a series circuit of RLC, which its capacitor is parallel-plate and dielectricless, connected to a constant voltage $\mathcal{V}$. After connection of the switch in the time $t = 0$, the equation of the circuit will be

$$\mathcal{V} = RI + L\frac{dI}{dt} + \frac{1}{2C}\int_{t=0}^{t} I_{(t)}dt. \tag{10}$$

(We should notice that as it has been proven in the paper "Two kinds of potential difference for a capacitor", in this circuit we must consider the circuital potential difference of the capacitor, ie the third term of the right-hand side of (10), not as it is usual wrongly its electrostatic potential difference ie $\frac{1}{C}\int_{t=0}^{t} I_{(t)}dt$. There, also we shall see that what the current instruments measure as capacitance is in fact two times more than the capacitance. Another noticeable point being that as it has been explained in the paper "Electromagnetic theory without the Lorentz transformations", L in (10) is in fact equal to $\mu\epsilon' a' L_B^*$ not equal to only $d\Phi_B^*/dI(= L_B^*)$ according to its usual definition. But since the current instruments for measuring L work based on the formula $\mathcal{E} = -LdI/dt$, they are in fact measuring $\mu\epsilon' a' L_B^*$ as L because as we can see in that article the correct relation is in fact $\mathcal{E} = -\mu\epsilon' a' L_B^* dI/dt$.)

With one time differentiation of this equation with respect to time, the following equation will be obtained considering that $\mathcal{V}$ is a constant: $Ld^2I/dt^2 + RdI/dt + I/(2c) = 0$. If $R/(2L) < (2LC)^{-1/2}$, this equation will be solved as $I = ae^{-Rt/(2L)}\cos(\omega_n t - \theta)$ in which

$$\omega_n = \sqrt{\frac{1}{2LC} - \frac{R^2}{4L^2}} \tag{11}$$

and a and θ are two arbitrary constants. Since in $t = 0$ we have $I = 0$ and then also from Eq. (10) we have $dI/dt = \mathcal{V}/L$, we conclude that $a = \mathcal{V}/(\omega_n L)$ and $\theta = \pi/2$, and then

$$I = \frac{\mathcal{V}}{\omega_n L}e^{-\frac{Rt}{2L}}\sin\omega_n t. \tag{12}$$

For calculating the voltage drop in the capacitor we should calculate the third term of the right-hand side of Eq. (10):

$$\frac{1}{2C}\int_{t=0}^{t}\frac{\mathcal{V}}{\omega_n L}e^{-\frac{Rt}{2L}}\sin(\omega_n t)dt = \mathcal{V}(1 - e^{-\frac{Rt}{2L}}(\cos\omega_n t + \frac{R}{2\omega_n L}\sin\omega_n t)) \tag{13}$$

Now, if the space between the two plates of the capacitor (without any change in the configuration of the plates) is to be filled by a linear dielectric with the permittivity ϵ, we must multiply the negative of the voltage drop in the capacitor ((13)) by $(1-\epsilon_0/\epsilon)$ in order that according to Eq. (5) the potential difference which we must attribute to the dielectric as source of potential is obtained. We then should add this source to the previous constant source and equate the sum to the right-hand side of Eq. (10):

$$\mathcal{V}+\mathcal{V}[e^{-\frac{Rt}{2L}}(\cos\omega_n t+\frac{R}{2\omega_n L}\sin\omega_n t)-1](1-\frac{\epsilon_0}{\epsilon}) = RI+L\frac{dI}{dt}+\frac{1}{2C}\int_{t=0}^{t} I(t)dt \tag{14}$$

With one time differentiation of this equation with respect to time the following equation will be obtained:

$$L\frac{d^2I}{dt^2}+R\frac{dI}{dt}+\frac{1}{2C}I = \mathcal{V}(1-\frac{\epsilon_0}{\epsilon})\frac{2\omega_n L}{R^2C-2L}e^{-Rt/(2L)}\sin(\omega_n t)$$

Particular solution of this equation is

$$\frac{V}{2L-R^2C}(1-\frac{\epsilon_0}{\epsilon})te^{-Rt/(2L)}\cos(\omega_n t),$$

and general solution of its corresponding homogeneous equation is $ae^{-Rt/(2L)}\cos(\omega_n t-\theta)$ with the two arbitrary constants a and θ. Then general solution of this equation is

$$I = ae^{-Rt/(2L)}\cos(\omega_n t-\theta)+\frac{\mathcal{V}}{2L-R^2C}(1-\frac{\epsilon_0}{\epsilon})te^{-Rt/(2L)}\cos(\omega_n t)$$

with the two arbitrary constants a and θ. For obtaining a and θ by means of the initial conditions, we should be careful that initial conditions must be fit, ie $t=0$ should be the same moment that, without dielectric, the current in the circuit was zero and we had $dI/dt=\mathcal{V}/L$; and now, when the dielectric has been inserted, we should see how the conditions change, and in this moment ($t=0$) what the current and its time derivative are as initial conditions. The physics of the problem says that we have in this state $I=0$ in this moment too, and then also it is clear from Eq. (14) that in this moment we have $dI/dt=\mathcal{V}/L$ too. Then

$$a=\frac{\mathcal{V}}{\omega_n L}+\frac{\mathcal{V}}{\omega_n(R^2C-2L)}(1-\frac{\epsilon_0}{\epsilon}) = \frac{L(1+\epsilon_0/\epsilon)-R^2C}{2L-R^2C}\frac{\mathcal{V}}{\omega_n L}$$

and $\theta=\pi/2$.

Thus

$$I=\frac{L(1+\epsilon_0/\epsilon)-R^2C}{2L-R^2C}\frac{\mathcal{V}}{\omega_n L}e^{-Rt/2L}\sin\omega_n t+\frac{\mathcal{V}}{2L-R^2C}(1-\frac{\epsilon_0}{\epsilon})te^{-Rt/2L}\cos\omega_n t \tag{15}$$

(It is noticeable that when $\omega=\omega_0$ the same Eq. (12) will be obtained from this equation.) We obtained Eq. (15) for the current of the circuit, while what is current at present is that inserting the linear dielectric (with the permittivity ϵ) between the plates of the capacitor only the capacitor's capacitance changes from C to KC where $K=\epsilon/\epsilon_0$ (without any addition of new source of potential to the circuit), and then the circuit's current has the same form of Eq. (12) with this only difference that in the equation related to ω_n (Eq. (11)) we must write KC instead of C.

Now suppose that instead of the constant voltage $\mathcal{V}$ we have an alternating voltage in the form of $\mathcal{V}(t) = \mathcal{V}_0 \sin(\omega t - \theta')$(in which θ' is a constant value) as the main source of potential in the series circuit of RLC which its parallel-plate capacitor is dielectricless. In such a case we have

$$\mathcal{V}_0 \sin(\omega_t - \theta') = RI + L\frac{dI}{dt} + \frac{1}{2C}\int_{t=0}^{t} I(t)dt \tag{16}$$

and then $Ld^2I/dt^2 + RdI/dt + I/(2C) = \mathcal{V}_0\omega\cos(\omega t - \theta')$. Particular solution of this equation is $a_1\cos(\omega - \theta' - \theta_1)$ in which

$$a_1 = \mathcal{V}_0 / \sqrt{(\frac{1}{2C\omega} - L\omega)^2 + R^2} \tag{17}$$

and

$$\theta_1 = \cot^{-1}\frac{(2\omega C)^{-1} - L}{R}. \tag{18}$$

Since solution of its corresponding homogeneous equation is

$$ae^{-Rt/(2L)}\cos(\omega_n t - \theta),$$

the general solution of this equation is

$$I = ae^{-Rt/(2L)}\cos(\omega_n t - \theta) + a_1\cos(\omega t - \theta' - \theta_1) \tag{19}$$

with the two arbitrary constants a and θ (of course assuming that$R/(2L) < (2LC)^{-1/2}$).

We suppose that we have $I = 0$ in $t = 0$ and from Eq. (16) we have $dI/dt = -\mathcal{V}_0\sin\theta'/L$ in this moment. Having these initial values we can obtain a and θ, but since the first term of the right-hand side of Eq. (19) is transient, this act is of no importance for us. (Nevertheless, they should be obtained by solving the system of

$$\begin{cases} a\cos\theta = -a_1\cos(\theta' + \theta_1) \\ a\sin\theta = -\frac{1}{\omega_n}(\frac{\mathcal{V}_0\sin\theta'}{L} + \frac{Ra_1}{2L}\cos(\theta' + \theta_1) + a_1\omega\sin(\theta' + \theta_1)) \end{cases}$$

for a and θ.)

Now, as before, having the form of current (Eq. (19)) we obtain voltage drop in the capacitor:

$$\frac{1}{2C}\int_{t=0}^{t}(ae^{-Rt/2L}\cos(\omega_n t - \theta) + a_1\cos(\omega t - \theta' - \theta_1))dt$$

$$= a[e^{-Rt/2L}(\omega_n L\sin(\omega_n t - \theta) - \frac{R}{2}\cos(\omega_n t - \theta)) + \omega_n L\sin\theta + \frac{R}{2}\cos\theta] +$$
$$\frac{a_1}{2\omega C}[\sin(\omega t - \theta' - \theta_1) + \sin(\theta' + \theta_1)]. \tag{20}$$

And now, as before, if the space between the two plates of the capacitor is to be filled by a linear dielectric with the permittivity ϵ (without any change in the plates' configuration), in order to obtain the potential difference that we must attribute to the dielectric as a source of potential in the circuit, according to Eq. (5) we should multiply the negative of the potential drop in the capacitor (20) by $(1-\epsilon_0/\epsilon)$. We then must add this source to the initial alternating source and equate the sum to the right-hand side of Eq. (16):

$$\mathcal{V}_0\sin(\omega t-\theta')+a(1-\frac{\epsilon_0}{\epsilon})[e^{-Rt/2L}(\frac{R}{2}\cos(\omega_n t-\theta)-\omega_n L\sin(\omega_n t-\theta))$$

$$-\omega_n L\sin\theta-\frac{R}{2}\cos\theta]-\frac{a_1}{2\omega C}(1-\frac{\epsilon_0}{\epsilon})[\sin(\omega t-\theta'-\theta_1)+\sin(\theta'+\theta_1)]$$

$$=RI+L\frac{dI}{dt}+\frac{1}{2C}\int_{t=0}^{t}I_{(t)}dt.$$

With one time differentiation of this equation with respect to time the following equation will be obtained:

$$\frac{Ld^2}{dt^2}+R\frac{dI}{dt}+\frac{1}{2C}I=\mathcal{V}_0\omega\cos(\omega t-\theta')$$

$$-a(1-\frac{\epsilon_0}{\epsilon})(\frac{R^2}{4L}+\omega_n^2 L)e^{-Rt/2L}\cos(\omega_n t-\theta)$$

$$-\frac{a_1}{2C}(1-\frac{\epsilon_0}{\epsilon})\cos(\omega t-\theta'-\theta_1) \qquad (21)$$

For obtaining the particular solution of this equation we must add up particular solutions of the following equations (for reason see Differential Equations with Application and Historical Notes by Simmons, McGraw-Hill Inc., 1972):

$$\frac{Ld^2I}{dt^2}+\frac{RdI}{dt}+\frac{1}{2C}I=\mathcal{V}_0\omega\cos(\omega t-\theta') \qquad (22)$$

$$\frac{Ld^2I}{dt^2}+\frac{RdI}{dt}+\frac{1}{2C}I=-a(1-\frac{\epsilon_0}{\epsilon})(\frac{R^2}{4L}+\omega_n^2 L)e^{-Rt/2L}\cos(\omega_n t-\theta) \qquad (23)$$

$$\frac{Ld^2I}{dt^2}+\frac{RdI}{dt}+\frac{1}{2C}I=-\frac{a_1}{2C}(1-\frac{\epsilon_0}{\epsilon})\cos(\omega t-\theta'-\theta_1) \qquad (24)$$

We then must add the obtained particular solution to the general solution of the corresponding homogeneous equation to obtain the general solution of Eq. (21).

Both the general solution of the homogeneous equation and particular solution of Eq. (23) are (trigonometric) multiples of $e^{-Rt/(2L)}$, thus these two terms in the general solution of Eq. (21) are transient and then unimportant for us. Thus, for obtaining the nontransient part of the general solution of Eq. (21) we should obtain the particular solution of the equations (22) and (24) and then add them up.

Particular solution of Eq. (22) is

$$\frac{2\mathcal{V}_0 C\omega}{4L^2C^2\omega^4 + 4(R^2C - L)C\omega^2 + 1}[(1-2LC\omega^2)\cos(\omega t-\theta') + 2RC\omega\sin(\omega t-\theta')]$$

and particular solution of Eq. (24) is

$$\frac{-a_1}{4L^2C^2\omega^4 + 4(R^2C - L)C\omega^2 + 1}(1 - \frac{\epsilon_0}{\epsilon})[(1 - 2LC\omega^2)\cos(\omega t - \theta' - \theta_1)$$

$$+2RC\omega\sin(\omega t - \theta' - \theta_1)].$$

If we write the trigonometric terms in the recent solution in terms of the sine and cosine of the arguments $(\omega t - \theta')$ and θ_1, and add up the particular solutions obtained for the equations (22) and (24), and equate the sum to the expression $a_2\cos(\omega t - \theta' - \theta_2)$, and use the equations (17) and (18), we shall finally obtain:

$$a_2\cos\theta_2 = 2\mathcal{V}_0 C\omega$$

$$\cdot\frac{(1 - 2LC\omega^2)(4R^2C^2\omega^2 + (1 - 2LC\omega^2)^2) + (1 - \epsilon_0/\epsilon)(4R^2C^2\omega^2 - (1 - 2LC\omega^2)^2)}{(4R^2C^2\omega^2 + (1 - 2LC\omega^2)^2)^2} \tag{25}$$

and

$$a_2\sin\theta_2 = 4\mathcal{V}_0 RC^2\omega^2\cdot\frac{4R^2C^2\omega^2 + (1 - 2LC\omega^2)^2 - 2(1 - \epsilon_0/\epsilon)(1 - 2LC\omega^2)}{(4R^2C^2\omega^2 + (1 - 2LC\omega^2)^2)^2} \tag{26}$$

We can solve these equations to obtain a_2 and θ_2 in order that the nontransient solution $a_2\cos(\omega t - \theta' - \theta_2)$ for the circuit current will be obtained unambiguously. The value which is obtained for the amplitude a_2 from these equations is

$$a_2 = \frac{2\mathcal{V}_0 C\omega\sqrt{4R^2C^2\omega^2 + (K^{-1} - 2LC\omega^2)^2}}{4R^2C^2\omega^2 + (1 - 2LC\omega^2)^2} \tag{27}$$

in which $K = \epsilon/\epsilon_0$. (It is easily seen that for $K = 1$ the same amplitude a_1 presented in Eq. (17) will be obtained from a_2.)

Now if, as it is thought at present, after inserting the dielectric between the capacitor's plates its capacitance is to increase to KC and no more, then we must conclude that the amplitude of the (nontransient) current is in the same form shown in Eq. (17) except that KC must be substituted for C in this equation. Namely, the magnitude of such an amplitude will be:

$$(\frac{\mathcal{V}_0}{\sqrt{((2KC\omega)^{-1} - L\omega)^2 + R^2}} =)\frac{2\mathcal{V}_0 C\omega}{\sqrt{4R^2C^2\omega^2 + (K^{-1} - 2LC\omega^2)^2}}$$

$$(= \frac{2\mathcal{V}_0 C\omega\sqrt{4R^2C^2\omega^2 + (K^{-1} - 2LC\omega^2)^2}}{4R^2C^2\omega^2 + (K^{-1} - 2LC\omega^2)^2}). \tag{28}$$

A comparison between (27) and (28) shows that their variations with K is opposite to each other, ie if (27) increases with increase of K, (28) will decrease with increase of K, and if (27) decreases with increase of K, (28) will increase with increase of K, and vice versa. For example on condition that ω^2 being greater than or equal to $1/(2LC)$ the expression (27) indicates that the current's amplitude increases by inserting the dielectric, while the expression (28) says that this amplitude must decrease under the same condition. Investigating that whether or not experiment shows that provided that ω^2 being greater than or equal to $1/(2LC)$ current intensity of the circuit increases by inserting dielectric between the capacitor's plates is a good test for accepting the theory presented here and rejecting the current one or vice versa.

To find the resonance frequency of the circuit it is sufficient to differentiate from the right-hand side of Eq. (27) with respect to ω and then to equate the obtained result to zero and to solve the obtained equation for ω. By doing this act we obtain the following result for the square of the resonance frequency ω_r:

$$\omega_r^2 = \frac{2(K-1) + \sqrt{4(K-1)^2 + 1}}{2LCK} \tag{29}$$

(It is seen that for $K = 1$, square of the resonance frequency is $1/(2LC)$ which is just the same result which Eq. (17) predicts for the square of the resonance frequency. (Reminding of this point is necessary that as we said we have $L = \mu\epsilon' a' L_B^*$ here.))

Now, let's see what the prediction of the present current belief is for the resonance frequency of the circuit. It says that since inserting the dielectric (according to its belief) the amplitude of the current is $\mathcal{V}_0/\sqrt{((2KC\omega)^{-1} - L\omega)^2 + R^2}$ (see Eq. (28)), the square of the resonance frequency will be:

$$\omega_r^2 = \frac{1}{2LCK} \tag{30}$$

A simple mathematical try shows that the coefficient of $1/(2LC)$ in (29) (ie $(2(K-1) + \sqrt{4(K-1)^2 + 1})/K$) is an ascending function of K, while the coefficient of $1/(2LC)$ in (30) (ie K^{-1}) is a descending function of K. Namely, the analysis presented here shows that by inserting dielectric between the capacitor's plates the resonance frequency increases, while according to the current belief this frequency must decrease.

NOTE:

That actually whether or not the resonance frequency of the circuit increases with inserting dielectric between the plates of the capacitor (without any change in the plates' configuration) is a quite practical test for establishing the validity of the theory presented in this article and invalidity of the current belief in this respect, or vice versa. Recently this experiment has been performed with a brilliant success for the theory presented in this article showing specifically increase of the resonance frequency when inserting the dielectric. Here is the report of an electronics engineer who could not believe the result of his experiments in this respect:

"Oh, yes, indeed the resonant frequencies do change as drastically as you suggest if you put a dielectric with high dielectric constant between the parallel plates of a capacitor. I've put an example at the end of this posting.

Example of capacitor with high-K dielectric.... You can buy "disc ceramic" capacitors with about $1.0nF$ capacitance. These are nominally 1cm diameter, with nominally $0.5mm$ plate separation, with dielectric only between the conductive plates. The dielectric has a very high dielectric constant. If you resonant such a capacitor with, say, a $5\mu H$ inductor, you will find its resonant frequency will be about $70kHz$. You can replace that capacitor with one with the same plate size and spacing but air dielectric, resulting in roughly $0.5pF$ capacitance. Then you will find that the measured resonant frequency depends on the self-resonance of the inductor, because you will be very hard-pressed to make a $5\mu H$ inductor with self-capacitance as low as $0.5pF$. If you choose an inductor of, say, $1\mu H$, properly constructed, then you might reasonably see the effects of $0.5pF$, but now you will be dealing with much more awkward (especially if you have limited access to good test equipment) resonant frequencies in the hundreds of MHz. You will indeed find that the resonant frequency of that inductor with the nominal $1.0nF$ ceramic-dielectric capacitor will be on the order of $5MHz$. The Q in each case should be high enough (with a well-constructed inductor) to give an easily measured resonant frequency. I -could- do the experiment to specifically demonstrate the -dramatic- shift in resonance, and even use other dielectrics less extreme, but I feel no need to: as I told you before, I -routinely- design resonant circuits and filters, even taking into account the effects of stray capacitance and inductance and the resistances of things like circuit board traces where appropriate, and within my understanding of the tolerances of the parts and the effects of the strays, I'm never surprised. I am CERTAINLY never surprised by a resonance shifting higher as I increase capacitance so long as I'm within the practical range of the parts I'm using.

Note on $1\mu H$ coil: If you make a coil with $\#18AWG$ wire, which is about $1.0mm$ diameter, and make that coil with uniformly spaced turns, about $2.6cm$ diameter turns, spaced out $2.5cm$ total coil length, it will have an inductance about $1.0\mu H$, and its first parallel self-resonance at about $190MHz$. That implies about $0.7pF$ effective self-capacitance. Adding an external $0.5pF$ capacitance would drop the resonant frequency to about $145MHz$."

(It is probable that the instrument by which one measures resonance frequency needs to obtain the capacitance of the capacitor before calculating the resonance frequency based on the formula $\omega_r = 1/(2LC)$. If so, there are two errors in such a measurement:

1. The process in which the current instruments measure capacitance of a capacitor is not accurate, because as we explained at the end of Section

3.3 (in these instruments) this capacitance is defined (wrongly) as the charge accumulated on the capacitor divided by the potential difference between the two conductors of the capacitor.
2. As it has been proven (Eq. (29)), the above formula is not correct.)

At present dielectric constant is determined in one of the two following manners:
1. A parallel-plate capacitor is connected to a constant voltage two times: (first) when its dielectric is vacuum (or air), and (second) when its dielectric is the substance under measurement. The geometry of the conductors remains unchanged. Since it is supposed that the capacitance of the capacitor is increased with the dielectric, the ratio of the gathered charge in the second state to the gathered charge in the first state is the dielectric constant of the substance.
2. Instead of retaining the voltage unchanged, we put a unique charge on the capacitor two times: (first) when its dielectric is vacuum, and (second) when its dielectric is the substance under measurement. The geometry of the conductors remains unchanged. Since it is supposed that the capacitance of the dielectric is increased with the dielectric, the ratio of the potential difference between the plates in the first state to the potential difference between them in the second state is the dielectric constant of the substance.

Certainly the above methods don't give the dielectric constant according to the contents of this article in which it has been proven that the capacitance of a capacitor depends only on the geometry of the conductors and not also on its dielectric. Thus, what are in fact those measured as dielectric constant by these methods?

At the beginning of Section 3.4 it has been proven that the charge accumulated on a parallel-plate capacitor with area A and plates' separation d and a linear dielectric with the permittivity ϵ, which the potential difference between its plates is V, is: (Eq. (6))

$$Q = (2 - \frac{\epsilon_0}{\epsilon})\epsilon_0 \frac{A}{d} V$$

What has been done in the first method above is in fact calculating

$$\frac{(2 - \epsilon_0/\epsilon)\epsilon_0(A/d)V}{\epsilon_0(A/d)V} = 2 - \frac{\epsilon_0}{\epsilon} = 2 - \frac{1}{K}$$

as the dielectric constant (K). And what has been done in the second method is in fact calculating

$$\frac{Q/(\epsilon_0 \frac{A}{d})}{Q/((2 - \frac{\epsilon_0}{\epsilon})\epsilon_0 \frac{A}{d})} = 2 - \frac{\epsilon_0}{\epsilon} = 2 - \frac{1}{K}$$

as the dielectric constant (K). Anyhow, what is at present considered as K is indeed $2 - (1/K)$. Since ideally K at least is 1 (for vacuum) and at most is infinity, what is measured as K at present (ie in fact $2 - 1/K$) can be at least 1 and at most, for the best linear dielectrics, 2.

Reviewing different tables of the dielectric constants in different texts shows that these constants scarcely exceed 2 or 3 for the best linear dielectrics, although for some materials this constant even exceeds 100. (For example it is 2 for paper, paraffin, mineral oil, Indian rubber, ebonite, benzene, teflon, mica, wood, polyethylene, liquid CCl4 and CS2 (while for liquid O2 and A is 1.5), ..., but is suddenly near 100 for water.) In addition, there is notable difference between the constants registered in different texts. It seems that there is a drastic uncertainty in the results obtained by the above-mentioned methods (esp when there is a huge difference from 2). The cause of this uncertainty should be searched (maybe in the nonlinearity of the dielectrics), but anyway it can be said that for almost all of the best linear dielectrics (the permittivity of which can be taken infinity) the constant registered as (wrong) dielectric constant, as the above reasoning predicts, is about 2 (indeed the (true) dielectric constant of these good dielectrics is infinity).

Separate from the theory, now let's prove physically that the above-mentioned ratio of the gathered charges in the method 1 can not exceed 2: Suppose that a parallel-plate capacitor, connected to a constant voltage, when is dielectricless, gathers a charge Q. In this state suppose we insert an ideal linear dielectric, with an infinite permittivity, between its plates. When this linear dielectric is set in the field between the plates it begins to become polarized, ie by ordering the molecular electric dipoles of the dielectric the charges of the capacitor begin to be cancelled, but the potential source to which the capacitor is connected compensates for the cancelled charges of the capacitor in such a manner that the dielectric is always in a constant electric field which its presence is essential for the linear dielectric to maintain the polarization. (Notice the relation $\mathbf{P} = (\epsilon - \epsilon_0)\mathbf{E}$ for a linear dielectric in which when $\epsilon = \infty$ we shall have $\mathbf{E} = \mathbf{0}$ where $\mathbf{E}$ is arising from both polarized charge and that part of the conductors' charges which are gathered by these polarized charges. (If we wish to consider $\mathbf{E}$ as the field arising from the polarized charges and the whole charge of the capacitor, then the ϵ won't be infinity (because indeed in such a case it is not related to only the dielectric but the role of the conductors (or capacitor) has been added to it).)) Thus, the dielectric can attract, onto the capacitor, some additional charge at most equal to the original charge of the capacitor (related to when there is no dielectric). Then, the ratio of the charge of the capacitor with dielectric to one without dielectric is at most $2Q/Q = 2$ (and at least is $Q/Q = 1$ when there is no order for the molecular electric dipoles even in the electric field between the plates).

Surely there are some persons reckoning these reasonings as fantasy. The following material may help them not to think so: The current usual prediction for the resonance frequency of a series RLC circuit which its dielectricless capacitor is parallel-plate, when its capacitor is filled with a linear dielectric having dielectric constant K, is that square of the resonance frequency drops by $1/K$. If, in addition, the limitation of 2 is also a fantasy for K in the above-mentioned $1/K$, and K, depending on the used dielectric, can take amounts like 20, 30, 40, 80, 100, 200, 300, ..., then we should conclude that the resonance frequency becomes almost zero when these dielectrics are used (since eg square root of 1/300 is about zero). A question: Is this the case or not? And in principle, is this reasonable? But as we saw in this article the coefficient by which the square of the resonance frequency, when the dielectric is inserted, increases is:

$$\frac{2(K-1)+\sqrt{4(K-1)^2+1}}{K} = 2(1-\frac{1}{K}) + \sqrt{4-\frac{8}{K}+\frac{5}{K^2}}$$

It is seen when $K=1$, square of the resonance frequency is 1, and when K is infinity, square of the resonance frequency is 4. This means that by inserting a linear dielectric we expect that the resonance frequency will become double at most (when we have an ideal linear dielectric with infinite permittivity). That ratio of the resonance frequency with dielectric to the one without dielectric is a number between 1 and 2 is analogous to that the ratio of the charge gathered in the capacitor with dielectric to the one without dielectric is a number between 1 and 2.

3.5 Again parallel-plate capacitor as another test

Now we obtain the electrostatic potential energy of the parallel-plate capacitor mentioned at the end of the section 3.2 by two methods. First, using the relation $U = 1/2C(\Delta\phi)^2_Q$ we obtain $U = 1/2(\epsilon_0 A/d)(\Delta\phi)^2_Q$.

In the second method we use the relation (2), ie $U = 1/2\int_{V_h} \mathbf{D}\cdot\mathbf{E_Q}dv$ in which $\mathbf{E_Q}$ is the field arising from Q and $-Q$ (and not also from the polarization charges). We have the following relation:

$$\mathbf{D} = \epsilon\mathbf{E} = \epsilon(\mathbf{E}_Q + \mathbf{E_P}) = \epsilon\mathbf{E}_Q + \epsilon\mathbf{E_P} \tag{31}$$

in which $\mathbf{E_P}$ is the field arising only from the polarization charges of the dielectric set between the two plates. Let's obtain $\mathbf{E_P}$ in terms of $\mathbf{D}$. Suppose that $\mathbf{P}$ is the polarization of the dielectric and $\hat{n}$ is the unit vector in the direction of $\mathbf{E}$. We know $\mathbf{P}\cdot(-\hat{n})$ is the polarization charge surface density formed adjacent to the plate bearing the (positive) charge Q, and $\mathbf{P}\cdot\hat{n}$ is the polarization charge surface density formed adjacent to the plate bearing the charge $-Q$. Since $\mathbf{P} = (\epsilon-\epsilon_0)\mathbf{E}$, we have $\mathbf{P}\cdot(-\hat{n}) = (\epsilon_0-\epsilon)E$ and $\mathbf{P}\cdot\hat{\mathbf{n}} = (\epsilon-\boldsymbol{\epsilon_0})\mathbf{E}$ which the first is negative and the second is positive obviously. Then, the electrostatic field arising from these (polarization) charges in the dielectric is

$$\mathbf{E_P} = \frac{\mathbf{P}\cdot\hat{n}}{\epsilon_0}(-\hat{n}) = \frac{\epsilon_0-\epsilon}{\epsilon_0}\mathbf{E} \tag{32}$$

and since $\mathbf{D} = \epsilon\mathbf{E}$ we have $\mathbf{E_P} = (\epsilon_0-\epsilon)/(\epsilon_0\epsilon)\mathbf{D}$. Combining this result with the relation (31) yields

$$\mathbf{D} = \epsilon\mathbf{E}_Q + \frac{\epsilon_0-\epsilon}{\epsilon_0}\mathbf{D} \Rightarrow \mathbf{D} = \epsilon_0\mathbf{E}_Q. \tag{33}$$

Therefore, we have

$$U = \frac{1}{2}\int_{V_h} \mathbf{D}\cdot\mathbf{E}_Q dv = \frac{1}{2}\int_{V=Ad} \epsilon_0 E_Q^2 dv = \frac{1}{2}\epsilon_0 Ad E_Q^2 = \frac{1}{2}\epsilon_0 Ad((\Delta\phi)_Q/d)^2$$
$$= \frac{1}{2}(\epsilon_0 A/d)(\Delta\phi)_Q^2,$$

which is the same result obtained in the first method.

Now we proceed to another case. Consider the figure. The (unindicated) width of the plates is w. A linear dielectric block is along the l-dimension and only the length x is between the plates. Potential difference between the two plates is constant (equal to $(\Delta\phi)_Q$; we proved this fact beforehand). It is clear that the charges on that part of a plate of the capacitor which is in the empty part of the capacitor exert an attractive force on the polarization charges adjacent to that plate and a repulsive force on the polarization charges adjacent to the other plate, while the charges on the empty part of the other plate act a similar work, and the resultant force of all of these forces is an inward force along the l-dimension magnitude of which must approach zero when d approaches zero. Now let's try to obtain this force from the energy method. First of all, according to what said so far, it is obvious that with the dielectric displacement the electrostatic potential energy of the capacitor being only of the capacitor charge (Q and $-Q$) does not alter. Thus, only the electrostatic potential energy of the dielectric and its alteration must be considered.

We know that the surface density of polarization charge of the dielectric in the capacitor is $+P$ or $-P$ and then the electrostatic field arising from it is $\mathbf{E}_P = -\mathbf{P}/\epsilon_{\mathbf{0}}$. On the other hand, by using each of the relations (1) and (2) we obtain a unique expression for the electrostatic potential energy of only the polarization charges of the dielectric:

$$(1) \Longrightarrow U_{\mathbf{P}} = \frac{1}{2}\int_{V_h} \rho\phi dv = \frac{1}{2}((-\frac{Pd}{2\epsilon_0}+0)(-Q_{\mathbf{P}}) + (-\frac{-Pd}{2\epsilon_0}+0)(Q_{\mathbf{P}}))$$
$$= \frac{Pd}{2\epsilon_0}Q_{\mathbf{P}} = \frac{Pd}{2\epsilon_0}P(wx) = \frac{P^2 d}{2\epsilon_0}wx$$

considering that the potential arising from an infinite charged plate with the surface charge density σ is $-\sigma/(2\epsilon_0)d$ at the (nonnegative) distance d from the plate, and

$$(2) \Longrightarrow U_{\mathbf{P}} = \frac{1}{2}\int_{V_h} \epsilon_0 \mathbf{E_P}\cdot\mathbf{E_P} dv = \frac{\epsilon_0}{2}\int_{V_h} E_{\mathbf{P}}^2 dv = \frac{\epsilon_0}{2}\int_{V_h} (\frac{P}{\epsilon_0})^2 dv$$
$$= \frac{\epsilon_0}{2}\frac{P^2}{\epsilon_0^2}wxd = \frac{P^2 d}{2\epsilon_0}wx.$$

We have also $\mathbf{P} = -\epsilon_0\mathbf{E_P}$ from $\mathbf{E_P} = -\mathbf{P}/\epsilon_0$. If in addition we apply the relations (32), (33) and (31), we shall obtain

$$\mathbf{P} = -\epsilon_0\mathbf{E_P} = (\epsilon-\epsilon_0)\mathbf{E} = (\epsilon_0(\epsilon-\epsilon_0)/\epsilon)\mathbf{E}_Q$$

and consequently

$$U_{\mathbf{P}} = \frac{P^2 d}{2\epsilon_0} wx = \frac{\epsilon_0 E_{\mathbf{P}}^2 d}{2} wx = \frac{(\epsilon - \epsilon_0)^2 E^2 d}{2\epsilon_0} wx = \frac{\epsilon_0(\epsilon - \epsilon_0)^2 E_Q^2 d}{2\epsilon^2} wx.$$

Since with displacement of the dielectric only x is changed,

$$dU_{\mathbf{P}} = \frac{\epsilon_0 E_{\mathbf{P}}^2 d}{2} wdx = \frac{(\epsilon - \epsilon_0)^2 E^2 d}{2\epsilon_0} wdx = \frac{\epsilon_0(\epsilon - \epsilon_0)^2 E_Q^2 d}{2\epsilon^2} wdx. \tag{34}$$

We know that the above-mentioned force pulling the dielectric into the capacitor performs some work on the dielectric which, according to the conservation law of energy, this work must be conserved in some manner. By pulling inward, this force not only causes forming more polarization charges, but also alters (and in fact increases) the kinetic energy of the dielectric block. Thus, the above mentioned work is conserved both as the electrostatic potential energy of the formed polarization charges and as the alteration of the kinetic energy. We show this work as dW and the alteration of the electrostatic potential energy as $dU_{\mathbf{P}}$ and the alteration of the kinetic energy as dT. Therefore, we have:

$$\left.\begin{array}{l} dW = dU_{\mathbf{P}} + dT \\ dW = F_x dx \end{array}\right\} \Rightarrow \left.\begin{array}{r} F_x dx = dU_{\mathbf{P}} + dT \\ (34) \end{array}\right\} \Rightarrow F_x dx = \frac{\epsilon_0(\epsilon-\epsilon_0)^2 wd}{2\epsilon^2} E_Q^2 dx + dT$$

$$= \frac{\epsilon_0(\epsilon - \epsilon_0)^2}{2\epsilon^2} w \frac{(\Delta\phi)_Q^2}{d} dx + dT. \tag{35}$$

It is obvious that if in an especial case we have $dT = 0$ then we shall have

$$F_x = \frac{\epsilon_0(\epsilon - \epsilon_0)^2}{2\epsilon^2} E_Q^2 wd = \frac{\epsilon_0(\epsilon - \epsilon_0)^2}{2\epsilon^2} w \frac{(\Delta\phi)_Q^2}{d}. \tag{36}$$

(It is seen that as was predicted beforehand, this force will approach zero if d approaches zero.)

Observing the present current mistakes (including what we saw about the capacitance and electrostatic potential energy of a capacitor) we see the following relation instead of Eq. (35) in the present books of Electricity and Magnetism or Electromagnetism:

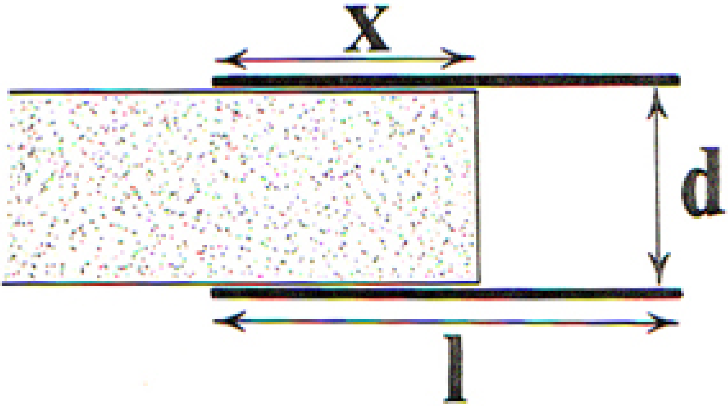
$$F_x dx = \frac{1}{2}(\epsilon - \epsilon_0) w \frac{(\Delta\phi)_Q^2}{d} dx = \frac{1}{2}(K-1)\epsilon_0 E_Q^2 (wd) dx \tag{37}$$

where it is supposed that $(\Delta\phi)_Q$ remains constant. (How? I don't know(!) because even by connecting the two plates to a battery the voltage of the battery is equal to the sum of $(\Delta\phi)_Q$ and the potential difference caused by the dielectric (also see the beginning part of Section 3.4).)

And also by mistake the following general result (instead of the especial result (36)) is inferred from the relation (37):

$$F_x = \frac{1}{2}(\epsilon - \epsilon_0) w \frac{(\Delta\phi)_Q^2}{d} = \frac{1}{2}(K-1)\epsilon_0 E_Q^2 wd$$

Practical comparison of the above relations for experimental testing of the truth of Eq. (35) should be possible by preparing ideal conditions and regarding fringing effects at the edges of the capacitor and considering the real value of K (see Note section in the previous section).



x
d
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