TGD: PHYSICS AS INFINITE-DIMENSIONAL GEOMETRY

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TOPOLOGICAL GEOMETRODYNAMICS: PHYSICS AS INFINITE-DIMENSIONAL GEOMETRY

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Preface

This book belongs to a series of online books summarizing the recent state Topological Geometrodynamics (TGD) and its applications. TGD can be regarded as a unified theory of fundamental interactions but is not the kind of unified theory as so called GUTs constructed by graduate students at seventies and eighties using detailed recipes for how to reduce everything to group theory. Nowadays this activity has been completely computerized and it probably takes only a few hours to print out the predictions of this kind of unified theory as an article in the desired format. TGD is something different and I am not ashamed to confess that I have devoted the last 37 years of my life to this enterprise and am still unable to write The Rules.

If I remember correctly, I got the basic idea of Topological Geometrodynamics (TGD) during autumn 1977, perhaps it was October. What I realized was that the representability of physical space-times as 4-dimensional surfaces of some higher-dimensional space-time obtained by replacing the points of Minkowski space with some very small compact internal space could resolve the conceptual difficulties of general relativity related to the definition of the notion of energy. This belief was too optimistic and only with the advent of what I call zero energy ontology the understanding of the notion of Poincare invariance has become satisfactory. This required also the understanding of the relationship to General Relativity.

It soon became clear that the approach leads to a generalization of the notion of space-time with particles being represented by space-time surfaces with finite size so that TGD could be also seen as a generalization of the string model. Much later it became clear that this generalization is consistent with conformal invariance only if space-time is 4-dimensional and the Minkowski space factor of imbedding space is 4-dimensional. During last year it became clear that 4-D Minkowski space and 4-D complex projective space CP_2 are completely unique in the sense that they allow twistor space with Kähler structure.

It took some time to discover that also the geometrization of also gauge interactions and elementary particle quantum numbers could be possible in this framework: it took two years to find the unique internal space (CP_2) providing this geometrization involving also the realization that family replication phenomenon for fermions has a natural topological explanation in TGD framework and that the symmetries of the standard model symmetries are much more profound than pragmatic TOE builders have believed them to be. If TGD is correct, main stream particle physics chose the wrong track leading to the recent deep crisis when people decided that quarks and leptons belong to same multiplet of the gauge group implying instability of proton.

There have been also longstanding problems.

- Gravitational energy is well-defined in cosmological models but is not conserved. Hence the conservation of the inertial energy does not seem to be consistent with the Equivalence Principle. Furthermore, the imbeddings of Robertson-Walker cosmologies turned out to be vacuum extremals with respect to the inertial energy. About 25 years was needed to realize that the sign of the inertial energy can be also negative and in cosmological scales the density of inertial energy vanishes: physically acceptable universes are creatable from vacuum. Eventually this led to the notion of zero energy ontology (ZEO) which deviates dramatically from the standard ontology being however consistent with the crossing symmetry of quantum field theories. In this framework the quantum numbers are assigned with zero energy states located at the boundaries of so called causal diamonds defined as intersections of future and past directed light-cones. The notion of energy-momentum becomes length scale dependent since one has a scale hierarchy for causal diamonds. This allows to understand the non-conservation of energy as apparent.

  Equivalence Principle as it is expressed by Einstein's equations follows from Poincare invariance once it is realized that GRT space-time is obtained from the many-sheeted space-time of TGD by lumping together the space-time sheets to a regionof Minkowski space and endowing it with an effective metric given as a sum of Minkowski metric and deviations of the metrices of space-time sheets from Minkowski metric. Similar description relates classical gauge potentials identified as components of induced spinor connection to Yang-Mills gauge potentials in GRT space-time. Various topological inhomogenities below resolution scale identified as particles are described using energy momentum tensor and gauge currents.

- From the beginning it was clear that the theory predicts the presence of long ranged classical electro-weak and color gauge fields and that these fields necessarily accompany classical electromagnetic fields.

  It took about 26 years to gain the maturity to admit the obvious: these fields are classical correlates for long range color and weak interactions assignable to dark matter. The only possible conclusion is that TGD physics is a fractal consisting of an entire hierarchy of fractal copies of standard model physics. Also the understanding of electro-weak massivation and screening of weak charges has been a long standing problem, and 32 years was needed to discover that what I call weak form of electric-magnetic duality gives a satisfactory solution of the problem and provides also surprisingly powerful insights to the mathematical structure of quantum TGD.

  The latest development was the realization that the well- definedness of electromagnetic charge as quantum number for the modes of the induced spinors field requires that the CP_2 projection of the region in which they are non-vanishing carries vanishing W boson field and is 2-D. This implies in the generic case their localization to 2-D surfaces: string world sheets and possibly also partonic 2-surfaces. This localization applies to all modes except covariantly constant right handed neutrino generating supersymmetry and mplies that string model in 4-D space-time is part of TGD. Localization is possible only for Kähler-Dirac assigned with Kähler action defining the dynamics of space-time surfaces. One must however leave open the question whether W field might vanish for the space-time of GRT if related to many-sheeted space-time in the proposed manner even when they do not vanish for space-time sheets.

I started the serious attempts to construct quantum TGD after my thesis around 1982. The original optimistic hope was that path integral formalism or canonical quantization might be enough to construct the quantum theory but the first discovery made already during first year of TGD was that these formalisms might be useless due to the extreme non-linearity and enormous vacuum degeneracy of the theory. This turned out to be the case.

- It took some years to discover that the only working approach is based on the generalization of Einstein's program. Quantum physics involves the geometrization of the infinite-dimensional "world of classical worlds" (WCW) identified as 3-dimensional surfaces. Still few years had to pass before I understood that general coordinate invariance leads to a more or less unique solution of the problem and in positive energyontology implies that space-time surfaces are analogous to Bohr orbits. This in positive energy ontology in which space-like 3-surface is basic object. It is not clear whether Bohr orbitology is necessary also in ZEO in which space-time surfaces connect space-like 3-surfaces at the light-like boundaries of causal diamond CD obtained as intersection of future and past directed light-cones (with CP_2 factor included). The reason is that the pair of 3-surfaces replaces the boundary conditions at single 3-surface involving also time derivatives. If one assumes Bohr orbitology then strong correlations between the 3-surfaces at the ends of CD follow. Still a couple of years and I discovered that quantum states of the Universe can be identified as classical spinor fields in WCW. Only quantum jump remains the genuinely quantal aspect of quantum physics.

- During these years TGD led to a rather profound generalization of the space-time concept. Quite general properties of the theory led to the notion of many-sheeted space-time with sheets representing physical subsystems of various sizes. At the beginning of 90s I became dimly aware of the importance of p-adic number fields and soon ended up with the idea that p-adic thermodynamics for a conformally invariant system allows to understand elementary particle massivation with amazingly few input assumptions. The attempts to understand p-adicity from basic principles led gradually to the vision about physics as a generalized number theory as an approach complementary to the physics as an infinite-dimensional spinor geometry of WCW approach. One of its elements was a generalization of the number concept obtained by fusing real numbers and various p-adic numbers along common rationals. The number theoretical trinity involves besides p-adic number fields also quaternions and octonions and the notion of infinite prime.

- TGD inspired theory of consciousness entered the scheme after 1995 as I started to write a book about consciousness. Gradually it became difficult to say where physics ends and consciousness theory begins since consciousness theory could be seen as a generalization of quantum measurement theory by identifying quantum jump as a moment of consciousness and by replacing the observer with the notion of self identified as a system which is conscious as long as it can avoid entanglement with environment. The somewhat cryptic statement "Everything is conscious and consciousness can be only lost" summarizes the basic philosophy neatly.

The idea about p-adic physics as physics of cognition and intentionality emerged also rather naturally and implies perhaps the most dramatic generalization of the space-time concept in which most points of p-adic space-time sheets are infinite in real sense and the projection to the real imbedding space consists of discrete set of points. One of the most fascinating outcomes was the observation that the entropy based on p-adic norm can be negative. This observation led to the vision that life can be regarded as something in the intersection of real and p-adic worlds. Negentropic entanglement has interpretation as a correlate for various positively colored aspects of conscious experience and means also the possibility of strongly correlated states stable under state function reduction and different from the conventional bound states and perhaps playing key role in the energy metabolism of living matter.

If one requires consistency of Negentropy Mazimization Principle with standard measurement theory, negentropic entanglement defined in terms of number theoretic negentropy is necessarily associated with a density matrix proportional to unit matrix and is maximal and is characterized by the dimension n of the unit matrix. Negentropy is positive and maximal for a p-adic unique prime dividing n.

- One of the latest threads in the evolution of ideas is not more than nine years old. Learning about the paper of Laurent Nottale about the possibility to identify planetary orbits as Bohr orbits with a gigantic value of gravitational Planck constant made once again possible to see the obvious. Dynamical quantized Planck constant is strongly suggested by quantum classical correspondence and the fact that space-time sheets identifiable as quantum coherence regions can have arbitrarily large sizes. Second motivation for the hierarchy of Planck constants comes from bio-electromagnetism suggesting that in living systems Planck constant could have large values making macroscopic quantum coherence possible. The interpretation of dark matter as a hierarchy of phases of ordinary matter characterized by the value of Planck constant is very natural.

  During summer 2010 several new insights about the mathematical structure and interpretation of TGD emerged. One of these insights was the realization that the postulated hierarchy of Planck constants might follow from the basic structure of quantum TGD. The point is that due to the extreme non-linearity of the classical action principle the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is one-to-many and the natural description of the situation is in terms of local singular covering spaces of the imbedding space. One could speak about effective value of Planck constant $h_{eff} = n \times h$ coming as a multiple of minimal value of Planck constant. Quite recently it became clear that the non-determinism of Kähler action is indeed the fundamental justification for the hierarchy: the integer n can be also interpreted as the integer characterizing the dimension of unit matrix characterizing negentropic entanglement made possible by the many-sheeted character of the space-time surface.

  Due to conformal invariance acting as gauge symmetry the n degenerate space-time sheets must be replaced with conformal equivalence classes of space-time sheets and conformal transformations correspond to quantum critical deformations leaving the ends of space-time surfaces invariant. Conformal invariance would be broken: only the sub-algebra for which conformal weights are divisible by n act as gauge symmetries. Thus deep connections between conformal invariance related to quantum criticality, hierarchy of Planck constants, negentropic entanglement, effective p-adic topology, and non-determinism of Kähler action perhaps reflecting p-adic non-determinism emerges.

  The implications of the hierarchy of Planck constants are extremely far reaching so that the significance of the reduction of this hierarchy to the basic mathematical structure distinguishing between TGD and competing theories cannot be under-estimated.

From the point of view of particle physics the ultimate goal is of course a practical construction recipe for the S-matrix of the theory. I have myself regarded this dream as quite too ambitious taking into account how far reaching re-structuring and generalization of the basic mathematical structure of quantum physics is required. It has indeed turned out that the dream about explicit formula is unrealistic before one has understood what happens in quantum jump. Symmetries and general physical principles have turned out to be the proper guide line here. To give some impressions about what is required some highlights are in order.

- With the emergence of ZEO the notion of S-matrix was replaced with M-matrix defined between positive and negative energy parts of zero energy states. M-matrix can be interpreted as a complex square root of density matrix representable as a diagonal and positive square root of density matrix and unitary S-matrix so that quantum theory in ZEO can be said to define a square root of thermodynamics at least formally. M-matrices in turn bombine to form the rows of unitary U-matrix defined between zero energy states.

- A decisive step was the strengthening of the General Coordinate Invariance to the requirement that the formulations of the theory in terms of light-like 3-surfaces identified as 3-surfaces at which the induced metric of space-time surfaces changes its signature and in terms of space-like 3-surfaces are equivalent. This means effective 2-dimensionality in the sense that partonic 2-surfaces defined as intersections of these two kinds of surfaces plus 4-D tangent space data at partonic 2-surfaces code for the physics. Quantum classical correspondence requires the coding of the quantum numbers characterizing quantum states assigned to the partonic 2-surfaces to the geometry of space-time surface. This is achieved by adding to the modified Dirac action a measurement interaction term assigned with light-like 3-surfaces.

- The replacement of strings with light-like 3-surfaces equivalent to space-like 3-surfaces means enormous generalization of the super conformal symmetries of string models. A further generalization of these symmetries to non-local Yangian symmetries generalizing the recently discovered Yangian symmetry of $\mathcal{N} = 4$ supersymmetric Yang-Mills theories is highly suggestive. Here the replacement of point like particles with partonic 2-surfaces means the replacement of conformal symmetry of Minkowski space with infinite-dimensional super-conformal algebras. Yangian symmetry provides also a further refinement to the notion of conserved quantum numbers allowing to define them for bound states using non-local energy conserved currents.

- A further attractive idea is that quantum TGD reduces to almost topological quantum field theory. This is possible if the Kähler action for the preferred extremals defining WCW Kähler function reduces to a 3-D boundary term. This takes place if the conserved currents are so called Beltrami fields with the defining property that the coordinates associated with flow lines extend to single global coordinate variable. This ansatz together with the weak form of electric-magnetic duality reduces the Kähler action to Chern-Simons term with the condition that the 3-surfaces are extremals of Chern-Simons action subject to the constraint force defined by the weak form of electric magnetic duality. It is the latter constraint which prevents the trivialization of the theory to a topological quantum field theory. Also the identification of the Kähler function of WCW as Dirac determinant finds support as well as the description of the scattering amplitudes in terms of braids with interpretation in terms of finite measurement resolution coded to the basic structure of the solutions of field equations.

- In standard QFT Feynman diagrams provide the description of scattering amplitudes. The beauty of Feynman diagrams is that they realize unitarity automatically via the so called Cutkosky rules. In contrast to Feynman's original beliefs, Feynman diagrams and virtual particles are taken only as a convenient mathematical tool in quantum field theories. QFT approach is however plagued by UV and IR divergences and one must keep mind open for the possibility that a genuine progress might mean opening of the black box of the virtual particle.

  In TGD framework this generalization of Feynman diagrams indeed emerges unavoidably. Light-like 3-surfaces replace the lines of Feynman diagrams and vertices are replaced by 2-D partonic 2-surfaces. Zero energy ontology and the interpretation of parton orbits as light-like "wormhole throats" suggests that virtual particle do not differ from on mass shell particles only in that the four- and three- momenta of wormhole throats fail to be parallel. The two throats of the wormhole contact defining virtual particle would contact carry on mass shell quantum numbers but for virtual particles the four-momenta need not be parallel and can also have opposite signs of energy.

The localization of the nodes of induced spinor fields to 2-D string world sheets (and possibly also to partonic 2-surfaces) implies a stringy formulation of the theory analogous to stringy variant of twistor formalism with string world sheets having interpretation as 2-braids. In TGD framework fermionic variant of twistor Grassmann formalism leads to a stringy variant of twistor diagrammatics in which basic fermions can be said to be on mass-shell but carry non-physical helicities in the internal lines. This suggests the generalization of the Yangian symmetry to infinite-dimensional super-conformal algebras.

What I have said above is strongly biased view about the recent situation in quantum TGD. This vision is single man's view and doomed to contain unrealistic elements as I know from experience. My dream is that young critical readers could take this vision seriously enough to try to demonstrate that some of its basic premises are wrong or to develop an alternative based on these or better premises. I must be however honest and tell that 32 years of TGD is a really vast bundle of thoughts and quite a challenge for anyone who is not able to cheat himself by taking the attitude of a blind believer or a light-hearted debunker trusting on the power of easy rhetoric tricks.

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Neither TGD nor these books would exist without the help and encouragement of many people. The friendship with Heikki and Raija Haila and their family have been kept me in contact with the everyday world and without this friendship I would not have survived through these lonely 32 years most of which I have remained unemployed as a scientific dissident. I am happy that my children have understood my difficult position and like my friends have believed that what I am doing is something valuable although I have not received any official recognition for it.

During last decade Tapio Tammi has helped me quite concretely by providing the necessary computer facilities and being one of the few persons in Finland with whom to discuss about my work. I have had also stimulating discussions with Samuli Penttinen who has also helped to get through the economical situations in which there seemed to be no hope. The continual updating of fifteen online books means quite a heavy bureaucracy at the level of bits and without a systemization one ends up with endless copying and pasting and internal consistency is soon lost. Pekka Rapinoja has offered his help in this respect and I am especially grateful for him for my Python skills. Also Matti Vallinkoski has helped me in computer related problems.

The collaboration with Lian Sidorov was extremely fruitful and she also helped me to survive economically through the hardest years. The participation to CASYS conferences in Liege has been an important window to the academic world and I am grateful for Daniel Dubois and Peter Marcer for making this participation possible. The discussions and collaboration with Eduardo de Luna and Istvan Dienes stimulated the hope that the communication of new vision might not be a mission impossible after all. Also blog discussions have been very useful. During these years I have received innumerable email contacts from people around the world. In particualr, I am grateful for Mark McWilliams and Ulla Matfolk for providing links to possibly interesting web sites and articles. These contacts have helped me to avoid the depressive feeling of being some kind of Don Quixote of Science and helped me to widen my views: I am grateful for all these people.

In the situation in which the conventional scientific communication channels are strictly closed it is important to have some loop hole through which the information about the work done can at least in principle leak to the publicity through the iron wall of the academic censorship. Without any exaggeration I can say that without the world wide web I would not have survived as a scientist nor as individual. Homepage and blog are however not enough since only the formally published

result is a result in recent day science. Publishing is however impossible without a direct support from power holders- even in archives like arXiv.org.

Situation changed for five years ago as Andrew Adamatsky proposed the writing of a book about TGD when I had already got used to the thought that my work would not be published during my life time. The Prespacetime Journal and two other journals related to quantum biology and consciousness - all of them founded by Huping Hu - have provided this kind of loop holes. In particular, Dainis Zeps, Phil Gibbs, and Arkadiusz Jadczyk deserve my gratitude for their kind help in the preparation of an article series about TGD catalyzing a considerable progress in the understanding of quantum TGD. Also the viXra archive founded by Phil Gibbs and its predecessor Archive Freedom have been of great help: Victor Christianto deserves special thanks for doing the hard work needed to run Archive Freedom. Also the Neuroquantology Journal founded by Sultan Tarlaci deserves a special mention for its publication policy. And last but not least: there are people who experience as a fascinating intellectual challenge to spoil the practical working conditions of a person working with something which might be called unified theory: I am grateful for the people who have helped me to survive through the virus attacks, an activity which has taken roughly one month per year during the last half decade and given a strong hue of grey to my hair.

For a person approaching his sixty year birthday it is somewhat easier to overcome the hard feelings due to the loss of academic human rights than for an inpatient youngster. Unfortunately the economic situation has become increasingly difficult during the twenty years after the economic depression in Finland which in practice meant that Finland ceased to be a constitutional state in the strong sense of the word. It became possible to depose people like me from the society without fear about public reactions and the classification as dropout became a convenient tool of ridicule to circumvent the ethical issues. During last few years when the right wing has held the political power this trend has been steadily strengthening. In this kind of situation the concrete help from individuals has been and will be of utmost importance. Against this background it becomes obvious that this kind of work is not possible without the support from outside and I apologize for not being able to mention all the people who have helped me during these years.

Matti Pitkänen

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Introduction

1.1 Basic Ideas of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged 37 years ago - would emerge now it would be seen as an attempt trying to solve the difficulties of these approaches to unification.

The basic physical picture behind TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. The CMAP files at my homepage provide an overview about ideas and evolution of TGD and make easier to understand what TGD and its applications are about (`http://www.tgdtheory.fi/cmaphtml.html` [L13]).

1.1.1 Basic vision very briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K1].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional imbedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).

2. Induction procedure allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections ofCP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface.

3. Geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and

lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. Imbedding space H has a number theoretic interpretation as 8-D space allowing octonionic tangent space structure. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of imbedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

   Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particle in space-time can be identified as a topological inhomogenuity in background space-time surface which looks like the space-time of general relativity in long length scales.

   One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distance of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

   This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which standard model and general relativity follow as a topological simplification however forcing to increase dramatically the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. The resolution of problem is implied by the condition that the modes of the induced spinor fields have well-defined electromagnetic charge. This forces their localization to 2-D string world sheets in the generic case having vanishing weak gauge fields so that parity breaking effects emerge just as they do in standard model. Also string model like picture emerges from TGD and one ends up with a rather concrete view about generalized Feynman diagrammatics.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last thirty seven years for the realization of this dream and this has resulted in eight online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

1.1.2 Two manners to see TGD and their fusion

As already mentioned, TGD can be interpreted both as a modification of general relativity and generalization of string models.

TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_\times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A59, A42, A54, A40].

The identification of the space-time as a sub-manifold [A36, A57] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces and string like objects (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possibly existence vapour phase.

What one obtains is what I have christened as many-sheeted space-time (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory system does not possess this kind of field identity. The notion of magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian and interpreted as lines of generalized Feynman diagrams. Also the Euclidian 4-D regions would have similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about strong form of holography.

1.1.3 Basic objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four imbedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particle topologically condenses to several space-time sheets simultaneously and experiences the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the imbeddability to 8-D imbedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation allows to understand the relationship to GRT space-time and how Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Topological field quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other thins this leads to models for cell membrane, nerve pulse, and EEG.

1.1.4 p-Adic variants of space-time surfaces

There is a further generalization of the space-time concept inspired by p-adic physics forcing a generalization of the number concept through the fusion of real numbers and various p-adic number fields. Also the hierarchy of Planck constants forces a generalization of the notion of space-time but this generalization can be understood in terms of the failure of strict determinism for Kähler action defining the fundamental variational principle behind the dynamics of space-time surfaces.

A very concise manner to express how TGD differs from Special and General Relativities could be following. Relativity Principle (Poincare Invariance), General Coordinate Invariance, and Equivalence Principle remain true. What is new is the notion of sub-manifold geometry: this allows to realize Poincare Invariance and geometrize gravitation simultaneously. This notion also allows a geometrization of known fundamental interactions and is an essential element of all applications of TGD ranging from Planck length to cosmological scales. Sub-manifold geometry is also crucial in the applications of TGD to biology and consciousness theory.

1.1.5 The threads in the development of quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

The theoretical framework involves several threads.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name 'TGD as a generalized number theory'. The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" thread.

3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to four.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The eight online books [K58, K42, K34, K75, K49, K74, K73, K48] about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology [K53, K7, K38, K6, K21, K25, K27, K47, K68] are warmly recommended to the interested reader.

Quantum TGD as spinor geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones:

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH ("world of classical worlds",WCW) consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [A51, A60, A61]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \to B+C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

2. During years this naive and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects unexpected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "configuration space". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

3. WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory [1]. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operator of WCW so that this classical free field theory would dictate M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. Given M-matrix in turn would decompose to a product of a hermitian density matrix and unitary S-matrix.

   M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the complex square roots of density matrices commuting with S-matrix means that they span infinite-dimensional Lie algebra acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in well-defined sense: its own symmetries would define the symmetries of the theory. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian

[1] There are four kinds of Dirac operators in TGD. WCW Dirac operator appearing in Super-Virasoro conditions, imbedding space Dirac operator whose modes define the ground states of Super-Virasoro representations, Kähler-Dirac operator at space-time surfaces, and the algebraic variant of M^4 Dirac operator appearing in propagators

square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible.

4. By quantum classical correspondence the construction of WCW spinor structure reduces to the second quantization of the induced spinor fields at space-time surface. The basic action is so called modified Dirac action (or Kähler-Dirac action) in which gamma matrices are replaced with the modified (Kähler-Dirac) gamma matrices defined as contractions of the canonical momentum currents with the imbedding space gamma matrices. In this manner one achieves super-conformal symmetry and conservation of fermionic currents among other things and consistent Dirac equation. The modified gamma matrices define as anti-commutators effective metric, which might provide geometrization for some basic observables of condensed matter physics. One might also talk about bosonic emergence in accordance with the prediction that the gauge bosons and graviton are expressible in terms of bound states of fermion and anti-fermion.

5. An important result relates to the notion of induced spinor connection. If one requires that spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrino generating super-symmetries forms an exception. The vanishing of also Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

   The localization simplifies enormously the mathematics and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

   At the light-like 3-surfaces at which the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes one can pose the condition that the algebraic analog of massless Dirac equation is satisfied by the nodes so that Kähler-Dirac action gives massless Dirac propagator localizable at the boundaries of the string world sheets.

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factorc coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

   Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The manner to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak

form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this manner almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space ("world of classical worldss", WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name 'TGD as a generalized number theory'. It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

1. p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduces the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.

2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that

clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

The notion of p-adic manifold [K79] identified as p-adic space-time surface solving p-adic analogs of field equations and having real space-time sheets as chart maps provides a possible solution of the basic challenge. One can also speak of real space-time surfaces having p-adic space-time surfaces as chart maps (cognitive maps, "thought bubbles"). Discretization required having interpretation in terms of finite measurement resolution is unavoidable in this approach.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structures. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of imbedding space and space-time concept and one can speak about real and p-adic space-time sheets. The quantum dynamics should be such that it allows quantum transitions transforming space-time sheets belonging to different number fields to each other. The space-time sheets in the intersection of real and p-adic worlds are of special interest and the hypothesis is that living matter resides in this intersection. This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book).

The basic principle is number theoretic universality stating roughly that the physics in various number fields can be obtained as completion of rational number based physics to various number fields. Rational number based physics would in turn describe physics in finite measurement resolution and cognitive resolution. The notion of finite measurement resolution has become one of the basic principles of quantum TGD and leads to the notions of braids as representatives of 3-surfaces and inclusions of hyper-finite factors as a representation for finite measurement resolution. The braids actually co-emerge with string world sheets implied by the condition that em charge is well-defined for spinor modes.

2. The role of classical number fields

The vision about the physical role of the classical number fields relies on certain speculative questions inspired by the idea that space-time dynamics could be reduced to associativity or co-associativity condition. Associativity means here associativity of tangent spaces of space-time region and co-associativity associativity of normal spaces of space-time region.

1. Could space-time surfaces X^4 be regarded as associative or co-associative ("quaternionic" is equivalent with "associative") surfaces of H endowed with octonionic structure in the sense that tangent space of space-time surface would be associative (co-associative with normal space associative) sub-space of octonions at each point of X^4 [K52]. This is certainly possible and an interesting conjecture is that the preferred extremals of Kähler action include associative and co-associative space-time regions.

2. Could the notion of compactification generalize to that of number theoretic compactification in the sense that one can map associative (co-associative) surfaces of M^8 regarded as octonionic linear space to surfaces in $M^4 \times CP_2$ [K52]? This conjecture - $M^8 - H$ duality - would give for $M^4 \times CP_2$ deep number theoretic meaning. CP_2 would parametrize associative planes of octonion space containing fixed complex plane $M^2 \subset M^8$ and CP_2 point would thus characterize the tangent space of $X^4 \subset M^8$. The point of M^4 would be obtained

by projecting the point of $X^4 \subset M^8$ to a point of M^4 identified as tangent space of X^4. This would guarantee that the dimension of space-time surface in H would be four. The conjecture is that the preferred extremals of Kähler action include these surfaces.

3. M^8-H duality can be generalized to a duality $H \to H$ if the images of the associative surface in M^8 is associative surface in H. One can start from associative surface of H and assume that it contains the preferred M^2 tangent plane in 8-D tangent space of H or integrable distribution $M^2(x)$ of them, and its points to H by mapping M^4 projection of H point to itself and associative tangent space to CP_2 point. This point need not be the original one! If the resulting surface is also associative, one can iterate the process indefinitely. WCW would be a category with one object.

4. G_2 defines the automorphism group of octonions, and one might hope that the maps of octonions to octonions such that the action of Jacobian in the tangent space of associative or co-associative surface reduces to that of G_2 could produce new associative/co-associative surfaces. The action of G_2 would be analogous to that of gauge group.

5. One can also ask whether the notions of commutativity and co-commutativity could have physical meaning. The well-definedness of em charge as quantum number for the modes of the induced spinor field requires their localization to 2-D surfaces (right-handed neutrino is an exception) - string world sheets and partonic 2-surfaces. This can be possible only for Kähler action and could have commutativity and co-commutativity as a number theoretic counterpart. The basic vision would be that the dynamics of Kähler action realizes number theoretical geometrical notions like associativity and commutativity and their co-notions.

The notion of number theoretic compactification stating that space-time surfaces can be regarded as surfaces of either M^8 or $M^4 \times CP_2$. As surfaces of M^8 identifiable as space of hyper-octonions they are hyper-quaternionic or co-hyper-quaternionic- and thus maximally associative or co-associative. This means that their tangent space is either hyper-quaternionic plane of M^8 or an orthogonal complement of such a plane. These surface can be mapped in natural manner to surfaces in $M^4 \times CP_2$ [K52] provided one can assign to each point of tangent space a hyper-complex plane $M^2(x) \subset M^4 \subset M^8$. One can also speak about $M^8 - H$ duality.

This vision has very strong predictive power. It predicts that the preferred extremals of Kähler action correspond to either hyper-quaternionic or co-hyper-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane $M^2(x) \subset M^4$. As a consequence, the M^4 projection of space-time surface at each point contains $M^2(x)$ and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets Y^2 and partonic 2-surfaces X^2. The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of $M^2(x)$ is as the space of non-physical polarizations and the plane of local 4-momentum.

Number theoretical compactification has inspired large number of conjectures. This includes dual formulations of TGD as Minkowskian and Euclidian string model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of WCW metric and spinor structure in terms of second quantized induced spinor fields with modified Dirac action defined by Kähler action realizing the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type II_1 about which Clifford algebra of WCW represents an example.

The two most important number theoretic conjectures relate to the preferred extremals of Kähler action. The general idea is that classical dynamics for the preferred extremals of Kähler action should reduce to number theory: space-time surfaces should be either associative or co-associative in some sense.

Associativity (co-associativity) would be that tangent (normal) spaces of space-time surfaces associative (co-associative) in some sense and thus quaternionic (co-quaternionic). This can be formulated in two manners.

1. One can introduce octonionic tangent space basis by assigning to the "free" gamma matrices octonion basis or in terms of octonionic representation of the imbedding space gamma matrices possible in dimension $D = 8$.

2. Associativity (quaternionicity) would state that the projections of octonionic basic vectors or induced gamma matrices basis to the space-time surface generates associative (quaternionic) sub-algebra at each space-time point. Co-associativity is defined in analogous manner and can be expressed in terms of the components of second fundamental form.

3. For gamma matrix option induced rather than modified gamma matrices must be in question since modified gamma matrices can span lower than 4-dimensional space and are not parallel to the space-time surfaces as imbedding space vectors.

3. Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially interesting is that p-adic and real regions of the space-time surface might aso emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

1.1.6 Hierarchy of Planck constants and dark matter hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark matter as large $\hbar$ phases

D. Da Rocha and Laurent Nottale [E2] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr}. Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K45] .

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Hierarchy of Planck constants from the anomalies of neuroscience and biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K39, K40, K66]) support the view that dark matter might be a key player in living matter.

Does the hierarchy of Planck constants reduce to the vacuum degeneracy of Kähler action?

This starting point led gradually to the recent picture in which the hierarchy of Planck constants is postulated to come as integer multiples of the standard value of Planck constant. Given integer multiple $\hbar = n\hbar_0$ of the ordinary Planck constant $\hbar_0$ is assigned with a multiple singular covering of the imbedding space [K17]. One ends up to an identification of dark matter as phases with non-standard value of Planck constant having geometric interpretation in terms of these coverings providing generalized imbedding space with a book like structure with pages labelled by Planck constants or integers characterizing Planck constant. The phase transitions changing the value of Planck constant would correspond to leakage between different sectors of the extended imbedding space. The question is whether these coverings must be postulated separately or whether they are only a convenient auxiliary tool.

The simplest option is that the hierarchy of coverings of imbedding space is only effective. Many-sheeted coverings of the imbedding space indeed emerge naturally in TGD framework. The huge vacuum degeneracy of Kähler action implies that the relationship between gradients of the imbedding space coordinates and canonical momentum currents is many-to-one: this was the very fact forcing to give up all the standard quantization recipes and leading to the idea about physics as geometry of the "world of classical worlds". If one allows space-time surfaces for which all sheets corresponding to the same values of the canonical momentum currents are present, one obtains effectively many-sheeted covering of the imbedding space and the contributions from sheets to the Kähler action are identical. If all sheets are treated effectively as one and the same sheet, the value of Planck constant is an integer multiple of the ordinary one. A natural boundary condition would be that at the ends of space-time at future and past boundaries of causal diamond containing the space-time surface, various branches co-incide. This would raise the ends of space-time surface in special physical role.

A more precise formulation is in terms of presence of large number of space-time sheets connecting given space-like 3-surfaces at the opposite boundaries of causal diamond. Quantum criticality presence of vanishing second variations of Kähler action and identified in terms of conformal invariance broken down to to sub-algebras of super-conformal algebras with conformal weights divisible by integer n is highly suggestive notion and would imply that n sheets of the effective covering are actually conformal equivalence classes of space-time sheets with same Kähler action and same values of conserved classical charges (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book). n would naturally correspond the value of h_{eff} and its factors negentropic entanglement with unit density matrix would be between the n sheets of two coverings of this kind. p-Adic prime would be largest prime power factor of n.

Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of biochemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff}.

1.2 Bird's eye of view about the topics of the book

The topics of this book are the purely geometric aspects of the vision about physics as an infinite-dimensional Kähler geometry of the "world of classical worlds", with " classical world" identified either as light-like 3-D surface of the unique Bohr orbit like 4-surface traversing through it. The non-determinism of Kähler action forces to generalize the notion of 3-surface so that unions of space-like surfaces with time like separations must be allowed. Zero energy ontology allows to formulate this picture elegantly in terms of causal diamonds defined as intersections of future and past directed light-cones. Also a a geometric realization of coupling constant evolution and finite measurement resolution emerges.

There are two separate tasks involved.

1. Provide configuration space of 3-surfaces with Kähler geometry which is consistent with 4-dimensional general coordinate invariance so that the metric is Diff^4 degenerate. General

coordinate invariance implies that the definition of metric must assign to a given light-like 3-surface X^3 a 4-surface as a kind of Bohr orbit $X^4(X^3)$.

2. Provide the configuration space with a spinor structure. The great idea is to identify configuration space gamma matrices in terms of super algebra generators expressible using second quantized fermionic oscillator operators for induced free spinor fields at the space-time surface assignable to a given 3-surface. The isometry generators and contractions of Killing vectors with gamma matrices would thus form a generalization of Super Kac-Moody algebra.

The condition of mathematical existence poses surprisingly strong conditions on configuration space metric and spinor structure.

1. From the experience with loop spaces one can expect that there is no hope about existence of well-defined Riemann connection unless this space is union of infinite-dimensional symmetric spaces with constant curvature metric and simple considerations requires that vacuum Einstein equations are satisfied by each component in the union. The coordinates labeling these symmetric spaces are zero modes having interpretation as genuinely classical variables which do not quantum fluctuate since they do not contribute to the line element of the configuration space.

2. The construction of the Kähler structure involves also the identification of complex structure. Direct construction of Kähler function as action associated with a preferred Bohr orbit like extremal for some physically motivated action action leads to a unique result. Second approach is group theoretical and is based on a direct guess of isometries of the infinite-dimensional symmetric space formed by 3-surfaces with fixed values of zero modes. The group of isometries is generalization of Kac-Moody group obtained by replacing finite-dimensional Lie group with the group of symplectic transformations of $\delta M^4_+ \times CP_2$, where δM^4_+ is the boundary of 4-dimensional future light-cone. A crucial role is played by the generalized conformal invariance assignable to light-like 3-surfaces and to the boundaries of causal diamond. Contrary to the original belief, the coset construction does not provide a realization of Equivalence Principle at quantum level. The proper realization of EP at quantum level seems to be based on the identification of classical Noether charges in Cartan algebra with the eigenvalues of their quantum counterparts assignable to Kähler-Dirac action. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to M^4 with effective metric satisfying Einstein's equations as a reflection of the underlying Poincare invariance.

3. Fermionic statistics and quantization of spinor fields can be realized in terms of configuration space spinors structure. Quantum criticality and the idea about space-time surfaces as analogs of Bohr orbits have served as basic guiding lines of Quantum TGD. These notions can be formulated more precisely in terms of the modified Dirac equation for induced spinor fields allowing also realization of super-conformal symmetries and quantum gravitational holography. A rather detailed view about how configuration space Kähler function emerges as Dirac determinant allowing a tentative identification of the preferred extremals of Kähler action as surface for which second variation of Kähler action vanishes for some deformations of the surface. The catastrophe theoretic analog for quantum critical space-time surfaces are the points of space spanned by behavior and control variables at which the determinant defined by the second derivatives of potential function with respect to behavior variables vanishes. Number theoretic vision leads to rather detailed view about preferred extremals of Kähler action. In particular, preferred extremals should be what I have dubbed as hyper-quaternionic surfaces. It it still an open question whether this characterization is equivalent with quantum criticality or not.

1.3 Sources

The eight online books about TGD [K58, K42, K75, K49, K34, K74, K73, K48] and nine online books about TGD inspired theory of consciousness and quantum biology [K53, K7, K38, K6, K21, K25, K27, K47, K68] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (http://www.tgdtheory.com/curri.html) contains a lot of material about TGD. In particular, there is summary about TGD and its applications using CMAP representation serving also as a TGD glossary [L13, L14] (see http://www.tgdtheory.fi/cmaphtml.html and http://www.tgdtheory.fi/tgdglossary.pdf).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (http://journals.sfu.ca/jnonlocality/index.php/jnonlocality founded by Lian Sidorov and in *Prespacetime Journal* (http://prespacetime.com), *Journal of Consciousness Research and Exploration* (https://www.createspace.com/4185546), and *DNA Decipher Journal* (http://dnadecipher.com), all of them founded by Huping Hu. One can find the list about the articles published at http://www.tgdtheory.com/curri.html. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

1.4 The contents of the book

In the following abstracts of various chapters of the book are given in order to provide overall view.

1.4.1 Identification of the Configuration Space Kähler Function

There are two basic approaches to quantum TGD. The first approach, which is discussed in this chapter, is a generalization of Einstein's geometrization program of physics to an infinite-dimensional context. Second approach is based on the identification of physics as a generalized number theory. The first approach relies on the vision of quantum physics as infinite-dimensional Kähler geometry for the "world of classical worlds" (WCW) identified as the space of 3-surfaces in in certain 8-dimensional space. There are three separate approaches to the challenge of constructing WCW Kähler geometry and spinor structure. The first approach relies on direct guess of Kähler function. Second approach relies on the construction of Kähler form and metric utilizing the huge symmetries of the geometry needed to guarantee the mathematical existence of Riemann connection. The third approach relies on the construction of spinor structure based on the hypothesis that complexified WCW gamma matrices are representable as linear combinations of fermionic oscillator operator for second quantized free spinor fields at space-time surface and on the geometrization of super-conformal symmetries in terms of WCW spinor structure.

In this chapter the proposal for Kähler function based on the requirement of 4-dimensional General Coordinate Invariance implying that its definition must assign to a given 3-surface a unique space-time surface. Quantum classical correspondence requires that this surface is a preferred extremal of some some general coordinate invariant action, and so called Kähler action is a unique candidate in this respect. The preferred extremal has intepretation as an analog of Bohr orbit so that classical physics becomes and exact part of WCW geometry and therefore also quantum physics.

The basic challenge is the explicit identification of WCW Kähler function K. Two assumptions lead to the identification of K as a sum of Chern-Simons type terms associated with the ends of causal diamond and with the light-like wormhole throats at which the signature of the induced metric changes. The first assumption is the weak form of electric magnetic duality. Second assumption is that the Kähler current for preferred extremals satisfies the condition $j_K \wedge dj_K = 0$ implying that the flow parameter of the flow lines of j_K defines a global space-time coordinate. This would mean that the vision about reduction to almost topological QFT would be realized.

Second challenge is the understanding of the space-time correlates of quantum criticality. Electric-magnetic duality helps considerably here. The realization that the hierarchy of Planck constant realized in terms of coverings of the imbedding space follows from basic quantum TGD leads to a further understanding. The extreme non-linearity of canonical momentum densities as functions of time derivatives of the imbedding space coordinates implies that the correspondence between these two variables is not 1-1 so that it is natural to introduce coverings of $CD \times CP_2$. This leads also to a precise geometric characterization of the criticality of the preferred extremals.

1.4.2 Construction of Configuration Space Kähler Geometry from Symmetry Principles

There are three separate approaches to the challenge of constructing WCW Kähler geometry and spinor structure. The first one relies on a direct guess of Kähler function. Second approach relies on the construction of Kähler form and metric utilizing the huge symmetries of the geometry needed to guarantee the mathematical existence of Riemann connection. The third approach relies on the construction of spinor structure assuming that complexified WCW gamma matrices are representable as linear combinations of fermionic oscillator operator for the second quantized free spinor fields at space-time surface and on the geometrization of super-conformal symmetries in terms of spinor structure.

In this chapter the construction of Kähler form and metric based on symmetries is discussed. The basic vision is that WCW can be regarded as the space of generalized Feynman diagrams with lines thickned to light-like 3-surfaces and vertices identified as partonic 2-surfaces. In zero energy ontology the strong form of General Coordinate Invariance (GCI) implies effective 2-dimensionality and the basic objects are taken to be pairs partonic 2-surfaces X^2 at opposite light-like boundaries of causal diamonds (CDs). This has turned out to be too strong formulation for effective 2-dimensionality.

The hypothesis is that WCW can be regarded as a union of infinite-dimensional symmetric spaces G/H labeled by zero modes having an interpretation as classical, non-quantum fluctuating variables. A crucial role is played by the metric 2-dimensionality of the light-cone boundary δM^4_+ and of light-like 3-surfaces implying a generalization of conformal invariance. The group G acting as isometries of WCW is tentatively identified as the symplectic group of $\delta M^4_+ \times CP_2$. H corresponds to sub-group acting as diffeomorphisms at preferred 3-surface, which can be taken to correspond to maximum of Kähler function.

An explicit construction for the Hamiltonians of WCW isometry algebra as so called flux Hamiltonians using Haltonians of light-cone boundary is proposed and also the elements of Kähler form can be constructed in terms of these. Explicit expressions for WCW flux Hamiltonians as functionals of complex coordinates of the Cartesian product of the infinite-dimensional symmetric spaces having as points the partonic 2-surfaces defining the ends of the the light 3-surface (line of generalized Feynman diagram) are proposed.

This construction suffers from some rather obvious defects. Effective 2-dimensionality is realized in too strong sense, only covariantly constant right-handed neutrino is involved, and WCW Hamiltonians do not directly reflect the dynamics of Kähler action. The construction however generalizes in very straightforward manner to a construction free of these problems. This however requires understanding of the dynamics of preferred extremals and modified Dirac action.

1.4.3 Configuration space spinor structure

There are three separate approaches to the challenge of constructing WCW Kähler geometry and spinor structure. The first approach relies on a direct guess of Kähler function. Second approach relies on the construction of Kähler form and metric utilizing the huge symmetries of the geometry needed to guarantee the mathematical existence of Riemann connection. The third approach discussed in this chapter relies on the construction of spinor structure based on the hypothesis that complexified WCW gamma matrices are representable as linear combinations of fermionic oscillator operator for the second quantized free spinor fields at space-time surface and on the geometrization of super-conformal symmetries in terms of spinor structure. This implies a geometrization of fermionic statistics.

The basic philosophy is that at fundamental level the construction of WCW geometry reduces to the second quantization of the induced spinor fields using Dirac action. This assumption is parallel with the bosonic emergence stating that all gauge bosons are pairs of fermion and antifermion at opposite throats of wormhole contact. An attractive conjecture is that vacuum functional corresponds to Dirac determinant and that it reduces to the exponent of Kähler function. In order to achieve internal consistency the induced gamma matrices appearing in Dirac operator must be replaced by the modified gamma matrices defined uniquely by Kähler action and one must also assume that extremals of Kähler action are in question so that the classical space-time dynamics reduces to a consistency condition. This implies also super-symmetries and the fermionic oscillator algebra at partonic 2-surfaces has intepretation as $\mathcal{N} = \infty$ generalization of space-time supersymmetry algebra different however from standard SUSY algebra in that Majorana spinors are not needed. This algebra serves as a building brick of various super-conformal algebras involved.

The requirement that there exist deformations giving rise to conserved Noether charges requires that the preferred extremals are critical in the sense that the second variation of the Kähler action vanishes for these deformations. Thus Bohr orbit property could correspond to criticality or at least involve it.

Quantum classical correspondence demands that quantum numbers are coded to the properties of the preferred extremals given by the Dirac determinant and this requires a linear coupling to the conserved quantum charges in Cartan algebra. Effective 2-dimensionality allows a measurement interaction term only in 3-D Chern-Simons Dirac action assignable to the wormhole throats and the ends of the space-time surfaces at the boundaries of CD. This allows also to have physical propagators reducing to Dirac propagator not possible without the measurement interaction term. An essential point is that the measurement interaction corresponds formally to a gauge transformation for the induced Kähler gauge potential. If one accepts the weak form of electric-magnetic duality Kähler function reduces to a generalized Chern-Simons term and the effect of measurement interaction term to Kähler function reduces effectively to the same gauge transformation.

The basic vision is that WCW gamma matrices are expressible as super-symplectic charges at the boundaries of CD. The basic building brick of WCW is the product of infinite-D symmetric spaces assignable to the ends of the propagator line of the generalized Feynman diagram. WCW Kähler metric has in this case "kinetic" parts associated with the ends and "interaction" part between the ends. General expressions for the super-counterparts of WCW flux Hamiltoniansand for the matrix elements of WCW metric in terms of their anticommutators are proposed on basis of this picture.

1.4.4 Does modified Dirac action define the fundamental action principle?

The construction of the spinor structure for the world of classical worlds (WCW) leads to the vision that second quantized modified Dirac equation codes for the entire quantum TGD. Among other things this would mean that Dirac determinant would define the vacuum functional of the theory having interpretation as the exponent of Kähler function of WCW and Kähler function would reduce to Kähler action for a preferred extremal of Kähler action. In this chapter the recent view about the modified Dirac action are explained in more detail.

1. Identification of the modified Dirac action

The most general form of the modified Dirac action action involves several terms. The first one is 4-dimensional assignable to Kähler action. Second term is instanton term reducible to an expression restricted to wormhole throats or any light-like 3-surfaces parallel to them in the slicing of space-time surface by light-like 3-surfaces. The third term is assignable to Chern-Simons term and has interpretation as a measurement interaction term linear in Cartan algebra of the isometry group of the imbedding space in order to obtain stringy propagators and also to realize coupling between the quantum numbers associated with super-conformal representations and space-time geometry required by quantum classical correspondence.

This means that 3-D light-like wormhole throats carry induced spinor field which can be regarded as independent degrees of freedom having the spinor fields at partonic 2-surfaces as sources and acting as 3-D sources for the 4-D induced spinor field. The most general measurement interaction would involve the corresponding coupling also for Kähler action but is not physically motivated. There are good arguments in favor of Chern-Simons Dirac action and corresponding measurement interaction.

1. A correlation between 4-D geometry of space-time sheet and quantum numbers is achieved by the identification of exponent of Kähler function as Dirac determinant making possible the entanglement of classical degrees of freedom in the interior of space-time sheet with quantum numbers.

2. Cartan algebra plays a key role not only at quantum level but also at the level of space-time geometry since quantum critical conserved currents vanish for Cartan algebra of isometries

and the measurement interaction terms giving rise to conserved currents are possible only for Cartan algebras. Furthermore, modified Dirac equation makes sense only for eigen states of Cartan algebra generators. The hierarchy of Planck constants realized in terms of the book like structure of the generalized imbedding space assigns to each CD (causal diamond) preferred Cartan algebra: in case of Poincare algebra there are two of them corresponding to linear and cylindrical M^4 coordinates.

3. Quantum holography and dimensional reduction hierarchy in which partonic 2-surface defined fermionic sources for 3-D fermionic fields at light-like 3-surfaces Y_l^3 in turn defining fermionic sources for 4-D spinors find an elegant realization. Effective 2-dimensionality is achieved if the replacement of light-like wormhole throat X_l^3 with light-like 3-surface Y_l^3 "parallel" with it in the definition of Dirac determinant corresponds to the $U(1)$ gauge transformation $K \to K + f + \overline{f}$ for Kähler function of WCW so that WCW Kähler metric is not affected. Here f is holomorphic function of WCW ("world of classical worlds") complex coordinates and arbitrary function of zero mode coordinates.

4. An elegant description of the interaction between super-conformal representations realized at partonic 2-surfaces and dynamics of space-time surfaces is achieved since the values of Cartan charges are feeded to the 3-D Dirac equation which also receives mass term at the same time. Almost topological QFT at wormhole throats results at the limit when four-momenta vanish: this is in accordance with the original vision about TGD as almost topological QFT.

5. A detailed view about the physical role of quantum criticality results. Quantum criticality fixes the values of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current except in the case of Cartan algebra of isometries. Quantum criticality allows to fix the values of couplings appearing in the measurement interaction by using the condition $K \to K + f + \overline{f}$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs).

6. The inclusion of imaginary instanton term to the definition of the modified gamma matrices is not consistent with the conjugation of the induced spinor fields. Measurement interaction can be however assigned to both Kähler action and its instanton term. CP breaking, irreversibility and the space-time description of dissipation are closely related and the CP and T oddness of the instanton part of the measurement interaction term could provide first level description for dissipative effects. It must be however emphasized that the mere addition of instanton term to Kähler function could be enough.

7. A radically new view about matter antimatter asymmetry based on zero energy ontology emerges and one could understand the experimental absence of antimatter as being due to the fact antimatter corresponds to negative energy states. The identification of bosons as wormhole contacts is the only possible option in this framework.

8. Almost stringy propagators and a consistency with the identification of wormhole throats as lines of generalized Feynman diagrams is achieved. The notion of bosonic emergence leads to a long sought general master formula for the M-matrix elements. The counterpart for fermionic loop defining bosonic inverse propagator at QFT limit is wormhole contact with fermion and cutoffs in mass squared and hyperbolic angle for loop momenta of fermion and antifermion in the rest system of emitting boson have precise geometric counterpart.

2. The exponent of Kähler function as Dirac determinant for the modified Dirac action

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography.

1. The Dirac determinant defined by the product of Dirac determinants associated with the light-like partonic 3-surfaces X_l^3 associated with a given space-time sheet X^4 is the simplest candidate for vacuum functional identifiable as the exponent of the Kähler function. Individual Dirac determinant is defined as the product of eigenvalues of the dimensionally reduced

modified Dirac operator $D_{K,3}$ and there are good arguments suggesting that the number of eigenvalues is finite. p-Adicization requires that the eigenvalues belong to a given algebraic extension of rationals. This restriction would imply a hierarchy of physics corresponding to different extensions and could automatically imply the finiteness and algebraic number property of the Dirac determinants if only finite number of eigenvalues would contribute. The regularization would be performed by physics itself if this were the case.

2. It remains to be proven that the product of eigenvalues gives rise to the exponent of Kähler action for the preferred extremal of Kähler action. At this moment the only justification for the conjecture is that this the only thing that one can imagine.

3. A long-standing conjecture has been that the zeros of Riemann Zeta are somehow relevant for quantum TGD. Rieman zeta is however naturally replaced Dirac zeta defined by the eigenvalues of $D_{K,3}$ and closely related to Riemann Zeta since the spectrum consists essentially for the cyclotron energy spectra for localized solutions region of non-vanishing induced Kähler magnetic field and hence is in good approximation integer valued up to some cutoff integer. In zero energy ontology the Dirac zeta function associated with these eigenvalues defines "square root" of thermodynamics assuming that the energy levels of the system in question are expressible as logarithms of the eigenvalues of the modified Dirac operator defining kind of fundamental constants. Critical points correspond to approximate zeros of Dirac zeta and if Kähler function vanishes at criticality as it indeed should, the thermal energies at critical points are in first order approximation proportional to zeros themselves so that a connection between quantum criticality and approximate zeros of Dirac zeta emerges.

4. The discretization induced by the number theoretic braids reduces the world of classical worlds to effectively finite-dimensional space and configuration space Clifford algebra reduces to a finite-dimensional algebra. The interpretation is in terms of finite measurement resolution represented in terms of Jones inclusion $\mathcal{M} \subset \mathcal{N}$ of HFFs with $\mathcal{M}$ taking the role of complex numbers. The finite-D quantum Clifford algebra spanned by fermionic oscillator operators is identified as a representation for the coset space $\mathcal{N}/\mathcal{M}$ describing physical states modulo measurement resolution. In the sectors of generalized imbedding space corresponding to non-standard values of Planck constant quantum version of Clifford algebra is in question.

1.4.5 The recent vision about preferred extremals and solutions of the modified Dirac equation

During years several approaches to what preferred extremals of Kähler action and solutions of the modified Dirac equation could be have been proposed and the challenge is to see whether at least some of these approaches are consistent with each other. It is good to list various approaches first.

1. For preferred extremals generalization of conformal invariance to 4-D situation is very attractive approach and leads to concrete conditions formally similar to those encountered in string model. The approach based on basic heuristics for massless equations, on effective 3-dimensionality, and weak form of electric magnetic duality is also promising. An alternative approach is inspired by number theoretical considerations and identifies space-time surfaces as associative or co-associative sub-manifolds of octonionic imbedding space.

2. There are also several approaches for solving the modified Dirac equation. The most promising approach is assumes that other than right-handed neutrino modes are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of number theoretic vision. The conditions stating that electric charge is conserved for preferred extremals is an alternative very promising approach.

In this chapter the question whether these various approaches are mutually consistent is discussed. It indeed turns out that the approach based on the conservation of electric charge leads under rather general assumptions to the proposal that solutions of the modified Dirac equation are localized on 2-dimensional string world sheets and/or partonic 2-surfaces. Einstein's equations are satisfied for the preferred extremals and this implies that the earlier proposal for the realization of Equivalence Principle is not needed. This leads to a considerable progress in the understanding of super Virasoro representations for super-symplectic and super-Kac-Moody algebra. In particular, the proposal is that super-Kac-Moody currents assignable to string world sheets define duals of gauge potentials and their generalization for gravitons: in the approximation that gauge group is Abelian - motivated by the notion of finite measurement resolution - the exponents for the sum of KM charges would define non-integrable phase factors. One can also identify Yangian as the algebra generated by these charges. The approach allows also to understand the special role of the right handed neutrino in SUSY according to TGD.

1.4.6 Recent View about Kähler Geometry and Spin Structure of "World of Classical Worlds"

The construction of Kähler geometry of WCW ("world of classical worlds") is fundamental to TGD program. I ended up with the idea about physics as WCW geometry around 1985 and made a breakthrough around 1990, when I realized that Kähler function for WCW could correspond to Kähler action for its preferred extremals defining the analogs of Bohr orbits so that classical theory with Bohr rules would become an exact part of quantum theory and path integral would be replaced with genuine integral over WCW. The motivating construction was that for loop spaces leading to a unique Kähler geometry. The geometry for the space of 3-D objects is even more complex than that for loops and the vision still is that the geometry of WCW is unique from the mere existence of Riemann connection.

This chapter represents the updated version of the construction providing a solution to the problems of the previous construction. The basic formulas remain as such but the expressions for WCW super-Hamiltonians defining WCW Hamiltonians (and matrix elements of WCW metric) as their anticommutator are replaced with those following from the dynamics of the modified Dirac action.

1.4.7 Unified Number Theoretical Vision

An updated view about M^8-H duality is discussed. M^8-H duality allows to deduce $M^4\times CP_2$ via number theoretical compactification. One important correction is that octonionic spinor structure makes sense only for M^8 whereas for $M^4\times CP_2$ complefixied quaternions characterized the spinor structure.

Octonions, quaternions, quaternionic space-time surfaces, octonionic spinors and twistors and twistor spaces are highly relevant for quantum TGD. In the following some general observations distilled during years are summarized.

There is a beautiful pattern present suggesting that $H = M^4 \times CP_2$ is completely unique on number theoretical grounds. Consider only the following facts. M^4 and CP_2 are the unique 4-D spaces allowing twistor space with Kähler structure. Octonionic projective space OP_2 appears as octonionic twistor space (there are no higher-dimensional octonionic projective spaces). Octotwistors generalise the twistorial construction from M^4 to M^8 and octonionic gamma matrices make sense also for H with quaternionicity condition reducing OP_2 to to 12-D $G_2/U(1)\times U(1)$ having same dimension as the the twistor space $CP_3 \times SU(3)/U(1)\times U(1)$ of H assignable to complexified quaternionic representation of gamma matrices.

A further fascinating structure related to octo-twistors is the non-associated analog of Lie group defined by automorphisms by octonionic imaginary units: this group is topologically six-sphere. Also the analogy of quaternionicity of preferred extremals in TGD with the Majorana condition central in super string models is very thought provoking. All this suggests that associativity indeed could define basic dynamical principle of TGD.

Number theoretical vision about quantum TGD involves both p-adic number fields and classical number fields and the challenge is to unify these approaches. The challenge is non-trivial since the p-adic variants of quaternions and octonions are not number fields without additional conditions. The key idea is that TGD reduces to the representations of Galois group of algebraic numbers realized in the spaces of octonionic and quaternionic adeles generalizing the ordinary adeles as Cartesian products of all number fields: this picture relates closely to Langlands program. Associativity would force sub-algebras of the octonionic adeles defining 4-D surfaces in the space of octonionic adeles so that 4-D space-time would emerge naturally. M^8-H correspondence in turn would map the space-time surface in M^8 to $M^4\times CP_2$.

1.4.8 Knots and TGD

Khovanov homology generalizes the Jones polynomial as knot invariant. The challenge is to find a quantum physical construction of Khovanov homology analous to the topological QFT defined by Chern-Simons action allowing to interpret Jones polynomial as vacuum expectation value of Wilson loop in non-Abelian gauge theory.

Witten's approach to Khovanov homology relies on fivebranes as is natural if one tries to define 2-knot invariants in terms of their cobordisms involving violent un-knottings. Despite the difference in approaches it is very useful to try to find the counterparts of this approach in quantum TGD since this would allow to gain new insights to quantum TGD itself as almost topological QFT identified as symplectic theory for 2-knots, braids and braid cobordisms. This comparison turns out to be extremely useful from TGD point of view.

1. A highly unique identification of string world sheets and therefore also of the braids whose ends carry quantum numbers of many particle states at partonic 2-surfaces emerges if one identifies the string word sheets as singular surfaces in the same manner as is done in Witten's approach. This identification need of course not be correct and later in the article a less ad hoc identification is proposed. Even more, the conjectured slicings of preferred extremals by 3-D surfaces and string world sheets central for quantum TGD can be identified uniquely if the identification is accepted. The slicing by 3-surfaces would be interpreted in gauge theory in terms of Higgs= constant surfaces with radial coordinate of CP_2 playing the role of Higgs. The slicing by string world sheets would be induced by different choices of $U(2)$ subgroup of $SU(3)$ leaving Higgs=constant surfaces invariant.

2. Also a physical interpretation of the operators Q, F, and P of Khovanov homology emerges. P would correspond to instanton number and F to the fermion number assignable to right handed neutrinos. The breaking of M^4 chiral invariance makes possible to realize Q physically. The finding that the generalizations of Wilson loops can be identified in terms of the gerbe fluxes $\int H_A J$ supports the conjecture that TGD as almost topological QFT corresponds essentially to a symplectic theory for braids and 2-knots.

The basic challenge of quantum TGD is to give a precise content to the notion of generalization Feynman diagram and the reduction to braids of some kind is very attractive possibility inspired by zero energy ontology. The point is that no $n > 2$-vertices at the level of braid strands are needed if bosonic emergence holds true.

1. For this purpose the notion of algebraic knot is introduce and the possibility that it could be applied to generalized Feynman diagrams is discussed. The algebraic structrures kei, quandle, rack, and biquandle and their algebraic modifications as such are not enough. The lines of Feynman graphs are replaced by braids and in vertices braid strands redistribute. This poses several challenges: the crossing associated with braiding and crossing occurring in non-planar Feynman diagrams should be integrated to a more general notion; braids are replaced with sub-manifold braids; braids of braids ....of braids are possible; the redistribution of braid strands in vertices should be algebraized. In the following I try to abstract the basic operations which should be algebraized in the case of generalized Feynman diagrams.

2. One should be also able to concretely identify braids and 2-braids (string world sheets) as well as partonic 2-surfaces and I have discussed several identifications during last years. Legendrian braids turn out to be very natural candidates for braids and their duals for the partonic 2-surfaces. String world sheets in turn could correspond to the analogs of Lagrangian sub-manifolds or two minimal surfaces of space-time surface satisfying the weak form of electric-magnetic duality. The latter opion turns out to be more plausible. This identification - if correct - would solve quantum TGD explicitly at string world sheet level which corresponds to finite measurement resolution.

3. Also a brief summary of generalized Feynman rules in zero energy ontology is proposed. This requires the identification of vertices, propagators, and prescription for integrating over al 3-surfaces. It turns out that the basic building blocks of generalized Feynman diagrams are well-defined.

4. The notion of generalized Feynman diagram leads to a beautiful duality between the descriptions of hadronic reactions in terms of hadrons and partons analogous to gauge-gravity duality and AdS/CFT duality but requiring no additional assumptions. The model of quark gluon plasma as s strongly interacting phase is proposed. Color magnetic flux tubes are responsible for the long range correlations making the plasma phase more like a very large hadron rather than a gas of partons. One also ends up with a simple estimate for the viscosity/entropy ratio using black-hole analogy.

1.4.9 Ideas emerging fromTGD

I have gathered to this chapter those ideas related to quantum TGD which are not absolutely central and whose status is not clear in the long run. I have represented earlier these ideas in chapters and the outcome was a total chaos and reader did not have a slightest idea what is they real message. I hope that this organization of material makes it easier for the reader to grasp the topology of TGD correctly.

Chapter 2

Identification of the Configuration Space Kähler Function

2.1 Introduction

The motivation or the construction of configuration space (”world of classical worlds” (WCW))geometry is the postulate that physics reduces to the geometry of classical spinor fields in the the ”world of the classical worlds” (WCW) identified as the infinite-dimensional WCW of 3-surfaces of some subspace of $M^4 \times CP_2$. The first candidates were $M^4_+ \times CP_2$ and $M^4 \times CP_2$, where M^4 and M^4_+ denote Minkowski space and its light cone respectively. The recent identification of WCW is as the the union of sub-WCWs consisting of light-like 3-surface representing generalized Feynman diagrams in $CD \times CP_2$, where CD is intersection of future and past directed light-cones of M^4. The details of this identification will be discussed later.

Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that WCW possesses Kähler geometry. One of the basic features of the Kähler geometry is that it is solely determined by the so called

Kähler function, which defines both the Kähler form J and the components of the Kähler metric g in complex coordinates via the formulas [A59]

$$\begin{aligned} J &= i\partial_k\partial_{\bar{l}}K dz^k \wedge d\bar{z}^l \ , \\ ds^2 &= 2\partial_k\partial_{\bar{l}}K dz^k d\bar{z}^l \ . \end{aligned} \tag{2.1.1}$$

Kähler form is covariantly constant two-form and can be regarded as a representation of imaginary unit in the tangent space of the WCW

$$J_{mr}J^{rn} = -g_m^{\ n} \ . \tag{2.1.2}$$

As a consequence Kähler form defines also symplectic structure in WCW.

2.1.1 WCW Kähler metric from Kähler function

The task of finding Kähler geometry for the WCW reduces to that of finding the Kähler function. The main constraints on the Kähler function result from the requirement of General Coordinate Invariance (GCI) -or more technically Diff^4 symmetry and Diff degeneracy. GCI requires that the definition of the Kähler function assigns to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$, the generalized Bohr orbit defining the classical physics associated with X^3. The natural guess inspired by quantum classical correspondence is that Kähler function is defined by what might be called Kähler action, which is essentially Maxwell action with Maxwell field expressible in terms of CP_2 coordinates and that the space-time surface corresponds to a preferred extremal of Kähler action.

One can end up with the identification of the preferred extremal via several routes. Kähler action contains Kähler coupling strength as a temperature like parameter and this leads to the idea of quantum criticality fixing this parameter. One could go even even further, and require that space-time surfaces are critical in the sense that there exist an infinite number of vanishing second variations of Kähler action defining conserved Noether charges. The approach based on the modified Dirac action (or Kähler-Dirac action) indeed leads naturally to this picture [K18] . Kähler coupling strength should be however visible in the solutions of field equations somehow before one can say that these two criticalities have something to do with each other. Since Kähler coupling strength does not appear in field equations it can make its way to field equations only via boundary conditions. This is achieved if one accepts the weak form of self-duality discussed in [K10] which roughly states that for the partonic 2-surfaces the induced Kähler electric field is proportional to the Kähler magnetic field strength. The proportionality constant turns out to be essentially the Kähler coupling strength. The simplest hypothesis is that Kähler coupling strength has single universal value for given value of Planck constant and the weak form of self-duality fixes it.

If Kähler action would define a strictly deterministic variational principle, Diff^4 degeneracy and invariance would be achieved by restricting the consideration to 3-surfaces Y^3 at the boundary of M^4_+ and by defining Kähler function for 3-surfaces X^3 at $X^4(Y^3)$ and diffeo-related to Y^3 as $K(X^3) = K(Y^3)$. This reduction might be called quantum gravitational holography. The classical non-determinism of the Kähler action introduces complications which might be overcome in zero energy ontology (ZEO). ZEO and strong from of GCI lead to the effective replacement of X^3 with partonic 2-surfaces at the ends of CD plus the 4-D tangent space distribution associated with them as basic geometric objects so that one can speak about effective 2-dimensionality and strong form of gravitational holography. In given resolution the effects of non-determinism might be expressible in terms dark matter hierarchy with levels characterized by $h_{eff} = n \times h$. The hierarchy would correspond to a hierarchy of sub-algebras of conformal algebra with conformal weights coming as multiples of n serving acting as gauge symmetries and defining what deformations at quantum critacility are.

2.1.2 WCW metric from symmetries

A complementary approach to the problem of constructing configuration space geometry is based on symmetries. The work of Dan [A37] [A37] has demonstrated that the Kähler geometry of loop spaces is unique from the existence of Riemann connection and fixed completely by the Kac Moody symmetries of the space. In 3-dimensional context one has even better reasons to expect uniqueness. The guess is that WCW is a union symmetric spaces labeled by zero modes not appearing in the line element as differentials and having interpretations as classical degrees providing a rigorous formulation of quantum measurement theory. The generalized conformal invariance of metrically 2-dimensional light like 3-surfaces acting as causal determinants is the corner stone of the construction. The construction works only for 4-dimensional space-time and imbedding space which is a product of four-dimensional Minkowski space or its future light cone with CP_2.

In the sequel I will first consider the basic properties of the WCW, propose an identification of the Kähler function and discuss various physical and mathematical motivations behind the proposed definition. The key feature of the Kähler action is the failure of classical determinism in its standard form, and various implications of the failure are discussed.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L13]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L14]. The topics relevant to this chapter are given by the following list.

- General Coordinate Invariance [L19]
- Weak form of electric-magnetic duality [L43]
- Geometry of WCW [L20]

- Structure of WCW [L33]
- Symmetries of WCW [L34]
- Vacuum functional in TGD [L40]

2.2 WCW

The view about configuration space ("world of classical worlds", WCW) has developed considerably during the last two decades. Here only the recent view is summarized in order to not load reader with unessential details.

2.2.1 Basic notions

The notions of imbedding space, 3-surface (and 4-surface), and WCW or "world of classical worlds" (WCW), are central to quantum TGD. The original idea was that 3-surfaces are space-like 3-surfaces of $H = M^4 \times CP_2$ or $H = M^4_+ \times CP_2$ (see figs. `http://www.tgdtheory.fi/appfigures/Hoo.jpg`, `http://www.tgdtheory.fi/appfigures/cp2.jpg`, `http://www.tgdtheory.fi/appfigures/Hoo.futurepast`,`http://www.tgdtheory.fi/appfigures/penrose.jpg`, which are also in the appendix of this book), and WCW consists of all possible 3-surfaces in H. The basic idea was that the definition of Kähler metric of WCW assigns to each X^3 a unique space-time surface $X^4(X^3)$ allowing in this manner to realize GCI. During years these notions have however evolved considerably.

The notion of imbedding space

Two generalizations of the notion of imbedding space were forced by number theoretical vision [K51, K52, K50] .

1. p-Adicization forced to generalize the notion of imbedding space by gluing real and p-adic variants of imbedding space together along rationals and common algebraic numbers. The generalized imbedding space has a book like structure with reals and various p-adic number fields (including their algebraic extensions) representing the pages of the book. As matter fact, this gluing idea generalizes to the level of WCW.

2. With the discovery of zero energy ontology [K9, K13] it became clear that the so called causal diamonds (CDs) interpreted as intersections $M^4_+ \cap M^4_-$ of future and past directed light-cones of $M^4 \times CP_2$ define correlates for the quantum states. The position of the "lower" tip of CD characterizes the position of CD in H. If the temporal distance between upper and lower tip of CD is quantized power of 2 multiples of CP_2 length, p-adic length scale hypothesis [K3] follows as a consequence. The upper *resp.* lower light-like boundary $\delta M^4_+ \times CP_2$ *resp.* $\delta M^4_- \times CP_2$ of CD can be regarded as the carrier of positive *resp.* negative energy part of the state. All net quantum numbers of states vanish so that everything is creatable from vacuum. Space-time surfaces assignable to zero energy states would would reside inside $CD \times CP_2$s and have their 3-D ends at the light-like boundaries of $CD \times CP_2$. Fractal structure is present in the sense that CDs can contains CDs within CDs, and measurement resolution dictates the length scale below which the sub-CDs are not visible.

3. The realization of the hierarchy of Planck constants [K17] led to a further generalization of the notion of imbedding space. Generalized imbedding space is obtained by gluing together Cartesian products of singular coverings and possibly also factor spaces of CD and CP_2 to form a book like structure. There are good physical and mathematical arguments suggesting that only the singular coverings should be allowed [K50] . The particles at different pages of this book behave like dark matter relative to each other. This generalization also brings in the geometric correlate for the selection of quantization axes in the sense that the geometry of the sectors of the generalized imbedding space with non-standard value of Planck constant involves symmetry breaking reducing the isometries to Cartan subalgebra. Roughly speaking, each CD and CP_2 is replaced with a union of CDs and CP_2s corresponding to different choices

of quantization axes so that no breaking of Poincare and color symmetries occurs at the level of entire WCW.

The notions of 3-surface and space-time surface

The question what one exactly means with 3-surface turned out to be non-trivial and the receont view is an outcome of a long and tedious process involving many hastily done mis-interpretations.

1. The original identification of 3-surfaces was as arbitrary space-like 3-surfaces subject to equivalence implied by GCI. There was a problem related to the realization of GCI since it was not at all obvious why the preferred extremal $X^4(Y^3)$ for Y^3 at $X^4(X^3)$ and Diff^4 related X^3 should satisfy $X^4(Y^3) = X^4(X^3)$.

2. Much later it became clear that light-like 3-surfaces have unique properties for serving as basic dynamical objects, in particular for realizing the GCI in 4-D sense (obviously the identification resolves the above mentioned problem) and understanding the conformal symmetries of the theory. Light-like 3-surfaces can be regarded as orbits of partonic 2-surfaces. Therefore it seems that one must choose between light-like and space-like 3-surfaces or assume generalized GCI requiring that equivalently either space-like 3-surfaces or light-like 3-surfaces at the ends of CDs can be identified as the fundamental geometric objects. General GCI requires that the basic objects correspond to the partonic 2-surfaces identified as intersections of these 3-surfaces plus common 4-D tangent space distribution.

   At the level of WCW metric this suggests that the components of the Kähler form and metric can be expressed in terms of data assignable to 2-D partonic surfaces. Since the information about normal space of the 2-surface is needed one has only effective 2-dimensionality. Weak form of self-duality [K10] however implies that the normal data (flux Hamiltonians associated with Kähler electric field) reduces to magnetic flux Hamiltonians. This is essential for conformal symmetries and also simplifies the construction enormously.

   It however turned out that this picture is too simplistic. It turned out that the solutions of the modified Dirac equation are localized at 2-D string world sheets, and this led to a generalization of the formulation of WCW geometry: given point of partonic 2-surface is effectively replaced with a string emanating from it and connecting it to another partonic 2-surface. Hence the formulation becomes 3-dimensional but thanks to super-conformal symmetries acting like gauge symmetries one obtains effective 2-dimensionality albeit in weaker sense [K80].

3. At some stage came the realization that light-like 3-surfaces can have singular topology in the sense that they are analogous to Feynman diagrams. This means that the light-like 3-surfaces representing lines of Feynman diagram can be glued along their 2-D ends playing the role of vertices to form what I call generalized Feynman diagrams. The ends of lines are located at boundaries of sub-CDs. This brings in also a hierarchy of time scales: the increase of the measurement resolution means introduction of sub-CDs containing sub-Feynman diagrams. As the resolution is improved, new sub-Feynman diagrams emerge so that effective 2-D character holds true in discretized sense and in given resolution scale only.

4. A further but inessential complication relates to the hierarchy of Planck constants forcing to generalize the notion of imbedding space and also to the fact that for non-standard values of Planck constant there is symmetry breaking due to preferred plane M^2 preferred homologically trivial geodesic sphere of CP_2 having interpretation as geometric correlate for the selection of quantization axis. For given sector of CH this means union over choices of this kind.

The basic vision forced by the generalization of GCI has been that space-time surfaces correspond to preferred extremals $X^4(X^3)$ of Kähler action and are thus analogous to Bohr orbits. Kähler function $K(X^3)$ defining the Kähler geometry of the world of classical worlds would correspond to the Kähler action for the preferred extremal. The precise identification of the preferred extremals actually has however remained open.

The study of the modified Dirac equation led to the realization that classical field equations for Kähler action can be seen as consistency conditions for the modified Dirac action and led to the identification of preferred extremals in terms of criticality. This identification which follows naturally also from quantum criticality.

1. The condition that electromagnetic charge is well-defined for the modes of Kähler-Dirac operator implies that in the generic case the modes are restricted to 2-D surfaces (string world sheets or possibly also partonic 2-surfaces) with vanishing W fields [K69]. Above weak scale at least one can also assume that Z^0 field vanishes. Also for space-time surfaces with 2-D CP_2 projection (cosmic strongs would be examples) the localization is expected to be possible. This localization is possible only for Kähler action and the set of these 2-surfaces is discrete except for the latter case. The stringy form of conformal invariance allows to solve Kähler-Dirac equation just like in string models and the solutions are labelled by integer valued conformal weights.

2. The next step of progress was the realization that the requirement that the conservation of the Noether currents associated with the modified Dirac equation requires that the second variation of the Kähler action vanishes. In strongest form this condition would be satisfied for all variations and in weak sense only for those defining dynamical symmetries. The interpretation is as a space-time correlate for quantum criticality and the vacuum degeneracy of Kähler action makes the criticality plausible.

   The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see fig. **??** in the appendix of this book).

   Weak form of electric-magnetic duality gives a precise formulation for how Kähler coupling strength is visible in the properties of preferred extremals. A generalization of the ideas of the catastrophe theory to infinite-dimensional context results. These conditions make sense also in p-adic context and have a number theoretical universal form.

The notion of number theoretical compactication led to important progress in the understanding of the preferred extremals and the conjectures were consistent with what is known about the known extremals.

1. The conclusion was that one can assign to the 4-D tangent space $T(X^4(X^3_l)) \subset M^8$ a subspace $M^2(x) \subset M^4$ having interpretation as the plane of non-physical polarizations. This in the case that the induced metric has Minkowskian signature. If not, and if co-hyper-quaternionic surface is in question, similar assigned should be possible in normal space. This means a close connection with super string models. Geometrically this would mean that the deformations of 3-surface in the plane of non-physical polarizations would not contribute to the line element of WCW. This is as it must be since complexification does not make sense in M^2 degrees of freedom.

2. In number theoretical framework $M^2(x)$ has interpretation as a preferred hyper-complex sub-space of hyper-octonions defined as 8-D subspace of complexified octonions with the property that the metric defined by the octonionic inner product has signature of M^8. The condition $M^2(x) \subset T(X^4(X^3_l)))$ in principle fixes the tangent space at X^3_l, and one has good hopes that the boundary value problem is well-defined and could fix $X^4(X^3)$ at least partially as a preferred extremal of Kähler action. This picture is rather convincing since the choice $M^2(x) \subset M^4$ plays also other important roles.

3. At the level of H the counterpart for the choice of $M^2(x)$ seems to be following. Suppose that $X^4(X^3_l)$ has Minkowskian signature. One can assign to each point of the M^4 projection $P_{M^4}(X^4(X^3_l))$ a sub-space $M^2(x) \subset M^4$ and its complement $E^2(x)$, and the distributions of these planes are integrable and define what I have called Hamilton-Jacobi coordinates which can be assigned to the known extremals of Kähler with Minkowskian signature. This decomposition allows to slice space-time surfaces by string world sheets and their 2-D partonic duals. Also a slicing to 1-D light-like surfaces and their 3-D light-like duals Y^3_l parallel to

X_l^3 follows under certain conditions on the induced metric of $X^4(X_l^3)$. This decomposition exists for known extremals and has played key role in the recent developments. Physically it means that 4-surface (3-surface) reduces effectively to 3-D (2-D) surface and thus holography at space-time level. A physically attractive realization of the slicings of space-time surface by 3-surfaces and string world sheets is discussed in [K23] by starting from the observation that TGD could define a natural realization of braids, braid cobordisms, and 2-knots.

4. The weakest form of number theoretic compactification [K52] states that light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^8$, where $X^4(X^3)$ hyper-quaternionic surface in hyper-octonionic M^8 can be mapped to light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^4 \times CP_2$, where $X^4(X^3)$ is now preferred extremum of Kähler action. The natural guess is that $X^4(X^3) \subset M^8$ is a preferred extremal of Kähler action associated with Kähler form of E^4 in the decomposition $M^8 = M^4 \times E^4$, where M^4 corresponds to hyper-quaternions. The conjecture would be that the value of the Kähler action in M^8 is same as in $M^4 \times CP_2$: in fact that 2-surface would have identical induced metric and Kähler form so that this conjecture would follow trivial. $M^8 - H$ duality would in this sense be Kähler isometry.

If one takes $M^- H$ duality seriously, one must conclude that one can choose any partonic 2-surface in the slicing of X^4 as a representative. This means gauge invariance reflect in the definition of Kähler function as $U(1)$ gauge transformation $K \to K + f + \overline{f}$ having no effect on Kähler metric and Kähler form.

Although the details of this vision might change it can be defended by its ability to fuse together all great visions about quantum TGD. In the sequel the considerations are restricted to 3-surfaces in $M^4_\pm \times CP_2$. The basic outcome is that Kähler metric is expressible using the data at partonic 2-surfaces $X^2 \subset \delta M^4_+ \times CP_2$. The generalization to the actual physical situation requires the replacement of $X^2 \subset \delta M^4_+ \times CP_2$ with unions of partonic 2-surfaces located at light-like boundaries of CDs and sub-CDs.

The notions of space-time sheet and many-sheeted space-time are basic pieces of TGD inspired phenomenology (see fig. **??** in the appendix of this book). Originally the space-time sheet was understood to have a boundary as "sheet" strongly suggests. It has however become clear that genuine boundaries are not allowed. Rather, space-time sheet is typically double (at least) covering of M^4. The light-like 3-surfaces separating space-time regions with Euclidian and Minkowskian signature are however very much like boundaries and define what I call generalized Feynman diagrams. A fascinating possibility is that every material object is accompanied by an Euclidian region representing the interior of the object and serving as TGD analog for blackhole like object. Space-time sheets suffer topological condensation (gluing by wormhole contacts or topological sum in more mathematical jargon) at larger space-time sheets. Space-time sheets form a length scale hierarchy. Quantitative formulation is in terms of p-adic length scale hypothesis and hierarchy of Planck constants proposed to explain dark matter as phases of ordinary matter.

The notion of WCW

From the beginning there was a problem related to the precise definition of WCW ("world of classical worlds" (WCW)). Should one regard CH as the space of 3-surfaces of $M^4 \times CP_2$ or $M^4_+ \times CP_2$ or perhaps something more delicate.

1. For a long time I believed that the basis question is "M^4_+ or M^4?" and that this question had been settled in favor of M^4_+ by the fact that M^4_+ has interpretation as empty Roberson-Walker cosmology. The huge conformal symmetries assignable to $\delta M^4_+ \times CP_2$ were interpreted as cosmological rather than laboratory symmetries. The work with the conceptual problems related to the notions of energy and time, and with the symmetries of quantum TGD, however led gradually to the realization that there are strong reasons for considering M^4 instead of M^4_+.

2. With the discovery of zero energy ontology it became clear that the so called causal diamonds (CDs) define excellent candidates for the fundamental building blocks of WCW or "world of classical worlds" (WCW). The spaces $CD \times CP_2$ regarded as subsets of H defined the sectors of WCW.

3. This framework allows to realize the huge symmetries of $\delta M^4_\pm \times CP_2$ as isometries of WCW. The gigantic symmetries associated with the $\delta M^4_\pm \times CP_2$ are also laboratory symmetries. Poincare invariance fits very elegantly with the two types of super-conformal symmetries of TGD. The first conformal symmetry corresponds to the light-like surfaces $\delta M^4_\pm \times CP_2$ of the imbedding space representing the upper and lower boundaries of CD. Second conformal symmetry corresponds to light-like 3-surface X^3_l, which can be boundaries of X^4 and light-like surfaces separating space-time regions with different signatures of the induced metric. This symmetry is identifiable as the counterpart of the Kac Moody symmetry of string models.

A rather plausible conclusion is that WCW (WCW) is a union of WCWs associated with the spaces $CD \times CP_2$. CDs can contain CDs within CDs so that a fractal like hierarchy having interpretation in terms of measurement resolution results. It must be however emphasized that Kähler function depends on partonic 2-surfaces at both ends of space-time surface so that WCW is topologically Cartesian product of corresponding symmetric spaces. WCW metric must therefore have parts corresponding to the partonic 2-surfaces (free part) and also an interaction term depending on the partonic 2-surface at the opposite ends of the light-like 3-surface. The conclusion is that geometrization reduces to that for single like of generalized Feynman diagram containing partonic 2-surfaces at its ends. Since the complications due to p-adic sectors and hierarchy of Planck constants are not relevant for the basic construction, it reduces to a high degree to a study of a simple special case corresponding to a line of generalized Feynman diagram. One can also deduce the free part of the metric by restricting the consideration to partonic 2-surfaces at single end of generalized Feynman diagram.

A further piece of understanding emerged from the following observations.

1. The induced Kähler form at the partonic 2-surface X^2 - the basic dynamical object if holography is accepted- can be seen as a fundamental symplectic invariant so that the values of $\epsilon^{\alpha\beta}J_{\alpha\beta}$ at X^2 define local symplectic invariants not subject to quantum fluctuations in the sense that they would contribute to the WCW metric. Hence only induced metric corresponds to quantum fluctuating degrees of freedom at WCW level and TGD is a genuine theory of gravitation at this level.

2. WCW can be divided into slices for which the induced Kähler forms of CP_2 and $\delta M^4_\pm$ at the partonic 2-surfaces X^2 at the light-like boundaries of CDs are fixed. The symplectic group of $\delta M^4_\pm \times CP_2$ parameterizes quantum fluctuating degrees of freedom in given scale (recall the presence of hierarchy of CDs).

3. This leads to the identification of the coset space structure of the sub-WCW associated with given CD in terms of the generalized coset construction for super-symplectic and super Kac-Moody type algebras (symmetries respecting light-likeness of light-like 3-surfaces). WCW in quantum fluctuating degrees of freedom for given values of zero modes can be regarded as being obtained by dividing symplectic group with Kac-Moody group. Equivalently, the local coset space $S^2 \times CP_2$ is in question: this was one of the first ideas about WCW which I gave up as too naive!

2.2.2 Constraints on WCW geometry

The constraints on the WCW result both from the infinite dimension of WCW and from physically motivated symmetry requirements. There are three basic physical requirements on the WCW geometry: namely four-dimensional GCI in strong form, Kähler property and the decomposition of WCW into a union $\cup_i G/H_i$ of symmetric spaces G/H_i, each coset space allowing G-invariant metric such that G is subgroup of some 'universal group' having natural action on 3-surfaces. Together with the infinite dimensionality of WCW these requirements pose extremely strong constraints on WCW geometry. In the following we shall consider these requirements in more detail.

Diff^4 invariance and Diff^4 degeneracy

Diff^4 plays fundamental role as the gauge group of General Relativity. In string models $Diff^2$ invariance ($Diff^2$ acts on the orbit of the string) plays central role in making possible the elimination of the time like and longitudinal vibrational degrees of freedom of string. Also in the present case the elimination of the tachyons (time like oscillatory modes of 3-surface) is a physical necessity and Diff^4 invariance provides an obvious manner to do the job.

In the standard path l integral formulation the realization of Diff^4 invariance is an easy task at the formal level. The problem is however that path integral over four-surfaces is plagued by divergences and doesn't make sense. In the present case WCW consists of 3-surfaces and only $Diff^3$ emerges automatically as the group of re-parameterizations of 3-surface. Obviously one should somehow define the action of Diff^4 in the space of 3-surfaces. Whatever the action of Diff^4 is it must leave the WCW metric invariant. Furthermore, the elimination of tachyons is expected to be possible only provided the time like deformations of the 3-surface correspond to zero norm vector fields of WCW so that 3-surface and its Diff^4 image have zero distance. The conclusion is that WCW metric should be both Diff^4 invariant and Diff^4 degenerate.

The problem is how to define the action of Diff^4 in $C(H)$. Obviously the only manner to achieve Diff^4 invariance is to require that the very definition of the WCW metric somehow associates a unique space time surface to a given 3-surface for Diff^4 to act on. The obvious physical interpretation of this space time surface is as "classical space time" so that "Classical Physics" would be contained in WCW geometry. In fact, this space-time surface is analogous to Bohr orbit so that semiclassical quantization rules become an exact part of the quantum theory. It is this requirement, which has turned out to be decisive concerning the understanding of the WCW geometry.

Decomposition of WCW into a union of symmetric spaces G/H

The extremely beautiful theory of finite-dimensional symmetric spaces constructed by Elie Cartan suggests that WCW should possess decomposition into a union of coset spaces $CH = \cup_i G/H_i$ such that the metric inside each coset space G/H_i is left invariant under the infinite dimensional isometry group G. The metric equivalence of surfaces inside each coset space G/H_i does not mean that 3-surfaces inside G/H_i are physically equivalent. The reason is that the vacuum functional is exponent of Kähler action which is not isometry invariant so that the 3-surfaces, which correspond to maxima of Kähler function for a given orbit, are in a preferred position physically. For instance, one can imagine of calculating functional integral around this maximum perturbatively. Symmetric space property actually allows also much more powerful non-perturbative approach based on harmonic analysis [K18] . The sum of over i means actually integration over the zero modes of the metric (zero modes correspond to coordinates not appearing as coordinate differentials in the metric tensor).

The coset space G/H is a symmetric space only under very special Lie-algebraic conditions. Denoting the decomposition of the Lie-algebra g of G to the direct sum of H Lie-algebra h and its complement t by $g = h \oplus t$, one has

$$[h, h] \subset h \ , \quad [h, t] \subset t \ , \quad [t, t] \subset h \ .$$

This decomposition turn out to play crucial role in guaranteeing that G indeed acts as isometries and that the metric is Ricci flat.

The four-dimensional $Diff$ invariance indeed suggests to a beautiful solution of the problem of identifying G. The point is that any 3-surface X^3 is $Diff^4$ equivalent to the intersection of $X^4(X^3)$ with the light cone boundary. This in turn implies that 3-surfaces in the space $\delta H = \delta M^4_+ \times CP_2$ should be all what is needed to construct WCW geometry. The group G can be identified as some subgroup of diffeomorphisms of δH and H_i contains that subgroup of G, which acts as diffeomorphisms of the 3-surface X^3. Since G preserves topology, WCW must decompose into union $\cup_i G/H_i$, where i labels 3-topologies and various zero modes of the metric. For instance, the elements of the Lie-algebra of G invariant under WCW complexification correspond to zero modes.

The reduction to the light cone boundary, identifiable as the moment of big bang, looks perhaps odd at first. In fact, it turns out that the classical non-determinism of Kähler action does not allow the complete reduction to the light cone boundary: physically this is a highly desirable implication but means a considerable mathematical challenge.

Kähler property

Kähler property implies that the tangent space of the configuration space allows complexification and that thereexists a covariantly constant two-form J_{kl}, which can be regarded as a representation of the imaginary unit in the tangent space of the WCW:

$$J_k{}^r J_{rl} = -G_{kl} \quad . \tag{2.2.1}$$

There are several physical and mathematical reasons suggesting that WCW metric should possess Kähler property in some generalized sense.

1. The deepest motivation comes from the need to geometrize hermitian conjugation which is basic mathematical operation of quantum theory.

2. Kähler property turns out to be a necessary prerequisite for defining divergence free WCW integration. We will leave the demonstration of this fact later although the argument as such is completely general.

3. Kähler property very probably implies an infinite-dimensional isometry loop groups $Map(S^1, G)$ [A37] shows that loop group allows only

   Riemann connection and this metric allows local G as its isometries!

   To see this consider the construction of Riemannian connection for $Map(X^3, H)$. The defining formula for the connection is given by the expression

$$\begin{aligned} 2(\nabla_X Y, Z) &= X(Y,Z) + Y(Z,X) - Z(X,Y) \\ &+ ([X,Y],Z) + ([Z,X],Y) - ([Y,Z],X) \end{aligned} \tag{2.2.2}$$

   X, Y, Z are smooth vector fields in $Map(X^3, G)$. This formula defines $\nabla_X Y$ uniquely provided the tangent space of Map is complete with respect to Riemann metric. In the finite-dimensional case completeness means that the inverse of the covariant metric tensor exists so that one can solve the components of connection from the conditions stating the covariant constancy of the metric. In the case of the loop spaces with Kähler metric this is however not the case.

   Now the symmetry comes into the game: if X, Y, Z are left (local gauge) invariant vector fields defined by the Lie-algebra of local G then the first three terms drop away since the scalar products of left invariant vector fields are constants. The expression for the covariant derivative is given by

$$\nabla_X Y = (Ad_X Y - Ad_X^* Y - Ad_Y^* X)/2 \tag{2.2.3}$$

   where Ad_X^* is the adjoint of Ad_X with respect to the metric of the loop space.

   At this point it is important to realize that Freed's argument does not force the isometry group of WCW to be $Map(X^3, M^4 \times SU(3))$! Any symmetry group, whose Lie algebra is complete with respect to the WCW metric (in the sense that any tangent space vector is expressible as superposition of isometry generators modulo a zero norm tangent vector) is an acceptable alternative.

   The Kähler property of the metric is quite essential in one-dimensional case in that it leads to the requirement of left invariance as a mathematical consistency condition and we expect that dimension three makes no exception in this respect. In 3-dimensional case the degeneracy of the metric turns out to be even larger than in 1-dimensional case due to the four-dimensional Diff degeneracy. So we expect that the metric ought to possess some infinite-dimensional isometry group and that the above formula generalizes also to the 3-dimensional case and to the case of local coset space. Note that in M^4 degrees of freedom $Map(X^3, M^4)$ invariance would imply the flatness of the metric in M^4 degrees of freedom.

   The physical implications of the above purely mathematical conjecture should not be underestimated. For example, one natural looking manner to construct physical theory would be based on the idea that configuration space geometry is dynamical and this approach is followed in the attempts to construct string theories [B9] . Various physical considerations (in particular the need to obtain oscillator operator algebra) seem to imply that WCW geometry is necessarily Kähler. The above result however states that WCW Kähler geometry cannot be dynamical quantity and is dictated solely by the requirement of internal consistency. This result is extremely nice since it has been already found that the definition of the WCW metric must somehow associate a unique classical space time and "classical physics" to a given 3-surface: uniqueness of the geometry implies the uniqueness of the "classical physics".

4. The choice of the imbedding space becomes highly unique. In fact, the requirement that WCW is not only symmetric space but also (contact) Kähler manifold inheriting its (degenerate) Kähler structure from the imbedding space suggests that spaces, which are products of four-dimensional Minkowski space with complex projective spaces CP_n, are perhaps the only possible candidates for H. The reason for the unique position of the four-dimensional Minkowski space turns out to be that the boundary of the light cone of D-dimensional Minkowski space is metrically a sphere S^{D-2} despite its topological dimension $D-1$: for $D=4$ one obtains two-sphere allowing Kähler structure and infinite parameter group of conformal symmetries!

5. It seems possible to understand the basic mathematical structures appearing in string model in terms of the Kähler geometry rather nicely.

   (a) The projective representations of the infinite-dimensional isometry group (not necessarily Map!) correspond to the ordinary representations of the corresponding centrally extended group [A41]. The representations of Kac Moody group indeed play central role in string models [B27, B20] and WCW approach would explain their occurrence, not as a result of some quantization procedure, but as a consequence of symmetry of the underlying geometric structure.

   (b) The bosonic oscillator operators of string models would correspond to centrally extended Lie-algebra generators of the isometry group acting on spinor fields of the WCW.

   (c) The "fermionic" fields (Ramond fields, Schwartz,Green) should correspond to gamma matrices of the WCW. Fermionic oscillator operators would correspond simply to contractions of isometry generators j_A^k with complexified gamma matrices of WCW

$$\begin{aligned}
\Gamma_A^{\pm} &= j_A^k \Gamma_k^{\pm} \\
\Gamma_k^{\pm} &= (\Gamma^k \pm J_l^k \Gamma^l)/\sqrt{2}
\end{aligned} \tag{2.2.4}$$

   (J_l^k is the Kähler form of WCW) and would create various spin excitations of WCW spinor field. $\Gamma_k^{\pm}$ are the complexified gamma matrices, complexification made possible by the Kähler structure of the WCW.

This suggests that some generalization of the so called Super Kac Moody algebra of string models [B27, B20] should be regarded as a spectrum generating algebra for the solutions of field equations in configuration space.

Although the Kähler structure seems to be physically well motivated there is a rather heavy counter argument against the whole idea. Kähler structure necessitates complex structure in the tangent space of WCW. In CP_2 degrees of freedom no obvious problems of principle are expected: WCW should inherit in some sense the complex structure of CP_2.

In Minkowski degrees of freedom the signature of the Minkowski metric seems to pose a serious obstacle for complexification: somehow one should get rid of two degrees of freedom so that only two Euclidian degrees of freedom remain. An analogous difficulty is encountered in quantum field theories: only two of the four possible polarizations of gauge boson correspond to physical degrees of freedom: mathematically the wrong polarizations correspond to zero norm states and transverse states span a complex Hilbert space with Euclidian metric. Also in string model analogous situation occurs: in case of D-dimensional Minkowski space only $D-2$ transversal degrees of freedom are physical. The solution to the problem seems therefore obvious: WCW metric must be degenerate so that each vibrational mode spans effectively a 2-dimensional Euclidian plane allowing complexification.

We shall find that the definition of Kähler function to be proposed indeed provides a solution to this problem and also to the problems listed before.

1. The definition of the metric doesn't differentiate between 1- and N-particle sectors, avoids spin statistics difficulty and has the physically appealing property that one can associate to each 3-surface a unique classical space time: classical physics is described by the geometry of WCW and d the geometry of WCW is determined uniquely by the requirement of mathematical consistency.

2. Complexification is possible only provided the dimension of the Minkowski space equals to four and is due to the effective 3-dimensionality of light-cone boundary.

3. It is possible to identify a unique candidate for the necessary infinite-dimensional isometry group G. G is subgroup of the diffeomorphism group of $\delta M^4_+ \times CP_2$. Essential role is played by the fact that the boundary of the four-dimensional light cone, which, despite being topologically 3-dimensional, is metrically two-dimensional Euclidian sphere, and therefore allows infinite-parameter groups of isometries as well as conformal and symplectic symmetries and also Kähler structure unlike the higher-dimensional light cone boundaries. Therefore WCW metric is Kähler only in the case of four-dimensional Minkowski space and allows symplectic $U(1)$ central extension without conflict with the no-go theorems about higher dimensional central extensions.

   The study of the vacuum degeneracy of Kähler function defined by Kähler action forces to conclude that the isometry group must consist of the symplectic transformations of $\delta H = \delta M^4_+ \times CP_2$. The corresponding Lie algebra can be regarded as a loop algebra associated with the symplectic group of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of light cone boundary. Thus the finite-dimensional group G defining loop group in case of string models extends to an infinite-dimensional group in TGD context. This group has a monstrous size. The radial Virasoro localized with respect to $S^2 \times CP_2$ defines naturally complexification for both G and H. The general form of the Kähler metric deduced on basis of this symmetry has same qualitative properties as that deduced from Kähler function identified as preferred extremal of Kähler action. Also the zero modes, among them isometry invariants, can be identified.

4. The construction of the WCW spinor structure is based on the identification of the WCW gamma matrices as linear superpositions of the oscillator operators associated with the second quantized induced spinor fields. The extension of the symplectic invariance to super symplectic invariance fixes the anti-commutation relations of the induced spinor fields, and WCW gamma matrices correspond directly to the super generators. Physics as number theory vision suggests strongly that WCW geometry exists for 8-dimensional imbedding space only and that the choice $M^4_+ \times CP_2$ for the imbedding space is the only possible one.

2.3 Identification of the Kähler function

There are three approaches to the construction of the WCW geometry: a direct physics based guess of the Kähler function, a group theoretic approach based on the hypothesis that CH can be regarded as a union of symmetric spaces, and the approach based on the construction of WCW spinor structure first by second quantization of induced spinor fields. Here the first approach is discussed.

2.3.1 Definition of Kähler function

Kähler metric in terms of Kähler function

Quite generally, Kähler function K defines Kähler metric in complex coordinates via the following formula

$$J_{k\bar{l}} = ig_{k\bar{l}} = i\partial_k\partial_{\bar{l}}K \ . \tag{2.3.1}$$

Kähler function is defined only modulo a real part of holomorphic function so that one has the gauge symmetry

$$K \rightarrow K + f + \overline{f} \ . \tag{2.3.2}$$

Let X^3 be a given 3-surface and let X^4 be any four-surface containing X^3 as a sub-manifold: $X^4 \supset X^3$. The 4-surface X^4 possesses in general boundary. If the 3-surface X^3 has nonempty boundary δX^3 then the boundary of X^3 belongs to the boundary of X^4: $\delta X^3 \subset \delta X^4$.

Induced Kähler form and its physical interpretation

Induced Kähler form defines a Maxwell field and it is important to characterize precisely its relationship to the gauge fields as they are defined in gauge theories. Kähler form J is related to the corresponding Maxwell field F via the formula

$$J = xF \ , \ x = \frac{g_K}{\hbar} \ . \tag{2.3.3}$$

Similar relationship holds true also for the other induced gauge fields. The inverse proportionality of J to $\hbar$ does not matter in the ordinary gauge theory context where one routinely choses units by putting $\hbar = 1$ but becomes very important when one considers a hierarchy of Planck constants [K17] .

Unless one has $J = (g_K/\hbar_0)$, where $\hbar_0$ corresponds to the ordinary value of Planck constant, $\alpha_K = g_K^2/4\pi\hbar$ together the large Planck constant means weaker interactions and convergence of the functional integral defined by the exponent of Kähler function and one can argue that the convergence of the functional integral is what forces the hierarchy of Planck constants. This is in accordance with the vision that Mother Nature likes theoreticians and takes care that the perturbation theory works by making a phase transition increasing the value of the Planck constant in the situation when perturbation theory fails. This leads to a replacement of the M^4 (or more precisely, causal diamond CD) and CP_2 factors of the imbedding space ($CD \times CP_2$) with its $r = \hbar/\hbar_0$-fold singular covering (one can consider also singular factor spaces). If the components of the space-time surfaces at the sheets of the covering are identical, one can interpret r-fold value of Kähler action as a sum of r identical contributions from the sheets of the covering with ordinary value of Planck constant and forget the presence of the covering. Physical states are however different even in the case that one assumes that sheets carry identical quantum states and anyonic phase could correspond to this kind of phase [K37] .

Kähler action

One can associate to Kähler form Maxwell action and also Chern-Simons anomaly term proportional to $\int_{X^4} J \wedge J$ in well known manner. Chern Simons term is purely topological term and well defined for orientable 4-manifolds, only. Since there is no deep reason for excluding non-orientable space-time surfaces it seems reasonable to drop Chern Simons term from consideration. Therefore Kähler action $S_K(X^4)$ can be defined as

$$S_K(X^4) = k_1 \int_{X^4;X^3 \subset X^4} J \wedge (*J) \ . \tag{2.3.4}$$

The sign of the square root of the metric determinant, appearing implicitly in the formula, is defined in such a manner that the action density is negative for the Euclidian signature of the induced metric and such that for a Minkowskian signature of the induced metric Kähler electric field gives a negative contribution to the action density.

The notational convention

$$k_1 \equiv \frac{1}{16\pi\alpha_K} , \tag{2.3.5}$$

where α_K will be referred as Kähler coupling strength will be used in the sequel. If the preferred extremals minimize/maximize [K52] the absolute value of the action in each region where action density has a definite sign, the value of α_K can depend on space-time sheet.

Kähler function

One can define the Kähler function in the following manner. Consider first the case $H = M^4_+ \times CP_2$ and neglect for a moment the non-determinism of Kähler action. Let X^3 be a 3-surface at the light-cone boundary $\delta M^4_+ \times CP_2$. Define the value $K(X^3)$ of Kähler function K as the value of the Kähler action for some preferred extremal in the set of four-surfaces containing X^3 as a sub-manifold:

$$K(X^3) = K(X^4_{pref}) , \quad X^4_{pref} \subset \{X^4|X^3 \subset X^4\} . \tag{2.3.6}$$

The most plausible identification of preferred extremals is in terms of quantum criticality in the sense that the preferred extremals allow an infinite number of deformations for which the second variation of Kähler action vanishes. Combined with the weak form of electric-magnetic duality forcing appearance of Kähler coupling strength in the boundary conditions at partonic 2-surfaces this condition might be enough to fix preferred extremals completely.

The precise formulation of Quantum TGD has developed rather slowly. Only quite recently-33 years after the birth of TGD - I have been forced to reconsider the question whether the precise identification of Kähler function. Should Kähler function actually correspond to the Kähler action for the space-time regions with Euclidian signature having interpretation as generalized Feynman graphs? If so what would be the interpretation for the Minkowskian contribution?

1. If one accepts just the formal definition for the square root of the metric determinant, Minkowskian regions would naturally give an imaginary contribution to the exponent defining the vacuum functional. The presence of the phase factor would give a close connection with the path integral approach of quantum field theories and the exponent of Kähler function would make the functional integral well-defined.

2. The weak form of electric magnetic duality would reduce the contributions to Chern-Simons terms from opposite sides of wormhole throats with degenerate four-metric with a constraint term guaranteeing the duality.

The motivation for this reconsideration came from the applications of ideas of Floer homology to TGD framework [K63]: the Minkowskian contribution to Kähler action for preferred extremals would define Morse function providing information about WCW homology. Both Kähler and Morse would find place in TGD based world order.

2.3.2 What are the values of the Kähler coupling strength?

Since the vacuum functional of the theory turns out to be essentially the exponent $exp(K)$ of the Kähler function, the dynamics depends on the normalization of the Kähler function. Since the Theory of Everything should be unique it would be highly desirable to find arguments fixing the normalization or equivalently the possible values of the Kähler coupling strength α_K. Also a discrete spectrum of values is acceptable.

The quantization of Kähler form could result in the following manner. It will be found that Abelian extension of the isometry group results by coupling spinors of the WCW to a multiple of Kähler potential. This means that Kähler potential plays role of gauge connection so that Kähler form must be integer valued by Dirac quantization condition for magnetic charge. So, if Kähler form is co-homologically nontrivial it is quantized.

Unfortunately, the exact definition of renormalization group concept is not at all obvious. There is however a much more general but more or less equivalent manner to formulate the condition fixing the value of α_K. Vacuum functional $exp(K)$ is analogous to the exponent $exp(-H/T)$ appearing in the definition of the partition function of a statistical system and S-matrix elements and other interesting physical quantities are integrals of type $\langle O\rangle = \int exp(K)O\sqrt{G}dV$ and therefore analogous to the thermal averages of various observables. α_K is completely analogous to temperature. The critical points of a statistical system correspond to critical temperatures T_c for which the partition function is non-analytic function of $T-T_c$ and according RGE hypothesis critical systems correspond to fixed points of renormalization group evolution. Therefore, a mathematically more precise manner to fix the value of α_K is to require that some integrals of type $\langle O\rangle$ (not necessary S-matrix elements) become non-analytic at $1/\alpha_K - 1/\alpha_K^c$.

This analogy suggests also a physical motivation for the unique value or value spectrum of α_K. Below the critical temperature critical systems suffer something analogous to spontaneous magnetization. At the critical point critical systems are characterized by long range correlations and arbitrarily large volumes of magnetized and non-magnetized phases are present. Spontaneous magnetization might correspond to the generation of Kähler magnetic fields: the most probable 3-surfaces are Kähler magnetized for subcritical values of α_K. At the critical values of α_K the most probable 3-surfaces contain regions dominated by either Kähler electric and or Kähler magnetic fields: by the compactness of CP_2 these regions have in general outer boundaries.

This suggests that 3-space has hierarchical, fractal like structure: 3-surfaces with all sizes (and with outer boundaries) are possible and they have suffered topological condensation on each other. Therefore the critical value of α_K allows the richest possible topological structure for the most probable 3-space. In fact, this hierarchical structure is in accordance with the basic ideas about renormalization group invariance. This hypothesis has highly nontrivial consequences even at the level of ordinary condensed matter physics.

Renormalization group invariance is closely related with criticality. The self duality of the Kähler form and Weyl tensor of CP_2 indeed suggest RG invariance. The point is that in $N = 4$ super-symmetric field theories duality transformation relates the strong coupling limit for ordinary particles with the weak coupling limit for magnetic monopoles and vice versa. If the theory is self-dual these limits must be identical so that action and coupling strength must be RG invariant quantities. This form of self-duality cannot hold true in TGD. The weak form of self-duality discussed in [K10] roughly states that for the partonic 2-surface the induce Kähler electric field is proportional to the Kähler magnetic field strength. The proportionality constant is essentially Kähler coupling strength. The simplest hypothesis is that Kähler coupling strength has single universal value and the weak form of self-duality fixes it. The proportionality $\alpha_K = g_K^2/4\pi\hbar$ and the proposed quantization of Planck constant requiring a generalization of the imbedding space imply that Kähler coupling strength varies but is constant at a given page of the "Big Book" defined by the generalized imbedding space [K17] .

2.3.3 What preferred extremal property means?

The requirement that the 4-surface having given 3-surface as its sub-manifold is absolute minimum of the Kähler action is the most obvious guess for the principle selecting the preferred extremals and has been taken as a working hypothesis for about one and half decades. Quantum criticality of Quantum TGD should have however led to the idea that preferred extremals are critical in the sense that space-time surface allows deformations for which second variation of Kähler action vanishes so that the corresponding Noether currents are conserved.

The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

Further insights emerged through the realization that Noether currents assignable to the modified Dirac equation are conserved only if the first variation of the modified Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X_l^3)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals.

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X_l^3)$ vanishing at the intersections of $X^4(X_l^3)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X_l^3 with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of WCW represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \rightarrow X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X_l^3)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X_l^3 involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

One must be very cautious with what one means with the preferred extremal property and criticality.

1. Does one assign criticality with the partonic 2-surfaces at the ends of CDs? Does one restrict it with the throats for which light-like 3-surface has also degenerate induced 4-metric? Or does one assume stronger form of holography requiring a slicing of space-time surface by partonic 2-surfaces and string world sheets and assign criticality to all partonic 2-surfaces. This kind of slicing is suggested by the study of the extremals [K5] , required by the number theoretic vision ($M^8 - H$ duality [K50]), and also by the purely physical condition that a stringy realization of GCI is possible.

2. What is the exact meaning of the preferred extremal property? The assumption that the variations of Kähler action leaving 3-surfaces at the ends of CDs invariant would not be consistent with the effective 2-dimensionality. The assumption that the critical deformations leave invariant only partonic 2-surfaces would imply genuine 2-dimensionality. Should one assume that critical deformations leave invariant partonic 2-surface and 3-D tangent space in the direction of space-like 3-surface or light-like 3-surface but not both. This would be consistent with effective 3-dimensionality and would explain why Kac-Moody symmetries

associated with the light-like 3-surfaces act as gauge symmetries. This is also essential for the realization of Poincare invariance since the quantization of the light-cone proper time distance between CDs implies that infinitesimal Poincare transformations lead out of CD unless compensated by Kac-Moody type transformations acting like gauge transformations. In the similar manner it would explain why symplectic transformations of δCD act like gauge transformations.

3. Could one pose the criticality condition for all partonic 2-surfaces in the slicing or only for the throats of light-like 3-surfaces? This hypothesis looks natural but is not necessary. Light-like throats are very singular objects criticality might apply only to their variations only in the limiting sense and it might be necessary to assume criticality for all partonic 2-surfaces.

2.3.4 Why non-local Kähler function?

Kähler function is non-local functional of 3-surface. Non-locality of the Kähler function seems to be at odds with basic assumptions of local quantum field theories. Why this rather radical departure from the basic assumptions of local quantum field theory? The answer is shortly given: WCW integration appears in the definition of the inner product for WCW spinor fields and this inner product must be free from perturbative divergences. Consider now the argument more closely.

In the case of finite-dimensional symmetric space with Kähler structure the representations of the isometry group necessitate the modification of the integration measure defining the inner product so that the integration measure becomes proportional to the exponent $exp(K)$ of the Kähler function [B28]. The generalization to infinite-dimensional case is obvious. Also the requirement of Kac-Moody symmetry leads to the presence of this kind of vacuum functional as will be found later. The exponent is in fact uniquely fixed by finiteness requirement. WCW integral is of the following form

$$\int \bar{S}_1 exp(K) S_1 \sqrt{g} dX \quad . \tag{2.3.7}$$

One can develop perturbation theory using local complex coordinates around a given 3-surface in the following manner. The $(1,1)$-part of the second variation of the Kähler function defines the metric and therefore propagator as contravariant metric and the remaining $(2,0)-$ and $(0,2)$-parts of the second variation are treated perturbatively. The most natural choice for the 3-surface are obviously the 3-surfaces, which correspond to extrema of the Kähler function.

When perturbation theory is developed around the 3-surface one obtains two ill-defined determinants.

1. The Gaussian determinant coming from the exponent, which is just the inverse square root for the matrix defined by the metric defining $(1,1)$-part of the second variation of the Kähler function in local coordinates.

2. The metric determinant. The matrix representing covariant metric is however same as the matrix appearing in Gaussian determinant by the defining property of the Kähler metric: in local complex coordinates the matrix defined by second derivatives is of type $(1,1)$. Therefore these two ill defined determinants (recall the presence of Diff degeneracy) cancel each other exactly for a unique choice of the vacuum functional!

Of course, the cancellation of the determinants is not enough. For an arbitrary local action one encounters the standard perturbative divergences. Since most local actions (Chern-Simons term is perhaps an exception [B17]) for induced geometric quantities are extremely nonlinear there is no hope of obtaining a finite theory. For non-local action the situation is however completely different. There are no local interaction vertices and therefore no products of delta functions in perturbation theory.

A further nice feature of the perturbation theory is that the propagator for small deformations is nothing but the contravariant metric. Also the various vertices of the theory are closely related to the metric of WCW since they are determined by the Kähler function so that perturbation theory would have a beautiful geometric interpretation. Furthermore, since four-dimensional Diff degeneracy implies that the propagator doesn't couple to un-physical modes.

It should be noticed that divergence cancellation arguments do not necessarily exclude Chern Simons term from vacuum functional defined as imaginary exponent of $exp(ik_2 \int_{X^4} J \wedge J)$. The term is not well defined for non-orientable space-time surfaces and one must assume that k_2 vanishes for these surfaces. The presence of this term might provide first principle explanation for CP breaking. If k_2 is integer multiple of $1/(8\pi)$ Chern Simons term gives trivial contribution for closed space-time surfaces since instanton number is in question. By adding a suitable boundary term of form $exp(ik_3 \int_{\delta X^3} J \wedge A)$ it is possible to guarantee that the exponent is integer valued for 4-surfaces with boundary, too.

There are two arguments suggesting that local Chern Simons term would not introduce divergences. First, 3-dimensional Chern Simons term for ordinary Abelian gauge field is known to define a divergence free field theory [B17] . The term doesn't depend at all on the induced metric and therefore contains no dimensional parameters (CP_2 radius) and its expansion in terms of CP_2 coordinate variables is of the form allowed by renormalizable field theory in the sense that only quartic terms appear. This is seen by noticing that there always exist symplectic coordinates, where the expression of the Kähler potential is of the form

$$A = \sum_k P_k dQ^k \quad . \tag{2.3.8}$$

The expression for Chern-Simons term in these coordinates is given by

$$k_2 \int_{X^3} \sum_{k,l} P_l dP_k \wedge dQ^k \wedge dQ^l \quad , \tag{2.3.9}$$

and clearly quartic CP_2 coordinates. A further nice property of the Chern Simons term is that this term is invariant under symplectic transformations of CP_2, which are realized as $U(1)$ gauge transformation for the Kähler potential.

2.4 Some properties of Kähler action

In this section some properties of Kähler action and Kähler function are discussed in light of experienced gained during about 15 years after the introduction of the notion.

2.4.1 Vacuum degeneracy and some of its implications

The vacuum degeneracy is perhaps the most characteristic feature of the Kähler action. Although it is not associated with the preferred extremals of Kähler action, there are good reasons to expect that it has deep consequences concerning the structure of the theory.

Vacuum degeneracy of the Kähler action

The basic reason for choosing Kähler action is its enormous vacuum degeneracy, which makes long range interactions possible (the well known problem of the membrane theories is the absence of massless particles [B25]). The Kähler form of CP_2 defines symplectic structure and any 4-surface for which CP_2 projection is so called Lagrangian manifold (at most two dimensional manifold with vanishing induced Kähler form), is vacuum extremal due to the vanishing of the induced Kähler form. More explicitly, in the local coordinates, where the vector potential A associated with the Kähler form reads as $A = \sum_k P_k dQ^k$. Lagrangian manifolds are expressible locally in the following form

$$P_k = \partial_k f(Q^i) \quad . \tag{2.4.1}$$

where the function f is arbitrary. Notice that for the general YM action surfaces with one-dimensional CP_2 projection are vacuum extremals but for Kähler action one obtains additional degeneracy.

There is also a second kind of vacuum degeneracy, which is relevant to the elementary particle physics. The so called CP_2 type vacuum extremals are warped imbeddings X^4 of CP_2 to H such that Minkowski coordinates are functions of a single CP_2 coordinate, and the one-dimensional projection of X^4 is random light like curve. These extremals have a non-vanishing action but vanishing Poincare charges. Their small deformations are identified as space-time counterparts of fermions and their super partners. Wormhole throats identified as pieces of these extremals are identified as bosons and their super partners.

The conditions stating light likeness are equivalent with the Virasoro conditions of string models and this actually led to the eventualo realization that conformal invariance is a basic symmetry of TGD and that WCW can be regarded as a union of symmetric spaces with isometry groups having identification as symplectic and Kac-Moody type groups assignable to the partonic 2-surfaces.

Approximate symplectic invariance

Vacuum extremals have diffeomorphisms of M^4_+ and M^4_+ local symplectic transformations as symmetries. For non-vacuum extremals these symmetries leave induced Kähler form invariant and only induced metric breaks these symmetries. Symplectic transformations of CP_2 act on the Maxwell field defined by the induced Kähler form in the same manner as ordinary $U(1)$ gauge symmetries. They are however not gauge symmetries since gauge invariance is still present. In fact, the construction of WCW geometry relies on the assumption that symplectic transformations of $\delta M^4_+ \times CP_2$ which infinitesimally correspond to combinations of M^4_+ local CP_2 symplectic and CP_2-local M^4_+ symplectic transformations act as isometries of WCW. In zero energy ontology these transformations act simultaneously on all partonic 2-surfaces characterizing the space-time sheet representing a generalized Feynman diagram inside CD.

The fact that CP_2 symplectic transformations do not act as genuine gauge transformations means that $U(1)$ gauge invariance is effectively broken. This has non-trivial implications. The field equations allow purely geometric vacuum 4-currents not possible in Maxwell's electrodynamics [K5] . For the known extremals (massless extremals) they are light-like and a possible interpretation is in terms of Bose-Einstein condensates of collinear massless bosons.

Spin glass degeneracy

Vacuum degeneracy means that all surfaces belonging to $M^4_+ \times Y^2$, Y^2 any Lagrangian sub-manifold of CP_2 are vacua irrespective of the topology and that symplectic transformations of CP_2 generate new surfaces Y^2. If preferred extremals are obtained as small deformations of vacuum extremals (for which the criticality is maximal), one expects therefore enormous ground state degeneracy, which could be seen as 4-dimensional counterpart of the spin glass degeneracy. This degeneracy corresponds to the hypothesis that WCW is a union of symmetric spaces labeled by zero modes which do not appear at the line-element of the WCW metric.

Zero modes define what might be called the counterpart of spin glass energy landscape and the maxima Kähler function as a function of zero modes define a discrete set which might be called reduced configuration space. Spin glass degeneracy turns out to be crucial element for understanding how macro-temporal quantum coherence emerges in TGD framework. One of the basic ideas about p-adicization is that the maxima of Kähler function define the TGD counterpart of spin glass energy landscape [K51, K20] . The hierarchy of discretizations of the symmetric spaces corresponding to a hierarchy of measurement resolutions [K18] could allow an identification in terms of a hierarchy spin glass energy landscapes so that the algebraic points of the WCW would correspond to the maxima of Kähler function. The hierarchical structure would be due to the failure of strict non-determinism of Kähler action allowing in zero energy ontology to add endlessly details to the space-time sheets representing zero energy states in shorter scale.

Generalized quantum gravitational holography

The original naive belief was that the construction of the configuration space geometry reduces to $\delta H = \delta M^4_+ \times CP_2$. An analogous idea in string model context became later known as quantum gravitational holography. The basic implication of the vacuum degeneracy is classical non-determinism, which is expected to reflect itself as the properties of the Kähler function and WCW geometry. Obviously classical non-determinism challenges the notion of quantum gravitational holography.

The hope was that a generalization of the notion of 3-surface is enough to get rid of the degeneracy and save quantum gravitational holography in its simplest form. This would mean that one just replaces space-like 3-surfaces with "association sequences" consisting of sequences of space-like 3-surfaces with time like separations as causal determinants. This would mean that the absolute minima of Kähler function would become degenerate: same space-like 3-surface at δH would correspond to several association sequences with the same value of Kähler function.

The life turned out to be more complex than this. CP_2 type extremals have Euclidian signature of the induced metric and therefore CP_2 type extremals glued to space-time sheet with Minkowskian signature of the induced metric are surrounded by light like surfaces X^3_l, which might be called elementary particle horizons. The non-determinism of the CP_2 type extremals suggests strongly that also elementary particle horizons behave non-deterministically and must be regarded as causal determinants having time like projection in M^4_+. Pieces of CP_2 type extremals are good candidates for the wormhole contacts connecting a space-time sheet to a larger space-time sheet and are also surrounded by an elementary particle horizons and non-determinism is also now present. That this non-determinism would allow the proposed simple description seems highly implausible.

Zero energy ontology realized in terms of a hierarchy of CDs seems to provide the most plausible treatment of the non-determinism and has indeed led to a breakthrough in the construction and understanding of quantum TGD. At the level of generalized Feynman diagrams sub-CDs containing zero energy states represent a hierarchy of radiative corrections so that the classical determinism is direct correlate for the quantum non-determinism. Determinism makes sense only when one has specified the length scale of measurement resolution. One can always add a CD containing a vacuum extremal to get a new zero energy state and a preferred extremal containing more details.

Classical non-determinism saves the notion of time

Although classical non-determinism represents a formidable mathematical challenge it is a must for several reasons. Quantum classical correspondence, which has become a basic guide line in the development of TGD, states that all quantum phenomena have classical space-time correlates. This is not new as far as properties of quantum states are considered. What is new that also quantum jumps and quantum jump sequences which define conscious existence in TGD Universe, should have classical space-time correlates: somewhat like written language is correlate for the contents of consciousness of the writer. Classical non-determinism indeed makes this possible. Classical non-determinism makes also possible the realization of statistical ensembles as ensembles formed by strictly deterministic pieces of the space-time sheet so that even thermodynamics has space-time representations. Space-time surface can thus be seen as symbolic representations for the quantum existence.

In canonically quantized general relativity the loss of time is fundamental problem. If quantum gravitational holography would work in the most strict sense, time would be lost also in TGD since all relevant information about quantum states would be determined by the moment of big bang. More precisely, geometro-temporal localization for the contents of conscious experience would not be possible. Classical non-determinism together with quantum-classical correspondence however suggests that it is possible to have quantum jumps in which non-determinism is concentrated in space-time region so that also conscious experience contains information about this region only.

2.4.2 Four-dimensional General Coordinate Invariance

The proposed definition of the Kähler function is consistent with GCI and implies also 4-dimensional Diff degeneracy of the Kähler metric. Zero energy ontology inspires strengthening of the GCI in the sense that space-like 3-surfaces at the boundaries of CD are physically equivalent with the light-like 3-surfaces connecting the ends. This implies that basic geometric objects are partonic 2-surfaces at the boundaries of CDs identified as the intersections of these two kinds of surfaces. Besides this the distribution of 4-D tangent planes at partonic 2-surfaces would code for physics so that one would have only effective 2-dimensionality. The failure of the non-determinism of Kähler action in the standard sense of the word affects the situation also and one must allow a fractal hierarchy of CDs inside CDs having interpretation in terms of radiative corrections.

Resolution of tachyon difficulty and absence of Diff anomalies

In TGD as in string models the tachyon difficulty is potentially present: unless the time like vibrational excitations possess zero norm they contribute tachyonic term to the mass squared operator of Super Kac Moody algebra. This difficulty is familiar already from string models [B27, B20] .

The degeneracy of the metric with respect to the time like vibrational excitations guarantees that time like excitations do not contribute to the mass squared operator so that mass spectrum is tachyon free. It also implies the decoupling of the tachyons from physical states: the propagator of the theory corresponds essentially to the inverse of the Kähler metric and therefore decouples from time like vibrational excitations. The experience with string model suggests that if metric is degenerate with respect to diffeomorphisms of $X^4(X^3)$ there are indeed good hopes that time like excitations possess vanishing norm with respect to WCW metric.

The four-dimensional Diff invariance of the Kähler function implies that Diff invariance is guaranteed in the strong sense since the scalar product of two Diff vector fields given by the matrix associated with $(1,1)$ part of the second variation of the Kähler action vanishes identically. This property gives hopes of obtaining theory, which is free from Diff anomalies: in fact loop space metric is not Diff degenerate and this might be the underlying reason to the problems encountered in string models [B27, B20] .

Complexification of WCW

Strong form of GCI plays a fundamental role in the complexification of WCW. GCI in strong form reduces the basic building brick of WCW to the pairs of partonic 2-surfaces and their 4-D tangent space data associated with ends of light-like 3-surface at light-like boundaries of CD. At boths end the imbedding space is effectively reduces to $\delta M^4_+ \times CP_2$ (forgetting the complications due to non-determinism of Kähler action). Light cone boundary in turn is metrically 2-dimensional Euclidian sphere allowing infinite-dimensional group of conformal symmetries and Kähler structure. Therefore one can say that in certain sense configuration space metric inherits the Kähler structure of $S^2 \times CP_2$. This mechanism works in case of four-dimensional Minkowski space only: higher-dimensional spheres do not possess even Kähler structure. In fact, it turns out that the quantum fluctuating degrees of freedom can be regarded in well-defined sense as a local variant of $S^2 \times CP_2$ and thus as an infinite-dimensional analog of symmetric space as the considerations of [K10] demonstrate.

The details of the complexification were understood only after the construction of WCW geometry and spinor structure in terms of second quantized induced spinor fields [K9] . This also allows to make detailed statements about complexification [K10] .

Contravariant metric and Diff^4 degeneracy

Diff degeneracy implies that the definition of the contravariant metric, which corresponds to the propagator associated to small deformations of minimizing surface is not quite straightforward. We believe that this problem is only technical. Certainly this problem is not new, being encountered in both GRT and gauge theories [B10, B16] . In TGD a solution of the problem is provided by the existence of infinite-dimensional isometry group. If the generators of this group form a complete set in the sense that any vector of the tangent space is expressible as as sum of these generators plus some zero norm vector fields then one can restrict the consideration to this subspace and in this subspace the matrix $g(X,Y)$ defined by the components of the metric tensor indeed indeed possesses well defined inverse $g^{-1}(X,Y)$. This procedure is analogous to gauge fixing conditions in gauge theories and coordinate fixing conditions in General Relativity.

It has turned that the representability of WCW as a union of symmetric spaces makes possible an approach to WCW integration based on harmonic analysis replacing the perturbative approach based on perturbative functional integral. This approach allows also a p-adic variant and leads an effective discretization in terms of discrete variants of WCW for which the points of symmetric space consist of algebraic points. There is an infinite number of these discretizations [K51] and the interpretation is in terms of finite measurement resolution. This gives a connection with the p-adicization program, infinite primes, inclusions of hyper-finite factors as representation of the finite measurement resolution, and the hierarchy of Planck constants [K50] so that various approaches to quantum TGD converge nicely.

General Coordinate Invariance and WCW spinor fields

GCI applies also at the level of quantum states. WCW spinor fields are Diff^4 invariant. This in fact fixes not only classical but also quantum dynamics completely. The point is that the values of the WCW spinor fields must be essentially same for all Diff^4 related 3-surfaces at the orbit X^4 associated with a given 3-surface. This would mean that the time development of Diff^4 invariant configuration spinor field is completely determined by its initial value at the moment of the big bang!

This is of course a naive over statement. The non-determinism of Kähler action and zero energy ontology force to take the causal diamond (CD) defined by the intersection of future and past directed light-cones as the basic structural unit of WCW, and there is fractal hierarchy of CDs within CDs so that the above statement makes sense only for giving CD in measurement resolution neglecting the presence of smaller CDs. Strong form of GCI also implies factorization of WCW spinor fields into a sum of products associated with various partonic 2-surfaces. In particular, one obtains time-like entanglement between positive and negative energy parts of zero energy states and entanglement coefficients define what can be identified as M-matrix expressible as a "complex square root" of density matrix and reducing to a product of positive definite diagonal square root of density matrix and unitary S-matrix. The collection of orthonormal M-matrices in turn define unitary U-matrix between zero energy states. M-matrix is the basic object measured in particle physics laboratory.

2.4.3 WCW geometry, generalized catastrophe theory, and phase transitions

The definition of WCW geometry has nice catastrophe theoretic interpretation. To understand the connection consider first the definition of the ordinary catastrophe theory [A65] .

1. In catastrophe theory one considers extrema of the potential function depending on dynamical variables x as function of external parameters c. The basic space decomposes locally into cartesian product $E = C \times X$ of control variables c, appearing as parameters in potential function $V(c, x)$ and of state variables x appearing as dynamical variables. Equilibrium states of the system correspond to the extrema of the potential $V(x, c)$ with respect to the variables x and in the absence of symmetries they form a sub-manifold of M with dimension equal to that of the parameter space C. In some regions of C there are several extrema of potential function and the extremum value of x as a function of c is multi-valued. These regions of $C \times X$ are referred to as catastrophes. The simplest example is cusp catastrophe (see Fig. 2.4.3) with two control parameters and one state variable.

2. In catastrophe regions the actual equilibrium state must be selected by some additional physical requirement. If system obeys flow dynamics defined by first order differential equations the catastrophic jumps take place along the folds of the cusp catastrophe (delay rule). On the other hand, the Maxwell rule obeyed by thermodynamic phase transitions states that the equilibrium state corresponds to the absolute minimum of the potential function and the state of system changes in discontinuous manner along the Maxwell line in the middle between the folds of the cusp (see Fig. 2.4.3).

3. As far as discontinuous behavior is considered fold catastrophe is the basic catastrophe: all catastrophes contain folds as there 'satellites' and one aim of the catastrophe theory is to derive all possible manners for the stable organization of folds into higher catastrophes. The fundamental result of the catastrophe theory is that for dimensions d of C smaller than 5 there are only 7 basic catastrophes and polynomial potential functions provide a canonical representation for the catastrophes: fold catastrophe corresponds to third order polynomial

(in fold the two real roots become a pair of complex conjugate roots), cusp to fourth order polynomial, etc.

Consider now the TGD counterpart of this. TGD allows allows two kinds of catastrophe theories.

1. The first one is related to Kähler action as a local functional of 4-surface. The nature of this catastrophe theory depends on what one means with the preferred extremals.

2. Second catastrophe theory corresponds to Kähler function a non-local functional of 3-surface. The maxima of the vacuum functional defined as the exponent of Kähler function define what might called effective space-times, and discontinuous jumps changing the values of the parameters characterizing the maxima are possible.

Consider first the option based on Kähler action.

1. Potential function corresponds to Kähler action restricted to the solutions of Euler Lagrange equations. Catastrophe surface corresponds to the four-surfaces found by extremizing Kähler action with respect to to the variables of X (time derivatives of coordinates of C specifying X^3 in H_a) keeping the variables of C specifying 3-surface X^3 fixed. Preferred extremal property is analogous to the Bohr quantization since canonical momenta cannot be chosen freely as in the ordinary initial value problems of the classical physics. Preferred extremals are by definition at criticality. Behavior variables correspond to the deformations of the 4-surface keeping partonic 2-surfaces and 3-D tangent space data fixed and preserving extremal property. Control variables would correspond to these data.

2. At criticality the rank of the infinite-dimensional matrix defined by the second functional derivatives of the Kähler action is reduced. Catastrophes form a hierarchy characterized by the reduction of the rank of this matrix and Thom's catastrophe theory generalizes to infinite-dimensional context. Criticality in this sense would be one aspect of quantum criticality having also other aspects. No discrete jumps would occur and system would only move along the critical surface becoming more or less critical.

3. There can exist however several critical extremals assignable to a given partonic 2-surface but have nothing to do with the catastrophes as defined in Thom's approach. In presence of degeneracy one should be able to choose one of the critical extremals or replace this kind of regions of WCW by their multiple coverings so that single partonic 2-surface is replaced with its multiple copy. The degeneracy of the preferred extremals could be actually a deeper reason for the hierarchy of Planck constants involving in its most plausible version n-fold singular coverings of CD and CP_2. This interpretation is very satisfactory since the generalization of the imbedding space and hierarchy of Planck constants would follow naturally from quantum criticality rather than as separate hypothesis.

4. The existence of the catastrophes is implied by the vacuum degeneracy of the Kähler action. For example, for pieces of Minkowski space in $M^4_+ \times CP_2$ the second variation of the Kähler action vanishes identically and only the fourth variation is non-vanishing: these 4-surfaces are analogous to the tip of the cusp catastrophe. There are also space-time surfaces for which the second variation is non-vanishing but degenerate and a hierarchy of subsets in the space of extremal 4-surfaces with decreasing degeneracy of the second variation defines the boundaries of the projection of the catastrophe surface to the space of 3-surfaces. The space-times for which second variation is degenerate contain as subset the critical and initial value sensitive absolute minimum space-times.

The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

Consider next the catastrophe theory defined by Kähler function.

1. In this case the most obvious identification for the behavior variables would be in terms of the space of all 3-surfaces in $CD \times CP_2$ - and if one believes in holography and zero energy ontology - the 2-surfaces assignable the boundaries of causal diamonds (CDs).

2. The natural control variables are zero modes whereas behavior variables would correspond to quantum fluctuating degrees of freedom contributing to the WCW metric. The induced Kähler form at partonic 2-surface would define infinitude of purely classical control variables. There is also a correlation between zero modes identified as degrees of freedom assignable to the interior of 3-surface and quantum fluctuating degrees of freedom assigned to the partonic 2-surfaces. This is nothing but holography and effective 2-dimensionality justifying the basic assumption of quantum measurement theory about the correspondence between classical and quantum variables. The absence of several maxima implies also the presence of saddle surfaces at which the rank of the matrix defined by the second derivatives is reduced. This could lead to a non-positive definite metric. It seems that it is possible to have maxima of Kähler function without losing positive definiteness of the metric since metric is defined as (1,1)-type derivatives with respect to complex coordinates. In case of CP_2 however Kähler function has single degenerate maximum corresponding to the homologically trivial geodesic sphere at $r = \infty$. It might happen that also in the case of infinite-D symmetric space finite maxima are impossible.

3. The criticality of Kähler function would be analogous to thermodynamical criticality and to the criticality in the sense of catastrophe theory. In this case Maxwell's rule is possible and even plausible since quantum jump replaces the dynamics defined by a continuous flow.

Cusp catastrophe provides a simple concretization of the situation for the criticality of Kähler action (as distinguished from that for Kähler function).

1. The set M of the critical 4-surfaces corresponds to the V-shaped boundary of the 2-D cusp catastrophe in 3-D space to plane. In general case it forms codimension one set in WCW. In TGD Universe physical system would reside at this line or its generalization to higher dimensional catastrophes. For the criticality associated with Kähler action the transitions would be smooth transitions between different criticalities characterized by the rank defined above: in the case of cusp from the tip of cusp to the vertex of cusp or vice versa. Evolution could mean a gradual increase of criticality in this sense. If preferred extremals are not unique, cusp catastrophe does not provide any analogy. The strong form of criticality would mean that the system would be always "at the tip of cusp" in metaphoric sense. Vacuum extremals are maximally critical in trivial sense, and the deformations of vacuum extremals could define the hierarchy of criticalities.

2. For the criticality of Kähler action Maxwell's rule stating that discontinuous jumps occur along the middle line of the cusp is in conflict with catastrophe theory predicting that jumps occurs along at criticality. For the criticality of Kähler function -if allowed at all by symmetric space property- Maxwell's rule can hold true but cannot be regarded as a fundamental law. It is of course known that phase transitions can occur in different manners (super heating and super cooling).

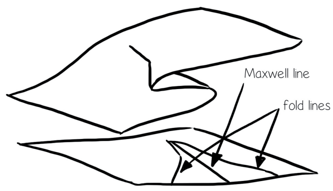

Figure 2.1: Cusp catastrophe

2.5 Weak form electric-magnetic duality and its implications

The notion of electric-magnetic duality [B2] was proposed first by Olive and Montonen and is central in $\mathcal{N}=4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K10] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2,-1,-1)$ and could be proportional to color hyper charge.

3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.

4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that

all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural. Also Chern-Simons Dirac equation implies the localization of solutions to flow lines, and this is consistent with the localization solutions of Kähler-Dirac equation to string world sheets.

2.5.1 Could a weak form of electric-magnetic duality hold true?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} \ . \tag{2.5.1}$$

A more general form of this duality is suggested by the considerations of [K22] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B1] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} \ . \tag{2.5.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1 + K)J_{12} \ , \tag{2.5.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L1] , [L1] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \tag{2.5.4}$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \tag{2.5.5}$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \tag{2.5.6}$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi}\times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \tag{2.5.7}$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the modified Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K37] supports this interpretation.

3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z \ . \tag{2.5.8}$$

This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{2.5.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the lightlikeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{2.5.10}$$

   Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L1]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K41]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [K56]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.

4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

2.5.2 Magnetic confinement, the short range of weak forces, and color confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L\overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L\nu_R$. $\nu_L\overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hypercharge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D3] .

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [K19] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_\pm$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

2. The addition of the particles $X^\pm$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

3. How should one describe the bound state formed by the fermion and $X^\pm$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K28] . If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K29] .

2.5.3 Could Quantum TGD reduce to almost topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the modified Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^\alpha A_\alpha$ plus and integral of the boundary term $J^{n\beta}A_\beta\sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta}A_\beta\sqrt{g_4}$ over the ends of the 3-surface.

2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K\epsilon^{n\beta\gamma\delta}J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K\epsilon^{0\beta\gamma\delta}J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $\hbar_0 \rightarrow r\hbar_0$ would effectively describe this. Boundary conditions would however give $1/r$ factor so that $\hbar$ would disappear from the Kähler function! The original attempt to realize quantum TGD as an almost topological QFT was in terms of Chern-Simons action but was given up. It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [K5]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

2. The original naive conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha(J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma}J_{\beta\ gamma})\sqrt{g_4}d^3x \quad . \tag{2.5.11}$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.

3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta}K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^\alpha \partial_\alpha \phi = -j^\alpha A_\alpha \ . \tag{2.5.12}$$

   This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j_K^\alpha$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j^K_\beta \partial_\gamma j^K_{delta} \ = \ 0 \ . \tag{2.5.13}$$

   j_K is a four-dimensional counterpart of Beltrami field [B19] and could be called generalized Beltrami field.

   The integrability conditions follow also from the construction of the extremals of Kähler action [K5] . The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^\alpha \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j_K^\alpha \phi$ and $j_I^\alpha \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \to A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha \phi) \ = \ 0 \ . \tag{2.5.14}$$

   As a consequence Coulomb term reduces to a difference of the conserved charges $Q^e_\phi = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q^m_\phi = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved

charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.

7. A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q^m_ϕ and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

8. Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to modified Dirac action as boundary term.

   Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

   One can assign to partonic orbits Chern-Simons Dirac action and now the condition would be that the action of C-S-D operator equals to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator. Twistor Grassmann approach suggests that also the virtual fermions reduce effectively to massless on-shell states but have non-physical helicity.

2.5.4 About the notion of measurement interaction

The notion of measurement has been central notion in quantum TGD but the precise definition of this notion is far from clear. In the following two possibly equivalent formulations are considered. The first formulation relies on the gauge transformations leaving Coulomb term of Kähler action unchanged and the second one to the interpretation of TGD as a square root of thermodynamics allowing to fix the values of conserved classical charges for zero energy energy state using Lagrange multipliers analogous to chemical potentials.

1. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \to A + \nabla\phi$ for which the scalar function the integral $\int j^\alpha_K \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha\phi) = 0 \ . \tag{2.5.15}$$

   As a consequence Coulomb term reduces to a difference of the conserved charges $Q^e_\phi = \int j^0\phi\sqrt{g_4}d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q^m_\phi = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

2. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action.

   The gauge transformed Kähler potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function. Kähler function is identified as a Dirac determinant of Chern-Simons Dirac operator (after many turns and twists) and the spectrum of this operator should not be invariant under these gauge transformations if this picture is correct. This is is achieved if the gauge transformation is carried only in the Dirac action corresponding to instanton term: this assumption is motivated by the breaking of time reversal invariance induced by quantum measurements. The modification of Kähler action can be guessed to correspond just to the Chern-Simons contribution from the instanton term.

3. A reasonable looking guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q_{ϕ}^m and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

In zero energy ontology (ZEO) TGD can be seen as square root of thermodynamics and this suggests an alternative manner to define what measurement interaction term means.

1. The condition that the space-time sheets appearing in superposition of space-time surfaces with given quantum numbers in Cartan algebra have same classical quantum numbers associated with Kähler action can be realized in terms of Lagrange multipliers in standard manner. These kind of terms would be analogous to various chemical potential terms in the partition function. One could call them measurement interaction terms. Measurement interaction terms would code the values of quantum charges to the space-time geometry.

   Kähler action contains also Chern-Simons term at partonic orbits compensating the Chern-Simons terms coming from Kähler action when weak form of electric-magnetic duality is assumed. This guarantees that Kähler action for preferred extremals reduces to Chern-Simons terms at the space-like ends of the spacetime surface and one obtains almost topological QFT.

2. If Kähler-Dirac action is constructed from Kähler action in super-symmetric manner by defining the modified gamma matrices in terms of canonical momentum densities one obtains also the fermionic counterparts of the Lagrange multiplier terms at partonic orbits and could call also them measurement interaction terms. Besides this one has also the Chern-Simons Dirac terms associated with the partonic orbits giving ordinary massless Dirac propagator. In presence of measurement interaction terms at the space-like ends of the space-time surface the boundary conditions $\Gamma^n\Psi = 0$ at the ends would be modified by the addition of term coming from the modified gamma matrix associated with the Lagrange multiplier terms. The original generalized massless generalized eigenvalue spectrum $p^k\gamma_k$ of Γ^n would be modified to massive spectrum given by the condition

$$(\Gamma^n + \sum_i \lambda_i \Gamma^{\alpha}_{Q_i} D_{\alpha})\Psi = 0 \ ,$$

   where Q_i refers to i:th conserved charge.

An interesting question is whether these two manners to introduce measurement interaction terms are actually equivalent.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

2.5.5 A general solution ansatz based on almost topological QFT property

The basic vision behind the ansatz is the reduction of quantum TGD to almost topological field theory. This requires that the flow parameters associated with the flow lines of isometry currents and Kähler current extend to global coordinates. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when weak electro-weak duality is applied as boundary conditions. The strongest form of the hydrodynamical interpretation requires that all conserved currents are parallel to Kähler current. In the more general case one would have several hydrodynamic flows. Also the braidings (several of them for the most general ansatz) assigned with the light-like 3-surfaces are naturally defined by the flow lines of conserved currents. The independent behavior of particles at different flow lines can be seen as a realization of the complete integrability of the theory. In free quantum field theories on mass shell Fourier components are in a similar role but the geometric interpretation in terms of flow is of course lacking. This picture should generalize also to the solution of the modified Dirac equation.

Basic field equations

Consider first the equations at general level.

1. The breaking of the Poincare symmetry due to the presence of monopole field occurs and leads to the isometry group $T \times SO(3) \times SU(3)$ corresponding to time translations, rotations, and color group. The Cartan algebra is four-dimensional and field equations reduce to the conservation laws of energy E, angular momentum J, color isospin I_3, and color hypercharge Y.

2. Quite generally, one can write the field equations as conservation laws for I, J, I_3, and Y.

$$D_\alpha \left[D_\beta (J^{\alpha\beta} H_A) - j_K^\alpha H^A + T^{\alpha\beta} j_A^l h_{kl} \partial_\beta h^l \right] \;\; = \;\; 0 \;\; . \tag{2.5.16}$$

The first term gives a contraction of the symmetric Ricci tensor with antisymmetric Kähler form and vanishes so that one has

$$D_\alpha \left[j_K^\alpha H^A - T^{\alpha\beta} j_A^k h_{kl} \partial_\beta h^l \right] \;\; = \;\; 0 \;\; . \tag{2.5.17}$$

For energy one has $H_A = 1$ and energy current associated with the flow lines is proportional to the Kähler current. Its divergence vanishes identically.

3. One can express the divergence of the term involving energy momentum tensor as as sum of terms involving $j_K^\alpha J_{\alpha\beta}$ and contraction of second fundamental form with energy momentum tensor so that one obtains

$$j_K^\alpha D_\alpha H^A = j_K^\alpha J_\alpha{}^\beta j_\beta^A + T^{\alpha\beta} H_{\alpha\beta}^k j_k^A \quad . \tag{2.5.18}$$

Hydrodynamical solution ansatz

The characteristic feature of the solution ansatz would be the reduction of the dynamics to hydrodynamics analogous to that for a continuous distribution of particles iniatially at the end of X^3 of the light-like 3-surface moving along flow lines defined by currents j_A satisfying the integrability condition $j_A \wedge dj_A = 0$. Field theory would reduce effectively to particle mechanics along flow lines with conserved charges defined by various isometry currents. The strongest condition is that all isometry currents j_A and also Kähler current j_K are proportional to the same current j. The more general option corresponds to multi-hydrodynamics.

1. Solution ansatz

Conserved currents are analogous to hydrodynamical currents in the sense that the flow parameter along flow lines extends to a global space-time coordinate. The conserved current is proportional to the gradient $\nabla\Phi$ of the coordinate varying along the flow lines: $J = \Psi\nabla\Phi$ and by a proper choice of Ψ one can allow to have conservation. The initial values of Ψ and Φ can be selected freely along the flow lines beginning from either the end of the space-time surface or from wormhole throats.

If one requires hydrodynamics also for Chern-Simons action (effective 2-dimensionality is required for preferred extremals), the initial values of scalar functions can be chosen freely only at the partonic 2-surfaces. The freedom to chose the intial values of the charges conserved along flow lines at the partonic 2-surfaces means the existence of an infinite number of conserved charges so that the theory would be integrable and even in two different coordinate directions. The basic difference as compared to ordinary conservation laws is that the conserved currents are parallel and their flow parameter extends to a global coordinate.

1. The most general assumption is that the conserved isometry currents

$$J_A^\alpha = j_K^\alpha H^A - T^{\alpha\beta} j_A^k h_{kl} \partial_\beta h^l \tag{2.5.19}$$

and Kähler current as well as instanto current are integrable in the sense that $J_A \wedge J_A = 0$ and $j_K \wedge j_K = 0$ hold true. One could imagine the possibility that the currents are not parallel. If instanton current and Kähler current are proportional to each other, Coulomb interaction term in the Kähler action vanishes and almost topological QFT property is achieved.

2. The integrability condition $dJ_A \wedge J_A = 0$ is satisfied if one one has

$$J_A = \Psi_A d\Phi_A \quad . \tag{2.5.20}$$

The ansatz allows a gauge transformation induced by a symplectic transformation of S^2.Φ_A is same for Kähler current and instanton current.

3. The conservation of J_A gives

$$d * (\Psi_A d\Phi_A) = 0 \quad . \tag{2.5.21}$$

This would mean separate hydrodynamics for each of the currents involved. In principle there is not need to assume any further conditions and one can imagine infinite basis of scalar function pairs (Ψ_A, Φ_A) since criticality implies infinite number deformations implying conserved Noether currents.

4. The conservation condition reduces to d'Alembert equation in the induced metric if one assumes that $\nabla\Psi_A$ is orthogonal with every $d\Phi_A$.

$$d * d\Phi_A = 0 \ , \ d\Psi_A \cdot d\Phi_A = 0 \ . \tag{2.5.22}$$

Taking $x = \Phi_A$ as a coordinate the orthogonality condition states $g^{xj}\partial_j\Psi_A = 0$ and in the general case one cannot solve the condition by simply assuming that Ψ_A depends on the coordinates transversal to Φ_A only. These conditions bring in mind $p \cdot p = 0$ and $p \cdot e$ condition for massless modes of Maxwell field having fixed momentum and polarization. $d\Phi_A$ would correspond to p and $d\Psi_A$ to polarization. The condition that each isometry current corresponds its own pair (Ψ_A, Φ_A) would mean that each isometry current corresponds to independent light-like momentum and polarization. Ordinary free quantum field theory would support this view whereas hydrodynamics and QFT limit of TGD would support single flow.

These are the most general hydrodynamical conditions that one can assume. One can consider also more restricted scenarios.

1. The strongest ansatz is inspired by the hydrodynamical picture in which all conserved isometry charges flow along same flow lines so that one would have

$$J_A = \Psi_A d\Phi \ . \tag{2.5.23}$$

In this case same Φ would satisfy simultaneously the d'Alembert type equations.

$$d * d\Phi = 0 \ , \ d\Psi_A \cdot d\Phi = 0. \tag{2.5.24}$$

This would mean that the massless modes associated with isometry currents move in parallel manner but can have different polarizations. The spinor modes associated with light-light like 3-surfaces carry parallel four-momenta, which suggest that this option is correct. This allows a very general family of solutions and one can have a complete 3-dimensional basis of functions Ψ_A with gradient orthogonal to $d\Phi$.

2. Isometry invariance under $T \times SO(3) \times SU(3)$ allows to consider the possibility that one has

$$J_A = k_A\Psi_A d\Phi_{G(A)} \ , \ d * (d\Phi_G(A)) = 0 \ , \ d\Psi_A \cdot d\Phi_G(A)) = 0 \ . \tag{2.5.25}$$

where $G(A)$ is T for energy current, $SO(3)$ for angular momentum currents and $SU(3)$ for color currents. Energy would thus flow along its own flux lines, angular momentum along its own flow lines, and color quantum numbers along their own flow lines. For instance, color currents would differ from each other only by a numerical constant. The replacement of Ψ_A with $\Psi_{G(A)}$ would be too strong a condition since Killing vector fields are not related by a constant factor.

To sum up, the most general option is that each conserved current J_A defines its own integrable flow lines defined by the scalar function pair (Ψ_A, Φ_A). A complete basis of scalar functions satisfying the d'Alembert type equation guaranteeing current conservation could be imagined with restrictions coming from the effective 2-dimensionality reducing the scalar function basis effectively to the partonic 2-surface. The diametrically opposite option corresponds to the basis obtained by assuming that only single Φ is involved.

The proposed solution ansatz can be compared to the earlier ansatz [K22] stating that Kähler current is topologized in the sense that for $D(CP_2) = 3$ it is proportional to the identically conserved instanton current (so that 4-D Lorentz force vanishes) and vanishes for $D(CP_2) = 4$ (Maxwell phase). This hypothesis requires that instanton current is Beltrami field for $D(CP_2) = 3$. In the recent case the assumption that also instanton current satisfies the Beltrami hypothesis in strong sense (single function Φ) generalizes the topologization hypothesis for $D(CP_2) = 3$ and guarantees that Coulomb term in Kähler action vanishes identically. A weaker form is obtained by replacing Kähler potential by its gauge transform in which case one also obtains a boundary term. As a matter fact, the topologization hypothesis applies to isometry currents also for $D(CP_2) = 4$ although instanton current is not conserved anymore. One can consider variants of instanton current since both (A_1, J_1) and (A, J) are available.

Can one require the extremal property in the case of Chern-Simons action?

Effective 2-dimensionality is achieved if the ends and wormhole throats are extremals of Chern-Simons action. The strongest condition would be that space-time surfaces allow orthogonal slicings by 3-surfaces which are extremals of Chern-Simons action.

Also in this case one can require that the flow parameter associated with the flow lines of the isometry currents extends to a global coordinate. Kähler magnetic field $B = *J$ defines a conserved current so that all conserved currents would flow along the field lines of B and one would have 3-D Beltrami flow. Note that in magnetohydrodynamics the standard assumption is that currents flow along the field lines of the magnetic field.

For wormhole throats light-likeness causes some complications since the induced metric is degenerate and the contravariant metric must be restricted to the complement of the light-like direction. This means that d'Alembert equation reduces to 2-dimensional Laplace equation. For space-like 3-surfaces one obtains the counterpart of Laplace equation with partonic 2-surfaces serving as sources. The interpretation in terms of analogs of Coulomb potentials created by 2-D charge distributions would be natural.

If $J + J1$ appears in Kähler action the extremals need not have 2-dimensional CP_2 projection as they must have for J option, and one can hope of obtaining large enough solution family consistent with effective 2-dimensionality. The field equations can be reduced to conservation conditions for the isometry currents for $SO(3) \times SU(3)$ along flow lines.

2.5.6 Holomorphic factorization of Kähler function

One can guess the general form of the core part of the Kähler function as function of complex coordinates assignable to the partonic surfaces at positive and negative energy ends of CD. It its convenient to restrict the consideration to the simplest possible non-trivial case which is represented by single propagator line connecting the ends of CD.

1. The propagator line corresponds to a symmetric space defined as a coset space G/H of the symplectic group and Kac-Moody group. This coset space is as a manifold Cartesian product $(G/H) \times (G/H)$ of symmetric spaces G/H associated with ends of the line. Kähler metric contains also an interaction term between the factors of the Cartesian product so that Kähler function can be said to reduce to a sum of "kinetic" terms and interaction term.

2. The exponent of Kähler function depends on both ends of the line and this means that the geometries at the ends are correlated in the sense that that Kähler form contains interaction terms between the line ends. It is however not quite clear whether it contains separate "kinetic" or self interaction terms assignable to the line ends. For Kähler function the kinetic and interaction terms should have the following general expressions as functions of complex WCW coordinates:

$$
\begin{aligned}
K_{kin,i} &= \sum_n f_{i,n}(Z_i)\overline{f_{i,n}(Z_i)} + c.c \ , \\
K_{int} &= \sum_n g_{1,n}(Z_1)\overline{g_{2,n}(Z_2)} + c.c \ , i = 1, 2 \ .
\end{aligned} \tag{2.5.26}
$$

Here $K_{kin,i}$ define "kinetic" terms and K_{int} defines interaction term. One would have what might be called holomorphic factorization suggesting a connection with conformal field theories. K_{kin} would correspond to the Chern-Simons term assignable to the ends of the line and K_{int} to the Chern-Simons terms assignable to the wormhole throats.

2.5.7 Could the dynamics of Kähler action predict the hierarchy of Planck constants?

The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. The formula for the Planck constant involves heuristic guess work and physical plausibility arguments. There are good arguments in favor of the hypothesis that only coverings are possible. Only a finite number of pages of the Big Book correspond to a given value of Planck constant, biological evolution corresponds to a gradual dispersion to the pages of the Big Book with larger Planck constant, and a connection with the hierarchy of infinite primes and p-adicization program based on the mathematical realization of finite measurement resolution emerges.

One can however ask whether this hierarchy could emerge directly from the basic quantum TGD rather than as a separate hypothesis. The following arguments suggest that this might be possible. One finds also a precise geometric interpretation of preferred extremal property interpreted as criticality in zero energy ontology.

1-1 correspondence between canonical momentum densities and time derivatives fails for Kähler action

The basic motivation for the geometrization program was the observation that canonical quantization for TGD fails. To see what is involved let us try to perform a canonical quantization in zero energy ontology at the 3-D surfaces located at the light-like boundaries of $CD \times CP_2$.

1. In canonical quantization canonical momentum densities $\pi_k^0 \equiv \pi_k = \partial L_K/\partial(\partial_0 h^k)$, where $\partial_0 h^k$ denotes the time derivative of imbedding space coordinate, are the physically natural quantities in terms of which to fix the initial values: once their value distribution is fixed also conserved charges are fixed. Also the weak form of electric-magnetic duality given by $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and a mild generalization of this condition to be discussed below can be interpreted as a manner to fix the values of conserved gauge charges (not Noether charges) to their quantized values since Kähler magnetic flux equals to the integer giving the homology class of the (wormhole) throat. This condition alone need not characterize criticality, which requires an infinite number of deformations of X^4 for which the second variation of the Kähler action vanishes and implies infinite number conserved charges. This in fact gives hopes of replacing π_k with these conserved Noether charges.

2. Canonical quantization requires that $\partial_0 h^k$ in the energy is expressed in terms of π_k. The equation defining π_k in terms of $\partial_0 h^k$ is however highly non-linear although algebraic. By taking squares the equations reduces to equations for rational functions of $\partial_0 h^k$. $\partial_0 h^k$ appears in contravariant and covariant metric at most quadratically and in the induced Kähler electric field linearly and by multiplying the equations by $det(g_4)^3$ one can transform the equations to a polynomial form so that in principle $\partial_0 h^k$ can obtained as a solution of polynomial equations.

3. One can always eliminate one half of the coordinates by choosing 4 imbedding space coordinates as the coordinates of the space-time surface so that the initial value conditions reduce to those for the canonical momentum densities associated with the remaining four coordinates. For instance, for space-time surfaces representable as map $M^4 \to CP_2$ M^4 coordinates are natural and the time derivatives $\partial_0 s^k$ of CP_2 coordinates are multi-valued. One would obtain four polynomial equations with $\partial_0 s^k$ as unknowns. In regions where CP_2 projection is 4-dimensional -in particular for the deformations of CP_2 vacuum extremals the natural coordinates are CP_2 coordinates and one can regard $\partial_0 m^k$ as unknowns. For the deformations of cosmic strings, which are of form $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, one can use coordinates of $M^2 \times S^2$, where S^2 is geodesic sphere as natural coordinates and regard as unknowns E^2 coordinates and remaining CP_2 coordinates.

4. One can imagine solving one of the four polynomials equations for time derivaties in terms of other obtaining N roots. Then one would substitute these roots to the remaining 3 conditions to obtain algebraic equations from which one solves then second variable. Obviously situation is very complex without additional symmetries. The criticality of the preferred extremals might however give additional conditions allowing simplifications. The reasons for giving up the canonical quantization program was following. For the vacuum extremals of Kähler action π_k are however identically vanishing and this means that there is an infinite number of value distributions for $\partial_0 h^k$. For small deformations of vacuum extremals one might however hope a finite number of solutions to the conditions and thus finite number of space-time surfaces carrying same conserved charges.

If one assumes that physics is characterized by the values of the conserved charges one must treat the the many-valuedness of $\partial_0 h^k$. The most obvious guess is that one should replace the space of space-like 4-surfaces corresponding to different roots $\partial_0 h^k = F^k(\pi_l)$ with four-surfaces in the covering space of $CD \times CP_2$ corresponding to different branches of the many-valued function $\partial_0 h^k = F(\pi_l)$ co-inciding at the ends of CD.

Do the coverings forces by the many-valuedness of $\partial_0 h^k$ correspond to the coverings associated with the hierarchy of Planck constants?

The obvious question is whether this covering space actually corresponds to the covering spaces associated with the hierarchy of Planck constants. This would conform with quantum classical correspondence. The hierarchy of Planck constants and hierarchy of covering spaces was introduced to cure the failure of the perturbation theory at quantum level. At classical level the multi-valuedness of $\partial_0 h^k$ means a failure of perturbative canonical quantization and forces the introduction of the covering spaces. The interpretation would be that when the density of matter becomes critical the space-time surface splits to several branches so that the density at each branches is sub-critical. It is of course not at all obvious whether the proposed structure of the Big Book is really consistent with this hypothesis and one also consider modifications of this structure if necessary. The manner to proceed is by making questions.

1. The proposed picture would give only single integer characterizing the covering. Two integers assignable to CD and CP_2 degrees of freedom are however needed. How these two coverings could emerge?

   (a) One should fix also the values of $\pi_k^n = \partial L_K/\partial h_n^k$, where n refers to space-like normal coordinate at the wormhole throats. If one requires that charges do not flow between regions with different signatures of the metric the natural condition is $\pi_k^n = 0$ and allows also multi-valued solution. Since wormhole throats carry magnetic charge and since weak form of electric-magnetic duality is assumed, one can assume that CP_2 projection is four-dimensional so that one can use CP_2 coordinates and regard $\partial_0 m^k$ as un-knows. The basic idea about topological condensation in turn suggests that M^4 projection can be assumed to be 4-D inside space-like 3-surfaces so that here $\partial_0 s^k$ are the unknowns. At partonic 2-surfaces one would have conditions for both π_k^0 and π_k^n. One might hope that the numbers of solutions are finite for preferred extremals because of their symmetries and given by n_a for $\partial_0 m^k$ and by n_b for $\partial_0 s^k$. The optimistic guess is that n_a and n_b corresponds to the numbers of sheets for singular coverings of CD and CP_2. The covering could be visualized as replacement of space-time surfaces with space-time surfaces which have $n_a n_b$ branches. n_b branches would degenerate to single branch at the ends of diagrams of the generaled Feynman graph and n_a branches would degenerate to single one at wormhole throats.

(b) This picture is not quite correct yet. The fixing of π_k^0 and π_k^n should relate closely to the effective 2-dimensionality as an additional condition perhaps crucial for criticality. One could argue that both π_k^0 and π_k^n must be fixed at X^3 and X_l^3 in order to effectively bring in dynamics in two directions so that X^3 could be interpreted as a an orbit of partonic 2-surface in space-like direction and X_l^3 as its orbit in light-like direction. The additional conditions could be seen as gauge conditions made possible by symplectic and Kac-Moody type conformal symmetries. The conditions for π_0^k would give n_b branches in CP_2 degrees of freedom and the conditions for π_k^n would split each of these branches to n_a branches.

(c) The existence of these two kinds of conserved charges (possibly vanishing for π_k^n) could relate also very closely to the slicing of the space-time sheets by string world sheets and partonic 2-surfaces.

2. Should one then treat these branches as separate space-time surfaces or as a single space-time surface? The treatment as a single surface seems to be the correct thing to do. Classically the conserved changes would be $n_a n_b$ times larger than for single branch. Kähler action need not (but could!) be same for different branches but the total action is $n_a n_b$ times the average action and this effectively corresponds to the replacement of the $\hbar_0/g_K^2$ factor of the action with $\hbar/g_K^2$, $r \equiv \hbar/\hbar_0 = n_a n_b$. Since the conserved quantum charges are proportional to $\hbar$ one could argue that $r = n_a n_b$ tells only that the charge conserved charge is $n_a n_b$ times larger than without multi-valuedness. $\hbar$ would be only effectively $n_a n_b$ fold. This is of course poor man's argument but might catch something essential about the situation.

3. How could one interpret the condition $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and its generalization to be discussed below in this framework? The first observation is that the total Kähler electric charge is by $\alpha_K \propto 1/(n_a n_b)$ same always. The interpretation would be in terms of charge fractionization meaning that each branch would carry Kähler electric charge $Q_K = n g_K/n_a n_b$. I have indeed suggested explanation of charge fractionization and quantum Hall effect based on this picture.

4. The vision about the hierarchy of Planck constants involves also assumptions about imbedding space metric. The assumption that the M^4 covariant metric is proportional to $\hbar^2$ follows from the physical idea about $\hbar$ scaling of quantum lengths as what Compton length is. One can always introduce scaled M^4 coordinates bringing M^4 metric into the standard form by scaling up the M^4 size of CD. It is not clear whether the scaling up of CD size follows automatically from the proposed scenario. The basic question is why the M^4 size scale of the critical extremals must scale like $n_a n_b$? This should somehow relate to the weak self-duality conditions implying that Kähler field at each branch is reduced by a factor $1/r$ at each branch. Field equations should posses a dynamical symmetry involving the scaling of CD by integer k and $J^{0\beta}\sqrt{g_4}$ and $J^{n\beta}\sqrt{g_4}$ by $1/k$. The scaling of CD should be due to the scaling up of the M^4 time interval during which the branched light-like 3-surface returns back to a non-branched one.

5. The proposed view about hierarchy of Planck constants is that the singular coverings reduce to single-sheeted coverings at $M^2 \subset M^4$ for CD and to $S^2 \subset CP_2$ for CP_2. Here S^2 is any homologically trivial geodesic sphere of CP_2 and has vanishing Kähler form. Weak self-duality condition is indeed consistent with any value of $\hbar$ and impies that the vacuum property for the partonic 2-surface implies vacuum property for the entire space-time sheet as holography indeed requires. This condition however generalizes. In weak self-duality conditions the value of $\hbar$ is free for any 2-D Lagrangian sub-manifold of CP_2.

The branching along M^2 would mean that the branches of preferred extremals always collapse to single branch when their M^4 projection belongs to M^2. Magnetically charged light-light-like throats cannot have M^4 projection in M^2 so that self-duality conditions for different values of $\hbar$ do not lead to inconsistencies. For space-like 3-surfaces at the boundaries of CD the condition would mean that the M^4 projection becomes light-like geodesic. Straight cosmic strings would have M^2 as M^4 projection. Also CP_2 type vacuum extremals for which the random light-like projection in M^4 belongs to M^2 would represent this of situation. One can ask whether the degeneration of branches actually takes place along any string like object $X^2 \times Y^2$, where X^2 defines a minimal surface in M^4. For these the weak self-duality condition would imply $\hbar = \infty$ at the ends of the string. It is very plausible that string like objects feed their magnetic fluxes to larger space-times sheets through wormhole contacts so that these conditions are not encountered.

Connection with the criticality of preferred extremals

Also a connection with quantum criticality and the criticality of the preferred extremals suggests itself. Criticality for the preferred extremals must be a property of space-like 3-surfaces and light-like 3-surfaces with degenerate 4-metric and the degeneration of the $n_a n_b$ branches of the space-time surface at the its ends and at wormhole throats is exactly what happens at criticality. For instance, in catastrophe theory roots of the polynomial equation giving extrema of a potential as function of control parameters co-incide at criticality. If this picture is correct the hierarchy of Planck constants would be an outcome of criticality and of preferred extremal property and preferred extremals would be just those multi-branched space-time surfaces for which branches co-incide at the the boundaries of $CD \times CP_2$ and at the throats.

2.6 Does the exponent of Chern-Simons action reduce to the exponent of the area of minimal surfaces?

As I scanned of hep-th I found an interesting article by Giordano, Peschanski, and Seki [B22] based on AdS/CFT correspondence. What is studied is the high energy behavior of the gluon-gluon and quark-quark scattering amplitudes of $\mathcal{N} = 4$ SUSY.

1. The proposal made earlier by Aldaya and Maldacena [B5] is that gluon-gluon scattering amplitudes are proportional to the imaginary exponent of the area of a minimal surface in AdS_5 whose boundary is identified as *momentum space*. The boundary of the minimal surface would be polygon with light-like edges: this polygon and its dual are familiar from twistor approach.

2. Giordano, Peschanski, and Seki claim that quark-quark scattering amplitude for heavy quarks corresponds to the exponent of the area for a minimal surface in the *Euclidian* version of AdS_5 which is hyperbolic space (space with a constant negative curvature): it is interpreted as a counterpart of WCW rather than momentum space and amplitudes are obtained by analytic continuation. For instance, a universal Regge behavior is obtained. For general amplitudes the exponent of the area alone is not enough since it does not depend on gluon quantum numbers and vertex operators at the edges of the boundary polygon are needed.

In the following my intention is to consider the formulation of this conjecture in quantum TGD framework. I hasten to inform that I am not a specialist in AdS/CFT and can make only general comments inspired by analogies with TGD.

2.6.1 Why Chern-Simons action should reduce to area for minimal surfaces?

The minimal surface conjectures are highly interesting from TGD point of view. The weak form of electric magnetic duality implies the reduction of Kähler action to 3-D Chern-Simons terms. Effective 2-dimensionality implied by the strong form of General Coordinate Invariance suggests a further reduction of Chern-Simons terms to 2-D terms and the areas of string world sheet and of partonic 2-surface are the only non-topological options that one can imagine. Skeptic could of course argue that the exponent of the minimal surface area results as a characterizer of the quantum state rather than vacuum functional. In the following I defend the minimal interpretation as Chern-Simons terms.

Let us look this conjecture in more detail.

1. In zero energy ontology twistor approach is very natural since all physical states are bound states of massless particles. Also virtual particles are composites of massless states. The possibility to have both signs of energy makes possible space-like momenta for wormhole contacts. Mass shell conditions at internal lines imply extremely strong constraints on the virtual momenta and both UV and IR finiteness are expected to hold true.

2. The weak form of electric magnetic duality [K18] implies that the exponent of Kähler action reduces to the exponent of Chern-Simons term for 3-D space-like surfaces at the ends of space-time surface inside CD and for light-like 3-surfaces. The coefficient of this term is complex since the contribution of Minkowskian regions of the space-time surface is imaginary ($\sqrt{g_4}$ is imaginary) and that of Euclidian regions (generalized Feynman diagrams) real. The Chern-Simons term from Minkowskian regions is like Morse function and that from Euclidian regions defines Kähler function and stationary phase approximation makes sense. The two contributions are different since the space-like 3-surfaces contributing to Kähler function and Morse function are different.

3. Electric magnetic duality [K18] leads also to the conclusion that wormhole throats carrying elementary particle quantum numbers are Kähler magnetic monopoles. This forces to identify elementary particles as string like objects with ends having opposite monopole charges. Also more complex configurations are possible.

   It is not quite clear what the scale of the stringyness is. The natural first guess inspired by quantum classical correspondence is that it corresponds to the p-adic length scale of the particle characterizing its Compton length. Second possibility is that it corresponds to electroweak scale. For leptons stringyness in Compton length scale might not have any fatal implications since the second end of string contains only neutrinos neutralizing the weak isospin of the state. This kind of monopole pairs could appear even in condensed matter scales: in particular if the proposed hierarchy of Planck constants [K17] is realized.

4. Strong form of General Coordinate Invariance requires effective 2-dimensionality. In given UV and IR resolutions either partonic 2-surfaces or string world sheets form a finite hierarchy of CDs inside CDs with given CD characterized by a discrete scale coming as an integer multiple of a fundamental scale (essentially CP_2 size). The string world sheets have boundaries consisting of either light-like curves in induced metric at light-like wormhole throats and space-like curves at the ends of CD whose M^4 projections are light-like. These braids intersect partonic 2-surfaces at discrete points carrying fermionic quantum numbers.

   This implies a rather concrete analogy with $AdS_5 \times S_5$ duality, which describes gluons as open strings. In zero energy ontology (ZEO) string world sheets are indeed a fundamental notion and the natural conjecture is that these surfaces are minimal surfaces whose area by quantum classical correspondence depends on the quantum numbers of the external particles. String tension in turn should depend on gauge couplings -perhaps only Kähler coupling strength- and geometric parameters like the size scale of CD and the p-adic length scale of the particle.

5. Are the minimal surfaces in question minimal surfaces of the imbedding space $M^4 \times CP_2$ or of the space-time surface X^4? All possible 2-surfaces at the boundary of CD must be allowed so that they cannot correspond to minimal surfaces in $M^4 \times CP_2$ unless one assumes that they emerge in stationary phase approximation only. The boundary conditions at the ends of CD could however be such that *any* partonic 2-surface correspond to a minimal surfaces in X^4. Same applies to string world sheets. One might even hope that these conditions combined with the weak form of electric magnetic duality fixes completely the boundary conditions at wormhole throats and space-like ends of space-time surface.

   The trace of the second fundamental form orthogonal to the string world sheet/partonic 2-surface as sub-manifold of space-time surface would vanish: this is nothing but a generalization of the geodesic motion obtained by replacing word line with a 2-D surface. It does not imply the vanishing of the trace of the second fundamental form in $M^4 \times CP_2$ having interpretation as a generalization of particle acceleration [K56]. Effective 2-dimensionality would be realized if Chern-Simons terms reduce to a sum of the areas of these minimal surfaces.

These arguments suggest that scattering amplitudes are proportional to the product of exponents of 2-dimensional actions which can be either imaginary or real. Imaginary exponent would be proportional to the total area of string world sheets and the imaginary unit would come naturally from $\sqrt{g_2}$. Teal exponent proportional to the total area of partonic 2-surfaces. The coefficient of these areas would not in general be same.

The equality of the Minkowskian and Euclidian Chern-Simons terms is suggestive but not necessarily true since there could be also other Chern-Simons contributions than those assignable to wormhole throats and the ends of space-time. The equality would imply that the total area of string world sheets equals to the total area of partonic 2-surfaces suggesting strongly a duality meaning that either Euclidian or Minkowskian regions carry the needed information.

2.6.2 IR cutoff and connection with p-adic physics

In twistor approach the IR cutoff is necessary to get rid of IR divergences. Also in the AdS_5 approach the condition that the minimal surface area is finite requires an IR cutoff. The problem is that there is no natural IR cutoff. In TGD framework zero energy ontology brings in a natural IR cutoff via the finite and quantized size scale of CD guaranteeing that the minimal surfaces involved have a finite area. This implies that also particles usually regarded as massless have a small mass characterized by the size of CD. The size scale of CD would correspond to the scale parameter R assigned with the metric of AdS_5.

1. String tension relates in AdS_5 approach to the gauge coupling g_{YM} and to the number N_c of colors by the formula

$$\lambda = g_{YM}^2 N_c = \frac{R^2}{\alpha'} \ . \tag{2.6.1}$$

$1/N_c$-expansion is in terms of $1/\sqrt{\lambda}$. The formula has an alternative form as an expression for the string tension

$$\alpha' = \frac{R^2}{\sqrt{g_{YM}^2 N_c}} \ . \tag{2.6.2}$$

The analog this formula in TGD framework suggests an connection with p-adic length scale hypothesis.

1. As already noticed, the natural counterpart for the scale R could be the discrete value of the size scale of CD. Since the symplectic group assignable to $\delta M^4_\pm \times CP_2$ (or the upper or lower boundary of CD) is the natural generalization of the gauge group, it would seem that $N_c = \infty$ holds true in the absence of cutoff. At the limit $N_c = \infty$ only planar diagrams would contribute to YM scattering amplitudes. Finite measurement resolution must make the effective value of N_c finite so that also λ would be finite. String tension would depend on both the size of CD and the effective number of symplectic colors.

2. If α' is characterized by the square of the Compton length of the particle, λ would be essentially the square of the ratio of CD size scale given by secondary p-adic lengths and of the primary p-adic length scale associated with the particle: $\lambda = g_{YM}^2\sqrt{p}$, where p is the p-adic prime characterizing the particle. Favored values of the p-adic prime correspond to primes near powers of two. The effective number of symplectic colors would be $N_c = \sqrt{p}/g_{YM}^2$ and the expansion would come in powers of $g_{YM}^2/\sqrt{p}$. For electron one would have $p = M_{127} = 2^{127-1}$ so that the expansion would converge extremely fast. Together with the amazing success of the p-adic mass calculations based on p-adic thermodynamics for the scaling generator L_0 [K31] this suggests a deep connection with p-adic physics and number theoretic universality.

2.6.3 Could Kähler action reduce to Kähler magnetic flux over string world sheets and partonic 2-surfaces?

Can one consider alternative identifications of Kähler action for preferred extremals? The only alternative identification of Kähler function that I can imagine is that Kähler action proportional to the Kähler magnetic flux $\int_{Y^2} J$ or Kähler electric flux $\int_{Y^2} *J$ for string world sheets and possibly also partonic 2-surfaces. These fluxes are dimensionless numbers. If the weak form of electric-magnetic duality holds true also at string world sheets, the two options are equivalent apart from a proportionality constant.

1. For Kähler magnetic flux there would be no explicit dependence on the induced metric. This is in accordance with the almost topological QFT property.

2. Unless the weak form of electric-magnetic duality holds true, the Kähler electric flux has an explicit dependence on the induced metric but in a scaling invariant manner. The most obvious objection relates to the sign factor of the dual flux which depends on the orientation of the string world sheet and thus changes sign when the orientation of space-time sheet is changed by changing that of the string world sheet. This is in conflict with the independence of Kähler action on orientation. One can however argue that the orientation makes itself actually physically visible via the weak form of electric-magnetic duality and that the change of the orientation as a symmetry is dynamically broken. This breaking would be analogous to parity breaking at the level of imbedding space.

3. In [K23] it is proposed that braids defined by the boundaries of string world sheets could correspond to Legendrian sub-manifolds, whereas partonic 2-surfaces could the duals of Legendrian manifolds, so that braiding would take place dynamically. The identification of the Kähler action as Kähler magnetic flux associated with string world sheets and possibly also partonic 2-surfaces is consistent with the assumption that the extremal of Kähler action in question. Indeed, the Legendrian property says that the projection of the Kähler gauge potential on braid strand vanishes and this expresses the extremality of the Kähler magnetic flux.

The assumption that Kähler action is proportional to Kähler magnetic flux seems to be consistent with the minimal surface property. The weak form of electric-magnetic duality gives a constraint on the normal derivatives of imbedding space coordinates at the string world sheet and minimal surface property strengthens these constraints. One could perhaps say that space-time surface chooses its shape in such a manner that the string world sheet has a minimal area.

The open questions are following.

1. Does Kähler action for the preferred extremals reduce to the area of the string world sheet or to Kähler flux, or are the representations equivalent so that the induced Kähler form would effectively define area form? If the Kähler form form associated with the induced metric on string world sheet is proportional to the induced Kähler form the Kähler magnetic flux is proportional to the area and Kähler action reduces to genuine area. This condition looks like a natural additional constraint on string world sheets besides minimal surface property.

2. The proportionality of the induced Kähler form and Kähler form of the induced 2-metric implies as such only the extremal property against the symplectic variations so that one cannot have minimal surface property at imbedding space level. Minimality at space-time level is however possible since space-time surface itself can arrange the situation so that general variations deforming the string world sheet along space-time surface reduce to symplectic variations at the level of the imbedding space.

3. Does the situation depend on whether the string world sheet is in Minkowskian or Euclidian space-time region? The problem is that in Euclidian regions the value of Kähler action is positive definite and it is not obvious why the Kähler magnetic flux for Euclidian string world sheets should have a fixed sign. Could weak form of electric-magnetic duality fix the sign?

Irrespective whether the Kähler action is proportional to the total area or the Kähler electric flux over string world sheets, the theory would be exactly solvable at string world sheet level (finite measurement resolution).

2.6.4 What is the interpretation of Yangian duality in TGD framework?

Minimal surfaces in both WCW and momentum space are used in the above mentioned two articles [B5, B22]. The possibility of these two descriptions must reflect the Yangian symmetry unifying the conformal symmetries of Minkowski space and momentum space in twistorial approach.

The minimal surfaces in $X^4 \subset M^4 \times CP_2$ are natural in TGD framework. Could also the minimal surfaces in momentum space have some interpretation in TGD framework? Ore more generally, what could be the interpretation of the dual descriptions provided by twistor diagrams with light-like edges and dual twistor diagrams with light-like vertices? One can imagine many interpretations but zero energy ontology suggests an especially attractive and natural interpretation of this duality as the exchange of the roles of wormhole throats carrying always on mass shell massless momenta and wormhole contacts carrying in general off-mass shell momenta and massive momenta in incoming lines.

1. For WCW twistor diagrams vertices correspond to incoming and outgoing light-like momenta. The light-like momenta associated with the wormhole throats of the incoming and outgoing lines of generalized Feynman diagram could correspond to the light-like momenta associated with the vertices of the polygon. The internal lines defined by wormhole contacts carrying virtual off mass shell momenta would naturally correspond to to edges of the twistor diagram.

2. What about dual twistor diagrams in which light-like momenta correspond to lines? Zero energy ontology implies that virtual wormhole throats carry on mass shell massless momenta whereas incoming wormhole contacts in general carry massive particles: this guarantees the absence of IR divergences. Could one identify the momenta of internal wormhole throats as light-like momenta associated with the lines dual twistor diagrams and the incoming net momenta assignable to wormhole contacts as incoming and outgoing momenta.

Also the transition from Minkowskian to Euclidian signature by Wick rotation could have interpretation in TGD framework. Space-time surfaces decompose into Minkowskian and Euclidian regions. The latter ones represent generalized Feynman diagrams. This suggests a generalization of Wick rotation. The string world sheets in Euclidian regions would define the analogs of the minimal surfaces in Euclidian AdS_5 and the string world sheets in Minkowskian regions the analogs of Minkowskian AdS_5. The magnitudes of the areas would be identical so that they might be seen as analytical continuations of each other in some sense. Note that partonic 2-surfaces would belong to the intersection of Euclidian and Minkowskian space-time regions. This argument tells nothing about possible momentum space analog of $M^4 \times CP_2$.

Chapter 3

Construction of Configuration Space Kähler Geometry from Symmetry Principles

3.1 Introduction

The most general expectation is that configuration space ("world of classical worlds" (WCW)) can be regarded as a union of coset spaces which are infinite-dimensional symmetric spaces with Kähler structure: $C(H) = \cup_i G/H(i)$. Index i labels 3-topology and zero modes. The group G, which can depend on 3-surface, can be identified as a subgroup of diffeomorphisms of $\delta M^4_+ \times CP_2$ and H must contain as its subgroup a group, whose action reduces to $Diff(X^3)$ so that these transformations leave 3-surface invariant.

In zero energy ontology (ZEO) 3-surface corresponds to a pair of space-like 3-surfaces at the opposide boundaries of causal diamond (CD) and thus to a more or less unique extremal of Kähler action. The interpretation would be in terms of holography. One can also consider the inclusion of the light-like 3-surfaces at which the signature of the induced metric changes to the 3-surface so that it would become connected.

The task is to identify plausible candidate for G and H and to show that the tangent space of the WCW allows Kähler structure, in other words that the Lie-algebras of G and $H(i)$ allow complexification. One must also identify the zero modes and construct integration measure for the functional integral in these degrees of freedom. Besides this one must deduce information about the explicit form of WCW metric from symmetry considerations combined with the hypothesis that Kähler function is Kähler action for a preferred extremal of Kähler action. One must of course understand what "preferred" means.

3.1.1 General Coordinate Invariance and generalized quantum gravitational holography

The basic motivation for the construction of WCW geometry is the vision that physics reduces to the geometry of classical spinor fields in the infinite-dimensional WCW of 3-surfaces of $M^4_+ \times CP_2$ or of $M^4 \times CP_2$. Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that WCW possesses Kähler geometry. Kähler geometry is coded into Kähler function.

The original belief was that the four-dimensional general coordinate invariance of Kähler function reduces the construction of the geometry to that for the boundary of configuration space consisting of 3-surfaces on $\delta M^4_+ \times CP_2$, the moment of big bang. The proposal was that Kähler function $K(Y^3)$ could be defined as a preferred extremal of so called Kähler action for the unique space-time surface $X^4(Y^3)$ going through given 3-surface Y^3 at $\delta M^4_+ \times CP_2$. For Diff^4 transforms of Y^3 at $X^4(Y^3)$ Kähler function would have the same value so that Diff^4 invariance and degeneracy would be the outcome. The proposal was that the preferred extremals are absolute minima of Kähler action.

This picture turned out to be too simple.

1. I have already described the recent view about light-like 3-surfaces as generalized Feynman diagrams and space-time surfaces as preferred extremals of Kähler action and will not repeat what has been said. Note that the inclusion of space-like ends at boundaries of CD gives analog of Wilson loop.

2. It has also become obvious that the gigantic symmetries associated with $\delta M^4_\pm \times CP_2 \subset CD \times CP_2$ manifest themselves as the properties of propagators and vertices. Cosmological considerations, Poincare invariance, and the new view about energy favor the decomposition of the WCW to a union of configuration spaces assignable to causal diamonds CDs defined as intersections of future and past directed light-cones. The minimum assumption is that CDs label the sectors of CH: the nice feature of this option is that the considerations of this chapter restricted to $\delta M^4_+ \times CP_2$ generalize almost trivially. This option is beautiful because the center of mass degrees of freedom associated with the different sectors of CH would correspond to M^4 itself and its Cartesian powers.

The definition of the Kähler function requires that the many-to-one correspondence $X^3 \rightarrow X^4(X^3)$ must be replaced by a bijective correspondence in the sense that X^3_l as light-like 3-surface is unique among all its Diff^4 translates. This also allows physically preferred "gauge fixing" allowing to get rid of the mathematical complications due to Diff4 degeneracy. The internal geometry of the space-time sheet must define the preferred 3-surface X^3_l.

The realization of this vision means a considerable mathematical challenge. The effective metric 2-dimensionality of 3-dimensional light-like surfaces X^3_l of M^4 implies generalized conformal and symplectic symmetries allowing to generalize quantum gravitational holography from light like boundary so that the complexities due to the non-determinism can be taken into account properly.

3.1.2 Light like 3-D causal determinants and effective 2-dimensionality

The light like 3-surfaces X^3_l of space-time surface appear as 3-D causal determinants. Basic examples are boundaries and elementary particle horizons (parton orbits) at which Minkowskian signature of the induced metric transforms to Euclidian one. This brings in a second conformal symmetry related to the metric 2-dimensionality of the 3-D light-like 3-surface. This symmetry is analogous to TGD counterpart of the Kac Moody symmetry of string models and seems to be associated with quantum criticality implying non-uniqueness of the space-time surface with given space-like ends at boundaries of CD. Critical deformations would be Kac-Moody type transformation preserving the light-likeness of the parton orbits. The challenge is to understand the relationship of this symmetry to WCW geometry and the interaction between the two conformal symmetries.

1. Field-particle duality is realized. Light-like 3-surfaces X^3_l -generalized Feynman diagrams - correspond to the particle aspect of field-particle duality whereas the physics in the interior of space-time surface $X^4(X^3_l)$ would correspond to the field aspect. Generalized Feynman diagrams in 4-D sense could be identified as regions of space-time surface having Euclidian signature.

2. One could also say that light-like 3-surfaces X^3_l and the space-like 3-surfaces X^3 in the intersections of $X^4(X^3_l) \cap CD \times CP_2$ where the causal diamond CD is defined as the intersections of future and past directed light-cones provide dual descriptions.

3. Generalized coset construction implies that the differences of super-symplectic and Super Kac-Moody type Super Virasoro generators annihilated physical states. This construction in turn led to the realization that WCW for fixed values of zero modes - in particular the values of the induced Kähler form of $\delta M^4_\pm \times CP_2$ - allows identification as a coset space obtained by dividing the symplectic group of $\delta M^4_\pm \times CP_2$ with Kac-Moody group, whose generators vanish at $X^2 = X^3_l \times \delta M^4_\pm \times CP_2$. One can say that quantum fluctuating degrees of freedom in a very concrete sense correspond to the local variant of $S^2 \times CP_2$.

The analog of conformal invariance in the light-like direction of X_l^3 and in the light-like radial direction of $\delta M_\pm^4$ suggests that the data at either X^3 or X_l^3 should be enough to determine WCW geometry. This implies that the relevant data is contained to their intersection X^2 at least for finite regions of X^3. This is the case if the deformations of X_l^3 not affecting X^2 and preserving light likeness corresponding to zero modes or gauge degrees of freedom and induce deformations of X^3 also acting as zero modes. The outcome is effective 2-dimensionality. One must be however cautious in order to not make over-statements. The reduction to 2-D theory in global sense would trivialize the theory and the reduction to 2-D theory must takes places for finite region of X^3 only so one has in well defined sense three-dimensionality in discrete sense. A more precise formulation of this vision is in terms of hierarchy of CDs containing CDs containing.... The introduction of sub-CD:s brings in improved measurement resolution and means also that effective 2-dimensionality is realized in the scale of sub-CD only.

Experience has however taught to be extremely cautious: it could also be that in ZEO the unions of the space-like 3-surfaces at the ends of CD and of the light-like partonic orbits at which the signature of the induced metric changes are the basic objects analogous to Wilson loops. In this case the notion of effective 2-dimensionality is not so clear. Also in this case the Kac-Moody type symmetry preserving the light-likeness of partonic orbits could reduce the additional degrees of freedom to a finite number of conformal equivalence classes of partonic orbits for given pair of 3-surfaces.

One cannot over-emphasize the importance of the effective 2-dimensionality. It indeed simplifies dramatically the earlier formulas for WCW metric involving 3-dimensional integrals over $X^3 \subset M_+^4 \times CP_2$ reducing now to 2-dimensional integrals. Note that X^3 is determined by preferred extremal property of $X^4(X_l^3)$ once X_l^3 is fixed and one can hope that this mapping is one-to-one.

3.1.3 Magic properties of light cone boundary and isometries of WCW

The special conformal, metric and symplectic properties of the light cone of four-dimensional Minkowski space: δM_+^4, the boundary of four-dimensional light cone is metrically 2-dimensional(!) sphere allowing infinite-dimensional group of conformal transformations and isometries(!) as well as Kähler structure. Kähler structure is not unique: possible Kähler structures of light cone boundary are parameterized by Lobatchevski space $SO(3,1)/SO(3)$. The requirement that the isotropy group $SO(3)$ of S^2 corresponds to the isotropy group of the unique classical 3-momentum assigned to $X^4(Y^3)$ defined as a preferred extremum of Kähler action, fixes the choice of the complex structure uniquely. Therefore group theoretical approach and the approach based on Kähler action complement each other.

1. The allowance of an infinite-dimensional group of isometries isomorphic to the group of conformal transformations of 2-sphere is completely unique feature of the 4-dimensional light cone boundary. Even more, in case of $\delta M_+^4 \times CP_2$ the isometry group of δM_+^4 becomes localized with respect to CP_2! Furthermore, the Kähler structure of δM_+^4 defines also symplectic structure.

   Hence any function of $\delta M_+^4 \times CP_2$ would serve as a Hamiltonian transformation acting in both CP_2 and δM_+^4 degrees of freedom. These transformations obviously differ from ordinary local gauge transformations. This group leaves the symplectic form of $\delta M_+^4 \times CP_2$, defined as the sum of light cone and CP_2 symplectic forms, invariant. The group of symplectic transformations of $\delta M_+^4 \times CP_2$ is a good candidate for the isometry group of the WCW.

2. The approximate symplectic invariance of Kähler action is broken only by gravitational effects and is exact for vacuum extremals. If Kähler function were exactly invariant under the symplectic transformations of CP_2, CP_2 symplectic transformations wiykd correspond to zero modes having zero norm in the Kähler metric of WCW. This does not make sense since symplectic transformations of $\delta M^4 \times CP_2$ actually parameterize the quantum fluctuation degrees of freedom.

3. The groups G and H, and thus WCW itself, should inherit the complex structure of the light cone boundary. The diffeomorphims of M^4 act as dynamical symmetries of vacuum extremals. The radial Virasoro localized with respect to $S^2 \times CP_2$ could in turn act in zero

modes perhaps inducing conformal transformations: note that these transformations lead out from the symmetric space associated with given values of zero modes.

3.1.4 Symplectic transformations of $\delta M^4_+ \times CP_2$ as isometries of WCW

The symplectic transformations of $\delta M^4_+ \times CP_2$ are excellent candidates for inducing symplectic transformations of the WCW acting as isometries. There are however deep differences with respect to the Kac Moody algebras.

1. The conformal algebra of the WCW is gigantic when compared with the Virasoro + Kac Moody algebras of string models as is clear from the fact that the Lie-algebra generator of a symplectic transformation of $\delta M^4_+ \times CP_2$ corresponding to a Hamiltonian which is product of functions defined in δM^4_+ and CP_2 is sum of generator of δM^4_+-local symplectic transformation of CP_2 and CP_2-local symplectic transformations of δM^4_+. This means also that the notion of local gauge transformation generalizes.

2. The physical interpretation is also quite different: the relevant quantum numbers label the unitary representations of Lorentz group and color group, and the four-momentum labeling the states of Kac Moody representations is not present. Physical states carrying no energy and momentum at quantum level are predicted. The appearance of a new kind of angular momentum not assignable to elementary particles might shed some light to the longstanding problem of baryonic spin (quarks are not responsible for the entire spin of proton). The possibility of a new kind of color might have implications even in macroscopic length scales.

3. The central extension induced from the natural central extension associated with $\delta M^4_+ \times CP_2$ Poisson brackets is anti-symmetric with respect to the generators of the symplectic algebra rather than symmetric as in the case of Kac Moody algebras associated with loop spaces. At first this seems to mean a dramatic difference. For instance, in the case of CP_2 symplectic transformations localized with respect to δM^4_+ the central extension would vanish for Cartan algebra, which means a profound physical difference. For $\delta M^4_+ \times CP_2$ symplectic algebra a generalization of the Kac Moody type structure however emerges naturally.

   The point is that δM^4_+-local CP_2 symplectic transformations are accompanied by CP_2 local δM^4_+ symplectic transformations. Therefore the Poisson bracket of two δM^4_+ local CP_2 Hamiltonians involves a term analogous to a central extension term symmetric with respect to CP_2 Hamiltonians, and resulting from the δM^4_+ bracket of functions multiplying the Hamiltonians. This additional term could give the entire bracket of the WCW Hamiltonians at the maximum of the Kähler function where one expects that CP_2 Hamiltonians vanish and have a form essentially identical with Kac Moody central extension because it is indeed symmetric with respect to indices of the symplectic group.

The most natural option is that symplectic and Kac-Moody algebras together generate the isometry algebra and that the corresponding transformations leaving invariant the partonic 2-surfaces and their 4-D tangent space data act as gauge transformations and affect only zero modes.

3.1.5 Does the symmetric space property reduce to coset construction for Super Virasoro algebras?

The idea about symmetric space is extremely beautiful but it took a long time and several false alarms before the time was ripe for identifying the precise form of the Cartan decomposition $g = t + h$ satisfying the defining conditions

$$g = t + h \ , \quad [t,t] \subset h \ , \quad [h,t] \subset t \ . \tag{3.1.1}$$

The ultimate solution of the puzzle turned out to be amazingly simple and came only after quantum TGD was understood well enough.

WCW geometry allows two super-conformal symmetries assignable the coset space decomposition G/H for a sector of WCW with fixed values of zero moes. One can assign to the tangent space algebras g resp. h of G resp. H analogous to Kac-Moody algebras super Virasoro algebras and construct super-conformal representation as a coset representation meaning that the differences of super Virasoro generators annihilate the physical states. This obviously generalizes Goddard-Olive-Kent construction [A58].

The identification of the two algeras is not a mechanical task and has involved a lot of trial and erroring. The algebra g should be be spanned by the generators of super-symplectic algebra of light-cone boundary and by the Kac-Moody algebra acting on light-like orbits of partonic 2-surfaces. The sub-algebra h should be spanned by generators which vanish for a preferred point of WCW analogous to origin of $CP_2 = SU(3)/U(2)$. Now this point would correspond to maximum or minimum of Kähler function (no saddle points are allowed if the WCW metric has definite signature). In hindsight it is obvious that the generators of both symplectic and Kac-Moody algebras are needed to generate g and h: already the effective 2-dimensionality meaning that 4-D tangent space data of partonic surface matters requires this.

The maxima of Kähler function could correspond to this kind of points (pairs formed by 3-surfaces at different ends of CD in ZEO) and could play also an essential role in the integration over WCW by generalizing the Gaussian integration of free quantum field theories. It took quite a long time to realize that Kähler function must be identified as Kähler action for the Euclidian region of preferred extremal. Kähler action for Minkowskian regions gives imaginary contribution to the action exponential and has interpretation in terms of Morse function. This part of Kähler action can have and is expected to have saddle points and to define Hessian with signature which is not positive definite.

3.1.6 What effective 2-dimensionality and holography really mean?

Concerning the interpretation of Kac-Moody algebra there are some poorly understood points, which directly relate to what one means with holography.

1. Holography suggests that light-like 3-surfaces with fixed ends give rise to same WCW metric and the deformations of these surfaces by Kac-Moody algebra correspond to zero modes just like the interior degrees of freedom for space-like 3-surface do. The same would be true for space-like 3-surfaces at the ends of space-time surface with respect to symplectic transformations.

2. The non-trivial action of Kac-Moody algebra in the interior of X_l^3 together with effective 2-dimensionality and holography would encourage the interpretation of Kac-Moody symmetries acting trivially at X^2 as gauge symmetries. Light-like 3-surfaces having fixed partonic 2-surfaces at their ends would be equivalent physically and effective 2-dimensionality and holography would be realized modulo gauge transformations. As a matter fact, the action on WCW metric would be a change of zero modes so that one could identify it as analog of conformal scaling. The action of symplectic transformations vanishing in the interior of space-like 3-surface at the end of space-time surface affects only zero modes.

3.1.7 Attempts to identify WCW Hamiltonians

I have made several attempts to identify WCW Hamiltonians. The first two candidates referred to as magnetic and electric Hamiltonians, emerged in a relatively early stage. The third candidate is based on the formulation of quantum TGD using 3-D light-like surfaces identified as orbits of partons. The proposal is out-of-date but the most recent proposal is obtained by a very straightforward generalization from the proposal for magnetic Hamiltonians discussed below.

Magnetic Hamiltonians

Assuming that the elements of the radial Virasoro algebra of δM^4_+ have zero norm, one ends up with an explicit identification of the symplectic structures of WCW. There is almost unique identification for the symplectic structure. WCW counterparts of $\delta M^4 \times CP_2$ Hamiltonians are defined by the generalized signed and and unsigned Kähler magnetic fluxes

$$
\begin{aligned}
Q_m(H_A,X^2) &= Z\int_{X^2} H_A J\sqrt{g_2}d^2x \ \ ,\\
Q_m^+(H_A,r_M) &= Z\int_{X^2} H_A|J|\sqrt{g_2}d^2x \ \ ,\\
J &\equiv \epsilon^{\alpha\beta}J_{\alpha\beta} \ \ .
\end{aligned}
$$

H_A is CP_2 Hamiltonian multiplied by a function of coordinates of light cone boundary belonging to a unitary representation of the Lorentz group. Z is a conformal factor depending on symplectic invariants. The symplectic structure is induced by the symplectic structure of CP_2.

The most general flux is superposition of signed and unsigned fluxes Q_m and Q_m^+.

$$
Q_m^{\alpha,\beta}(H_A,X^2) = \alpha Q_m(H_A,X^2) + \beta Q_m^+(H_A,X^2) \ \ .
$$

Thus it seems that symmetry arguments fix the form of the WCW metric apart from the presence of a conformal factor Z multiplying the magnetic flux and the degeneracy related to the signed and unsigned fluxes.

Generalization

The generalization for definition WCW super-Hamiltonians defining WCW gamma matrices is discussed in detail in [K80] feeds in the wisdom gained about preferred extremals of Kähler action and solutions of the modified Dirac action: in particular, about their localization at string worlds sheets (right handed neutrino could be an exception).

The basic formulas generalize as such: the only modification is that the super-Hamiltonian of $\delta M^4_\pm \times CP_2$ at given point of partonic 2-surface is replaced with the Noether super charge associated with the Hamiltonian obtained by integrating the 1-D super current over string emanating from partonic 2-surface. Right handed neutrino spinor is replaced with any mode of the modified Dirac operator localized at string world sheet in the case of Kac-Moody sub-algebra of super-symplectic algebra corresponding to symplectic isometries at light-cone boundary and CP_2. In the case of right- handed neutrino one obtains entire super-symplectic algebra and the direct sum of these algebras is used to construct physical states. This step is analogous to the replacement of point like particle with string.

The resulting super Hamiltonians define WCW gamma matrices. They are labelled by two conformal weights. The first one is the conformal weight associated with the light-like coordinate of $\delta M^4_\pm \times CP_2$. Second conformal weight is associated with the spinor mode and the coordinate along stringy curve. One cannot exclude the possibility that the two conformal weights have same value. Effective 2-dimensionality and the fact that string coordinate cannot be always radial light-like coordinate would suggest that they are independent.

The presence of two conformal weights is in accordance with the idea that a generalization of conformal invariance to 4-D situation is in question. If Yangian extension of conformal symmetries is possible and would bring an additional integer n telling the degree of multilocality of Yangian generators defined as the number of partonic 2-surfaces at which the generator acts. For conformal algebra degree of multilocality equals to $n = 1$.

3.1.8 For the reader

Few words about the representation of ideas are in order. For a long time the books about TGD served as kind of lab note books - a bottom-up representation providing kind of a ladder making clear the evolution of ideas. This led gradually to a rather chaotic situation in which it was difficult for me to control the internal consistency and for the possible reader to distinguish between the big ideas and ad hoc guesses, most of them related to the detailed realization of big visions. Therefore I have made now and the the decision to clean up a lot of the ad hoc stuff. In this process I have also changed the representation so that it is more top-down and tries to achieve over-all views.

There are several visions about what TGD is and I have worked hardly to achieve a fusion of these visions. Hence simple linear representation in which reader climbs to a tree of wisdom is impossible. I must summarize overall view from the beginning and refer to the results deduced in chapters towards the end of the book and also to ideas discussed in other books. For instance, the construction of WCW ("world of classical worlds" (WCW)) spinor structure discussed in chapters [K9, K18, K69] provides the understanding necessary to make the construction of configuration space geometry more detailed. Also number theoretical vision discussed in another book [K49] is necessary. Somehow it seems that a graphic representation emphasizing visually the big picture should be needed to make the representation more comprehensible.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L13]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L14]. The topics relevant to this chapter are given by the following list.

- Geometry of WCW [L20]
- Zero Energy Ontology (ZEO) [L44]
- Symmetries of WCW [L34]
- TGD as ATQFT [L36]
- Vacuum functional in TGD [L40]

3.2 How to generalize the construction of WCW geometry to take into account the classical non-determinism?

If the imbedding space were $H_+ = M^4_+ \times CP_2$ and if Kähler action were deterministic, the construction of WCW geometry reduces to $\delta M^4_+ \times CP_2$. Thus in this limit quantum holography principle [B12, B23] would be satisfied also in TGD framework and actually reduce to the general coordinate invariance. The classical non-determinism of Kähler action however means that this construction is not quite enough and the challenge is to generalize the construction.

3.2.1 Quantum holography in the sense of quantum gravity theories

In string theory context quantum holography is more or less synonymous with Maldacena conjecture Maldacena which (very roughly) states that string theory in Anti-de-Sitter space AdS is equivalent with a conformal field theory at the boundary of AdS. In purely quantum gravitational context [B12] , quantum holography principle states that quantum gravitational interactions at high energy limit in AdS can be described using a topological field theory reducing to a conformal (and non-gravitational) field theory defined at the *time like* boundary of the AdS. Thus the time like boundary plays the role of a dynamical hologram containing all information about correlation functions of $d+1$ dimensional theory. This reduction also conforms with the fact that black hole entropy is proportional to the horizon area rather than the volume inside horizon.

Holography principle reduces to general coordinate invariance in TGD. If the action principle assigning space-time surface to a given 3-surface X^3 at light cone boundary were completely deterministic, four-dimensional general coordinate invariance would reduce the construction of the configuration geometry for the space of 3-surfaces in $M^4_+ \times CP_2$ to the construction of the geometry at the boundary of WCW consisting of 3-surfaces in $\delta M^4_+ \times CP_2$ (moment of big bang). Also the quantum theory would reduce to the boundary of the future light cone.

The classical non-determinism of Kähler action however implies that quantum holography in this strong form fails. This is very desirable from the point of view of both physics and consciousness theory. Classical determinism would also mean that time would be lost in TGD as it is lost in GRT. Classical non-determinism is also absolutely essential for quantum consciousness and makes possible conscious experiences with contents localized into finite time interval despite the fact that quantum jumps occur between WCW spinor fields defining what I have used to call quantum histories. Classical non-determinism makes it also possible to generalize quantum-classical correspondence in the sense that classical non-determinism at the space-time level provides correlate for quantum non-determinism. The failure of classical determinism is a difficult challenge for the construction of WCW geometry. One might however hope that the notion of quantum holography generalizes.

3.2.2 How the classical determinism fails in TGD?

One might hope that determinism in a generalized sense might be achieved by generalizing the notion of 3-surface by allowing unions of space-like 3-surfaces with time like separations with very strong but not complete correlations between the space-like 3-surfaces. In this case the non-determinism would mean that the 3-surfaces Y^3 at light cone boundary correspond to at most enumerable number of preferred extremals $X^4(Y^3)$ of Kähler action so that one would get finite or at most enumerably infinite number of replicas of a given WCW region and the construction would still reduce to the light cone boundary.

1. This is probably quite too simplistic view. Any 4-surface which has CP_2 projection which belongs to so called Lagrange manifold of CP_2 having by definition vanishing induced Kähler form is vacuum extremal. Thus there is an infinite variety of 6-dimensional sub-manifolds of H for which all extremals of Kähler action are vacua.

2. CP_2 type vacuum extremals are different since they possess non-vanishing Kähler form and Kähler action. They are identifiable as classical counterparts of elementary particles have M^4_+ projection which is a random light like curve (this in fact gives rise to conformal invariance identifiable as counterpart of quaternion conformal invariance). Thus there are good reasons to suspect that classical non-determinism might destroy the dream about complete reduction to the light cone boundary.

3. The wormhole contacts connecting different space-time sheets together can be seen as pieces of CP_2 type extremals and one expects that the non-determinism is still there and that the metrically 2-dimensional elementary particle horizons (light like 3-surfaces of H surrounding wormhole contacts and having time-like M^4_+ projection) might be a crucial element in the understanding of quantum TGD. The non-determinism of CP_2 type extremals is absolutely crucial for the ordinary elementary particle physics. It seems that the conformal symmetries responsible for the ordinary elementary particle quantum numbers acting in these degrees of freedom do not contribute to the WCW metric line element.

4. The possibility of space-time sheets with a negative time orientation with ensuing negative sign of classical energy is a further blow against δM^4_+ reductionism. Space-time sheets can be created as pairs of positive and negative energy space-time sheet from vacuum and this forces to modify radically the ontology of physics. Crossing symmetry allows to interpret particle reactions as a creation of zero energy states from vacuum, and the identification of the gravitational energy as the difference between positive and negative energies of matter supports the view that the net inertial (conserved Poincare-) energy of the universe vanishes both in quantal and classical sense. This option resolves unpleasant questions about net conserved quantum numbers of Universe, and provides an elegant interpretation of the vacuum extremals as correlates for systems with vanishing Poincare energy. This option is the only possible alternative from the point of view of TGD inspired cosmology where Robertson-Walker metrics are vacuum extremals with respect to inertial energy. In particular, super-symplectic invariance transforms to a fundamental symmetry of elementary particle physics besides the conformal symmetry associated with 3-D light like causal determinants which means a dramatic departure from string models unless it is somehow equivalent with the super-symplectic symmetry.

The treatment of the non-determinism in a framework in which the prediction of time evolution is seen as initial value problem, seems to be difficult. Also the notion of WCW becomes a messy concept. Zero energy ontology changes the situation completely. Light-like 3-surfaces become representations of generalized Feynman diagrams and brings in the notion of finite time resolution. One obtains a direct connection with the concepts of quantum field theory with path integral with cutoff replaced with a sum over various preferred extremals with cutoff in time resolution.

3.2.3 The notions of imbedding space, 3-surface, and configuration space

The notions of imbedding space, 3-surface (and 4-surface), and configuration space ("world of classical worlds", WCW) are central to quantum TGD. The original idea was that 3-surfaces are space-like 3-surfaces of $H = M^4 \times CP_2$ or $H = M^4_+ \times CP_2$, and WCW consists of all possible 3-surfaces in H. The basic idea was that the definition of Kähler metric of WCW assigns to each X^3 a unique space-time surface $X^4(X^3)$ allowing in this manner to realize general coordinate invariance. During years these notions have however evolved considerably. Therefore it seems better to begin directly from the recent picture.

The notion of imbedding space

Two generalizations of the notion of imbedding space were forced by number theoretical vision [K51, K52, K50] .

1. p-Adicization forced to generalize the notion of imbedding space by gluing real and p-adic variants of imbedding space together along rationals and common algebraic numbers. The generalized imbedding space has a book like structure with reals and various p-adic number fields (including their algebraic extensions) representing the pages of the book.

2. With the discovery of zero energy ontology [K9, K13] it became clear that the so called causal diamonds (CDs) interpreted as intersections $M^4_+ \cap M^4_-$ of future and past directed light-cones of $M^4 \times CP_2$ define correlates for the quantum states. The position of the "lower" tip of CD characterizes the position of CD in H. If the temporal distance between upper and lower tip of CD is quantized power of 2 multiples of CP_2 length, p-adic length scale hypothesis [K33] follows as a consequence. The upper *resp.* lower light-like boundary $\delta M^4_+ \times CP_2$ *resp.* $\delta M^4_- \times CP_2$ of CD can be regarded as the carrier of positive *resp.* negative energy part of the state. All net quantum numbers of states vanish so that everything is creatable from vacuum. Space-time surfaces assignable to zero energy states would would reside inside $CD \times CP_2$s and have their 3-D ends at the light-like boundaries of $CD \times CP_2$. Fractal structure is present in the sense that CDs can contains CDs within CDs, and measurement resolution dictates the length scale below which the sub-CDs are not visible.

3. The realization of the hierarchy of Planck constants [K17] led to a further generalization of the notion of imbedding space. Generalized imbedding space is obtained by gluing together Cartesian products of singular coverings and factor spaces of CD and CP_2 to form a book like structure. The particles at different pages of this book behave like dark matter relative to each other. This generalization also brings in the geometric correlate for the selection of quantization axes in the sense that the geometry of the sectors of the generalized imbedding space with non-standard value of Planck constant involves symmetry breaking reducing the isometries to Cartan subalgebra. Roughly speaking, each CD and CP_2 is replaced with a union of CDs and CP_2s corresponding to different choices of quantization axes so that no breaking of Poincare and color symmetries occurs at the level of entire WCW.

4. The construction of quantum theory at partonic level brings in very important delicacies related to the Kähler gauge potential of CP_2. Kähler gauge potential must have what one might call pure gauge parts in M^4 in order that the theory does not reduce to mere topological quantum field theory. Hence the strict Cartesian product structure $M^4 \times CP_2$ breaks down in a delicate manner. These additional gauge components -present also in CP_2- play key role in the model of anyons, charge fractionization, and quantum Hall effect [K37] .

The notions of 3-surface and space-time surface

The question what one exactly means with 3-surface turned out to be non-trivial.

1. The original identification of 3-surfaces was as arbitrary space-like 3-surfaces subject to Equivalence implied by General Coordinate Invariance. There was a problem related to the realization of General Coordinate Invariance since it was not at all obvious why the preferred extremal $X^4(Y^3)$ for Y^3 at $X^4(X^3)$ and Diff^4 related X^3 should satisfy $X^4(Y^3) = X^4(X^3)$.

2. Much later it became clear that light-like 3-surfaces have unique properties for serving as basic dynamical objects, in particular for realizing the General Coordinate Invariance in 4-D sense (obviously the identification resolves the above mentioned problem) and understanding

the conformal symmetries of the theory. On basis of these symmetries light-like 3-surfaces can be regarded as orbits of partonic 2-surfaces so that the theory is locally 2-dimensional. It is however important to emphasize that this indeed holds true only locally. At the level of WCW metric this means that the components of the Kähler form and metric can be expressed in terms of data assignable to 2-D partonic surfaces. It is however essential that information about normal space of the 2-surface is needed.

3. At some stage came the realization that light-like 3-surfaces can have singular topology in the sense that they are analogous to Feynman diagrams. This means that the light-like 3-surfaces representing lines of Feynman diagram can be glued along their 2-D ends playing the role of vertices to form what I call generalized Feynman diagrams. The ends of lines are located at boundaries of sub-CDs. This brings in also a hierarchy of time scales: the increase of the measurement resolution means introduction of sub-CDs containing sub-Feynman diagrams. As the resolution is improved, new sub-Feynman diagrams emerge so that effective 2-D character holds true in discretized sense and in given resolution scale only.

4. A further complication relates to the hierarchy of Planck constants forcing to generalize the notion of imbedding space and also to the fact that for non-standard values of Planck constant there is symmetry breaking due to preferred plane M^2 preferred homologically trivial geodesic sphere of CP_2 having interpretation as geometric correlate for the selection of quantization axis. For given sector of CH this means union over choices of this kind.

The basic vision forced by the generalization of General Coordinate Invariance has been that space-time surfaces correspond to preferred extremals $X^4(X^3)$ of Kähler action and are thus analogous to Bohr orbits. Kähler function $K(X^3)$ defining the Kähler geometry of the world of classical worlds would correspond to the Kähler action for the preferred extremal. The precise identification of the preferred extremals actually has however remained open.

The obvious but rather ad hoc guess motivated by physical intuition was that preferred extremals correspond to the absolute minima of Kähler action for space-time surfaces containing X^3. This choice has some nice implications. For instance, one can develop an argument for the existence of an infinite number of conserved charges. If X^3 is light-like surface- either light-like boundary of X^4 or light-like 3-surface assignable to a wormhole throat at which the induced metric of X^4 changes its signature- this identification circumvents the obvious objections. This option however failed to have a direct analog in the p-adic sectors of the world of classical worlds (WCW). The reason is that minimization does not make sense for the p-adic valued counterpart of Kähler action since it is not even well-defined although the field equations make sense p-adically. Therefore, if absolute minimization makes sense it must have expression as purely algebraic conditions.

For this reason it is better to talk just about preferred extremals of Kähler action and accept as the fact that there are several proposals for what this notion could mean. For instance, one can consider the identification of space-time surface as quaternionic sub-manifold meaning that tangent space of space-time surface can be regarded as quaternionic sub-manifold of complexified octonions defining tangent space of imbedding space. One manner to define "quaternionic sub-manifold" is by introducing octonionic representation of imbedding space gamma matrices identified as tangent space vectors. It must be also assumed that the tangent space contains a preferred complex (commutative) sub-space at each point and defining an integrable distribution having identification as string world sheet (also slicing of space-time sheet by string world sheets can be considered). Associativity and commutativity would define the basic dynamical principle. A closely related approach is based on so called Hamilton-Jacobi structure [K5] defining also this kind of slicing and the approaches could be equivalent. A further approach is based on the identification of preferred extremal property as quantum criticality [K5].

The notion of number theoretical compactification led to important progress in the understanding of the preferred extremals and the conjectures were consistent with what is known about the known extremals.

1. The conclusion was that one can assign to the 4-D tangent space $T(X^4(X_l^3)) \subset M^8$ a subspace $M^2(x) \subset M^4$ having interpretation as the plane of non-physical polarizations. This in the case that the induced metric has Minkowskian signature. If not, and if co-hyper-quaternionic surface is in question, similar assigned should be possible in normal space. This means a close

connection with super string models. Geometrically this would mean that the deformations of 3-surface in the plane of non-physical polarizations would not contribute to the line element of WCW. This is as it must be since complexification does not make sense in M^2 degrees of freedom.

2. In number theoretical framework $M^2(x)$ has interpretation as a preferred hyper-complex sub-space of hyper-octonions defined as 8-D subspace of complexified octonions with the property that the metric defined by the octonionic inner product has signature of M^8. The condition $M^2(x) \subset T(X^4(X_l^3)))$ in principle fixes the tangent space at X_l^3, and one has good hopes that the boundary value problem is well-defined and could fix $X^4(X^3)$ at least partially as a preferred extremal of Kähler action. This picture is rather convincing since the choice $M^2(x) \subset M^4$ plays also other important roles.

3. At the level of H the counterpart for the choice of $M^2(x)$ seems to be following. Suppose that $X^4(X_l^3)$ has Minkowskian signature. One can assign to each point of the M^4 projection $P_{M^4}(X^4(X_l^3))$ a sub-space $M^2(x) \subset M^4$ and its complement $E^2(x)$, and the distributions of these planes are integrable and define what I have called Hamilton-Jacobi coordinates which can be assigned to the known extremals of Kähler with Minkowskian signature. This decomposition allows to slice space-time surfaces by string world sheets and their 2-D partonic duals. Also a slicing to 1-D light-like surfaces and their 3-D light-like duals Y_l^3 parallel to X_l^3 follows under certain conditions on the induced metric of $X^4(X_l^3)$. This decomposition exists for known extremals and has played key role in the recent developments. Physically it means that 4-surface (3-surface) reduces effectively to 3-D (2-D) surface and thus holography at space-time level.

4. The weakest form of number theoretic compactification [K52] states that light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^8$, where $X^4(X^3)$ hyper-quaternionic surface in hyper-octonionic M^8 can be mapped to light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^4 \times CP_2$, where $X^4(X^3)$ is now preferred extremum of Kähler action. The natural guess is that $X^4(X^3) \subset M^8$ is a preferred extremal of Kähler action associated with Kähler form of E^4 in the decomposition $M^8 = M^4 \times E^4$, where M^4 corresponds to hyper-quaternions. The conjecture would be that the value of the Kähler action in M^8 is same as in $M^4 \times CP_2$: in fact that 2-surface would have identical induced metric and Kähler form so that this conjecture would follow trivial. $M^8 - H$ duality would in this sense be Kähler isometry.

The study of the modified Dirac equation meant further steps of progress and lead to a rather detailed view about what preferred extremals are.

1. The detailed construction of the generalized eigen modes of the modified Dirac operator D_K associated with Kähler action [K9] relies on the vision that the generalized eigenvalues of this operator code for information about preferred extremal of Kähler action. The view about TGD as almost topological QFT is realized if the eigenmodes correspond to the solutions of D_K, which are effectively 3-dimensional. Otherwise almost topological QFT property would require Chern-Simons action alone and this choice is definitely un-physical. The first guess was that the eigenmodes are restricted to X_l^3 and therefore analogous to spinorial shock waves. As I realized that number theoretical compactification requires the slicing of $X^4(X_l^3)$ by light-like 3-surfaces Y_l^3 parallel to X_l^3, it became clear that super-conformal gauge invariance with respect to the coordinate labeling the slices is a more natural manner to realized effective 3-dimensionality and guarantees that Y_l^3 is gauge equivalent with X_l^3 (General Coordinate Invariance).

2. The eigen modes of the modified Dirac operator D_K have the defining property that they are localized in regions of X_l^3, where the induced Kähler gauge field is non-vanishing. This guarantees that the number of generalized eigen modes is finite so that Dirac determinant is also finite and algebraic number if eigenvalues are algebraic numbers, and therefore makes sense also in p-adic context although Kähler action itself does not make sense p-adically.

3. The construction of WCW geometry in terms of modified Dirac action strengthens also the boundary conditions to the condition that there exists space-time coordinates in which the

induced CP_2 Kähler form and induced metric satisfy the conditions $J_{ni} = 0$, $g_{ni} = 0$ hold at X_l^3. One could say that at X_l^3 situation is static both metrically and for the Maxwell field defined by the induced Kähler form.

4. The final step in the rapid evolution of ideas that too place during three months - at least I hope so since I do not want to continue this updating endlessly - was the realization that the introduction of imaginary CP breaking instanton part to the Kähler action is possible and also necessary if one wants a stringy variant of Feynman rules. Imaginary part does not contribute to the WCW metric. This enriches the spectrum of the modified Dirac operator with an infinite number of conformal excitations breaking the effective 2-dimensionality of 3-surfaces and exact holography. Conformal excitations make possible stringy Feynman diagrammatics [K12] . A breaking of effective 3-dimensionality of space-time surface comes through the non-determinism of Kähler action which indeed is the mechanism breaking the effective 2-dimensionality. Dirac determinant can be defined in terms of zeta function regularization using Riemann Zeta. Finite measurement resolution realized in terms of braids defined on basis of purely physical criteria however forces a cutoff in conformal weight and finiteness so that number theoretical universality is not lost.

5. This picture relying crucially on the the slicing of $X^4(X^3)$ did not yet fix the definition of preferred extremals analytically at the level of field equations. The next step of progress was the realization that the requirement that the conservation of the Noether currents associated with the modified Dirac equation requires that the second variation of the Kähler action vanishes. In strongest form this condition would be satisfied for all variations and in weak sense only for those defining dynamical symmetries. The interpretation is as space-time correlate for quantum criticality and the vacuum degeneracy of Kähler action makes the criticality plausible. A generalization of the ideas of the catastrophe theory to infinite-dimensional context results [K22] . These conditions make sense also in p-adic context and have a number theoretical universal form.

Although the details of this vision might change it can be defended by its ability to fuse together all great visions about quantum TGD. In the sequel the considerations are restricted to 3-surfaces in $M^4_+ \times CP_2$. The basic outcome is that Kähler metric is expressible using the data at partonic 2-surfaces $X^2 \subset \delta M^4_+ \times CP_2$. The generalization to the actual physical situation requires the replacement of $X^2 \subset \delta M^4_\pm \times CP_2$ with unions of partonic 2-surfaces located at light-like boundaries of CDs and sub-CDs.

The notion of WCW

From the beginning there was a problem related to the precise definition of WCW ("world of classical worlds" (WCW)). Should one regard CH as the space of 3-surfaces of $M^4 \times CP_2$ or $M^4_+ \times CP_2$ or perhaps something more delicate.

1. For a long time I believed that the question "M^4_+ or M^4?" had been settled in favor of M^4_+ by the fact that M^4_+ has interpretation as empty Roberson-Walker cosmology. The huge conformal symmetries assignable to $\delta M^4_+ \times CP_2$ were interpreted as cosmological rather than laboratory symmetries. The work with the conceptual problems related to the notions of energy and time, and with the symmetries of quantum TGD, however led gradually to the realization that there are strong reasons for considering M^4 instead of M^4_+.

2. With the discovery of zero energy ontology it became clear that the so called causal diamonds (CDs) define excellent candidates for the fundamental building blocks of WCW or "world of classical worlds" (WCW). The spaces $CD \times CP_2$ regarded as subsets of H defined the sectors of WCW.

3. This framework allows to realize the huge symmetries of $\delta M^4_\pm \times CP_2$ as isometries of WCW. The gigantic symmetries associated with the $\delta M^4_\pm \times CP_2$ are also laboratory symmetries. Poincare invariance fits very elegantly with the two types of super-conformal symmetries of TGD. The first conformal symmetry corresponds to the light-like surfaces $\delta M^4_\pm \times CP_2$ of the imbedding space representing the upper and lower boundaries of CD. Second conformal

symmetry corresponds to light-like 3-surface X_l^3, which can be boundaries of X^4 and light-like surfaces separating space-time regions with different signatures of the induced metric. This symmetry is identifiable as the counterpart of the Kac Moody symmetry of string models.

A rather plausible conclusion is that WCW (WCW) is a union of WCWs associated with the spaces $CD \times CP_2$. CDs can contain CDs within CDs so that a fractal like hierarchy having interpretation in terms of measurement resolution results. Since the complications due to p-adic sectors and hierarchy of Planck constants are not relevant for the basic construction, it reduces to a high degree to a study of a simple special case $\delta M^4_+ \times CP_2$.

A further piece of understanding emerged from the following observations.

1. The induced Kähler form at the partonic 2-surface X^2 - the basic dynamical object if holography is accepted- can be seen as a fundamental symplectic invariant so that the values of $\epsilon^{\alpha\beta}J_{\alpha\beta}$ at X^2 define local symplectic invariants not subject to quantum fluctuations in the sense that they would contribute to the WCW metric. Hence only induced metric corresponds to quantum fluctuating degrees of freedom at WCW level and TGD is a genuine theory of gravitation at this level.

2. WCW can be divided into slices for which the induced Kähler forms of CP_2 and $\delta M^4_\pm$ at the partonic 2-surfaces X^2 at the light-like boundaries of CDs are fixed. The symplectic group of $\delta M^4_\pm \times CP_2$ parameterizes quantum fluctuating degrees of freedom in given scale (recall the presence of hierarchy of CDs).

3. This leads to the identification of the coset space structure of the sub-WCW associated with given CD in terms of the generalized coset construction for super-symplectic and super Kac-Moody type algebras (symmetries respecting light-likeness of light-like 3-surfaces). WCW in quantum fluctuating degrees of freedom for given values of zero modes can be regarded as being obtained by dividing symplectic group with Kac-Moody group. Equivalently, the local coset space $S^2 \times CP_2$ is in question: this was one of the first ideas about WCW which I gave up as too naive!

4. Generalized coset construction and coset space structure have very deep physical meaning since they realize Equivalence Principle at quantum level. Contrary to the original belief, this construction does not provide a realization of Equivalence Principle at quantum level. The proper realization of EP at quantum level seems to be based on the identification of classical Noether charges in Cartan algebra with the eigenvalues of their quantum counterparts assignable to Kähler-Dirac action. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to M^4 with effective metric satisfying Einstein's equations as a reflection of the underlying Poincare invariance.

3.2.4 The treatment of non-determinism of Kähler action in zero energy ontology

The non-determinism of Kähler action means that the reduction of the construction of WCW geometry to the light cone boundary fails. Besides degeneracy of the preferred extrema of Kähler action, the non-determinism should manifest itself as a presence of causal determinants also other than light cone boundary.

One can imagine two kinds of causal determinants.

1. Elementary particle horizons and light-like boundaries $X_l^3 \subset X^4$ of 4-surfaces representing wormhole throats act as causal determinants for the space-time dynamics defined by Kähler action. The boundary values of this dynamics have been already considered.

2. At imbedding space level causal determinants correspond to light like CD forming a fractal hierarchy of CDs within CDs. These causal determinants determine the dynamics of zero energy states having interpretation as pairs of initial and final states in standard quantum theory.

The manner to treat the classical non-determinism would be roughly following.

1. The replacement of space-like 3-surface X^3 with X_l^3 transforms initial value problem for X^3 to a boundary value problem for X_l^3. In principle one can also use the surfaces $X^3 \subset \delta CD \times CP_2$ if X_l^3 fixes $X^4(X_l^3)$ and thus X^3 uniquely. For years an important question was whether both X^3 and X_l^3 contribute separately to WCW geometry or whether they provide descriptions, which are in some sense dual. This lead to the notion of 7-3 duality and I even considered the possibility that $\delta M^4_+ \times CP_2$ could be replaced with a more general surface $X_l^3 \times CP_2$ allowing also generalized symplectic and conformal symmetries. 7-3 duality is not a good term since the actual duality actually relates descriptions based on space-like 3-surfaces X^3 and light-like 3-surfaces X_l^3. Hence it seems that the proper place for 7-3 duality is in paper basked.

2. Only Super-Kac-Moody type conformal algebra makes sense in the interior of X_l^3. In the 2-D intersections of X_l^3 with the boundary of causal diamond (CD) defined as intersection of future and past directed light-cones super-symplectic algebra makes sense. This implies effective two-dimensionality which is broken by the non-determinism represented using the hierarchy of CDs meaning that the data from these 2-D surfaces and their normal spaces at boundaries of CDs in various scales determine the WCW metric.

3. An important question has been whether Kac-Moody and super-symplectic algebras provide descriptions which are dual in some sense. At the level of Super-Virasoro algebras duality seems to be satisfied in the sense of generalized coset construction meaning that the differences of Super Virasoro generators of super-symplectic and super Kac-Moody algebras annihilate physical states. Among other things this means that four-momenta assignable to the two Super Virasoro representations are identical. T he interpretation is in terms of a generalization of Equivalence Principle [K9, K13] . This gives also a justification for p-adic thermodynamics applying only to Super Kac-Moody algebra.

4. Light-like 3-surfaces can be regarded also as generalized Feynman diagrams. The finite length resolution mean means also a cutoff in the number of generalized Feynman diagrams and this number remains always finite if the light-like 3-surfaces identifiable as maxima of Kähler function correspond to the diagrams. The finiteness of this number is also essential for number theoretic universality since it guarantees that the elements of M-matrix are algebraic numbers if momenta and other quantum numbers have this property. The introduction of new sub-CDs means also introduction of zero energy states in corresponding time scale.

5. The notion of finite measurement resolution expressed in terms of hierarchy of CDs within CDs is important for the treatment of classical non-determinism. In a given resolution the non-determinism of Kähler action remains invisible below the time scale assigned to the smallest CDs. One could also say that complete non-determinism characterized in terms path integral with cutoff is replaced in TGD framework with the partial failure of classical non-determinism leading to generalized Feynman diagrams. This gives rise to to discrete coupling constant evolution and avoids the mathematical ill-definedness and infinities plaguing path integral formalism since the functional integral over 3-surfaces is well defined.

6. Dirac determinant defining vacuum functional is assumed to correspond to exponent of Kähler action for its preferred extremal. Dirac determinant is defined as a product of finite number of eigenvalues of the transverse part $D_K(X^2)$ of the modified Dirac operator D_K assumed to have decomposition $D_K = D_K(X^2) + D_K(Y^2)$ reflecting the dual slicings of X^4 to string world sheets Y^2 and partonic 2-surfaces X^2. The existence of the slicing is supported by the properties of known extremals of Kähler action and strongly suggested by number theoretical compactification, and it implies among other things dimensional reduction to Minkowskian string model like theory and its Euclidian equivalent allowing to understand how Equivalence Principle is realized at space-time level. Finite number for the eigenvalues raises even hope that in a given resolution the functional integral reduces to Gaussian integral over a finite-dimensional space of logarithms of eigenvalues.

7. One can ask why Kähler action and playing with all these delicacies related to the failure of complete determinism. After all, one could formally replace Kähler action with 4-volume as in brane models. Space-time surfaces would be minimal surfaces and Dirac operator

would be standard Dirac operator for the induced metric. Dirac determinant would however become infinite since the modes would not be anymore analogs of cyclotron states necessarily localized to a finite region of X_l^3. Recall that for Kähler action X_l^3 indeed decomposes into patches inside with induced Kähler form is non-vanishing and Dirac determinant defining the exponent of Kähler function is well-defined and finite without any regularization procedure. Hence Kähler action is completely unique.

3.2.5 Category theory and WCW geometry

Due the effects caused by the classical non-determinism even classical TGD universes are very far from simple Cartesian clockworks, and the understanding of the general structure of WCW is a formidable challenge. Category theory is a branch of mathematics which is basically a theory about universal aspects of mathematical structures. Thus category theoretical thinking might help in disentangling the complexities of WCW geometry and the basic ideas of category theory are discussed in this spirit and as an innocent layman. It indeed turns out that the approach makes highly non-trivial predictions.

In zero energy ontology the effects of non-determinism are taken into account in terms of causal diamonds forming a hierarchical fractal structure. One must allow also the unions of CDs, CDs within CDs, and probably also overlapping of CDs, and there are good reasons to expert that CDs and corresponding algebraic structures could define categories. If one does not allow overlapping CDs then set theoretic inclusion map defines a natural arrow. If one allows both unions and intersections then CDs would form a structure analogous to the set of open sets used in set theoretic topology. One could indeed see CDs (or rather their Cartesian products with CP_2) as analogs of open sets in Minkowskian signature.

So called ribbon categories seem to be tailor made for the formulation of quantum TGD and allow to build bridge to topological and conformal field theories. This discussion based on standard ontology. In [K8] rather detailed category theoretical constructions are discussed. Important role is played by the notion of operad operad,operads : this structure can be assigned with both generalized Feynman diagrams and with the hierarchy of symplectic fusion algebras realizing symplectic analogs of the fusion rules of conformal field theories.

3.3 Identification of the symmetries and coset space structure of WCW

In this section the identification of the isometry group of the configuration ("world of classical worlds" or briefly WCW) will be discussed at general level.

3.3.1 Reduction to the light cone boundary

The reduction to the light cone boundary would occur exactly if Kähler action were strictly deterministic. This is not the case but it is possible to generalize the construction at light cone boundary to the general case if causal diamonds define the basic structural units of the WCW.

Old argument

The identification of WCW follows as a consequence of 4-dimensional Diff invariance. The right question to ask is the following one. How could one coordinatize the physical(!) vibrational degrees of freedom for 3-surfaces in Diff^4 invariant manner: coordinates should have same values for all Diff^4 related 3-surfaces belonging to the orbit of X^3? The answer is following:

1. Fix some 3-surface (call it Y^3) on the orbit of X^3 in Diff^4 invariant manner.
2. Use as WCW coordinates of X^3 and all its diffeomorphs the coordinates parameterizing small deformations of Y^3. This kind of replacement is physically acceptable since metrically the WCW is equivalent with $Map/Diff^4$.
3. Require that the fixing procedure is Lorentz invariant, where Lorentz transformations in question leave light M^4_+ invariant and thus act as isometries.

The simplest choice of Y^3 is the intersection of the orbit of 3-surface (X^4) with the set $\delta M^4_+ \times CP_2$, where δM^4_+ denotes the boundary of the light cone (moment of big bang):

$$Y^3 = X^4 \cap \delta M^4_+ \times CP_2 \tag{3.3.1}$$

Lorentz invariance allows also the choice $X \times CP_2$, where X corresponds to the hyperboloid $a = \sqrt{(m^0)^2 - r_M^2} = constant$ but only the proposed choice ($a = 0$) leads to a natural complexification in M^4 degrees of freedom. This choice is also cosmologically very natural and completely analogous to the quantum gravitational holography of string theories.

WCW has a fiber space structure. Base space consists of 3-surfaces $Y^3 \subset \delta M^4_+ \times CP_2$ and fiber consists of 3-surfaces on the orbit of Y^3 , which are Diff^4 equivalent with Y^3. The distance between the surfaces in the fiber is vanishing in WCW metric. An elegant manner to avoid difficulties caused by Diff^4 degeneracy in WCW integration is to *define* integration measure as integral over the reduced WCW consisting of 3-surfaces Y^3 at the light cone boundary.

Situation is however quite not so simple. The vacuum degeneracy of Kähler action suggests strongly classical non-determinism so that there are several, possibly, infinite number of preferred extremals $X^4(Y^3)$ associated with given Y^3 on light cone boundary. This implies additional degeneracy.

One might hope that the reduced WCW could be replaced by its covering space so that given Y^3 corresponds to several points of the covering space and WCW has many-sheeted structure. Obviously the copies of Y^3 have identical geometric properties. WCW integral would decompose into a sum of integrals over different sheets of the reduced WCW. Note that WCW spinor fields are in general different on different sheets of the reduced WCW.

Even this is probably not enough: it is quite possible that all light like surfaces of M^4 possessing Hamilton Jacobi structure (and thus interpretable as light fronts) are involved with the construction of the WCW geometry. Because of their metric two-dimensionality the proposed construction should generalize. This would mean that WCW geometry has also local laboratory scale aspects and that the general ideas might allow testing.

New version of the argument

The above summary was the basic argument for two decades ago. A more elegant formulation would in terms of light-like 3-surfaces connecting the boundaries of causal diamond taken as basic geometric objects and identified as generalized Feynman diagrams so that they are singular as manifolds at the vertices.

If both formulations are required to be correct, the only conclusion is that effective 2-dimensionality must hold true in the scale of given CD. In other words, the intersection $X^2 = X^3_l \cap X^3$ at the boundary of CD is effectively the basic dynamical unit. The failure of strict non-determinism however forces to introduce entire hierarchy of CDs responsible also for coupling constant evolution defined in terms of the measurement resolution identified as the size of the smallest CD present.

3.3.2 WCW as a union of symmetric spaces

In finite-dimensional context globally symmetric spaces are of form G/H and connection and curvature are independent of the metric, provided it is left invariant under G. The hope is that same holds true in infinite-dimensional context. The most one can hope of obtaining is the decomposition $C(H) = \cup_i G/H_i$ over orbits of G. One could allow also symmetry breaking in the sense that G and H depend on the orbit: $C(H) = \cup_i G_i/H_i$ but it seems that G can be chosen to be same for all orbits. What is essential is that these groups are infinite-dimensional. The basic properties of the coset space decomposition give very strong constraints on the group H, which certainly contains the subgroup of G, whose action reduces to diffeomorphisms of X^3.

Consequences of the decomposition

If the decomposition to a union of coset spaces indeed occurs, the consequences for the calculability of the theory are enormous since it suffices to find metric and curvature tensor for single representative 3-surface on a given orbit (contravariant form of metric gives propagator in perturbative calculation of matrix elements as functional integrals over the WCW). The representative surface can be chosen to correspond to the maximum of Kähler function on a given orbit and one obtains perturbation theory around this maximum (Kähler function is not isometry invariant).

The task is to identify the infinite-dimensional groups G and H and to understand the zero mode structure of the WCW. Almost twenty (seven according to long held belief!) years after the discovery of the candidate for the Kähler function defining the metric, it became finally clear that these identifications follow quite nicely from $Diff^4$ invariance and $Diff^4$ degeneracy as well as special properties of the Kähler action.

The guess (not the first one!) would be following. G corresponds to the symplectic transformations of $\delta M^4_\pm \times CP_2$ leaving the induced Kähler form invariant. If G acts as isometries the values of Kähler form at partonic 2-surfaces (remember effective 2-dimensionality realized in simplistic manner) are zero modes and WCW allows slicing to symplectic orbits of the partonic 2-surface with fixed induced Kähler form. Quantum fluctuating degrees of freedom would correspond to symplectic group and to the fluctuations of the induced metric. The group H dividing G would act as diffeomorphisms at the preferred 3-surface X^3 and leaving X^3 itself invariant. Therefore the identification of g and h would be in terms of tangent space algebra of WCW sector realized as coset space G/H.

Coset space structure of WCW and Equivalence Principle

The realization of WCW sectors with fixed values of zero modes as symmetric spaces G/H (analogous to $CP_2 = SU(3)/U(2)$) suggests that one can assign super-Virasoro algebras with G. What the two algebras g and h are is however difficult question. The following vision is only one of the many (the latest one).

1. Symplectic algebra g generates isometries and h is identified as algebra, whose generators generate diffeormorphisms at preferred X^3.

2. The original long-held belief was that the Super Kac-Moody symmetry corresponds to local imbedding space isometries for light-like 3-surfaces X^3_l, which might be boundaries of X^4 (probably not: it seems that boundary conditions cannot be satisfied so that space-time surfaces must consists of regions defining at least double coverings of M^4) and light-like surfaces separating space-time regions with different signatures of the induced metric. This symmetry would be identifiable as the counterpart of the Kac Moody symmetry of string models.

   It has turned out that one can assume Kac-Moody algebra to be sub-algebra of symplectic algebra consisting of the symplectic isometries of imbedding space. This Super Kac-Moody algebra is generated by super-currents assignable to the modes of induced spinor fields other than right-handed neutrino and localized at string world sheets. The entire symplectic algebra would correspond to the modes of right-handed neutrino and the entire algebra one would be direct sum of these two algebras so that the number of tensor factors would be indeed 5. The beauty of this option is that localization would be for both algebras inherent and with respect to the light-like coordinate of light-cone boundary rather than forced by hand.

3. p-Adic mass calculations require that symplectic and Kac-Moody algebras together generate the entire algebra. In this situation strong form of holography implies that transformations located to the interior of space-like 3-surface and light-like partonic orbit define zero modes and act like gauge symmetries. The physically non-trivial transformations correspond to transformations acting non-trivially at partonic 2-surfaces. g corresponds to the algebra generated by these transformations and for preferred 3-surface - identified as (say) maximum of Kähler function - h corresponds to the elements of this algebra generating diffeomorphisms of X^3. Super-conformal representation has five tensor factors corresponding to color algebra, two factors from electroweak $U(2)$, one factor from transversal M^4 translations and one factor from symplectic algebra (note that also Hamiltonians which are products of δM^4_+ and CP_2 Hamiltonians are possible.

Equivalence Principle (EP) has been a longstanding problem for TGD although the recent stringy view about graviton mediated scattering makes it can be argued to reduce to a tautology. I have considered several explanations for EP and coset representation has been one of them.

1. Coset representation associated with the super Virasoro algebra is defined by the condition that the differences of super Virasoro generators for g and h annihilate the physical. The original proposal for the realization of EP was that this condition implies that the four-momenta associated with g and h are identical and identifiable as inertial and gravitational four-momenta. Translations however lead out from CD boundary and cannot leave 3-surface invariant. Hence the Virasoro generators for h should not carry four-momentum. Therefore EP cannot be understood in terms of coset representations.

2. The equivalence of classical Noether momentum associated with Kähler action with eigenvalues of the corresponding quantal momentum for modified Dirac action certainly realizes quantum classical correspondence (QCC) EP could correspond to QCC.

3. A further option is that EP reduces to the identification of the four momenta for Super Virasoro representations assignable to space-like and light-like 3-surfaces and therefore become part of strong form of holography in turn implied by strong form of GCI! It seems that this option is the most plausible one found hitherto.

WCW isometries as a subgroup of $Diff(\delta M^4_+ \times CP_2)$

The reduction to light cone boundary leads to the identification of the isometry group as some subgroup of for the group G for the diffeomorphisms of $\delta M^4_+ \times CP_2$. These diffeomorphisms indeed act in a natural manner in δCH, the the space of 3-surfaces in $\delta M^4_+ \times CP_2$. WCW is expected to decompose to a union of the coset spaces G/H_i, where H_i corresponds to some subgroup of G containing the transformations of G acting as diffeomorphisms for given X^3. Geometrically the vector fields acting as diffeomorphisms of X^3 are tangential to the 3-surface. H_i could depend on the topology of X^3 and since G does not change the topology of 3-surface each 3-topology defines separate orbit of G. Therefore, the union involves sum over all topologies of X^3 plus possibly other 'zero modes'. Different topologies are naturally glued together since singular 3-surfaces intermediate between two 3-topologies correspond to points common to the two sectors with different topologies.

3.3.3 Isometries of WCW geometry as symplectic transformations of $\delta M^4_+ \times CP_2$

During last decade I have considered several candidates for the group G of isometries of WCW as the sub-algebra of the subalgebra of $Diff(\delta M^4_+ \times CP_2)$. To begin with let us write the general decomposition of $diff(\delta M^4_+ \times CP_2)$:

$$diff(\delta M^4_+ \times CP_2) \quad = \quad S(CP_2) \times diff(\delta M^4_+) \oplus S(\delta M^4_+) \times diff(CP_2) \ . \tag{3.3.2}$$

Here $S(X)$ denotes the scalar function basis of space X. This Lie-algebra is the direct sum of light cone diffeomorphisms made local with respect to CP_2 and CP_2 diffeomorphisms made local with respect to light cone boundary.

The idea that entire diffeomorphism group would act as isometries looks unrealistic since the theory should be more or less equivalent with topological field theory in this case. Consider now the various candidates for G.

1. The fact that symplectic transformations of CP_2 and M^4_+ diffeomorphisms are dynamical symmetries of the vacuum extremals suggests the possibility that the diffeomorphisms of the light cone boundary and symplectic transformations of CP_2 could leave Kähler function invariant and thus correspond to zero modes. The symplectic transformations of CP_2 localized with respect to light cone boundary acting as symplectic transformations of CP_2 have interpretation as local color transformations and are a good candidate for the isometries. The fact that local color transformations are not even approximate symmetries of Kähler action is not a problem: if they were exact symmetries, Kähler function would be invariant and zero modes would be in question.

2. CP_2 local conformal transformations of the light cone boundary act as isometries of δM^4_+. Besides this there is a huge group of the symplectic symmetries of $\delta M^4_+ \times CP_2$ if light cone boundary is provided with the symplectic structure. Both groups must be considered as candidates for groups of isometries. $\delta M^4_+ \times CP_2$ option exploits fully the special properties of $\delta M^4_+ \times CP_2$, and one can develop simple argument demonstrating that $\delta M^4_+ \times CP_2$ symplectic invariance is the correct option. Also the construction of WCW gamma matrices as super-symplectic charges supports $\delta M^4_+ \times CP_2$ option.

WCW as a union of symmetric spaces

The idea about symmetric space is extremely beautiful but it took a long time and several false alarms before the time was ripe for identifying the precise form of the Cartan decomposition $g = t + h$ satisfying the defining conditions

$$g = t + h \ , \quad [t,t] \subset h \ , \quad [h,t] \subset t \ . \tag{3.3.3}$$

The ultimate solution of the puzzle turned out to be amazingly simple and came only after quantum TGD was understood well enough. $[t,t] \subset h$ condition is highly nontrivial and equivalent with the existence of involution. Inversion in the light-like radial coordinate of δM^4 is a natural guess for this involution and induces complex conjugation in super-conformal algebras mapping positive and negative conformal weights to each other.

WCW geometry allows two super-conformal symmetries. The first one corresponds to super-symplectic transformations acting at the level of imbedding space. The second one corresponds to super Kac-Moody symmetry. The original identification of Kac-Moody was in terms of deformations of light-like 3-surfaces respecting their light-likeness. This not wrong as such: also entire symplectic algebra can be assigned with light-like surfaces and the theory can be constructed using also these conformal algebras. This identification however makes it very difficult to see how Kac-Moody could act as isometry: in particular, the localization with respect to internal coordinates of 3-surface produces technical problems since symplectic algebra is localized with respect to the light-like radial coordinate of light-cone boundary.

The more plausible identification is as the sub-algebra of symplectic algebra realized as isometries of δCD so that localization is inherent and in terms of the radial light-like coordinate of light-like boundary [K80]. This identification is made possible by the wisdom gained from the solutions of the modified Dirac equations predicting the localization of its modes (except right-handed neutrino) to string world sheets.

1. g would thus correspond to a direct sum of super-symplectic algebra and super Kac-Moody algebra defined by its isometry sub-algebra but represented in different manner (this is absolutely essential!). More concretely, neutrino modes defined super Hamiltonians associated with the super symplectic algebra and other modes of induced spinor field the super Hamiltonians associated with the super Kac-Moody algebra. The maxima of Kähler function could be chosen as natural candidates for the preferred points and could play also an essential role in WCW integration by generalizing the Gaussian integration of free quantum field theories.

2. These super-conformal algebra representations form a direct sum. p-Adic mass calculations require five super-conformal tensor factors and the number of tensor factors would be indeed this.

3. This algebra has as sub-algebra the algebra for which generators leave 3-surface invariant - in other words, induce its diffeomorphism. Quantum states correspond to the coset representations for entire algebra and this algebra so that differences of the corresponding super-Virasoro generators annihilate physical states. This obviously generalizes Goddard-Olive-Kent construction [A58]. It seems now clear that coset representation does not imply EP: the four-momentum simply does not appear in the representation of the isotropy sub-algebra since translations lead out of CD boundary.

To minimize confusions it must be emphasized that only the contribution of the symplectic algebra realized in terms of single right-handed neutrino mode is discussed in this chapter and the WCW Hamiltonians have 2-dimensional representation. Also the direct connection with the dynamics of Kähler action is lacking. A more realistic construction [K80] uses 3-dimensional representations of Hamiltonians and requires all modes of right-handed neutrino for symplectic algebra and the modes of induced spinor field carrying electroweak quantum numbers in the case of Kac-Moody algebra.

3.4 Complexification

A necessary prerequisite for the Kähler geometry is the complexification of the tangent space in vibrational degrees of freedom. What this means in recent context is non-trivial.

3.4.1 Why complexification is needed?

The Minkowskian signature of M^4 metric seems however to represent an insurmountable obstacle for the complexification of M^4 type vibrational degrees of freedom. On the other hand, complexification seems to have deep roots in the actual physical reality.

1. In the perturbative quantization of gauge fields one associates to each gauge field excitation polarization vector e and massless four-momentum vector p ($p^2 = 0$, $p \cdot e = 0$). These vectors define the decomposition of the tangent space of M^4: $M^4 = M^2 \times E^2$, where M^2 type polarizations correspond to zero norm states and E^2 type polarizations correspond to physical states with non-vanishing norm. Same type of decomposition occurs also in the linearized theory of gravitation. The crucial feature is that E^2 allows complexification! The general conclusion is that the modes of massless, linear, boson fields define always complexification of M^4 (or its tangent space) by effectively reducing it to E^2. Also in string models similar situation is encountered. For a string in D-dimensional space only D-2 transversal Euclidian degrees of freedom are physical.

2. Since symplectically extended isometry generators are expected to create physical states in TGD approach same kind of physical complexification should take place for them, too: this indeed takes place in string models in critical dimension. Somehow one should be able to associate polarization vector and massless four momentum vector to the deformations of a given 3-surface so that these vectors define the decomposition $M^4 = M^2 \times E^2$ for each mode. Configuration space metric should be degenerate: the norm of M^2 deformations should vanish as opposed to the norm of E^2 deformations.

Consider now the implications of this requirement.

1. In order to associate four-momentum and polarization (or at least the decomposition $M^4 = M^2 \times E^2$) to the deformations of the 3-surface one should have field equations, which determine the time development of the 3-surface uniquely. Furthermore, the time development for small deformations should be such that it makes sense to associate four momentum and polarization or at least the decomposition $M^4 = M^2 \times E^2$ to the deformations in suitable basis.

   The solution to this problem is afforded by the proposed definition of the Kähler function. The definition of the Kähler function indeed associates to a given 3-surface a unique four-surface as the preferred extremal of the Kähler action. Therefore one can associate a unique time development to the deformations of the surface X^3 and if TGD describes the observed world this time development should describe the evolution of photon, gluon, graviton, etc. states and so we can hope that tangent space complexification could be defined.

2. We have found that M^2 part of the deformation should have zero norm. In particular, the time like vibrational modes have zero norm in WCW metric. This is true if Kähler function is not only $Diff^3$ invariant but also Diff^4 invariant in the sense that Kähler function has same value for all 3-surfaces belonging to the orbit of X^3 and related to X^3 by diffeomorphism of X^4. This is indeed the case.

3. Even this is not enough. One expects the presence of massive modes having also longitudinal polarization and for these states the number of physical vibrational degrees of freedom is 3 so that complexification seems to be impossible by odd dimension.

The reduction to the light cone boundary implied by $Diff^4$ invariance makes possible to identify the complexification. Crucial role is played by the special properties of the boundary of 4-dimensional light cone, which is metrically two-sphere and thus allows generalized complex and Kähler structure.

3.4.2 The metric, conformal and symplectic structures of the light cone boundary

The special metric properties of the light cone boundary play a crucial role in the complexification. The point is that the boundary of the light cone has degenerate metric: although light cone boundary is topologically 3-dimensional it is metrically 2-dimensional: effectively sphere. In standard spherical Minkowski coordinates light cone boundary is defined by the equation $r_M = m^0$ and induced metric reads

$$ds^2 = -r_M^2 d\Omega^2 = -r_M^2 dz d\bar{z}/(1+z\bar{z})^2 \ , \tag{3.4.1}$$

and has Euclidian signature. Since S^2 allows complexification and thus also Kähler structure (and as a by-product also symplectic structure) there are good hopes of obtaining just the required type of complexification in non-degenerate M^4 degrees of freedom: WCW would effectively inherit its Kähler structure from $S^2 \times CP_2$.

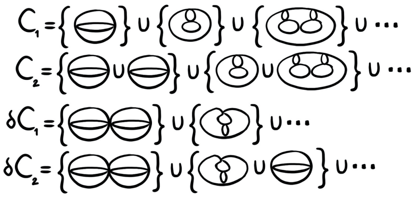

Figure 3.1: Conformal symmetry preserves angles in complex plane

By its effective two-dimensionality the boundary of the four-dimensional light cone has infinite-dimensional group of (local) conformal transformations. Using complex coordinate z for S^2 the general local conformal transformation reads

$$\begin{aligned} r &\rightarrow f(r_M, z, \bar{z}) \ , \\ z &\rightarrow g(z) \ , \end{aligned} \tag{3.4.2}$$

where f is an arbitrary real function and g is an arbitrary analytic function with a finite number of poles. The infinitesimal generators of this group span an algebra, call it C, analogous to Virasoro algebra. This algebra is semidirect sum of two algebras L and R given by

$$\begin{aligned} C &= L \oplus R \ , \\ [L,R] &\subset R \ , \end{aligned} \tag{3.4.3}$$

where L denotes standard Virasoro algebra of the two- sphere generated by the generators

$$L_n = z^{n+1} d/dz \tag{3.4.4}$$

and R denotes the algebra generated by the vector fields

$$R_n \quad = \quad f_n(z,\bar{z},r_M)\partial_{r_M} \quad , \tag{3.4.5}$$

where $f(z,\bar{z},r_M)$ forms complete real scalar function basis for light cone boundary. The vector fields of R have the special property that they have vanishing norm in M^4 metric.

This modification of conformal group implies that the Virasoro generator L_0 becomes $L_0 = zd/dz - r_M d/dr_M$ so that the scaling momentum becomes the difference $n-m$ or S^2 and radial scaling momenta. One could achieve conformal invariance by requiring that S^2 and radial scaling quantum numbers compensate each other.

Of crucial importance is that light cone boundary allows infinite dimensional group of isometries! An arbitrary conformal transformation $z \rightarrow f(z)$ induces to the metric a conformal factor given by $|df/dz|^2$. The compensating radial scaling $r_M \rightarrow r_M/|df/dz|$ compensates this factor so that the line element remains invariant.

The Kähler structure of light cone boundary defines automatically symplectic structure. The symplectic form is degenerate and just the area form of S^2 given by

$$J = r_M^2 sin(\theta) d\theta \wedge d\phi,$$

in standard spherical coordinates, there is infinite-dimensional group of symplectic transformations leaving the symplectic form of the light cone boundary (that is S^2) invariant. These transformations are local with respect to the radial coordinate r_M. The symplectic and Kähler structures of light cone boundary are not unique: different structures are labeled by the coset space $SO(3,1)/SO(3)$. One can however associate with a given 3-surface Y^3 a unique structure by requiring that the the corresponding subgroup $SO(3)$ of Lorentz group acts as the isotropy group of the conserved classical four-momentum assigned to Y^3 by the preferred extremal property.

In the case of $\delta M^4_+ \times CP_2$ both the conformal transformations, isometries and symplectic transformations of the light cone boundary can be made local also with respect to CP_2. The idea that the infinite-dimensional algebra of symplectic transformations of $\delta M^4_+ \times CP_2$ act as isometries of WCW and that radial vector fields having zero norm in the metric of light cone boundary possess zero norm also in WCW metric, looks extremely attractive.

In the case of $\delta M^4_+ \times CP_2$ one could combine the symplectic and Kähler structures of δM^4_+ and CP_2 to single symplectic/Kähler structure. The symplectic transformations leaving this symplectic structure invariant would be generated by the function algebra of $\delta M^4_+ \times CP_2$ such that a arbitrary function serves as a Hamiltonian of a symplectic transformation. This group serves as a candidate for the isometry group of WCW. An alternative identification for the isometry algebra is as symplectic symmetries of CP_2 localized with respect to the light cone boundary. Hamiltonians would be also now elements of the function algebra of $\delta M^4_+ \times CP_2$ but their Poisson brackets would be defined using only CP_2 symplectic form.

The problem is to decide which option is correct. There is a simple argument fixing the latter option. The symplecticly imbedded CP_2 would be left invariant under δM^4_+ local symplectic transformations of CP_2. This seems strange. Under symplectic algebra of $\delta M^4_+ \times CP_2$ also symplecticly imbedded CP_2 is deformed and this sounds more realistic. The isometry algebra is therefore assumed to be the group $can(\delta M^4_+ \times CP_2)$ generated by the scalar function basis $S(\delta M^4_+ \times CP_2) = S(\delta M^4_+) \times S(CP_2)$ of the light cone boundary using the Poisson brackets to be discussed in more detail later.

There are some no-go theorems associated with higher-dimensional Abelian extensions [A49] , and although the contexts are quite different, it is interesting to consider the recent situation in light of these theorems.

1. Conformal invariance is an essentially 2-dimensional notion. Light cone boundary is however metrically and conformally 2-sphere, and therefore the conformal algebra is effectively that associated with the 2-sphere. In the same manner, the quaternion conformal algebra associated with the metrically 2-dimensional elementary particle horizons surrounding wormhole contacts allows the usual Kac Moody algebra and actually also contributes to the WCW metric.

2. In dimensions $D > 2$ Abelian extensions of the gauge algebra are extensions by an infinite-dimensional Abelian group rather than central extensions by the group $U(1)$. This result has an analog at the level of WCW geometry. The extension associated with the symplectic algebra of CP_2 localized with respect to the light cone boundary is analogous a symplectic extension defined by Poisson bracket $\{p, q\} = 1$. The central extension is the function space associated with δM^4_+ and indeed infinite-dimensional if only only CP_2 symplectic structure induces the Poisson bracket but one-dimensional if $\delta M^4_+ \times CP_2$ Poisson bracket induces the extension. In the latter case the symmetries fix the metric completely at the point corresponding to the origin of symmetric space (presumably the maximum of Kähler function for given values of zero modes).

3. $D > 2$ extensions possess no unitary faithful representations (satisfying certain well motivated physical constraints) [A49] . It might be that the degeneracy of the WCW metric is the analog for the loss of faithful representations.

3.4.3 Complexification and the special properties of the light cone boundary

In case of Kähler metric G and H Lie-algebras must allow complexification so that the isometries can act as holomorphic transformations. Since G and H can be regarded as subalgebras of the vector fields of $\delta M^4_+ \times CP_2$, they inherit in a natural manner the complex structure of the light cone boundary.

There are two candidates for WCW complexification. The simplest, and also the correct, alternative is that complexification is induced by natural complexification of vector field basis on $\delta M^4_+ \times CP_2$. In CP_2 degrees of freedom there is natural complexification

$$\xi \rightarrow \bar{\xi} \ .$$

In δM^4_+ degrees of freedom this could involve the transformation

$$z \rightarrow \bar{z}$$

and certainly involves complex conjugation for complex scalar function basis in the radial direction:

$$f(r_M) \rightarrow \overline{f(r_M)} \ ,$$

which turns out to play same role as the function basis of circle in the Kähler geometry of loop groups [A37] .

The requirement that the functions are eigen functions of radial scalings favors functions $(r_M/r_0)^k$, where k is in general a complex number. The function can be expressed as a product of real power of r_M and logarithmic plane wave. It turns out that the radial complexification alternative is the correct manner to obtain Kähler structure. The reason is that symplectic transformations leave the value of r_M invariant. Radial Virasoro invariance plays crucial role in making the complexification possible.

One could consider also a second alternative assumed in the earlier formulation of the WCW geometry. The close analogy with string models and conformal field theories suggests that for Virasoro generators the complexification must reduce to the hermitian conjugation of the conformal field theories: $L_n \rightarrow L_{-n} = L_n^\dagger$. Clearly this complexification is induced from the transformation $z \rightarrow \frac{1}{z}$ and differs from the complexification induced by complex conjugation $z \rightarrow \bar{z}$. The basis would be polynomial in z and $\bar{z}$. Since radial algebra could be also seen as Virasoro algebra localized with respect to $S^2 \times CP_2$ one could consider the possibility that also in radial direction the inversion $r_M \rightarrow \frac{1}{r_M}$ is involved.

In fact, the complexification changing the signs of radial conformal weights is induced from inversion $r_M/r_0 \rightarrow r_0/r_M$. This transformation is also an excellent candidate for the involution necessary for obtaining the structure of symmetric space implying among other things the covariant constancy of the curvature tensor, which is of special importance in infinite-D context.

The essential prerequisite for the Kähler structure is that both G and H allow same complexification so that the isometries in question can be regarded as holomorphic transformations. In finite-dimensional case this essentially what is needed since metric can be constructed by parallel translation along the orbit of G from H-invariant Kähler metric at a representative point. The requirement of H-invariance forces the radial complexification based on complex powers r_M^k: radial complexification works since symplectic transformations leave r_M invariant.

Some comments on the properties of the proposed complexification are in order.

1. The proposed complexification, which is analogous to the choice of gauge in gauge theories is not Lorentz invariant unless one can fix the coordinates of the light cone boundary apart from $SO(3)$ rotation not affecting the value of the radial coordinate r_M (if the imaginary part of k in r_M^k is always non-vanishing). This is possible as will be explained later.

2. It turns out that the function basis of light-cone boundary multiplying CP_2 Hamiltonians corresponds to unitary representations of the Lorentz group at light cone boundary so that the Lorentz invariance is rather manifest.

3. There is a nice connection with the proposed physical interpretation of the complexification. At the moment of the big bang all particles move with the velocity of light and therefore behave as massless particles. To a given point of the light cone boundary one can associate a unique direction of massless four-momentum by semiclassical considerations: at the point $m^k = (m^0, m^i)$ momentum is proportional to the vector $(m^0, -m^i)$. Since the particles are massless only two polarization vectors are possible and these correspond to the tangent vectors to the sphere $m^0 = r_M$. Of course, one must always fix polarizations at some point of tangent space but since massless polarization vectors are not physical this doesn't imply difficulties: different choices correspond to different gauges.

4. Complexification in the proposed manner is not possible except in the case of four-dimensional Minkowski space. Non-zero norm deformations correspond to vector fields of the light cone boundary acting on the sphere S^{D-2} and the decomposition to $(1,0)$ and $(0,1)$ parts is possible only when the sphere in question is two-dimensional since other spheres do allow neither complexification nor Kähler structure.

3.4.4 How to fix the complex and symplectic structures in a Lorentz invariant manner?

One can assign to light-cone boundary a symplectic structure since it reduces effectively to S^2. The possible symplectic structures of δM_+^4 are parameterized by the coset space $SO(3,1)/SO(3))$, where H is the isotropy group $SO(3)$ of a time like vector. Complexification also fixes the choice of the spherical coordinates apart from rotations around the quantization axis of angular momentum.

The selection of some preferred symplectic structure in an ad hoc manner breaks manifest Lorentz invariance but is possible if physical theory remains Lorentz invariant. The more natural possibility is that 3-surface Y^3 itself fixes in some natural manner the choice of the symplectic structure so that there is unique subgroup $SO(3)$ of $SO(3,1)$ associated with Y^3. If WCW Kähler function corresponds to a preferred extremal of Kähler action, this is indeed the case. One can associate unique conserved four-momentum $P^k(Y^3)$ to the preferred extremal $X^4(Y^3)$ of the Kähler action and the requirement that the rotation group $SO(3)$ leaving the symplectic structure invariant leaves also $P^k(Y^3)$ invariant, fixes the symplectic structure associated with Y^3 uniquely.

Therefore WCW decomposes into a union of symplectic spaces labeled by $SO(3,1)/SO(3)$ isomorphic to $a = constant$ hyperboloid of light cone. The direction of the classical angular momentum vector $w^k = \epsilon^{klmn} P_l J_{mn}$ determined by the classical angular momentum tensor of associated with Y^3 fixes one coordinate axis and one can require that $SO(2)$ subgroup of $SO(3)$ acting as rotation around this coordinate axis acts as phase transformation of the complex coordinate z of S^2. Other rotations act as nonlinear holomorphic transformations respecting the complex structure.

Clearly, the coordinates are uniquely fixed modulo $SO(2)$ rotation acting as phase multiplication in this case. If $P^k(Y^3)$ is light like, one can only require that the rotation group $SO(2)$ serving as the isotropy group of 3-momentum belongs to the group $SO(3)$ characterizing the symplectic structure and it seems that symplectic structure cannot be uniquely fixed without additional constraints in this case. Probably this has no practical consequences since the 3-surfaces considered have actually infinite size and 4-momentum is most probably time like for them. Note however that the direction of 3-momentum defines unique axis such that $SO(2)$ rotations around this axis are represented as phase multiplication.

Similar almost unique frame exists also in CP_2 degrees of freedom and corresponds to the complex coordinates transforming linearly under $U(2)$ acting as isotropy group of the Lie-algebra element defined by classical color charges Q_a of Y^3. One can fix unique Cartan subgroup of $U(2)$ by noticing that $SU(3)$ allows completely symmetric structure constants d_{abc} such that $R_a = {d_a}^{bc}Q_bQ_c$ defines Lie-algebra element commuting with Q_a. This means that R_a and Q_a span in generic case $U(1) \times U(1)$ Cartan subalgebra and there are unique complex coordinates for which this subgroup acts as phase multiplications. The space of nonequivalent frames is isomorphic with $CP(2)$ so that one can say that cm degrees of freedom correspond to Cartesian product of $SO(3,1)/SO(3)$ hyperboloid and CP_2 whereas coordinate choices correspond to the Cartesian product of $SO(3,1)/SO(2)$ and $SU(3)/U(1) \times U(1)$.

Symplectic transformations leave the value of δM^4_+ radial coordinate r_M invariant and this implies large number of additional zero modes characterizing the size and shape of the 3-surface. Besides this Kähler magnetic fluxes through the $r_M = constant$ sections of X^3 as a function of r_M provide additional invariants, which are functions rather than numbers. The Fourier components for the magnetic fluxes provide infinite number of symplectic invariants. The presence of these zero modes imply that 3-surfaces behave much like classical objects in the sense that neither their shape nor form nor classical Kähler magnetic fields, are subject to Gaussian fluctuations. Of course, quantum superpositions of 3-surfaces with different values of these invariants are possible.

There are reasons to expect that at least certain infinitesimal symplectic transformations correspond to zero modes of the Kähler metric (symplectic transformations act as dynamical symmetries of the vacuum extremals of the Kähler action). If this is indeed the case, one can ask whether it is possible to identify an integration measure for them.

If one can associate symplectic structure with zero modes, the symplectic structure defines integration measure in a standard manner (for 2n-dimensional symplectic manifold the integration measure is just the n-fold wedge power $J \wedge J ... \wedge J$ of the symplectic form J). Unfortunately, in infinite-dimensional context this is not enough since divergence free functional integral analogous to a Gaussian integral is needed and it seems that it is not possible to integrate in zero modes and that this relates in a deep manner to state function reduction. If all symplectic transformations of $\delta M^4_+ \times CP_2$ are represented as symplectic transformations of the configuration space, then the existence of symplectic structure decomposing into Kähler (and symplectic) structure in complexified degrees of freedom and symplectic (but not Kähler) structure in zero modes, is an automatic consequence.

3.4.5 The general structure of the isometry algebra

There are three options for the isometry algebra of WCW.

1. Isometry algebra as the algebra of CP_2 symplectic transformations leaving invariant the symplectic form of CP_2 localized with respect to δM^4_+.

2. Certainly the WCW metric in δM^4_+ must be non-trivial and actually given by the magnetic flux Hamiltonians defining symplectic invariants. Furthermore, the super-symplectic generators constructed from quarks automatically give as anti-commutators this part of the WCW metric. One could interpret these symplectic invariants as WCW Hamiltonians for δM^4_+ symplectic transformations obtained when CP_2 Hamiltonian is constant.

3. Isometry algebra consists of $\delta M^4_+ \times CP_2$ symplectic transformations. In this case a local color transformation involves necessarily a local S^2 transformation. Unfortunately, it is difficult to decide at this stage which of these options is correct.

The eigen states of the rotation generator and Lorentz boost in the same direction defining a unitary representation of the Lorentz group at light cone boundary define the most natural function basis for the light cone boundary. The elements of this bases have also well defined scaling quantum numbers and define also a unitary representation of the conformal algebra. The product of the basic functions is very simple in this basis since various quantum numbers are additive.

Spherical harmonics of S^2 provide an alternative function basis for the light cone boundary:

$$H^m_{jk} \equiv Y_{jm}(\theta,\phi)r_M^k \ . \tag{3.4.6}$$

One can criticize this basis for not having nice properties under Lorentz group.

The product of basis functions is given by Glebch-Gordan coefficients for symmetrized tensor product of two representation of the rotation group. Poisson bracket in turn reduces to the Glebch-Gordans of anti-symmetrized tensor product. The quantum numbers m and k are additive. The basis is eigen-function basis for the imaginary part of the Virasoro generator L_0 generating rotations around quantization axis of angular momentum. In fact, only the imaginary part of the Virasoro generator $L_0 = zd/dz = \rho\partial_\rho - \frac{2}{2}\partial_\phi$ has global single valued Hamiltonian, whereas the corresponding representation for the transformation induced by the real part of L_0, with a compensating radial scaling added, cannot be realized as a global symplectic transformation.

The Poisson bracket of two functions $H^m_{j_1k_1}$ and $H^m_{j_2k_2}$ can be calculated and is of the general form

$$\{H^{m_1}_{j_1k}, H^{m_2}_{j_2k_2}\} \equiv C(j_1m_1j_2m_2|j, m_1+m_2)_A H^{m_1+m_2}_{j,k_1+k_2} \ . \tag{3.4.7}$$

The coefficients are Glebch-Gordan coefficients for the anti-symmetrized tensor product for the representations of the rotation group.

The isometries of the light cone boundary correspond to conformal transformations accompanied by a local radial scaling compensating the conformal factor coming from the conformal transformations having parametric dependence of radial variable and CP_2 coordinates. It seems however that isometries cannot in general be realized as symplectic transformations. The first difficulty is that symplectic transformations cannot affect the value of the radial coordinate. For rotation algebra the representation as symplectic transformations is however possible.

In CP_2 degrees of freedom scalar function basis having definite color transformation properties is desirable. Scalar function basis can be obtained as the algebra generated by the Hamiltonians of color transformations by multiplication. The elements of basis can be typically expressed as monomials of color Hamiltonians H^A_c

$$H^A_D = \sum_{\{B_j\}} C^A_{DB_1B_2....B_N} \prod_{B_i} H^{B_i}_c \ , \tag{3.4.8}$$

where summation over all index combinations $\{B_i\}$ is understood. The coefficients $C^A_{DB_1B_2....B_N}$ are Glebch-Gordan coefficients for completely symmetric N:th power $8\otimes 8 ... \otimes 8$ of octet representations. The representation is not unique since $\sum_A H^A_c H^A_c = 1$ holds true. One can however find for each representation D some minimum value of N.

The product of Hamiltonians $H^{D_1}_A$ and $H^B_{D_2}$ can be decomposed by Glebch-Gordan coefficients of the symmetrized representation $(D_1 \otimes D_2)_S$ as

$$H^A_{D_1} H^B_{D_2} = C^{ABD}_{D_1D_2DC}(S) H^C_D \ , \tag{3.4.9}$$

where $'S'$ indicates that the symmetrized representation is in question. In the similar manner one can decompose the Poisson bracket of two Hamiltonians

$$\{H^A_{D_1}, H^B_{D_2}\} = C^{ABD}_{D_1D_2DC}(A) H^C_D \ . \tag{3.4.10}$$

Here $'A'$ indicates that Glebch-Gordan coefficients for the anti-symmetrized tensor product of the representations D_1 and D_2 are in question.

One can express the infinitesimal generators of CP_2 symplectic transformations in terms of the color isometry generators J^B_c using the expansion of the Hamiltonian in terms of the monomials of color Hamiltonians:

$$\begin{aligned} j^A_{DN} &= F^A_{DB} J^B_c \ , \\ F^A_{DB} &= N \sum_{\{B_j\}} C^A_{DB_1B_2...B_{N-1}B} \prod_j H^{B_j}_c \ , \end{aligned} \tag{3.4.11}$$

where summation over all possible $\{B_j\}$:s appears. Therefore, the interpretation as a color group localized with respect to CP_2 coordinates is valid in the same sense as the interpretation of space-time diffeomorphism group as local Poincare group. Thus one can say that TGD color is localized with respect to the entire $\delta M^4_+ \times CP_2$.

A convenient basis for the Hamiltonians of $\delta M^4_+ \times CP_2$ is given by the functions

$$H^{mA}_{jkD} = H^m_{jk} H^A_D \ .$$

The symplectic transformation generated by H^{mA}_{jkD} acts both in M^4 and CP_2 degrees of freedom and the corresponding vector field is given by

$$J^r = H^A_D J^{rl}(\delta M^4_+)\partial_l H^m_{jk} + H^m_{jk} J^{rl}(CP_2)\partial_l H^A_D \ . \tag{3.4.12}$$

The general form for their Poisson bracket is:

$$\begin{aligned} &\{H^{m_1A_1}_{j_1k_1D_1}, H^{m_2A_2}_{j_2k_2D_2}\} = H^{A_1}_{D_1} H^{A_2}_{D_2} \{H^{m_1}_{j_1k_1}, H^{m_2}_{j_2k_2}\} + H^{m_1}_{j_1k_1} H^{m_2}_{j_2k_2} \{H^{A_1}_{D_1}, H^{A_2}_{D_2}\} \\ &= \left[C^{A_1A_2A}_{D_1D_2D}(S) C(j_1m_1j_2m_2|jm)_A + C^{A_1A_2A}_{D_1D_2D}(A) C(j_1m_1j_2m_2|jm)_S \right] H^{mA}_{j,k_1+k_2,D} \ . \end{aligned} \tag{3.4.13}$$

What is essential that radial 'momenta' and angular momentum are additive in δM^4_+ degrees of freedom and color quantum numbers are additive in CP_2 degrees of freedom.

3.4.6 Representation of Lorentz group and conformal symmetries at light cone boundary

A guess deserving testing is that the representations of the Lorentz group at light cone boundary might provide natural building blocks for the construction of the WCW Hamiltonians. In the following the explicit representation of the Lorentz algebra at light cone boundary is deduced, and a function basis giving rise to the representations of Lorentz group and having very simple properties under modified Poisson bracket of δM^4_+ is constructed.

Explicit representation of Lorentz algebra

It is useful to write the explicit expressions of Lorentz generators using complex coordinates for S^2. The expression for the $SU(2)$ generators of the Lorentz group are

$$\begin{aligned} J_x &= (z^2-1)d/dz + c.c. = L_1 - L_{-1} + c.c. \ , \\ J_y &= (iz^2+1)d/dz + c.c. = iL_1 + iL_{-1} + c.c. \ , \\ J_z &= iz\frac{d}{dz} + c.c. = iL_z + c.c. \ . \end{aligned} \tag{3.4.14}$$

The expressions for the generators of Lorentz boosts can be derived easily. The boost in m^3 direction corresponds to an infinitesimal transformation

$$\begin{aligned} \delta m^3 &= -\varepsilon r_M \ , \\ \delta r_M &= -\varepsilon m^3 = -\varepsilon\sqrt{r_M^2 - (m^1)^2 - (m^2)^2} \ . \end{aligned} \tag{3.4.15}$$

The relationship between complex coordinates of S^2 and M^4 coordinates m^k is given by stereographic projection

$$\begin{aligned} z &= \frac{(m^1+im^2)}{(r_M-\sqrt{r_M^2-(m^1)^2-(m^2)^2})} \\ &= \frac{sin(\theta)(cos\phi+isin\phi)}{(1-cos\theta)} \ , \\ cot(\theta/2) &= \rho=\sqrt{z\bar{z}} \ , \\ tan(\phi) &= \frac{m^2}{m^1} \ . \end{aligned} \tag{3.4.16}$$

This implies that the change in z coordinate doesn't depend at all on r_M and is of the following form

$$\delta z = -\frac{\varepsilon}{2}(1+\frac{z(z+\bar{z})}{2})(1+z\bar{z}) \ . \tag{3.4.17}$$

The infinitesimal generator for the boosts in z-direction is therefore of the following form

$$L_z = [\frac{2z\bar{z}}{(1+z\bar{z})}-1]r_M\frac{\partial}{\partial_{r_M}}-iJ_z \ . \tag{3.4.18}$$

Generators of L_x and L_y are most conveniently obtained as commutators of $[L_z,J_y]$ and $[L_z,J_x]$. For L_y one obtains the following expression:

$$L_y = 2\frac{(z\bar{z}(z+\bar{z})+i(z-\bar{z}))}{(1+z\bar{z})^2}r_M\frac{\partial}{\partial_{r_M}}-iJ_y \ , \tag{3.4.19}$$

For L_x one obtains analogous expressions. All Lorentz boosts are of the form $L_i=-iJ_i+$ *local radial scaling* and of zeroth degree in radial variable so that their action on the general generator $X^{klm}\propto z^k\bar{z}^l r_M^m$ doesn't change the value of the label m being a mere local scaling transformation in radial direction. If radial scalings correspond to zero norm isometries this representation is metrically equivalent with the representations of Lorentz boosts as Möbius transformations.

Representations of the Lorentz group reduced with respect to $SO(3)$

The ordinary harmonics of S^2 define in a natural manner infinite series of representation functions transformed to each other in Lorentz transformations. The inner product defined by the integration measure $r_M^2 d\Omega dr_M/r_M$ remains invariant under Lorentz boosts since the scaling of r_M induced by the Lorentz boost compensates for the conformal scaling of $d\Omega$ induced by a Lorentz transformation represented as a Möbius transformation. Thus unitary representations of Lorentz group are in question.

The unitary main series representations of the Lorentz group are characterized by half-integer m and imaginary number $k_2=i\rho$, where ρ is any real number [A45] . A natural guess is that $m=0$ holds true for all representations realizable at the light cone boundary and that radial waves are of form r_M^k, $k=k_1+ik_2=-1+i\rho$ and thus eigen states of the radial scaling so that the action of Lorentz boosts is simple in the angular momentum basis. The inner product in radial degrees of freedom reduces to that for ordinary plane waves when $log(r_M)$ is taken as a new integration variable. The complexification is well-defined for non-vanishing values of ρ.

It is also possible to have non-unitary representations of the Lorentz group and the realization of the symmetric space structure suggests that one must have $k=k_1+ik_2$, k_1 half-integer. For these representations unitarity fails because the inner product in the radial degrees of freedom is non-unitary. A possible physical interpretation consistent with the general ideas about conformal invariance is that the representations $k=-1+i\rho$ correspond to the unitary ground state representations and $k=-1+n/2+i\rho$, $n=\pm1,\pm2,...$, to non-unitary representations. The general view about conformal invariance suggests that physical states constructed as tensor products satisfy the condition $\sum_i n_i=0$ completely analogous to Virasoro conditions.

Representations of the Lorentz group with $E^2 \times SO(2)$ as isotropy group

One can construct representations of Lorentz group and conformal symmetries at the light cone boundary. Since $SL(2,C)$ is the group generated by the generators L_0 and $L_\pm$ of the conformal algebra, it is clear that infinite-dimensional representations of Lorentz group can be also regarded as representations of the conformal algebra. One can require that the basis corresponds to eigen functions of the rotation generator J_z and corresponding boost generator L_z. For functions which do not depend on r_M these generators are completely analogous to the generators L_0 generating scalings and iL_0 generating rotations. Also the generator of radial scalings appears in the formulas and one must consider the possibility that it corresponds to the generator L_0.

In order to construct scalar function eigen basis of L_z and J_z, one can start from the expressions

$$\begin{aligned} L_3 &\equiv i(L_z + L_{\bar{z}}) = 2i[\frac{2z\bar{z}}{(1+z\bar{z})} - 1]r_M\frac{\partial}{\partial_{r_M}} + i\rho\partial_\rho \ , \\ J_3 &\equiv iL_z - iL_{\bar{z}} = i\partial_\phi \ . \end{aligned} \tag{3.4.20}$$

If the eigen functions do not depend on r_M, one obtains the usual basis z^n of Virasoro algebra, which however is not normalizable basis. The eigenfunctions of the generators L_3, J_3 and $L_0 = ir_M d/dr_M$ satisfying

$$\begin{aligned} J_3 f_{m,n,k} &= m f_{m,n,k} \ , \\ L_3 f_{m,n,k} &= n f_{m,n,k} \ , \\ L_0 f_{m,n,k} &= k f_{m,n,k} \ . \end{aligned} \tag{3.4.21}$$

are given by

$$f_{m,n,k} = e^{im\phi}\frac{\rho^{n-k}}{(1+\rho^2)^k} \times (\frac{r_M}{r_0})^k \ . \tag{3.4.22}$$

$n = n_1 + in_2$ and $k = k_1 + ik_2$ are in general complex numbers. The condition

$$n_1 - k_1 \geq 0$$

is required by regularity at the origin of S^2 The requirement that the integral over S^2 defining norm exists (the expression for the differential solid angle is $d\Omega = \frac{\rho}{(1+\rho^2)^2}d\rho d\phi$) implies

$$n_1 < 3k_1 + 2 \ .$$

From the relationship $(cos(\theta), sin(\theta)) = (\rho^2 - 1)/(\rho^2 + 1), 2\rho/(\rho^2 + 1))$ one can conclude that for $n_2 = k_2 = 0$ the representation functions are proportional to f $sin(\theta)^{n-k}(cos(\theta) - 1)^{n-k}$. Therefore they have in their decomposition to spherical harmonics only spherical harmonics with angular momentum $l < 2(n-k)$. This suggests that the condition

$$|m| \leq 2(n-k) \tag{3.4.23}$$

is satisfied quite generally.

The emergence of the three quantum numbers (m, n, k) can be understood. Light cone boundary can be regarded as a coset space $SO(3,1)/E^2 \times SO(2)$, where $E^2 \times SO(2)$ is the group leaving the light like vector defined by a particular point of the light cone invariant. The natural choice of the Cartan group is therefore $E^2 \times SO(2)$. The three quantum numbers (m, n, k) have interpretation as quantum numbers associated with this Cartan algebra.

The representations of the Lorentz group are characterized by one half-integer valued and one complex parameter. Thus k_2 and n_2, which are Lorentz invariants, might not be independent parameters, and the simplest option is $k_2 = n_2$.

The nice feature of the function basis is that various quantum numbers are additive under multiplication:

$$f(m_a, n_a, k_a) \times f(m_b, n_b, k_b) = f(m_a + m_b, n_a + n_b, k_a + k_b) \quad .$$

These properties allow to cast the Poisson brackets of the symplectic algebra of WCW into an elegant form.

The Poisson brackets for the δM^4_+ Hamiltonians defined by f_{mnk} can be written using the expression $J^{\rho\phi} = (1 + \rho^2)/\rho$ as

$$\begin{aligned} \{f_{m_a,n_a,k_a}, f_{m_b,n_b,k_b}\} &= i\,[(n_a - k_a)m_b - (n_b - k_b)m_a] \times f_{m_a+m_b,n_a+n_b-2,k_a+k_b} \\ &+ 2i\,[(2 - k_a)m_b - (2 - k_b)m_a] \times f_{m_a+m_b,n_a+n_b-1,k_a+k_b-1} \quad . \end{aligned} \tag{3.4.24}$$

Can one find unitary light-like representations of Lorentz group?

It is interesting to compare the representations in question to the unitary representations Gelfand .

1. The unitary representations discussed in [A45] are characterized by are constructed by deducing the explicit representations for matrix elements of the rotation generators J_x, J_y, J_z and boost generators L_x, L_y, L_z by decomposing the representation into series of representations of $SU(2)$ defining the isotropy subgroup of a time like momentum. Therefore the states are labeled by eigenvalues of J_z. In the recent case the isotropy group is $E^2 \times SO(2)$ leaving light like point invariant. States are therefore labeled by three different quantum numbers.

2. The representations of [A45] are realized the space of complex valued functions of complex coordinates ξ and $\overline{\xi}$ labeling points of complex plane. These functions have complex degrees $n_+ = m/2 - 1 + l_1$ with respect to ξ and $n_- = -m/2 - 1 + l_1$ with respect to $\overline{\xi}$. l_0 is complex number in the general case but for unitary representations of main series it is given by $l_1 = i\rho$ and for the representations of supplementary series l_1 is real and satisfies $0 < |l_1| < 1$. The main series representation is derived from a representation space consisting of homogenous functions of variables z^0, z^1 of degree n_+ and of $\overline{z}^0$ and $\overline{z}^1$ of degrees $n_\pm$. One can separate express these functions as product of $(z^1)^{n^+}$ $(\overline{z}^1)^{n_-}$ and a polynomial of $\xi = z^1/z^2$ and $\overline{\xi}$ with degrees n_+ and n_-. Unitarity reduces to the requirement that the integration measure of complex plane is invariant under the Lorentz transformations acting as Moebius transformations of the complex plane. Unitarity implies $l_1 = -1 + i\rho$.

3. For the representations at δM^4_+ formal unitarity reduces to the requirement that the integration measure of $r_M^2 d\Omega dr_M/r_M$ of δM^4_+ remains invariant under Lorentz transformations. The action of Lorentz transformation on the complex coordinates of S^2 induces a conformal scaling which can be compensated by an S^2 local radial scaling. At least formally the function space of δM^4_+ thus defines a unitary representation. For the function basis f_{mnk} $k = -1 + i\rho$ defines a candidate for a unitary representation since the logarithmic waves in the radial coordinate are completely analogous to plane waves for $k_1 = -1$. This condition would be completely analogous to the vanishing of conformal weight for the physical states of super conformal representations. The problem is that for $k_1 = -1$ guaranteeing square integrability in S^2 implies $-2 < n_1 < -2$ so that unitarity is possible only for the function basis consisting of spherical harmonics.

   There is no deep reason against non-unitary representations and symmetric space structure indeed requires that k_1 is half-integer valued. First of all, WCW spinor fields are analogous to ordinary spinor fields in M^4, which also define non-unitary representations of Lorentz group. Secondly, if 3-surfaces at the light cone boundary are finite-sized, the integrals defined by f_{mnk} over 3-surfaces Y^3 are always well-defined. Thirdly, the continuous spectrum of k_2 could be transformed to a discrete spectrum when k_1 becomes half-integer valued.

Hermitian form for light cone Hamiltonians involves also the integration over S^2 degrees of freedom and the non-unitarity of the inner product reflects itself as non-orthogonality of the the eigen function basis. Introducing the variable $u = \rho^2 + 1$ as a new integration variable, one can express the inner product in the form

$$\begin{aligned} \langle m_a, n_a, k_a | m_b, n_b, k_b \rangle &= \pi\delta(k_{2a} - k_{2b}) \times \delta_{m_1,m_2} \times I \ , \\ I &= \int_1^\infty f(u) du \ , \\ f(u) &= \frac{(u-1)^{\frac{(N-K)+i\Delta}{2}}}{u^{K+2}} \ . \end{aligned} \tag{3.4.25}$$

The integrand has cut from $u = 1$ to infinity along real axis. The first thing to observe is that for $N = K$ the exponent of the integral reduces to very simple form and integral exists only for $K = k_{1a} + k_{1b} > -1$. For $k_{1i} = -1/2$ the integral diverges.

The discontinuity of the integrand due to the cut at the real axis is proportional to the integrand and given by

$$\begin{aligned} f(u) - f(e^{i2\pi}u) &= \left[1 - e^{-\pi\Delta}\right] f(u) \ , \\ \Delta &= n_{1a} - k_{1a} - n_{1b} + k_{1b} \ . \end{aligned} \tag{3.4.26}$$

This means that one can transform the integral to an integral around the cut. This integral can in turn completed to an integral over closed loop by adding the circle at infinity to the integration path. The integrand has $K+1$-fold pole at $u = 0$.

Under these conditions one obtains

$$\begin{aligned} I &= \frac{2\pi i}{1 - e^{-\pi\Delta}} \times R \times (R-1) .... \times (R-K-1) \times (-1)^{\frac{N-K}{2} - K - 1} \ , \\ R &\equiv \frac{N-K}{2} + i\Delta \ . \end{aligned} \tag{3.4.27}$$

This expression is non-vanishing for $\Delta \neq 0$. Thus it is not possible to satisfy orthogonality conditions without the un-physical $n = k, k_1 = 1/2$ constraint. The result is finite for $K > -1$ so that $k_1 > -1/2$ must be satisfied and if one allows only half-integers in the spectrum, one must have $k_1 \geq 0$, which is very natural if real conformal weights which are half integers are allowed.

3.4.7 How the complex eigenvalues of the radial scaling operator relate to symplectic conformal weights?

3.4.8 How the complex eigenvalues of the radial scaling operator relate to symplectic conformal weights?

Complexified Hamiltonians can be chosen to be eigenmodes of the radial scaling operator $r_M d/dr_M$, and the first guess was that the correct interpretation is as conformal weights. The problem is however that the eigenvalues are complex. Second problem is that general arguments are not enough to fix the spectrum of eigenvalues. There should be a direct connection to the dynamics defined by Kähler action with instanton term included and the modified Dirac action defined by it.

The construction of WCW spinor structure in terms of second quantized induced spinor fields [K9] leads to the conclusion that the modes of induced spinor fields must be restricted at surfaces with 2-D CP_2 projection to guarantee vanishing W fields and well-defined em charge for them. In the generic case these surfaces are 2-D string world sheets (or possibly also partonic 2-surfaces) and in the non-generic case can be chosen to be such. The modes are labeled by generalized conformal weights assignable to complex or hypercomplex string coordinate. Conformal weights are expected to be integers from the experience with string models.

It is an open question whether these conformal weights are independent of the symplectic formal weights or not but on can consider also the possibility that they are dependent. Note hovewer that string coordinate is not reducible to the light-like radial coordinate in the generic case and one can imagine situations in which r_M is constant although string coordinate varies. Dependency would be achieved if the Hamiltonians are generalized eigen modes of $D = \gamma^x d/dx$, $x = log(r/r_0)$, satisfying $DH = \lambda\gamma^x H$ and thus of form $exp(\lambda x) = (r/r_0)^\lambda$ with the same spectrum of eigenvalues λ as associated with the modified Dirac operator. That $log(r/r_0)$ naturally corresponds to the coordinate u assignable to the generalized eigen modes of modified Dirac operator supports this interpretation.

3.5 Magnetic and electric representations of the configuration space Hamiltonians

Symmetry considerations lead to the hypothesis that WCW Hamiltonians are apart from a factor depending on symplectic invariants equal to magnetic flux Hamiltonians. On the other hand, the hypothesis that Kähler function corresponds to a preferred extremal of Kähler action leads to the hypothesis that WCW Hamiltonians corresponds to classical charges associated with the Hamiltonians of the light cone boundary. These charges are very much like electric charges. The requirement that two approaches are equivalent leads to the hypothesis that magnetic and electric Hamiltonians are identical apart from a factor depending on isometry invariants. At the level of CP_2 corresponding duality corresponds to the self-duality of Kähler form stating that the magnetic and electric parts of Kähler form are identical.

3.5.1 Radial symplectic invariants

All $\delta M^4_+ \times CP_2$ symplectic transformations leave invariant the value of the radial coordinate r_M. Therefore the radial coordinate r_M of X^3 regarded as a function of $S^2 \times CP_2$ coordinates serves as height function. The number, type, ordering and values for the extrema for this height function in the interior and boundary components are isometry invariants. These invariants characterize not only the topology but also the size and shape of the 3-surface. The result implies that WCW metric indeed differentiates between 3-surfaces with the size of Planck length and with the size of galaxy. The characterization of these invariants reduces to Morse theory. The extrema correspond to topology changes for the two-dimensional (one-dimensional) $r_M = constant$ section of 3-surface (boundary of 3-surface). The height functions of sphere and torus serve as a good illustrations of the situation. A good example about non-topological extrema is provided by a sphere with two horns.

There are additional symplectic invariants. The 'magnetic fluxes' associated with the δM^4_+ symplectic form

$$J_{S^2} = r_M^2 sin(\theta) d\theta \wedge d\phi$$

over any $X^2 \subset X^3$ are symplectic invariants. In particular, the integrals over $r_M = constant$ sections (assuming them to be 2-dimensional) are symplectic invariants. They give simply the solid angle $\Omega(r_M)$ spanned by $r_M = constant$ section and thus $r_M^2\Omega(r_M)$ characterizes transversal geometric size of the 3-surface. A convenient manner to discretize these invariants is to consider the Fourier components of these invariants in radial logarithmic plane wave basis discussed earlier:

$$\Omega(k) \quad = \quad \int_{r_{min}}^{r_{max}} (r_M/r_{max})^k \Omega(r_M) \frac{dr_M}{r_M} \quad , \quad k = k_1 + ik_2 \quad , \quad perk_1 \geq 0 \quad . \tag{3.5.1}$$

One must take into account that for each section in which the topology of $r_M = constant$ section remains constant one must associate invariants with separate components of the two-dimensional section. For a given value of r_M, r_M constant section contains several components (to visualize the situation consider torus as an example).

Also the quantities

$$\Omega^+(X^2) = \int_{X^2} |J| \equiv \int |\epsilon^{\alpha\beta} J_{\alpha\beta}| \sqrt{g_2} d^2x$$

are symplectic invariants and provide additional geometric information about 3-surface. These fluxes are non-vanishing also for closed surfaces and give information about the geometry of the boundary components of 3-surface (signed fluxes vanish for boundary components unless they enclose the dip of the light cone).

Since zero norm generators remain invariant under complexification, their contribution to the Kähler metric vanishes. It is not at all obvious whether WCW integration measure in these degrees of freedom exists at all. A localization in zero modes occurring in each quantum jump seems a more plausible and under suitable additional assumption it would have interpretation as a state function reduction. In string model similar situation is encountered; besides the functional integral determined by string action, one has integral over the moduli space.

If the effective 2-dimensionality implied by the strong form of general coordinate invariance discussed in the introduction is accepted, there is no need to integrate over the variable r_M and just the fluxes over the 2-surfaces X_i^2 identified as intersections of light like 3-D causal determinants with X^3 contain the data relevant for the construction of the WCW geometry. Also the symplectic invariants associated with these surfaces are enough.

3.5.2 Kähler magnetic invariants

The Kähler magnetic fluxes defined both the normal component of the Kähler magnetic field and by its absolute value

$$\begin{aligned} Q_m(X^2) &= \int_{X^2} J_{CP_2} = J_{\alpha\beta}\epsilon^{\alpha\beta}\sqrt{g_2}d^2x \ , \\ Q_m^+(X^2) &= \int_{X^2} |J_{CP_2}| \equiv \int_{X^2} |J_{\alpha\beta}\epsilon^{\alpha\beta}|\sqrt{g_2}d^2x \ , \end{aligned} \tag{3.5.2}$$

over suitably defined 2-surfaces are invariants under both Lorentz isometries and the symplectic transformations of CP_2 and can be calculated once X^3 is given.

For a closed surface $Q_m(X^2)$ vanishes unless the homology equivalence class of the surface is nontrivial in CP_2 degrees of freedom. In this case the flux is quantized. $Q_M^+(X^2)$ is non-vanishing for closed surfaces, too. Signed magnetic fluxes over non-closed surfaces depend on the boundary of X^2 only:

$$\begin{array}{l} \int_{X^2} J = \int_{\delta X^2} A \ . \\ J = dA \ . \end{array}$$

Un-signed fluxes can be written as sum of similar contributions over the boundaries of regions of X^2 in which the sign of J remains fixed.

$$\begin{aligned} Q_m(X^2) &= \int_{X^2} J_{CP_2} = J_{\alpha\beta}\epsilon^{\alpha\beta}\sqrt{g_2}d^2x \ , \\ Q_m^+(X^2) &= \int_{X^2} |J_{CP_2}| \equiv \int_{X^2} |J_{\alpha\beta}\epsilon^{\alpha\beta}|\sqrt{g_2}d^2x \ , \end{aligned} \tag{3.5.3}$$

There are also symplectic invariants, which are Lorentz covariants and defined as

$$\begin{aligned} Q_m(K,X^2) &= \int_{X^2} f_K J_{CP_2} \ , \\ Q_m^+(K,X^2) &= \int_{X^2} f_K |J_{CP_2}| \ , \\ f_{K\equiv(s,n,k)} &= e^{is\phi}\times\frac{\rho^{n-k}}{(1+\rho^2)^k}\times(\frac{r_M}{r_0})^k \end{aligned} \tag{3.5.4}$$

These symplectic invariants transform like an infinite-dimensional unitary representation of Lorentz group.

There must exist some minimal number of symplectically non-equivalent 2-surfaces of X^3, and the magnetic fluxes over the representatives these surfaces give thus good candidates for zero modes.

1. If effective 2-dimensionality is accepted, the surfaces X_i^2 defined by the intersections of light like 3-D causal determinants X_l^3 and X^3 provide a natural identification for these 2-surfaces.

2. Without effective 2-dimensionality the situation is more complex. Since symplectic transformations leave r_M invariant, a natural set of 2-surfaces X^2 appearing in the definition of fluxes are separate pieces for $r_M = constant$ sections of 3-surface. For a generic 3-surface, these surfaces are 2-dimensional and there is continuum of them so that discrete Fourier transforms of these invariants are needed. One must however notice that $r_M = constant$ surfaces could be be 3-dimensional in which case the notion of flux is not well-defined.

3.5.3 Isometry invariants and spin glass analogy

The presence of isometry invariants implies coset space decomposition $\cup_i G/H$. This means that quantum states are characterized, not only by the vacuum functional, which is just the exponential $exp(K)$ of Kähler function (Gaussian in lowest approximation) but also by a wave function in vacuum modes. Therefore the functional integral over the WCW decomposes into an integral over zero modes for approximately Gaussian functionals determined by $exp(K)$. The weights for the various vacuum mode contributions are given by the probability density associated with the zero modes. The integration over the zero modes is a highly problematic notion and it could be eliminated if a localization in the zero modes occurs in quantum jumps. The localization would correspond to a state function reduction and zero modes would be effectively classical variables correlated in one-one manner with the quantum numbers associated with the quantum fluctuating degrees of freedom.

For a given orbit K depends on zero modes and thus one has mathematical similarity with spin glass phase for which one has probability distribution for Hamiltonians appearing in the partition function $exp(-H/T)$. In fact, since TGD Universe is also critical, exact similarity requires that also the temperature is critical for various contributions to the average partition function of spin glass phase. The characterization of isometry invariants and zero modes of the Kähler metric provides a precise characterization for how TGD Universe is quantum analog of spin glass.

The spin glass analogy has been the basic starting point in the construction of p-adic field theory limit of TGD. The ultra-metric topology for the free energy minima of spin glass phase motivates the hypothesis that effective quantum average space-time possesses ultra-metric topology. This approach leads to excellent predictions for elementary particle masses and predicts even new branches of physics [K29, K55] . As a matter fact, an entire fractal hierarchy of copies of standard physics is predicted.

3.5.4 Magnetic flux representation of the symplectic algebra

Accepting the strong form of general coordinate invariance implying effective two-dimensionality WCW Hamiltonians correspond to the fluxes associated with various 2-surfaces X_i^2 defined by the intersections of light-like light-like 3-surfaces $X_{l,i}^3$ with X^3 at the boundaries of CD considered. Bearing in mind that zero energy ontology is the correct approach, one can restrict the consideration on fluxes at $\delta M_+^4 \times CP_2$ One must also remember that if the proposed symmetries hold true, it is in principle choose any partonic 2-surface in the conjectured slicing of the Minkowskian space-time sheet to partonic 2-surfaces parametrized by the points of stringy world sheets.vA physically attractive realization of the slicings of space-time surface by 3-surfaces and string world sheets is discussed in [K23] by starting from the observation that TGD could define a natural realization of braids, braid cobordisms, and 2-knots.

Generalized magnetic fluxes

Isometry invariants are just special case of the fluxes defining natural coordinate variables for WCW. Symplectic transformations of CP_2 act as $U(1)$ gauge transformations on the Kähler potential of CP_2 (similar conclusion holds at the level of $\delta M_+^4 \times CP_2$).

One can generalize these transformations to local symplectic transformations by allowing the Hamiltonians to be products of the CP_2 Hamiltonians with the real and imaginary parts of the functions $f_{m,n,k}$ (see Eq. 3.4.22) defining the Lorentz covariant function basis H_A, $A \equiv (a, m, n, k)$ at the light cone boundary: $H_A = H_a \times f(m, n, k)$, where a labels the Hamiltonians of CP_2.

One can associate to any Hamiltonian H^A of this kind both signed and unsigned magnetic flux via the following formulas:

$$\begin{aligned} Q_m(H_A|X^2) &= \int_{X^2} H_A J \ , \\ Q_m^+(H_A|X^2) &= \int_{X^2} H_A|J| \ . \end{aligned} \tag{3.5.5}$$

Here X^2 corresponds to any surface X_i^2 resulting as intersection of X^3 with $X_{l,i}^3$. Both signed and unsigned magnetic fluxes and their superpositions

$$Q_m^{\alpha,\beta}(H_A|X^2) = \alpha Q_m(H_A|X^2) + \beta Q_m^+(H_A|X^2) \ , \ A \equiv (a,s,n,k) \tag{3.5.6}$$

provide representations of Hamiltonians. Note that symplectic invariants $Q_m^{\alpha,\beta}$ correspond to $H^A = 1$ and $H^A = f_{s,n,k}$. $H^A = 1$ can be regarded as a natural central term for the Poisson bracket algebra. Therefore, the isometry invariance of Kähler magnetic and electric gauge fluxes follows as a natural consequence.

The obvious question concerns about the correct values of the parameters α and β. One possibility is that the flux is an unsigned flux so that one has $\alpha = 0$. This option is favored by the construction of the WCW spinor structure involving the construction of the fermionic super charges anti-commuting to WCW Hamiltonians: super charges contain the square root of flux, which must be therefore unsigned. Second possibility is that magnetic fluxes are signed fluxes so that β vanishes.

One can define also the electric counterparts of the flux Hamiltonians by replacing J in the defining formulas with its dual $*J$

$$*J_{\alpha\beta} = \epsilon_{\alpha\beta}^{\ \ \gamma\delta} J_{\gamma\delta}.$$

For $H_A = 1$ these fluxes reduce to ordinary Kähler electric fluxes. These fluxes are however not symplectic covariants since the definition of the dual involves the induced metric, which is not symplectic invariant. The electric gauge fluxes for Hamiltonians in various representations of the color group ought to be important in the description of hadrons, not only as string like objects, but quite generally. These degrees of freedom would be identifiable as non-perturbative degrees of freedom involving genuinely classical Kähler field whereas quarks and gluons would correspond to the perturbative degrees of freedom, that is the interactions between CP_2 type extremals.

Poisson brackets

From the symplectic invariance of the radial component of Kähler magnetic field it follows that the Lie-derivative of the flux $Q_m^{\alpha,\beta}(H_A)$ with respect to the vector field $X(H_B)$ is given by

$$X(H_B)\cdot Q_m^{\alpha,\beta}(H_A) = Q_m^{\alpha,\beta}(\{H_B,H_A\}) \ . \tag{3.5.7}$$

The transformation properties of $Q_m^{\alpha,\beta}(H_A)$ are very nice if the basis for H_B transforms according to appropriate irreducible representation of color group and rotation group. This in turn implies that the fluxes $Q_m^{\alpha,\beta}(H_A)$ as functionals of 3-surface on given orbit provide a representation for the Hamiltonian as a functional of 3-surface. For a given surface X^3, the Poisson bracket for the two fluxes $Q_m^{\alpha,\beta}(H_A)$ and $Q_m^{\alpha,\beta}(H_B)$ can be defined as

$$\begin{aligned} \{Q_m^{\alpha,\beta}(H_A),Q_m^{\alpha,\beta}(H_B)\} &\equiv X(H_B)\cdot Q_m^{\alpha,\beta}(H_A) \\ &= Q_m^{\alpha,\beta}(\{H_A,H_B\}) = Q_m^{\alpha,\beta}(\{H_A,H_B\}) \ . \end{aligned} \tag{3.5.8}$$

The study of WCW gamma matrices identifiable as symplectic super charges demonstrates that the supercharges associated with the radial deformations vanish identically so that radial deformations correspond to zero norm degrees of freedom as one might indeed expect on physical grounds. The reason is that super generators involve the invariants $j^{ak}\gamma_k$ which vanish by $\gamma_{r_M}=0$.

The natural central extension associated with the symplectic group of CP_2 ($\{p,q\}=1!$) induces a central extension of this algebra. The central extension term resulting from $\{H_A,H_B\}$ when CP_2 Hamiltonians have $\{p,q\}=1$ equals to the symplectic invariant $Q_m^{\alpha,\beta}(f(m_a+m_b,n_a+n_b,k_a+k_b))$ on the right hand side. This extension is however anti-symmetric in symplectic degrees of freedom rather than in loop space degrees of freedom and therefore does not lead to the standard Kac Moody type algebra.

Quite generally, the Virasoro and Kac Moody algebras of string models are replaced in TGD context by much larger symmetry algebras. Kac Moody algebra corresponds to the the deformations of light-like 3-surfaces respecting their light-likeness and leaving partonic 2-surfaces at δCD intact and are highly relevant to the elementary particle physics. This algebra allows a representation in terms of X_l^3 local Hamiltonians generating isometries of $\delta M_\pm^4\times CP_2$. Hamiltonian representation is essential for super-symmetrization since fermionic super charges anti-commute to Hamiltonians rather than vector fields: this is one of the deep differences between TGD and string models. Kac-Moody algebra does not contribute to WCW metric since by definition the generators vanish at partonic 2-surfaces. This is essential for the coset space property.

A completely new algebra is the CP_2 symplectic algebra localized with respect to the light cone boundary and relevant to the configuration space geometry. This extends to $S^2\times CP_2$ -or rather $\delta M_\pm^4\times CP_2$ symplectic algebra and this gives the strongest predictions concerning WCW metric. The local radial Virasoro localized with respect to $S^2\times CP_2$ acts in zero modes and has automatically vanishing norm with respect to WCW metric defined by super charges.

3.5.5 Symplectic transformations of $\delta M_\pm^4\times CP_2$ as isometries and electric-magnetic duality

According to the construction of Kähler metric, symplectic transformations of $\delta M_\pm^4\times CP_2$ act as isometries whereas radial Virasoro algebra localized with respect to CP_2 has zero norm in the WCW metric.

Hamiltonians can be organized into light like unitary representations of $so(3,1)\times su(3)$ and the symmetry condition $Zg(X,Y)=0$ requires that the component of the metric is $so(3,1)\times su(3)$ invariant and this condition is satisfied if the component of metric between two different representations D_1 and D_2 of $so(3,1)\times su(3)$ is proportional to Glebch-Gordan coefficient $C_{D_1D_2,D_S}$ between $D_1\otimes D_2$ and singlet representation D_S. In particular, metric has components only between states having identical $so(3,1)\times su(3)$ quantum numbers.

Magnetic representation of WCW Hamiltonians means the action of the symplectic transformations of the light cone boundary as WCW isometries is an intrinsic property of the light cone boundary. If electric-magnetic duality holds true, the preferred extremal property only determines the conformal factor of the metric depending on zero modes. This is precisely as it should be if the group theoretical construction works. Hence it should be possible by a direct calculation check whether the metric defined by the magnetic flux Hamiltonians as half Poisson brackets in complex coordinates is invariant under isometries. Symplectic invariance of the metric means that matrix elements of the metric are left translates of the metric along geodesic lines starting from the origin of coordinates, which now naturally corresponds to the preferred extremal of Kähler action. Since metric derives from symplectic form this means that the matrix elements of symplectic form given by Poisson brackets of Hamiltonians must be left translates of their values at origin along geodesic line. The matrix elements in question are given by flux Hamiltonians and since symplectic transforms of flux Hamiltonian is flux Hamiltonian for the symplectic transform of Hamiltonian, it seems that the conditions are satisfied.

3.6 General expressions for the symplectic and Kähler forms

One can derive general expressions for symplectic and Kähler forms as well as Kähler metric of WCW. The fact that these expressions involve only first variation of the Kähler action implies huge simplification of the basic formulas. Duality hypothesis leads to further simplifications of the formulas.

3.6.1 Closedness requirement

The fluxes of Kähler magnetic and electric fields for the Hamiltonians of $\delta M^4_+ \times CP_2$ suggest a general representation for the components of the symplectic form of the WCW. The basic requirement is that Kähler form satisfies the defining condition

$$X \cdot J(Y,Z) + J([X,Y],Z) + J(X,[Y,Z]) \quad = \quad 0 \ , \tag{3.6.1}$$

where X, Y, Z are now vector fields associated with Hamiltonian functions defining WCW coordinates.

3.6.2 Matrix elements of the symplectic form as Poisson brackets

Quite generally, the matrix element of $J(X(H_A), X(H_B))$ between vector fields $X(H_A))$ and $X(H_B))$ defined by the Hamiltonians H_A and H_B of $\delta M^4_+ \times CP_2$ isometries is expressible as Poisson bracket

$$J^{AB} \quad = \quad J(X(H_A), X(H_B)) = \{H_A, H_B\} \ . \tag{3.6.2}$$

J^{AB} denotes contravariant components of the symplectic form in coordinates given by a subset of Hamiltonians. The magnetic flux Hamiltonians $Q_m^{\alpha,\beta}(H_{A,k})$ of Eq. 4.4.1 provide an explicit representation for the Hamiltonians at the level of WCW so that the components of the symplectic form of the WCW are expressible as classical charges for the Poisson brackets of the Hamiltonians of the light cone boundary:

$$J(X(H_A), X(H_B)) \quad = \quad Q_m^{\alpha,\beta}(\{H_A, H_B\}) \ . \tag{3.6.3}$$

Recall that the superscript α, β refers the coefficients of J and $|J|$ in the superposition of these Kähler magnetic fluxes. Note that $Q_m^{\alpha,\beta}$ contains unspecified conformal factor depending on symplectic invariants characterizing Y^3 and is unspecified superposition of signed and unsigned magnetic fluxes.

This representation does not carry information about the tangent space of space-time surface at the partonic 2-surface, which motivates the proposal that also electric fluxes are present and proportional to magnetic fluxes with a factor K, which is symplectic invariant so that commutators of flux Hamiltonians come out correctly. This would give

$$Q_m^{\alpha,\beta}(H_A)_{em} \quad = \quad Q_e^{\alpha,\beta}(H_A) + Q_m^{\alpha,\beta}(H_A) = (1+K)Q_m^{\alpha,\beta}(H_A) \ . \tag{3.6.4}$$

Since Kähler form relates to the standard field tensor by a factor $e/\hbar$, flux Hamiltonians are dimensionless so that commutators do not involve $\hbar$. The commutators would come as

$$Q_{em}^{\alpha,\beta}(\{H_A, H_B\}) \rightarrow (1+K)Q_m^{\alpha,\beta}(\{H_A, H_B\}) \ . \tag{3.6.5}$$

The factor $1+K$ plays the same role as Planck constant in the commutators.

WCW Hamiltonians vanish for the extrema of the Kähler function as variational derivatives of the Kähler action. Hence Hamiltonians are good candidates for the coordinates appearing as coordinates in the perturbative functional integral around extrema (with maxima giving dominating contribution). It is clear that WCW coordinates around a given extremum include only those Hamiltonians, which vanish at extremum (that is those Hamiltonians which span the tangent space of G/H) In Darboux coordinates the Poisson brackets reduce to the symplectic form

$$\begin{aligned} \{P^I, Q^J\} \quad &= \quad J^{IJ} = J_I \delta^{I,J} \ . \\ J_I \quad &= \quad 1 \ . \end{aligned} \tag{3.6.6}$$

It is not clear whether Darboux coordinates with $J_I = 1$ are possible in the recent case: probably the unit matrix on right hand side of the defining equation is replaced with a diagonal matrix depending on symplectic invariants so that one has $J_I \neq 1$. The integration measure is given by the symplectic volume element given by the determinant of the matrix defined by the Poisson brackets of the Hamiltonians appearing as coordinates. The value of the symplectic volume element is given by the matrix formed by the Poisson brackets of the Hamiltonians and reduces to the product

$$Vol = \prod_I J_I$$

in generalized Darboux coordinates.

Kähler potential (that is gauge potential associated with Kähler form) can be written in Darboux coordinates as

$$A = \sum_I J_I P_I dQ^I \ . \tag{3.6.7}$$

3.6.3 General expressions for Kähler form, Kähler metric and Kähler function

The expressions of Kähler form and Kähler metric in complex coordinates can obtained by transforming the contravariant form of the symplectic form from symplectic coordinates provided by Hamiltonians to complex coordinates:

$$J^{Z^i\bar{Z}^j} = iG^{Z^i\bar{Z}^j} = \partial_{H^A} Z^i \partial_{H^B} \bar{Z}^j J^{AB} \ , \tag{3.6.8}$$

where J^{AB} is given by the classical Kahler charge for the light cone Hamiltonian $\{H^A, H^B\}$. Complex coordinates correspond to linear coordinates of the complexified Lie-algebra providing exponentiation of the isometry algebra via exponential mapping. What one must know is the precise relationship between allowed complex coordinates and Hamiltonian coordinates: this relationship is in principle calculable. In Darboux coordinates the expressions become even simpler:

$$J^{Z^i\bar{Z}^j} = iG^{Z^i\bar{Z}^j} = \sum_I J(I)(\partial_{P^i} Z^i \partial_{Q^I} \bar{Z}^j - \partial_{Q^I} Z^i \partial_{P^I} \bar{Z}^j) \ . \tag{3.6.9}$$

Kähler function can be formally integrated from the relationship

$$\begin{aligned} A_{Z^i} &= i\partial_{Z^i} K \ , \\ A_{\bar{Z}^i} &= -i\partial_{Z^i} K \ . \end{aligned} \tag{3.6.10}$$

holding true in complex coordinates. Kähler function is obtained formally as integral

$$K = \int_0^Z (A_{Z^i} dZ^i - A_{\bar{Z}^i} d\bar{Z}^i) \ . \tag{3.6.11}$$

3.6.4 $Diff(X^3)$ invariance and degeneracy and conformal invariances of the symplectic form

$J(X(H_A), X(H_B))$ defines symplectic form for the coset space G/H only if it is $Diff(X^3)$ degenerate. This means that the symplectic form $J(X(H_A), X(H_B))$ vanishes whenever Hamiltonian H_A or H_B is such that it generates diffeomorphism of the 3-surface X^3. If effective 2-dimensionality holds true, $J(X(H_A), X(H_B))$ vanishes if H_A or H_B generates two-dimensional diffeomorphism $d(H_A)$ at the surface X^2_i.

One can always write

$$J(X(H_A),X(H_B)) = X(H_A)Q(H_B|X_i^2) \ .$$

If H_A generates diffeomorphism, the action of $X(H_A)$ reduces to the action of the vector field X_A of some X_i^2-diffeomorphism. Since $Q(H_B|r_M)$ is manifestly invariant under the diffemorphisms of X^2, the result is vanishing:

$$X_AQ(H_B|X_i^2) = 0 \ ,$$

so that $Diff^2$ invariance is achieved.

The radial diffeomorphisms possibly generated by the radial Virasoro algebra do not produce trouble. The change of the flux integrand X under the infinitesimal transformation $r_M \rightarrow r_M + \epsilon r_M^n$ is given by $r_M^n dX/dr_M$. Replacing r_M with $r_M^{-n+1}/(-n+1)$ as variable, the integrand reduces to a total divergence dX/du the integral of which vanishes over the closed 2-surface X_i^2. Hence radial Virasoro generators having zero norm annihilate all matrix elements of the symplectic form. The induced metric of X_i^2 induces a unique conformal structure and since the conformal transformations of X_i^2 can be interpreted as a mere coordinate changes, they leave the flux integrals invariant.

3.6.5 Complexification and explicit form of the metric and Kähler form

The identification of the Kähler form and Kähler metric in symplectic degrees of freedom follows trivially from the identification of the symplectic form and definition of complexification. The requirement that Hamiltonians are eigen states of angular momentum (and possibly Lorentz boost generator), isospin and hypercharge implies physically natural complexification. In order to fix the complexification completely one must introduce some convention fixing which states correspond to 'positive' frequencies and which to 'negative frequencies' and which to zero frequencies that is to decompose the generators of the symplectic algebra to three sets Can_+, Can_- and Can_0. One must distinguish between Can_0 and zero modes, which are not considered here at all. For instance, CP_2 Hamiltonians correspond to zero modes.

The natural complexification relies on the imaginary part of the radial conformal weight whereas the real part defines the $g = t + h$ decomposition naturally. The wave vector associated with the radial logarithmic plane wave corresponds to the angular momentum quantum number associated with a wave in S^1 in the case of Kac Moody algebra. One can imagine three options.

1. It is quite possible that the spectrum of k_2 does not contain $k_2 = 0$ at all so that the sector Can_0 could be empty. This complexification is physically very natural since it is manifestly invariant under $SU(3)$ and $SO(3)$ defining the preferred spherical coordinates. The choice of $SO(3)$ is unique if the classical four-momentum associated with the 3-surface is time like so that there are no problems with Lorentz invariance.

2. If $k_2 = 0$ is possible one could have

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k=k1+ik_2}, k_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = 0\} \ . \end{aligned} \tag{3.6.12}$$

3. If it is possible to $n_2 \neq 0$ for $k_2 = 0$, one could define the decomposition as

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k}, k_2 > 0 \ \text{ or } \ k_2 = 0, n_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0 \ \text{ or} k_2 = 0, n_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = n_2 = 0\} \ . \end{aligned} \tag{3.6.13}$$

   In this case the complexification is unique and Lorentz invariance guaranteed if one can fix the $SO(2)$ subgroup uniquely. The quantization axis of angular momentum could be chosen to be the direction of the classical angular momentum associated with the 3-surface in its rest system.

The only thing needed to get Kähler form and Kähler metric is to write the half Poisson bracket defined by Eq. 3.9.15

$$\begin{aligned} J_f(X(H_A), X(H_B)) &= 2Im\,(iQ_f(\{H_A, H_B\}_{-+}))\ , \\ G_f(X(H_A), X(H_B)) &= 2Re\,(iQ_f(\{H_A, H_B\}_{-+}))\ . \end{aligned} \tag{3.6.14}$$

Symplectic form, and thus also Kähler form and Kähler metric, could contain a conformal factor depending on the isometry invariants characterizing the size and shape of the 3-surface. At this stage one cannot say much about the functional form of this factor.

3.6.6 Comparison of CP_2 Kähler geometry with configuration space geometry

The explicit discussion of the role of $g = t + h$ decomposition of the tangent space of WCW provides deep insights to the metric of the symmetric space. There are indeed many questions to be answered. To what point of WCW (that is 3-surface) the proposed $g = t + h$ decomposition corresponds to? Can one derive the components of the metric and Kähler form from the Poisson brackets of complexified Hamiltonians? Can one characterize the point in question in terms of the properties of WCW Hamiltonians? Does the central extension of WCW reduce to the symplectic central extension of the symplectic algebra or can one consider also other options?

Cartan decomposition for CP_2

A good manner to gain understanding is to consider the CP_2 metric and Kähler form at the origin of complex coordinates for which the sub-algebra $h = u(2)$ defines the Cartan decomposition.

1. $g = t + h$ decomposition depends on the point of the symmetric space in general. In case of CP_2 $u(2)$ sub-algebra transforms as $g \circ u(2) \circ g^{-1}$ when the point s is replaced by gsg^{-1}. This is expected to hold true also in case of WCW (unless it is flat) so that the task is to identify the point of WCW at which the proposed decomposition holds true.

2. The Killing vector fields of h sub-algebra vanish at the origin of CP_2 in complex coordinates. The corresponding Hamiltonians need not vanish but their Poisson brackets must vanish. It is possible to add suitable constants to the Hamiltonians in order to guarantee that they vanish at origin.

3. It is convenient to introduce complex coordinates and decompose isometry generators to holomorphic components $J^a_+ = j^{ak}\partial_k$ and $j^a_- = j^{a\bar{k}}\partial_{\bar{k}}$. One can introduce what might be called half Poisson bracket and half inner product defined as

$$\begin{aligned} \{H^a, H^b\}_{-+} &\equiv \partial_{\bar{k}} H^a J^{\bar{k}l} \partial_l H^b \\ &= j^{ak} J_{k\bar{l}} j^{b\bar{l}} = -i(j^a_+, j^b_-)\ . \end{aligned} \tag{3.6.15}$$

One can express Poisson bracket of Hamiltonians and the inner product of the corresponding Killing vector fields in terms of real and imaginary parts of the half Poisson bracket:

$$\begin{aligned} \{H^a, H^b\} &= 2Im\left(i\{H^a, H^b\}_{-+}\right)\ , \\ (j^a, j^b) &= 2Re\left(i(j^a_+, j^b_-)\right) = 2Re\left(i\{H^a, H^b\}_{-+}\right)\ . \end{aligned} \tag{3.6.16}$$

What this means that Hamiltonians and their half brackets code all information about metric and Kähler form. Obviously this is of utmost importance in the case of the WCW metric whose symplectic structure and central extension are derived from those of CP_2.

Consider now the properties of the metric and Kähler form at the origin.

1. The relations satisfied by the half Poisson brackets can be written symbolically as

$$\begin{aligned} &\{h,h\}_{-+} = 0 \ , \\ &Re\,(i\{h,t\}_{-+}) = 0 \ , \quad Im\,(i\{h,t\}_{-+}) = 0 \ , \\ &Re\,(i\{t,t\}_{-+}) \neq 0 \ , \quad Im\,(i\{t,t\}_{-+}) \neq 0 \ . \end{aligned} \tag{3.6.17}$$

2. The first two conditions state that h vector fields have vanishing inner products at the origin. The first condition states also that the Hamiltonians for the commutator algebra $[h,h] = SU(2)$ vanish at origin whereas the Hamiltonian for $U(1)$ algebra corresponding to the color hyper charge need not vanish although it can be made vanishing. The third condition implies that the Hamiltonians of t vanish at origin.

3. The last two conditions state that the Kähler metric and form are non-vanishing between the elements of t. Since the Poisson brackets of t Hamiltonians are Hamiltonians of h, the only possibility is that $\{t,t\}$ Poisson brackets reduce to a non-vanishing $U(1)$ Hamiltonian at the origin or that the bracket at the origin is due to the symplectic central extension. The requirement that all Hamiltonians vanish at origin is very attractive aesthetically and forces to interpret $\{t,t\}$ brackets at origin as being due to a symplectic central extension. For instance, for S^2 the requirement that Hamiltonians vanish at origin would mean the replacement of the Hamiltonian $H = cos(\theta)$ representing a rotation around z-axis with $H_3 = cos(\theta) - 1$ so that the Poisson bracket of the generators H_1 and H_2 can be interpreted as a central extension term.

4. The conditions for the Hamiltonians of $u(2)$ sub-algebra state that their variations with respect to g vanish at origin. Thus $u(2)$ Hamiltonians have extremum value at origin.

5. Also the Kähler function of CP_2 has extremum at the origin. This suggests that in the case of the WCW the counterpart of the origin corresponds to the maximum of the Kähler function.

Cartan algebra decomposition at the level of WCW

The discussion of the properties of CP_2 Kähler metric at origin provides valuable guide lines in an attempt to understand what happens at the level of WCW. The use of the half bracket for WCW Hamiltonians in turn allows to calculate the matrix elements of the WCW metric and Kähler form explicitly in terms of the magnetic or electric flux Hamiltonians.

The earlier construction was rather tricky and formula-rich and not very convincing physically. Cartan decomposition had to be assigned with something and in lack of anything better it was assigned with Super Virasoro algebra, which indeed allows this kind of decompositions but without any strong physical justification. The realization that super-symplectic and super Kac-Moody symmetries define coset construction at the level of basic quantum TGD. The wrong conclusions were that this construction provides a realization of Equivalence Principle (EP) at microscopic level that the coset space decomposition of WCW realizes EP geometrically. At quantum level the EP reduces to Quantum Classical Correspondence (QCC). At classical level EP reduces to the fact that GRT space-time follows naturally as an effective description of many-sheeted space-time [K56] (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book).

It must be however emphasized that holography implying effective 2-dimensionality of 3-surfaces in some length scale resolution is absolutely essential for this construction since it allows to effectively reduce Kac-Moody generators associated with X_l^3 to $X^2 = X_l^3 \cap \delta M_\pm^4 \times CP_2$. In the similar manner super-symplectic generators can be dimensionally reduced to X^2. Number theoretical compactification forces the dimensional reduction and the known extremals are consistent with it [K5] . The construction of WCW spinor structure and metric in terms of the second quantized spinor fields [K9] relies to this picture as also the recent view about M-matrix [K12] .

In this framework the coset space decomposition becomes trivial.

1. The algebra g is labeled by color quantum numbers of CP_2 Hamiltonians and by the label (m, n, k) labeling the function basis of the light cone boundary. Also a localization with respect to X^2 is needed. This is a new element as compared to the original view.

2. Super Kac-Moody algebra is labeled by color octet Hamiltonians and function basis of X^2. Since Lie-algebra action does not lead out of irreps, this means that Cartan algebra decomposition is satisfied.

3.6.7 Comparison with loop groups

It is useful to compare the recent approach to the geometrization of the loop groups consisting of maps from circle to Lie Freed , which served as the inspirer of the WCW geometry approach but later turned out to not apply as such in TGD framework.

In the case of loop groups the tangent space T corresponds to the local Lie-algebra $T(k, A) = exp(ik\phi)T_A$, where T_A generates the finite-dimensional Lie-algebra g and ϕ denotes the angle variable of circle; k is integer. The complexification of the tangent space corresponds to the decomposition

$$T = \{X(k > 0, A)\} \oplus \{X(k < 0, A)\} \oplus \{X(k = 0, A)\} = T_+ \oplus T_- \oplus T_0$$

of the tangent space. Metric corresponds to the central extension of the loop algebra to Kac Moody algebra and the Kähler form is given by

$$J(X(k_1 < 0, A), X(k_2 > 0, B)) = k_2 \delta(k_1 + k_2)\delta(A, B) \ .$$

In present case the finite dimensional Lie algebra g is replaced with the Lie-algebra of the symplectic transformations of $\delta M^4_+ \times CP_2$ centrally extended using symplectic extension. The scalar function basis on circle is replaced with the function basis on an interval of length Δr_M with periodic boundary conditions; effectively one has circle also now.

The basic difference is that one can consider two kinds of central extensions now.

1. Central extension is most naturally induced by the natural central extension ($\{p, q\} = 1$) defined by Poisson bracket. This extension is anti-symmetric with respect to the generators of the symplectic group: in the case of the Kac Moody central extension it is symmetric with respect to the group G. The symplectic transformations of CP_2 might correspond to non-zero modes also because they are not exact symmetries of Kähler action. The situation is however rather delicate since $k = 0$ light cone harmonic has a diverging norm due to the radial integration unless one poses both lower and upper radial cutoffs although the matrix elements would be still well defined for typical 3-surfaces. For Kac Moody group $U(1)$ transformations correspond to the zero modes. Light cone function algebra can be regarded as a local $U(1)$ algebra defining central extension in the case that only CP_2 symplectic transformations local with respect to δM^4_+ act as isometries: for Kac Moody algebra the central extension corresponds to an ordinary $U(1)$ algebra. In the case that entire light cone symplectic algebra defines the isometries the central extension reduces to a $U(1)$ central extension.

3.7 Magnetic and electric representations of the configuration space Hamiltonians

Symmetry considerations lead to the hypothesis that WCW Hamiltonians are apart from a factor depending on symplectic invariants equal to magnetic flux Hamiltonians. On the other hand, the hypothesis that Kähler function corresponds to a preferred extremal of Kähler action leads to the hypothesis that WCW Hamiltonians corresponds to classical charges associated with the Hamiltonians of the light cone boundary. These charges are very much like electric charges. The requirement that two approaches are equivalent leads to the hypothesis that magnetic and electric Hamiltonians are identical apart from a factor depending on isometry invariants. At the level of CP_2 corresponding duality corresponds to the self-duality of Kähler form stating that the magnetic and electric parts of Kähler form are identical.

3.7.1 Radial symplectic invariants

All $\delta M_+^4 \times CP_2$ symplectic transformations leave invariant the value of the radial coordinate r_M. Therefore the radial coordinate r_M of X^3 regarded as a function of $S^2 \times CP_2$ coordinates serves as height function. The number, type, ordering and values for the extrema for this height function in the interior and boundary components are isometry invariants. These invariants characterize not only the topology but also the size and shape of the 3-surface. The result implies that WCW metric indeed differentiates between 3-surfaces with the size of Planck length and with the size of galaxy. The characterization of these invariants reduces to Morse theory. The extrema correspond to topology changes for the two-dimensional (one-dimensional) $r_M = constant$ section of 3-surface (boundary of 3-surface). The height functions of sphere and torus serve as a good illustrations of the situation. A good example about non-topological extrema is provided by a sphere with two horns.

There are additional symplectic invariants. The 'magnetic fluxes' associated with the δM_+^4 symplectic form

$$J_{S^2} = r_M^2 sin(\theta) d\theta \wedge d\phi$$

over any $X^2 \subset X^3$ are symplectic invariants. In particular, the integrals over $r_M = constant$ sections (assuming them to be 2-dimensional) are symplectic invariants. They give simply the solid angle $\Omega(r_M)$ spanned by $r_M = constant$ section and thus $r_M^2 \Omega(r_M)$ characterizes transversal geometric size of the 3-surface. A convenient manner to discretize these invariants is to consider the Fourier components of these invariants in radial logarithmic plane wave basis discussed earlier:

$$\Omega(k) = \int_{r_{min}}^{r_{max}} (r_M/r_{max})^k \Omega(r_M) \frac{dr_M}{r_M} \ , \ k = k_1 + ik_2 \ , \ perk_1 \geq 0 \ . \tag{3.7.1}$$

One must take into account that for each section in which the topology of $r_M = constant$ section remains constant one must associate invariants with separate components of the two-dimensional section. For a given value of r_M, r_M constant section contains several components (to visualize the situation consider torus as an example).

Also the quantities

$$\Omega^+(X^2) = \int_{X^2} |J| \equiv \int |\epsilon^{\alpha\beta} J_{\alpha\beta}| \sqrt{g_2} d^2x$$

are symplectic invariants and provide additional geometric information about 3-surface. These fluxes are non-vanishing also for closed surfaces and give information about the geometry of the boundary components of 3-surface (signed fluxes vanish for boundary components unless they enclose the dip of the light cone).

Since zero norm generators remain invariant under complexification, their contribution to the Kähler metric vanishes. It is not at all obvious whether WCW integration measure in these degrees of freedom exists at all. A localization in zero modes occurring in each quantum jump seems a more plausible and under suitable additional assumption it would have interpretation as a state function reduction. In string model similar situation is encountered; besides the functional integral determined by string action, one has integral over the moduli space.

If the effective 2-dimensionality implied by the strong form of general coordinate invariance discussed in the introduction is accepted, there is no need to integrate over the variable r_M and just the fluxes over the 2-surfaces X_i^2 identified as intersections of light like 3-D causal determinants with X^3 contain the data relevant for the construction of the WCW geometry. Also the symplectic invariants associated with these surfaces are enough.

3.7.2 Kähler magnetic invariants

The Kähler magnetic fluxes defined both the normal component of the Kähler magnetic field and by its absolute value

$$\begin{aligned} Q_m(X^2) &= \int_{X^2} J_{CP_2} = J_{\alpha\beta} \epsilon^{\alpha\beta} \sqrt{g_2} d^2x \ , \\ Q_m^+(X^2) &= \int_{X^2} |J_{CP_2}| \equiv \int_{X^2} |J_{\alpha\beta} \epsilon^{\alpha\beta}| \sqrt{g_2} d^2x \ , \end{aligned} \tag{3.7.2}$$

over suitably defined 2-surfaces are invariants under both Lorentz isometries and the symplectic transformations of CP_2 and can be calculated once X^3 is given.

For a closed surface $Q_m(X^2)$ vanishes unless the homology equivalence class of the surface is nontrivial in CP_2 degrees of freedom. In this case the flux is quantized. $Q_M^+(X^2)$ is non-vanishing for closed surfaces, too. Signed magnetic fluxes over non-closed surfaces depend on the boundary of X^2 only:

$$\begin{array}{l} \int_{X^2} J = \int_{\delta X^2} A \ . \\ J = dA \ . \end{array}$$

Un-signed fluxes can be written as sum of similar contributions over the boundaries of regions of X^2 in which the sign of J remains fixed.

$$\begin{array}{lll} Q_m(X^2) & = & \int_{X^2} J_{CP_2} = J_{\alpha\beta}\epsilon^{\alpha\beta}\sqrt{g_2}d^2x \ , \\ Q_m^+(X^2) & = & \int_{X^2} |J_{CP_2}| \equiv \int_{X^2} |J_{\alpha\beta}\epsilon^{\alpha\beta}|\sqrt{g_2}d^2x \ , \end{array} \tag{3.7.3}$$

There are also symplectic invariants, which are Lorentz covariants and defined as

$$\begin{array}{lll} Q_m(K,X^2) & = & \int_{X^2} f_K J_{CP_2} \ , \\ Q_m^+(K,X^2) & = & \int_{X^2} f_K |J_{CP_2}| \ , \\ f_{K\equiv(s,n,k)} & = & e^{is\phi} \times \frac{\rho^{n-k}}{(1+\rho^2)^k} \times (\frac{r_M}{r_0})^k \end{array} \tag{3.7.4}$$

These symplectic invariants transform like an infinite-dimensional unitary representation of Lorentz group.

There must exist some minimal number of symplectically non-equivalent 2-surfaces of X^3, and the magnetic fluxes over the representatives these surfaces give thus good candidates for zero modes.

1. If effective 2-dimensionality is accepted, the surfaces X_i^2 defined by the intersections of light like 3-D causal determinants X_l^3 and X^3 provide a natural identification for these 2-surfaces.

2. Without effective 2-dimensionality the situation is more complex. Since symplectic transformations leave r_M invariant, a natural set of 2-surfaces X^2 appearing in the definition of fluxes are separate pieces for $r_M = constant$ sections of 3-surface. For a generic 3-surface, these surfaces are 2-dimensional and there is continuum of them so that discrete Fourier transforms of these invariants are needed. One must however notice that $r_M = constant$ surfaces could be be 3-dimensional in which case the notion of flux is not well-defined.

3.7.3 Isometry invariants and spin glass analogy

The presence of isometry invariants implies coset space decomposition $\cup_i G/H$. This means that quantum states are characterized, not only by the vacuum functional, which is just the exponential $exp(K)$ of Kähler function (Gaussian in lowest approximation) but also by a wave function in vacuum modes. Therefore the functional integral over the WCW decomposes into an integral over zero modes for approximately Gaussian functionals determined by $exp(K)$. The weights for the various vacuum mode contributions are given by the probability density associated with the zero modes. The integration over the zero modes is a highly problematic notion and it could be eliminated if a localization in the zero modes occurs in quantum jumps. The localization would correspond to a state function reduction and zero modes would be effectively classical variables correlated in one-one manner with the quantum numbers associated with the quantum fluctuating degrees of freedom.

For a given orbit K depends on zero modes and thus one has mathematical similarity with spin glass phase for which one has probability distribution for Hamiltonians appearing in the partition function $exp(-H/T)$. In fact, since TGD Universe is also critical, exact similarity requires that also the temperature is critical for various contributions to the average partition function of spin glass phase. The characterization of isometry invariants and zero modes of the Kähler metric provides a precise characterization for how TGD Universe is quantum analog of spin glass.

The spin glass analogy has been the basic starting point in the construction of p-adic field theory limit of TGD. The ultra-metric topology for the free energy minima of spin glass phase motivates the hypothesis that effective quantum average space-time possesses ultra-metric topology. This approach leads to excellent predictions for elementary particle masses and predicts even new branches of physics [K29, K55] . As a matter fact, an entire fractal hierarchy of copies of standard physics is predicted.

3.7.4 Magnetic flux representation of the symplectic algebra

Accepting the strong form of general coordinate invariance implying effective two-dimensionality WCW Hamiltonians correspond to the fluxes associated with various 2-surfaces X^2_i defined by the intersections of light-like light-like 3-surfaces $X^3_{l,i}$ with X^3 at the boundaries of CD considered. Bearing in mind that zero energy ontology is the correct approach, one can restrict the consideration on fluxes at $\delta M^4_+ \times CP_2$ One must also remember that if the proposed symmetries hold true, it is in principle choose any partonic 2-surface in the conjectured slicing of the Minkowskian space-time sheet to partonic 2-surfaces parametrized by the points of stringy world sheets.vA physically attractive realization of the slicings of space-time surface by 3-surfaces and string world sheets is discussed in [K23] by starting from the observation that TGD could define a natural realization of braids, braid cobordisms, and 2-knots.

Generalized magnetic fluxes

Isometry invariants are just special case of the fluxes defining natural coordinate variables for WCW. Symplectic transformations of CP_2 act as $U(1)$ gauge transformations on the Kähler potential of CP_2 (similar conclusion holds at the level of $\delta M^4_+ \times CP_2$).

One can generalize these transformations to local symplectic transformations by allowing the Hamiltonians to be products of the CP_2 Hamiltonians with the real and imaginary parts of the functions $f_{m,n,k}$ (see Eq. 3.4.22) defining the Lorentz covariant function basis H_A, $A \equiv (a, m, n, k)$ at the light cone boundary: $H_A = H_a \times f(m, n, k)$, where a labels the Hamiltonians of CP_2.

One can associate to any Hamiltonian H^A of this kind both signed and unsigned magnetic flux via the following formulas:

$$\begin{aligned} Q_m(H_A|X^2) &= \int_{X^2} H_A J \ , \\ Q^+_m(H_A|X^2) &= \int_{X^2} H_A |J| \ . \end{aligned} \tag{3.7.5}$$

Here X^2 corresponds to any surface X^2_i resulting as intersection of X^3 with $X^3_{l,i}$. Both signed and unsigned magnetic fluxes and their superpositions

$$Q^{\alpha,\beta}_m(H_A|X^2) = \alpha Q_m(H_A|X^2) + \beta Q^+_m(H_A|X^2) \ , \ A \equiv (a, s, n, k) \tag{3.7.6}$$

provide representations of Hamiltonians. Note that symplectic invariants $Q^{\alpha,\beta}_m$ correspond to $H^A = 1$ and $H^A = f_{s,n,k}$. $H^A = 1$ can be regarded as a natural central term for the Poisson bracket algebra. Therefore, the isometry invariance of Kähler magnetic and electric gauge fluxes follows as a natural consequence.

The obvious question concerns about the correct values of the parameters α and β. One possibility is that the flux is an unsigned flux so that one has $\alpha = 0$. This option is favored by the construction of the WCW spinor structure involving the construction of the fermionic super charges anti-commuting to WCW Hamiltonians: super charges contain the square root of flux, which must be therefore unsigned. Second possibility is that magnetic fluxes are signed fluxes so that β vanishes.

One can define also the electric counterparts of the flux Hamiltonians by replacing J in the defining formulas with its dual $*J$

$$*J_{\alpha\beta} = \epsilon_{\alpha\beta}^{\ \ \gamma\delta} J_{\gamma\delta} .$$

For $H_A = 1$ these fluxes reduce to ordinary Kähler electric fluxes. These fluxes are however not symplectic covariants since the definition of the dual involves the induced metric, which is not symplectic invariant. The electric gauge fluxes for Hamiltonians in various representations of the color group ought to be important in the description of hadrons, not only as string like objects, but quite generally. These degrees of freedom would be identifiable as non-perturbative degrees of freedom involving genuinely classical Kähler field whereas quarks and gluons would correspond to the perturbative degrees of freedom, that is the interactions between CP_2 type extremals.

Poisson brackets

From the symplectic invariance of the radial component of Kähler magnetic field it follows that the Lie-derivative of the flux $Q_m^{\alpha,\beta}(H_A)$ with respect to the vector field $X(H_B)$ is given by

$$X(H_B)\cdot Q_m^{\alpha,\beta}(H_A) \quad = \quad Q_m^{\alpha,\beta}(\{H_B,H_A\}) \ . \tag{3.7.7}$$

The transformation properties of $Q_m^{\alpha,\beta}(H_A)$ are very nice if the basis for H_B transforms according to appropriate irreducible representation of color group and rotation group. This in turn implies that the fluxes $Q_m^{\alpha,\beta}(H_A)$ as functionals of 3-surface on given orbit provide a representation for the Hamiltonian as a functional of 3-surface. For a given surface X^3, the Poisson bracket for the two fluxes $Q_m^{\alpha,\beta}(H_A)$ and $Q_m^{\alpha,\beta}(H_B)$ can be defined as

$$\begin{aligned} \{Q_m^{\alpha,\beta}(H_A),Q_m^{\alpha,\beta}(H_B)\} &\equiv X(H_B)\cdot Q_m^{\alpha,\beta}(H_A) \\ &= Q_m^{\alpha,\beta}(\{H_A,H_B\}) = Q_m^{\alpha,\beta}(\{H_A,H_B\}) \ . \end{aligned} \tag{3.7.8}$$

The study of WCW gamma matrices identifiable as symplectic super charges demonstrates that the supercharges associated with the radial deformations vanish identically so that radial deformations correspond to zero norm degrees of freedom as one might indeed expect on physical grounds. The reason is that super generators involve the invariants $j^{ak}\gamma_k$ which vanish by $\gamma_{r_M} = 0$.

The natural central extension associated with the symplectic group of CP_2 ($\{p,q\} = 1!$) induces a central extension of this algebra. The central extension term resulting from $\{H_A,H_B\}$ when CP_2 Hamiltonians have $\{p,q\} = 1$ equals to the symplectic invariant $Q_m^{\alpha,\beta}(f(m_a+m_b,n_a+n_b,k_a+k_b))$ on the right hand side. This extension is however anti-symmetric in symplectic degrees of freedom rather than in loop space degrees of freedom and therefore does not lead to the standard Kac Moody type algebra.

Quite generally, the Virasoro and Kac Moody algebras of string models are replaced in TGD context by much larger symmetry algebras. Kac Moody algebra corresponds to the the deformations of light-like 3-surfaces respecting their light-likeness and leaving partonic 2-surfaces at δCD intact and are highly relevant to the elementary particle physics. This algebra allows a representation in terms of X_l^3 local Hamiltonians generating isometries of $\delta M_\pm^4 \times CP_2$. Hamiltonian representation is essential for super-symmetrization since fermionic super charges anti-commute to Hamiltonians rather than vector fields: this is one of the deep differences between TGD and string models. Kac-Moody algebra does not contribute to WCW metric since by definition the generators vanish at partonic 2-surfaces. This is essential for the coset space property.

A completely new algebra is the CP_2 symplectic algebra localized with respect to the light cone boundary and relevant to the configuration space geometry. This extends to $S^2 \times CP_2$ -or rather $\delta M_\pm^4 \times CP_2$ symplectic algebra and this gives the strongest predictions concerning WCW metric. The local radial Virasoro localized with respect to $S^2 \times CP_2$ acts in zero modes and has automatically vanishing norm with respect to WCW metric defined by super charges.

3.7.5 Symplectic transformations of $\delta M^4_\pm \times CP_2$ as isometries and electric-magnetic duality

According to the construction of Kähler metric, symplectic transformations of $\delta M^4_\pm \times CP_2$ act as isometries whereas radial Virasoro algebra localized with respect to CP_2 has zero norm in the WCW metric.

Hamiltonians can be organized into light like unitary representations of $so(3,1) \times su(3)$ and the symmetry condition $Zg(X,Y) = 0$ requires that the component of the metric is $so(3,1) \times su(3)$ invariant and this condition is satisfied if the component of metric between two different representations D_1 and D_2 of $so(3,1) \times su(3)$ is proportional to Glebch-Gordan coefficient $C_{D_1 D_2, D_S}$ between $D_1 \otimes D_2$ and singlet representation D_S. In particular, metric has components only between states having identical $so(3,1) \times su(3)$ quantum numbers.

Magnetic representation of WCW Hamiltonians means the action of the symplectic transformations of the light cone boundary as WCW isometries is an intrinsic property of the light cone boundary. If electric-magnetic duality holds true, the preferred extremal property only determines the conformal factor of the metric depending on zero modes. This is precisely as it should be if the group theoretical construction works. Hence it should be possible by a direct calculation check whether the metric defined by the magnetic flux Hamiltonians as half Poisson brackets in complex coordinates is invariant under isometries. Symplectic invariance of the metric means that matrix elements of the metric are left translates of the metric along geodesic lines starting from the origin of coordinates, which now naturally corresponds to the preferred extremal of Kähler action. Since metric derives from symplectic form this means that the matrix elements of symplectic form given by Poisson brackets of Hamiltonians must be left translates of their values at origin along geodesic line. The matrix elements in question are given by flux Hamiltonians and since symplectic transforms of flux Hamiltonian is flux Hamiltonian for the symplectic transform of Hamiltonian, it seems that the conditions are satisfied.

3.8 General expressions for the symplectic and Kähler forms

One can derive general expressions for symplectic and Kähler forms as well as Kähler metric of WCW. The fact that these expressions involve only first variation of the Kähler action implies huge simplification of the basic formulas. Duality hypothesis leads to further simplifications of the formulas.

3.8.1 Closedness requirement

The fluxes of Kähler magnetic and electric fields for the Hamiltonians of $\delta M^4_+ \times CP_2$ suggest a general representation for the components of the symplectic form of the WCW. The basic requirement is that Kähler form satisfies the defining condition

$$X \cdot J(Y,Z) + J([X,Y],Z) + J(X,[Y,Z]) \quad = \quad 0 \ , \tag{3.8.1}$$

where X, Y, Z are now vector fields associated with Hamiltonian functions defining WCW coordinates.

3.8.2 Matrix elements of the symplectic form as Poisson brackets

Quite generally, the matrix element of $J(X(H_A), X(H_B))$ between vector fields $X(H_A))$ and $X(H_B))$ defined by the Hamiltonians H_A and H_B of $\delta M^4_+ \times CP_2$ isometries is expressible as Poisson bracket

$$J^{AB} \quad = \quad J(X(H_A), X(H_B)) = \{H_A, H_B\} \ . \tag{3.8.2}$$

J^{AB} denotes contravariant components of the symplectic form in coordinates given by a subset of Hamiltonians. The magnetic flux Hamiltonians $Q^{\alpha,\beta}_m(H_{A,k})$ of Eq. 4.4.1 provide an explicit representation for the Hamiltonians at the level of WCW so that the components of the symplectic form of the WCW are expressible as classical charges for the Poisson brackets of the Hamiltonians of the light cone boundary:

$$J(X(H_A),X(H_B)) = Q_m^{\alpha,\beta}(\{H_A,H_B\}) \ . \tag{3.8.3}$$

Recall that the superscript α,β refers the coefficients of J and $|J|$ in the superposition of these Kähler magnetic fluxes. Note that $Q_m^{\alpha,\beta}$ contains unspecified conformal factor depending on symplectic invariants characterizing Y^3 and is unspecified superposition of signed and unsigned magnetic fluxes.

This representation does not carry information about the tangent space of space-time surface at the partonic 2-surface, which motivates the proposal that also electric fluxes are present and proportional to magnetic fluxes with a factor K, which is symplectic invariant so that commutators of flux Hamiltonians come out correctly. This would give

$$Q_m^{\alpha,\beta}(H_A)_{em} = Q_e^{\alpha,\beta}(H_A)+Q_m^{\alpha,\beta}(H_A)=(1+K)Q_m^{\alpha,\beta}(H_A) \ . \tag{3.8.4}$$

Since Kähler form relates to the standard field tensor by a factor $e/\hbar$, flux Hamiltonians are dimensionless so that commutators do not involve $\hbar$. The commutators would come as

$$Q_{em}^{\alpha,\beta}(\{H_A,H_B\}) \rightarrow (1+K)Q_m^{\alpha,\beta}(\{H_A,H_B\}) \ . \tag{3.8.5}$$

The factor $1+K$ plays the same role as Planck constant in the commutators.

WCW Hamiltonians vanish for the extrema of the Kähler function as variational derivatives of the Kähler action. Hence Hamiltonians are good candidates for the coordinates appearing as coordinates in the perturbative functional integral around extrema (with maxima giving dominating contribution). It is clear that WCW coordinates around a given extremum include only those Hamiltonians, which vanish at extremum (that is those Hamiltonians which span the tangent space of G/H) In Darboux coordinates the Poisson brackets reduce to the symplectic form

$$\begin{aligned} \{P^I,Q^J\} &= J^{IJ}=J_I\delta^{I,J} \ . \\ J_I &= 1 \ . \end{aligned} \tag{3.8.6}$$

It is not clear whether Darboux coordinates with $J_I=1$ are possible in the recent case: probably the unit matrix on right hand side of the defining equation is replaced with a diagonal matrix depending on symplectic invariants so that one has $J_I\neq 1$. The integration measure is given by the symplectic volume element given by the determinant of the matrix defined by the Poisson brackets of the Hamiltonians appearing as coordinates. The value of the symplectic volume element is given by the matrix formed by the Poisson brackets of the Hamiltonians and reduces to the product

$$Vol=\prod_I J_I$$

in generalized Darboux coordinates.

Kähler potential (that is gauge potential associated with Kähler form) can be written in Darboux coordinates as

$$A = \sum_I J_IP_IdQ^I \ . \tag{3.8.7}$$

3.8.3 General expressions for Kähler form, Kähler metric and Kähler function

The expressions of Kähler form and Kähler metric in complex coordinates can obtained by transforming the contravariant form of the symplectic form from symplectic coordinates provided by Hamiltonians to complex coordinates:

$$J^{Z^i\bar{Z}^j} = iG^{Z^i\bar{Z}^j} = \partial_{H^A}Z^i\partial_{H^B}\bar{Z}^j J^{AB} \ , \tag{3.8.8}$$

where J^{AB} is given by the classical Kahler charge for the light cone Hamiltonian $\{H^A, H^B\}$. Complex coordinates correspond to linear coordinates of the complexified Lie-algebra providing exponentiation of the isometry algebra via exponential mapping. What one must know is the precise relationship between allowed complex coordinates and Hamiltonian coordinates: this relationship is in principle calculable. In Darboux coordinates the expressions become even simpler:

$$J^{Z^i\bar{Z}^j} = iG^{Z^i\bar{Z}^j} = \sum_I J(I)(\partial_{P^i}Z^i\partial_{Q^I}\bar{Z}^j - \partial_{Q^I}Z^i\partial_{P^I}\bar{Z}^j) \ . \tag{3.8.9}$$

Kähler function can be formally integrated from the relationship

$$\begin{aligned} A_{Z^i} &= i\partial_{Z^i}K \ , \\ A_{\bar{Z}^i} &= -i\partial_{Z^i}K \ . \end{aligned} \tag{3.8.10}$$

holding true in complex coordinates. Kähler function is obtained formally as integral

$$K = \int_0^Z (A_{Z^i}dZ^i - A_{\bar{Z}^i}d\bar{Z}^i) \ . \tag{3.8.11}$$

3.8.4 $Diff(X^3)$ invariance and degeneracy and conformal invariances of the symplectic form

$J(X(H_A), X(H_B))$ defines symplectic form for the coset space G/H only if it is $Diff(X^3)$ degenerate. This means that the symplectic form $J(X(H_A), X(H_B))$ vanishes whenever Hamiltonian H_A or H_B is such that it generates diffeomorphism of the 3-surface X^3. If effective 2-dimensionality holds true, $J(X(H_A), X(H_B))$ vanishes if H_A or H_B generates two-dimensional diffeomorphism $d(H_A)$ at the surface X_i^2.

One can always write

$$J(X(H_A), X(H_B)) = X(H_A)Q(H_B|X_i^2) \ .$$

If H_A generates diffeomorphism, the action of $X(H_A)$ reduces to the action of the vector field X_A of some X_i^2-diffeomorphism. Since $Q(H_B|r_M)$ is manifestly invariant under the diffemorphisms of X^2, the result is vanishing:

$$X_AQ(H_B|X_i^2) = 0 \ ,$$

so that $Diff^2$ invariance is achieved.

The radial diffeomorphisms possibly generated by the radial Virasoro algebra do not produce trouble. The change of the flux integrand X under the infinitesimal transformation $r_M \rightarrow r_M + \epsilon r_M^n$ is given by $r_M^n dX/dr_M$. Replacing r_M with $r_M^{-n+1}/(-n+1)$ as variable, the integrand reduces to a total divergence dX/du the integral of which vanishes over the closed 2-surface X_i^2. Hence radial Virasoro generators having zero norm annihilate all matrix elements of the symplectic form. The induced metric of X_i^2 induces a unique conformal structure and since the conformal transformations of X_i^2 can be interpreted as a mere coordinate changes, they leave the flux integrals invariant.

3.8.5 Complexification and explicit form of the metric and Kähler form

The identification of the Kähler form and Kähler metric in symplectic degrees of freedom follows trivially from the identification of the symplectic form and definition of complexification. The requirement that Hamiltonians are eigen states of angular momentum (and possibly Lorentz boost generator), isospin and hypercharge implies physically natural complexification. In order to fix the complexification completely one must introduce some convention fixing which states correspond to 'positive' frequencies and which to 'negative frequencies' and which to zero frequencies that is to decompose the generators of the symplectic algebra to three sets Can_+, Can_- and Can_0. One must distinguish between Can_0 and zero modes, which are not considered here at all. For instance, CP_2 Hamiltonians correspond to zero modes.

The natural complexification relies on the imaginary part of the radial conformal weight whereas the real part defines the $g = t + h$ decomposition naturally. The wave vector associated with the radial logarithmic plane wave corresponds to the angular momentum quantum number associated with a wave in S^1 in the case of Kac Moody algebra. One can imagine three options.

1. It is quite possible that the spectrum of k_2 does not contain $k_2 = 0$ at all so that the sector Can_0 could be empty. This complexification is physically very natural since it is manifestly invariant under $SU(3)$ and $SO(3)$ defining the preferred spherical coordinates. The choice of $SO(3)$ is unique if the classical four-momentum associated with the 3-surface is time like so that there are no problems with Lorentz invariance.

2. If $k_2 = 0$ is possible one could have

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k=k1_+ik_2}, k_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = 0\} \ . \end{aligned} \tag{3.8.12}$$

3. If it is possible to $n_2 \neq 0$ for $k_2 = 0$, one could define the decomposition as

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k}, k_2 > 0 \ \text{ or } \ k_2 = 0, n_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0 \ \text{ or} k_2 = 0, n_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = n_2 = 0\} \ . \end{aligned} \tag{3.8.13}$$

In this case the complexification is unique and Lorentz invariance guaranteed if one can fix the $SO(2)$ subgroup uniquely. The quantization axis of angular momentum could be chosen to be the direction of the classical angular momentum associated with the 3-surface in its rest system.

The only thing needed to get Kähler form and Kähler metric is to write the half Poisson bracket defined by Eq. 3.9.15

$$\begin{aligned} J_f(X(H_A), X(H_B)) &= 2Im\,(iQ_f(\{H_A, H_B\}_{-+})) \ , \\ G_f(X(H_A), X(H_B)) &= 2Re\,(iQ_f(\{H_A, H_B\}_{-+})) \ . \end{aligned} \tag{3.8.14}$$

Symplectic form, and thus also Kähler form and Kähler metric, could contain a conformal factor depending on the isometry invariants characterizing the size and shape of the 3-surface. At this stage one cannot say much about the functional form of this factor.

3.8.6 Comparison of CP_2 Kähler geometry with configuration space geometry

The explicit discussion of the role of $g = t + h$ decomposition of the tangent space of WCW provides deep insights to the metric of the symmetric space. There are indeed many questions to be answered. To what point of WCW (that is 3-surface) the proposed $g = t + h$ decomposition corresponds to? Can one derive the components of the metric and Kähler form from the Poisson brackets of complexified Hamiltonians? Can one characterize the point in question in terms of the properties of WCW Hamiltonians? Does the central extension of WCW reduce to the symplectic central extension of the symplectic algebra or can one consider also other options?

Cartan decomposition for CP_2

A good manner to gain understanding is to consider the CP_2 metric and Kähler form at the origin of complex coordinates for which the sub-algebra $h = u(2)$ defines the Cartan decomposition.

1. $g = t + h$ decomposition depends on the point of the symmetric space in general. In case of CP_2 $u(2)$ sub-algebra transforms as $g \circ u(2) \circ g^{-1}$ when the point s is replaced by gsg^{-1}. This is expected to hold true also in case of WCW (unless it is flat) so that the task is to identify the point of WCW at which the proposed decomposition holds true.

2. The Killing vector fields of h sub-algebra vanish at the origin of CP_2 in complex coordinates. The corresponding Hamiltonians need not vanish but their Poisson brackets must vanish. It is possible to add suitable constants to the Hamiltonians in order to guarantee that they vanish at origin.

3. It is convenient to introduce complex coordinates and decompose isometry generators to holomorphic components $J_+^a = j^{ak}\partial_k$ and $j_-^a = j^{a\bar{k}}\partial_{\bar{k}}$. One can introduce what might be called half Poisson bracket and half inner product defined as

$$\begin{aligned} \{H^a, H^b\}_{-+} &\equiv \partial_{\bar{k}} H^a J^{\bar{k}l} \partial_l H^b \\ &= j^{ak} J_{k\bar{l}} j^{b\bar{l}} = -i(j_+^a, j_-^b) \ . \end{aligned} \tag{3.8.15}$$

One can express Poisson bracket of Hamiltonians and the inner product of the corresponding Killing vector fields in terms of real and imaginary parts of the half Poisson bracket:

$$\begin{aligned} \{H^a, H^b\} &= 2Im\left(i\{H^a, H^b\}_{-+}\right) \ , \\ (j^a, j^b) &= 2Re\left(i(j_+^a, j_-^b)\right) = 2Re\left(i\{H^a, H^b\}_{-+}\right) \ . \end{aligned} \tag{3.8.16}$$

What this means that Hamiltonians and their half brackets code all information about metric and Kähler form. Obviously this is of utmost importance in the case of the WCW metric whose symplectic structure and central extension are derived from those of CP_2.

Consider now the properties of the metric and Kähler form at the origin.

1. The relations satisfied by the half Poisson brackets can be written symbolically as

$$\begin{aligned} &\{h, h\}_{-+} = 0 \ , \\ &Re\left(i\{h, t\}_{-+}\right) = 0 \ , \quad Im\left(i\{h, t\}_{-+}\right) = 0 \ , \\ &Re\left(i\{t, t\}_{-+}\right) \neq 0 \ , \quad Im\left(i\{t, t\}_{-+}\right) \neq 0 \ . \end{aligned} \tag{3.8.17}$$

2. The first two conditions state that h vector fields have vanishing inner products at the origin. The first condition states also that the Hamiltonians for the commutator algebra $[h, h] = SU(2)$ vanish at origin whereas the Hamiltonian for $U(1)$ algebra corresponding to the color hyper charge need not vanish although it can be made vanishing. The third condition implies that the Hamiltonians of t vanish at origin.

3. The last two conditions state that the Kähler metric and form are non-vanishing between the elements of t. Since the Poisson brackets of t Hamiltonians are Hamiltonians of h, the only possibility is that $\{t, t\}$ Poisson brackets reduce to a non-vanishing $U(1)$ Hamiltonian at the origin or that the bracket at the origin is due to the symplectic central extension. The requirement that all Hamiltonians vanish at origin is very attractive aesthetically and forces to interpret $\{t, t\}$ brackets at origin as being due to a symplectic central extension. For instance, for S^2 the requirement that Hamiltonians vanish at origin would mean the replacement of the Hamiltonian $H = cos(\theta)$ representing a rotation around z-axis with $H_3 = cos(\theta) - 1$ so that the Poisson bracket of the generators H_1 and H_2 can be interpreted as a central extension term.

4. The conditions for the Hamiltonians of $u(2)$ sub-algebra state that their variations with respect to g vanish at origin. Thus $u(2)$ Hamiltonians have extremum value at origin.

5. Also the Kähler function of CP_2 has extremum at the origin. This suggests that in the case of the WCW the counterpart of the origin corresponds to the maximum of the Kähler function.

Cartan algebra decomposition at the level of WCW

The discussion of the properties of CP_2 Kähler metric at origin provides valuable guide lines in an attempt to understand what happens at the level of WCW. The use of the half bracket for WCW Hamiltonians in turn allows to calculate the matrix elements of the WCW metric and Kähler form explicitly in terms of the magnetic or electric flux Hamiltonians.

The earlier construction was rather tricky and formula-rich and not very convincing physically. Cartan decomposition had to be assigned with something and in lack of anything better it was assigned with Super Virasoro algebra, which indeed allows this kind of decompositions but without any strong physical justification. The realization that super-symplectic and super Kac-Moody symmetries define coset construction at the level of basic quantum TGD. Contrary to the original belief, this construction does not provide a realization of Equivalence Principle at quantum level. The proper realization of EP at quantum level seems to be based on the identification of classical Noether charges in Cartan algebra with the eigenvalues of their quantum counterparts assignable to Kähler-Dirac action. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to M^4 with effective metric satisfying Einstein's equations as a reflection of the underlying Poincare invariance.

It must be however emphasized that holography implying effective 2-dimensionality of 3-surfaces in some length scale resolution is absolutely essential for this construction since it allows to effectively reduce Kac-Moody generators associated with X_l^3 to $X^2 = X_l^3 \cap \delta M_\pm^4 \times CP_2$. In the similar manner super-symplectic generators can be dimensionally reduced to X^2. Number theoretical compactification forces the dimensional reduction and the known extremals are consistent with it [K5] . The construction of WCW spinor structure and metric in terms of the second quantized spinor fields [K9] relies to this picture as also the recent view about M-matrix [K12] .

In this framework the coset space decomposition becomes trivial.

1. The algebra g is labeled by color quantum numbers of CP_2 Hamiltonians and by the label (m, n, k) labeling the function basis of the light cone boundary. Also a localization with respect to X^2 is needed. This is a new element as compared to the original view.

2. Super Kac-Moody algebra is labeled by color octet Hamiltonians and function basis of X^2. Since Lie-algebra action does not lead out of irreps, this means that Cartan algebra decomposition is satisfied.

3.8.7 Comparison with loop groups

It is useful to compare the recent approach to the geometrization of the loop groups consisting of maps from circle to Lie group G [A37] , which served as the inspirer of the WCW geometry approach but later turned out to not apply as such in TGD framework.

In the case of loop groups the tangent space T corresponds to the local Lie-algebra $T(k, A) = exp(ik\phi)T_A$, where T_A generates the finite-dimensional Lie-algebra g and ϕ denotes the angle variable of circle; k is integer. The complexification of the tangent space corresponds to the decomposition

$$T = \{X(k > 0, A)\} \oplus \{X(k < 0, A)\} \oplus \{X(k = 0, A)\} = T_+ \oplus T_- \oplus T_0$$

of the tangent space. Metric corresponds to the central extension of the loop algebra to Kac Moody algebra and the Kähler form is given by

$$J(X(k_1 < 0, A), X(k_2 > 0, B)) = k_2\delta(k_1 + k_2)\delta(A, B) \ \ .$$

In present case the finite dimensional Lie algebra g is replaced with the Lie-algebra of the symplectic transformations of $\delta M^4_+ \times CP_2$ centrally extended using symplectic extension. The scalar function basis on circle is replaced with the function basis on an interval of length Δr_M with periodic boundary conditions; effectively one has circle also now.

The basic difference is that one can consider two kinds of central extensions now.

1. Central extension is most naturally induced by the natural central extension ($\{p, q\} = 1$) defined by Poisson bracket. This extension is anti-symmetric with respect to the generators of the symplectic group: in the case of the Kac Moody central extension it is symmetric with respect to the group G. The symplectic transformations of CP_2 might correspond to non-zero modes also because they are not exact symmetries of Kähler action. The situation is however rather delicate since $k = 0$ light cone harmonic has a diverging norm due to the radial integration unless one poses both lower and upper radial cutoffs although the matrix elements would be still well defined for typical 3-surfaces. For Kac Moody group $U(1)$ transformations correspond to the zero modes. Light cone function algebra can be regarded as a local $U(1)$ algebra defining central extension in the case that only CP_2 symplectic transformations local with respect to δM^4_+ act as isometries: for Kac Moody algebra the central extension corresponds to an ordinary $U(1)$ algebra. In the case that entire light cone symplectic algebra defines the isometries the central extension reduces to a $U(1)$ central extension.

3.8.8 Symmetric space property implies Ricci flatness and isometric action of symplectic transformations

The basic structure of symmetric spaces is summarized by the following structural equations

$$\begin{aligned} &g = h + t \ \ , \\ &[h, h] \subset h \ \ , \quad [h, t] \subset t \ \ , \quad [t, t] \subset h \ \ . \end{aligned} \tag{3.8.18}$$

In present case the equations imply that all commutators of the Lie-algebra generators of $Can(\neq 0)$ having non-vanishing integer valued radial quantum number n_2, possess zero norm. This condition is extremely strong and guarantees isometric action of $Can(\delta M^4_+ \times CP_2)$ as well as Ricci flatness of the WCW metric.

The requirement $[t, t] \subset h$ and $[h, t] \subset t$ are satisfied if the generators of the isometry algebra possess generalized parity P such that the generators in t have parity $P = -1$ and the generators belonging to h have parity $P = +1$. Conformal weight n must somehow define this parity. The first possibility to come into mind is that odd values of n correspond to $P = -1$ and even values to $P = 1$. Since n is additive in commutation, this would automatically imply $h \oplus t$ decomposition with the required properties. This assumption looks however somewhat artificial. TGD however forces a generalization of Super Algebras and N-S and Ramond type algebras can be combined to a larger algebra containing also Virasoro and Kac Moody generators labeled by half-odd integers. This suggests strongly that isometry generators are labeled by half integer conformal weight and that half-odd integer conformal weight corresponds to parity $P = -1$ whereas integer conformal weight corresponds to parity $P = 1$. Coset space would structure would state conformal invariance of the theory since super-symplectic generators with integer weight would correspond to zero modes.

Quite generally, the requirement that the metric is invariant under the flow generated by vector field X leads together with the covariant constancy of the metric to the Killing conditions

$$X \cdot g(Y,Z) \;\; = \;\; 0 = g([X,Y],Z) + g(Y,[X,Z]) \;\; . \tag{3.8.19}$$

If the commutators of the complexified generators in $Can(\neq 0)$ have zero norm then the two terms on the right hand side of Eq. (3.9.19) vanish separately. This is true if the conditions

$$Q_m^{\alpha,\beta}(\{H^A,\{H^B,H^C\}\}) \;\; = \;\; 0 \;\; , \tag{3.8.20}$$

are satisfied for all triplets of Hamiltonians in $Can_{\neq 0}$. These conditions follow automatically from the $[t,t] \subset h$ property and guarantee also Ricci flatness as will be found later.

It must be emphasized that for Kähler metric defined by purely magnetic fluxes, one cannot pose the conditions of Eq. (3.9.20) as consistency conditions on the initial values of the time derivatives of imbedding space coordinates whereas in general case this is possible. If the consistency conditions are satisfied for a single surface on the orbit of symplectic group then they are satisfied on the entire orbit. Clearly, isometry and Ricci flatness requirements and the requirement of time reversal invariance might well force Kähler electric alternative.

3.8.9 How to find Kähler function?

If one has found the expansion of WCW Kähler form in terms of electric fluxes one can solve also the Kähler function from the defining partial differential equations $J_{k\bar{l}} = \partial k \partial_{\bar{l}} K$. The solution is not unique since the equation allows the symmetry

$$K \rightarrow K + f(z^k) + \overline{f(z^k)} \;\; ,$$

where f is arbitrary holomorphic function of z^k. This non-uniqueness is probably eliminated by the requirement that Kähler function vanishes for vacuum extremals. This in turn makes in principle possible to find the maxima of Kähler function and to perform functional integration perturbatively around them.

Electric-magnetic duality implies that, apart from conformal factor depending on isometry invariants, one can solve Kähler metric without any knowledge on the initial values of the time derivatives of the imbedding space coordinates. Apart from conformal factor the resulting geometry is purely intrinsic to δCH. The role of Kähler action is only to to define $Diff^4$ invariance and give the rule how the metric is translated to metric on arbitrary point of CH. The degeneracy of the preferred extrema also implies that configuration space has multi-sheeted structure analogous to that encountered in case of Riemann surfaces.

As shown in [K22] , very general assumptions inspired by the light-likeness of Kähler current for the known extremals combined with electric-magnetic duality imply the reduction of Kähler action for the preferred extremals to Chern-Simons terms at the ends of CD and at wormhole throats plus boundary term depending on induced metric so that one has almost topological QFT.

If Dirac determinant equals to the exponent of Kähler action, one might try to construct it in terms of Kähler-Dirac operator [K9]. Since Kähler action reduces to Chern-Simons term the result should be finite. Kähler action contains Chern-Simons action at partonic orbits as analog of boundary term and compensating the Chern-Simons term coming from Kähler action at partonic orbits so that only the contributions from the space-like ends of space-time surface remain. Byt superconformal symmetry Kähler Dirac action contains also Chern-Simons-Dirac term and the generalized eigenvalues of C-S-D operator identifiable as virtual four-momenta allow to have non-trivial fermionic propagator assignable to the boundaries of string world sheets and also define Dirac determinant as a square root of the product of mass squared eigenvalues.

If the virtual four-momenta are identified as hyper-quaternions, one can define even their product to get quaternionion valued determinant actually reducing to real number. Also the product of

$p^k\gamma_k$ makes sense. If the conjecture that Dirac determinant coincides with the exponent of Kähler action for a preferred extremal is correct, the value of the Kähler coupling strength follows as a prediction of the theory. The size of causal diamond (CD) induces IR cutoff and the smallest size for the sup-CDs induces UV cutoff. Hence Dirac determinant involves only a finite number of eigenvalues of the modified Dirac operator and can thus be an algebraic or even rational number if eigenvalues have this property.

3.9 General expressions for the symplectic and Kähler forms

One can derive general expressions for symplectic and Kähler forms as well as Kähler metric of WCW. The fact that these expressions involve only first variation of the Kähler action implies huge simplification of the basic formulas. Duality hypothesis leads to further simplifications of the formulas.

3.9.1 Closedness requirement

The fluxes of Kähler magnetic and electric fields for the Hamiltonians of $\delta M^4_+ \times CP_2$ suggest a general representation for the components of the symplectic form of the WCW. The basic requirement is that Kähler form satisfies the defining condition

$$X\cdot J(Y,Z)+J([X,Y],Z)+J(X,[Y,Z]) \;=\; 0 \ , \tag{3.9.1}$$

where X,Y,Z are now vector fields associated with Hamiltonian functions defining WCW coordinates.

3.9.2 Matrix elements of the symplectic form as Poisson brackets

Quite generally, the matrix element of $J(X(H_A),X(H_B))$ between vector fields $X(H_A))$ and $X(H_B))$ defined by the Hamiltonians H_A and H_B of $\delta M^4_+ \times CP_2$ isometries is expressible as Poisson bracket

$$J^{AB} \;=\; J(X(H_A),X(H_B)) = \{H_A,H_B\} \ . \tag{3.9.2}$$

J^{AB} denotes contravariant components of the symplectic form in coordinates given by a subset of Hamiltonians. The magnetic flux Hamiltonians $Q_m^{\alpha,\beta}(H_{A,k})$ of Eq. 4.4.1 provide an explicit representation for the Hamiltonians at the level of WCW so that the components of the symplectic form of the WCW are expressible as classical charges for the Poisson brackets of the Hamiltonians of the light cone boundary:

$$J(X(H_A),X(H_B)) \;=\; Q_m^{\alpha,\beta}(\{H_A,H_B\}) \ . \tag{3.9.3}$$

Recall that the superscript α,β refers the coefficients of J and $|J|$ in the superposition of these Kähler magnetic fluxes. Note that $Q_m^{\alpha,\beta}$ contains unspecified conformal factor depending on symplectic invariants characterizing Y^3 and is unspecified superposition of signed and unsigned magnetic fluxes.

This representation does not carry information about the tangent space of space-time surface at the partonic 2-surface, which motivates the proposal that also electric fluxes are present and proportional to magnetic fluxes with a factor K, which is symplectic invariant so that commutators of flux Hamiltonians come out correctly. This would give

$$Q_m^{\alpha,\beta}(H_A)_{em} \;=\; Q_e^{\alpha,\beta}(H_A)+Q_m^{\alpha,\beta}(H_A) = (1+K)Q_m^{\alpha,\beta}(H_A) \ . \tag{3.9.4}$$

Since Kähler form relates to the standard field tensor by a factor $e/\hbar$, flux Hamiltonians are dimensionless so that commutators do not involve $\hbar$. The commutators would come as

$$Q_{em}^{\alpha,\beta}(\{H_A, H_B\}) \to (1+K)Q_m^{\alpha,\beta}(\{H_A, H_B\}) \ . \tag{3.9.5}$$

The factor $1+K$ plays the same role as Planck constant in the commutators.

WCW Hamiltonians vanish for the extrema of the Kähler function as variational derivatives of the Kähler action. Hence Hamiltonians are good candidates for the coordinates appearing as coordinates in the perturbative functional integral around extrema (with maxima giving dominating contribution). It is clear that WCW coordinates around a given extremum include only those Hamiltonians, which vanish at extremum (that is those Hamiltonians which span the tangent space of G/H) In Darboux coordinates the Poisson brackets reduce to the symplectic form

$$\begin{aligned} \{P^I, Q^J\} &= J^{IJ} = J_I \delta^{I,J} \ . \\ J_I &= 1 \ . \end{aligned} \tag{3.9.6}$$

It is not clear whether Darboux coordinates with $J_I = 1$ are possible in the recent case: probably the unit matrix on right hand side of the defining equation is replaced with a diagonal matrix depending on symplectic invariants so that one has $J_I \neq 1$. The integration measure is given by the symplectic volume element given by the determinant of the matrix defined by the Poisson brackets of the Hamiltonians appearing as coordinates. The value of the symplectic volume element is given by the matrix formed by the Poisson brackets of the Hamiltonians and reduces to the product

$$Vol = \prod_I J_I$$

in generalized Darboux coordinates.

Kähler potential (that is gauge potential associated with Kähler form) can be written in Darboux coordinates as

$$A = \sum_I J_I P_I dQ^I \ . \tag{3.9.7}$$

3.9.3 General expressions for Kähler form, Kähler metric and Kähler function

The expressions of Kähler form and Kähler metric in complex coordinates can obtained by transforming the contravariant form of the symplectic form from symplectic coordinates provided by Hamiltonians to complex coordinates:

$$J^{Z^i \bar{Z}^j} = iG^{Z^i \bar{Z}^j} = \partial_{H^A} Z^i \partial_{H^B} \bar{Z}^j J^{AB} \ , \tag{3.9.8}$$

where J^{AB} is given by the classical Kahler charge for the light cone Hamiltonian $\{H^A, H^B\}$. Complex coordinates correspond to linear coordinates of the complexified Lie-algebra providing exponentiation of the isometry algebra via exponential mapping. What one must know is the precise relationship between allowed complex coordinates and Hamiltonian coordinates: this relationship is in principle calculable. In Darboux coordinates the expressions become even simpler:

$$J^{Z^i \bar{Z}^j} = iG^{Z^i \bar{Z}^j} = \sum_I J(I)(\partial_{P^i} Z^i \partial_{Q^I} \bar{Z}^j - \partial_{Q^I} Z^i \partial_{P^I} \bar{Z}^j) \ . \tag{3.9.9}$$

Kähler function can be formally integrated from the relationship

$$\begin{aligned} A_{Z^i} &= i\partial_{Z^i} K \ , \\ A_{\bar{Z}^i} &= -i\partial_{Z^i} K \ . \end{aligned} \tag{3.9.10}$$

holding true in complex coordinates. Kähler function is obtained formally as integral

$$K = \int_0^Z (A_{Z^i} dZ^i - A_{\bar{Z}^i} d\bar{Z}^i) \ . \tag{3.9.11}$$

3.9.4 $Diff(X^3)$ invariance and degeneracy and conformal invariances of the symplectic form

$J(X(H_A), X(H_B))$ defines symplectic form for the coset space G/H only if it is $Diff(X^3)$ degenerate. This means that the symplectic form $J(X(H_A), X(H_B))$ vanishes whenever Hamiltonian H_A or H_B is such that it generates diffeomorphism of the 3-surface X^3. If effective 2-dimensionality holds true, $J(X(H_A), X(H_B))$ vanishes if H_A or H_B generates two-dimensional diffeomorphism $d(H_A)$ at the surface X_i^2.

One can always write

$$J(X(H_A), X(H_B)) = X(H_A)Q(H_B|X_i^2) \ .$$

If H_A generates diffeomorphism, the action of $X(H_A)$ reduces to the action of the vector field X_A of some X_i^2-diffeomorphism. Since $Q(H_B|r_M)$ is manifestly invariant under the diffemorphisms of X^2, the result is vanishing:

$$X_A Q(H_B|X_i^2) = 0 \ ,$$

so that $Diff^2$ invariance is achieved.

The radial diffeomorphisms possibly generated by the radial Virasoro algebra do not produce trouble. The change of the flux integrand X under the infinitesimal transformation $r_M \rightarrow r_M + \epsilon r_M^n$ is given by $r_M^n dX/dr_M$. Replacing r_M with $r_M^{-n+1}/(-n+1)$ as variable, the integrand reduces to a total divergence dX/du the integral of which vanishes over the closed 2-surface X_i^2. Hence radial Virasoro generators having zero norm annihilate all matrix elements of the symplectic form. The induced metric of X_i^2 induces a unique conformal structure and since the conformal transformations of X_i^2 can be interpreted as a mere coordinate changes, they leave the flux integrals invariant.

3.9.5 Complexification and explicit form of the metric and Kähler form

The identification of the Kähler form and Kähler metric in symplectic degrees of freedom follows trivially from the identification of the symplectic form and definition of complexification. The requirement that Hamiltonians are eigen states of angular momentum (and possibly Lorentz boost generator), isospin and hypercharge implies physically natural complexification. In order to fix the complexification completely one must introduce some convention fixing which states correspond to 'positive' frequencies and which to 'negative frequencies' and which to zero frequencies that is to decompose the generators of the symplectic algebra to three sets Can_+, Can_- and Can_0. One must distinguish between Can_0 and zero modes, which are not considered here at all. For instance, CP_2 Hamiltonians correspond to zero modes.

The natural complexification relies on the imaginary part of the radial conformal weight whereas the real part defines the $g = t + h$ decomposition naturally. The wave vector associated with the radial logarithmic plane wave corresponds to the angular momentum quantum number associated with a wave in S^1 in the case of Kac Moody algebra. One can imagine three options.

1. It is quite possible that the spectrum of k_2 does not contain $k_2 = 0$ at all so that the sector Can_0 could be empty. This complexification is physically very natural since it is manifestly invariant under $SU(3)$ and $SO(3)$ defining the preferred spherical coordinates. The choice of $SO(3)$ is unique if the classical four-momentum associated with the 3-surface is time like so that there are no problems with Lorentz invariance.

2. If $k_2 = 0$ is possible one could have

$$\begin{aligned}
Can_+ &= \{H^a_{m,n,k=k1_+ik_2}, k_2 > 0\} \ , \\
Can_- &= \{H^a_{m,n,k}, k_2 < 0\} \ , \\
Can_0 &= \{H^a_{m,n,k}, k_2 = 0\} \ .
\end{aligned} \qquad (3.9.12)$$

3. If it is possible to $n_2 \neq 0$ for $k_2 = 0$, one could define the decomposition as

$$
\begin{aligned}
Can_+ &= \{H^a_{m,n,k}, k_2 > 0 \ \text{ or } \ k_2 = 0, n_2 > 0\} \ , \\
Can_- &= \{H^a_{m,n,k}, k_2 < 0 \ \text{ or} k_2 = 0, n_2 < 0\} \ , \\
Can_0 &= \{H^a_{m,n,k}, k_2 = n_2 = 0\} \ .
\end{aligned} \tag{3.9.13}
$$

In this case the complexification is unique and Lorentz invariance guaranteed if one can fix the $SO(2)$ subgroup uniquely. The quantization axis of angular momentum could be chosen to be the direction of the classical angular momentum associated with the 3-surface in its rest system.

The only thing needed to get Kähler form and Kähler metric is to write the half Poisson bracket defined by Eq. 3.9.15

$$
\begin{aligned}
J_f(X(H_A), X(H_B)) &= 2Im\,(iQ_f(\{H_A, H_B\}_{-+})) \ , \\
G_f(X(H_A), X(H_B)) &= 2Re\,(iQ_f(\{H_A, H_B\}_{-+})) \ .
\end{aligned} \tag{3.9.14}
$$

Symplectic form, and thus also Kähler form and Kähler metric, could contain a conformal factor depending on the isometry invariants characterizing the size and shape of the 3-surface. At this stage one cannot say much about the functional form of this factor.

3.9.6 Comparison of CP_2 Kähler geometry with configuration space geometry

The explicit discussion of the role of $g = t + h$ decomposition of the tangent space of WCW provides deep insights to the metric of the symmetric space. There are indeed many questions to be answered. To what point of WCW (that is 3-surface) the proposed $g = t + h$ decomposition corresponds to? Can one derive the components of the metric and Kähler form from the Poisson brackets of complexified Hamiltonians? Can one characterize the point in question in terms of the properties of WCW Hamiltonians? Does the central extension of WCW reduce to the symplectic central extension of the symplectic algebra or can one consider also other options?

Cartan decomposition for CP_2

A good manner to gain understanding is to consider the CP_2 metric and Kähler form at the origin of complex coordinates for which the sub-algebra $h = u(2)$ defines the Cartan decomposition.

1. $g = t + h$ decomposition depends on the point of the symmetric space in general. In case of CP_2 $u(2)$ sub-algebra transforms as $g \circ u(2) \circ g^{-1}$ when the point s is replaced by gsg^{-1}. This is expected to hold true also in case of WCW (unless it is flat) so that the task is to identify the point of WCW at which the proposed decomposition holds true.

2. The Killing vector fields of h sub-algebra vanish at the origin of CP_2 in complex coordinates. The corresponding Hamiltonians need not vanish but their Poisson brackets must vanish. It is possible to add suitable constants to the Hamiltonians in order to guarantee that they vanish at origin.

3. It is convenient to introduce complex coordinates and decompose isometry generators to holomorphic components $J^a_+ = j^{ak}\partial_k$ and $j^a_- = j^{a\bar{k}}\partial_{\bar{k}}$. One can introduce what might be called half Poisson bracket and half inner product defined as

$$
\begin{aligned}
\{H^a, H^b\}_{-+} &\equiv \partial_{\bar{k}} H^a J^{\bar{k}l} \partial_l H^b \\
&= j^{ak} J_{k\bar{l}} j^{b\bar{l}} = -i(j^a_+, j^b_-) \ .
\end{aligned} \tag{3.9.15}
$$

One can express Poisson bracket of Hamiltonians and the inner product of the corresponding Killing vector fields in terms of real and imaginary parts of the half Poisson bracket:

$$\begin{aligned} \{H^a, H^b\} &= 2Im\left(i\{H^a, H^b\}_{-+}\right) \ , \\ (j^a, j^b) &= 2Re\left(i(j^a_+, j^b_-)\right) = 2Re\left(i\{H^a, H^b\}_{-+}\right) \ . \end{aligned} \tag{3.9.16}$$

What this means that Hamiltonians and their half brackets code all information about metric and Kähler form. Obviously this is of utmost importance in the case of the WCW metric whose symplectic structure and central extension are derived from those of CP_2.

Consider now the properties of the metric and Kähler form at the origin.

1. The relations satisfied by the half Poisson brackets can be written symbolically as

$$\begin{aligned} &\{h, h\}_{-+} = 0 \ , \\ &Re\left(i\{h, t\}_{-+}\right) = 0 \ , \quad Im\left(i\{h, t\}_{-+}\right) = 0 \ , \\ &Re\left(i\{t, t\}_{-+}\right) \neq 0 \ , \quad Im\left(i\{t, t\}_{-+}\right) \neq 0 \ . \end{aligned} \tag{3.9.17}$$

2. The first two conditions state that h vector fields have vanishing inner products at the origin. The first condition states also that the Hamiltonians for the commutator algebra $[h, h] = SU(2)$ vanish at origin whereas the Hamiltonian for $U(1)$ algebra corresponding to the color hyper charge need not vanish although it can be made vanishing. The third condition implies that the Hamiltonians of t vanish at origin.

3. The last two conditions state that the Kähler metric and form are non-vanishing between the elements of t. Since the Poisson brackets of t Hamiltonians are Hamiltonians of h, the only possibility is that $\{t, t\}$ Poisson brackets reduce to a non-vanishing $U(1)$ Hamiltonian at the origin or that the bracket at the origin is due to the symplectic central extension. The requirement that all Hamiltonians vanish at origin is very attractive aesthetically and forces to interpret $\{t, t\}$ brackets at origin as being due to a symplectic central extension. For instance, for S^2 the requirement that Hamiltonians vanish at origin would mean the replacement of the Hamiltonian $H = cos(\theta)$ representing a rotation around z-axis with $H_3 = cos(\theta) - 1$ so that the Poisson bracket of the generators H_1 and H_2 can be interpreted as a central extension term.

4. The conditions for the Hamiltonians of $u(2)$ sub-algebra state that their variations with respect to g vanish at origin. Thus $u(2)$ Hamiltonians have extremum value at origin.

5. Also the Kähler function of CP_2 has extremum at the origin. This suggests that in the case of the WCW the counterpart of the origin corresponds to the maximum of the Kähler function.

Cartan algebra decomposition at the level of WCW

The discussion of the properties of CP_2 Kähler metric at origin provides valuable guide lines in an attempt to understand what happens at the level of WCW. The use of the half bracket for WCW Hamiltonians in turn allows to calculate the matrix elements of the WCW metric and Kähler form explicitly in terms of the magnetic or electric flux Hamiltonians.

The earlier construction was rather tricky and formula-rich and not very convincing physically. Cartan decomposition had to be assigned with something and in lack of anything better it was assigned with Super Virasoro algebra, which indeed allows this kind of decompositions but without any strong physical justification.

It must be however emphasized that holography implying effective 2-dimensionality of 3-surfaces in some length scale resolution is absolutely essential for this construction since it allows to effectively reduce Kac-Moody generators associated with X_l^3 to $X^2 = X_l^3 \cap \delta M_\pm^4 \times CP_2$. In the similar manner super-symplectic generators can be dimensionally reduced to X^2. Number theoretical compactification forces the dimensional reduction and the known extremals are consistent with it [K5] . The construction of WCW spinor structure and metric in terms of the second quantized spinor fields [K9] relies to this picture as also the recent view about M-matrix [K12] .

In this framework the coset space decomposition becomes trivial.

1. The algebra g is labeled by color quantum numbers of CP_2 Hamiltonians and by the label (m, n, k) labeling the function basis of the light cone boundary. Also a localization with respect to X^2 is needed. This is a new element as compared to the original view.

2. Super Kac-Moody algebra is labeled by color octet Hamiltonians and function basis of X^2. Since Lie-algebra action does not lead out of irreps, this means that Cartan algebra decomposition is satisfied.

3.9.7 Comparison with loop groups

It is useful to compare the recent approach to the geometrization of the loop groups consisting of maps from circle to Lie group G [A37] , which served as the inspirer of the WCW geometry approach but later turned out to not apply as such in TGD framework.

In the case of loop groups the tangent space T corresponds to the local Lie-algebra $T(k, A) = exp(ik\phi)T_A$, where T_A generates the finite-dimensional Lie-algebra g and ϕ denotes the angle variable of circle; k is integer. The complexification of the tangent space corresponds to the decomposition

$$T = \{X(k > 0, A)\} \oplus \{X(k < 0, A)\} \oplus \{X(k = 0, A)\} = T_+ \oplus T_- \oplus T_0$$

of the tangent space. Metric corresponds to the central extension of the loop algebra to Kac Moody algebra and the Kähler form is given by

$$J(X(k_1 < 0, A), X(k_2 > 0, B)) = k_2\delta(k_1 + k_2)\delta(A, B) \ .$$

In present case the finite dimensional Lie algebra g is replaced with the Lie-algebra of the symplectic transformations of $\delta M_+^4 \times CP_2$ centrally extended using symplectic extension. The scalar function basis on circle is replaced with the function basis on an interval of length Δr_M with periodic boundary conditions; effectively one has circle also now.

The basic difference is that one can consider two kinds of central extensions now.

1. Central extension is most naturally induced by the natural central extension ($\{p, q\} = 1$) defined by Poisson bracket. This extension is anti-symmetric with respect to the generators of the symplectic group: in the case of the Kac Moody central extension it is symmetric with respect to the group G. The symplectic transformations of CP_2 might correspond to non-zero modes also because they are not exact symmetries of Kähler action. The situation is however rather delicate since $k = 0$ light cone harmonic has a diverging norm due to the radial integration unless one poses both lower and upper radial cutoffs although the matrix elements would be still well defined for typical 3-surfaces. For Kac Moody group $U(1)$ transformations correspond to the zero modes. Light cone function algebra can be regarded as a local $U(1)$ algebra defining central extension in the case that only CP_2 symplectic transformations local with respect to δM_+^4 act as isometries: for Kac Moody algebra the central extension corresponds to an ordinary $U(1)$ algebra. In the case that entire light cone symplectic algebra defines the isometries the central extension reduces to a $U(1)$ central extension.

3.9.8 Symmetric space property implies Ricci flatness and isometric action of symplectic transformations

The basic structure of symmetric spaces is summarized by the following structural equations

$$\begin{aligned} &g = h + t \ , \\ &[h,h] \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ . \end{aligned} \tag{3.9.18}$$

In present case the equations imply that all commutators of the Lie-algebra generators of $Can(\neq 0)$ having non-vanishing integer valued radial quantum number n_2, possess zero norm. This condition is extremely strong and guarantees isometric action of $Can(\delta M^4_+ \times CP_2)$ as well as Ricci flatness of the WCW metric.

The requirement $[t,t] \subset h$ and $[h,t] \subset t$ are satisfied if the generators of the isometry algebra possess generalized parity P such that the generators in t have parity $P = -1$ and the generators belonging to h have parity $P = +1$. Conformal weight n must somehow define this parity. The first possibility to come into mind is that odd values of n correspond to $P = -1$ and even values to $P = 1$. Since n is additive in commutation, this would automatically imply $h \oplus t$ decomposition with the required properties. This assumption looks however somewhat artificial. TGD however forces a generalization of Super Algebras and N-S and Ramond type algebras can be combined to a larger algebra containing also Virasoro and Kac Moody generators labeled by half-odd integers. This suggests strongly that isometry generators are labeled by half integer conformal weight and that half-odd integer conformal weight corresponds to parity $P = -1$ whereas integer conformal weight corresponds to parity $P = 1$. Coset space would structure would state conformal invariance of the theory since super-symplectic generators with integer weight would correspond to zero modes.

Quite generally, the requirement that the metric is invariant under the flow generated by vector field X leads together with the covariant constancy of the metric to the Killing conditions

$$X \cdot g(Y,Z) \quad = \quad 0 = g([X,Y],Z) + g(Y,[X,Z]) \ . \tag{3.9.19}$$

If the commutators of the complexified generators in $Can(\neq 0)$ have zero norm then the two terms on the right hand side of Eq. (3.9.19) vanish separately. This is true if the conditions

$$Q_m^{\alpha,\beta}(\{H^A,\{H^B,H^C\}\}) \quad = \quad 0 \ , \tag{3.9.20}$$

are satisfied for all triplets of Hamiltonians in $Can_{\neq 0}$. These conditions follow automatically from the $[t,t] \subset h$ property and guarantee also Ricci flatness as will be found later.

It must be emphasized that for Kähler metric defined by purely magnetic fluxes, one cannot pose the conditions of Eq. (3.9.20) as consistency conditions on the initial values of the time derivatives of imbedding space coordinates whereas in general case this is possible. If the consistency conditions are satisfied for a single surface on the orbit of symplectic group then they are satisfied on the entire orbit. Clearly, isometry and Ricci flatness requirements and the requirement of time reversal invariance might well force Kähler electric alternative.

3.9.9 How to find Kähler function?

If one has found the expansion of WCW Kähler form in terms of electric fluxes one can solve also the Kähler function from the defining partial differential equations $J_{k\bar{l}} = \partial k \partial_{\bar{l}} K$. The solution is not unique since the equation allows the symmetry

$$K \to K + f(z^k) + \overline{f(z^k)} \ ,$$

where f is arbitrary holomorphic function of z^k. This non-uniqueness is probably eliminated by the requirement that Kähler function vanishes for vacuum extremals. This in turn makes in principle possible to find the maxima of Kähler function and to perform functional integration perturbatively around them.

Electric-magnetic duality implies that, apart from conformal factor depending on isometry invariants, one can solve Kähler metric without any knowledge on the initial values of the time derivatives of the imbedding space coordinates. Apart from conformal factor the resulting geometry is purely intrinsic to δCH. The role of Kähler action is only to to define $Diff^4$ invariance and give the rule how the metric is translated to metric on arbitrary point of CH. The degeneracy of the preferred extrema also implies that configuration space has multi-sheeted structure analogous to that encountered in case of Riemann surfaces.

1. As shown in [K22] , very general assumptions inspired by the light-likeness of Kähler current for the known extremals combined with electric-magnetic duality imply the reduction of Kähler action for the preferred extremals to Chern-Simons terms at the ends of CD and at wormhole throats plus boundary term depending on induced metric so that one has almost topological QFT.

2. In order to obtain non-trivial fermion propagator one must add to Kähler-Dirac action Chern-Simons Dirac term located at partonic orbits at which the signature of the induced metric changes. The modes of induced spinor field can be required to be generalized eigenmodes of C-S-D operator with generalized eigenvalue $p^k\gamma_k$ with p^k identified as virtual momentum so that massless Dirac propagator is obtained. By super-symmetry one must add to Kähler action Chern-Simons term located at partonic orbits and this term must cancel the Chern-Simons term coming from Kähler action by weak form of electric-magnetic duality so that only the Chern-Simons terms associated with space-like ends of the space-time surface remain. These terms reduce to Chern-Simons terms only if one poses weak form of electric magnetic duality also here. This is not necessary.

3. The quantum numbers characterizing zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse.

Also a promising concrete construction recipe for Kähler function is in terms of the modified Dirac operator [K9]. The modes of Kähler-Dirac operator (modified Dirac operator) are localized at string world sheets and are holomorphic spinors. K-D operator annihilates these modes so that Dirac determinant must be assigned with the Chern-Simons Dirac term associated with the light-like partonic orbits with vanishing metric determinant g_4. Spinor modes at partonic orbits are assumed to be generalized eigen modes of C-S-D operator with eigenvalues $ip^k\gamma_k$, with p^k interpreted as virtual momentum of the fermion propagating along lined defined by the string world sheet boundary. Therefore C-S-D term acts effectively as massless Dirac action in perturbation theory.

The spectrum of p^k is determined by the boundary conditions for C-S-D operator at the ends of CD and periodic boundary conditions is one natural possibility. As in massless QFTs Dirac determinant could be identified as a square root of the product of mass squared eigenvalues p^2. If the spectrum is unbounded, a regularization must be used. Finite measurement resolution means UV and IR cutoffs and would make Dirac determinant finite. Finite IR resolution would be due to the fact that only space-time surfaces within CD and thus having finite size scale are considered. UV resolution would be due to the lower limit on the size of sub-CDs.

One can however define Dirac determinant directly as the product of the generalized eigenvalues $p^k\gamma_k$ or as product of hyper-quaternions defined by p^k. By symmetry arguments the outcome must be real.

The full Dirac determinant would be product of Dirac determinants associated with various string world sheets. Needless to say that this is an enormous calculational advantage. If Dirac determinant identified in this manner reduces to exponent of Kähler action for preferred extremal this definition of Dirac determinant should give exponent of Kähler function reducing by weak form of electric-magnetic duality to exponent of Chern-Simons terms associated with the space-like ends of the space-time surface. Euclidian and Minkowskian regions would give contributions different by a phase factor $\sqrt{-1}$. The reduction of determinant to exponent of Chern-Simons terms would guarantee its finiteness.

3.10 Consistency conditions on metric

In this section various consistency conditions on the configuration space metric are discussed. In particular, it will be found that the conditions guaranteeing the existence of Riemann connection in the set of all(!) vector fields (including zero norm vector fields) gives very strong constraints on the general form of the metric and that these constraints are indeed satisfied for the proposed metric.

3.10.1 Consistency conditions on Riemann connection

To study the consequences of the consistency conditions, it is most convenient to consider matrix elements of the metric in the basis formed by the isometry generators themselves. The consistency conditions state the covariant constancy of the metric tensor

$$\nabla_Z g(X,Y) = g(\nabla_Z X,Y) + g(X,\nabla_Z Y) = Z\cdot g(X,Y) \ . \tag{3.10.1}$$

$Z\cdot g(X,Y)$ vanishes, when Z generates isometries so that conditions state the covariant constancy of the matrix elements in this case. It must be emphasized that the ill defined-ness of the inner products of form $g(\nabla_Z X,Y)$ is just the reason for requiring infinite-dimensional isometry group. The point is that $\nabla_Z X$ need not to belong to the Hilbert space spanned by the tangent vector fields since the terms of type $Zg(X,Y)$ do not necessarily exist mathematically [A37] . The elegant solution to the problem is that all tangent space vector fields act as isometries so that these quantities vanish identically.

The conditions of Eq. (3.10.1) can be written explicitly by using the general expression for the covariant derivative

$$\begin{aligned} g(\nabla_Z X,Y) &= [Zg(X,Y) + Xg(Z,Y) - Yg(Z,X) \\ &+ g(Ad_Z X - Ad_Z^* X - Ad_X^* Z, Y)]/2 \ . \end{aligned} \tag{3.10.2}$$

What happens is that the terms depending on the derivatives of the matrix elements (terms of type $Zg(X,Y)$) cancel each other (these terms vanish for the metric invariant under isometries), and one obtains the following consistency conditions

$$g(Ad_Z X - Ad_Z^* X - Ad_X^* Z, Y) + g(X, Ad_Z Y - Ad_Z^* Y - Ad_Y^* Z) = 0 \ . \tag{3.10.3}$$

Using the explicit representations of $Ad_Z X$ and $Ad *_Z X$ in terms of structure constants

$$\begin{aligned} Ad_Z X &= [Z,X] = C_{Z,X:U} U \ . \\ Ad_Z^* X &= C_{Z,U:V} g(X,V) g^{-1}(U,W) W = g(X,[Z,U]) g^{-1}(U,W) W \ . \end{aligned} \tag{3.10.4}$$

where the summation over repeated "indices" is performed, one finds that consistency conditions are identically satisfied provided the generators X and Y have a non-vanishing norm. The reason is that the contributions coming from $\nabla_Z X$ and $\nabla_Z Y$ cancel each other.

When one of the generators, say X, appearing in the inner product has a vanishing norm so that one has $g(X,Y)=0$, for any generator Y, situation changes! The contribution of $\nabla_Z Y$ term to the consistency conditions drops away and using Eqs. (3.10.3) and (3.10.4) one obtains the following consistency conditions

$$C_{Z,X:U} g(U,Y) + C_{X,Y:U} g(U,Z) = -X\cdot g(Z,Y) \ . \tag{3.10.5}$$

Note that summation over U is carried out. If X is isometry generator (this need not be the case always) the condition reduces to a simpler form:

$$C_{X,Z:U} g(U,Y) + C_{X,Y:U} g(Z,U) = g([X,Z].Y) + g(Z,[X,Y]) = 0 \ . \tag{3.10.6}$$

These conditions have nice geometric interpretation. If the matrix elements are regarded as ordinary Hilbert space products between the isometry generators the conditions state that the metric defining the inner product behaves as a scalar in the general case.

3.10.2 Consistency conditions for the radial Virasoro algebra

The action of the radial Virasoro in nontrivial manner in the zero modes. Therefore isometry interpretation is excluded and consistency conditions do not make sense in this case. One can however consider the possibility that metric is invariant or suffers only an overall scaling under the action of the radial scaling generated by $L_0 = r_M d/dr_M$. Since the radial integration measure is scaling invariant and only powers of r_M/r_0 appear in Hamiltonians, the effect of the scaling $r_M \rightarrow \lambda r_M$ on the matrix elements of the metric is a scaling by $\lambda^{k_a+\bar{k}_b}$). One can interpret this by saying that scaling changes the values of zero modes and hence leads outside the symmetric space in question.

Invariance of reduced matrix element obtained by dividing away the powers of the scaling factor is achieved if the metric contains the conformal factor

$$S = \frac{1}{\Delta u} f(\frac{r_i}{r_j}) \ , \tag{3.10.7}$$

where r_i are the extrema of r_M interpreted as height function of X^3 and f is a priori arbitrary positive definite function. Since the presence of f presumably gives rise to renormalization corrections depending on the size and shape of 3-surface by scaling the propagator defined by the contravariant metric, the dependence on the ratios r_i/r_j should be slow, logarithmic dependence. Also the dependence on the Fourier components of the solid angles $\Omega(r_M)$ associated with the $r_M = constant$ sections is possible.

3.10.3 Explicit conditions for the isometry invariance

The identification of the Lie-algebra of isometry generators has been proposed but cannot provide any proof for the existence of the infinite parameter symmetry group at this stage. What one can do at this stage is to formulate explicitly the conditions guaranteeing isometry invariance of the metric and try to see whether there are any hopes that these conditions are satisfied. It has been already found that the expression of the metric reduces for light cone alternative to the sum of two boundary terms coming from infinite future and from the boundary of the light cone. If the contribution from infinitely distant future vanishes, as one might expect, then only the contribution from the boundary of the light cone remains.

A tedious but straightforward evaluation of the second variation (see Appendix of the book) for Kähler action implies the following form for the second variation of the Kähler action

$$\delta^2 S = \int_{a=0}^{a=\infty} I^{n\beta}_{kl} \delta h^k D_\beta \delta h^l \ , \tag{3.10.8}$$

where the tensor $I^{\alpha\beta}_{kl}$ is defined as partial derivatives of the Kähler Lagrangian with respect to the derivatives $\partial_\alpha h^k$

$$I^{\alpha\beta}_{kl} = \partial_{\partial_\alpha h^k} \partial_{\partial_\beta h^l} L_M \ . \tag{3.10.9}$$

If the upper limit $a = \sqrt{(m^0)^2 - r_M^2} = \infty$ in the substitution vanishes then one can calculate second variation and therefore metric from the knowledge of the time derivatives $\partial_n h^k$ and $\partial_n \delta h^k$ on the boundary of the light cone only.

Kähler metric can be identified as the $(1,1)$ part of the second variation. This means that one can express the deformation as an element of the isometry algebra plus a arbitrary deformation in radial direction of the light cone boundary interpretable as conformal transformation of the light cone boundary. Radial contributions to the second variation are dropped (by definition of Kähler metric) and what remains is essentially a deformation in S^2 degrees of freedom.

The left invariance of the metric under the deformations of the isometry algebra implies an infinite number of conditions of the form

$$J^C g(J^A, J^B) = 0 \ , \tag{3.10.10}$$

where J^A, J^B and J^C denote the generators of the isometry group. These conditions ought to fix completely the time derivatives of the coordinates h^k for each 3-surface at light cone boundary and therefore in principle the whole minimizing four-surface provided the initial value problem associated with the Kähler action possesses a unique solution. What is nice that the requirement of isometry invariance in principle would provides solution to the problem of finding preferred extremals of the Kähler action.

These conditions, when written explicitly give infinite number of conditions for the time derivative of the generator J^C (we assume for a moment that C is held fixed and let A and B run) at the boundary of the light cone. Time derivatives are in principle determined also by the requirement that deformed surface corresponds to an absolute minimum of the Kähler action. The basis of δH scalar functions respecting color and rotational symmetries is the most promising one.

3.10.4 Direct consistency checks

If duality holds true, the most general form of WCW metric is defined by the fluxes $Q_m^{\alpha,\beta}$, where α and β are the coefficients of signed and unsigned magnetic fluxes. Present is also a conformal factor depending on those zero modes, which do not appear in the symplectic form and which characterize the size and shape of the 3-surface. $[t,t] \subset h$ property implying Ricci flatness and isometry property of symplectic transformations, requires the vanishing of the fluxes $Q_m^{\alpha,\beta}(\{H_{A,m\neq 0}, \{H_{B,n\neq 0}, H_{C,p\neq 0}\}\})$ associated with double commutators and poses strong consistency conditions on the metric. If n labelling symplectic generators has half integer values then the conditions simply state conformal invariance: generators labelled by integers have vanishing norm whereas half-odd integers correspond to non-vanishing norm. Isometry invariance gives additional conditions on fluxes $Q_m^{\alpha,\beta}$. Lorentz invariance strengthens these conditions further. It could be that these conditions fix the initial values of the imbedding space coordinates completely.

Chapter 4

Configuration Space Spinor Structure

4.1 Introduction

Quantum TGD should be reducible to the classical spinor geometry of the configuration space ("world of classical worlds", WCW). In particular, physical states should correspond to the modes of the configuration space spinor fields. The immediate consequence is that configuration space spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the WCW spinor structure there are some important clues.

4.1.1 Geometrization of fermionic statistics in terms of WCW spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the configuration space spinor structure. [B27] has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the configuration space and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their naturally derives from the anti-commutativity of the fermionic oscillator operators.

   As a consequence, WCW spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of WCW spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the 'orbital' degrees of freedom of the ordinary spinor field.

2. The classical theory for the bosonic fields is an essential part of the WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. Contrary to the long held belief it seems that covariantly constant right handed neutrino does not generate . The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and its boundaries. There is actually no deep reason forbidding the gamma matrices of the WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with

$$\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB} \ ,$$

where J_{AB} denotes the matrix elements of the Kähler form of the WCW. The presence of the Hermitian conjugation is necessary because WCW gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the WCW Dirac operator comes out correctly.

5. TGD as a generalized number theory vision leads to the understanding of how the second quantization of the induced spinor fields should be carried out and space-time conformal symmetries allow to explicitly solve the Dirac equation associated with the modified Dirac action in the interior and at the 3-D light like causal determinants. An essentially new element is the notion of number theoretic braid forced by the fact that the modified Dirac operator allows only finite number of generalized eigen modes so that the number of fermionic oscillator operators is finite. As a consequence, anti-commutation relations can be satisfied only for a finite set of points defined by the number theoretic braid, which is uniquely identifiable. The interpretation is in terms of finite measurement resolution. The finite Clifford algebra spanned by the fermionic oscillator operators is interpreted as the factor space $\mathcal{M}/\mathcal{N}$ of infinite hyper-finite factors of type II_1 defined by WCW Clifford algebra $\mathcal{N}$ and included Clifford algebra $\mathcal{M} \subset \mathcal{N}$ interpreted as the characterizer of the finite measurement resolution. Note that the finite number of eigenvalues guarantees that Dirac determinant identified as the exponent of Kähler function is finite. Finite number of eigenvalues is also essential for number theoretic universality.

4.1.2 Modified Dirac equation for induced classical spinor fields

It is now clear that Kähler-Dirac action with measurement interaction terms as boundary term is the unique choice for the Dirac action.

There are several approaches for solving the modified Dirac (or Kähler-Dirac) equation.

1. The most promising approach assumes that the solutions are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of both strong form of holography and of number theoretic vision, and also follows from the notion of finite measurement resolution having discretization at partonic 2-surfaces as a geometric correlate. Furthermore, the conditions stating that electric charge is well-defined for preferred extremals forces the localization of the modes to 2-D surfaces in the generic case. This also resolves the interpretational problems related to possibility of strong parity breaking effects since induce W fields and possibly also Z^0 field above weak scale, vahish at these surfaces.

2. One expects that stringy approach based on 4-D generalization of conformal invariance or its 2-D variant at 2-D preferred surfaces should also allow to understand the modified Dirac

equation. Conformal invariance indeed allows to write the solutions explicitly using formulas similar to encountered in string models. In accordance with the earlier conjecture, all modes of the modified Dirac operator generate badly broken super-symmetries.

3. Covariantly constant right-handed neutrino certainly defines solutions de-localized inside entire space-time sheet. This need not be the case if right-handed neutrino is not covarianty constant since the non-vanishing CP_2 part for the induced gamma matrices mixes it with left-handed neutrino. For massless extremals (at least) the CP_2 part however vanishes and right-handed neutrino allows also massless holomorphic modes de-localized at entire space-time surface and the de-localization inside Euclidian region defining the line of generalized Feynman diagram is a good candidate for the right-handed neutrino generating the least broken super-symmetry.This super-symmetry seems however to differ from the ordinary one in that ν_R is expected to behave like a passive spectator in the scattering. Also for the left-handed neutrino solutions localized inside string world sheet the condition that coupling to right-handed neutrino vanishes is guaranteed if gamma matrices are either purely Minkowskian or CP_2 like inside the world sheet.

4.1.3 Identification of WCW gamma matrices as super Hamiltonians

The basic super-algebra corresponds to the fermionic oscillator operators and can be regarded as a generalization $\mathcal{N}$ super algebras by replacing $\mathcal{N}$ with the number of solutions of the modified Dirac equation which can be infinite. This leads to QFT SUSY limit of TGD different in many respects crucially from standard SUSYs.

WCW gamma matrices are identified as super generators of super-symplectic and are expressible in terms of these oscillator operators. In the original proposal super-symplectic and super charges were assumed to be expressible as integrals over 2-dimensional partonic surfaces X^2 and interior degrees of freedom of X^4 can be regarded as zero modes representing classical variables in one-one correspondence with quantal degrees of freedom at X_l^3 as indeed required by quantum measurement theory.

Quite recently (at the end of 2013) it became clear that one must perform a generalization analogous to a transition from field theory to string models requiring the replacement of points of partonic 2-surfaces with stringy curves connecting the points of two partonic 2-surfaces. This does not mean loss of effective 2-dimensionality implied by strong form of general coordinate invariance but allows genuine generalization of super-conformal invariance in 4-D context.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found here [L13]. Another glossary type representation involving both pdf and html files can be found at `http://www.tgdtheory.fi/tgdglossary.pdf`. The topics relevant to this chapter are given by the following list.

- WCW gamma matrices [L41]
- WCW spinor fields [L42]

4.2 WCW spinor structure: general definition

The basic problem in constructing WCW spinor structure is clearly the construction of the explicit representation for the gamma matrices of WCW. One should be able to identify the space, where these gamma matrices act as well as the counterparts of the "free" gamma matrices, in terms of which the gamma matrices would be representable using generalized vielbein coefficients.

4.2.1 Defining relations for gamma matrices

The ordinary definition of the gamma matrix algebra is in terms of the anti-commutators

$$\{\gamma_A, \gamma_B\} = 2g_{AB} \quad .$$

This definition served implicitly also as a basic definition of the gamma matrix algebra in TGD context until the difficulties related to the understanding of WCW d'Alembertian defined in terms of the square of the Dirac operator forced to reconsider the definition. If WCW allows Kähler structure, the most general definition allows to replace the metric any covariantly constant Hermitian form. In particular, g_{AB} can be replaced with

$$\{\Gamma_A^\dagger, \Gamma_B\} = iJ_{AB} \quad , \tag{4.2.1}$$

where J_{AB} denotes the matrix element of the Kähler form of WCW. The reason is that gamma matrices carry fermion number and are non-hermitian in all coordinate systems. This definition is numerically equivalent with the standard one in the complex coordinates but in arbitrary coordinates situation is different since in general coordinates iJ_{kl} is a nontrivial positive square root of g_{kl}. The realization of this delicacy is necessary in order to understand how the square of WCW Dirac operator comes out correctly. Obviously, what one must do is the equivalent of replacing $D^2 = (\Gamma^k D_k)^2$ with $D\hat{D}$ with $\hat{D}$ defined as

$$\hat{D} = iJ^{kl}\Gamma_l^\dagger D_k \quad .$$

4.2.2 General vielbein representations

There are two ideas, which make the solution of the problem obvious.

1. Since the classical time development in bosonic degrees of freedom (induced gauge fields) is coded into the geometry of WCW it seems natural to expect that same applies in the case of the spinor structure. The time development of the induced spinor fields dictated by TGD counterpart of the massless Dirac action should be coded into the definition of the WCW spinor structure. This leads to the challenge of defining what classical spinor field means.

2. Since classical scalar field in WCW corresponds to second quantized boson fields of the imbedding space same correspondence should apply in the case of the fermions, too. The spinor fields of WCW should correspond to second quantized fermion field of the imbedding space and the space of the configuration space spinors should be more or less identical with the Fock space of the second quantized fermion field of imbedding space or $X^4(X^3)$. Since classical spinor fields at space-time surface are obtained by restricting the spinor structure to the space-time surface, one might consider the possibility that life is really simple: the second quantized spinor field corresponds to the free spinor field of the imbedding space satisfying the counterpart of the massless Dirac equation and more or less standard anti-commutation relations. Unfortunately life is not *so* simple as the construction of WCW spinor structure demonstrates: second quantization must be performed for induced spinor fields.

It is relatively simple to fill in the details once these basic ideas are accepted.

1. The only natural candidate for the second quantized spinor field is just the on X^4. Since this field is free field, one can indeed perform second quantization and construct fermionic oscillator operator algebra with unique anti-commutation relations. The space of WCW spinors can be identified as the associated with these oscillator operators. This space depends on 3-surface and strictly speaking one should speak of the Fock bundle having WCW as its base space.

2. The gamma matrices of WCW (or rather fermionic Kac Moody generators) are representable as super positions of the fermionic oscillator algebra generators:

$$\begin{aligned}
\Gamma_A^+ &= E_A^n a_n^\dagger \\
\Gamma_A^- &= \bar{E}_A^n a_n \\
iJ_{A\bar{B}} &= \sum_n E_A^n \bar{E}_B^n
\end{aligned} \tag{4.2.2}$$

where E^n_A are the vielbein coefficients. Induced spinor fields can possess zero modes and there is no oscillator operators associated with these modes. Since oscillator operators are spin 1/2 objects, WCW gamma matrices are analogous to spin 3/2 spinor fields (in a very general sense). Therefore the generalized vielbein and WCW metric is analogous to the pair of spin 3/2 and spin 2 fields encountered in super gravitation! Notice that the contractions $j^{Ak}\Gamma_k$ of the complexified gamma matrices with the isometry generators are genuine spin 1/2 objects labeled by the quantum numbers labeling isometry generators. In particular, in CP_2 degrees of freedom these fermions are color octets.

3. A further great idea inspired by the symplectic and Kähler structures of WCW is that configuration gamma matrices are actually generators of super-symplectic symmetries. This simplifies enormously the construction allows to deduce explicit formulas for the gamma matrices.

4.2.3 Inner product for WCW spinor fields

The conjugation operation for WCW spinor s corresponds to the standard $ket \rightarrow bra$ operation for the states of the Fock space:

$$\begin{aligned} \Psi &\leftrightarrow |\Psi\rangle \\ \bar{\Psi} &\leftrightarrow \langle\Psi| \end{aligned} \tag{4.2.3}$$

The inner product for WCW spinor s at a given point of WCW is just the standard Fock space inner product, which is unitary.

$$\bar{\Psi}_1(X^3)\Psi_2(X^3) = \langle\Psi_1|\Psi_2\rangle_{|X^3} \tag{4.2.4}$$

WCW inner product for two WCW spinor fields is obtained as the integral of the Fock space inner product over the whole WCW using the vacuum functional $exp(K)$ as a weight factor

$$\langle\Psi_1|\Psi_2\rangle = \int \langle\Psi_1|\Psi_2\rangle_{|X^3} exp(K)\sqrt{G}dX^3 \tag{4.2.5}$$

This inner product is obviously unitary. A modified form of the inner product is obtained by including the factor $exp(K/2)$ in the definition of the spinor field. In fact, the construction of the central extension for the isometry algebra leads automatically to the appearance of this factor in vacuum spinor field.

The inner product differs from the standard inner product for, say, Minkowski space spinors in that integration is over the entire WCW rather than over a time= constant slice of the WCW. Also the presence of the vacuum functional makes it different from the finite dimensional inner product. These are not un-physical features. The point is that (apart from classical non-determinism forcing to generalized the concept of 3-surface) Diff^4 invariance dictates the behavior of WCW spinor field completely: it is determined form its values at the moment of the big bang. Therefore there is no need to postulate any Dirac equation to determine the behavior and therefore no need to use the inner product derived from dynamics.

4.2.4 Holonomy group of the vielbein connection

Generalized vielbein allows huge gauge symmetry. An important constraint on physical observables is that they do not depend at all on the gauge chosen to represent the gamma matrices. This is indeed achieved using vielbein connection, which is now quadratic in fermionic oscillator operators. The holonomy group of the vielbein connection is the WCW counterpart of the electro-weak gauge group and its algebra is expected to have same general structure as the algebra of the WCW isometries. In particular, the generators of this algebra should be labeled by conformal weights like the elements of Kac Moody algebras. In present case however conformal weights are complex as the construction of WCW geometry demonstrates.

4.2.5 Realization of WCW gamma matrices in terms of super symmetry generators

In string models super symmetry generators behave effectively as gamma matrices and it is very tempting to assume that WCW gamma matrices can be regarded as generators of the symplectic algebra extended to super-symplectic Kac Moody type algebra. The experience with string models suggests also that radial Virasoro algebra extends to Super Virasoro algebra. There are good reasons to expect that WCW Dirac operator and its square give automatically a realization of this algebra. It this is indeed the case, then WCW spinor structure as well as Dirac equation reduces to mere group theory.

One can actually guess the general form of the super-symplectic algebra. The form is a direct generalization of the ordinary super Kac Moody algebra. The complexified super generators S_A are identifiable as WCW gamma matrices:

$$\Gamma_A = S_A \ . \tag{4.2.6}$$

The anti-commutators $\{\Gamma_A^\dagger, \Gamma_B\}_+ = i2J_{A,B}$ define a Hermitian matrix, which is proportional to the Kähler form of the configuration space rather than metric as usually. Only in complex coordinates the anti-commutators equal to the metric numerically. This is, apart from the multiplicative constant n, is expressible as the Poisson bracket of the WCW Hamiltonians H_A and H_B. Therefore one should be able to identify super generators $S_A(r_M)$ for each values of r_M as the counterparts of fluxes. The anti-commutators between the super generators S_A and their Hermitian conjugates should read as

$$\{S_A, S_B^\dagger\}_+ = iQ_m(H_{[A,B]}) \ . \tag{4.2.7}$$

and should be induced directly from the anti-commutation relations of free second quantized spinor fields of the imbedding space restricted to the light cone boundary.

The commutation relations between s and super generators follow solely from the transformation properties of the super generators under symplectic transformations, which are same as for the Hamiltonians themselves

$$\{H_{Am}, S_{Bn}\}_- = S_{[Am,Bn]} \ , \tag{4.2.8}$$

and are of the same form as in the case of Super-Kac-Moody algebra.

The task is to derive an explicit representation for the super generators S_A in both cases. For obvious reason the spinor fields restricted to the 3-surfaces on the light cone boundary $\delta M^4_+ \times CP_2$ can be used. Leptonic/quark like oscillator operators are used to construct Ramond/NS type algebra.

What is then the strategy that one should follow?

1. WCW Hamiltonians correspond to either magnetic or electric flux Hamiltonians and the conjecture is that these representations are equivalent. It turns out that this electric-magnetic duality generalizes to the level of super charges. It also turns out that quark representation is the only possible option whereas leptonic super charges super-symmetrize the ordinary function algebra of the light cone boundary.

2. The simplest option would be that second quantized imbedding space spinors could be used in the definition of super charges. This turns out to not work and one must second quantize the induced spinor fields.

3. The task is to identify a super-symmetric variational principle for the induced spinors: ordinary Dirac action does not work. It turns out that in the most plausible scenario the modified Dirac action varied with respect to *both* imbedding space coordinates and spinor fields is the fundamental action principle. The c-number parts of the conserved symplectic charges associated with this action give rise to bosonic conserved charges defining WCW Hamiltonians. The second quantization of the spinor fields reduces to the requirement that super charges and Hamiltonians generate super-symplectic algebra determining the anti-commutation relations for the induced spinor fields.

4.2.6 Central extension as symplectic extension at configuration space level

The earlier attempts to understand the emergence of central extension of super-symplectic algebra were based on the notion of symplectic extension. This general view is not given up although it seems that this abstract approach is not very practical. Symplectic extension emerged originally in the attempts to construct formal expression for the WCW Dirac equation. The rather obvious idea was that the Dirac equation reduces to super Virasoro conditions with Super Virasoro generators involving the Dirac operator of the imbedding space. The basic difficulty was the necessity to assign to the gamma matrices of the imbedding space fermion number. In the recent formulation the Dirac operator of H does not appear in in the Super Virasoro conditions so that this problem disappears.

The proposal that Super Virasoro conditions should replaced with conditions stating that the commutator of super-symplectic and super Kac-Moody algebras annihilates physical states, looks rather feasible. One could call these conditions as WCW Dirac equation but at this moment I feel that this would be just play with words and mask the group theoretical content of these conditions. In any case, the formulas for the symplectic extension and action of isometry generators on WCW spinor deserve to be summarized.

Symplectic extension

The Abelian extension of the super-symplectic algebra is obtained by an extremely simple trick. Replace the ordinary derivatives appearing in the definition of, say spinorial isometry generator, by the covariant derivatives defined by a coupling to a multiple of the Kähler potential.

$$\begin{aligned} j^{Ak}\partial_k &\rightarrow j^{Ak}D_k \ , \\ D_k &= \partial_k + ikA_k/2 \ . \end{aligned} \tag{4.2.9}$$

where A_k denotes Kähler potential. The reality of the parameter k is dictated by the Hermiticity requirement and also by the requirement that Abelian extension reduces to the standard form in Cartan algebra. k is expected to be integer also by the requirement that covariant derivative corresponds to connection (quantization of magnetic charge).

The commutation relations for the centrally extended generators J^A read:

$$[J^A, J^B] = J^{[A,B]} + ikj^{Ak}J_{kl}j^{Bl} \equiv J^{[A,B]} + ikJ_{AB} \ . \tag{4.2.10}$$

Since Kähler form defines symplectic structure in WCW one can express Abelian extension term as a Poisson bracket of two Hamiltonians

$$J_{AB} \equiv j^{Ak}J_{kl}j^{Bl} = \{H^A, H^B\} \ . \tag{4.2.11}$$

Notice that Poisson bracket is well defined also when Kähler form is degenerate.

The extension indeed has acceptable properties:

1. Jacobi-identities reduce to the form

$$\sum_{cyclic} H^{[A,[B,C]]} = 0 \ , \tag{4.2.12}$$

and therefore to the Jacobi identities of the original Lie- algebra in Hamiltonian representation.

2. In the Cartan algebra Abelian extension reduces to a constant term since the Poisson bracket for two commuting generators must be a multiple of a unit matrix. This feature is clearly crucial for the non-triviality of the Abelian extension and is encountered already at the level of ordinary (q,p) Poisson algebra: although the differential operators ∂_p and ∂_q commute the Poisson bracket of the corresponding Hamiltonians p and q is nontrivial: $\{p,q\} = 1$. Therefore the extension term commutes with the generators of the Cartan subalgebra. Extension is also local $U(1)$ extension since Poisson algebra differs from the Lie-algebra of the vector fields in that it contains constant Hamiltonian ("1" in the commutator), which commutes with all other Hamiltonians and corresponds to a vanishing vector field.

3. For the generators not belonging to Cartan sub-algebra of CH isometries Abelian extension term is not annihilated by the generators of the original algebra and in this respect the extension differs from the standard central extension for the loop algebras. It must be however emphasized that for the super-symplectic algebra generators correspond to products of δM^4_+ and CP_2 Hamiltonians and this means that generators of say δM^4_+-local $SU(3)$ Cartan algebra are non-commuting and the commutator is completely analogous to central extension term since it is symmetric with respect to $SU(3)$ generators.

4. The proposed method yields a trivial extension in the case of Diff^4. The reason is the (four-dimensional!) Diff degeneracy of the Kähler form. Abelian extension term is given by the contraction of the Diff^4 generators with the Kähler potential

$$j^{Ak}J_{kl}j^{Bl} \quad = \quad 0 \ , \tag{4.2.13}$$

which vanishes identically by the Diff degeneracy of the Kähler form. Therefore neither 3- or 4-dimensional Diff invariance is not expected to cause any difficulties. Recall that 4-dimensional Diff degeneracy is what is needed to eliminate time like vibrational excitations from the spectrum of the theory. By the way, the fact that the loop space metric is not Diff degenerate makes understandable the emergence of Diff anomalies in string models [B27, B20]
.

5. The extension is trivial also for the other zero norm generators of the tangent space algebra, in particular for the $k_2 = Im(k) = 0$ symplectic generators possible present so that these generators indeed act as genuine $U(1)$ transformations.

6. Concerning the solution of WCW Dirac equation the maximum of Kähler function is expected to be special, much like origin of Minkowski space and symmetric space property suggests that the construction of solutions reduces to this point. At this point the generators and Hamiltonians of the algebra h in the defining Cartan decomposition $g = h + t$ should vanish. h corresponds to integer values of $k_1 = Re(k)$ for Cartan algebra of super-symplectic algebra and integer valued conformal weights n for Super Kac-Moody algebra. The algebra reduces at the maximum to an exceptionally simple form since only central extension contributes to the metric and Kähler form. In the ideal case the elements of the metric and Kähler form could be even diagonal. The degeneracy of the metric might of course pose additional complications.

Super symplectic action on WCW spinor s

The generators of symplectic transformations are obtained in the spinor representation of the isometry group of WCW by the following formal construction. Take isometry generator in the spinor representation and add to the covariant derivative D_k defined by vielbein connection the coupling to the multiple of the Kähler potential: $D_k \rightarrow D_k + ikAk/2$.

$$\begin{aligned} J^A &= j^{Ak}D_k + D_l j_k \Sigma^{kl}/2 \ , \\ &\rightarrow \hat{J}^A = j^{Ak}(D_k + ikA_k/2) + D_l j^A_k \Sigma^{kl}/2 \ , \end{aligned} \tag{4.2.14}$$

This induces the required central term to the commutation relations. Introduce complex coordinates and define bosonic creation and annihilation operators as $(1,0)$ and $(0,1)$ parts of the modified isometry generators

$$\begin{aligned} B_A^{\dagger} &= J_+^A = j^{Ak}(D_k + ... \ , \\ B_A &= J_-^A = j^{A\bar{k}}(D_{\bar{k}} + ... \ . \end{aligned} \tag{4.2.15}$$

where "k" refers now to complex coordinates and "$\bar{k}$" to their conjugates.

Fermionic generators are obtained as the contractions of the complexified gamma matrices with the isometry generators

$$\begin{aligned} \Gamma_A^{\dagger} &= j^{Ak}\Gamma_k \ , \\ \Gamma_A &= j^{A\bar{k}}\Gamma_{\bar{k}} \ . \end{aligned} \tag{4.2.16}$$

Notice that the bosonic Cartan algebra generators obey standard oscillator algebra commutation relations and annihilate fermionic Cartan algebra generators. Hermiticity condition holds in the sense that creation type generators are hermitian conjugates of the annihilation operator type generators. There are two kinds of representations depending on whether one uses leptonic or quark like oscillator operators to construct the gammas. These will be assumed to correspond to Ramond and NS type generators with the radial plane waves being labeled by integer and half odd integer indices respectively.

The non-vanishing commutators between the Cartan algebra bosonic generators are given by the matrix elements of the Kähler form in the basis of formed by the isometry generators

$$[B_A^{\dagger}, B_B] = J(j^{A\dagger}, j^B) \equiv J_{\bar{A}B} \ . \tag{4.2.17}$$

and are isometry invariant quantities. The commutators between local $SU(3)$ generators not belonging to Cartan algebra are just those of the local gauge algebra with Abelian extension term added.

The anti-commutators between the fermionic generators are given by the elements of the metric (as opposed to Kähler form in the case of bosonic generators) in the basis formed by the isometry generators

$$\{\Gamma_A\dagger, \Gamma_B\} = 2g(j^{A\dagger}, j^B) \equiv 2g_{\bar{A}B} \ . \tag{4.2.18}$$

and are invariant under isometries. Numerically the commutators and anti-commutators differ only the presence of the imaginary unit and the scale factor R relating the metric and Kähler form to each other (the factor R is same for CP_2 metric and Kähler form).

The commutators between bosonic and fermionic generators are given by

$$[B_A, \Gamma_B] = \Gamma_{[A,B]} \ . \tag{4.2.19}$$

The presence of vielbein and rotation terms in the representation of the isometry generators is essential for obtaining these nice commutations relations. The commutators vanish identically for Cartan algebra generators. From the commutation relations it is clear that Super Kac Moody algebra structure is directly related to the Kähler structure of WCW: the anti-commutator of fermionic generators is proportional to the metric and the commutator of the bosonic generators is proportional to the Kähler form. It is this algebra, which should generate the solutions of the field equations of the theory.

The vielbein and rotational parts of the bosonic isometry generators are quadratic in the fermionic oscillator operators and this suggests the interpretation as the fermionic contribution to the isometry currents. This means that the action of the bosonic generators is essentially non-perturbative since it creates fermion anti-fermion pairs besides exciting bosonic degrees of freedom.

4.2.7 WCW Clifford algebra as a hyper-finite factor of type II_1

The naive expectation is that the trace of the unit matrix associated with the Clifford algebra spanned by WCW sigma matrices is infinite and thus defines an excellent candidate for a source of divergences in perturbation theory. This potential source of infinities remained un-noticed until it became clear that there is a connection with von Neumann algebras [A35] . In fact, for a separable Hilbert space defines a standard representation for so called [A46] . This guarantees that the trace of the unit matrix equals to unity and there is no danger about divergences.

Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [A46] .

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless.

von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac based on the notion of delta function, plus the emergence of Feynman graphs, the possibility to formulate the notion of delta function rigorously in terms of distributions, and the emergence of path integral approach meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [A56, A62] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [A47, A33] relate closely to type II_1 factors. In topological quantum computation [B21] based on braid groups [A64] modular S-matrices they play an especially important role.

Clifford algebra of WCW as von Neumann algebra

The Clifford algebra of WCW provides a school example of a hyper-finite factor of type II_1, which means that fermionic sector does not produce divergence problems. Super-symmetry means that also "orbital" degrees of freedom corresponding to the deformations of 3-surface define similar factor. The general theory of hyper-finite factors of type II_1 is very rich and leads to rather detailed understanding of the general structure of S-matrix in TGD framework. For instance, there is a unitary evolution operator intrinsic to the von Neumann algebra defining in a natural manner single particle time evolution. Also a connection with 3-dimensional topological quantum field theories and knot theory, conformal field theories, braid groups, quantum groups, and quantum counterparts of quaternionic and octonionic division algebras emerges naturally. These aspects are discussed in detail in [K60] .

4.3 An attempt to understand preferred extremals of Kähler action

Preferred extremal of Kähler action is one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what "preferred" really means. For instance, the conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [K61]. The problem is however how to assign a complex coordinate with the string world sheet having Minkowskian signature of metric. One can hope that the understanding of preferred extremals could allow to identify two preferred complex coordinates whose existence is also suggested by number theoretical vision giving preferred role for the rational points of partonic 2-surfaces in preferred coordinates. The best one could hope is a general solution of field equations in accordance with the hints that TGD is integrable quantum theory.

4.3.1 What "preferred" could mean?

The first question is what preferred extremal could mean.

1. In positive energy ontology preferred extremal would be a space-time surface assignable to given 3-surface and unique in the ideal situation: since one cannot pose conditions to the normal derivatives of imbedding space coordinates at 3-surface, there is infinity of extremals. Some additional conditions are required and space-time surface would be analogous to Bohr orbit : hence the attribute "preferred". The problem would be to understand what "preferred" could mean. The non-determinism of Kähler action however destroyed this dream in its original form and led to zero energy ontology (ZEO).

2. In ZEO one considers extremals as space-time surfaces connecting two space-like 3-surfaces at the boundaries. One might hope that these 4-surfaces are unique. The non-determinism of Kähler action suggests that this is not the case. At least there is conformal invariance respecting the light-likeness of the 3-D parton orbits at which the signature of the induced metric changes: the conformal transformations would leave the space-like 3-D ends or at least partonic 2-surfaces invariant. This non-determinism would correspond to quantum criticality.

3. Effective 2-dimensionality follows from strong form of general coordinate invariance (GCI) stating that light-like partonic orbits and space-like 3-surfaces at the ends of space-time surface are equivalent physically: partonic 2-surfaces and their 4-D tangent space data would determine everything. One can however worry about how effective 2-dimensionality relates to the the fact that the modes of the induced spinor field are localized at string world sheets and partonic 2-surface. Are the tangent space data equivalent with the data characterizing string world sheets as surfaces carrying vanishing electroweak fields?

   There is however a problem: the hierarchy of Planck constants (dark matter) requires that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom so that either space-like or light-like surfaces do not seem to be quite enough.

Should one then include also the light-like partonic orbits to the what one calls 3-surface? The resulting connected 3-surfaces would define analogs of Wilson loops. Could the conformal equivalence class of the preferred extremal be unique without any additional conditions? If so, one could get rid of the attribute "preferred". The fractal character of the many-sheeted space-time however suggests that one can have this kind of uniqueness only in given length scale resolution and that "radiative corrections" due to the non-determinism are always present.

These considerations show that the notion of preferred extremal is still far from being precisely defined and it is not even clear whether the attribute "preferred" is needed. If not then the question is what are the extremals of Kähler action.

4.3.2 What is known about extremals?

A lot is is known about properties of extremals and just by trying to integrate all this understanding, one might gain new visions. The problem is that all these arguments are heuristic and rely heavily on physical intuition. The following considerations relate to the space-time regions having Minkowskian signature of the induced metric. The attempt to generalize the construction also to Euclidian regions could be very rewarding. Only a humble attempt to combine various ideas to a more coherent picture is in question.

The core observations and visions are following.

1. Hamilton-Jacobi coordinates for M^4 (discussed in this chapter) define natural preferred coordinates for Minkowskian space-time sheet and might allow to identify string world sheets for X^4 as those for M^4. Hamilton-Jacobi coordinates consist of light-like coordinate m and its dual defining local 2-plane $M^2 \subset M^4$ and complex transversal complex coordinates $(w, \overline{w})$ for a plane E^2_x orthogonal to M^2_x at each point of M^4. Clearly, hyper-complex analyticity and complex analyticity are in question.

2. Space-time sheets allow a slicing by string world sheets (partonic 2-surfaces) labelled by partonic 2-surfaces (string world sheets).

3. The quaternionic planes of octonion space containing preferred hyper-complex plane are labelled by CP_2, which might be called CP_2^{mod} [K52]. The identification $CP_2 = CP_2^{mod}$ motivates the notion of $M^8 - -M^4 \times CP_2$ duality [K13]. It also inspires a concrete solution ansatz assuming the equivalence of two different identifications of the quaternionic tangent space of the space-time sheet and implying that string world sheets can be regarded as strings in the 6-D coset space $G_2/SU(3)$. The group G_2 of octonion automorphisms has already earlier appeared in TGD framework.

4. The duality between partonic 2-surfaces and string world sheets in turn suggests that the $CP_2 = CP_2^{mod}$ conditions reduce to string model for partonic 2-surfaces in $CP_2 = SU(3)/U(2)$. String model in both cases could mean just hypercomplex/complex analyticity for the coordinates of the coset space as functions of hyper-complex/complex coordinate of string world sheet/partonic 2-surface.

The considerations of this section lead to a revival of an old very ambitious and very romantic number theoretic idea.

1. To begin with express octonions in the form $o = q_1 + Iq_2$, where q_i is quaternion and I is an octonionic imaginary unit in the complement of fixed a quaternionic sub-space of octonions. Map preferred coordinates of $H = M^4 \times CP_2$ to octonionic coordinate, form an arbitrary octonion analytic function having expansion with real Taylor or Laurent coefficients to avoid problems due to non-commutativity and non-associativity. Map the outcome to a point of H to get a map $H \rightarrow H$. This procedure is nothing but a generalization of Wick rotation to get an 8-D generalization of analytic map.

2. Identify the preferred extremals of Kähler action as surfaces obtained by requiring the vanishing of the imaginary part of an octonion analytic function. Partonic 2-surfaces and string

world sheets would correspond to commutative sub-manifolds of the space-time surface and of imbedding space and would emerge naturally. The ends of braid strands at partonic 2-surface would naturally correspond to the poles of the octonion analytic functions. This would mean a huge generalization of conformal invariance of string models to octonionic conformal invariance and an exact solution of the field equations of TGD and presumably of quantum TGD itself.

4.3.3 Basic ideas about preferred extremals

The slicing of the space-time sheet by partonic 2-surfaces and string world sheets

The basic vision is that space-time sheets are sliced by partonic 2-surfaces and string world sheets. The challenge is to formulate this more precisely at the level of the preferred extremals of Kähler action.

1. Almost topological QFT property means that the Kähler action reduces to Chern-Simons terms assignable to 3-surfaces. This is guaranteed by the vanishing of the Coulomb term in the action density implied automatically if conserved Kähler current is proportional to the instanton current with proportionality coefficient some scalar function.

2. The field equations reduce to the conservation of isometry currents. An attractive ansatz is that the flow lines of these currents define global coordinates. This means that these currents are Beltrami flows [B19] so that corresponding 1-forms J satisfy the condition $J \wedge dJ = 0$. These conditions are satisfied if

$$J = \Phi \nabla \Psi$$

   hold true for conserved currents. From this one obtains that Ψ defines global coordinate varying along flow lines of J.

3. A possible interpretation is in terms of local polarization and momentum directions defined by the scalar functions involved and natural additional conditions are that the gradients of Ψ and Φ are orthogonal:

$$\nabla \Phi \cdot \nabla \Psi = 0 \ ,$$

   and that the Ψ satisfies massless d'Alembert equation

$$\nabla^2 \Psi = 0$$

   as a consequence of current conservation. If Ψ defines a light-like vector field - in other words

$$\nabla \Psi \cdot \nabla \Psi = 0 \ ,$$

   the light-like dual of Φ -call it Φ_c- defines a light-like like coordinate and Φ and Φ_c defines a light-like plane at each point of space-time sheet.

   If also Φ satisfies d'Alembert equation

$$\nabla^2 \Phi = 0 \ ,$$

   also the current

$$K = \Psi \nabla \Phi$$

   is conserved and its flow lines define a global coordinate in the polarization plane orthogonal to time-lik plane defined by local light-like momentum direction.

If Φ allows a continuation to an analytic function of the transversal complex coordinate, one obtains a coordinatization of space-time surface by Ψ and its dual (defining hyper-complex coordinate) and $w, \overline{w}$. Complex analyticity and its hyper-complex variant would allow to provide space-time surface with four coordinates very much analogous with Hamilton-Jacobi coordinates of M^4.

This would mean a decomposition of the tangent space of space-time surface to orthogonal planes defined by light-like momentum and plane orthogonal to it. If the flow lines of J defined Beltrami flow it seems that the distribution of momentum planes is integrable.

4. General arguments suggest that the space-time sheets allow a slicing by string world sheets parametrized by partonic 2-surfaces or vice versa. This would mean a intimate connection with the mathematics of string models. The two complex coordinates assignable to the Yangian of affine algebra would naturally relate to string world sheets and partonic 2-surfaces and the highly non-trivial challenge is to identify them appropriately.

Hamilton-Jacobi coordinates for M^4

The earlier attempts to construct preferred extremals [K5] led to the realization that so called Hamilton-Jacobi coordinates (m, w) for M^4 define its slicing by string world sheets parametrized by partonic 2-surfaces. m would be pair of light-like conjugate coordinates associated with an integrable distribution of planes M^2 and w would define a complex coordinate for the integrable distribution of 2-planes E^2 orthogonal to M^2. There is a great temptation to assume that these coordinates define preferred coordinates for M^4.

1. The slicing is very much analogous to that for space-time sheets and the natural question is how these slicings relate. What is of special interest is that the momentum plane M^2 can be defined by massless momentum. The scaling of this vector does not matter so that these planes are labelled by points z of sphere S^2 telling the direction of the line $M^2 \cap E^3$, when one assigns rest frame and therefore S^2 with the preferred time coordinate defined by the line connecting the tips of CD. This direction vector can be mapped to a twistor consisting of a spinor and its conjugate. The complex scalings of the twistor $(u, \overline{u}) \rightarrow \lambda u, \overline{u}/\lambda)$ define the same plane. Projective twistor like entities defining CP_1 having only one complex component instead of three are in question. This complex number defines with certain prerequisites a local coordinate for space-time sheet and together with the complex coordinate of E^2 could serve as a pair of complex coordinates (z, w) for space-time sheet. This brings strongly in mind the two complex coordinates appearing in the expansion of the generators of quantum Yangian of quantum affine algebra [K61].

2. The coordinate Ψ appearing in Beltrami flow defines the light-like vector field defining M^2 distribution. Its hyper-complex conjugate would define Ψ_c and conjugate light-like direction. An attractive possibility is that Φ allows analytic continuation to a holomorphic function of w. In this manner one would have four coordinates for M^4 also for space-time sheet.

3. The general vision is that at each point of space-time surface one can decompose the tangent space to $M^2(x) \subset M^4 = M^2_x \times E^2_x$ representing momentum plane and polarization plane $E^2 \subset E^2_x \times T(CP_2)$. The moduli space of planes $E^2 \subset E^6$ is 8-dimensional and parametrized by $SO(6)/SO(2) \times SO(4)$ for a given E^2_x. How can one achieve this selection and what conditions it must satisfy? Certainly the choice must be integrable but this is not the only condition.

Space-time surfaces as associative/co-associative surfaces

The idea that number theory determines classical dynamics in terms of associativity condition means that space-time surfaces are in some sense quaternionic surfaces of an octonionic space-time. It took several trials before the recent form of this hypothesis was achieved.

1. Octonionic structure is defined in terms of the octonionic representaton of gamma matrices of the imbedding space existing only in dimension $D = 8$ since octonion units are in one-one correspondence with tangent vectors of the tangent space. Octonionic real unit corresponds

to a preferred time axes (and rest frame) identified naturally as that connecting the tips of CD. What modified gamma matrices mean depends on variational principle for space-time surface. For volume action one would obtain induced gamma matrices. For Kähler action one obtains something different. In particular, the modified gamma matrices do not define vector basis identical with tangent vector basis of space-time surface.

2. Quaternionicity means that the modified gamma matrices defined as contractions of gamma matrices of H with canonical momentum densities for Kähler action span quaternionic sub-space of the octonionic tangent space [K18]. A further condition is that each quaternionic space defined in this manner contains a preferred hyper-complex subspace of octonions.

3. The sub-space defined by the modified gamma matrices does not co-incide with the tangent space of space-time surface in general so that the interpretation of this condition is far from obvious. The canonical momentum densities need not define four independent vectors at given point. For instance, for massless extremals these densities are proportional to light-like vector so that the situation is degenerate and the space in question reduces to 2-D hyper-complex sub-space since light-like vector defines plane M^2.

The obvious questions are following.

1. Does the analog of tangent space defined by the octonionic modified gammas contain the local tangent space $M^2 \subset M^4$ for preferred extremals? For massless extremals [K5] this condition would be true. The orthogonal decomposition $T(X^4) = M^2 \oplus_\perp E^2$ can be defined at each point if this is true. For massless extremals also the functions Ψ and Φ can be identified.

2. One should answer also the following delicate question. Can M^2 really depend on point x of space-time? CP_2 as a moduli space of quaternionic planes emerges naturally if M^2 is *same* everywhere. It however seems that one should allow an integrable distribution of M^2_x such that M^2_x is same for all points of a given partonic 2-surface.

   How could one speak about fixed CP_2 (the imbedding space) at the entire space-time sheet even when M^2_x varies?

   (a) Note first that G_2 defines the Lie group of octonionic automorphisms and G_2 action is needed to change the preferred hyper-octonionic sub-space. Various $SU(3)$ subgroups of G_2 are related by G_2 automorphism. Clearly, one must assign to each point of a string world sheet in the slicing parameterizing the partonic 2-surfaces an element of G_2. One would have Minkowskian string model with G_2 as a target space. As a matter fact, this string model is defined in the target space $G_2/SU(3)$ having dimension $D = 6$ since $SU(3)$ automorphisms leave given $SU(3)$ invariant.

   (b) This would allow to identify at each point of the string world sheet standard quaternionic basis - say in terms of complexified basis vectors consisting of two hyper-complex units and octonionic unit q_1 with "color isospin" $I_3 = 1/2$ and "color hypercharge" $Y = -1/3$ and its conjugate $\overline{q}_1$ with opposite color isospin and hypercharge.

   (c) The CP_2 point assigned with the quaternionic basis would correspond to the $SU(3)$ rotation needed to rotate the standard basis to this basis and would actually correspond to the first row of $SU(3)$ rotation matrix. Hyper-complex analyticity is the basic property of the solutions of the field equations representing Minkowskian string world sheets. Also now the same assumption is highly natural. In the case of string models in Minkowski space, the reduction of the induced metric to standard form implies Virasoro conditions and similar conditions are expected also now. There is no need to introduce action principle -just the hyper-complex analycitity is enough-since Kähler action already defines it.

3. The WZW model inspired approach to the situation would be following. The parameterization corresponds to a map $g : X^2 \to G_2$ for which g defines a flat G_2 connection at string world sheet. WZW type action would give rise to this kind of situation. The transition $G_2 \to G_2/SU(3)$ would require that one gauges $SU(3)$ degrees of freedom by bringing in $SU(3)$ connection. Similar procedure for $CP_2 = SU(3)/U(2)$ would bring in $SU(3)$ valued

chiral field and $U(2)$ gauge field. Instead of introducing these connections one can simply introduce $G_2/SU(3)$ and $SU(3)/U(2)$ valued chiral fields. What this observation suggests that this ansatz indeed predicts gluons and electroweak gauge bosons assignable to string like objects so that the mathematical picture would be consistent with physical intuition.

The two interpretations of CP_2

An old observation very relevant for what I have called M^8-H duality [K13] is that the moduli space of quaternionic sub-spaces of octonionic space (identifiable as M^8) containing preferred hyper-complex plane is CP_2. Or equivalently, the space of two planes whose addition extends hyper-complex plane to some quaternionic subspace can be parametrized by CP_2. This CP_2 can be called it CP_2^{mod} to avoid confusion. In the recent case this would mean that the space $E^2(x) \subset E_x^2 \times T(CP_2)$ is represented by a point of CP_2^{mod}. On the other hand, the imbedding of space-time surface to H defines a point of "real" CP_2. This gives two different CP_2s.

1. The highly suggestive idea is that the identification $CP_2^{mod} = CP_2$ (apart from isometry) is crucial for the construction of preferred extremals. Indeed, the projection of the space-time point to CP_2 would fix the local polarization plane completely. This condition for $E^2(x)$ would be purely local and depend on the values of CP_2 coordinates only. Second condition for $E^2(x)$ would involve the gradients of imbedding space coordinates including those of CP_2 coordinates.

2. The conditions that the planes M_x^2 form an integrable distribution at space-like level and that M_x^2 is determined by the modified gamma matrices. The integrability of this distribution for M^4 could imply the integrability for X^2. X^4 would differ from M^4 only by a deformation in degrees of freedom transversal to the string world sheets defined by the distribution of M^2s.

   Does this mean that one can begin from vacuum extremal with constant values of CP_2 coordinates and makes them non-constant but allows to depend only on transversal degrees of freedom? This condition is too strong even for simplest massless extremals for which CP_2 coordinates depend on transversal coordinates defined by $\epsilon \cdot m$ and $\epsilon \cdot k$. One could however allow dependence of CP_2 coordinates on light-like M^4 coordinate since the modification of the induced metric is light-like so that light-like coordinate remains light-like coordinate in this modification of the metric.

   Therefore, if one generalizes directly what is known about massless extremals, the most general dependence of CP_2 points on the light-like coordinates assignable to the distribution of M_x^2 would be dependence on either of the light-like coordinates of Hamilton-Jacobi coordinates but not both.

4.3.4 What could be the construction recipe for the preferred extremals assuming $CP_2 = CP_2^{mod}$ identification?

The crucial condition is that the planes $E^2(x)$ determined by the point of $CP_2 = CP_2^{mod}$ identification and by the tangent space of $E_x^2 \times CP_2$ are same. The challenge is to transform this condition to an explicit form. $CP_2 = CP_2^{mod}$ identification should be general coordinate invariant. This requires that also the representation of E^2 as (e^2, e^3) plane is general coordinate invariant suggesting that the use of preferred CP_2 coordinates - presumably complex Eguchi-Hanson coordinates - could make life easy. Preferred coordinates are also suggested by number theoretical vision. A careful consideration of the situation would be required.

The modified gamma matrices define a quaternionic sub-space analogous to tangent space of X^4 but not in general identical with the tangent space: this would be the case only if the action were 4-volume. I will use the notation $T_x^m(X^4)$ about the modified tangent space and call the vectors of $T_x^m(X^4)$ modified tangent vectors. I hope that this would not cause confusion.

$CP_2 = CP_2^{mod}$ condition

Quaternionic property of the counterpart of $T_x^m(X^4)$ allows an explicit formulation using the tangent vectors of $T_x^m(X^4)$.

1. The unit vector pair (e_2, e_3) should correspond to a unique tangent vector of H defined by the coordinate differentials dh^k in some natural coordinates used. Complex Eguchi-Hanson coordinates [L1] are a natural candidate for CP_2 and require complexified octonionic imaginary units. If octonionic units correspond to the tangent vector basis of H uniquely, this is possible.

2. The pair (e_2, e_3) as also its complexification $(q_1 = e_2 + ie_3, \overline{q}_1 = e_2 - ie_3)$ is expressible as a linear combination of octonionic units $I_2, ...I_7$ should be mapped to a point of $CP_2^{mod} = CP_2$ in canonical manner. This mapping is what should be expressed explicitly. One should express given (e_2, e_3) in terms of $SU(3)$ rotation applied to a standard vector. After that one should define the corresponding CP_2 point by the bundle projection $SU(3) \rightarrow CP_2$.

3. The tangent vector pair

$$(\partial_w h^k, \partial_{\overline{w}} h^k)$$

defines second representation of the tangent space of $E^2(x)$. This pair should be equivalent with the pair $(q_1, \overline{q}_1)$. Here one must be however very cautious with the choice of coordinates. If the choice of w is unique apart from constant the gradients should be unique. One can use also real coordinates (x, y) instead of $(w = x + iy, \overline{w} = x - iy)$ and the pair (e_2, e_3). One can project the tangent vector pair to the standard vielbein basis which must correspond to the octonionic basis

$$(\partial_x h^k, \partial_y h^k) \rightarrow (\partial_x h^k e_k^A e_A, \partial_y h^k e_k^A) e_A) \leftrightarrow (e_2, e_3) \ ,$$

where the e_A denote the octonion units in 1-1 correspondence with vielbein vectors. This expression can be compared to the expression of (e_2, e_3) derived from the knowledge of CP_2 projection.

Formulation of quaternionicity condition in terms of octonionic structure constants

One can consider also a formulation of the quaternionic tangent planes in terms of (e_2, e_3) expressed in terms of octonionic units deducible from the condition that unit vectors obey quaternionic algebra. The expressions for octonionic *resp.* quaternionic structure constants can be found at [A17] *resp.* [A20].

1. The ansatz is

$$\begin{aligned} \{E_k\} &= \{1, I_1, E_2, E_3\} \ , \\ E_2 &= E_{2k} e^k \equiv \sum_{k=2}^{7} E_{2k} e^k \ , \ E_3 = E_{3k} e^k \equiv \sum_{k=2}^{7} E_{3k} e^k \ , \\ |E_2| &= 1 \ , \ |E_3| = 1 \ . \end{aligned} \tag{4.3.1}$$

2. The multiplication table for octonionic units expressible in terms of octonionic triangle [A17] gives

$$f^{1kl} E_{2k} = E_{3l} \ , \ f^{1kl} E_{3k} = -E_{2l} \ , \ f^{klr} E_{2k} E_{3l} = \delta_1^r \ . \tag{4.3.2}$$

Here the indices are raised by unit metric so that there is no difference between lower and upper indices. Summation convention is assumed. Also the contribution of the real unit is present in the structure constants of third equation but this contribution must vanish.

3. The conditions are linear and quadratic in the coefficients E_{2k} and E_{3k} and are expected to allow an explicit solution. The first two conditions define homogenous equations which must allow solution. The coefficient matrix acting on (E_2, E_3) is of the form

$$\begin{pmatrix} f_1 & 1 \\ -1 & f_1 \end{pmatrix} ,$$

where 1 denotes unit matrix. The vanishing of the determinant of this matrix should be due to the highly symmetric properties of the structure constants. In fact the equations can be written as eigen conditions

$$f_1 \circ (E_2 \pm iE_3) = \mp i(E_2 \pm iE_3) \ ,$$

and one can say that the structure constants are eigenstates of the hermitian operator defined by I_1 analogous to color hyper charge. Both values of color hyper charged are obtained.

Explicit expression for the $CP_2 = CP_2^{mod}$ conditions

The symmetry under $SU(3)$ allows to construct the solutions of the above equations directly.

1. One can introduce complexified basis of octonion units transforming like $(1,1,3,\overline{3})$ under SU(3). Note the analogy of triplet with color triplet of quarks. One can write complexified basis as $(1, e_1, (q_1, q_2, q_3), (\overline{q}_1\overline{q}_2, \overline{q}_3))$. The expressions for complexified basis elements are

$$(q_1, q_2, q_3) = \frac{1}{\sqrt{2}}(e_2 + ie_3, e_4 + ie_5, e_6 + ie_7) \ .$$

These options can be seen to be possible by studying octonionic triangle in which all lines containing 3 units defined associative triple: any pair of octonion units at this kind of line can be used to form pair of complexified unit and its conjugate. In the tangent space of $M^4 \times CP_2$ the basis vectors q_1, and q_2 are mixtures of E_x^2 and CP_2 tangent vectors. q_3 involves only CP_2 tangent vectors and there is a temptation to interpret it as the analog of the quark having no color isospin.

2. The quaternionic basis is real and must transform like $(1, 1, q_1, \overline{q}_1)$, where q_1 is any quark in the triplet and $\overline{q}_1$ its conjugate in antitriplet. Having fixed some basis one can perform SU(3) rotations to get a new basis. The action of the rotation is by 3×3 special unitary matrix. The over all phases of its rows do not matter since they induce only a rotation in (e_2, e_3) plane not affecting the plane itself. The action of $SU(3)$ on q_1 is simply the action of its first row on (q_1, q_2, q_3) triplet:

$$\begin{aligned} q_1 &\rightarrow (Uq)_1 = U_{11}q_1 + U_{12}q_2 + U_{13}q_3 \equiv z_1q_1 + z_2q_2 + z_3q_3 \\ &= z_1(e_2 + ie_3) + z_2(e_4 + ie_5) + z_3(e_6 + ie_7) \ . \end{aligned} \tag{4.3.3}$$

The triplets (z_1, z_2, z_3) defining a complex unit vector and point of S^5. Since overall phase does not matter a point of CP_2 is in question. The new real octonion units are given by the formulas

$$\begin{aligned} e_2 &\rightarrow Re(z_1)e_2 + Re(z_2)e_4 + Re(z_3)e_6 - Im(z_1)e_3 - Im(z_2)e_5 - Im(z_3)e_7 \ , \\ e_3 &\rightarrow Im(z_1)e_2 + Im(z_2)e_4 + Im(z_3)e_6 + Re(z_1)e_3 + Re(z_2)e_5 + Re(z_3)e_7 \ . \end{aligned} \tag{4.3.4}$$

For instance the CP_2 coordinates corresponding to the coordinate patch (z_1, z_2, z_3) with $z_3 \neq 0$ are obtained as $(\xi_1, \xi_2) = (z_1/z_3, z_2/z_3)$.

Using these expressions the equations expressing the conjecture $CP_2 = CP_2^{mod}$ equivalence can be expressed explicitly as first order differential equations. The conditions state the equivalence

$$(e_2, e_3) \quad \leftrightarrow \quad (\partial_x h^k e_k^A e_A, \partial_y h^k e_k^A e_A) \ , \tag{4.3.5}$$

where e_A denote octonion units. The comparison of two pairs of vectors requires normalization of the tangent vectors on the right hand side to unit vectors so that one takes unit vector in the direction of the tangent vector. After this the vectors can be equated. This allows to expresses the contractions of the partial derivatives with vielbein vectors with the 6 components of e_2 and e_3. Each condition gives 6+6 first order partial differential equations which are non-linear by the presence of the overal normalization factor for the right hand side. The equations are invariant under scalings of (x, y). The very special form of these equations suggests that some symmetry is involved.

It must be emphasized that these equations make sense only in preferred coordinates: ordinary Minkowski coordinates and Hamilton-Jacobi coordinates for M^4 and Eguchi-Hanson complex coordinates in which $SU(2) \times U(1)$ is represented linearly for CP_2. These coordinates are preferred because they carry deep physical meaning.

Does TGD boil down to two string models?

It is good to look what have we obtained. Besides Hamilton-Jacobi conditions, and $CP_2 = CP_2^{mod}$ conditions one has what one might call string model with 6-dimensional $G_2/SU(3)$ as targent space. The orbit of string in $G_2/SU(3)$ allows to deduce the G_2 rotation identifiable as a point of $G_2/SU(3)$ defining what one means with standard quaternionic plane at given point of string world sheet. The hypothesis is that hyper-complex analyticity solves these equations.

The conjectured electric-magnetic duality implies duality between string world sheet and partonic 2-surfaces central for the proposed mathematical applications of TGD [K23, K24, K50, K63]. This duality suggests that the solutions to the $CP_2 = CP_2^{mod}$ conditions could reduce to holomorphy with respect to the coordinate w for partonic 2-surface plus the analogs of Virasoro conditions. The dependence on light-like coordinate would appear as a parametric dependence.

If this were the case, TGD would reduce at least partially to what might be regarded as dual string models in $G_2/SU(3)$ and $SU(3)/U(2)$ and also to string model in M^4 and X^4! In the previous arguments one ends up to string models in moduli spaces of string world sheets and partonic 2-surfaces. TGD seems to yield an inflation of string models! This not actually surprising since the slicing of space-time sheets by string world sheets and partonic 2-surfaces implies automatically various kinds of maps having interpretation in terms of string orbits.

4.4 Representations for WCW gamma matrices in terms of super-symplectic charges at light cone boundary

During years I have considered several variants for the representation of WCW gamma matrices and each of these proposals has had some weakness.

1. One question has been whether the Noether currents assignable to WCW Hamiltonians should play any role in the construction or whether one can use only the generalization of flux Hamiltonians. Magnetic flux Hamiltonians do not refer to the space-time dynamics implying genuine 2-dimensionality, which is a catastrophe. If the sum of the magnetic and electric flux Hamiltonians and the weak form of self duality is assumed effective 2-dimensionality is achieved. The challenge is to identify the super-partners of the flux Hamiltonians and postulate correct anti-commutation relations for the induced spinor fields to achieve anti-commutation to flux Hamiltonians.

2. In the original proposal for WCW gamma matrices the covariantly constant right handed spinors played a key role. This led to interpretational problems with quarks. Are they needed at all or do leptons and quarks define somehow equivalent representations? I discovered only recently a brutally simple but deadly objection against this approach: the resulting WCW

gamma matrices do not generate all WCW spinors from Fock vacuum. Therefore all modes of the induced spinor fields must be used.

The latter objection forced to realize that nothing is changed if one replaces the covariantly constant right handed neutrino with the collection of quark spinor modes q_n *resp.* leptonic spinor modes L_n multiplied by the contractions $J_{A+} = j^{Ak}\Gamma_k$ *resp.* its conjugate $J_{A-} = j^{A\overline{k}}\Gamma_{\overline{k}}$. It is essential that only of these contractions is used for a given H-chirality.

1. If the anti-commutator of the spinor fields is or form $J = J_{\alpha\beta}\epsilon^{\alpha\beta}\delta^2(x,y)$ at X^2 for magnetic flux Hamiltonians and appropriate generalization of this fro the sum of magnetic and electric flux Hamiltonians, the "half-Poisson bracket" $\partial_k H_A J^{k\bar{l}}\partial_{\bar{l}} H_B$ from the quark spinor field and its conjugate as anti-commutator from the leptonic spinor field can combine to the full Poisson bracket if the remaining factors are identical.

2. This happens if the quark modes and lepton-like modes are in 1-1 correspondence and the contractions of the eigenmodes resulting in the contraction satisfy $\overline{q}_m\gamma^0 q_n = \overline{L}_m\gamma^0 L_n = \Phi_{mn}$. The resulting Hamiltonians define an X^2-local algebra: that this extension is needed became obvious already earlier. A stronger condition is that the spinors can be expressed in terms of scalar function bases $\{\Phi_m\}$ so that one would have $q_{m,i} = \{\Phi_m\}q_i$ and $L_{m,i} = \{\Phi_m\}L_i$ so that one would assign to the super-currents the local Hamiltonians $\Phi_m H_A$.

3. One could of course still argue that it is questionable to use sum of quark and lepton gamma matrices since this the resulting objects to not have a well defined fermion number and cannot be used to generate physical states from vacuum. How seriously this argument should be taken is not clear to me at this moment. One could of course consider also a scenario in which one divides leptonic (or quark) modes to two classes analogous to quark and lepton modes and uses J_{A_+} *resp.* J_{A_-} for these two classes.

In any case, the recent view is that all modes of the induced spinor fields must be used, that lepton-quark degeneracy is absolutely essential for the construction of WCW geometry, and that the original super-symmetrization of the flux Hamiltonians combined with weak electric-magnetic duality is the correct approach. There are also fermionic Noether charges and their super counterparts implied by the criticality but these can be assigned with zero modes.

This section represents both the earlier version of the construction of WCW gamma matrices and the construction introducing explicitly the notion of finite measurement resolution. The motivation for the latter option is that if the number the modes of modified Dirac operator is finite, strictly local anti-commutation relations fail unless one restricts the set of points included to that corresponding to number theoretic braid. In the following integral expressions for WCW Hamiltonians and their super-counterparts are derived first. After that the motivations for replacing integrals with sums are discussed and the expressions for Hamiltonians and super Hamiltonians are derived.

4.4.1 Magnetic flux representation of the super-symplectic algebra

In order to derive representation of WCW gamma matrices and super charges it is good to restate the basic facts about the magnetic flux representation of WCW gamma matrices using the original approach based on 2-dimensional integrals.

4.4.2 Quantization of the modified Dirac action and configuration space geometry

The quantization of the modified Dirac action involves a fusion of various number theoretical ideas. The naive approach would be based on standard canonical quantization of induced spinor fields by posing anti-commutation relations between Ψ and canonical momentum density $\partial L/\partial(\partial_t\Psi)$.

Generalized magnetic and electric fluxes

Isometry invariants are just a special case of fluxes defining natural coordinate variables for WCW. Canonical transformations of CP_2 act as $U(1)$ gauge transformations on the Kähler potential of CP_2 (similar conclusion holds at the level of $\delta M^4_+ \times CP_2$).

One can generalize these transformations to local symplectic transformations by allowing the Hamiltonians to be products of the CP_2 Hamiltonians with the real and imaginary parts of the functions $f_{s,n,k}$ defining the Lorentz covariant function basis H_A, $A \equiv (a,s,n,k)$ at the light cone boundary: $H_A = H_a \times f(s,n,k)$, where a labels the Hamiltonians of CP_2.

One can associate to any Hamiltonian H^A of this kind magnetic or electric flux via the following formulas:

$$Q_{m/e}(H_A|X^2) = \int_{X^2} H_A J_{m/e} \quad . \tag{4.4.1}$$

Here the magnetic (electric) flux J_m (J_e) denotes the flux associated with induced Kähler field and its dual which is well-defined since X^2 is part of 4-D space-time surface.

The flux Hamiltonians

$$Q_i(H_A|X^2) = Q_i(H_A|X^2) \quad , \quad A \equiv (a,s,n,k) \tag{4.4.2}$$

provide a representation of WCW Hamiltonians as far as the "kinetic" part of Kähler form is considered.

Anti-commutation relations between oscillator operators associated with same partonic 2-surface

The construction of WCW gamma matrices leads to the anti-commutation relations given by

$$\begin{aligned} \{\overline{\Psi}(x)\gamma^0, \Psi(x)\} &= [J_e + J_m)\delta^2_{x,y} \quad , \\ J_e &= \int J^{03}\sqrt{g_4} \quad . \end{aligned} \tag{4.4.3}$$

Kähler magnetic flux $J_m = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ has no dependence on the induced metric.

If the weak- form of the electric-magnetic duality holds true, Kähler electric flux relates to it via the formula

$$J^{03}\sqrt{g_4} = K J_{12} \quad ,$$

where K is symplectic invariant and identifiable in terms of Kähler coupling strength from classical charge quantization condition for Kähler electric flux. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K gives the condition $K = g_K^2/\hbar = 4\pi\alpha_K$, where g_K is Kähler coupling constant. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant. The arguments leading to the identification $\epsilon \pm 1$ at the opposite boundaries of CD are discussed in [K22] , [L4] . An alternative identification is as $\epsilon = 0$ but predicts that WCW is trivial in M^4 degrees of freedom if Kähler function reduces to Chern-Simons terms.

The general form of the anti-commutation relations is therefore

$$\{\overline{\Psi}(x)\gamma^0, \Psi(x)\} = (1+K)J\delta^2_{x,y} \quad . \tag{4.4.4}$$

What is nice that at the limit of vacuum extremals the right hand side vanishes when both J and J^1 vanish so that spinor fields become non-dynamical. One can criticize the non-vanishing of the anti-commutator for vacuum extremals of Kähler action.

For the latter option the fermionic counterparts of local flux Hamiltonians can be written in the form

$$
\begin{aligned}
H_{A,\pm,n} &= \epsilon_q(A,\mp,n)H_{A,\pm,q,n}+\epsilon_L(A,\pm)H_{A,\mp,L,n} \quad , \\
H_{A,+,q,n} &= \oint \overline{\Psi}J_+^A q_n d^2x \ , \\
H_{A,-,q,n} &= \oint \overline{q}_n J_-^A \Psi d^2x \ , \\
H_{A,-,L,n} &= \oint \overline{\Psi}J_+^A L_n d^2x \ , \\
H_{A,+,L,n} &= \oint \overline{L}_n J_-^A \Psi d^2x \ , \\
J_+^A &= j^{Ak}\Gamma_k \ , \ J_-^A = j^{A\overline{k}}\Gamma_{\overline{k}} \ .
\end{aligned} \tag{4.4.5}
$$

The commutative parameters $\epsilon_q(A,\pm,n)$ *resp.* $\epsilon_L(A,\pm,n)$ are assumed to carry quark *resp.* lepton number opposite to that of $H_{A,\mp,q,n}$ *resp.* $H_{A,\mp,L,n}$ and satisfy $\epsilon_i(A,+,n)\epsilon_i(A,-,n)=1$. One encounters a hierarchy discrete algebras satisfying this condition in the construction of a symplectic analog of conformal quantum field theory required by the construction of quantum TGD [K43] . Associativity condition fixes uniquely the commutative multiplication of these units and analogs of plane waves with discrete momentum are in question.

Suppose that there is a one-one correspondence between quark modes and leptonic modes is satisfied and the label n decomposes as $n=(m,i)$, where n labels a scalar function basis and i labels spinor components. This would give

$$
\begin{aligned}
q_n = q_{m,i} &= \Phi_m q_i \ , \\
L_n = L_{m,i} &= \Phi_m L_i \ , \\
\overline{q}_i\gamma^0 q_j &= \overline{L}_i\gamma^0 L_j = g_{ij} \ .
\end{aligned} \tag{4.4.6}
$$

Suppose that the inner products g_{ij} are constant. The simplest possibility is $g_{ij}=\delta_{ij}$ Under these assumptions the anti-commutators of the super-symmetric flux Hamiltonians give flux Hamiltonians.

$$
\{H_{A,+,n},H_{A,-,n}\} = g_{ij}\oint \overline{\Phi}_m\Phi_n H_A J d^2x \ . \tag{4.4.7}
$$

The product of scalar functions can be expressed as

$$
\overline{\Phi}_m\Phi_n = c_{mn}^{\ k}\Phi_k \ . \tag{4.4.8}
$$

Note that the notion of symplectic QFT [K12] led to a scalar function algebra of similar kind consisting of phase factors and there excellent reasons to consider the possibility that there is a deep connection with this approach.

One expects that the symplectic algebra is restricted to a direct sum of symplectic algebras localized to the regions where the induced Kähler form is non-vanishing implying that the algebras associated with different region form to a direct sum. Also the contributions to WCW metric are direct sums. The symplectic algebras associated with different region can be truncated to finite-dimensional spaces of symplectic algebras associated with the regions in question. As far as coordinatization of the reduced WCW is considered, these symplectic sub-spaces are enough. These truncated algebras naturally correspond to the hyper-finite factor property of the Clifford algebra of WCW.

Generalization of WCW Hamiltonians and anti-commutation relations between flux Hamiltonians belonging to different ends of CD

This picture requires a generalization of the view about configuration space Hamiltonians since also the interaction term between the ends of the line is present not taken into account in the previous approach.

1. The proposed representation of WCW Hamiltonians as flux Hamiltonians [K10, K9] , [L5]

$$Q(H_A) = \int H_A J d^2x \ . \tag{4.4.9}$$

works for the kinetic terms only since J is not expectred to be the same at the ends of the line.

The assumption that Poisson bracket of WCW Hamiltonians reduces to the level of imbedding space - in other words $\{Q(H_A), Q(H_B\} = Q(\{H_A, H_B\})$ - can be justified. One starts from the representation in terms of say flux Hamiltonians $Q(H_A)$ and *defines* $J_{A,B}$ as $J_{A,B} \equiv Q(\{H_A, H_B\})$. One has $\partial H_A/\partial t_B = \{H_B, H_A\}$, where t_B is the parameter associated with the exponentiation of H_B. The inverse J^{AB} of $J_{A,B} = \partial H_B/\partial t_A$ is expressible as $J^{A,B} = \partial t_A/\partial H_B$. From these formulas one can deduce by using chain rule that the bracket $\{Q(H_A), Q(H_B\} = \partial t_C Q(H_A) J^{CD} \partial t_D Q(H_B)$ of flux Hamiltonians equals to the flux Hamiltonian $Q(\{H_A, H_B\})$.

2. One should be able to assign to WCW Hamiltonians also a part corresponding to the interaction term. The symplectic conjugation associated with the interaction term permutes the WCW coordinates assignable to the ends of the line. One should reduce this apparently non-local symplectic conjugation (if one thinks the ends of line as separate objects) to a non-local symplectic conjugation for $\delta CD \times CP_2$ by identifying the points of lower and upper end of CD related by time reflection and assuming that conjugation corresponds to time reflection. Formally this gives a well defined generalization of the local Poisson brackets between time reflected points at the boundaries of CD. The connection of Hermitian conjugation and time reflection in quantum field theories is is in accordance with this picture.

3. Perhaps the only manner to proceed is to assign to the flux Hamiltonian also a part obtained by the replacement of the flux integral over X^2 with an integral over the projection of X^2 to a sphere S^2 assignable to the light-cone boundary or to a geodesic sphere of CP_2, which come as two varieties corresponding to homologically trivial and non-trivial spheres. The projection is defined as by the geodesic line orthogonal to S^2 and going through the point of X^2. The hierarchy of Planck constants assigns to CD a preferred geodesic sphere of CP_2 as well as a unique sphere S^2 as a sphere for which the radial coordinate r_M or the light-cone boundary defined uniquely is constant: this radial coordinate corresponds to spherical coordinate in the rest system defined by the time-like vector connecting the tips of CD. Either spheres or possibly both of them could be relevant.

Recall that also the construction of number theoretic braids and symplectic QFT [K12] led to the proposal that braid diagrams and symplectic triangulations could be defined in terms of projections of braid strands to one of these spheres. One could also consider a weakening for the condition that the points of the number theoretic braid are algebraic by requiring only that the S^2 coordinates of the projection are algebraic and that these coordinates correspond to the discretization of S^2 in terms of the phase angles associated with θ and ϕ.

This gives for the corresponding contribution of the WCW Hamiltonian the expression

$$Q(H_A)_{int} = (1+K)\int_{S^2_\pm} H_A X \delta^2(s_+, s_-) d^2 s_\pm = (1+K)\int_{P(X^2_+)\cap P(X^2_-)} \frac{\partial(s^1, s^2)}{\partial(x^1_\pm, x^2_\pm)} (d^2 x_\pm) \tag{4.4.10}$$

Here the Poisson brackets between ends of the line using the rules involve delta function $\delta^2(s_+,s_-)$ at S^2 and the resulting Hamiltonians can be expressed as a similar integral of $H_{[A,B]}$ over the upper or lower end since the integral is over the intersection of S^2 projections.

The expression must vanish when the induced Kähler form vanishes for either end. This is achieved by identifying the scalar X in the following manner:

$$\begin{aligned} X &= J^+_{kl} + J^-_{kl} \ , \\ J^{kl}_{\pm} &= \partial_\alpha s^k \partial_\beta s^l J^{\alpha\beta}_{\pm} \ . \end{aligned} \tag{4.4.11}$$

The tensors are lifts of the induced Kähler form of $X^2_{\pm}$ to S^2 (not CP_2).

4. One could of course ask why these Hamiltonians could not contribute also to the kinetic terms and why the brackets with flux Hamiltonians should vanish. This relate to how one *defines* the Kähler form. It was shown above that in case of flux Hamiltonians the definition of Kähler form as brackets gives the basic formula $\{Q(H_A),Q(H_B)\} = Q(\{H_A,H_B\}$ and same should hold true now. In the recent case $J_{A,B}$ would contain an interaction term defined in terms of flux Hamiltonians and the previous argument should go through also now by identifying Hamiltonians as sums of two contributions and by introducing the doubling of the coordinates t_A.

5. The quantization of the modified Dirac operator must be reconsidered. It would seem that one must add to the super-Hamiltonian completely analogous term obtained by replacing J with $X\partial(s^1,s^2)/\partial(x^1_{\pm},x^2_{\pm})$. Besides the anti-commutation relations defining correct anti-commutators to flux Hamiltonians, one should pose anti-commutation relations consistent with the anti-commutation relations of super Hamiltonians. In these anti-commutation relations $J\delta^2(x,y)$ would be replaced with $X\delta^2(s^+,s^-)$. This would guarantee that the oscillator operators at the ends of the line are not independent and that the resulting Hamiltonian reduces to integral over either end for $H_{[A,B]}$.

4.4.3 Expressions for WCW super-symplectic generators in finite measurement resolution

The expressions of WCW Hamiltonians and their super counterparts just discussed were based on 2-dimensional integrals. This is problematic for several reasons.

1. In p-adic context integrals do not makes sense so that this representation fails in p-adic context (for pe-adic numbers see [A31]). Sums would be more appropriate if one wants number theoretic universality at the level of basic formulas.

2. The use of sums would also conform with the notion of finite measurement resolution having discretization in terms of intersections of X^2 with number theoretic braids as a space-time correlate.

3. Number theoretic duality suggests a unique realization of the discretization in the sense that only the points of partonic 2-surface X^2 whose $\delta M^4_{\pm}$ projections commute in hyper-octonionic sense and thus belong to the intersections of the projection $P_{M^4}(X^2)$ with radial light-like geodesics $M_{\pm}$ representing intersections of $M^2 \subset M^4 \subset M^8$ with $\delta M^4_{\pm} \times CP_2$ contribute to WCW Hamiltonians and super Hamiltonians and therefore to the WCW metric.

Clearly, finite measurement resolution seems to be an unavoidable aspect of the geometrization of WCW as one can expect on basis of the fact that WCW Clifford algebra provides representation for hyper-finite factors of type II_1 whose inclusions provide a representation for the finite measurement resolution. This means that WCW can be represented as a finite-dimensional space in arbitrary precise approximation so that also also configuration Clifford algebra and WCW spinor fields becomes finite-dimensional.

The modification of anti-commutation relations to this case is

$$\{\overline{\Psi}(x_m)\gamma^0, \Psi(x_n)\} = (1+K)J\delta_{x_m,x_n} \ . \quad (4.4.12)$$

Note that the constancy of γ^0 implies a complete symmetry between the two points. The number of points must be the maximal one consistent with the Kronecker delta type anti-commutation relations so that information is not lost.

The question arises about the choice of the points x_m. This choice should general coordinate invariant. The number theoretic vision leads to the notion of number theoretic braid defined as the set of points common to real and p-adic variant of X^2. The points of the number theoretic braid are excellent candidates for points x_n. The p-adic variant exists only if X^2 is defined by rational functions with coefficients which are possibly algebraic and thus make sense both in real and p-adic sense. These points belong to the algebraic extension of rational numbers appearing in the representation of X^2 as an algebraic surface but one can consider quite generally the possibility that the points of the number theoretic braid are rational or in a finite algebraic extension of rationals. What is important that if one restricts the consideration to rational points this criterion makes sense even if X^2 is not algebraic. In the generic case one can expect that the number of these points is finite.

4.4.4 WCW geometry and hierarchy of inclusions of hyper-finite factors of type II_1

The WCW metric defined as anti-commutators of the WCW gamma matrices is extremely degenerate since it effectively corresponds to a quadratic form in N-dimensional space, where N_m is the total number of the eigenmodes of D_K. Since two Hamiltonians whose values and corresponding Killing vector fields co-incide at the points of B are equivalent for given ray $M_\pm$, it is natural to pose a cutoff in the number of Hamiltonians used for the representation of reduced WCW in given region inside which induced Kähler form is non-vanishing. The natural manner to pose this cutoff is by ordering the representations with respect to dimension and eigenvalue of Casimir operator for the irreducible representations of $SO(3) \times SO(4)$ in case of M^8 and for the representations of $SO(3) \times SU(3)$ in case of H.

This boils down to a hierarchy of approximate representations of the WCW as Kähler manifold with spinor structure with a truncation of the Clifford algebra to a finite dimensional Clifford algebra. This is in spirit with the proposed interpretation of the inclusion sequence of hyper-finite factors of type II_1 and with the very notion of hyper-finiteness.

A rather concrete connection of WCW geometry with generalized eigenvalue spectrum of the Kähler-Dirac (K-D) operator and basic quantum physics suggests itself if the Dirac determinant can be identified as exponent of Kähler action. One must however be however aware of following points.

1. It would be exaggeration to say that Kähler function emerges from K-D action. The reason is that K-D gamma matrices appear in K-D action and internal consistency requires that an extremal of K-D action is in question. Hence it seems that Khler action and K-D action are in completely democratic position and one can wonder whether the possible connection actually gives any profound insights or means anything practical. It could only create technical challenges and one can claim that the definition of exponent of vacuum functional reducing to exponent of Chern-Simons terms looks much more practical and elegant.

2. Kähler function corresponds to Kähler action in Euclidian space-time regions assignable to the lines of generalized Feynman diagrams. It is not clear whether one represent also the Kähler action from Minkowskian regions in this manner.

3. The definition of the Dirac determinant is far from obvious. The spectrum of the Kähler Dirac (KD) operator was originally identified in terms of generalized eigenvalues. The identification coming first in mind would be in terms of conformal weights assignable to the modes of KD operator. The experience with the string models suggests that these conformal weights are integer valued, which would mean that the multiplicative contribution from given string world sheet is constant and cannot depend on 3-surface at all!

The boundary conditions at the string curves at the space-like ends of space-time surface however give algebraic form of Dirac equation with the analog of Higgs coupling in algebraic form $(p^k\gamma_k + \Gamma^n)\Psi = 0$, with p^k identifiable as four-momentum of fermionic line emanating from partonic 2-surface. The normal component Γ^n (in time direction) of the vector defined by K-D gamma matrices defines the analog of Higgs vacuum expectation value, and could be covariantly constant along string curve for a suitable choice of string coordinates. $h^2 \equiv (\Gamma^n)^2$ could be interpreted as ground state conformal weight. In p-adic mass calculations ground state conformal weight must be negative half-odd integer and the time-like character of Γ^n could explain this. h^2 could have p-adically small deviation from half-odd integer value and give rise to a Higgs like additional contribution to the conformal weights.

Since spinor modes effectively propagate as particles with momentum p^k along braid strands one could argue that one must include h^2 to the integer valued conformal weight so that the square of Dirac determinant would be defined as as the product of conformal weights $h(n) = h^2 + nM_0^2$, M_0 the mass scale determined by CP_2 radius.

The resulting determinant - if well-defined - would depend on space-time surface and would be obtained as a perturbation from the determinant assignable to Riemann Zeta. Modulus squared for the exponent of vacuum functional would be analogous to the square of Dirac determinant associated with a massless fermion with eigenvalues of m^2 replaced with $h(n)$. The overall determinant would be product over the determinants coming from various strings and possibly also from he partonic 2-surfaces.

If one accepts this questionable proposal, one can relate WCW geometry directly to elementary particle physics. For instance, from the general expression of Kähler metric in terms of Kähler function

$$G_{k\bar{l}} = \partial_k\partial_{\bar{l}}K = \frac{\partial_k\partial_{\bar{l}}exp(K)}{exp(K)} - \frac{\partial_k exp(K)}{exp(K)}\frac{\partial_{\bar{l}}exp(K)}{exp(K)} \ , \tag{4.4.13}$$

and from the expression of $exp(K) = \prod_i \lambda_i$ as the product of of finite number of eigenvalues of $D_K(X^3)$, the expression

$$G_{k\bar{l}} = \sum_i \frac{\partial_k\partial_{\bar{l}}\lambda_i}{\lambda_i} - \frac{\partial_k\lambda_i}{\lambda_i}\frac{\partial_{\bar{l}}\ lambda_i}{\lambda_i} \tag{4.4.14}$$

for the WCW metric follows. Here complex coordinates refer to the complex coordinates of WCW.

A good candidate for these complex coordinates are the complex coordinates of $S^2\times S$, $S = CP_2$ or E^4, for the points of B so that a close connection with the geometry of imbedding space is obtained. Once these coordinates have been specified G can be contracted with the Killing vector fields of WCW isometries defining the coordinates for the truncated WCW. By studying the behavior of eigenvalue spectrum under small deformations of X_l^3 by symplectic transformations of $\delta CD \times S$ the components of G can be estimated.

Chapter 5

Does the Modified Dirac Equation Define the Fundamental Action Principle?

5.1 Introduction

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The Dirac determinant associated with the Kähler-Dirac action is an excellent candidate in this respect.

The original working hypothesis was that Dirac determinant defines the vacuum functional of the theory having interpretation as product of the exponent of Kähler function of world of classical worlds (WCW) identified as Kähler action coming from Euclidian space-time regions and the exponent of imaginary contribution identified as Kähler action from Minkowskian regions. It seems however that the most one can demand is that Dirac determinant equals to the exponent of Kähler action. The reason is that Kähler-Dirac gamma matrices involving canonical momentum densities for Kähler action appear in modified (Kähler-Dirac) action.

5.1.1 What are the basic equations of quantum TGD?

A good place to start is to as what might the basic equations of quantum TGD. There are two kinds of equations at the level of space-time surfaces.

1. Purely classical equations define the dynamics of the space-time sheets as preferred extremals of Kähler action. Preferred extremals are quantum critical in the sense that second variation vanishes for critical deformations representing zero modes. This condition guarantees that corresponding fermionic currents linear in deformations are conserved. There is infinite hierarchy of these currents and they define fermionic counterparts for zero modes.

   Zero energy ontology (ZEO) was motivated by the non-determinism of Kähler action suggesting that it difficult to assign unique preferred extremal to given 3-surface in positive energy ontology. In ZEO one can consider the possibility that the attribute "preferred" is not needed in given measurement resolution since the basic objects are now either pairs of space-like 3-surfaces at the ends of CD or these plus parton orbits (light-like 3-surfaces at which the signature of the induced metric changes).

2. The purely quantal equations are associated with the representations of various super-conformal algebras and with the Kähler-Dirac equation. The requirement that there are deformations of the space-time surface - actually infinite number of them - giving rise to conserved fermionic charges implies quantum criticality at the level of Kähler action in the sense of critical deformations.

3. The precise forms of Kähler action and Kähler Dirac equation at effective and real boundaries (boundary conditions) are not completely fixed without further input. For Kähler action the inputs are Lagrange multiplier terms at boundary like 3-surfaces expressing weak form of electric-magnetic duality and the equality of quantal and classical charges in Cartan algebra required by quantum classical correspondence (QCC). These states with well-defined classical charges might correspond to outcomes of state function reduction implying localization in WCW.

   The condition that fermionic propagator is non-trivial forces the addition of Chern-Simons Dirac term at the partonic orbits at which the signature of the induced metric changes. Supersymmetry requires the addition of Chern-Simons term at partonic orbits to Kähler action. This means explicit breaking of CP and T. The effective reduction of both Kähler and Kähler-Dirac equation to boundary terms means enormous calculational simplification and is consistent with the vision inspired by twistor approach [K44].

4. At the level of WCW spinor fields describing zero energy states quantal equations involve also generalized Feynman rules for M-matrix generalizing S-matrix to a "complex square root" of density matrix and defined by time-like entanglement coefficients between positive and negative energy parts of zero energy states is certainly the basic goal of quantum TGD.

5. The notion of weak electric-magnetic duality leads to a detailed understanding of how TGD reduces to almost topological quantum field theory. If Kähler current defines 4-D Beltrami flow, it is possible to find a gauge in which Coulomb contribution to Kähler action vanishes so that it reduces to Chern-Simons term. If light-like 3-surfaces and ends of space-time surface are extremals of Chern-Simons action also effective 2-dimensionality is realized. The condition that the theory reduces to almost topological QFT and the hydrodynamical character of field equations leads to a detailed ansatz for the general solution of field equations and also for the solutions of the modified Dirac equation relying on the notion of Beltrami flow for which the flow parameter associated with the flow lines defined by a conserved current extends to a global coordinate. This makes the theory is in well-defined sense completely integrable. Direct connection with massless theories emerges: every conserved Beltrami currents corresponds to a pair of scalar functions with the first one satisfying massless d'Alembert equation in the induced metric. The orthogonality of the gradients of these functions allows interpretation in terms of polarization and momentum directions. The Beltrami flow property can be also seen as one aspect of quantum criticality since the conserved currents associated with critical deformations define this kind of pairs.

6. The hierarchy of Planck constants provides also a fresh view to the quantum criticality. The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. It however seems that TGD actually predicts this hierarchy of covering spaces. The extreme non-linearity of the field equations defined by Kähler action means that the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is 1-to-many. This leads naturally to the introduction of the covering space of $CD \times CP_2$, where CD denotes causal diamond defined as intersection of future and past directed light-cones.

At the level of WCW there is the generalization of the Dirac equation, which can be regarded as a purely classical Dirac equation. The modified Dirac operators associated with quarks and leptons carry fermion number but the Dirac equations are well-defined. An orthogonal basis of solutions of these Dirac operators define in zero energy ontology a basis of zero energy states. The M-matrices defining entanglement between positive and negative energy parts of the zero energy state define what can be regarded as analogs of thermal S-matrices. The M-matrices associated with the solution basis of the WCW Dirac equation define by their orthogonality unitary U-matrix between zero energy states. This matrix finds the proper interpretation in TGD inspired theory of consciousness. WCW Dirac equation as the analog of super-Virasoro conditions for the "gamma fields" of superstring models defining super counterparts of Virasoro generators was the main focus during earlier period of quantum TGD but has not received so much attention lately and will not be discussed in this chapter.

Quantum classical correspondence (QCC) requires a coupling between quantum and classical and this coupling should also give rise to a generalization of quantum measurement theory. The big question mark is how to realize this coupling.

1. The proposal discussed in this chapter is that the addition of a measurement interaction term to the Kähler-Dirac action could do the job, solve a handful of problems of quantum TGD and unify various visions about the physics predicted by quantum TGD. This proposal implies QCC at the level of Kähler-Dirac action and Kähler action.

2. Another possibility is that QCC is realized at the level of WCW Dirac operator and Kähler-Dirac operator contains only interior term. The vanishing of the normal component of fermion current replaces Chern-Simons Dirac operator at various boundary like surfaces. I have proposed that WCW spinor fields with given quantum charges in Cartan algebra are superpositions of space-time surfaces with same classical charges. A stronger form of QCC at the level of WCW would be that classical correlation functions for various geometric observables are identical with quantal correlation functions.

   QCC could be realized at the level of WCW by putting it in by hand. One can of course consider also the possibility that the equality of quantal and classical Cartan charges is realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system with Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD in zero energy ontology (ZEO) can be regarded as square root of thermodynamics, the procedure looks logically sound.

5.1.2 Kähler-Dirac equation for induced classical spinor fields

The basic vision is that WCW geometry reduces to the second quantization of induced spinor fields. This means that WCW gamma matrices are linear combinations of fermionic oscillator operators and the Dirac determinant equals to vacuum functional of the theory. An unproven conjecture is that this determinant equals to the exponent of Kähler action for its preferred extremal.

The motivation for the Kähler-Dirac action came from the observation that the counterpart of the ordinary Dirac equation is internally consistent only if the space-time surfaces are minimal surfaces. One can however assign to any general coordinate invariant action principle for space-time surfaces a unique Kähler-Dirac action, which is internally consistent and super-symmetric. By quantum-classical correspondence space-time geometry must carry information about conserved quantum charges assignable to partonic 2-surfaces and it took considerable to to realize that this is achieved via measurement interaction terms realized as Lagrangian multiplier terms stating that classical conserved charges belonging to Cartan algebra are equal to their quantum counterparts for the space-time surfaces in quantum suerposition.

Second key idea [K69, K80] is that the well-definedness of em charge eigenvalue for spinor modes requires their localization to 2-D string world sheets and possibly also partonic 2-surfaces at which induced W boson field and possibly also Z^0 field vanish. Due to the presence of classical W boson fields this is possible only if localization takes plce at 2-D string world sheets and partonic 2-surfaces. Therefore string theory like structure emerges as part of TGD. The super Hamiltoanians defined in terms fluxes of Hammiltonians over partonic 2-surfaces are modified: a super-Hamiltonian at point of partonic 2-surface is replaced with an integral over stringy curve connecting points of two partonic 2-surfaces. Boundary conditions for the modes of induced spinor field can be interpreted as classical correlate for the stringy mass formula.

Preferred extremals as critical extremals

The study of the Kähler-Dirac equation leads to a detailed view about criticality. Quantum criticality [D5] fixes the values of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current except in the case of Cartan algebra of isometries. Quantum criticality allows to fix the values of couplings appearing in the measurement interaction by using the condition $K \rightarrow K + f + \overline{f}$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs).

The discovery that the hierarchy of Planck constants realized in terms of singular covering spaces of $CD \times CP_2$ can be understood in terms of the extremely non-linear dynamics of Kähler action implying 1-to-many correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates led to a further very concrete understanding of the criticality at space-time level and its relationship to zero energy ontology [K22] .

Criticality is accompanied by conformal invariance and this leads to the proposal that critical deformations correspond to Kac-Moody type conformal algebra respecting the light-likeness of the partonic orbits and acting trivially at partonic 2-surfaces. Sub-algebras of conformal algebras with conformal weights divisible by integer n would act as gauge symmetries and these algebras would form an inclusion hierarchy defining hierarchy of symmetry breakings. n would also characterize the value of Planck constant $h_{eff} = n \times h$ assignable to various phases of dark matter.

Inclusion of the Chern-Simons Dirac term

Kähler action contains Chern-Simons term cancelling the Chern-Simons contribution of Kähler action at space-time interior at partonic orbit reducing to Chern-Simons terms so that only the contribution at space-like ends of space-time surface at the boundaries of causal diamond (CD) remains.

By supersymmetry also Kähler-Dirac action contains Chern-Simons Dirac term at partonic orbits implying non-trivial fermionic propagator at the boundaries of string world sheets at which the spinor modes are localized. The generalized eigenvalues $ip^k\gamma_k$ of C-S-D operator correspond to virtual four-momenta.

The inclusion of Chern-Simons term localized at partonic orbits to the definition of Kähler action and Chern-Simons-Dirac term to the definition Kähler-Dirac action at partonic orbits implies explicit breaking of CP and T. This term should explain the CP breaking associated with the CKM matrix of quarks.

5.1.3 Dirac determinant as exponent of Kähler action?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. An obvious guess is that Dirac determinant equals to the vacuum functional identified as exponent of Kähler function from Euclidian space-time regions and its its imaginary counterpart from Minkowskian space-time regions. This does not mean that Kähler-Dirac action would be alone enough as the original dream was. The reason is simple: Kähler-Dirac gamma matrices are defined in terms of canonical momentum densities of Kähler action.

1. The natural definition of Dirac determinant is as the product of the generalized eigenvalues. This product makes sense in Clifford algebra and by symmetries must be equal proportional to unit matrix. One can defined the product also as product of hyper-quaternionic numbers. The product contains natural IR cutoff posed by the size of the CD involved and UV cutoff defined by the size of the smalles sub-CD. The hypohtesis that the determinant equals to exponent of Kähler action forces its finiteness. Dirac determinant depends on string world sheet. For instance, if one poses periodic boundary conditions the generalized eigenvalues of C-S-D operator depend on the length of the fermion line measured using the metric defined by the anticommutators of C-S-D gamma matrices.

2. One can also add to Kähler action 3-D boundary terms defining measurement interaction. In particular, fixing the classical conserved charges of the space-time surfaces in the quantum superposition. Also Kähler-Dirac action contains measurement interaction term coming from these terms. In absence of measurement interaction terms Kähler-Dirac equation gives boundary term $\Gamma^n\Psi = 0$. This equation is satisfied if one has $\Gamma^n\Psi = p^k\gamma_k\Psi = 0$ where

p^k is light-like incoming four-momentum. Space-like boundaries correspond to on-mass-shell states and do not contribute to Dirac determinant.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found here [L13]. Another glossary type representation involving both pdf and html files can be found at `http://www.tgdtheory.fi/tgdglossary.pdf`. The topics relevant to this chapter are given by the following list.

- TGD as infinite-dimensional geometry [L37]
- WCW spinor fields [L42]
- KD equation [L25]
- Kaehler-Dirac action [L24]

5.2 Weak form electric-magnetic duality and its implications

The notion of electric-magnetic duality [B2] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K10] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.

3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.

4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

   The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural. Also Chern-Simons Dirac equation implies the localization of solutions to flow lines, and this is consistent with the localization solutions of Kähler-Dirac equation to string world sheets.

5.2.1 Could a weak form of electric-magnetic duality hold true?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

4. To formulate a weaker form of the condition let us introduce coordinates (x^0,x^3,x^1,x^2) such (x^1,x^2) define coordinates for the partonic 2-surface and (x^0,x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} \ . \tag{5.2.1}$$

A more general form of this duality is suggested by the considerations of [K22] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B1] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon\times\epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} \ . \tag{5.2.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1+K)J_{12} \ , \tag{5.2.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K=0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L1] , [L1] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \tag{5.2.4}$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \tag{5.2.5}$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ \ p = sin^2(\theta_W) \ . \end{aligned} \tag{5.2.6}$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi}\times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \tag{5.2.7}$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the modified Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K37] supports this interpretation.

3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z \ . \tag{5.2.8}$$

This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{5.2.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{5.2.10}$$

   Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L1]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K41]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [K56]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.

4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

5.2.2 Magnetic confinement, the short range of weak forces, and color confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L\overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L\nu_R$. $\nu_L\overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hypercharge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D3] .

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [K19] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_\pm$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

2. The addition of the particles $X^\pm$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

3. How should one describe the bound state formed by the fermion and $X^\pm$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K28] . If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K29] .

5.2.3 Could Quantum TGD reduce to almost topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the modified Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^\alpha A_\alpha$ plus and integral of the boundary term $J^{n\beta}A_\beta\sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta}A_\beta\sqrt{g_4}$ over the ends of the 3-surface.

2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K\epsilon^{n\beta\gamma\delta}J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K\epsilon^{0\beta\gamma\delta}J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $\hbar_0 \rightarrow r\hbar_0$ would effectively describe this. Boundary conditions would however give $1/r$ factor so that $\hbar$ would disappear from the Kähler function! The original attempt to realize quantum TGD as an almost topological QFT was in terms of Chern-Simons action but was given up. It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [K5]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

2. The original naive conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha(J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma}J_{\beta\ gamma})\sqrt{g_4}d^3x \quad . \tag{5.2.11}$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.

3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta}K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^{\alpha}\partial_{\alpha}\phi = -j^{\alpha}A_{\alpha} \ . \tag{5.2.12}$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^{\alpha}/dt = j_K^{\alpha}$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta}j_{\beta}^{K}\partial_{\gamma}j_{delta}^{K} = 0 \ . \tag{5.2.13}$$

j_K is a four-dimensional counterpart of Beltrami field [B19] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [K5] . The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^{\alpha}\partial_{\alpha}\phi = \partial_{\alpha}j^{\alpha}\phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j_K^{\alpha}\phi$ and $j_I^{\alpha}\phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^{\alpha}\partial_{\alpha}\phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_{\alpha}(j^{\alpha}\phi) = 0 \ . \tag{5.2.14}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_{\phi}^{e} = \int j^0\phi\sqrt{g_4}d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q_{\phi}^{m} = \sum\int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved

charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.

7. A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q^m_ϕ and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

8. Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to modified Dirac action as boundary term.

   Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

   One can assign to partonic orbits Chern-Simons Dirac action and now the condition would be that the action of C-S-D operator equals to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator. Twistor Grassmann approach suggests that also the virtual fermions reduce effectively to massless on-shell states but have non-physical helicity.

5.2.4 About the notion of measurement interaction

The notion of measurement has been central notion in quantum TGD but the precise definition of this notion is far from clear. In the following two possibly equivalent formulations are considered. The first formulation relies on the gauge transformations leaving Coulomb term of Kähler action unchanged and the second one to the interpretation of TGD as a square root of thermodynamics allowing to fix the values of conserved classical charges for zero energy energy state using Lagrange multipliers analogous to chemical potentials.

1. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j^\alpha_K \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha\phi) = 0 . \tag{5.2.15}$$

   As a consequence Coulomb term reduces to a difference of the conserved charges $Q^e_\phi = \int j^0\phi\sqrt{g_4}d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q^m_\phi = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

2. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action.

   The gauge transformed Kähler potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function. Kähler function is identified as a Dirac determinant of Chern-Simons Dirac operator (after many turns and twists) and the spectrum of this operator should not be invariant under these gauge transformations if this picture is correct. This is is achieved if the gauge transformation is carried only in the Dirac action corresponding to instanton term: this assumption is motivated by the breaking of time reversal invariance induced by quantum measurements. The modification of Kähler action can be guessed to correspond just to the Chern-Simons contribution from the instanton term.

3. A reasonable looking guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q^m_ϕ and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

In zero energy ontology (ZEO) TGD can be seen as square root of thermodynamics and this suggests an alternative manner to define what measurement interaction term means.

1. The condition that the space-time sheets appearing in superposition of space-time surfaces with given quantum numbers in Cartan algebra have same classical quantum numbers associated with Kähler action can be realized in terms of Lagrange multipliers in standard manner. These kind of terms would be analogous to various chemical potential terms in the partition function. One could call them measurement interaction terms. Measurement interaction terms would code the values of quantum charges to the space-time geometry.

   Kähler action contains also Chern-Simons term at partonic orbits compensating the Chern-Simons terms coming from Kähler action when weak form of electric-magnetic duality is assumed. This guarantees that Kähler action for preferred extremals reduces to Chern-Simons terms at the space-like ends of the spacetime surface and one obtains almost topological QFT.

2. If Kähler-Dirac action is constructed from Kähler action in super-symmetric manner by defining the modified gamma matrices in terms of canonical momentum densities one obtains also the fermionic counterparts of the Lagrange multiplier terms at partonic orbits and could call also them measurement interaction terms. Besides this one has also the Chern-Simons Dirac terms associated with the partonic orbits giving ordinary massless Dirac propagator. In presence of measurement interaction terms at the space-like ends of the space-time surface the boundary conditions $\Gamma^n\Psi = 0$ at the ends would be modified by the addition of term coming from the modified gamma matrix associated with the Lagrange multiplier terms. The original generalized massless generalized eigenvalue spectrum $p^k\gamma_k$ of Γ^n would be modified to massive spectrum given by the condition

$$(\Gamma^n + \sum_i \lambda_i \Gamma^\alpha_{Q_i} D_\alpha)\Psi = 0 \ ,$$

   where Q_i refers to i:th conserved charge.

An interesting question is whether these two manners to introduce measurement interaction terms are actually equivalent.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

5.2.5 Kähler action for Euclidian regions as Kähler function and Kähler action for Minkowskian regions as Morse function?

One of the nasty questions about the interpretation of Kähler action relates to the square root of the metric determinant. If one proceeds completely straightforwardly, the only reason conclusion is that the square root is imaginary in Minkowskian space-time regions so that Kähler action would be complex. The Euclidian contribution would have a natural interpretation as positive definite Kähler function but how should one interpret the imaginary Minkowskian contribution? Certainly the path integral approach to quantum field theories supports its presence. For some mysterious reason I was able to forget this nasty question and serious consideration of the obvious answer to it. Only when I worked between possibile connections between TGD and Floer homology [K63] I realized that the Minkowskian contribution is an excellent candidate for Morse function whose critical points give information about WCW homology. This would fit nicely with the vision about TGD as almost topological QFT.

Euclidian regions would guarantee the convergence of the functional integral and one would have a mathematically well-defined theory. Minkowskian contribution would give the quantal interference effects and stationary phase approximation. The analog of Floer homology would represent quantum superpositions of critical points identifiable as ground states defined by the extrema of Kähler action for Minkowskian regions. Perturbative approach to quantum TGD would rely on functional integrals around the extrema of Kähler function. One would have maxima also for the Kähler function but only in the zero modes not contributing to the WCW metric.

There is a further question related to almost topological QFT character of TGD. Should one assume that the reduction to Chern-Simons terms occurs for the preferred extremals in *both* Minkowskian and Euclidian regions or only in Minkowskian regions?

1. All arguments for this have been represented for Minkowskian regions [K18] involve local light-like momentum direction which does not make sense in the Euclidian regions. This does not however kill the argument: one can have non-trivial solutions of Laplacian equation in the region of CP_2 bounded by wormhole throats: for CP_2 itself only covariantly constant right-handed neutrino represents this kind of solution and at the same time supersymmetry. In the general case solutions of Laplacian represent broken super-symmetries and should be in one-one correspondences with the solutions of the modified Dirac equation. The interpretation for the counterparts of momentum and polarization would be in terms of classical representation of color quantum numbers.

2. If the reduction occurs in Euclidian regions, it gives in the case of CP_2 two 3-D terms corresponding to two 3-D gluing regions for three coordinate patches needed to define coordinates and spinor connection for CP_2 so that one would have two Chern-Simons terms. I have earlier claimed that without any other contributions the first term would be identical with that from Minkowskian region apart from imaginary unit and different coefficient. This statement is wrong since the space-like parts of the corresponding 3-surfaces are discjoint for Euclidian and Minkowskian regions.

3. There is also an argument stating that Dirac determinant for Chern-Simons Dirac action equals to Kähler function, which would be lost if Euclidian regions would not obey holography. The argument obviously generalizes and applies to both Morse and Kähler function which are definitely not proportional to each other.

CP breaking and ground state degeneracy

The Minkowskian contribution of Kähler action is imaginary due to the negativity of the metric determinant and gives a phase factor to vacuum functional reducing to Chern-Simons terms at wormhole throats. Ground state degeneracy due to the possibility of having both signs for Minkowskian contribution to the exponent of vacuum functional provides a general view about the description of CP breaking in TGD framework.

1. In TGD framework path integral is replaced by inner product involving integral over WCV. The vacuum functional and its conjugate are associated with the states in the inner product so that the phases of vacuum functionals cancel if only one sign for the phase is allowed. Minkowskian contribution would have no physical significance. This of course cannot be the case. The ground state is actually degenerate corresponding to the phase factor and its complex conjugate since $\sqrt{g}$ can have two signs in Minkowskian regions. Therefore the inner products between states associated with the two ground states define 2×2 matrix and non-diagonal elements contain interference terms due to the presence of the phase factor. At the limit of full CP_2 type vacuum extremal the two ground states would reduce to each other and the determinant of the matrix would vanish.

2. A small mixing of the two ground states would give rise to CP breaking and the first principle description of CP breaking in systems like $K-\overline{K}$ and of CKM matrix should reduce to this mixing. K^0 mesons would be CP even and odd states in the first approximation and correspond to the sum and difference of the ground states. Small mixing would be present having exponential sensitivity to the actions of CP_2 type extremals representing wormhole throats. This might allow to understand qualitatively why the mixing is about 50 times larger than expected for B^0 mesons.

3. There is a strong temptation to assign the two ground states with two possible arrows of geometric time. At the level of M-matrix the two arrows would correspond to state preparation at either upper or lower boundary of CD. Do long- and shortlived neutral K mesons correspond to almost fifty-fifty orthogonal superpositions for the two arrow of geometric time or almost completely to a fixed arrow of time induced by environment? Is the dominant part of the arrow same for both or is it opposite for long and short-lived neutral measons? Different lifetimes would suggest that the arrow must be the same and apart from small leakage that induced by environment. CP breaking would be induced by the fact that CP is performed only K^0 but not for the environment in the construction of states. One can probably imagine also alternative interpretations.

5.2.6 A general solution ansatz based on almost topological QFT property

The basic vision behind the ansatz is the reduction of quantum TGD to almost topological QFT. This requires that the flow parameters associated with the flow lines of isometry currents and Kähler current extend to global coordinates. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when weak electro-weak duality is applied as boundary conditions. The strongest form of the hydrodynamical interpretation requires that all conserved currents are parallel to Kähler current. In the more general case one would have several hydrodynamic flows. Also the braidings (several of them for the most general ansatz) assigned with the light-like 3-surfaces are naturally defined by the flow lines of conserved currents. The independent behavior of particles at different flow lines can be seen as a realization of the complete integrability of the theory. In free quantum field theories on mass shell Fourier components are in a similar role but the geometric interpretation in terms of flow is of course lacking. This picture should generalize also to the solution of the modified Dirac equation.

Basic field equations

Consider first the equations at general level.

1. The breaking of the Poincare symmetry due to the presence of monopole field occurs and leads to the isometry group $T\times SO(3)\times SU(3)$ corresponding to time translations, rotations, and color group. The Cartan algebra is four-dimensional and field equations reduce to the conservation laws of energy E, angular momentum J, color isospin I_3, and color hypercharge Y.

2. Quite generally, one can write the field equations as conservation laws for I, J, I_3, and Y.

$$D_\alpha\left[D_\beta(J^{\alpha\beta}H_A)-j_K^\alpha H^A+T^{\alpha\beta}j_A^l h_{kl}\partial_\beta h^l\right] \;=\; 0 \ . \tag{5.2.16}$$

   The first term gives a contraction of the symmetric Ricci tensor with antisymmetric Kähler form and vanishes so that one has

$$D_\alpha\left[j_K^\alpha H^A-T^{\alpha\beta}j_A^k h_{kl}\partial_\beta h^l\right] \;=\; 0 \ . \tag{5.2.17}$$

   For energy one has $H_A=1$ and energy current associated with the flow lines is proportional to the Kähler current. Its divergence vanishes identically.

3. One can express the divergence of the term involving energy momentum tensor as as sum of terms involving $j_K^\alpha J_{\alpha\beta}$ and contraction of second fundamental form with energy momentum tensor so that one obtains

$$j_K^\alpha D_\alpha H^A \;=\; j_K^\alpha J_\alpha{}^\beta j_\beta^A+T^{\alpha\beta}H_{\alpha\beta}^k j_k^A \ . \tag{5.2.18}$$

Hydrodynamical solution ansatz

The characteristic feature of the solution ansatz would be the reduction of the dynamics to hydrodynamics analogous to that for a continuous distribution of particles initially at the end of X^3 of the light-like 3-surface moving along flow lines defined by currents j_A satisfying the integrability condition $j_A\wedge dj_A=0$. Field theory would reduce effectively to particle mechanics along flow lines with conserved charges defined by various isometry currents. The strongest condition is that all isometry currents j_A and also Kähler current j_K are proportional to the same current j. The more general option corresponds to multi-hydrodynamics.

Conserved currents are analogous to hydrodynamical currents in the sense that the flow parameter along flow lines extends to a global space-time coordinate. The conserved current is proportional to the gradient $\nabla\Phi$ of the coordinate varying along the flow lines: $J=\Psi\nabla\Phi$ and by a proper choice of Ψ one can allow to have conservation. The initial values of Ψ and Φ can be selected freely along the flow lines beginning from either the end of the space-time surface or from wormhole throats.

If one requires hydrodynamics also for Chern-Simons action (effective 2-dimensionality is required for preferred extremals), the initial values of scalar functions can be chosen freely only at the partonic 2-surfaces. The freedom to chose the initial values of the charges conserved along flow lines at the partonic 2-surfaces means the existence of an infinite number of conserved charges so that the theory would be integrable and even in two different coordinate directions. The basic difference as compared to ordinary conservation laws is that the conserved currents are parallel and their flow parameter extends to a global coordinate.

1. The most general assumption is that the conserved isometry currents

$$J_A^\alpha = j_K^\alpha H^A - T^{\alpha\beta} j_A^k h_{kl} \partial_\beta h^l \quad (5.2.19)$$

and Kähler current are integrable in the sense that $J_A \wedge J_A = 0$ and $j_K \wedge j_K = 0$ hold true. One could imagine the possibility that the currents are not parallel.

2. The integrability condition $dJ_A \wedge J_A = 0$ is satisfied if one one has

$$J_A = \Psi_A d\Phi_A \ . \quad (5.2.20)$$

The conservation of J_A gives

$$d*(\Psi_A d\Phi_A) = 0 \ . \quad (5.2.21)$$

This would mean separate hydrodynamics for each of the currents involved. In principle there is not need to assume any further conditions and one can imagine infinite basis of scalar function pairs (Ψ_A, Φ_A) since criticality implies infinite number deformations implying conserved Noether currents.

3. The conservation condition reduces to d'Alembert equation in the induced metric if one assumes that $\nabla\Psi_A$ is orthogonal with every $d\Phi_A$.

$$d*d\Phi_A = 0 \ , \ d\Psi_A \cdot d\Phi_A = 0 \ . \quad (5.2.22)$$

Taking $x = \Phi_A$ as a coordinate the orthogonality condition states $g^{xj}\partial_j\Psi_A = 0$ and in the general case one cannot solve the condition by simply assuming that Ψ_A depends on the coordinates transversal to Φ_A only. These conditions bring in mind $p \cdot p = 0$ and $p \cdot e$ condition for massless modes of Maxwell field having fixed momentum and polarization. $d\Phi_A$ would correspond to p and $d\Psi_A$ to polarization. The condition that each isometry current corresponds its own pair (Ψ_A, Φ_A) would mean that each isometry current corresponds to independent light-like momentum and polarization. Ordinary free quantum field theory would support this view whereas hydrodynamics and QFT limit of TGD would support single flow.

These are the most general hydrodynamical conditions that one can assume. One can consider also more restricted scenarios.

1. The strongest ansatz is inspired by the hydrodynamical picture in which all conserved isometry charges flow along same flow lines so that one would have

$$J_A = \Psi_A d\Phi \ . \quad (5.2.23)$$

In this case same Φ would satisfy simultaneously the d'Alembert type equations.

$$d*d\Phi = 0 \ , \ d\Psi_A \cdot d\Phi = 0. \quad (5.2.24)$$

This would mean that the massless modes associated with isometry currents move in parallel manner but can have different polarizations. The spinor modes associated with light-light like 3-surfaces carry parallel four-momenta, which suggest that this option is correct. This allows a very general family of solutions and one can have a complete 3-dimensional basis of functions Ψ_A with gradient orthogonal to $d\Phi$.

2. Isometry invariance under $T \times SO(3) \times SU(3)$ allows to consider the possibility that one has

$$J_A = k_A \Psi_A d\Phi_{G(A)} \ , \ d*(d\Phi_G(A)) = 0 \ , \ d\Psi_A \cdot d\Phi_G(A)) = 0 \ . \tag{5.2.25}$$

where $G(A)$ is T for energy current, $SO(3)$ for angular momentum currents and $SU(3)$ for color currents. Energy would thus flow along its own flux lines, angular momentum along its own flow lines, and color quantum numbers along their own flow lines. For instance, color currents would differ from each other only by a numerical constant. The replacement of Ψ_A with $\Psi_{G(A)}$ would be too strong a condition since Killing vector fields are not related by a constant factor.

To sum up, the most general option is that each conserved current J_A defines its own integrable flow lines defined by the scalar function pair (Ψ_A, Φ_A). A complete basis of scalar functions satisfying the d'Alembert type equation guaranteeing current conservation could be imagined with restrictions coming from the effective 2-dimensionality reducing the scalar function basis effectively to the partonic 2-surface. The diametrically opposite option corresponds to the basis obtained by assuming that only single Φ is involved.

The proposed solution ansatz can be compared to the earlier ansatz [K22] stating that Kähler current is topologized in the sense that for $D(CP_2) = 3$ it is proportional to the identically conserved instanton current (so that 4-D Lorentz force vanishes) and vanishes for $D(CP_2) = 4$ (Maxwell phase). This hypothesis requires that instanton current is Beltrami field for $D(CP_2) = 3$. In the recent case the assumption that also instanton current satisfies the Beltrami hypothesis in strong sense (single function Φ) generalizes the topologization hypothesis for $D(CP_2) = 3$. As a matter fact, the topologization hypothesis applies to isometry currents also for $D(CP_2) = 4$ although instanton current is not conserved anymore.

Can one require the extremal property in the case of Chern-Simons action?

Effective 2-dimensionality is achieved if the ends and wormhole throats are extremals of Chern-Simons action. The strongest condition would be that space-time surfaces allow orthogonal slicings by 3-surfaces which are extremals of Chern-Simons action.

Also in this case one can require that the flow parameter associated with the flow lines of the isometry currents extends to a global coordinate. Kähler magnetic field $B = *J$ defines a conserved current so that all conserved currents would flow along the field lines of B and one would have 3-D Beltrami flow. Note that in magnetohydrodynamics the standard assumption is that currents flow along the field lines of the magnetic field.

For wormhole throats light-likeness causes some complications since the induced metric is degenerate and the contravariant metric must be restricted to the complement of the light-like direction. This means that d'Alembert equation reduces to 2-dimensional Laplace equation. For space-like 3-surfaces one obtains the counterpart of Laplace equation with partonic 2-surfaces serving as sources. The interpretation in terms of analogs of Coulomb potentials created by 2-D charge distributions would be natural.

5.2.7 Hydrodynamic picture in fermionic sector

Super-symmetry inspires the conjecture that the hydrodynamical picture applies also to the solutions of the modified Dirac equation. This would mean that the solutions of Dirac equation can be localized to lower-dimensional surface or even flow lines.

Basic objection

The obvious objection against the localization to sub-manifolds is that it is not consistent with uncertainty principle in transversal degrees of freedom. More concretely, the assumption that the mode is localized to a lower-dimensional surface of X^4 implies that the action of the transversal part of Dirac operator in question acts on delta function and gives something singular.

The situation changes if the Dirac operator in question has vanishing transversal part at the lower-dimensional surface. This is not possible for the Dirac operator defined by the induced metric but is quite possible in the case of Kähler-Dirac operator. For instance, in the case of massless extremals Kähler-Dirac gamma matrices are non-vanishing in single direction only and the solution modes could be one-dimensional. For more general preferred extremals such as cosmic strings this is not the case.

In fact, there is a strong physical argument in favor of the localization of spinor modes at 2-D string world sheets so that hydrodynamical picture would result but with flow lines replaced with fermionic string world sheets.

1. Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

2. The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action and requires that the part of the Kähler-Dirac operator transversal to 2-surface vanishes.

3. This localization does not hold for cosmic string solutions which however have 2-D CP_2 projection which should have vanishing weak fields so that 4-D spinor modes with well-defined em charge are possible.

4. A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

4-dimensional modified Dirac equation and hydrodynamical picture

In following consideration is restricted to preferred extremals for which one has decomposition to regions characterized by local light-like vector and polarization direction. In this case one has good hopes that the modes can be restricted to 1-D light-like geodesics.

Consider first the solutions of of the induced spinor field in the interior of space-time surface.

1. The local inner products of the modes of the induced spinor fields define conserved currents

$$\begin{aligned} D_\alpha J^\alpha_{mn} &= 0 \ , \\ J^\alpha_{mn} &= \overline{u}_m \hat{\Gamma}^\alpha u_n \ , \\ \hat{\Gamma}^\alpha &= \frac{\partial L_K}{\partial(\partial_\alpha h^k)}\Gamma_k \ . \end{aligned} \tag{5.2.26}$$

The conjecture is that the flow parameters of also these currents extend to a global coordinate so that one would have in the completely general case the condition

$$\begin{aligned} J^{\alpha}_{mn} &= \Phi_{mn} d\Psi_{mn} \ , \\ d*(d\Phi_{mn}) &= 0 \ , \ \nabla\Psi_{mn}\cdot\Phi_{mn} = 0 \ . \end{aligned} \tag{5.2.27}$$

The condition $\Phi_{mn} = \Phi$ would mean that the massless modes propagate in parallel manner and along the flow lines of Kähler current. The conservation condition along the flow line implies tht the current component J_{mn} is constant along it. Everything would reduce to initial values at the ends of the space-time sheet boundaries of CD and 3-D modified Dirac equation would reduce everything to initial values at partonic 2-surfaces.

2. One might hope that the conservation of these super currents for all modes is equivalent with the modified Dirac equation. The modes u_n appearing in Ψ in quantized theory would be kind of "square roots" of the basis Φ_{mn} and the challenge would be to deduce the modes from the conservation laws.

3. The quantization of the induced spinor field in 4-D sense would be fixed by those at 3-D space-like ends by the fact that the oscillator operators are carried along the flow lines as such so that the anti-commutator of the induced spinor field at the opposite ends of the flow lines at the light-like boundaries of CD is in principle fixed by the anti-commutations at the either end. The anti-commutations at 3-D surfaces cannot be fixed freely since one has 3-D Chern-Simons flow reducing the anti-commutations to those at partonic 2-surfaces.

The following argument suggests that induced spinor fields are in a suitable gauge simply constant along the flow lines of the Kähler current just as massless spinor modes are constant along the geodesic in the direction of momentum.

1. The modified gamma matrices are of form $T^{\alpha}_k\Gamma^k$, $T^{\alpha}_k = \partial L_K/\partial(\partial_\alpha h^k)$. The H-vectors T^{α}_k can be expressed as linear combinations of a subset of Killing vector fields j^k_A spanning the tangent space of H. For CP_2 the natural choice are the 4 Lie-algebra generators in the complement of $U(2)$ sub-algebra. For CD one can used generator time translation and three generators of rotation group SO(3). The completeness of the basis defined by the subset of Killing vector fields gives completeness relation $h^k_l = j^{Ak}j_{Ak}$. This implies $T^{\alpha k} = T^{\alpha k}j^A_k j^k_A = T^{\alpha A}j^k_A$. One can defined gamma matrices Γ_A as $\Gamma_k j^k_A$ to get $T^{\alpha}_k\Gamma^k = T^{\alpha A}\Gamma_A$.

2. This together with the condition that all isometry currents are proportional to the Kähler current (or if this vanishes to same conserved current- say energy current) satisfying Beltrami flow property implies that one can reduce the modified Dirac equation to an ordinary differential equation along flow lines. The quantities T^{tA} are constant along the flow lines and one obtains

$$T^{tA}j_A D_t\Psi = 0 \ . \tag{5.2.28}$$

By choosing the gauge suitably the spinors are just constant along flow lines so that the spinor basis reduces by effective 2-dimensionality to a complete spinor basis at partonic 2-surfaces.

5.3 An attempt to understand preferred extremals of Kähler action

Preferred extremal of Kähler action is one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what "preferred" really means. For instance, the conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [K61]. The problem is however how to assign a complex coordinate with the string world sheet having Minkowskian signature of metric. One can hope that the understanding of preferred extremals could allow to identify two preferred complex coordinates whose existence is also suggested by number theoretical vision giving preferred role for the rational points of partonic 2-surfaces in preferred coordinates. The best one could hope is a general solution of field equations in accordance with the hints that TGD is integrable quantum theory.

5.3.1 What "preferred" could mean?

The first question is what preferred extremal could mean.

1. In positive energy ontology preferred extremal would be a space-time surface assignable to given 3-surface and unique in the ideal situation: since one cannot pose conditions to the normal derivatives of imbedding space coordinates at 3-surface, there is infinity of extremals. Some additional conditions are required and space-time surface would be analogous to Bohr orbit : hence the attribute "preferred". The problem would be to understand what "preferred" could mean. The non-determinism of Kähler action however destroyed this dream in its original form and led to zero energy ontology (ZEO).

2. In ZEO one considers extremals as space-time surfaces connecting two space-like 3-surfaces at the boundaries. One might hope that these 4-surfaces are unique. The non-determinism of Kähler action suggests that this is not the case. At least there is conformal invariance respecting the light-likeness of the 3-D parton orbits at which the signature of the induced metric changes: the conformal transformations would leave the space-like 3-D ends or at least partonic 2-surfaces invariant. This non-determinism would correspond to quantum criticality.

3. Effective 2-dimensionality follows from strong form of general coordinate invariance (GCI) stating that light-like partonic orbits and space-like 3-surfaces at the ends of space-time surface are equivalent physically: partonic 2-surfaces and their 4-D tangent space data would determine everything. One can however worry about how effective 2-dimensionality relates to the the fact that the modes of the induced spinor field are localized at string world sheets and partonic 2-surface. Are the tangent space data equivalent with the data characterizing string world sheets as surfaces carrying vanishing electroweak fields?

   There is however a problem: the hierarchy of Planck constants (dark matter) requires that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom so that either space-like or light-like surfaces do not seem to be quite enough.

   Should one then include also the light-like partonic orbits to the what one calls 3-surface? The resulting connected 3-surfaces would define analogs of Wilson loops. Could the conformal equivalence class of the preferred extremal be unique without any additional conditions? If so, one could get rid of the attribute "preferred". The fractal character of the many-sheeted space-time however suggests that one can have this kind of uniqueness only in given length scale resolution and that "radiative corrections" due to the non-determinism are always present.

These considerations show that the notion of preferred extremal is still far from being precisely defined and it is not even clear whether the attribute "preferred" is needed. If not then the question is what are the extremals of Kähler action.

5.3.2 What is known about extremals?

A lot is is known about properties of extremals and just by trying to integrate all this understanding, one might gain new visions. The problem is that all these arguments are heuristic and rely heavily on physical intuition. The following considerations relate to the space-time regions having Minkowskian signature of the induced metric. The attempt to generalize the construction also to Euclidian regions could be very rewarding. Only a humble attempt to combine various ideas to a more coherent picture is in question.

The core observations and visions are following.

1. Hamilton-Jacobi coordinates for M^4 (discussed in this chapter) define natural preferred coordinates for Minkowskian space-time sheet and might allow to identify string world sheets for X^4 as those for M^4. Hamilton-Jacobi coordinates consist of light-like coordinate m and its dual defining local 2-plane $M^2 \subset M^4$ and complex transversal complex coordinates $(w, \overline{w})$ for a plane E^2_x orthogonal to M^2_x at each point of M^4. Clearly, hyper-complex analyticity and complex analyticity are in question.

2. Space-time sheets allow a slicing by string world sheets (partonic 2-surfaces) labelled by partonic 2-surfaces (string world sheets).

3. The quaternionic planes of octonion space containing preferred hyper-complex plane are labelled by CP_2, which might be called CP_2^{mod} [K52]. The identification $CP_2 = CP_2^{mod}$ motivates the notion of $M^8 - -M^4 \times CP_2$ duality [K13]. It also inspires a concrete solution ansatz assuming the equivalence of two different identifications of the quaternionic tangent space of the space-time sheet and implying that string world sheets can be regarded as strings in the 6-D coset space $G_2/SU(3)$. The group G_2 of octonion automorphisms has already earlier appeared in TGD framework.

4. The duality between partonic 2-surfaces and string world sheets in turn suggests that the $CP_2 = CP_2^{mod}$ conditions reduce to string model for partonic 2-surfaces in $CP_2 = SU(3)/U(2)$. String model in both cases could mean just hypercomplex/complex analyticity for the coordinates of the coset space as functions of hyper-complex/complex coordinate of string world sheet/partonic 2-surface.

The considerations of this section lead to a revival of an old very ambitious and very romantic number theoretic idea.

1. To begin with express octonions in the form $o = q_1 + Iq_2$, where q_i is quaternion and I is an octonionic imaginary unit in the complement of fixed a quaternionic sub-space of octonions. Map preferred coordinates of $H = M^4 \times CP_2$ to octonionic coordinate, form an arbitrary octonion analytic function having expansion with real Taylor or Laurent coefficients to avoid problems due to non-commutativity and non-associativity. Map the outcome to a point of H to get a map $H \rightarrow H$. This procedure is nothing but a generalization of Wick rotation to get an 8-D generalization of analytic map.

2. Identify the preferred extremals of Kähler action as surfaces obtained by requiring the vanishing of the imaginary part of an octonion analytic function. Partonic 2-surfaces and string world sheets would correspond to commutative sub-manifolds of the space-time surface and of imbedding space and would emerge naturally. The ends of braid strands at partonic 2-surface would naturally correspond to the poles of the octonion analytic functions. This would mean a huge generalization of conformal invariance of string models to octonionic conformal invariance and an exact solution of the field equations of TGD and presumably of quantum TGD itself.

5.3.3 Basic ideas about preferred extremals

The slicing of the space-time sheet by partonic 2-surfaces and string world sheets

The basic vision is that space-time sheets are sliced by partonic 2-surfaces and string world sheets. The challenge is to formulate this more precisely at the level of the preferred extremals of Kähler action.

1. Almost topological QFT property means that the Kähler action reduces to Chern-Simons terms assignable to 3-surfaces. This is guaranteed by the vanishing of the Coulomb term in the action density implied automatically if conserved Kähler current is proportional to the instanton current with proportionality coefficient some scalar function.

2. The field equations reduce to the conservation of isometry currents. An attractive ansatz is that the flow lines of these currents define global coordinates. This means that these currents

are Beltrami flows [B19] so that corresponding 1-forms J satisfy the condition $J \wedge dJ = 0$. These conditions are satisfied if

$$J = \Phi \nabla \Psi$$

hold true for conserved currents. From this one obtains that Ψ defines global coordinate varying along flow lines of J.

3. A possible interpretation is in terms of local polarization and momentum directions defined by the scalar functions involved and natural additional conditions are that the gradients of Ψ and Φ are orthogonal:

$$\nabla \Phi \cdot \nabla \Psi = 0 \ ,$$

and that the Ψ satisfies massless d'Alembert equation

$$\nabla^2 \Psi = 0$$

as a consequence of current conservation. If Ψ defines a light-like vector field - in other words

$$\nabla \Psi \cdot \nabla \Psi = 0 \ ,$$

the light-like dual of Φ -call it Φ_c- defines a light-like like coordinate and Φ and Φ_c defines a light-like plane at each point of space-time sheet.

If also Φ satisfies d'Alembert equation

$$\nabla^2 \Phi = 0 \ ,$$

also the current

$$K = \Psi \nabla \Phi$$

is conserved and its flow lines define a global coordinate in the polarization plane orthogonal to time-lik plane defined by local light-like momentum direction.

If Φ allows a continuation to an analytic function of the transversal complex coordinate, one obtains a coordinatization of space-time surface by Ψ and its dual (defining hyper-complex coordinate) and $w, \overline{w}$. Complex analyticity and its hyper-complex variant would allow to provide space-time surface with four coordinates very much analogous with Hamilton-Jacobi coordinates of M^4.

This would mean a decomposition of the tangent space of space-time surface to orthogonal planes defined by light-like momentum and plane orthogonal to it. If the flow lines of J defined Beltrami flow it seems that the distribution of momentum planes is integrable.

4. General arguments suggest that the space-time sheets allow a slicing by string world sheets parametrized by partonic 2-surfaces or vice versa. This would mean a intimate connection with the mathematics of string models. The two complex coordinates assignable to the Yangian of affine algebra would naturally relate to string world sheets and partonic 2-surfaces and the highly non-trivial challenge is to identify them appropriately.

Hamilton-Jacobi coordinates for M^4

The earlier attempts to construct preferred extremals [K5] led to the realization that so called Hamilton-Jacobi coordinates (m, w) for M^4 define its slicing by string world sheets parametrized by partonic 2-surfaces. m would be pair of light-like conjugate coordinates associated with an integrable distribution of planes M^2 and w would define a complex coordinate for the integrable distribution of 2-planes E^2 orthogonal to M^2. There is a great temptation to assume that these coordinates define preferred coordinates for M^4.

1. The slicing is very much analogous to that for space-time sheets and the natural question is how these slicings relate. What is of special interest is that the momentum plane M^2 can be defined by massless momentum. The scaling of this vector does not matter so that these planes are labelled by points z of sphere S^2 telling the direction of the line $M^2 \cap E^3$, when one assigns rest frame and therefore S^2 with the preferred time coordinate defined by the line connecting the tips of CD. This direction vector can be mapped to a twistor consisting of a spinor and its conjugate. The complex scalings of the twistor $(u, \overline{u}) \to \lambda u, \overline{u}/\lambda)$ define the same plane. Projective twistor like entities defining CP_1 having only one complex component instead of three are in question. This complex number defines with certain prerequisites a local coordinate for space-time sheet and together with the complex coordinate of E^2 could serve as a pair of complex coordinates (z, w) for space-time sheet. This brings strongly in mind the two complex coordinates appearing in the expansion of the generators of quantum Yangian of quantum affine algebra [K61].

2. The coordinate Ψ appearing in Beltrami flow defines the light-like vector field defining M^2 distribution. Its hyper-complex conjugate would define Ψ_c and conjugate light-like direction. An attractive possibility is that Φ allows analytic continuation to a holomorphic function of w. In this manner one would have four coordinates for M^4 also for space-time sheet.

3. The general vision is that at each point of space-time surface one can decompose the tangent space to $M^2(x) \subset M^4 = M^2_x \times E^2_x$ representing momentum plane and polarization plane $E^2 \subset E^2_x \times T(CP_2)$. The moduli space of planes $E^2 \subset E^6$ is 8-dimensional and parametrized by $SO(6)/SO(2) \times SO(4)$ for a given E^2_x. How can one achieve this selection and what conditions it must satisfy? Certainly the choice must be integrable but this is not the only condition.

Space-time surfaces as associative/co-associative surfaces

The idea that number theory determines classical dynamics in terms of associativity condition means that space-time surfaces are in some sense quaternionic surfaces of an octonionic space-time. It took several trials before the recent form of this hypothesis was achieved.

1. Octonionic structure is defined in terms of the octonionic representaton of gamma matrices of the imbedding space existing only in dimension $D = 8$ since octonion units are in one-one correspondence with tangent vectors of the tangent space. Octonionic real unit corresponds to a preferred time axes (and rest frame) identified naturally as that connecting the tips of CD. What modified gamma matrices mean depends on variational principle for space-time surface. For volume action one would obtain induced gamma matrices. For Kähler action one obtains something different. In particular, the modified gamma matrices do not define vector basis identical with tangent vector basis of space-time surface.

2. Quaternionicity means that the modified gamma matrices defined as contractions of gamma matrices of H with canonical momentum densities for Kähler action span quaternionic sub-space of the octonionic tangent space [K18]. A further condition is that each quaternionic space defined in this manner contains a preferred hyper-complex subspace of octonions.

3. The sub-space defined by the modified gamma matrices does not co-incide with the tangent space of space-time surface in general so that the interpretation of this condition is far from obvious. The canonical momentum densities need not define four independent vectors at given point. For instance, for massless extremals these densities are proportional to light-like

vector so that the situation is degenerate and the space in question reduces to 2-D hyper-complex sub-space since light-like vector defines plane M^2.

The obvious questions are following.

1. Does the analog of tangent space defined by the octonionic modified gammas contain the local tangent space $M^2 \subset M^4$ for preferred extremals? For massless extremals [K5] this condition would be true. The orthogonal decomposition $T(X^4) = M^2 \oplus_\perp E^2$ can be defined at each point if this is true. For massless extremals also the functions Ψ and Φ can be identified.

2. One should answer also the following delicate question. Can M^2 really depend on point x of space-time? CP_2 as a moduli space of quaternionic planes emerges naturally if M^2 is *same* everywhere. It however seems that one should allow an integrable distribution of M^2_x such that M^2_x is same for all points of a given partonic 2-surface.

   How could one speak about fixed CP_2 (the imbedding space) at the entire space-time sheet even when M^2_x varies?

   (a) Note first that G_2 defines the Lie group of octonionic automorphisms and G_2 action is needed to change the preferred hyper-octonionic sub-space. Various $SU(3)$ subgroups of G_2 are related by G_2 automorphism. Clearly, one must assign to each point of a string world sheet in the slicing parameterizing the partonic 2-surfaces an element of G_2. One would have Minkowskian string model with G_2 as a target space. As a matter fact, this string model is defined in the target space $G_2/SU(3)$ having dimension $D = 6$ since $SU(3)$ automorphisms leave given $SU(3)$ invariant.

   (b) This would allow to identify at each point of the string world sheet standard quaternionic basis - say in terms of complexified basis vectors consisting of two hyper-complex units and octonionic unit q_1 with "color isospin" $I_3 = 1/2$ and "color hypercharge" $Y = -1/3$ and its conjugate $\overline{q}_1$ with opposite color isospin and hypercharge.

   (c) The CP_2 point assigned with the quaternionic basis would correspond to the $SU(3)$ rotation needed to rotate the standard basis to this basis and would actually correspond to the first row of $SU(3)$ rotation matrix. Hyper-complex analyticity is the basic property of the solutions of the field equations representing Minkowskian string world sheets. Also now the same assumption is highly natural. In the case of string models in Minkowski space, the reduction of the induced metric to standard form implies Virasoro conditions and similar conditions are expected also now. There is no need to introduce action principle -just the hyper-complex analycitity is enough-since Kähler action already defines it.

3. The WZW model inspired approach to the situation would be following. The parameterization corresponds to a map $g : X^2 \rightarrow G_2$ for which g defines a flat G_2 connection at string world sheet. WZW type action would give rise to this kind of situation. The transition $G_2 \rightarrow G_2/SU(3)$ would require that one gauges $SU(3)$ degrees of freedom by bringing in $SU(3)$ connection. Similar procedure for $CP_2 = SU(3)/U(2)$ would bring in $SU(3)$ valued chiral field and $U(2)$ gauge field. Instead of introducing these connections one can simply introduce $G_2/SU(3)$ and $SU(3)/U(2)$ valued chiral fields. What this observation suggests that this ansatz indeed predicts gluons and electroweak gauge bosons assignable to string like objects so that the mathematical picture would be consistent with physical intuition.

The two interpretations of CP_2

An old observation very relevant for what I have called $M^8 - H$ duality [K13] is that the moduli space of quaternionic sub-spaces of octonionic space (identifiable as M^8) containing preferred hyper-complex plane is CP_2. Or equivalently, the space of two planes whose addition extends hyper-complex plane to some quaternionic subspace can be parametrized by CP_2. This CP_2 can be called it CP_2^{mod} to avoid confusion. In the recent case this would mean that the space $E^2(x) \subset E^2_x \times T(CP_2)$ is represented by a point of CP_2^{mod}. On the other hand, the imbedding of space-time surface to H defines a point of "real" CP_2. This gives two different CP_2s.

1. The highly suggestive idea is that the identification $CP_2^{mod} = CP_2$ (apart from isometry) is crucial for the construction of preferred extremals. Indeed, the projection of the space-time point to CP_2 would fix the local polarization plane completely. This condition for $E^2(x)$ would be purely local and depend on the values of CP_2 coordinates only. Second condition for $E^2(x)$ would involve the gradients of imbedding space coordinates including those of CP_2 coordinates.

2. The conditions that the planes M^2_x form an integrable distribution at space-like level and that M^2_x is determined by the modified gamma matrices. The integrability of this distribution for M^4 could imply the integrability for X^2. X^4 would differ from M^4 only by a deformation in degrees of freedom transversal to the string world sheets defined by the distribution of M^2s.

   Does this mean that one can begin from vacuum extremal with constant values of CP_2 coordinates and makes them non-constant but allows to depend only on transversal degrees of freedom? This condition is too strong even for simplest massless extremals for which CP_2 coordinates depend on transversal coordinates defined by $\epsilon \cdot m$ and $\epsilon \cdot k$. One could however allow dependence of CP_2 coordinates on light-like M^4 coordinate since the modification of the induced metric is light-like so that light-like coordinate remains light-like coordinate in this modification of the metric.

   Therefore, if one generalizes directly what is known about massless extremals, the most general dependence of CP_2 points on the light-like coordinates assignable to the distribution of M^2_x would be dependence on either of the light-like coordinates of Hamilton-Jacobi coordinates but not both.

5.3.4 What could be the construction recipe for the preferred extremals assuming $CP_2 = CP_2^{mod}$ identification?

The crucial condition is that the planes $E^2(x)$ determined by the point of $CP_2 = CP_2^{mod}$ identification and by the tangent space of $E^2_x \times CP_2$ are same. The challenge is to transform this condition to an explicit form. $CP_2 = CP_2^{mod}$ identification should be general coordinate invariant. This requires that also the representation of E^2 as (e^2, e^3) plane is general coordinate invariant suggesting that the use of preferred CP_2 coordinates - presumably complex Eguchi-Hanson coordinates - could make life easy. Preferred coordinates are also suggested by number theoretical vision. A careful consideration of the situation would be required.

The modified gamma matrices define a quaternionic sub-space analogous to tangent space of X^4 but not in general identical with the tangent space: this would be the case only if the action were 4-volume. I will use the notation $T^m_x(X^4)$ about the modified tangent space and call the vectors of $T^m_x(X^4)$ modified tangent vectors. I hope that this would not cause confusion.

$CP_2 = CP_2^{mod}$ condition

Quaternionic property of the counterpart of $T^m_x(X^4)$ allows an explicit formulation using the tangent vectors of $T^m_x(X^4)$.

1. The unit vector pair (e_2, e_3) should correspond to a unique tangent vector of H defined by the coordinate differentials dh^k in some natural coordinates used. Complex Eguchi-Hanson coordinates [L1] are a natural candidate for CP_2 and require complexified octonionic imaginary units. If octonionic units correspond to the tangent vector basis of H uniquely, this is possible.

2. The pair (e_2, e_3) as also its complexification $(q_1 = e_2 + ie_3, \overline{q}_1 = e_2 - ie_3)$ is expressible as a linear combination of octonionic units $I_2, ...I_7$ should be mapped to a point of $CP_2^{mod} = CP_2$ in canonical manner. This mapping is what should be expressed explicitly. One should express given (e_2, e_3) in terms of $SU(3)$ rotation applied to a standard vector. After that one should define the corresponding CP_2 point by the bundle projection $SU(3) \to CP_2$.

3. The tangent vector pair

$$(\partial_w h^k, \partial_{\overline{w}} h^k)$$

defines second representation of the tangent space of $E^2(x)$. This pair should be equivalent with the pair $(q_1, \overline{q}_1)$. Here one must be however very cautious with the choice of coordinates. If the choice of w is unique apart from constant the gradients should be unique. One can use also real coordinates (x, y) instead of $(w = x + iy, \overline{w} = x - iy)$ and the pair (e_2, e_3). One can project the tangent vector pair to the standard vielbein basis which must correspond to the octonionic basis

$$(\partial_x h^k, \partial_y h^k) \rightarrow (\partial_x h^k e_k^A e_A, \partial_y h^k e_k^A)e_A) \leftrightarrow (e_2, e_3) \ ,$$

where the e_A denote the octonion units in 1-1 correspondence with vielbein vectors. This expression can be compared to the expression of (e_2, e_3) derived from the knowledge of CP_2 projection.

Formulation of quaternionicity condition in terms of octonionic structure constants

One can consider also a formulation of the quaternionic tangent planes in terms of (e_2, e_3) expressed in terms of octonionic units deducible from the condition that unit vectors obey quaternionic algebra. The expressions for octonionic *resp.* quaternionic structure constants can be found at [A17] *resp.* [A20].

1. The ansatz is

$$\begin{aligned} \{E_k\} &= \{1, I_1, E_2, E_3\} \ , \\ E_2 &= E_{2k}e^k \equiv \sum_{k=2}^{7} E_{2k}e^k \ , \ E_3 = E_{3k}e^k \equiv \sum_{k=2}^{7} E_{3k}e^k \ , \\ |E_2| &= 1 \ , \ |E_3| = 1 \ . \end{aligned} \tag{5.3.1}$$

2. The multiplication table for octonionic units expressible in terms of octonionic triangle [A17] gives

$$f^{1kl}E_{2k} = E_{3l} \ , \ f^{1kl}E_{3k} = -E_{2l} \ , \ f^{klr}E_{2k}E_{3l} = \delta_1^r \ . \tag{5.3.2}$$

Here the indices are raised by unit metric so that there is no difference between lower and upper indices. Summation convention is assumed. Also the contribution of the real unit is present in the structure constants of third equation but this contribution must vanish.

3. The conditions are linear and quadratic in the coefficients E_{2k} and E_{3k} and are expected to allow an explicit solution. The first two conditions define homogenous equations which must allow solution. The coefficient matrix acting on (E_2, E_3) is of the form

$$\begin{pmatrix} f_1 & 1 \\ -1 & f_1 \end{pmatrix} \ ,$$

where 1 denotes unit matrix. The vanishing of the determinant of this matrix should be due to the highly symmetric properties of the structure constants. In fact the equations can be written as eigen conditions

$$f_1 \circ (E_2 \pm iE_3) = \mp i(E_2 \pm iE_3) \ ,$$

and one can say that the structure constants are eigenstates of the hermitian operator defined by I_1 analogous to color hyper charge. Both values of color hyper charged are obtained.

Explicit expression for the $CP_2 = CP_2^{mod}$ conditions

The symmetry under $SU(3)$ allows to construct the solutions of the above equations directly.

1. One can introduce complexified basis of octonion units transforming like $(1, 1, 3, \overline{3})$ under SU(3). Note the analogy of triplet with color triplet of quarks. One can write complexified basis as $(1, e_1, (q_1, q_2, q_3), (\overline{q}_1\overline{q}_2, \overline{q}_3))$. The expressions for complexified basis elements are

$$(q_1, q_2, q_3) = \frac{1}{\sqrt{2}}(e_2 + ie_3, e_4 + ie_5, e_6 + ie_7) \ .$$

   These options can be seen to be possible by studying octonionic triangle in which all lines containing 3 units defined associative triple: any pair of octonion units at this kind of line can be used to form pair of complexified unit and its conjugate. In the tangent space of $M^4 \times CP_2$ the basis vectors q_1, and q_2 are mixtures of E^2_x and CP_2 tangent vectors. q_3 involves only CP_2 tangent vectors and there is a temptation to interpret it as the analog of the quark having no color isospin.

2. The quaternionic basis is real and must transform like $(1, 1, q_1, \overline{q}_1)$, where q_1 is any quark in the triplet and $\overline{q}_1$ its conjugate in antitriplet. Having fixed some basis one can perform SU(3) rotations to get a new basis. The action of the rotation is by 3×3 special unitary matrix. The over all phases of its rows do not matter since they induce only a rotation in (e_2, e_3) plane not affecting the plane itself. The action of $SU(3)$ on q_1 is simply the action of its first row on (q_1, q_2, q_3) triplet:

$$\begin{aligned} q_1 \quad &\rightarrow \quad (Uq)_1 = U_{11}q_1 + U_{12}q_2 + U_{13}q_3 \equiv z_1q_1 + z_2q_2 + z_3q_3 \\ &= \quad z_1(e_2 + ie_3) + z_2(e_4 + ie_5) + z_3(e_6 + ie_7) \ . \end{aligned} \tag{5.3.3}$$

   The triplets (z_1, z_2, z_3) defining a complex unit vector and point of S^5. Since overall phase does not matter a point of CP_2 is in question. The new real octonion units are given by the formulas

$$\begin{aligned} e_2 \quad &\rightarrow \quad Re(z_1)e_2 + Re(z_2)e_4 + Re(z_3)e_6 - Im(z_1)e_3 - Im(z_2)e_5 - Im(z_3)e_7 \ , \\ e_3 \quad &\rightarrow \quad Im(z_1)e_2 + Im(z_2)e_4 + Im(z_3)e_6 + Re(z_1)e_3 + Re(z_2)e_5 + Re(z_3)e_7 \ . \end{aligned} \tag{5.3.4}$$

   For instance the CP_2 coordinates corresponding to the coordinate patch (z_1, z_2, z_3) with $z_3 \neq 0$ are obtained as $(\xi_1, \xi_2) = (z_1/z_3, z_2/z_3)$.

Using these expressions the equations expressing the conjecture $CP_2 = CP_2^{mod}$ equivalence can be expressed explicitly as first order differential equations. The conditions state the equivalence

$$(e_2, e_3) \quad \leftrightarrow \quad (\partial_x h^k e^A_k e_A, \partial_y h^k e^A_k e_A) \ , \tag{5.3.5}$$

where e_A denote octonion units. The comparison of two pairs of vectors requires normalization of the tangent vectors on the right hand side to unit vectors so that one takes unit vector in the direction of the tangent vector. After this the vectors can be equated. This allows to expresses the contractions of the partial derivatives with vielbein vectors with the 6 components of e_2 and e_3. Each condition gives 6+6 first order partial differential equations which are non-linear by the presence of the overal normalization factor for the right hand side. The equations are invariant under scalings of (x, y). The very special form of these equations suggests that some symmetry is involved.

It must be emphasized that these equations make sense only in preferred coordinates: ordinary Minkowski coordinates and Hamilton-Jacobi coordinates for M^4 and Eguchi-Hanson complex coordinates in which $SU(2) \times U(1)$ is represented linearly for CP_2. These coordinates are preferred because they carry deep physical meaning.

Does TGD boil down to two string models?

It is good to look what have we obtained. Besides Hamilton-Jacobi conditions, and $CP_2 = CP_2^{mod}$ conditions one has what one might call string model with 6-dimensional $G_2/SU(3)$ as targent space. The orbit of string in $G_2/SU(3)$ allows to deduce the G_2 rotation identifiable as a point of $G_2/SU(3)$ defining what one means with standard quaternionic plane at given point of string world sheet. The hypothesis is that hyper-complex analyticity solves these equations.

The conjectured electric-magnetic duality implies duality between string world sheet and partonic 2-surfaces central for the proposed mathematical applications of TGD [K23, K24, K50, K63]. This duality suggests that the solutions to the $CP_2 = CP_2^{mod}$ conditions could reduce to holomorphy with respect to the coordinate w for partonic 2-surface plus the analogs of Virasoro conditions. The dependence on light-like coordinate would appear as a parametric dependence.

If this were the case, TGD would reduce at least partially to what might be regarded as dual string models in $G_2/SU(3)$ and $SU(3)/U(2)$ and also to string model in M^4 and X^4! In the previous arguments one ends up to string models in moduli spaces of string world sheets and partonic 2-surfaces. TGD seems to yield an inflation of string models! This not actually surprising since the slicing of space-time sheets by string world sheets and partonic 2-surfaces implies automatically various kinds of maps having interpretation in terms of string orbits.

5.4 Handful of problems with a common resolution

Theory building could be compared to pattern recognition or to a solving a crossword puzzle. It is essential to make trials, even if one is aware that they are probably wrong. When stares long enough to the letters which do not quite fit, one suddenly realizes what one particular crossword must actually be and it is soon clear what those other crosswords are. In the following I describe an example in which this analogy is rather concrete.

I will first summarize the problems of ordinary Dirac action based on induced gamma matrices and propose modified Dirac action (or Kähler Dirac action as solution). After that I will describe the general structures of Kähler action and Kähler Dirac action. The non-trivial terms are associated to 3-D boundary like surfaces - that is ends of space-time surface inside CD and light-like 3-surfaces at which the signature of the induced metric changes. These terms are induced as Lagrange multiplier terms guaranteeing weak form of E-M duality and quantum classical correspondence (QCC) between classical and quantal Cartan charges. The condition guaranteeing that Chern-Simons Dirac propagator reduces to ordinary massless Dirac propagator must be however assumed as a property of the modes of Kähler Dirac equation rather than forced by a separate term in the Kähler-Dirac action as thought originally.

5.4.1 Why modified Dirac action?

Problems associated with the ordinary Dirac action

In the following the problems of the ordinary Dirac action are discussed and the notion of modified Dirac action is introduced.

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or some other general principle selecting preferred extremals as Bohr orbits [K10, K52] .

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the WCW geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of WCW geometry so that there is internal inconsistency.

Super-symmetry forces modified Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$\begin{aligned} D_\alpha T^\alpha_k &= 0 \ , \\ T^\alpha_k &= \frac{\partial}{\partial h^k_{\ \alpha}} L_K \ . \end{aligned} \tag{5.4.1}$$

If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l \Psi \ , \\ D_\alpha J^{\alpha k} &= 0 \ . \end{aligned} \tag{5.4.2}$$

having a vanishing divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^\alpha = \overline{\nu_R} T^\alpha_l \Gamma^l \Psi \tag{5.4.3}$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_\alpha J^{\alpha k} = \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l D_\alpha \Psi \ . \tag{5.4.4}$$

The requirement that this current vanishes is guaranteed if one assumes that modified Dirac equation

$$\begin{aligned} \hat{\Gamma}^\alpha D_\alpha \Psi &= 0 \ , \\ \hat{\Gamma}^\alpha &= T^\alpha_l \Gamma^l \ . \end{aligned} \tag{5.4.5}$$

This equation must be derivable from a modified Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi} \hat{\Gamma}^\alpha D_\alpha \Psi \ . \tag{5.4.6}$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with effective induced gamma matrices and the requirement

$$D_\mu \hat{\Gamma}^\mu = 0 \tag{5.4.7}$$

guaranteeing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

How can one avoid minimal surface property?

These observations suggest how to avoid the emergence of the minimal surface property as a consequence of field equations. It is not induced metric which appears in field equations. Rather, the effective metric appearing in the field equations is defined by the anti-commutators of $\hat{\gamma}_\mu$

$$\hat{g}_{\mu\nu} = \{\hat{\Gamma}_\mu, \hat{\Gamma}_\nu\} = 2T_\mu^k T_{\nu k} \ . \tag{5.4.8}$$

Here the index raising and lowering is however performed by using the induced metric so that the problems resulting from the non-invertibility of the effective metric are avoided. It is this dynamically generated effective metric which must appear in the number theoretic formulation of the theory.

Field equations state that space-time surface is minimal surface with respect to the effective metric. Note that a priori the choice of the bosonic action principle is arbitrary. The requirement that effective metric defined by energy momentum tensor has only non-diagonal components except in the case of non-light-like coordinates, is satisfied for the known solutions of field equations.

Does the modified Dirac action define the fundamental action principle?

There is quite fundamental and elegant interpretation of the modified Dirac action as a fundamental action principle discussed also in [K52] . In this approach vacuum functional can be defined as the Grassmannian functional integral associated with the exponent of the modified Dirac action. This definition is invariant with respect to the scalings of the Dirac action so that theory contains no free parameters.

An alternative definition is as a Dirac determinant which might be calculated in TGD framework without applying the poorly defined functional integral. There are good reasons to expect that the Dirac determinant equals to the exponent of Kähler function for a preferred Bohr orbit like extremal of the Kähler action with the value of Kähler coupling strength coming out as a prediction. Hence the dynamics of the modified Dirac action at light-like partonic 3-surfaces X_l^3, even when restricted to almost-topological dynamics induced by Chern-Simons action, would dictate the dynamics at the interior of the space-time sheet.

The knowledge of the symplectic currents and super-currents, together with the anti-commutation relations stating that the fermionic super-currents S_A and S_B associated with Hamiltonians H_A and H_B anti-commute to a bosonic current $H_{[A,B]}$, allows in principle to deduce the anti-commutation relations satisfied by the induced spinor field. In fact, these conditions replace the usual anti-commutation relations used to quantize free spinor field. Since the normal ordering of the Dirac action would give Kähler action,

Kähler coupling strength would be determined completely by the anti-commutation relations of the super-symplectic algebra. Kähler coupling strength would be dynamical and the selection of preferred extremals of Kähler action would be more or less equivalent with quantum criticality because criticality corresponds to conformal invariance and the hyper-quaternionic version of the super-conformal invariance results only for the extrema of Kähler action. p-Adic (or possibly more general) coupling constant evolution and quantum criticality would come out as a prediction whereas in the case that Kähler action is introduced as primary object, the value of Kähler coupling strength must be fixed by quantum criticality hypothesis.

The mixing of the M^4 chiralities of the imbedding space spinors serves as a signal for particle massivation and breaking of super-conformal symmetry. The induced gamma matrices for the space-time surfaces which are deformations of M^4 indeed contain a small contribution from CP_2 gamma matrices: this implies a mixing of M^4 chiralities even for the modified Dirac action so that there is no need to introduce this mixing by hand.

5.4.2 Overall view about Kähler action and Kähler Dirac action

In the following the most recent view about Kähler action and the modified Dirac action (Kähler-Dirac action) is explained in more detail.

1. The minimal formulation involves in the bosonic case only 4-D Kähler action with Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term is chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD).

   There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

2. By supersymmetry requirement the modified Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with imbedding space gamma matrices to obtain modified gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. At partonic orbits one only assumes that spinors are generalized eigen modes of Chern-Simons Dirac operator with generalized eigenvalues $p^k\gamma_k$ identified as virtual four-momenta so that C-S-D term gives fermionic propagators. At the ends of space-time surface one obtains boundary conditions stating in absence of measurement interaction terms that fundamental fermions are massless on-mass-shell states.

Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

1. QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

2. The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.

3. The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface.

Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

1. What one wants?

It is could to make first clear what one really wants.

1. What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi = 0$$

   at the space-like ends of space-time surface. The general idea is that the space-time geometry near the fermion line would *define* the on mass shell massless four-momentum propagating along the line and quantum classical correspondence would be realized.

   The basic condition is thus that $\sqrt{g_4}\Gamma^n$ is constant at the space-like boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant.

   Partonic orbits are not boundaries in the usual sense of the word and this condition is not elegant at them since g_4 vanishes at them. The assignement of Chern-Simons Dirac action to partonic orbits required to be continuous at them solves the problems. One can require that the induced spinors are generalized eigenstates of C-S-D operator with eigenvalues with correspond to virtual four-moment. This guarantees that one obtains massless Dirac propagator from C-S-D action. Note that the localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

2. If p^k associated with the partonic orbit is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram since the fermion current in the normal direction vanishes. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

3. One can wonder what the spectrum of p_k could be. If the identification of p^k as virtual momentum is correct, continuous mass spectrum suggests itself. Boundary conditions at the ends of CD might imply quantized mass spectrum and the study of C-S-D equation indeed suggets this if periodic boundary conditions are assumed. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation.

2. Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If C-S-D and C-S terms are assigned also with the space-like ends of space-time surface, Kähler action and Kähler function vanish identically if the weak form of em duality holds true. Hence C-S-D and C-S terms can be assigned only with partonic orbits. If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \tag{5.4.9}$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = -\gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. The physical interpretation would be as incoming massless fermions.

Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for the space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n\Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

Some details about Chern-Simons Dirac equation

To avoid confusion some general comments are in order. Only the Chern-Simons Dirac operator will be considered. Modified gamma matrices contain also the contribution from the Lagrange multiplier term stating weak form of electric-magnetic duality. At space-like 3-surface one has also the contribution coming from the Lagrange multiplier terms identifying classical and quantal charges in Cartan algebra.

When C-S-D action at partonic orbits is included, one obtains what I have called generalized eigenvalue equation introduced in ad hoc manner in order to define Dirac determinant. Now Dirac determinant at least formally reduces to the same expression as in massless gauge theories. Dirac determinant could be also defined directly as the product of generalized eigenvalues $p^k\gamma_k$ defining virtual momenta propagating in fermion lines. Also the identification as hyperquaternions makes sense and the outcome is by symmetries real number or perhaps complex number.

One can of course wonder whether the Dirac determinant has anything to do with the exponent of Kähler action! Measurement interaction term states that the action of D_{C-S} modified by the contribution from em-duality constraint is identical with that of the Dirac operator of M^4 regarded as algebraic multiplication with $p^k\gamma_k$, where p^k is the four-momentum associated with the propagator line defined by the light-like orbit of parton. This simplifies the formalism enormously and gives a direct connection with similar condition posed independently in twistorial approach [K44].

One can require that the modes annihilated by Kähler-Dirac operator are eigenstates of C-S-D oeprator with generalized eigenvalues $p^k\gamma_k$ giving rise to fermion propagator Consider now the properties of eigenmodes of D_{C-S}.

1. For $p^k = 0$ there is vacuum avoidance in the sense that Ψ must vanish in the regions where the modified gamma matrices vanish.

2. If only CP_2 Kähler form appears in the Kähler action, the modified Dirac action defined by the Chern-Simons term is non-vanishing only when the dimension of the CP_2 projection of the 3-surface is $D(CP_2) \geq 2$ and the induced Kähler field is non-vanishing. This conforms with the properties of Kähler action.

   $D(CP_2) \leq 2$ is inconsistent with the weak form of electric-magnetic duality. The extrema of Chern-Simons action have $D(CP_2) \leq 2$ and vanishing Chern-Simons density so that they would naturally represent on mass shell particles appearing as incoming and outgoing particles. This conforms with the interpretation of the basic extremals as free particles (massless extremals and cosmic strings with 2-D CP_2 projection). One could say that CP breaking is not present for free particles but unavoidably accompanies the propagator lines.

The explicit expression of D_{C-S} without constraint terms from the weak form of electric-magnetic duality is given by

$$\begin{aligned}
D &= \hat{\Gamma}^{\mu}D_{\mu} + \frac{1}{2}D_{\mu}\hat{\Gamma}^{\mu} \ , \\
\hat{\Gamma}^{\mu} &= \frac{\partial L_{C-S}}{\partial_{\mu}h^k}\Gamma_k = \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_{\alpha}h^l A_{\beta} + J_{\alpha\beta}A_k\right]\Gamma^k D_{\mu} \ , \\
D_{\mu}\hat{\Gamma}^{\mu} &= B^{\alpha}_K(J_{k\alpha} + \partial_{\alpha}A_k) \ , \\
B^{\alpha}_K &= \epsilon^{\alpha\beta\gamma}J_{\beta\gamma} \ , \quad J_{k\alpha} = J_{kl}\partial_{\alpha}s^l \ , \quad \hat{\epsilon}^{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma}\sqrt{g_3} \ .
\end{aligned} \tag{5.4.10}$$

Note $\hat{\epsilon}^{\alpha\beta\gamma}$ does not depend on the induced metric.

The extremals of Chern-Simons action satisfy

$$B^{\alpha}_K(J_{kl} + \partial_l A_k)\partial_{\alpha}h^l = 0 \ , \quad B^{\alpha}_K = \epsilon^{\alpha\beta\gamma}J_{\beta\gamma} \ . \tag{5.4.11}$$

For non-vanishing Kähler magnetic field B^{α} these equations hold true when CP_2 projection is 2-dimensional and S^2 projection is 1-dimensional or vice versa. This implies a vanishing of Chern-Simons action for both options. Consider for the simplicity the case when S^2 projection is 1-dimensional.

1. Suppose that one can assign a global coordinate to the flow lines of the Kähler magnetic field. In this case one might hope that ordinary intuitions about motion in constant magnetic field might be helpful. The repetition of the discussion of [K22] leads to the condition $B \wedge dB = 0$ implying that a Beltrami flow for which current flows along the field lines and Lorentz forces vanishes is in question. This need not be the generic case.

2. With this assumption the Chern-Simons Dirac operator reduces to a one-dimensional Dirac operator

$$D = \hat{\epsilon}^{r\alpha\beta}\left[2J_{kl}\partial_{\alpha}h^l A_{\beta} + J_{\alpha\beta}A_k\right]\Gamma^k D_r \ . \tag{5.4.12}$$

3. Consider first the general solutions of the modified Dirac equation when M^4 Dirac operator $p^k\gamma_k$ annhilates the spinor so that on mass shell massless fermion is in question. The spinor is covariantly constant with respect to the coordinate r:

$$D_r\Psi = 0 \ . \tag{5.4.13}$$

The solution to this condition can be written immediately in terms of a non-integrable phase factor $Pexp(i\int A_r dr)$, where integration is along curve with constant transversal coordinates. If $\hat{\Gamma^v}$ is light-like vector field also $\hat{\Gamma^v}\Psi_0$ defines a solution of D_{C-S}. This solution corresponds to a zero mode for D_{C-S} and does not contribute to the Dirac determinant. Note that the dependence of these solutions on transversal coordinates of X^3_l is arbitrary.

4. For internal lines $p^k\gamma_k$ does not annihilate the spinor although four-momentum can be still on mass shell if the spinor has unphysical helicity. In this case the equation is modified. Again the modes can be localized to 1-D curves.

5. The formal solution associated with a general eigenvalue can be constructed by integrating the eigenvalue equation separately along all coordinate curves. This makes sense if r indeed assigned to light-like curves indeed defines a global coordinate.

The localization is of utmost importance since and is consistent with the localization of the modes (other than right-handed neutrino) of Kähler Dirac equation at string world sheets discussed in chapter [K69]. String ends would thus define braid strands. The absence of correlation between the behaviors with respect longitudinal coordinate and transversal coordinates looked very strange at first glance. System looked like a collection of totally uncorrelated point like particles reflecting the flow of the current along flux lines.

5.4.3 A connection with quantum measurement theory

It is encouraging that isometry charges and also other charges could make themselves visible in the geometry of space-time surface as they should by quantum classical correspondence. This suggests an interpretation in terms of quantum measurement theory.

1. The interpretation resolves the problem caused by the fact that the choice of the commuting isometry charges is not unique. Cartan algebra corresponds naturally to the measured observables. For instance, one could choose the Cartan algebra of Poincare group to consist of energy and momentum, angular momentum and boost (velocity) in particular direction as generators of the Cartan algebra of Poincare group. In fact, the choices of a preferred plane $M^2 \subset M^4$ and geodesic sphere $S^2 \subset CP_2$ allowing to fix the measurement sub-algebra to a high degree are implied by the replacement of the imbedding space with a book like structure forced by the hierarchy of Planck constants. Therefore the hierarchy of Planck constants seems to be required by quantum measurement theory. One cannot overemphasize the importance of this connection.

2. One can add similar couplings of the net values of the measured observables to the currents whose existence and conservation is guaranteed by quantum criticality. It is essential that one maps the observables to Cartan algebra coupled to critical current characterizing the observable in question. The coupling should have interpretation as a replacement of the induced Kähler gauge potential with its gauge transform. Quantum classical correspondence encourages the identification of the classical charges associated with Kähler action with quantal Cartan charges. This would support the interpretation in terms of a measurement interaction feeding information to classical space-time physics about the eigenvalues of the observables of the measured system. The resulting field equations remain second order partial differential equations since the second order partial derivatives appear only linearly in the added terms.

3. What about the space-time correlates of electro-weak charges? The earlier proposal explains this correlation in terms of the properties of quantum states: the coupling of electro-weak charges to Chern-Simons term could give the correlation in stationary phase approximation. It would be however very strange if the coupling of electro-weak charges with the geometry of the space-time sheet would not have the same universal description based on quantum measurement theory as isometry charges have.

   (a) The hint as how this description could be achieved comes from a long standing unanswered question motivated by the fact that electro-weak gauge group identifiable as the holonomy group of CP_2 can be identified as $U(2)$ subgroup of color group. Could the electro-weak charges be identified as classical color charges? This might make sense since the color charges have also identification as fermionic charges implied by quantum criticality. Or could electro-weak charges be only represented as classical color charges by mapping them to classical color currents in the measurement interaction term in the modified Dirac action? At least this question might make sense.

(b) It does not make sense to couple both electro-weak and color charges to the same fermion current. There are also other fundamental fermion currents which are conserved. All the following currents are conserved.

$$\begin{aligned} J^\alpha &= \overline{\Psi} O \hat{\Gamma}^\alpha \Psi \\ O &\in \{1 \ , \ J \equiv J_{kl}\Sigma^{kl} \ , \ \Sigma_{AB} \ , \ \Sigma_{AB}J\} \ . \end{aligned} \quad (5.4.14)$$

Here J_{kl} is the covariantly constant CP_2 Kähler form and Σ_{AB} is the (also covariantly) constant sigma matrix of M^4 (flatness is absolutely essential).

(c) Electromagnetic charge can be expressed as a linear combination of currents corresponding to $O = 1$ and $O = J$ and vectorial isospin current corresponds to J. It is natural to couple of electromagnetic charge to the the projection of Killing vector field of color hyper charge and coupling it to the current defined by $O_{em} = a + bJ$. This allows to interpret the puzzling finding that electromagnetic charge can be identified as anomalous color hyper-charge for induced spinor fields made already during the first years of TGD. There exist no conserved axial isospin currents in accordance with CVC and PCAC hypothesis which belong to the basic stuff of the hadron physics of old days.

(d) Color charges would couple naturally to lepton and quark number current and the $U(1)$ part of electro-weak charges to the $n = 1$ multiple of quark current and $n = 3$ multiple of the lepton current (note that leptons *resp.* quarks correspond to $t = 0$ *resp.* $t = \pm 1$ color partial waves). If electro-weak *resp.* couplings to H-chirality are proportional to 1 *resp.* Γ_9, the fermionic currents assigned to color and electro-weak charges can be regarded as independent. This explains why the possibility of both vectorial and axial couplings in 8-D sense does not imply the doubling of gauge bosons.

(e) There is also an infinite variety of conserved currents obtained as the quantum critical deformations of the basic fermion currents identified above. This would allow in principle to couple an arbitrary number of observables to the geometry of the space-time sheet by mapping them to Cartan algebras of Poincare and color group for a particular conserved quantum critical current. Quantum criticality would therefore make possible classical space-time correlates of observables necessary for quantum measurement theory.

(f) The coupling constants associated with the deformations would appear in the couplings. Quantum criticality ($K \rightarrow K + f + \overline{f}$ condition) should predict the spectrum of these couplings. In the case of momentum the coupling would be proportional to $\sqrt{G/\hbar_0} = kR/\hbar_0$ and $k \sim 2^{11}$ should follow from quantum criticality. p-Adic coupling constant evolution should follow from the dependence on the scale of CD coming as powers of 2.

4. Quantum criticality implies fluctuations in long length and time scales and it is not surprising that quantum criticality is needed to produce a correlation between quantal degrees of freedom and macroscopic degrees of freedom. Note that quantum classical correspondence can be regarded as an abstract form of entanglement induced by the entanglement between quantum charges Q_A and fermion number type charges assignable to zero modes.

5. Space-time sheets can have an arbitrary number of wormhole contacts so that the interpretation in terms of measurement theory coupling short and long length scales suggests that the measurement interaction terms are localizable at the wormhole throats. This would favor Chern-Simons term or possibly instanton term if reducible to Chern-Simons terms. The breaking of CP and T might relate to the fact that state function reductions performed in quantum measurements indeed induce dissipation and breaking of time reversal invariance.

The formulation of quantum TGD in terms of the modified Dirac action requires the addition of CP and T breaking Chern-Simons term and corresponding Chern-Simons Dirac term to partonic orbits such that it cancels the similar contribution coming from Kähler action. Chern-Simons Dirac term fixed by superconformal symmetry and gives rise to massless fermionic propagators at the boundaries of string world sheets. This seems to be a natural first principle explanation for the CP breaking as it manifests at the level of CKM matrix and perhaps also in breaking of matter antimatter asymmetry.

6. The experimental arrangement quite concretely splits the quantum state to a quantum superposition of space-time sheets such that each eigenstate of the measured observables in the superposition corresponds to different space-time sheet already before the realization of state function reduction. This relates interestingly to the question whether state function reduction really occurs or whether only a branching of wave function defined by WCW spinor field takes place as in multiverse interpretation in which different branches correspond to different observers. TGD inspired theory consciousness requires that state function reduction takes place. Maybe multiversalist might be able to find from this picture support for his own beliefs.

7. One can argue that "free will" appears not only at the level of quantum jumps but also as the possibility to select the observables appearing in the modified Dirac action dictating in turn the Kähler function defining the Kähler metric of WCW representing the "laws of physics". This need not to be the case. The choice of CD fixes M^2 and the geodesic sphere S^2: this does not fix completely the choice of the quantization axis but by isometry invariance rotations and color rotations do not affect Kähler function for given CD and for a given type of Cartan algebra. In M^4 degrees of freedom the possibility to select the observables in two manners corresponding to linear and cylindrical Minkowski coordinates could imply that the resulting Kähler functions are different. The corresponding Kähler metrics do not differ if the real parts of the Kähler functions associated with the two choices differ by a term $f(Z)+\overline{f(Z)}$, where Z denotes complex coordinates of WCW, the Kähler metric remains the same. The function f can depend also on zero modes. If this is the case then one can allow in given CD superpositions of WCW spinor fields for which the measurement interactions are different. This condition is expected to pose non-trivial constraints on the measurement action and quantize coupling parameters appearing in it.

5.4.4 How to calculate Dirac determinant?

If the modes of the modified Dirac equation (or Kähler-Dirac equation) are localized to 2-D string world sheets as the well-definedness of em charge eigenvalue for the modes of induced spinor field strongly suggests, the definition of Dirac determinant could be rather simple as following argument shows.

The modes of Kähler-Dirac operator (modified Dirac operator) are localized at string world sheets and are holomorphic spinors. K-D operator annihilates these modes so that Dirac determinant must be assigned with the Chern-Simons Dirac term associated with the light-like partonic orbits with vanishing metric determinant g_4. Spinor modes at partonic orbits are assumed to be generalized eigen modes of C-S-D operator with eigenvalues $ip^k\gamma_k$, with p^k interpreted as virtual momentum of the fermion propagating along lined defined by the string world sheet boundary. Therefore C-S-D term acts effectively as massless Dirac action in perturbation theory.

The spectrum of p^k is determined by the boundary conditions for C-S-D operator at the ends of CD and periodic boundary conditions is one natural possibility. As in massless QFTs Dirac determinant could be identified as a square root of the product of mass squared eigenvalues p^2. If the spectrum is unbounded, a regularization must be used. Finite measurement resolution means UV and IR cutoffs and would make Dirac determinant finite. Finite IR resolution would be due to the fact that only space-time surfaces within CD and thus having finite size scale are considered. UV resolution would be due to the lower limit on the size of sub-CDs.

One can however define Dirac determinant directly as the product of the generalized eigenvalues $p^k\gamma_k$ or as product of hyper-quaternions defined by p^k. By symmetry arguments the outcome must be real.

The full Dirac determinant would be product of Dirac determinants associated with various string world sheets. Needless to say that this is an enormous calculational advantage. If Dirac determinant identified in this manner reduces to exponent of Kähler action for preferred extremal this definition of Dirac determinant should give exponent of Kähler function reducing by weak form of electric-magnetic duality to exponent of Chern-Simons terms associated with the space-like ends of the space-time surface. Euclidian and Minkowskian regions would give contributions different by a phase factor $\sqrt{-1}$. The reduction of determinant to exponent of Chern-Simons terms would guarantee its finiteness.

Before trying to calculate Dirac determinant it is good to try to guess what the reduction to Chern Simons action could give as a result. This kind of guesses are of course highly speculative but nothing prevents from trying.

1. Chern Simons action to which Kähler action is expected to reduce for the preferred extremals should be expressible in terms of invariants associated with string world sheets. The only invariant, which comes in mind is Kähler magnetic flux, which is zero mode and by general vision quantized as integer, rational or even algebraic number for surfaces for which parameters in their defining representations correspond to finite algebraic extensions of rationals. For instance, fluxes could belong to rationals with p-adic norm not larger than p^n and allowing realization as flux.

2. Finite measurement resolution suggests that the Kähler magnetic fluxes defined by $J\sqrt{g_2}$, which is constant in preferred coordinates by the internal consistency of quantization of induced spinors, are quantized as integer multiplies or rationals or even algebraic numbers corresponding to the hierarchy of algebraic extensions assignable to the parameters characterizing space-time surfaces (say the coefficients of polynomials defining the space-time sheet). Therefore space-time surface itself would realize the finite measurement resolution in their dynamics as the finiteness for the number of string world sheets and natural cutoffs for the generalized eigenvalue spectrum of C-S-D operator, and the calculation of Dirac determinant using finite number of string world sheets would not be an approximation. Finite measurement resolution would be also a property of state.

3. The value of k could depend on string world sheet so that one would obtain $K(X^3) \propto \sum_i k_i$, where the sum is sum over fluxes associated with string world sheets. Kähler function would be equal to Chern-Simons term in turn equal to the sum of Kähler fluxes over all alowed string world sheets: this looks indeed geometrically attractive.

4. The reduction of Chern-Simons action to a sum of terms proportional to Kähler fluxes takes place if Chern-Simons action is apart from a vanishing integral of divergence proportional to the sum $\sum_i \oint_{C_i} A_\mu dx^\nu$ around the string world sheet. This form would have interpretation in terms of a coupling of charged particles at braid strands to Kähler potential so that particle picture would emerge.

5. Since magnetic flux is conserved, one can argue that Chern-Simons term reduces to an integral of constant magnetic flux J over transverse degrees of freedom multiplied by integral over the boundary of string world sheet given by $\oint_C A_\mu (dx^\mu/ds)ds$ so that one indeed obtains the desired result. The result is non-vanishing only for monopole flux. Elementary particles indeed correspond to throats carrying monopole flux.

6. The arguent about finite measurement resolution can be of course criticized. An alternative argument relies on idea that the sum over logarithms of eigenvalues reduces to integral using as measure the transversal induced Kähler form J_T and the magnetic flux J over string world sheet. This conforms with the existence of slicing by string world sheets labelled by points of partonic 2-surface.

   The formula would be

$$K \propto \oint J(x,y) J_T dx^1 \wedge dx^2 \quad . \tag{5.4.15}$$

This would be non-local analog for the local quadratic dependence of Kähler action on Kähler form. This decomposition might have interpretation in terms of intersections of 2-D surfaces in relative homology.

5.5 Quantum criticality and modified Dirac action

The precise mathematical formulation of quantum criticality has remained one of the basic challenges of quantum TGD. The belief has been that the existence of conserved current for modified Dirac equation are possible if Kähler action is critical for the 3-surface in question in the sense that the deformation in question corresponds to vanishing of second variation of Kähler action. The vanishing of the second variation states that the deformation of the modified gamma matrix is divergence free just like the modified gamma matrix itself and is therefore very natural.

2-D conformal invariance accompanies 2-D criticality and allows to satisfy these conditions for spinor modes localized at 2-D surfaces - string world sheets and possibly also partonic 2-surfaces. This localization is in the generic case forced by the conditions that em charge is well-defined for the spinor modes: this requires that classical W fields vanish and also the vanishing of classical Z^0 field is natural -at least above weak scale. Only 2 modified gamma matrices can be non-vanishing and this is possible only for Kähler-Dirac action.

5.5.1 What quantum criticality could mean?

Quantum criticality is one of the basic guiding principles of Quantum TGD. What it means mathematically is however far from clear and one can imagine several meanings for it.

1. What is obvious is that quantum criticality implies quantization of Kähler coupling strength as a mathematical analog of critical temperature so that the theory becomes mathematically unique if only single critical temperature is possible. Physically this means the presence of long range fluctuations characteristic for criticality and perhaps assignable to the effective hierarchy of Planck constants having explanation in terms of effective covering spaces of the imbedding space. This hierarchy follows from the vacuum degeneracy of Kähler action, which in turn implies 4-D spin-glass degeneracy. It is easy to interpret the degeneracy in terms of criticality.

2. At more technical level one would expect criticality to correspond to deformations of a given preferred extremal defining a vanishing second variation of Kähler Khler function or Kähler action.

   (a) For Kähler function this criticality is analogous to thermodynamical criticality. The Hessian matrix defined by the second derivatives of free energy or potential function becomes degenerate at criticality as function of control variables which now would be naturally zero modes not contribution to Kähler metric of WCW but appearing as parameters in it. The bevavior variables correspond to quantum fluctuating degrees of freedom and according to catastrophe theory a big change can in quantum fluctuating degrees of freedom at criticality for zero modes. This would be control of quantum state by varying classical variables. Cusp catastrophe is standard example of this. One can imagined also a situation in which the roles of zero modes and behavior variables change and big jump in the values of zero modes is induced by small variation in behavior variables. This would mean quantum control of classical variables.

   (b) Zero modes controlling quantum fluctuating variables in Kähler function would correspond to vanishing of also second derivatives of potential function at extremum in certain directions so that the matrix defined by second derivatives does not have maximum rank. Entire hierarchy of criticalities is expected and a good finite-dimensional model is provided by the catastrophe theory of Thom [?]. Cusp catastrophe [A2] is the simplest catastrophe one can think of, and here the folds of cusp where discontinuous jump occurs correspond to criticality with respect to one control variable and the tip to criticality with respect to both control variables.

3. Quantum criticality makes sense also for Kähler action.

   (a) Now one considers space-time surface connecting which 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K17] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

   (b) Also now one expects a hierarchy of criticalities and and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

   (c) The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. I have discussed what criticality could mean for modified Dirac action [K18] .

   (a) I have conjectured that it leads to the existence of additional conserved currents defined by the variations which do not affect the value of Kähler action. These arguments are far from being mathematically rigorous and the recent view about the solutions of the modified Dirac equation predicting that the spinor modes are restricted to 2-D string world sheets requires a modification of these arguments.

   (b) The basic challenge is to understand the mechanism making this kind of currents conserved: the same challenge is met already in the case of isometries since imbedding space coordinates appear as parameters in modified Dirac action. Modified Dirac equation is satisfied if the first variation of the canonical momentum densities contracted with the imbedding space gamma matrices annihilates the spinor mode. Situation is analogous to massless Dirac equation: it does not imply the vanishing of four-momentum, only the vanishing of mass. One obtains conserved fermion current associated with deformations only if the deformation of the modified Gamma matrix is divergenceless just like the modified gamma matrix itself. This conditions requires the vanishing of the second variation of Kähler action.

   (c) It is far from obvious that these conditions can be satisfied. The localization of the spinor modes to string world sheets or partonic 2-surfaces guaranteeing in the generic case that em charge is well-defined for spinor modes implies holomorphy allowing to formulate current conservation for the deformations of the space-time surface for second quantized induced spinor field. The crux is that the deformation respects the holomorphy properties of the modified gamma matrices at string world sheet and thus does not mix Γ^z with $\Gamma^{\overline{z}}$. The deformation of Γ^z has only z-component and also annihilates the holomorphic spinor.

       This mechanism is possible only for Kähler-Dirac action since the Kähler-Dirac gamma matrices in directions orthogonal to the 2-surface must vanish and this is not possible for other actions. This also means that energy momentum tensor has rank 2 as a matrix. Cosmic string solutions are an exception since in this case CP_2 projection of space-time surface is 2-D and conditions guaranteing vanishing of classical W fields can be satisfied without the restriction to 2-surface.

The vacuum degeneracy of Kähler action strongly suggests that the number of critical deformations is always infinite and that these deformations define an infinite inclusion hierarchy of super-conformal algebras. This inclusion hierarchy would correspond to a fractal hierarchy of breakings of super-conformal symmetry generalizing the symmetry breaking hierarchies of gauge theories. These super-conformal inclusion hierarchies would realize the inclusion hierarchies for hyper-finite factors of type II_1.

5.5.2 Quantum criticality and fermionic representation of conserved charges associated with second variations of Kähler action

It is rather obvious that TGD allows a huge generalizations of conformal symmetries. The development of the understanding of conservation laws has been however slow. Modified Dirac action provides excellent candidates for quantum counterparts of Noether charges. The problem is that the imbedding space coordinates are in the role of classical external fields and induces spinor fields are second quantized so that it is not at all clear whether one obtains conserved charges.

What does the conservation of the fermionic Noether current require?

The obvious anser to the question of the title is that the conservation of the fermionic current requires the vanishing of the first variation of Kähler-Dirac action with respect to imbedding space coordinates. This is certainly true but need not mean vanishing of the second variation of Kähler action as thought first. Hence fermionic conserved currents might be obtained for much more general variations than critical ones.

1. The modified Dirac action assigns to a deformation of the space-time surface a conserved charge expressible as bilinears of fermionic oscillator operators only if the first variation of the modified Dirac action under this deformation vanishes.

   The vanishing of the first variation for the modified Dirac action is equivalent with the vanishing of the second variation for the Kähler action. This can be seen by the explicit calculation of the second variation of the modified Dirac action and by performing partial integration for the terms containing derivatives of Ψ and $\overline{\Psi}$ to give a total divergence representing the difference of the charge at upper and lower boundaries of the causal diamond plus a four-dimensional integral of the divergence term defined as the integral of the quantity

$$\begin{aligned} \Delta S_D &= \overline{\Psi}\Gamma^k D_\alpha J^\alpha_k \Psi \ , \\ J^\alpha_k &= \frac{\partial^2 L_K}{\partial h^k_\alpha \partial h^l_\beta}\delta h^k_\beta + \frac{\partial^2 L_K}{\partial h^k_\alpha \partial h^l}\delta h^l \ . \end{aligned} \tag{5.5.1}$$

   Here h^k_β denote partial derivative of the imbedding space coordinates with respect to space-time coordinates. ΔS_D vanishes if this term vanishes:

$$D_\alpha J^\alpha_k = 0 \ .$$

   The condition states the vanishing of the second variation of Kähler action. This can of course occur only for preferred deformations of X^4. One could consider the possibility that these deformations vanish at light-like 3-surfaces or at the boundaries of CD. Note that covariant divergence is in question so that J^α_k does not define conserved classical charge in the general case.

2. This condition is however un-necessarily strong. It is enough that that the deformation of Dirac operator anihilates the spinor mode, which can also change in the deformation. It must be possible to compensate the change of the covariant derivative in the deformation by a gauge transformation which requires that deformations act as gauge transformations on

induce gauge potentials. This gives additional constraint and strongly suggests Kac-Moody type algebra for the deformations. Conformal transformations would satisfy this constraint and are suggested by quantum criticality.

3. It is essential that the modified Dirac equation holds true so that the modified Dirac action vanishes: this is needed to cancel the contribution to the second variation coming from the determinant of the induced metric. The condition that the modified Dirac equation is satisfied for the deformed space-time surface requires that also Ψ suffers a transformation determined by the deformation. This gives

$$\delta\Psi = -\frac{1}{D} \times \Gamma^k J_k^\alpha \Psi \ . \tag{5.5.2}$$

Here $1/D$ is the inverse of the modified Dirac operator defining the counterpart of the fermionic propagator.

4. The fermionic conserved currents associated with the deformations are obtained from the standard conserved fermion current

$$J^\alpha = \overline{\Psi}\Gamma^\alpha \Psi \ . \tag{5.5.3}$$

Note that this current is conserved only if the space-time surface is extremal of Kähler action: this is also needed to guarantee Hermiticity and same form for the modified Dirac equation for Ψ and its conjugate as well as absence of mass term essential for super-conformal invariance. Note also that ordinary divergence rather only covariant divergence of the current vanishes.

The conserved currents are expressible as sums of three terms. The first term is obtained by replacing modified gamma matrices with their increments in the deformation keeping Ψ and its conjugate constant. Second term is obtained by replacing Ψ with its increment $\delta\Psi$. The third term is obtained by performing same operation for $\delta\overline{\Psi}$.

$$J^\alpha = \overline{\Psi}\Gamma^k J_k^\alpha \Psi + \overline{\Psi}\hat{\Gamma}^\alpha \delta\Psi + \delta\overline{\Psi}\hat{\Gamma}^\alpha \Psi \ . \tag{5.5.4}$$

These currents provide a representation for the algebra defined by the conserved charges analogous to a fermionic representation of Kac-Moody algebra.

5. Also conserved super charges corresponding to super-conformal invariance are obtained. The first class of super currents are obtained by replacing Ψ or $\overline{\Psi}$ right handed neutrino spinor or its conjugate in the expression for the conserved fermion current and performing the above procedure giving two terms since nothing happens to the covariantly constant right handed-neutrino spinor. Second class of conserved currents is defined by the solutions of the modified Dirac equation interpreted as c-number fields replacing Ψ or $\overline{\Psi}$ and the same procedure gives three terms appearing in the super current.

6. The existence of vanishing of second variations is analogous to criticality in systems defined by a potential function for which the rank of the matrix defined by second derivatives of the potential function vanishes at criticality. Quantum criticality becomes the prerequisite for the existence of quantum theory since fermionic anti-commutation relations in principle can be fixed from the condition that the algebra in question is equivalent with the algebra formed by the vector fields defining the deformations of the space-time surface defining second variations. Quantum criticality in this sense would also select preferred extremals of Kähler action as analogs of Bohr orbits and the the spectrum of preferred extremals would be more or less equivalent with the expected existence of infinite-dimensional symmetry algebras.

It is far from obvious that the criticality conditions or even the weaker conditions guaranteeing the existence of (say) isometry charges can be satisfied. It seems that the restriction of spinor modes to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces - implied by the condition that em charge is well-define for them, is the manner to achieve this. The reason is that conformal invariance allows complexification of the modified gamma matrices and allows to construct spinor modes as holomorphic modes and their conjugates. Holomorphy reduces K-D equation to algebraic condition that Γ^z annihilates the spinor mode. If this is true also the deformation of Γ^z then the existince of conserved current follows. It is essential that only two modified gamma matrices are non-vanishing and this is possible only for Kähler-Dirac action.

About the general structure of the algebra of conserved charges

Some general comments about the structure of the algebra of conserved charges are in order.

1. Any Cartan algebra of the isometry group $P \times SU(3)$ (there are two types of them for P corresponding to linear and cylindrical Minkowski coordinates) defines critical deformations (one could require that the isometries respect the geometry of CD). The corresponding second order charges for Kähler action are conserved but vanish since the corresponding conjugate coordinates are cyclic for the Kähler metric and Kähler form so that the conserved current is proportional to the gradient of a Killing vector field which is constant in these coordinates.

2. Contrary to the original conclusion, the corresponding fermionic charges expressible as fermionic bilinears are first order in deformation and do not vanish! Four-momentum and color quantum numbers are defined for Kähler action as classical conserved quantities and for Kähler-Dirac action as quantal charges.

Critical manifold is infinite-dimensional for Kähler action

Some examples might help to understand what is involved.

1. The action defined by four-volume gives a first glimpse about what one can expect. In this case modified gamma matrices reduce to the induced gamma matrices. Second variations satisfy d'Alembert type equation in the induced metric so that the analogs of massless fields are in question. Mass term is present only if some dimensions are compact. The vanishing of excitations at light-like boundaries is a natural boundary condition and might well imply that the solution spectrum could be empty. Hence it is quite possible that four-volume action leads to a trivial theory.

2. For the vacuum extremals of Kähler action the situation is different. There exists an infinite number of second variations and the classical non-determinism suggests that deformations vanishing at the light-like boundaries exist. For the canonical imbedding of M^4 the equation for second variations is trivially satisfied. If the CP_2 projection of the vacuum extremal is one-dimensional, the second variation contains a non-vanishing term and an equation analogous to massless d'Alembert equation for the increments of CP_2 coordinates is obtained. Also for the vacuum extremals of Kähler action with 2-D CP_2 projection all terms involving induced Kähler form vanish and the field equations reduce to d'Alembert type equations for CP_2 coordinates. A possible interpretation is as the classical analog of Higgs field. For the deformations of non-vacuum extremals this would suggest the presence of terms analogous to mass terms: these kind of terms indeed appear and are proportional to δs_k. M^4 degrees of freedom decouple completely and one obtains QFT type situation.

3. The physical expectation is that at least for the vacuum extremals the critical manifold is infinite-dimensional. The notion of finite measurement resolution suggests infinite hierarchies of inclusions of hyper-finite factors of type II_1 possibly having interpretation in terms of inclusions of the super conformal algebras defined by the critical deformations.

4. The properties of Kähler action give support for this expectation. The critical manifold is infinite-dimensional in the case of vacuum extremals. Canonical imbedding of M^4 would correspond to maximal criticality analogous to that encountered at the tip of the cusp catastrophe. The natural guess would be that as one deforms the vacuum extremal the previously

critical degrees of freedom are transformed to non-critical ones. The dimension of the critical manifold could remain infinite for all preferred extremals of the Kähler action. For instance, for cosmic string like objects any complex manifold of CP_2 defines cosmic string like objects so that there is a huge degeneracy is expected also now. For CP_2 type vacuum extremals M^4 projection is arbitrary light-like curve so that also now infinite degeneracy is expected for the deformations.

This leads to the conjecture that the critical deformations correspond to sub-algebras of super-conformal algebras with conformal weights coming as integer multiples of fixed integer m. One would have infinite hierarchy of breakings of conformal symmetry labelled by m. The super-conformal algebras would be effectively m-dimensional. Since all commutators with the critical sub-algebra would create zero energy states. In ordinary conformal field theory one have maximal criticality corresponding to $m = 1$.

Critical super-algebra and zero modes

The relationship of the critical super-algebra to WCW geometry is interesting.

1. The vanishing of the second variation plus the identification of Kähler function as a Kähler action for preferred extremals means that the critical variations are orthogonal to all deformations of the space-time surface with respect to the WCW metric.

   The original expectation was that critical deformations correspond to zero modes but this interpretation need not be correct since critical deformations can leave 3-surface invariant but affect corresponding preferred extremal: this would conform with the non-deterministic character of the dynamics which is indeed the basic signature of criticality. Rather, critical deformations are limiting cases of ordinary deformations acting in quantum fluctuating degrees of freedom.

   This conforms with the fact that WCW metric vanishes identically for canonically imbedded M^4 and that Kähler action has fourth order terms as first non-vanishing terms in perturbative expansion (for modified Dirac the expansion is quadratic in deformation).

   Therefore the super-conformal algebra associated with the critical deformations has genuine physical content.

2. Since the action of X^4 local Hamiltonians of $\delta M^4_\times CP_2$ corresponds to the action in quantum fluctuating degrees of freedom, critical deformations cannot correspond to this kind of Hamiltonians.

3. The notion of finite measurement resolution suggests that the degrees of freedom which are below measurement resolution correspond to vanishing gauge charges. The sub-algebras of critical super-conformal algebra for which charges annihilate physical states could correspond to this kind of gauge algebras.

4. The conserved super charges associated with the vanishing second variations cannot give WCW metric as their anti-commutator. This would also lead to a conflict with the effective 2-dimensionality stating that WCW line-element is expressible as sum of contribution coming from partonic 2-surfaces as also with fermionic anti-commutation relations.

Connection with quantum criticality

The notion of quantum criticality of TGD Universe was originally inspired by the question how to make TGD unique if Kähler function for WCW is defined by the Kähler action for a preferred extremal assignable to a given 3-surface. Vacuum functional defined by the exponent of Kähler function is analogous to thermodynamical weight and the obviou idea with Kähler coupling strength taking the role of temperature. The obvious idea was that the value of Kähler coupling strength is analogous to critical temperature so that TGD would be more or less uniquely defined.

To understand the delicacies it is convenient to consider various variations of Kähler action first.

1. The variation can leave 3-surface invariant but modify space-time surface such that Kähler action remains invariant. In this case infinitesimal deformation reduces to a diffeomorphism at space-like 3-surface and perhaps also at light-like 3-surfaces. In this case the correspondence between X^3 and $X^4(X^3)$ would not be unique and one would have non-deterministic dynamics characteristic for critical systems. This criticality would correspond to criticality of Kähler action at X^3. Note that the original working hypothesis was that $X^4(X^3)$ is unique. The failure of the strict classical determinism implying spin glass type vacuum degeneracy indeed suggets that this is the case.

2. The variation could act on zero modes which do not affect Kähler metric which corresponds to (1,1) part of Hessian in complex coordinates for WCW. Only the zero modes characterizing 3-surface appearing as parameters in the metric WCW would be affected and the result would be a generalization of conformal transformation. Kähler function would change but only due to the change in zero modes. These transformations do not seem to correspond to critical transformations since Kähler function changes.

3. The variation could act on 3-surface both in zero modes and dynamical degrees of freedom represented by complex coordinates. It would of course affect also the space-time surface. Criticality for Kähler function would mean that Kähler metric has zero modes at X^3 meaning that (1,1) part of Hessian is degenerate. This could mean that in the vicinity of X^3 the Kähler form has non-definite signature: physically this is unacceptable since inner product in Hilbert space would not be positive definite.

Critical transformations might relate closely to the coset space decomposition of WCW to a union of coset spaces G/H labelled by zero modes.

1. The critical deformations leave 3-surface X^3 invariant as do also the transformations of H associated with X^3. If H affects $X^4(X^3)$ and corresponds to critical transformations then critical transformation would extend WCW to a bundle for which 3-surfaces would be base points and preferred extremals $X^4(X^3)$ would define the fiber. Gauge invariance with respect to H would generalize the assumption that $X^4(X^3)$ is unique.

2. Critical deformations could correspond to H or sub-group of H (which dependes on X^3). For other 3-surfaces than X^3 the action of H is non-trivial as the case of $CP_2 = SU(3)/U(2)$ makes easy to understand.

3. A possible identification of Lie-algebra of H is as a sub-algebra of Virasoro algebra associated with the symplectic transformations of $\delta M^4 \times CP_2$ and acting as diffeomorphisms for the light-like radial coordinate of δM^4_+. The sub-algebras of Virasoro algebra have conformal weights coming as integer multiplies of a given conformal weight m and form inclusion hierarchies suggesting a direct connection with finite measurement resolution realized in terms of inclusions of hyperfinite factors of type II_1. For $m > 1$ one would have breaking of maximal conformal symmetry. The action of these Virasoro algebra on symplectic algebra would make the corresponding sub-algebras gauge degrees of freedom so that the number of symplectic generators generating non-gauge transformations would be finite. This result is not surprising since also for 2-D critical systems criticality corresponds to conformal invariance acting as local scalings.

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical. Basic example of criticality is bifurcation diagram for cusp catastrophe. Quantum criticality realized as the vanishing of the second variation gives hopes about a more or less unique identification of preferred extremals and considered alternative identifications such as absolute minimization of Kähler action which is just the opposite of criticality.

One must be very cautious here: there are two criticalities: one for the extremals of Kähler action with respect to deformations of four-surface and second for the Kähler function itself with respect to deformations of 3-surface: these criticalities are not equivalent since in the latter case variation respects preferred extremal property unlike in the first case.

1. The criticality for preferred extremals would make 4-D criticality a property of all physical systems.

2. The criticality for Kähler function would be 3-D and might hold only for very special systems. In fact, the criticality means that some eigenvalues for the Hessian of Kähler function vanish and for nearby 3-surfaces some eigenvalues are negative. On the other hand the Kähler metric defined by (1,1) part of Hessian in complex coordinates must be positive definite. Thus criticality might imply problems.

   This allows and suggests non-criticality of Kähler function coming from Kähler action for Euclidian space-time regions: this is mathematically the simplest situation since in this case there are no zero modes causing troubles in Gaussian approximation to functional integral. The Morse function coming from Kähler action in Minkowskian as imaginary contribution analogous to that appearing in path integral could be critical and allow non-definite signature in principle. In fact this is expected by the defining properties of Morse function.

3. The almost 2-dimensionality implied by strong form of holography suggests that the interior degrees of freedom of 3-surface can be regarded almost gauge degrees of freedom and that this relates directly to generalised conformal symmetries associated with symplectic isometries of WCW. These degrees of freedom are not critical in the sense inspired by G/H decomposition. The only plausible interaction seems to be that these degrees of freedom correspond to deformations in zero modes.

Both the super-symmetry of D_K and conservation Dirac Noether currents for modified Dirac action have thus a connection with quantum criticality.

1. Finite-dimensional critical systems defined by a potential function $V(x^1, x^2, ..)$ are characterized by the matrix defined by the second derivatives of the potential function and the rank of system classifies the levels in the hierarchy of criticalities. Maximal criticality corresponds to the complete vanishing of this matrix. Thom's catastrophe theory classifies these hierarchies, when the numbers of behavior and control variables are small (smaller than 5). In the recent case the situation is infinite-dimensional and the criticality conditions give additional field equations as existence of vanishing second variations of Kähler action.

2. The vacuum degeneracy of Kähler action allows to expect that this kind infinite hierarchy of criticalities is realized. For a general vacuum extremal with at most 2-D CP_2 projection the matrix defined by the second variation vanishes because $J_{\alpha\beta} = 0$ vanishes and also the matrix $(J^{\alpha}_{\ k} + J_k^{\ \alpha})(J^{\beta}_{\ l} + J_l^{\ \beta})$ vanishes by the antisymmetry $J^{\alpha}_{\ k} = -J_k^{\ \alpha}$.

   The formulation of quantal version of Equivalence Principle (EP) in string picture demonstrates that the conservation of of fermionic Noether currents defining gravitational four-momentum and other Poincare quantum numbers requires that the deformation of the Kähler-Dirac equation obtained by replacing Kähler-Dirac gamma matrices with their deformations is also satisfied. Holomorphy can guarantee this. The original wrong conclusion was that this condition is equivalent with much stronger condition stating the vanishing of the second variation of Kähler action, which it is not. There is analogy for this: massless Dirac equation does not imply the vanishing of four-momentum.

3. Conserved bosonic and fermionic Noether charges would characterize quantum criticality. In particular, the isometries of the imbedding space define conserved currents represented in terms of the fermionic oscillator operators if the second variations defined by the infinitesimal isometries vanish for the modified Dirac action. For vacuum extremals the dimension of the critical manifold is infinite: maybe there is hierarchy of quantum criticalities for which this dimension decreases step by step but remains always infinite. This hierarchy could closely relate to the hierarchy of inclusions of hyper-finite factors of type II_1. Also the conserved charges associated with super-symplectic and Super Kac-Moody algebras would require infinite-dimensional critical manifold defined by the spectrum of second variations.

4. Phase transitions are characterized by the symmetries of the phases involved with the transitions, and it is natural to expect that dynamical symmetries characterize the hierarchy of quantum criticalities. The notion of finite quantum measurement resolution based on the hierarchy of Jones inclusions indeed suggests the existence of a hierarchy of dynamical gauge symmetries characterized by gauge groups in ADE hierarchy [K17] with degrees of freedom below the measurement resolution identified as gauge degrees of freedom.

5. Does this criticality have anything to do with the criticality against the phase transitions changing the value of Planck constant? If the geodesic sphere S^2_I for which induced Kähler form vanishes corresponds to the back of the CP_2 book (as one expects), this could be the case. The homologically non-trivial geodesic sphere S^12_{II} is as far as possible from vacuum extremals. If it corresponds to the back of CP_2 book, cosmic strings would be quantum critical with respect to phase transition changing Planck constant. They cannot however correspond to preferred extremals.

5.5.3 Preferred extremal property as classical correlate for quantum criticality, holography, and quantum classical correspondence

The Noether currents assignable to the modified Dirac equation are conserved only if the first variation of the modified Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X^3_l)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago! The question whether these extremals correspond to absolute minima remains however open.

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X^3_l)$ vanishing at the intersections of $X^4(X^3_l)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X^3_l with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of WCW represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \rightarrow X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X^3_l)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X^3_l involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

5. There is a possible connection with the notion of self-organized criticality [B4] introduced to explain the behavior of systems like sand piles. Self-organization in these systems tends to lead "to the edge". The challenge is to understand how system ends up to a critical state, which by definition is unstable. Mechanisms for this have been discovered and based on phase transitions occurring in a wide range of parameters so that critical point extends to a critical manifold. In TGD Universe quantum criticality suggests a universal mechanism of this kind. The criticality for the preferred extremals of Kähler action would mean that classically all systems are critical in well-defined sense and the question is only about the degree of criticality. Evolution could be seen as a process leading gradually to increasingly critical systems. One must however distinguish between the criticality associated with the preferred extremals of Kähler action and the criticality caused by the spin glass like energy landscape like structure for the space of the maxima of Kähler function.

Chapter 6

The recent vision about preferred extremals and solutions of the modified Dirac equation

6.1 Introduction

During years several approaches to what preferred extremals of Kähler action and solutions of the modified Dirac equation could be have been proposed and the challenge is to see whether at least some of these approaches are consistent with each other.

The notion of preferred extremal emerged when I still lived in positive energy ontology. In zero energy ontology (ZEO) situation changes since 3-surfaces are now unions of space-like 3-surfaces at the opposite boundaries of causal diamond (CD). If Kähler action were deterministic, the attribute "preferred" would become obsolete. One of the most important outcomes of non-determinism is quantum criticality realized as a conformal invariance acting as gauge symmetries. The transformations in question are Kac-Moody type symmetries respecting the light-likeness of partonic orbits identified as surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. The orbits can be grouped to conformal equivalence classes and their number n would define in a natural manner the value of the effective Planck constant $h_{eff} = n \times h$.

One might hope that in finite measurement resolution the attribute "preferred" would not be needed. Bohr orbitology in ZEO would mean that one has Bohr orbits connecting 3-surfaces at boundaries of CD and this would give strong correlations between these 3-surfaces. Not all of them could be connected. Despite these uncertainties, I will talk in the following about preferred extremals. This means no loss since what is known recently is known for extremals.

It is good to list various approaches first.

6.1.1 Construction of preferred extremals

There has been considerable progress in the understanding of both preferred extremals and Kähler-Dirac equation.

1. For preferred extremals the generalization of conformal invariance to 4-D situation is very attractive idea and leads to concrete conditions formally similar to those encountered in string model [K5]. In particular, Einstein's equations with cosmological constant would solve consistency conditions and field equations would reduce to a purely algebraic statements analogous to those appearing in equations for minimal surfaces if one assumes that space-time surface has Hermitian structure or its Minkowskian variant Hamilton-Jacobi structure (Appendix). The older approach based on basic heuristics for massless equations, on effective 3-dimensionality, weak form of electric magnetic duality, and Beltrami flows is also promising. An alternative approach is inspired by number theoretical considerations and identifies space-time surfaces as associative or co-associative sub-manifolds of octonionic imbedding space [K52].

The basic step of progress was the realization that the known extremals of Kähler action - certainly limiting cases of more general extremals - can be deformed to more general extremals having interpretation as preferred extremals.

(a) The generalization boils down to the condition that field equations reduce to the condition that the traces $Tr(TH^k)$ for the product of energy momentum tensor and second fundamental form vanish. In string models energy momentum tensor corresponds to metric and one obtains minimal surface equations. The equations reduce to purely algebraic conditions stating that T and H^k have no common components. Complex structure of string world sheet makes this possible.

Stringy conditions for metric stating $g_{zz} = g_{\overline{z}\overline{z}} = 0$ generalize. The condition that field equations reduce to $Tr(TH^k) = 0$ requires that the terms involving Kähler gauge current in field equations vanish. This is achieved if Einstein's equations hold true (one can consider also more general manners to satisfy the conditions). The conditions guaranteeing the vanishing of the trace in turn boil down to the existence of Hermitian structure in the case of Euclidian signature and to the existence of its analog - Hamilton-Jacobi structure - for Minkowskian signature (Appendix). These conditions state that certain components of the induced metric vanish in complex coordinates or Hamilton-Jacobi coordinates.

In string model the replacement of the imbedding space coordinate variables with quantized ones allows to interpret the conditions on metric as Virasoro conditions. In the recent case a generalization of classical Virasoro conditions to four-dimensional ones would be in question. An interesting question is whether quantization of these conditions could make sense also in TGD framework at least as a useful trick to deduce information about quantum states in WCW degrees of freedom.

The interpretation of the extended algebra as Yangian [A27] [B18] suggested previously [K61] to act as a generalization of conformal algebra in TGD Universe is attractive. There is also the conjecture that preferred extremals could be interpreted as quaternionic of co-quaternionic 4-surface of the octonionic imbedding space with octonionic representation of the gamma matrices defining the notion of tangent space quanternionicity.

6.1.2 Understanding Kähler-Dirac equation

There are several approaches for solving the modified Dirac (or Kähler-Dirac) equation.

(a) The most promising approach is discussed in this chapter. It assumes that the solutions are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of both strong form of holography and of number theoretic vision, and also follows from the notion of finite measurement resolution having discretization at partonic 2-surfaces as a geometric correlate. Furthermore, the conditions stating that electric charge is well-defined for preferred extremals forces the localization of the modes to 2-D surfaces in the generic case. This also resolves the interpretational problems related to possibility of strong parity breaking effects since induce W fields and possibly also Z^0 field above weak scale, vahish at these surfaces.

(b) One expects that stringy approach based on 4-D generalization of conformal invariance or its 2-D variant at 2-D preferred surfaces should also allow to understand the modified Dirac equation. Conformal invariance indeed allows to write the solutions explicitly using formulas similar to encountered in string models. In accordance with the earlier conjecture, all modes of the modified Dirac operator generate badly broken super-symmetries.

(c) Covariantly constant right-handed neutrino certainly defines solutions de-localized inside entire space-time sheet. This need not be the case if right-handed neutrino is not covarianty constant since the non-vanishing CP_2 part for the induced gamma matrices mixes it with left-handed neutrino. For massless extremals (at least) the CP_2 part however vanishes and right-handed neutrino allows also massless holomorphic modes de-localized at entire space-time surface and the de-localization inside Euclidian region defining the line of generalized Feynman diagram is a good candidate for the right-handed neutrino generating the least broken super-symmetry.This super-symmetry seems however to differ from the ordinary one in that ν_R is expected to behave like a passive spectator in the scattering. Also for the left-handed neutrino solutions localized inside string world sheet the condition that coupling to right-handed neutrino vanishes is guaranteed if gamma matrices are either purely Minkowskian or CP_2 like inside the world sheet.

6.1.3 Measurement interaction term and boundary conditions

Quantum classical correspondence (QCC) requires a coupling between quantum and classical and this coupling should also give rise to a generalization of quantum measurement theory. The big question is how to realize this coupling.

(a) The proposal discussed in previous chapter was that the addition of a measurement interaction term to the modified Dirac action could do the job and solve a handful of problems of quantum TGD and unify various visions about the physics predicted by quantum TGD. This proposal implies QCC at the level of modified Dirac action and Kähler action. The simplest form of this term is completely analogous to algebraic form of Dirac action in M^4 but with integration measure $det(g_4)^{1/2}d^3x$ restricted to the 3-D surface in question.

(b) Another possibility consistent with the considerations of this chapter is that QCC is realized at the level of WCW Dirac operator and modified Dirac operator contains only interior term. I have indeed proposed that WCW spinor fields with given quantum charges in Cartan algebra are superpositions of space-time surfaces with same classical charges. A stronger form of QCC at the level of WCW would be that classical correlation functions for various geometric observables are identical with quantal correlation functions.

The boundary conditions for modified Dirac equation at space-like 3-surfaces are determined by the sum the analog of algebraic massless Dirac operator $p^k\gamma_k$ in M^4 coupled to the formal analog of Higgs field defined by the normal component Γ^n of the Kähler-Dirac gamma matrix. Higgs field is not in question. Rather the equation allows to formulate space-time correlate for stringy mass formula and also to understand how the ground state conformal weight can be negative half-integer as required by the p-adic mass calculations. At lightlike 3-surfaces Γ^n must vanish and the measurement interaction involving $p^k\gamma_k$ vanishes identically.

6.1.4 Progress in the understanding of super-conformal symmetries

The considerations in the sequel lead to a considerable progress in the understanding of super Virasoro representations for super-symplectic and super-Kac-Moody algebra. In particular, the proposal is that super-Kac-Moody currents assignable to string world sheets define duals of gauge potentials and their generalization for gravitons: in the approximation that gauge group is Abelian - motivated by the notion of finite measurement resolution - the exponents for the sum of KM charges would define non-integrable phase factors. One can also identify Yangian as the algebra generated by these charges. The approach allows also to understand the special role of the right handed neutrino in SUSY according to TGD. It must be however emphasized that also a weaker form of Einstein's equations can be considered solving the condition that the energy momentum tensor for Kähler action has vanishing divergence [K78] implying Einstein's equations with cosmological constant in general relativity. The weaker form involves several non-constant parameters analogous to cosmological constant.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http:`

//www.tgdtheory.fi/cmaphtml.html [L13]. Pdf representation of same files serving as a kind of glossary can be found at http://www.tgdtheory.fi/tgdglossary.pdf [L14]. The topics relevant to this chapter are given by the following list.

- TGD as infinite-dimensional geometry [L37]
- WCW spinor fields [L42]
- KD equation [L25]
- Kaehler-Dirac action [L24]

6.2 About deformations of known extremals of Kähler action

I have done a considerable amount of speculative guesswork to identify what I have used to call preferred extremals of Kähler action. The difficulty is that the mathematical problem at hand is extremely non-linear and that I do not know about existing mathematical literature relevant to the situation. One must proceed by trying to guess the general constraints on the preferred extremals which look physically and mathematically plausible. The hope is that this net of constraints could eventually chrystallize to Eureka! Certainly the recent speculative picture involves also wrong guesses. The need to find explicit ansatz for the deformations of known extremals based on some common principles has become pressing. The following considerations represent an attempt to combine the existing information to achieve this.

6.2.1 What might be the common features of the deformations of known extremals

The dream is to discover the deformations of all known extremals by guessing what is common to all of them. One might hope that the following list summarizes at least some common features.

Effective three-dimensionality at the level of action

(a) Holography realized as effective 3-dimensionality also at the level of action requires that it reduces to 3-dimensional effective boundary terms. This is achieved if the contraction $j^\alpha A_\alpha$ vanishes. This is true if j^α vanishes or is light-like, or if it is proportional to instanton current in which case current conservation requires that CP_2 projection of the space-time surface is 3-dimensional. The first two options for j have a realization for known extremals. The status of the third option - proportionality to instanton current - has remained unclear.

(b) As I started to work again with the problem, I realized that instanton current could be replaced with a more general current $j = *B \wedge J$ or concretely: $j^\alpha = \epsilon^{\alpha\beta\gamma\delta} B_\beta J_{\gamma\delta}$, where B is vector field and CP_2 projection is 3-dimensional, which it must be in any case. The contractions of j appearing in field equations vanish automatically with this ansatz.

(c) Almost topological QFT property in turn requires the reduction of effective boundary terms to Chern-Simons terms: this is achieved by boundary conditions expressing weak form of electric magnetic duality. If one generalizes the weak form of electric-magnetic duality to $J = \Phi * J$ one has $B = d\Phi$ and j has a vanishing divergence for 3-D CP_2 projection. This is clearly a more general solution ansatz than the one based on proportionality of j with instanton current and would reduce the field equations in concise notation to $Tr(TH^k) = 0$.

(d) Any of the alternative properties of the Kähler current implies that the field equations reduce to $Tr(TH^k) = 0$, where T and H^k are shorthands for Maxwellian energy momentum tensor and second fundamental form and the product of tensors is obvious generalization of matrix product involving index contraction.

Could Einstein's equations emerge dynamically?

For j^α satisfying one of the three conditions, the field equations have the same form as the equations for minimal surfaces except that the metric g is replaced with Maxwell energy momentum tensor T.

(a) This raises the question about dynamical generation of small cosmological constant Λ: $T = \Lambda g$ would reduce equations to those for minimal surfaces. For $T = \Lambda g$ modified gamma matrices would reduce to induced gamma matrices and the modified Dirac operator would be proportional to ordinary Dirac operator defined by the induced gamma matrices. One can also consider weak form for $T = \Lambda g$ obtained by restricting the consideration to a sub-space of tangent space so that space-time surface is only "partially" minimal surface but this option is not so elegant although necessary for other than CP_2 type vacuum extremals.

(b) What is remarkable is that $T = \Lambda g$ implies that the divergence of T which in the general case equals to $j^\beta J^\alpha_\beta$ vanishes. This is guaranteed by one of the conditions for the Kähler current. Since also Einstein tensor has a vanishing divergence, one can ask whether the condition to $T = \kappa G + \Lambda g$ could the general condition. This would give Einstein's equations with cosmological term besides the generalization of the minimal surface equations. GRT would emerge dynamically from the non-linear Maxwell's theory although in slightly different sense as conjectured [K56]! Note that the expression for G involves also second derivatives of the imbedding space coordinates so that actually a partial differential equation is in question. If field equations reduce to purely algebraic ones, as the basic conjecture states, it is possible to have $Tr(GH^k) = 0$ and $Tr(gH^k) = 0$ separately so that also minimal surface equations would hold true.

What is amusing that the first guess for the action of TGD was curvature scalar. It gave analogs of Einstein's equations as a definition of conserved four-momentum currents. The recent proposal would give the analog of ordinary Einstein equations as a dynamical constraint relating Maxwellian energy momentum tensor to Einstein tensor and metric.

(c) Minimal surface property is physically extremely nice since field equations can be interpreted as a non-linear generalization of massless wave equation: something very natural for non-linear variant of Maxwell action. The theory would be also very "stringy" although the fundamental action would not be space-time volume. This can however hold true only for Euclidian signature. Note that for CP_2 type vacuum extremals Einstein tensor is proportional to metric so that for them the two options are equivalent. For their small deformations situation changes and it might happen that the presence of G is necessary. The GRT limit of TGD discussed in [K56] [L12] indeed suggests that CP_2 type solutions satisfy Einstein's equations with large cosmological constant and that the small observed value of the cosmological constant is due to averaging and small volume fraction of regions of Euclidian signature (lines of generalized Feynman diagrams).

(d) For massless extremals and their deformations $T = \Lambda g$ cannot hold true. The reason is that for massless extremals energy momentum tensor has component T^{vv} which actually quite essential for field equations since one has $H^k_{vv} = 0$. Hence for massless extremals and their deformations $T = \Lambda g$ cannot hold true if the induced metric has Hamilton-Jacobi structure meaning that g^{uu} and g^{vv} vanish. A more general relationship of form $T = \kappa G + \Lambda G$ can however be consistent with non-vanishing T^{vv} but require that deformation has at most 3-D CP_2 projection (CP_2 coordinates do not depend on v).

(e) The non-determinism of vacuum extremals suggest for their non-vacuum deformations a conflict with the conservation laws. In, also massless extremals are characterized by a non-determinism with respect to the light-like coordinate but like-likeness saves the situation. This suggests that the transformation of a properly chosen time coordinate of vacuum extremal to a light-like coordinate in the induced metric combined with Einstein's equations in the induced metric of the deformation could allow to handle the non-determinism.

Are complex structure of CP_2 and Hamilton-Jacobi structure of M^4 respected by the deformations?

The complex structure of CP_2 and Hamilton-Jacobi structure of M^4 could be central for the understanding of the preferred extremal property algebraically.

(a) There are reasons to believe that the Hermitian structure of the induced metric ((1,1) structure in complex coordinates) for the deformations of CP_2 type vacuum extremals could be crucial property of the preferred extremals. Also the presence of light-like direction is also an essential elements and 3-dimensionality of M^4 projection could be essential. Hence a good guess is that allowed deformations of CP_2 type vacuum extremals are such that (2,0) and (0,2) components the induced metric and/or of the energy momentum tensor vanish. This gives rise to the conditions implying Virasoro conditions in string models in quantization:

$$g_{\xi^i\xi^j} = 0 \ , \quad g_{\overline{\xi}^i\overline{\xi}^j} = 0 \ , \quad i,j = 1,2 \ . \tag{6.2.1}$$

Holomorphisms of CP_2 preserve the complex structure and Virasoro conditions are expected to generalize to 4-dimensional conditions involving two complex coordinates. This means that the generators have two integer valued indices but otherwise obey an algebra very similar to the Virasoro algebra. Also the super-conformal variant of this algebra is expected to make sense.

These Virasoro conditions apply in the coordinate space for CP_2 type vacuum extremals. One expects similar conditions hold true also in field space, that is for M^4 coordinates.

(b) The integrable decomposition $M^4(m) = M^2(m)+E^2(m)$ of M^4 tangent space to longitudinal and transversal parts (non-physical and physical polarizations) - Hamilton-Jacobi structure- could be a very general property of preferred extremals and very natural since non-linear Maxwellian electrodynamics is in question. This decomposition led rather early to the introduction of the analog of complex structure in terms of what I called Hamilton-Jacobi coordinates $(u, v, w, \overline{w})$ for M^4. (u, v) defines a pair of light-like coordinates for the local longitudinal space $M^2(m)$ and $(w, \overline{w})$ complex coordinates for $E^2(m)$. The metric would not contain any cross terms between $M^2(m)$ and $E^2(m)$: $g_{uw} = g_{vw} = g_{u\overline{w}} = g_{v\overline{w}} = 0$.

A good guess is that the deformations of massless extremals respect this structure. This condition gives rise to the analog of the constraints leading to Virasoro conditions stating the vanishing of the non-allowed components of the induced metric. $g_{uu} = g_{vv} = g_{ww} = g_{\overline{w}\overline{w}} = g_{uw} = g_{vw} = g_{u\overline{w}} = g_{v\overline{w}} = 0$. Again the generators of the algebra would involve two integers and the structure is that of Virasoro algebra and also generalization to super algebra is expected to make sense. The moduli space of Hamilton-Jacobi structures would be part of the moduli space of the preferred extremals and analogous to the space of all possible choices of complex coordinates. The analogs of infinitesimal holomorphic transformations would preserve the modular parameters and give rise to a 4-dimensional Minkowskian analog of Virasoro algebra. The conformal algebra acting on CP_2 coordinates acts in field degrees of freedom for Minkowskian signature.

Field equations as purely algebraic conditions

If the proposed picture is correct, field equations would reduce basically to purely algebraically conditions stating that the Maxwellian energy momentum tensor has no common index pairs with the second fundamental form. For the deformations of CP_2 type vacuum extremals T is a complex tensor of type (1,1) and second fundamental form H^k a tensor of type (2,0) and (0,2) so that $Tr(TH^k) =$ is true. This requires that second light-like coordinate of M^4 is constant so that the M^4 projection is 3-dimensional. For Minkowskian signature of the induced metric Hamilton-Jacobi structure replaces conformal structure. Here the dependence of CP_2 coordinates on second light-like coordinate of $M^2(m)$ only plays a fundamental role. Note that now T^{vv} is non-vanishing (and light-like). This picture generalizes to the deformations of cosmic strings and even to the case of vacuum extremals.

6.2.2 What small deformations of CP_2 type vacuum extremals could be?

I was led to these arguments when I tried find preferred extremals of Kähler action, which would have 4-D CP_2 and M^4 projections - the Maxwell phase analogous to the solutions of Maxwell's equations that I conjectured long time ago. It however turned out that the dimensions of the projections can be ($D_{M^4} \leq 3, D_{CP_2} = 4$) or ($D_{M^4} = 4, D_{CP_2} \leq 3$). What happens is essentially breakdown of linear superposition so that locally one can have superposition of modes which have 4-D wave vectors in the same direction. This is actually very much like quantization of radiation field to photons now represented as separate space-time sheets and one can say that Maxwellian superposition corresponds to union of separate photonic space-time sheets in TGD.

Approximate linear superposition of fields is fundamental in standard physics framework and is replaced in TGD with a linear superposition of effects of classical fields on a test particle topologically condensed simultaneously to several space-time sheets. One can say that linear superposition is replaced with a disjoint union of space-time sheets. In the following I shall restrict the consideration to the deformations of CP_2 type vacuum extremals.

Solution ansatz

I proceed by the following arguments to the ansatz.

(a) Effective 3-dimensionality for action (holography) requires that action decomposes to vanishing $j^\alpha A_\alpha$ term + total divergence giving 3-D "boundary" terms. The first term certainly vanishes (giving effective 3-dimensionality) for

$$D_\beta J^{\alpha\beta} = j^\alpha = 0 \ .$$

Empty space Maxwell equations, something extremely natural. Also for the proposed GRT limit these equations are true.

(b) How to obtain empty space Maxwell equations $j^\alpha = 0$? The answer is simple: assume self duality or its slight modification:

$$J = *J$$

holding for CP_2 type vacuum extremals or a more general condition

$$J = k * J \ ,$$

In the simplest situation k is some constant not far from unity. * is Hodge dual involving 4-D permutation symbol. $k = constant$ requires that the determinant of the induced metric is apart from constant equal to that of CP_2 metric. It does not require that the induced metric is proportional to the CP_2 metric, which is not possible since M^4 contribution to metric has Minkowskian signature and cannot be therefore proportional to CP_2 metric.

One can consider also a more general situation in which k is scalar function as a generalization of the weak electric-magnetic duality. In this case the Kähler current is non-vanishing but divergenceless. This also guarantees the reduction to $Tr(TH^k) = 0$.

In this case however the proportionality of the metric determinant to that for CP_2 metric is not needed. This solution ansatz becomes therefore more general.

(c) Field equations reduce with these assumptions to equations differing from minimal surfaces equations only in that metric g is replaced by Maxwellian energy momentum tensor T. Schematically:

$$Tr(TH^k) = 0 \ ,$$

where T is the Maxwellian energy momentum tensor and H^k is the second fundamental form - asymmetric 2-tensor defined by covariant derivative of gradients of imbedding space coordinates.

How to satisfy the condition $Tr(TH^k) = 0$?

It would be nice to have minimal surface equations since they are the non-linear generalization of massless wave equations. It would be also nice to have the vanishing of the terms involving Kähler current in field equations as a consequence of this condition. Indeed, $T = \kappa G + \Lambda g$ implies this. In the case of CP_2 vacuum extremals one cannot distinguish between these options since CP_2 itself is constant curvature space with $G \propto g$. Furthermore, if G and g have similar tensor structure the algebraic field equations for G and g are satisfied separately so that one obtains minimal surface property also now. In the following minimal surface option is considered.

(a) The first opton is achieved if one has

$$T = \Lambda g \ .$$

Maxwell energy momentum tensor would be proportional to the metric! One would have dynamically generated cosmological constant! This begins to look really interesting since it appeared also at the proposed GRT limit of TGD [L12]. Note that here also non-constant value of Λ can be considered and would correspond to a situation in which k is scalar function: in this case the the determinant condition can be dropped and one obtains just the minimal surface equations.

(b) Very schematically and forgetting indices and being sloppy with signs, the expression for T reads as

$$T = JJ - g/4Tr(JJ) \ .$$

Note that the product of tensors is obtained by generalizing matrix product. This should be proportional to metric.

Self duality implies that $Tr(JJ)$ is just the instanton density and does not depend on metric and is constant.

For CP_2 type vacuum extremals one obtains

$$T = -g + g = 0 \ .$$

Cosmological constant would vanish in this case.

(c) Could it happen that for deformations a small value of cosmological constant is generated?

The condition would reduce to

$$JJ = (\Lambda - 1)g \ .$$

Λ must relate to the value of parameter k appearing in the generalized self-duality condition. For the most general ansatz Λ would not be constant anymore.

This would generalize the defining condition for Kähler form

$$JJ = -g \ \ (i^2 = -1 \ \ geometrically)$$

stating that the square of Kähler form is the negative of metric. The only modification would be that index raising is carried out by using the induced metric containing also M^4 contribution rather than CP_2 metric.

(d) Explicitly:

$$J_{\alpha\mu}J^{\mu}_{\ \beta} = (\Lambda - 1)g_{\alpha\beta} \ \ .$$

Cosmological constant would measure the breaking of Kähler structure. By writing $g = s + m$ and defining index raising of tensors using CP_2 metric and their product accordingly, this condition can be also written as

$$Jm = (\Lambda - 1)mJ \ \ .$$

If the parameter k is constant, the determinant of the induced metric must be proportional to the CP_2 metric. If k is scalar function, this condition can be dropped. Cosmological constant would not be constant anymore but the dependence on k would drop out from the field equations and one would hope of obtaining minimal surface equations also now. It however seems that the dimension of M^4 projection cannot be four. For 4-D M^4 projection the contribution of the M^2 part of the M^4 metric gives a non-holomorphic contribution to CP_2 metric and this spoils the field equations.

For $T = \kappa G + \Lambda g$ option the value of the cosmological constant is large - just as it is for the proposed GRT limit of TGD [K56] [L12]. The interpretation in this case is that the average value of cosmological constant is small since the portion of space-time volume containing generalized Feynman diagrams is very small.

More detailed ansatz for the deformations of CP_2 type vacuum extremals

One can develop the ansatz to a more detailed form. The most obvious guess is that the induced metric is apart from constant conformal factor the metric of CP_2. This would guarantee self-duality apart from constant factor and $j^\alpha = 0$. Metric would be in complex CP_2 coordinates tensor of type (1,1) whereas CP_2 Riemann connection would have only purely holomorphic or anti-holomorphic indices. Therefore CP_2 contributions in $Tr(TH^k)$ would vanish identically. M^4 degrees of freedom however bring in difficulty. The M^4 contribution to the induced metric should be proportional to CP_2 metric and this is impossible due to the different signatures. The M^4 contribution to the induced metric breaks its Kähler property but would preserve Hermitian structure.

A more realistic guess based on the attempt to construct deformations of CP_2 type vacuum extremals is following.

(a) Physical intuition suggests that M^4 coordinates can be chosen so that one has integrable decomposition to longitudinal degrees of freedom parametrized by two light-like coordinates u and v and to transversal polarization degrees of freedom parametrized by complex coordinate w and its conjugate. M^4 metric would reduce in these coordinates to a direct sum of longitudinal and transverse parts. I have called these coordinates Hamilton-Jacobi coordinates.

(b) w would be holomorphic function of CP_2 coordinates and therefore satisfy the analog of massless wave equation. This would give hopes about rather general solution ansatz. u and v cannot be holomorphic functions of CP_2 coordinates. Unless wither u or v is constant, the induced metric would receive contributions of type (2,0) and (0,2) coming from u and v which would break Kähler structure and complex structure. These contributions would give no-vanishing contribution to all minimal surface equations. Therefore either u or v is constant: the coordinate line for non-constant coordinate -say u- would be analogous to the M^4 projection of CP_2 type vacuum extremal.

(c) With these assumptions the induced metric would remain $(1,1)$ tensor and one might hope that $Tr(TH^k)$ contractions vanishes for all variables except u because the there are no common index pairs (this if non-vanishing Christoffel symbols for H involve only holomorphic or anti-holomorphic indices in CP_2 coordinates). For u one would obtain massless wave equation expressing the minimal surface property.

(d) If the value of k is constant the determinant of the induced metric must be proportional to the determinant of CP_2 metric. The induced metric would contain only the contribution from the transversal degrees of freedom besides CP_2 contribution. Minkowski contribution has however rank 2 as CP_2 tensor and cannot be proportional to CP_2 metric. It is however enough that its determinant is proportional to the determinant of CP_2 metric with constant proportionality coefficient. This condition gives an additional non-linear condition to the solution. One would have wave equation for u (also w and its conjugate satisfy massless wave equation) and determinant condition as an additional condition.

The determinant condition reduces by the linearity of determinant with respect to its rows to sum of conditions involved 0,1,2 rows replaced by the transversal M^4 contribution to metric given if M^4 metric decomposes to direct sum of longitudinal and transversal parts. Derivatives with respect to derivative with respect to particular CP_2 complex coordinate appear linearly in this expression they can depend on u via the dependence of transversal metric components on u. The challenge is to show that this equation has (or does not have) non-trivial solutions.

(e) If the value of k is scalar function the situation changes and one has only the minimal surface equations and Virasoro conditions.

What makes the ansatz attractive is that special solutions of Maxwell empty space equations are in question, equations reduces to non-linear generalizations of Euclidian massless wave equations, and possibly space-time dependent cosmological constant pops up dynamically. These properties are true also for the GRT limit of TGD [L12].

6.2.3 Hamilton-Jacobi conditions in Minkowskian signature

The maximally optimistic guess is that the basic properties of the deformations of CP_2 type vacuum extremals generalize to the deformations of other known extremals such as massless extremals, vacuum extremals with 2-D CP_2 projection which is Lagrangian manifold, and cosmic strings characterized by Minkowskian signature of the induced metric. These properties would be following.

(a) The recomposition of M^4 tangent space to longitudinal and transversal parts giving Hamilton-Jacobi structure. The longitudinal part has hypercomplex structure but the second light-like coordinate is constant: this plays a crucial role in guaranteeing the vanishing of contractions in $Tr(TH^k)$. It is the algebraic properties of g and T which are crucial. T can however have light-like component T^{vv}. For the deformations of CP_2 type vacuum extremals $(1,1)$ structure is enough and is guaranteed if second light-like coordinate of M^4 is constant whereas w is holomorphic function of CP_2 coordinates.

(b) What could happen in the case of massless extremals? Now one has 2-D CP_2 projection in the initial situation and CP_2 coordinates depend on light-like coordinate u and single real transversal coordinate. The generalization would be obvious: dependence on single light-like coordinate u and holomorphic dependence on w for complex CP_2 coordinates. The constraint is $T = \Lambda g$ cannot hold true since T^{vv} is non-vanishing (and light-like). This property restricted to transversal degrees of freedom could reduce the field equations to minimal surface equations in transversal degrees of freedom. The transversal part of energy momentum tensor would be proportional to metric and hence covariantly constant. Gauge current would remain light-like but would not be given by $j = *d\phi \wedge J$. $T = \kappa G + \Lambda g$ seems to define the attractive option.

It therefore seems that the essential ingredient could be the condition

$$T = \kappa G + \lambda g \ ,$$

which has structure (1,1) in both $M^2(m)$ and $E^2(m)$ degrees of freedom apart from the presence of T^{vv} component with deformations having no dependence on v. If the second fundamental form has (2,0)+(0,2) structure, the minimal surface equations are satisfied provided Kähler current satisfies on of the proposed three conditions and if G and g have similar tensor structure.

One can actually pose the conditions of metric as complete analogs of stringy constraints leading to Virasoro conditions in quantization to give

$$g_{uu} = 0 \ , \quad g_{vv} = 0 \ , \quad g_{ww} = 0 \ , \quad g_{\overline{w}\overline{w}} = 0 \ . \tag{6.2.2}$$

This brings in mind the generalization of Virasoro algebra to four-dimensional algebra for which an identification in terms of non-local Yangian symmetry has been proposed [K61]. The number of conditions is four and the same as the number of independent field equations. One can consider similar conditions also for the energy momentum tensor T but allowing non-vanishing component T^{vv} if deformations has no v-dependence. This would solve the field equations if the gauge current vanishes or is light-like. On this case the number of equations is 8. First order differential equations are in question and they can be also interpreted as conditions fixing the coordinates used since there is infinite number of manners to choose the Hamilton-Jacobi coordinates.

One can can try to apply the physical intuition about general solutions of field equations in the linear case by writing the solution as a superposition of left and right propagating solutions:

$$\xi^k = f^k_+(u,w) + f^k_+(v,w) \ . \tag{6.2.3}$$

This could guarantee that second fundamental form is of form (2,0)+(0,2) in both M^2 and E^2 part of the tangent space and these terms if $Tr(TH^k)$ vanish identically. The remaining terms involve contractions of T^{uw}, $T^{u\overline{w}}$ and T^{vw}, $T^{v\overline{w}}$ with second fundamental form. Also these terms should sum up to zero or vanish separately. Second fundamental form has components coming from f^k_+ and f^k_-

Second fundamental form H^k has as basic building bricks terms $\hat{H}^k$ given by

$$\hat{H}^k_{\alpha\beta} \ = \ \partial_\alpha\partial_\beta h^k + \left({k \atop l\ m}\right) \partial_\alpha h^l \partial_\beta h^m \ . \tag{6.2.4}$$

For the proposed ansatz the first terms give vanishing contribution to H^k_{uv}. The terms containing Christoffel symbols however give a non-vanishing contribution and one can allow only f^k_+ or f^k_- as in the case of massless extremals. This reduces the dimension of CP_2 projection to $D = 3$.

What about the condition for Kähler current? Kähler form has components of type $J_{w\overline{w}}$ whose contravariant counterpart gives rise to space-like current component. J_{uw} and $J_{u\overline{w}}$ give rise to light-like currents components. The condition would state that the $J^{w\overline{w}}$ is covariantly constant. Solutions would be characterized by a constant Kähler magnetic field. Also electric field is represent. The interpretation both radiation and magnetic flux tube makes sense.

6.2.4 Deformations of cosmic strings

In the physical applications it has been assumed that the thickening of cosmic strings to Kähler magnetic flux tubes takes place. One indeed expects that the proposed construction generalizes also to the case of cosmic strings having the decomposition $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, where X^2 is minimal surface and Y^2 a complex homologically non-trivial sub-manifold of CP_2. Now the starting point structure is Hamilton-Jacobi structure for $M^2_m \times Y^2$ defining the coordinate space.

(a) The deformation should increase the dimension of either CP_2 or M^4 projection or both. How this thickening could take place? What comes in mind that the string orbits X^2 can be interpreted as a distribution of longitudinal spaces $M^2(x)$ so that for the deformation w coordinate becomes a holomorphic function of the natural Y^2 complex coordinate so that M^4 projection becomes 4-D but CP_2 projection remains 2-D. The new contribution to the X^2 part of the induced metric is vanishing and the contribution to the Y^2 part is of type $(1,1)$ and the the ansatz $T = \kappa G + \Lambda g$ might be needed as a generalization of the minimal surface equations The ratio of κ and G would be determined from the form of the Maxwellian energy momentum tensor and be fixed at the limit of undeformed cosmic strong to $T = (ag(Y^2) - bg(Y^2)$. The value of cosmological constant is now large, and overall consistency suggests that $T = \kappa G + \Lambda g$ is the correct option also for the CP_2 type vacuum extremals.

(b) One could also imagine that remaining CP_2 coordinates could depend on the complex coordinate of Y^2 so that also CP_2 projection would become 4-dimensional. The induced metric would receive holomorphic contributions in Y^2 part. As a matter fact, this option is already implied by the assumption that Y^2 is a complex surface of CP_2.

6.2.5 Deformations of vacuum extremals?

What about the deformations of vacuum extremals representable as maps from M^4 to CP_2?

(a) The basic challenge is the non-determinism of the vacuum extremals. One should perform the deformation so that conservation laws are satisfied. For massless extremals there is also non-determinism but it is associated with the light-like coordinate so that there are no problems with the conservation laws. This would suggest that a properly chosen time coordinate consistent with Hamilton-Jacobi decomposition becomes light-like coordinate in the induced metric. This poses a conditions on the induced metric.

(b) Physical intuition suggests that one cannot require $T = \Lambda g$ since this would mean that the rank of T is maximal whereas the original situation corresponds to the vanishing of T. For small deformations rank two for T looks more natural and one could think that T is proportional to a projection of metric to a 2-D subspace. The vision about the long length scale limit of TGD is that Einstein's equations are satisfied and this would suggest $T = kG$ or $T = \kappa G + \Lambda g$. The rank of T could be smaller than four for this ansatz and this conditions binds together the values of κ and G.

(c) These extremals have CP_2 projection which in the generic case is 2-D Lagrangian sub-manifold Y^2. Again one could assume Hamilton-Jacobi coordinates for X^4. For CP_2 one could assume Darboux coordinates (P_i, Q_i), $i = 1, 2$, in which one has $A = P_i dQ^i$, and that $Y^2 \subset CP_2$ corresponds to $Q_i = constant$. In principle P_i would depend on arbitrary manner on M^4 coordinates. It might be more convenient to use as coordinates (u, v) for M^2 and (P_1, P_2) for Y^2. This covers also the situation when M^4 projection is not 4-D. By its 2-dimensionality Y^2 allows always a complex structure defined by its induced metric: this complex structure is not consistent with the complex structure of CP_2 (Y^2 is not complex sub-manifold).

Using Hamilton-Jacobi coordinates the pre-image of a given point of Y^2 is a 2-dimensional sub-manifold X^2 of X^4 and defines also 2-D sub-manifold of M^4. The following picture suggests itself. The projection of X^2 to M^4 can be seen for a suitable choice of Hamilton-Jacobi coordinates as an analog of Lagrangian sub-manifold in M^4 that is as surface for which v and $Im(w)$ vary and u and $Re(w)$ are constant. X^2 would be obtained by allowing u and $Re(w)$ to vary: as a matter fact, (P_1, P_2) and $(u, Re(w))$ would be related to each other. The induced metric should be consistent with this picture. This would requires $g_{uRe(w)} = 0$.

For the deformations Q_1 and Q_2 would become non-constant and they should depend on the second light-like coordinate v only so that only g_{uu} and g_{uw} and $gu\overline{w}$ $g_{w,w}$ and $g_{\overline{w},\overline{w}}$ receive contributions which vanish. This would give rise to the analogs of Virasoro conditions guaranteeing that T is a tensor of form $(1,1)$ in both M^2 and E^2 indices and that there are no cross components in the induced metric. A more general formulation states that energy momentum tensor satisfies these conditions. The conditions on T might be equivalent with the conditions for g and G separately.

(d) Einstein's equations provide an attractive manner to achieve the vanishing of effective 3-dimensionality of the action. Einstein equations would be second order differential equations and the idea that a deformation of vacuum extremal is in question suggests that the dynamics associated with them is in directions transversal to Y^2 so that only the deformation is dictated partially by Einstein's equations.

(e) Lagrangian manifolds do not involve complex structure in any obvious manner. One could however ask whether the deformations could involve complex structure in a natural manner in CP_2 degrees of freedom so that the vanishing of g_{ww} would be guaranteed by holomorphy of CP_2 complex coordinate as function of w.

One should get the complex structure in some natural manner: in other words, the complex structure should relate to the geometry of CP_2 somehow. The complex coordinate defined by say $z = P_1 + iQ^1$ for the deformation suggests itself. This would suggest that at the limit when one puts $Q_1 = 0$ one obtains $P_1 = P_1(Re(w))$ for the vacuum extremals and the deformation could be seen as an analytic continuation of real function to region of complex plane. This is in spirit with the algebraic approach. The vanishing of Kähler current requires that the Kähler magnetic field is covariantly constant: $D_z J^{z\overline{z}} = 0$ and $D_{\overline{z}} J^{z\overline{z}} = 0$.

(f) One could consider the possibility that the resulting 3-D sub-manifold of CP_2 can be regarded as contact manifold with induced Kähler form non-vanishing in 2-D section with natural complex coordinates. The third coordinate variable- call it s- of the contact manifold and second coordinate of its transversal section would depend on time spacetime coordinates for vacuum extremals. The coordinate associated with the transversal section would be continued to a complex coordinate which is holomorphic function of w and u.

(g) The resulting thickened magnetic flux tubes could be seen as another representation of Kähler magnetic flux tubes: at this time as deformations of vacuum flux tubes rather than cosmic strings. For this ansatz it is however difficult to imagine deformations carrying Kähler electric field.

6.2.6 About the interpretation of the generalized conformal algebras

The long-standing challenge has been finding of the direct connection between the superconformal symmetries assumed in the construction of the geometry of the "world of classical worlds" (WCW) and possible conformal symmetries of field equations. 4-dimensionality and Minkowskian signature have been the basic problems. The recent construction provides new insights to this problem.

(a) In the case of string models the quantization of the Fourier coefficients of coordinate variables of the target space gives rise to Kac-Moody type algebra and Virasoro algebra generators are quadratic in these. Also now Kac-Moody type algebra is expected. If one were to perform a quantization of the coefficients in Laurents series for complex CP_2 coordinates, one would obtain interpretation in terms of $su(3) = u(2)+t$ decomposition, where t corresponds to CP_3: the oscillator operators would correspond to generators in t

and their commutator would give generators in $u(2)$. SU(3)/SU(2) coset representation for Kac-Moody algebra would be in question. Kac-Moody algebra would be associated with the generators in both M^4 and CP_2 degrees of freedom. This kind of Kac-Moody algebra appears in quantum TGD.

(b) The constraints on induced metric imply a very close resemblance with string models and a generalization of Virasoro algebra emerges. An interesting question is how the two algebras acting on coordinate and field degrees of freedom relate to the super-conformal algebras defined by the symplectic group of $\delta M^4_+ \times CP_2$ acting on space-like 3-surfaces at boundaries of CD and to the Kac-Moody algebras acting on light-like 3-surfaces. It has been conjectured that these algebras allow a continuation to the interior of space-time surface made possible by its slicing by 2-surfaces parametrized by 2-surfaces. The proposed construction indeed provides this kind of slicings in both M^4 and CP_2 factor.

(c) In the recent case, the algebras defined by the Fourier coefficients of field variables would be Kac-Moody algebras. Virasoro algebra acting on preferred coordinates would be expressed in terms of the Kac-Moody algebra in the standard Sugawara construction applied in string models. The algebra acting on field space would be analogous to the conformal algebra assignable to the symplectic algebra so that also symplectic algebra is present. Stringy pragmatist could imagine quantization of symplectic algebra by replacing CP_2 coordinates in the expressions of Hamiltonians with oscillator operators. This description would be counterpart for the construction of spinor harmonics in WCW and might provide some useful insights.

(d) For given type of space-time surface either CP_2 or M^4 corresponds to Kac-Moody algebra but not both. From the point of view of quantum TGD it looks as that something were missing. An analogous problem was encountered at GRT limit of TGD [L12]. When Euclidian space-time regions are allowed Einstein-Maxwell action is able to mimic standard model with a surprising accuracy but there is a problem: one obtains either color charges or M^4 charges but not both. Perhaps it is not enough to consider either CP_2 type vacuum extremal or its exterior but both to describe particle: this would give the direct product of the Minkowskian and Euclidian algebras acting on tensor product. This does not however seem to be consistent with the idea that the two descriptions are duality related (the analog of T-duality).

6.3 Under what conditions electric charge is conserved for the modified Dirac equation?

One might think that talking about the conservation of electric charge at 21st century is a waste of time. In TGD framework this is certainly not the case.

(a) In quantum field theories there are two manners to define em charge: as electric flux over 2-D surface sufficiently far from the source region or in the case of spinor field quantum mechanically as combination of fermion number and vectorial isospin. The latter definition is quantum mechanically more appropriate.

(b) There is however a problem. In standard approach to gauge theory Dirac equation in presence of charged classical gauge fields does not conserve electric charge as quantum number: electron is transformed to neutrino and vice versa. Quantization solves the problem since the non-conservation can be interpreted in terms of emission of gauge bosons. In TGD framework this does not work since one does not have path integral quantization anymore. Preferred extremals carry classical gauge fields and the question whether em charge is conserved arises. Heuristic picture suggests that em charge must be conserved.

It seems that one should pose the well-definedness of spinorial em charge as an additional condition. Well-definedness of em charge is not the only problem. How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? Does this require that classical weak fields vanish in the regions where the modes of induced spinor fields are non-vanishing?

This condition might be one of the conditions defining what it is to be a preferred extremal/solution of Kähler Dirac equation. It is not however trivial whether this kind of additional condition can be posed unless it follows automatically from the recent formulation for Kähler action and Kähler Dirac action. The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string/parton picture part of TGD. The vanishing of classical weak fields has also number theoretic interpretation: space-time surfaces would have quaternionic (hyper-complex) tangent space and the 2-surfaces carrying spinor fields complex (hyper-complex) tangent space.

6.3.1 Conservation of em charge for Kähler Dirac equation

What does the conservation of em charge imply in the case of the modified Dirac equation? The obvious guess that the em charged part of the modified Dirac operator must annihilate the solutions, turns out to be correct as the following argument demonstrates.

(a) Em charge as coupling matrix can be defined as a linear combination $Q = aI + bI_3$, $I_3 = J_{kl}\Sigma^{kl}$, where I is unit matrix and I_3 vectorial isospin matrix, J_{kl} is the Kähler form of CP_2, Σ^{kl} denotes sigma matrices, and a and b are numerical constants different for quarks and leptons. Q is covariantly constant in $M^4 \times CP_2$ and its covariant derivatives at space-time surface are also well-defined and vanish.

(b) The modes of the modified Dirac equation should be eigen modes of Q. This is the case if the modified Dirac operator D commutes with Q. The covariant constancy of Q can be used to derive the condition

$$
\begin{aligned}
[D,Q]\,\Psi &= D_1\Psi = 0 \ , \\
D &= \hat{\Gamma}^\mu D_\mu \ , \ D_1 = [D,Q] = \hat{\Gamma}_1^\mu D_\mu \ , \ \hat{\Gamma}_1^\mu = \left[\hat{\Gamma}^\mu, Q\right] \ .
\end{aligned}
\tag{6.3.1}
$$

Covariant constancy of J is absolutely essential: without it the resulting conditions would not be so simple.

It is easy to find that also $[D_1,Q]\Psi = 0$ and its higher iterates $[D_n,Q]\Psi = 0$, $D_n = [D_{n-1},Q]$ must be true. The solutions of the modified Dirac equation would have an additional symmetry.

(c) The commutator $D_1 = [D,Q]$ reduces to a sum of terms involving the commutators of the vectorial isospin $I_3 = J_{kl}\Sigma^{kl}$ with the CP_2 part of the gamma matrices:

$$
D_1 = [Q,D] = [I_3,\Gamma_r]\partial_\mu s^r T^{\alpha\mu} D_\alpha \ . \tag{6.3.2}
$$

In standard complex coordinates in which $U(2)$ acts linearly the complexified gamma matrices can be chosen to be eigenstates of vectorial isospin. Only the charged flat space complexified gamma matrices Γ^A denoted by Γ^+ and Γ^- possessing charges +1 and -1 contribute to the right hand side. Therefore the additional Dirac equation $D_1\Psi = 0$ states

$$
\begin{aligned}
D_1\Psi &= [Q,D]\Psi = I_3(A)e_{Ar}\Gamma^A\partial_\mu s^r T^{\alpha\mu} D_\alpha\Psi \\
&= (e_{+r}\Gamma^+ - e_{-r}\Gamma^-)\partial_\mu s^r T^{\alpha\mu} D_\alpha\Psi = 0 \ .
\end{aligned}
\tag{6.3.3}
$$

The next condition is

$$
D_2\Psi = [Q,D]\Psi = (e_{+r}\Gamma^+ + e_{-r}\Gamma^-)\partial_\mu s^r T^{\alpha\mu} D_\alpha\Psi = 0 \ . \tag{6.3.4}
$$

Only the relative sign of the two terms has changed. The remaining conditions give nothing new.

(d) These equations imply two separate equations for the two charged gamma matrices

$$\begin{aligned} D_+\Psi &= T^{\alpha}_{+}\Gamma^{+}D_{\alpha}\Psi = 0 \ , \\ D_-\Psi &= T^{\alpha}_{-}\Gamma^{-}D_{\alpha}\Psi = 0 \ , \\ T^{\alpha}_{\pm} &= e_{\pm r}\partial_{\mu}s^{r}T^{\alpha\mu} \ . \end{aligned} \tag{6.3.5}$$

These conditions state what one might have expected: the charged part of the modified Dirac operator annihilates separately the solutions. The reason is that the classical W fields are proportional to $e_{r\pm}$.

The above equations can be generalized to define a decomposition of the energy momentum tensor to charged and neutral components in terms of vierbein projections. The equations state that the analogs of the modified Dirac equation defined by charged components of the energy momentum tensor are satisfied separately.

(e) In complex coordinates one expects that the two equations are complex conjugates of each other for Euclidian signature. For the Minkowskian signature an analogous condition should hold true. The dynamics enters the game in an essential manner: whether the equations can be satisfied depends on the coefficients a and b in the expression $T = aG + bg$ implied by Einstein's equations in turn guaranteeing that the solution ansatz generalizing minimal surface solutions holds true [K5].

(f) As a result one obtains three separate Dirac equations corresponding to the the neutral part $D_0\Psi = 0$ and charged parts $D_\pm\Psi = 0$ of the modified Dirac equation. By acting on the equations with these Dirac operators one obtains also that the commutators $[D_+, D_-]$, $[D_0, D_\pm]$ and also higher commutators obtained from these annihilate the induced spinor field model. Therefore entire -possibly- infinite-dimensional algebra would annihilate the induced spinor fields. In string model the counterpart of Dirac equation when quantized gives rise to Super-Virasoro conditions. This analogy would suggest that modified Dirac equation gives rise to the analog of Super-Virasoro conditions in 4-D case. But what the higher conditions mean? Could they relate to the proposed generalization to Yangian algebra? Obviously these conditions resemble structurally Virasoro conditions $L_n|phys\rangle = 0$ and their supersymmetric generalizations, and might indeed correspond to a generalization of these conditions just as the field equations for preferred extremals could correspond to the Virasoro conditions if one takes seriously the analogy with the quantized string.

What could this additional symmetry mean from the point of view of the solutions of the modified Dirac equation? The field equations for the preferred extremals of Kähler action reduce to purely algebraic conditions in the same manner as the field equations for the minimal surfaces in string model. Could this happen also for the modified Dirac equation and could the condition on charged part of the Dirac operator help to achieve this?

This argument was very general and one can ask for simple manners to realize these conditions. Obviously the vanishing of classical W fields in the region where the spinor mode is non-vanishing defines this kind of condition.

6.3.2 About the solutions of Kähler Dirac equation for known extremals

To gain perpective consider first Dirac equation in in H. Quite generally, one can construct the solutions of the ordinary Dirac equation in H from covariantly constant right-handed neutrino spinor playing the role of fermionic vacuum annihilated by the second half of complexified gamma matrices. Dirac equation reduces to Laplace equation for a scalar function and solution can be constructed from this "vacuum" by multiplying with the spherical harmonics of CP_2 and applying Dirac operator [K26]. Similar construction works quite generally thanks to the existence of covariantly constant right handed neutrino spinor. Spinor harmonics of CP_2 are only replaced with those of space-time surface possessing either hermitian structure of Hamilton-Jacobi structure (corresponding to Euclidian and Minkowskian signatures of the induced metric [K5, K69]). What is remarkable is that these solutions possess well-defined em charge although classical W boson fields are present.

This in sense that H d'Alembertian commutes with em charge matrix defined as a linear combination of unit matrix and the covariantly constant matrix $J^{kl}\Sigma_{kl}$ since the commutators of the covariant derivatives give constant Ricci scalar and $J^{kl}\Sigma_{kl}$ term to the d'Alembertian besides scalar d'Alembertian commuting with em charge. Dirac operator itself does not commute with em charge matrix since gamma matrices not commute with em charge matrix.

Consider now Kähler Dirac operator. The square of Kähler Dirac operator contains commutator of covariant derivatives which contains contraction $[\Gamma^\mu, \Gamma^\nu]\, F^{weak}_{\mu\nu}$ which is quadratic in sigma matrices of $M^4 \times CP_2$ and does not reduce to a constant term commuting which em charge matrix. Therefore additional condition is required even if one is satisfies with the commutativity of d'Alembertian with em charge. Stronger condition would be commutativity with the Kähler Dirac operator and this will be considered in the following.

To see what happens one must consider space-time regions with Minkowskian and Euclidian signature. What will be assumed is the existence of Hamilton-Jacobi structure [K5] meaning complex structure in Euclidian signature and hyper-complex plus complex structure in Minkowskian signature. The goal is to get insights about what the condition that spinor modes have a well-defined em charge eigenvalue requires. Or more concretely: is the localization at string world sheets guaranteeing well-defined value of em charge predicted by Kähler Dirac operator or must one introduce this condition separately? One can also ask whether this condition reduces to commutativity/co-commutativity in number theoretic vision.

(a) CP_2 type vacuum extremals serve as a convenient test case for the Euclidian signature. In this case the modified Dirac equation reduces to the massless ordinary Dirac equation in CP_2 allowing only covariantly constant right-handed neutrino as solution. Only part of CP_2 so that one give up the constraint that the solution is defined in the entire CP_2. In this case holomorphic solution ansatz obtained by assuming that solutions depend on the coordinates ξ^i, $i = 1, 2$ but not on their conjugates and that the gamma matrices $\Gamma^{\bar{i}}$, $i = 1, 2$, annihilate the solutions, works. The solutions ansatz and its conjugate are of exactly the same form as in case string models where one considers string world sheets instead of CP_2 region.

The solutions are not restricted to 2-D string world sheets and it is not clear whether one can assign to them a well-defined em charge in any sense. Note that for massless Dirac equation in H one obtains all CP_2 harmonics as solutions, and it is possible to talk about em charge of the solution although solution itself is not restricted to 2-D surface of CP_2.

(b) For massless extremals and a very wide class of solutions produced by Hamilton-Jacobi structure - perhaps all solutions representable locally as graphs for map $M^4 \to CP_2$ - canonical momentum densities are light-like and solutions are hyper-holomorphic in the coordinates associated with with string world sheet and annihilated by the conjugate gamma and arbitrary functions in transversal coordinates. This allows localization to string world sheets. The localization is now strictly dynamical and implied by the properties of Kähler Dirac operator.

(c) For string like objects one obtains massless Dirac equation in $X^2 \times Y^2 \subset M^4 \times Y^2$, Y^2 a complex 2-surface in CP_2. Homologically trivial geodesic sphere corresponds to the simplest choice for Y^2. Modified Dirac operator reduces to a sum of massless Dirac operators associated with X^2 and Y^2. The most general solutions would have Y^2 mass. Holomorphic solutions reduces to product of hyper-holomorphic and holomorphic solutions and massless 2-D Dirac equation is satisfied in both factors.

For instance, for S^2 a geodesic sphere and $X^2 = M^2$ one obtains M^2 massivation with mass squared spectrum given by Laplace operator for S^2. Conformal and hyper-conformal symmetries are lost, and one might argue that this is quite not what one wants. One must be however resist the temptation to make too hasty conclusions since the massivation of string like objects is expected to take place. The question is whether it takes place already at the level of fundamental spinor fields or only at the level of elementary particles constructed as many-fermion states of them as twistor Grassmann approach assuming massless M^4 propagators for the fundamental fermions strongly suggests [K44].

(d) For vacuum extremals the Kähler Dirac operator vanishes identically so that it does not make sense to speak about solutions.

What can one conclude from these observations?

(a) The localization of solutions to 2-D string world sheets follows from Kähler Dirac equation only for the Minkowskian regions representable as graphs of map $M^4 \rightarrow CP_2$ locally. For string like objects and deformations of CP_2 type vacuum extremals this is not expected to take place.

(b) It is not clear whether one can speak about well-defined em charge for the holomorphic spinors annihilated by the conjugate gamma matrices or their conjugates. As noticed, for imbedding space spinor harmonics this is however possible.

(c) Strong form of conformal symmetry and the condition that em charge is well-defined for the nodes suggests that the localization at 2-D surfaces at which the charged parts of induced electroweak gauge fields vanish must be assumed as an additional condition. Number theoretic vision would suggest that these surfaces correspond to 2-D commutative or co-commutative surfaces. The string world sheets inside space-time surfaces would not emerge from theory but would be defined as basic geometric objects.

    This kind of condition would also allow duals of string worlds sheets as partonic 2-surfaces identified number theoretically as co-commutative surfaces. Commutativity and co-commutativity would become essential elements of the number theoretical vision.

(d) The localization of solutions of the modified Dirac action at string world sheets and partonic 2-surfaces as a constraint would mean induction procedure for Kähler-Dirac matrices from SX^4 to X^2 - that is projection. The resulting em neutral gamma matrices would correspond to tangent vectors of the string world sheet. The vanishing of the projections of charged parts of energy momentum currents would define these surfaces. The conditions would apply both in Minkowskian and Euclidian regions. An alternative interpretation would be number theoretical: these surface would be commutative or co-commutative.

6.3.3 Concrete realization of the conditions guaranteeing well-defined em charge

Well-definedness of the em charge is the fundamental condiiton on spinor modes. Physical intuition suggests that also classical Z^0 field should vanish - at least in scales longer than weak scale. Above the condition guaranteeing vanishing of em charge has been discussed at very general level. It has however turned out that one can understand situation by simply posing the simplest condition that one can imagine: the vanishing of classical W and possibly also Z^0 fields inducing mixing of different charge states.

(a) Induced W fields mean that the modes of Kähler-Dirac equation do not in general have well-defined em charge. The problem disappears if the induced W gauge fields vanish. This does not yet guarantee that couplings to classical gauge fields are physical in long scales. Also classical Z^0 field should vanish so that the couplings would be purely vectorial. Vectoriality might be true in long enough scales only. If W and Z^0 fields vanish in all scales then electroweak forces are due to the exchanges of corresponding gauge bosons described as string like objects in TGD and represent non-trivial space-time geometry and topology at microscopic scale.

(b) The conditions solve also another long-standing interpretational problem. Color rotations induce rotations in electroweak-holonomy group so that the vanishing of all induced weak fields also guarantees that color rotations do not spoil the property of spinor modes to be eigenstates of em charge.

One can study the conditions quite concretely by using the formulas for the components of spinor curvature [L1] (`http://www.tgdtheory.fi/public_html/pdfpool/append.pdf`).

(a) The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{lllllll} R_{01} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , & R_{23} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , \\ R_{02} & = & e^0\wedge e^2 - e^3\wedge e^1 \ , & R_{31} & = & -e^0\wedge e^2 + e^3\wedge e^1 \ , \\ R_{03} & = & 4e^0\wedge e^3 + 2e^1\wedge e^2 \ , & R_{12} & = & 2e^0\wedge e^3 + 4e^1\wedge e^2 \ . \end{array} \tag{6.3.6}$$

$R_{01} = R_{23}$ and $R_{03} = -R_{31}$ combine to form purely left handed classical W boson fields and Z^0 field corresponds to $Z^0 = 2R_{03}$.
Kähler form is given by

$$J \ = \ 2(e^0\wedge e^3 + e^1\wedge e^2) \ . \tag{6.3.7}$$

(b) The vanishing of classical weak fields is guaranteed by the conditions

$$\begin{array}{c} e^0\wedge e^1 - e^2\wedge e^3 = 0 \ , \\ e^0\wedge e^2 - e^3\wedge e^1 \ , \\ 4e^0\wedge e^3 + 2e^1\wedge e^2 \ . \end{array} \tag{6.3.8}$$

(c) There are many manners to satisfy these conditions. For instance, the condition $e^1 = a\times e^0$ and $e^2 = -a\times e^3$ with arbitrary a which can depend on position guarantees the vanishing of classical W fields. The CP_2 projection of the tangent space of the region carrying the spinor mode must be 2-D.
Also classical Z^0 vanishes if $a^2 = 2$ holds true. This guarantees that the couplings of induced gauge potential are purely vectorial. One can consider other alternaties. For instance, one could require that only classical Z^0 field or induced Kähler form is non-vanishing and deduce similar condition.

(d) The vanishing of the weak part of induced gauge field implies that the CP_2 projection of the region carrying spinor mode is 2-D. Therefore the condition that the modes of induced spinor field are restricted to 2-surfaces carrying no weak fields sheets guarantees well-definedness of em charge and vanishing of classical weak couplings. This condition does not imply string world sheets in the general case since the CP_2 projection of the space-time sheet can be 2-D.

How string world sheets could emerge?

(a) Additional consistency condition to neutrality of string world sheets is that Kähler-Dirac gamma matrices have no components orthogonal to the 2-surface in question. Hence various fermionic would flow along string world sheet.

(b) If the Kähler-Dirac gamma matrices at string world sheet are expressible in terms of two non-vanishing gamma matrices parallel to string world sheet and sheet and thus define an integrable distribution of tangent vectors, this is achieved. What is important that modified gamma matrices can indeed span lower than 4-D space and often do so as already described. Induced gamma matrices defined always 4-D space so that the restriction of the modes to string world sheets is not possible.

(c) String models suggest that string world sheets are minimal surfaces of space-time surface or of imbedding space but it might not be necessary to pose this condition separately.

In the proposed scenario string world sheets emerge rather than being postulated from beginning.

(a) The vanishing conditions for induced weak fields allow also 4-D spinor modes if they are true for entire spatime surface. This is true if the space-time surface has 2-D projection. One can expect that the space-time surface has foliation by string world sheets and the general solution of K-D equation is continuous superposition of the 2-D modes in this case and discrete one in the generic case.

(b) If the CP_2 projection of space-time surface is homologically non-trivial geodesic sphere S^2, the field equations reduce to those in $M^4 \times S^2$ since the second fundamental form for S^2 is vanishing. It is possible to have geodesic sphere for which induced gauge field has only em component?

(c) If the CP_2 projection is complex manifold as it is for string like objects, the vanishing of weak fields might be also achieved.

(d) Does the phase of cosmic strings assumed to dominate primordial cosmology correspond to this phase with no classical weak fields? During radiation dominated phase 4-D string like objects would transform to string world sheets.Kind of dimensional transmutation would occur.

Right-handed neutrino has exceptional role in K-D action.

(a) Electroweak gauge potentials do not couple to ν_R at all. Therefore the vanishing of W fields is un-necessary if the induced gamma matrices do not mix right handed neutrino with left-handed one. This is guaranteed if M^4 and CP_2 parts of Kähler-Dirac operator annihilate separately right-handed neutrino spinor mode. Also ν_R modes can be interpreted as continuous superpositions of 2-D modes and this allows to define overlap integrals for them and induced spinor fields needed to define WCW gamma matrices and super-generators.

(b) For covariantly constant right-handed neutrino mode defining a generator of supersymmetries is certainly a solution of K-D. Whether more general solutions of K-D exist remains to be checked out.

6.3.4 Connection with number theoretic vision?

The interesting potential connection of the Hamilton-Jacobi vision to the number theoretic vision about field equations has been already mentioned.

(a) The vision that associativity/co-associativity defines the dynamics of space-time surfaces boils down to $M^8 - H$ duality stating that space-time surfaces can be regarded as associative/co-associative surfaces either in M^8 or H [K77]. Associativity reduces to hyper-quaternionicity implying that that the tangent/normal space of space-time surface at each point contains preferred sub-space $M^2(x) \subset M^8$ and these sub-spaces forma an integrable distribution. An analogous condition is involved with the definition of Hamilton-Jacobi structure.

(b) The octonionic representation of the tangent space of M^8 and H effectively replaces $SO(7,1)$ as tangent space group with its octonionic analog obtained by the replacement of sigma matrices with their octonionic counterparts defined by anti-commutators of gamma matrices. By non-associativity the resulting algebra is not ordinary Lie-algebra and exponentiates to a non-associative analog of Lie group. The original wrong belief was that the reduction takes place to the group G_2 of octonionic automorphisms acting as a subgroup of $SO(7)$. One can ask whether the conditions on the charged part of energy momentum tensor could relate to the reduction of $SO(7,1)$

(c) What puts bells ringing is that the modified Dirac equation for the octonionic representation of gamma matrices allows the conservation of electromagnetic charge in the proposed sense. The reason is that the left handed sigma matrices (W charges are left-handed) in the octonionic representation of gamma matrices vanish identically! What remains are vectorial=right-handed em and Z^0 charge which becomes proportional to em charge since its left-handed part vanishes. All spinor modes have a well-defined em charge in the octonionic sense defined by replacing imbedding space spinor locally by its octonionic variant? Maybe this could explain why H spinor modes can have well-defined em charge contrary to the naive expectations.

(d) The non-associativity of the octonionic spinors is however a problem. Even non-commutativity poses problems - also at space-time level if one assumes quaternion-real analyticity for the spinor modes. Could one assume commutativity or co-commutativity for the induced spinor modes? This would mean restriction to associative or co-associative 2-surfaces and (hyper-)holomorphic depends on its (hyper-)complex coordinate. The outcome would be a localization to a hyper-commutative of commutative 2-surface, string world sheet or partonic 2-surface.

(e) These conditions could also be interpreted by saying that for the Kähler Dirac operator the octonionic induced spinors assumed to be commutative/co-commutative are equivalent with ordinary induced spinors. The well-definedness of em charge for ordinary spinors would correspond to commutativity/co-commutativity for octonionic spinors. Even the Dirac equations based on induced and modified gamma matrices could be equivalent since it is essentially holomorphy which matters.

To sum up, these considerations inspire to ask whether the associativity/co-associativity of the space-time surface is equivalent with the reduction of the field equations to stringy field equations stating that certain components of the induced metric in complex/Hamilton-Jacobi coordinates vanish in turn guaranteeing that field equations reduce to algebraic identifies following from the fact that energy momentum tensor and second fundamental form have no common components? Commutativity/co-commutativity would characterize fermionic dynamics and would have physical representation as possibility to have em charge eigenspinors. This should be the case if one requires that the two solution ansätze are equivalent.

6.4 Preferred extremals and solutions of the modified Dirac equation and super-conformal symmetries

The previous considerations concerning super-conformal symmetries and space-time SUSY have been based on general arguments. The new vision about preferred extremals and modified Dirac equation [K69] however leads to a detailed understanding of super-conformal symmetries at the level of field equations and is bound to modify the existing vision about super-conformal symmetries. One important discovery is that Einstein's equations imply the vanishing of terms proportional to Kähler current in field equations for preferred extremals and Equivalence Principle at the classical level could be realized automatically in all scales in contrast to the earlier belief. This obviously must have implications to the general vision about Super-Virasoro representations and one must be ready to modify the existing picture based on the assumption that quantum version of Equivalence Principle is realized in terms coset representations.

The very special role of right handed neutrino is also bound to have profound implications. A further important outcome is the identification of gauge potentials as duals of Kac-Moody currents at the boundaries of string world sheets: quantum gauge potentials are defined only where they are needed that is the curves defining the non-integrable phase factors. This gives also rise to the realization of the conjecture Yangian in terms of the Kac-Moody charges and commutators in accordance with the earlier conjecture.

6.4.1 Super-conformal symmetries

It is good to summarize first the basic ideas about Super-Virasoro representations. TGD allows two kinds of super-conformal symmetries.

(a) The first super-conformal symmetry is associated with $\delta M^4_\pm \times CP_2$ and corresponds to symplectic symmetries of $\delta M^4_\pm \times CP_2$. The reason for extension of conformal symmetries is metric 2-dimensionality of the light-like boundary $\delta M^4_\pm$ defining upper/lower boundary of causal diamond (CD). This super-conformal symmetry is something new and corresponds to replacing finite-dimensional Lie-group G for Kac-Moody symmetry with infinite-dimensional symplectic group. The light-like radial coordinate of $\delta M^4_\pm$ takes the role of the real part of complex coordinate z for ordinary conformal symmetry. Together with complex coordinate of S^2 it defines 3-D restriction of Hamilton-Jacobi variant of 4-D super-conformal symmetries. One can continue the conformal symmetries from light-cone boundary to CD by forming a slicing by parallel copies of $\delta M^4_\pm$. There are two possible slicings corresponding to the choices δM^4_+ and δM^4_- assignable to the upper and lower boundaries of CD. These two choices correspond to two arrows of geometric time for the basis of zero energy states in ZEO.

(b) Super-symplectic degrees of freedom determine the electroweak and color quantum numbers of elementary particles. Bosonic emergence implies that ground states assignable to partonic 2-surfaces correspond to partial waves in $\delta M^4_\pm$ and one obtains color partial waves in particular. These partial waves correspond to the solutions for the Dirac equation in imbedding space and the correlation between color and electroweak quantum numbers is not quite correct. Super-Kac-Moody generators give the compensating color for massless states obtained from tachyonic ground states guaranteeing that standard correlation is obtained. Super-symplectic degrees are therefore directly visible in particle spectrum. One can say that at the point-like limit the WCW spinors reduce to tensor products of imbedding space spinors assignable to the center of mass degrees of freedom for the partonic 2-surfaces defining wormhole throats.

I have proposed a physical interpretation of super-symplectic vibrational degrees of freedom in terms of degrees of freedom assignable to non-perturbative QCD. These degrees of freedom would be responsible for most of the baryon masses but their theoretical understanding is lacking in QCD framework.

(c) The second super-conformal symmetry is assigned light-like 3-surfaces and to the isometries and holonomies of the imbedding space and is analogous to the super-Kac-Moody symmetry of string models. Kac-Moody symmetries could be assigned to the light-like deformations of light-like 3-surfaces. Isometries give tensor factor $E^2 \times SU(3)$ and holonomies factor $SU(2)_L \times U(1)$. Altogether one has 5 tensor factors to super-conformal algebra. That the number is just five is essential for the success p-adic mass calculations [K31, K26].

The construction of solutions of the modified Dirac equation suggests strongly that the fermionic representation of the Super-Kac-Moody algebra can be assigned as conserved charges associated with the space-like braid strands at both the 3-D space-like ends of space-time surfaces and with the light-like (or space-like with a small deformation) associated with the light-like 3-surfaces. The extension to Yangian algebra involving higher multi-linears of super-Kac Moody generators is also highly suggestive. These charges would be non-local and assignable to several wormhole contacts simultaneously. The ends of braids would correspond points of partonic 2-surfaces defining a discretization of the partonic 2-surface having interpretation in terms of finite measurement resolution.

These symmetries would correspond to electroweak and strong gauge fields and to gravitation. The duals of the currents giving rise to Kac-Moody charges would define the counterparts of gauge potentials and the conserved Kac-Moody charges would define the counterparts of non-integrable phase factors in gauge theories. The higher Yangian charges would define generalization of non-integrable phase factors. This would suggest a rather direct connection with the twistorial program for calculating the scattering amplitudes implies also by zero energy ontology.

Quantization recipes have worked in the case of super-string models and one can ask whether the application of quantization to the coefficients of powers of complex coordinates or Hamilton-Jacobi coordinates could lead to the understanding of the 4-D variants of the conformal symmetries and give detailed information about the representations of the Kac-Moody algebra too.

6.4.2 What is the role of the right-handed neutrino?

A highly interesting aspect of Super-Kac-Moody symmetry is the special role of right handed neutrino.

(a) Only right handed neutrino allows besides the modes restricted to 2-D surfaces also the 4D modes de-localized to the entire space-time surface. The first ones are holomorphic functions of single coordinate and the latter ones holomorphic functions of two complex/Hamilton-Jacobi coordinates. Only ν_R has the full $D = 4$ counterpart of the conformal symmetry and the localization to 2-surfaces has interpretation as super-conformal symmetry breaking halving the number of super-conformal generators.

(b) This forces to ask for the meaning of super-partners. Are super-partners obtained by adding ν_R neutrino localized at partonic 2-surface or de-localized to entire space-time surface or its Euclidian or Minkowskian region accompanying particle identified as wormhole throat? Only the Euclidian option allows to assign right handed neutrino to a unique partonic 2-surface. For the Minkowskian regions the assignment is to many particle state defined by the partonic 2-surfaces associated with the 3-surface. Hence for spartners the 4-D right-handed neutrino must be associated with the 4-D Euclidian line of the generalized Feynman diagram.

(c) The orthogonality of the localized and de-localized right handed neutrino modes requires that 2-D modes correspond to higher color partial waves at the level of imbedding space. If color octet is in question, the 2-D right handed neutrino as the candidate for the generator of standard SUSY would combine with the left handed neutrino to form a massive neutrino. If 2-D massive neutrino acts as a generator of super-symmetries, it is is in the same role as badly broken super-symmeries generated by other 2-D modes of the induced spinor field (SUSY with rather large value of $\mathcal{N}$) and one can argue that the counterpart of standard SUSY cannot correspond to this kind of super-symmetries. The right-handed neutrinos de-localized inside the lines of generalized Feynman diagrams, could generate $\mathcal{N} = 2$ variant of the standard SUSY.

How particle and right handed neutrino are bound together?

Ordinary SUSY means that apart from kinematical spin factors sparticles and particles behave identically with respect to standard model interactions. These spin factors would allow to distinguish between particles and sparticles. But is this the case now?

(a) One can argue that 2-D particle and 4-D right-handed neutrino behave like independent entities, and because ν_R has no standard model couplings this entire structure behaves like a particle rather than sparticle with respect to standard model interactions: the kinematical spin dependent factors would be absent.

(b) The question is also about the internal structure of the sparticle. How the four-momentum is divided between the ν_R and and 2-D fermion. If ν_R carries a negligible portion of four-momentum, the four-momentum carried by the particle part of sparticle is same as that carried by particle for given four-momentum so that the distinctions are only kinematical for the ordinary view about sparticle and trivial for the view suggested by the 4-D character of ν_R.

Could sparticle character become manifest in the ordinary scattering of sparticle?

(a) If ν_R behaves as an independent unit not bound to the particle, it would continue in the original direction as particle scatters: sparticle would decay to particle and right-handed neutrino. If ν_R carries a non-negligible energy the scattering could be detected via a missing energy. If not, then the decay could be detected by the interactions revealing the presence of ν_R. ν_R can have only gravitational interactions. What these gravitational interactions are is not however quite clear since the proposed identification of gravitational gauge potentials is as duals of Kac-Moody currents analogous to gauge potentials located at the boundaries of string world sheets. Does this mean that 4-D right-handed neutrino has no quantal gravitational interactions? Does internal consistency require ν_R to have a vanishing gravitational and inertial masses and does this mean that this particle carries only spin?

(b) The cautious conclusion would be following: if de-localized ν_R and parton are uncorrelated particle and sparticle cannot be distinguished experimentally and one might perhaps understand the failure to detect standard SUSY at LHC. Note however that the 2-D fermionic oscillator algebra defines badly broken large $\mathcal{N}$ SUSY containing also massive (longitudinal momentum square is non-vanishing) neutrino modes as generators.

Taking a closer look on sparticles

It is good to take a closer look at the de-localized right handed neutrino modes.

(a) At imbedding space level that is in cm mass degrees of freedom they correspond to covariantly constant CP_2 spinors carrying light-like momentum which for causal diamond could be discretized. For non-vanishing momentum one can speak about helicity having opposite sign for ν_R and $\overline{\nu}_R$. For vanishing four-momentum the situation is delicate since only spin remains and Majorana like behavior is suggestive. Unless one has momentum continuum, this mode might be important and generate additional SUSY resembling standard $\mathcal{N}=1$ SUSY.

(b) At space-time level the solutions of modified Dirac equation are holomorphic or anti-holomorphic.

   i. For non-constant holomorphic modes these characteristics correlate naturally with fermion number and helicity of ν_R . One can assign creation/annihilation operator to these two kinds of modes and the sign of fermion number correlates with the sign of helicity.

   ii. The covariantly constant mode is naturally assignable to the covariantly constant neutrino spinor of imbedding space. To the two helicities one can assign also oscillator operators $\{a_\pm, a_\pm^\dagger\}$. The effective Majorana property is expressed in terms of non-orthogonality of ν_R and and $\overline{\nu}_R$ translated to the the non-vanishing of the anti-commutator $\{a_+^\dagger, a_-\} = \{a_-^\dagger, a_+\} = 1$. The reduction of the rank of the 4×4 matrix defined by anti-commutators to two expresses the fact that the number of degrees of freedom has halved. $a_+^\dagger = a_-$ realizes the conditions and implies that one has only $\mathcal{N}=1$ SUSY multiplet since the state containing both ν_R and $\overline{\nu}_R$ is same as that containing no right handed neutrinos.

   iii. One can wonder whether this SUSY is masked totally by the fact that sparticles with all possible conformal weights n for induced spinor field are possible and the branching ratio to $n=0$ channel is small. If momentum continuum is present, the zero momentum mode might be equivalent to nothing.

What can happen in spin degrees of freedom in super-symmetric interaction vertices if one accepts this interpretation? As already noticed, this depends solely on what one assumes about the correlation of the four-momenta of particle and ν_R.

(a) For SUSY generated by covariantly constant ν_R and $\overline{\nu}_R$ there is no neutrino four-momentum involved so that only spin matters. One cannot speak about the change of direction for ν_R. In the scattering of sparticle the direction of particle changes and introduces different spin quantization axes. ν_R retains its spin and in new system it is superposition of two spin projections. The presence of both helicities requires that the transformation $\nu_R \rightarrow \overline{\nu}_R$ happens with an amplitude determined purely kinematically by spin rotation matrices. This is consistent with fermion number conservation modulo 2. $\mathcal{N}=1$ SUSY based on Majorana spinors is highly suggestive.

(b) For SUSY generated by non-constant holomorphic and anti-holomorphic modes carrying fermion number the behavior in the scattering is different. Suppose that the sparticle does not split to particle moving in the new direction and ν_R moving in the original direction so that also ν_R or $\overline{\nu}_R$ carrying some massless fraction of four-momentum changes its direction of motion. One can form the spin projections with respect to the new spin axis but must drop the projection which does not conserve fermion number. Therefore the kinematics at the vertices is different. Hence $\mathcal{N}=2$ SUSY with fermion number conservation is suggestive when the momentum directions of particle and ν_R are completely correlated.

(c) Since right-handed neutrino has no standard model couplings, p-adic thermodynamics for 4-D right-handed neutrino must correspond to a very low p-adic temperature $T=1/n$. This implies that the excitations with non-vanishing conformal weights are effectively absent and one would have $\mathcal{N}=1$ SUSY effectively.

The simplest assumption is that particle and sparticle correspond to the same p-adic mass scale and have degenerate masses: it is difficult to imagine any good reason for why the p-adic mass scales should differ. This should have been observed -say in decay widths of weak bosons - unless the spartners correspond to large $\hbar$ phase and therefore to dark matter. Note that for the badly broken 2-D N=2 SUSY in fermionic sector this kind of almost degeneracy cannot be excluded and I have considered an explanation for the mysterious X and Y mesons in terms of this degeneracy [K29].

Why space-time SUSY is not possible in TGD framework?

LHC suggests that one does not have $\mathcal{N}=1$ SUSY in standard sense. Why one cannot have standard space-time SUSY in TGD framework. Let us begin by listing all arguments popping in mind.

(a) Could covariantly constant ν_R represents a gauge degree of freedom? This is plausible since the corresponding fermion current is non-vanishing.

(b) The original argument for absence of space-time SUSY years ago was indirect: $M^4 \times CP_2$ does not allow Majorana spinors so that $\mathcal{N}=1$ SUSY is excluded.

(c) One can however consider $\mathcal{N}=2$ SUSY by including both helicities possible for covariantly constant ν_R. For ν_R the four-momentum vanishes so that one cannot distinguish the modes assigned to the creation operator and its conjugate via complex conjugation of the spinor. Rather, one oscillator operator and its conjugate correspond to the two different helicities of right-handed neutrino with respect to the direction determined by the momentum of the particle. The spinors can be chosen to be real in this basis. This indeed gives rise to an irreducible representation of spin 1/2 SUSY algebra with right-handed neutrino creation operator acting as a ladder operator. This is however $\mathcal{N}=1$ algebra and right-handed neutrino in this particular basis behaves effectively like Majorana spinor. One can argue that the system is mathematically inconsistent. By choosing the spin projection axis differently the spinor basis becomes complex. In the new basis one would have $\mathcal{N}=2$, which however reduces to $\mathcal{N}=1$ in the real basis.

(d) Or could it be that fermion and sfermion do exist but cannot be related by SUSY? In standard SUSY fermions and sfermions forming irreducible representations of super Poincare algebra are combined to components of superfield very much like finite-dimensional representations of Lorentz group are combined to those of Poincare. In TGD framework ν_R generates in space-time interior generalization of 2-D super-conformal symmetry but covarianlty constant ν_R cannot give rise to space-time SUSY.

This would be very natural since right-handed neutrinos do not have any electroweak interactions and are are de-localized into the interior of the space-time surface unlike other particles localized at 2-surfaces. It is difficult to imagine how fermion and ν_R could behave as a single coherent unit reflecting itself in the characteristic spin and momentum dependence of vertices implied by SUSY. Rather, it would seem that fermion and sfermion should behave identically with respect to electroweak interactions.

The third argument looks rather convincing and can be developed to a precise argument.

(a) If sfermion is to represent elementary bosons, the products of fermionic oscillator operators with the oscillator operators assignable to the covariantly constant right handed neutrinos must define might-be bosonic oscillator operators as $b_n = a_n a$ and $b_n^\dagger = a_n^\dagger a^\dagger$ One can calculate the commutator for the product of operators. If fermionic oscillator operators commute, so do the corresponding bosonic operators. The commutator $[b_n, b_n^\dagger]$ is however proportional to occupation number for ν_R in $\mathcal{N} = 1$ SUSY representation and vanishes for the second state of the representation. Therefore $\mathcal{N} = 1$ SUSY is a pure gauge symmetry.

(b) One can however have both irreducible representations of SUSY: for them either fermion or sfermion has a non-vanishing norm. One would have both fermions and sfermions but they would not belong to the same SUSY multiplet, and one cannot expect SUSY symmetries of 3-particle vertices.

(c) For instance, γFF vertex is closely related to $\gamma\tilde{F}\tilde{\overline{F}}$ in standard SUSY. Now one expects this vertex to decompose to a product of $\gamma F\overline{F}$ vertex and amplitude for the creation of $\nu_R\tilde{\nu}_R$ from vacuum so that the characteristic momentum and spin dependent factors distinguishing between the couplings of photon to scalar and and fermion are absent. Both states behave like fermions. The amplitude for the creation of $\nu_R\tilde{\nu}_R$ from vacuum is naturally equal to unity as an occupation number operator by crossing symmetry. The presence of right-handed neutrinos would be invisible if this picture is correct. Whether this invisible label can have some consequences is not quite clear: one could argue that the decay rates of weak bosons to fermion pairs are doubled unless one introduces $1/\sqrt{2}$ factors to couplings.

Where the sfermions might make themselves visible are loops. What loops are? Consider boson line first. Boson line is replaced with a sum of two contributions corresponding to ordinary contribution with fermion and anti-fermion at opposite throats and second contribution with fermion and anti-fermion accompanied by right-handed neutrino ν_R and its antiparticle which now has opposite helicity to ν_R. The loop for ν_R decomposes to four pieces since also the propagation from wormhole throat to the opposite wormhole throat must be taken into account. Each of the four propagators equals to $a^\dagger_{1/2}a^\dagger_{-1/2}$ or its hermitian conjugate. The product of these is slashed between vacuum states and anti-commutations give imaginary unit per propagator giving $i^4 = 1$. The two contributions are therefore identical and the scaling $g \to g/\sqrt{2}$ for coupling constants guarantees that sfermions do not affect the scattering amplitudes at all. The argument is identical for the internal fermion lines.

6.4.3 WCW geometry and super-conformal symmetries

The vision about the geometry of WCW has been roughly the following and the recent steps of progress induce to it only small modifications if any.

(a) Kähler geometry is forced by the condition that hermitian conjugation allows geometrization. Kähler function is given by the Kähler action coming from space-time regions with Euclidian signature of the induced metric identifiable as lines of generalized Feynman diagrams. Minkowskian regions give imaginary contribution identifiable as the analog of Morse function and implying interference effects and stationary phase approximation. The vision about quantum TGD as almost topological QFT inspires the proposal that Kähler action reduces to 3-D terms reducing to Chern-Simons terms by the weak form of electric-magnetic duality. The recent proposal for preferred extremals is consistent with this property realizing also holography implied by general coordinate invariance.

Strong form of general coordinate invariance implying effective 2-dimensionality in turn suggests that Kähler action is expressible in terms of areas of partonic 2-surfaces and string world sheets.

(b) The complexified gamma matrices of WCW come as hermitian conjugate pairs and anti-commute to the Kähler metric of WCW. Also bosonic generators of symplectic transformations of $\delta M^4_\pm \times CP_2$ a assumed to act as isometries of WCW geometry can be complexified and appear as similar pairs. The action of isometry generators coincides with that of symplectic generators at partonic 2-surfaces and string world sheets but elsewhere inside the space-time surface it is expected to be deformed from the symplectic action. The super-conformal transformations of $\delta M^4_\pm \times CP_2$ acting on the light-like radial coordinate of $\delta M^4_\pm$ act as gauge symmetries of the geometry meaning that the corresponding WCW vector fields have zero norm.

(c) WCW geometry has also zero modes which by definition do not contribute to WCW metric expect possibly by the dependence of the elements of WCW metric on zero modes through a conformal factor. In particular, induced CP_2 Kähler form and its analog for sphere $r_M = constant$ of light cone boundary are symplectic invariants, and one can define an infinite number of zero modes as invariants defined by Kähler fluxes over partonic 2-surfaces and string world sheets. This requires however the slicing of CD parallel copies of δM^4_+ or δM^4_-. The physical interpretation of these non-quantum fluctuating degrees of freedom is as classical variables necessary for the interpretation of quantum measurement theory. Classical variable would metaphorically correspond the position of the pointer of the measurement instrument.

(d) The construction receives a strong philosophical inspiration from the geometry of loop spaces. Loop spaces allow a unique Kähler geometry with maximal isometry group identifiable as Kac-Moody group. The reason is that otherwise Riemann connection does not exist. The only problem is that curvature scalar diverges since the Riemann tensor is by constant curvature property proportional to the metric. In 3-D case one would have union of constant curvature spaces labelled by zero modes and the situation is expected to be even more restrictive. The conjecture indeed is that WCW geometry exists only for $H = M^4 \times CP_2$: infinite-D Kähler geometric existence and therefore physics would be unique. One can also hope that Ricci scalar is finite and therefore zero by the constant curvature property so that Einstein's equations are satisfied.

(e) WCW Hamiltonians determined the isometry currents and WCW metric is given in terms of the anti-commutators of the Killing vector fields associated with symplectic isometry currents. The WCW Hamiltonians generating symplectic isometries correspond to the Hamiltonians spanning the symplectic group of $\delta M^4_\pm \times CP_2$. One can say that the space of quantum fluctuating degrees of freedom is this symplectic group of $\delta M^4_\pm \times CP_2$ or its subgroup or coset space: this must have very deep implications for the structure of the quantum TGD.

(f) Zero energy ontology brings in additional delicacies. Basic objects are now unions of partonic 2-surfaces at the ends of CD. Also string world sheets would naturally contribute. One can generalize the expressions for the isometry generators in a straightforward manner by requiring that given isometry restricts to a symplectic transformation at partonic 2-surfaces and string world sheets.

(g) One could criticize the effective metric 2-dimensionality forced by general consistency arguments as something non-physical. The Hamiltonians are expressed using only the data at partonic 2-surfaces: this includes also 4-D tangent space data via the weak form of electric-magnetic duality so that one has only effective 2-dimensionality. Obviously WCW geometry must have large gauge symmetries besides zero modes. The super-conformal symmetries indeed represent gauge symmetries of this kind. Effective 2-dimensionality realizing strong form of holography in turn is induced by the strong form of general coordinate invariance. Light-like 3-surfaces at which the signature of the induced metric changes must be equivalent with the 3-D space-like ends of space-time surfaces at the light-boundaries of space-time surfaces as far as WCW geometry is considered. This requires that the data from their 2-D intersections defining partonic

2-surfaces should dictate the WCW geometry. Note however that Super-Kac-Moody charges giving information about the interiors of 3-surfaces appear in the construction of the physical states.

What is the role of the right handed neutrino in this construction?

(a) In the construction of components of WCW metric as anti-commutators of super-generators only the covariantly constant right-handed neutrino appears in the super-generators analogous to super-Kac-Moody generators. All holomorphic modes of right handed neutrino characterized by two integers could in principle contribute to the WCW gamma matrices identified as fermionic super-symplectic generators anti-commuting to the metric. At the space-like ends of space-time surface the holomorphic generators would restrict to symplectic generators since the radial light-like coordinate r_M identified and complex coordinate of CP_2 allowing identification as restrictions of two complex coordinates or Hamilton-Jacobi coordinates to light-like boundary.

(b) The non-covariantly constant modes could also correspond to purely super-conformal gauge degrees of freedom. Originally the restriction to right-handed neutrino looked somewhat un-satisfactory but the recent view about Super-Kac-Moody symmetries makes its special role rather natural. One could say that WCW geometry possesses the maximal $D = 4$ supersymmetry.

(c) One can of course ask whether the Super-Kac-Moody generators assignable to the isometries of H and expressible as conserved charges associated with the boundaries of string world sheets could contribute to the WCW geometry via the anti-commutators. This option cannot be excluded but in this case the interpretation in terms of Hamiltonians is not obvious.

6.4.4 The relationship between inertial gravitational masses

The relationship between inertial and gravitational masses and Equivalence Principle have been on of the longstanding problems in TGD. Not surprisingly, the realization how GRT space-time relates to the many-sheeted space-time of TGD finally allowed to solve the problem.

ZEO and non-conservation of Poincare charges in Poincare invariant theory of gravitation

In positive energy ontology the Poincare invarance of TGD is in sharpt contrast with the fact that GRT based cosmology predicts non-conservation of Poincare charges (as a matter fact, the definition of Poincare charges is very questionable for general solutions of field equations).

In zero energy ontology (ZEO) all conserved (that is Noether-) charges of the Universe vanish identically and their densities should vanish in scales below the scale defining the scale for observations and assignable to causal diamond (CD). This observation allows to imagine a ways out of what seems to be a conflict of Poincare invariance with cosmological facts.

ZEO would explain the local non-conservation of average energies and other conserved quantum numbers in terms of the contributions of sub-CDs analogous to quantum fluctuations. Classical gravitation should have a thermodynamical description if this interpretation is correct. The average values of the quantum numbers assignable to a space-time sheet would depend on the size of CD and possibly also its location in M^4. If the temporal distance between the tips of CD is interpreted as a quantized variant of cosmic time, the non-conservation of energy-momentum defined in this manner follows. One can say that conservation laws hold only true in given scale defined by the largest CD involved.

Equivalence Principle at quantum level

The interpretation of EP at quantum level has developed slowly and the recent view is that it reduces to quantum classical correspondence meaning that the classical charges of Kähler action can be identified with eigen values of quantal charges associated with Kähler-Dirac action.

(a) At quantum level I have proposed coset representations for the pair of super-symplectic algebras assignable to the light-like boundaries of CD and the Super Kac-Moody algebra assignable to the light-like 3-surfaces defining the orbits of partonic 2-surfaces as realization of EP. For coset representation the differences of super-conformal generators would annihilate the physical states so that one can argue that the corresponding four-momenta are identical. One could even say that one obtains coset representation for the "vibrational" parts of the super-conformal algebras in question. It is now clear that this idea does not work. Note however that coset representations occur naturally for the subalgebras of symplectic algebra and Super Kac-Moody algebra and are naturally induced by finite measurement resolution.

(b) The most recent view (2014) about understanding how EP emerges in TGD is described in [K56] and relies heavily on superconformal invariance and a detailed realisation of ZEO at quantum level. In this approach EP corresponds to quantum classical correspondence (QCC): four-momentum identified as classical conserved Noether charge for space-time sheets associated with Käbler action is identical with quantal four-momentum assignable to the representations of super-symplectic and super Kac-Moody algebras as in string models and having a realisation in ZEO in terms of wave functions in the space of causal diamonds (CDs).

(c) The latest realization is that the eigenvalues of quantal four-momentum can be identified as eigenvalues of the four-momentum operator assignable to the modified Dirac equation. This realisation seems to be consistent with the p-adic mass calculations requiring that the super-conformal algebra acts in the tensor product of 5 tensor factors.

Equivalence Principle at classical level

How Einstein's equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD.

The first proposal making sense even when one does not assume ZEO is that vacuum extremals are only approximate representations of the physical situation and that small fluctuations around them give rise to an inertial four-momentum identifiable as gravitational four-momentum identifiable in terms of Einstein tensor. EP would hold true in the sense that the average gravitational four-momentum would be determined by the Einstein tensor assignable to the vacuum extremal. This interpretation does not however take into account the many-sheeted character of TGD spacetime and is therefore questionable.

The resolution of the problem came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see fig. `http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg` or fig. 11 in the appendix of this book).

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

(c) Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

(d) The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K78].

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

6.4.5 Constraints from p-adic mass calculations and ZEO

A further important physical input comes from p-adic thermodynamics forming a core element of p-adic mass calculations.

(a) The first thing that one can get worried about relates to the extension of conformal symmetries. If the conformal symmetries generalize to $D = 4$, how can one take seriously the results of p-adic mass calculations based on 2-D conformal invariance? There is no reason to worry. The reduction of the conformal invariance to 2-D one for the preferred extremals takes care of this problem. This however requires that the fermionic contributions assignable to string world sheets and/or partonic 2-surfaces - Super- Kac-Moody contributions - should dictate the elementary particle masses. For hadrons also symplectic contributions should be present. This is a valuable hint in attempts to identify the mathematical structure in more detail.

(b) ZEO suggests that all particles, even virtual ones correspond to massless wormhole throats carrying fermions. As a consequence, twistor approach would work and the kinematical constraints to vertices would allow the cancellation of divergences. This would suggest that the p-adic thermal expectation value is for the longitudinal M^2 momentum squared (the definition of CD selects $M^1 \subset M^2 \subset M^4$ as also does number theoretic vision). Also propagator would be determined by M^2 momentum. Lorentz invariance would be obtained by integration of the moduli for CD including also Lorentz boosts of CD.

(c) In the original approach one allows states with arbitrary large values of L_0 as physical states. Usually one would require that L_0 annihilates the states. In the calculations however mass squared was assumed to be proportional L_0 apart from vacuum contribution. This is a questionable assumption. ZEO suggests that total mass squared vanishes and that one can decompose mass squared to a sum of longitudinal and transversal parts. If one can do the same decomposition to longitudinal and transverse parts also for the Super Virasoro algebra then one can calculate longitudinal mass squared as a p-adic thermal expectation in the transversal super-Virasoro algebra and only states with $L_0 = 0$ would contribute and one would have conformal invariance in the standard sense.

(d) In the original approach the assumption motivated by Lorentz invariance has been that mass squared is replaced with conformal weight in thermodynamics, and that one first calculates the thermal average of the conformal weight and then equates it with mass squared. This assumption is somewhat ad hoc. ZEO however suggests an alternative interpretation in which one has zero energy states for which longitudinal mass squared of positive energy state derive from p-adic thermodynamics. Thermodynamics - or rather, its square root - would become part of quantum theory in ZEO. M-matrix is indeed product of hermitian square root of density matrix multiplied by unitary S-matrix and defines the entanglement coefficients between positive and negative energy parts of zero energy state.

(e) The crucial constraint is that the number of super-conformal tensor factors is $N = 5$: this suggests that thermodynamics applied in Super-Kac-Moody degrees of freedom assignable to string world sheets is enough, when one is interested in the masses of fermions and gauge bosons. Super-symplectic degrees of freedom can also contribute and determine the dominant contribution to baryon masses. Should also this contribution obey p-adic thermodynamics in the case when it is present? Or does the very fact that this contribution need not be present mean that it is not thermal? The symplectic contribution should correspond to hadronic p-adic length prime rather the one assignable to (say) u quark. Hadronic p-adic mass squared and partonic p-adic mass squared cannot be summed since primes are different. If one accepts the basic rules [K32], longitudinal energy and momentum are additive as indeed assumed in perturbative QCD.

(f) Calculations work if the vacuum expectation value of the mass squared must be assumed to be tachyonic. There are two options depending on whether one whether p-adic thermodynamics gives total mass squared or longitudinal mass squared.

   i. One could argue that the total mass squared has naturally tachyonic ground state expectation since for massless extremals longitudinal momentum is light-like and transversal momentum squared is necessary present and non-vanishing by the localization to topological light ray of finite thickness of order p-adic length scale. Transversal degrees of freedom would be modeled with a particle in a box.

   ii. If longitudinal mass squared is what is calculated, the condition would require that transversal momentum squared is negative so that instead of plane wave like behavior exponential damping would be required. This would conform with the localization in transversal degrees of freedom.

6.4.6 The emergence of Yangian symmetry and gauge potentials as duals of Kac-Moody currents

Yangian symmetry plays a key role in $\mathcal{N} = 4$ super-symmetric gauge theories. What is special in Yangian symmetry is that the algebra contains also multi-local generators. In TGD framework multi-locality would naturally correspond to that with respect to partonic 2-surfaces and string world sheets and the proposal has been that the Super-Kac-Moody algebras assignable to string worlds sheets could generalize to Yangian.

Witten has written a beautiful exposition of Yangian algebras [B18]. Yangian is generated by two kinds of generators J^A and Q^A by a repeated formation of commutators. The number of commutations tells the integer characterizing the multi-locality and provides the Yangian algebra with grading by natural numbers. Witten describes a 2-dimensional QFT like situation in which one has 2-D situation and Kac-Moody currents assignable to real axis define the Kac-Moody charges as integrals in the usual manner. It is also assumed that the gauge potentials defined by the 1-form associated with the Kac-Moody current define a flat connection:

$$\partial_\mu j^A_\nu - \partial_\nu j^A_\nu + [j^A_\mu, j^A_\nu] = 0 \ . \tag{6.4.1}$$

This condition guarantees that the generators of Yangian are conserved charges. One can however consider alternative manners to obtain the conservation.

(a) The generators of first kind - call them J^A - are just the conserved Kac-Moody charges. The formula is given by

$$J_A = \int_{-\infty}^{\infty} dx j^{A0}(x,t) \ . \tag{6.4.2}$$

(b) The generators of second kind contain bi-local part. They are convolutions of generators of first kind associated with different points of string described as real axis. In the basic formula one has integration over the point of real axis.

$$Q^A = f^A_{BC} \int_{-\infty}^{\infty} dx \int_{x}^{\infty} dy j^{B0}(x,t) j^{C0}(y,t) - 2\int_{-\infty}^{\infty} j^A_x dx \ . \tag{6.4.3}$$

These charges are indeed conserved if the curvature form is vanishing as a little calculation shows.

How to generalize this to the recent context?

(a) The Kac-Moody charges would be associated with the braid strands connecting two partonic 2-surfaces - Strands would be located either at the space-like 3-surfaces at the ends of the space-time surface or at light-like 3-surfaces connecting the ends. Modified Dirac equation would define Super-Kac-Moody charges as standard Noether charges. Super charges would be obtained by replacing the second quantized spinor field or its conjugate in the fermionic bilinear by particular mode of the spinor field. By replacing both spinor field and its conjugate by its mode one would obtain a conserved c-number charge corresponding to an anti-commutator of two fermionic super-charges. The convolution involving double integral is however not number theoretically attractive whereas single 1-D integrals might make sense.

(b) An encouraging observation is that the Hodge dual of the Kac-Moody current defines the analog of gauge potential and exponents of the conserved Kac-Moody charges could be identified as analogs for the non-integrable phase factors for the components of this gauge potential. This identification is precise only in the approximation that generators commute since only in this case the ordered integral $P(exp(i\int Adx))$ reduces to $P(exp(i\int Adx))$.Partonic 2-surfaces connected by braid strand would be analogous to nearby points of space-time in its discretization implying that Abelian approximation works. This conforms with the vision about finite measurement resolution as discretization in terms partonic 2-surfaces and braids.

   This would make possible a direct identification of Kac-Moody symmetries in terms of gauge symmetries. For isometries one would obtain color gauge potentials and the analogs of gauge potentials for graviton field (in TGD framework the contraction with M^4 vierbein would transform tensor field to 4 vector fields). For Kac-Moody generators corresponding to holonomies one would obtain electroweak gauge potentials. Note that super-charges would give rise to a collection of spartners of gauge potentials automatically. One would obtain a badly broken SUSY with very large value of $\mathcal{N}$ defined by the number of spinor modes as indeed speculated earlier [K19].

(c) The condition that the gauge field defined by 1-forms associated with the Kac-Moody currents are trivial looks unphysical since it would give rise to the analog of topological QFT with gauge potentials defined by the Kac-Moody charges. For the duals of Kac-Moody currents defining gauge potentials only covariant divergence vanishes implying that curvature form is

$$F_{\alpha\beta} = \epsilon_{\alpha\beta}[j_\mu, j^\mu] \ , \tag{6.4.4}$$

so that the situation does not reduce to topological QFT unless the induced metric is diagonal. This is not the case in general for string world sheets.

(d) It seems however that there is no need to assume that j_μ defines a flat connection. Witten mentions that although the discretization in the definition of J^A does not seem to be possible, it makes sense for Q^A in the case of $G = SU(N)$ for any representation of G. For general G and its general representation there exists no satisfactory definition of Q. For certain representations, such as the fundamental representation of $SU(N)$, the definition of Q^A is especially simple. One just takes the bi-local part of the previous formula:

$$Q^A = f^A_{BC} \sum_{i<j} J^B_i J^C_j \ . \tag{6.4.5}$$

What is remarkable that in this formula the summation need not refer to a discretized point of braid but to braid strands ordered by the label i by requiring that they form a connected polygon. Therefore the definition of J^A could be just as above.

(e) This brings strongly in mind the interpretation in terms of twistor diagrams. Yangian would be identified as the algebra generated by the logarithms of non-integrable phase factors in Abelian approximation assigned with pairs of partonic 2-surfaces defined in terms of Kac-Moody currents assigned with the modified Dirac action. Partonic 2-surfaces connected by braid strand would be analogous to nearby points of space-time in its discretization. This would fit nicely with the vision about finite measurement resolution as discretization in terms partonic 2-surfaces and braids.

The resulting algebra satisfies the basic commutation relations

$$\left[J^A, J^B\right] = f^{AB}_C J^C \ , \ \left[J^A, Q^B\right] = f^{AB}_C Q^C \ . \tag{6.4.6}$$

plus the rather complex Serre relations described in [B18].

6.4.7 Quantum criticality and electroweak symmetries

In the following quantum criticali and electroweak symmetries are discussed for Kähler-Dirac action.

What does one mean with quantum criticality?

Quantum criticality is one of the basic guiding principles of Quantum TGD. What it means mathematically is however far from clear and one can imagine several meanings for it.

(a) What is obvious is that quantum criticality implies quantization of Kähler coupling strength as a mathematical analog of critical temperature so that the theory becomes mathematically unique if only single critical temperature is possible. Physically this means the presence of long range fluctuations characteristic for criticality and perhaps assignable to the effective hierarchy of Planck constants having explanation in terms of effective covering spaces of the imbedding space. This hierarchy follows from the vacuum degeneracy of Kähler action, which in turn implies 4-D spin-glass degeneracy. It is easy to interpret the degeneracy in terms of criticality.

(b) At more technical level one would expect criticality to corresponds to deformations of a given preferred extremal defining a vanishing second variation of Kähler Khler function or Kähler action.

i. For Kähler function this criticality is analogous to thermodynamical criticality. The Hessian matrix defined by the second derivatives of free energy or potential function becomes degenerate at criticality as function of control variables which now would be naturally zero modes not contribution to Kähler metric of WCW but appearing as parameters in it. The bevavior variables correspond to quantum fluctuating degrees of freedom and according to catastrophe theory a big change can in quantum fluctuating degrees of freedom at criticality for zero modes. This would be control of quantum state by varying classical variables. Cusp catastrophe is standard example of this. One can imagined also a situation in which the roles of zero modes and behavior variables change and big jump in the values of zero modes is induced by small variation in behavior variables. This would mean quantum control of classical variables.

ii. Zero modes controlling quantum fluctuating variables in Kähler function would correspond to vanishing of also second derivatives of potential function at extremum in certain directions so that the matrix defined by second derivatives does not have maximum rank. Entire hierarchy of criticalities is expected and a good finite-dimensional model is provided by the catastrophe theory of Thom [?]. Cusp catastrophe [A2] is the simplest catastrophe one can think of, and here the folds of cusp where discontinuous jump occurs correspond to criticality with respect to one control variable and the tip to criticality with respect to both control variables.

(c) Quantum criticality makes sense also for Kähler action.

i. Now one considers space-time surface connecting which 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K17] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

ii. Also now one expects a hierarchy of criticalities and and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

iii. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

(d) I have discussed what criticality could mean for modified Dirac action [K18] .

i. I have conjectured that it leads to the existence of additional conserved currents defined by the variations which do not affect the value of Kähler action. These arguments are far from being mathematically rigorous and the recent view about the solutions of the modified Dirac equation predicting that the spinor modes are restricted to 2-D string world sheets requires a modification of these arguments.

ii. The basic challenge is to understand the mechanism making this kind of currents conserved: the same challenge is met already in the case of isometries since imbedding space coordinates appear as parameters in modified Dirac action. The existence of conserved currents does not actually require the vanishing of the second variation of Kähler action as claimed earlier. It is enough that the first variation of the canonical momentum densities contracted with the imbedding space gamma matrices annihilates the spinor mode. Situation is analogous to massless Dirac equation:it does not imply the vanishing of four-momentum, only the vanishing of mass. Hence conserved currents are obtained also outside the quantum criticality.

iii. It is far from obvious that these conditions can be satisfied. The localization of the spinor modes to string world sheets or partonic 2-surfaces guaranteeing in the generaic case that em charge is well-defined for spinor modes implies holomorphy allowing to formulate current conservation for currents associated with the deformations of the space-time surface for second quantized induced spinor field. The crux is that the deformation respects the holomorphy properties of the modified gamma matrices at string world sheet and thus does not mix Γ^z with $\Gamma^{\overline{z}}$. The deformation of Γ^z has only z-component and also annihilates the holomorphic spinor. This mechanism is possible only for Kähler-Dirac action since the Kähler-Dirac gamma matrices in directions orthogonal to the 2-surface must vanish and this is not possible for other actions. This also means that energy momentum tensor has rank 2 as matrix. Cosmic string solutions are an exception since in this case CP_2 projection of space-time surface is 2-D and conditions guaranteing vanishing of classical W fields can be satisfied.

In the following these arguments are formulated more precisely. The unexpected result is that critical deformations induce conformal scalings of the modified metric and electro-weak gauge transformations of the induced spinor connection at X^2. Therefore holomorphy brings in the Kac-Moody symmetries associated with isometries of H (gravitation and color gauge group) and quantum criticality those associated with the holonomies of H (electro-weak-gauge group) as additional symmetries.

The variation of modes of the induced spinor field in a variation of space-time surface respecting the preferred extremal property

Consider first the variation of the induced spinor field in a variation of space-time surface respecting the preferred extremal property. The deformation must be such that the deformed modified Dirac operator D annihilates the modified mode. By writing explicitly the variation of the modified Dirac action (the action vanishes by modified Dirac equation) one obtains deformations and requiring its vanishing one obtains

$$\delta\Psi = D^{-1}(\delta D)\Psi \ . \tag{6.4.7}$$

D^{-1} is the inverse of the modified Dirac operator defining the analog of Dirac propagator and δD defines vertex completely analogous to $\gamma^k\delta A_k$ in gauge theory context. The functional integral over preferred extremals can be carried out perturbatively by expressing δD in terms of δh^k and one obtains stringy perturbation theory around X^2 associated with the preferred extremal defining maximum of Kähler function in Euclidian region and extremum of Kähler action in Minkowskian region (stationary phase approximation).

What one obtains is stringy perturbation theory for calculating n-points functions for fermions at the ends of braid strands located at partonic 2-surfaces and representing intersections of string world sheets and partonic 2-surfaces at the light-like boundaries of CDs. δD- or more precisely, its partial derivatives with respect to functional integration variables - appear atthe vertices located anywhere in the interior of X^2 with outgoing fermions at braid ends. Bosonic propagators are replaced with correlation functions for δh^k. Fermionic propagator is defined by D^{-1}.

After 35 years or hard work this provides for the first time a reasonably explicit formula for the N-point functions of fermions. This is enough since by bosonic emergence [K36] these N-point functions define the basic building blocks of the scattering amplitudes. Note that bosonic emergence states that bosons corresponds to wormhole contacts with fermion and anti-fermion at the opposite wormhole throats.

What critical modes could mean for the induced spinor fields?

What critical modes could mean for the induced spinor fields at string world sheets and partonic 2-surfaces. The problematic part seems to be the variation of the modified Dirac operator since it involves gradient. One cannot require that covariant derivative remains invariant since this would require that the components of the induced spinor connection remain invariant and this is quite too restrictive condition. Right handed neutrino solutions de-localized into entire X^2 are however an exception since they have no electro-weak gauge couplings and in this case the condition is obvious: modified gamma matrices suffer a local scaling for critical deformations:

$$\delta\Gamma^{\mu} = \Lambda(x)\Gamma^{\mu} . \tag{6.4.8}$$

This guarantees that the modified Dirac operator D is mapped to ΛD and still annihilates the modes of ν_R labelled by conformal weight, which thus remain unchanged.

What is the situation for the 2-D modes located at string world sheets? The condition is obvious. Ψ suffers an electro-weak gauge transformation as does also the induced spinor connection so that D_μ is not affected at all. Criticality condition states that the deformation of the space-time surfaces induces a conformal scaling of Γ^μ at X^2. It might be possible to continue this conformal scaling of the entire space-time sheet but this might be not necessary and this would mean that all critical deformations induced conformal transformations of the effective metric of the space-time surface defined by $\{\Gamma^\mu, \Gamma^\nu\} = 2G^{\mu\nu}$. Thus it seems that effective metric is indeed central concept (recall that if the conjectured quaternionic structure is associated with the effective metric, it might be possible to avoid problem related to the Minkowskian signature in an elegant manner).

In fact, one can consider even more general action of critical deformation: the modes of the induced spinor field would be mixed together in the infinitesimal deformation besides infinitesimal electroweak gauge transformation, which is same for all modes. This would extend electroweak gauge symmetry. Modified Dirac equation holds true also for these deformations. One might wonder whether the conjectured dynamically generated gauge symmetries assignable to finite measurement resolution could be generated in this manner.

The infinitesimal generator of a critical deformation J_M can be expressed as tensor product of matrix A_M acting in the space of zero modes and of a generator of infinitesimal electro-weak gauge transformation $T_M(x)$ acting in the same manner on all modes: $J_M = A_M \otimes T_M(x)$. A_M is a spatially constant matrix and $T_M(x)$ decomposes to a direct sum of left- and right-handed $SU(2) \times U(1)$ Lie-algebra generators. Left-handed Lie-algebra generator can be regarded as a quaternion and right handed as a complex number. One can speak of a direct sum of left-handed local quaternion $q_{M,L}$ and right-handed local complex number $c_{M,R}$. The commutator $[J_M, J_N]$ is given by $[J_M, J_N] = [A_M, A_N] \otimes \{T_M(x), T_N(x)\} + \{A_M, A_N\} \otimes [T_M(x), T_N(x)]$. One has $\{T_M(x), T_N(x)\} = \{q_{M,L}(x), q_{N,L}(x)\} \oplus \{c_{M,R}(x), c_{N,R}(x)\}$ and $[T_M(x), T_N(x)] = [q_{M,L}(x), q_{N,L}(x)]$. The commutators make sense also for more general gauge group but quaternion/complex number property might have some deeper role.

Thus the critical deformations would induce conformal scalings of the effective metric and dynamical electro-weak gauge transformations. Electro-weak gauge symmetry would be a dynamical symmetry restricted to string world sheets and partonic 2-surfaces rather than acting at the entire space-time surface. For 4-D de-localized right-handed neutrino modes the conformal scalings of the effective metric are analogous to the conformal transformations of M^4 for $\mathcal{N} = 4$ SYMs. Also ordinary conformal symmetries of M^4 could be present for string world sheets and could act as symmetries of generalized Feynman graphs since even virtual wormhole throats are massless. An interesting question is whether the conformal invariance associated with the effective metric is the analog of dual conformal invariance in $\mathcal{N} = 4$ theories.

Critical deformations of space-time surface are accompanied by conserved fermionic currents. By using standard Noetherian formulas one can write

$$J_i^\mu = \overline{\Psi}\Gamma^\mu \delta_i \Psi + \delta_i \overline{\Psi}\Gamma^\mu \Psi \ . \tag{6.4.9}$$

Here $\delta\Psi_i$ denotes derivative of the variation with respect to a group parameter labeled by i. Since $\delta\Psi_i$ reduces to an infinitesimal gauge transformation of Ψ induced by deformation, these currents are the analogs of gauge currents. The integrals of these currents along the braid strands at the ends of string world sheets define the analogs of gauge charges. The interpretation as Kac-Moody charges is also very attractive and I have proposed that the 2-D Hodge duals of gauge potentials could be identified as Kac-Moody currents. If so, the 2-D Hodge duals of J would define the quantum analogs of dynamical electro-weak gauge fields and Kac-Moody charge could be also seen as non-integral phase factor associated with the braid strand in Abelian approximation (the interpretation in terms of finite measurement resolution is discussed earlier).

One can also define super currents by replacing $\overline{\Psi}$ or Ψ by a particular mode of the induced spinor field as well as c-number valued currents by performing the replacement for both $\overline{\Psi}$ or Ψ. As expected, one obtains a super-conformal algebra with all modes of induced spinor fields acting as generators of super-symmetries restricted to 2-D surfaces. The number of the charges which do not annihilate physical states as also the effective number of fermionic modes could be finite and this would suggest that the integer $\mathcal{N}$ for the supersymmetry in question is finite. This would conform with the earlier proposal inspired by the notion of finite measurement resolution implying the replacement of the partonic 2-surfaces with collections of braid ends.

Note that Kac-Moody charges might be associated with "long" braid strands connecting different wormhole throats as well as short braid strands connecting opposite throats of wormhole contacts. Both kinds of charges would appear in the theory.

What is the interpretation of the critical deformations?

Critical deformations bring in an additional gauge symmetry. Certainly not all possible gauge transformations are induced by the deformations of preferred extremals and a good guess is that they correspond to holomorphic gauge group elements as in theories with Kac-Moody symmetry. What is the physical character of this dynamical gauge symmetry?

(a) Do the gauge charges vanish? Do they annihilate the physical states? Do only their positive energy parts annihilate the states so that one has a situation characteristic for the representation of Kac-Moody algebras. Or could some of these charges be analogous to the gauge charges associated with the constant gauge transformations in gauge theories and be therefore non-vanishing in the absence of confinement. Now one has electro-weak gauge charges and these should be non-vanishing. Can one assign them to deformations with a vanishing conformal weight and the remaining deformations to those with non-vanishing conformal weight and acting like Kac-Moody generators on the physical states?

(b) The simplest option is that the critical Kac-Moody charges/gauge charges with non-vanishing positive conformal weight annihilate the physical states. Critical degrees of freedom would not disappear but make their presence known via the states labelled by different gauge charges assignable to critical deformations with vanishing conformal weight. Note that constant gauge transformations can be said to break the gauge symmetry also in the ordinary gauge theories unless one has confinement.

(c) The hierarchy of quantum criticalities suggests however entire hierarchy of electro-weak Kac-Moody algebras. Does this mean a hierarchy of electro-weak symmetries breakings in which the number of Kac-Moody generators not annihilating the physical states gradually increases as also modes with a higher value of positive conformal weight fail to annihilate the physical state?

The only manner to have a hierarchy of algebras is by assuming that only the generators satisfying $n\ mod\ N = 0$ define the sub-Kac-Moody algebra annihilating the physical states so that the generators with $n\ mod\ N \neq 0$ would define the analogs of gauge charges. I have suggested for long time ago the relevance of kind of fractal hierarchy of Kac-Moody and Super-Virasoro algebras for TGD but failed to imagine any concrete realization.

A stronger condition would be that the algebra reduces to a finite dimensional algebra in the sense that the actions of generators Q_n and Q_{n+kN} are identical. This would correspond to periodic boundary conditions in the space of conformal weights. The notion of finite measurement resolution suggests that the number of independent fermionic oscillator operators is proportional to the number of braid ends so that an effective reduction to a finite algebra is expected.

Whatever the correct interpretation is, this would obviously refine the usual view about electro-weak symmetry breaking.

These arguments suggests the following overall view. The holomorphy of spinor modes gives rise to Kac-Moody algebra defined by isometries and includes besides Minkowskian generators associated with gravitation also SU(3) generators associated with color symmetries. Vanishing second variations in turn define electro-weak Kac-Moody type algebra.

Note that criticality suggests that one must perform functional integral over WCW by decomposing it to an integral over zero modes for which deformations of X^4 induce only an electro-weak gauge transformation of the induced spinor field and to an integral over moduli corresponding to the remaining degrees of freedom.

6.4.8 The importance of being light-like

The singular geometric objects associated with the space-time surface have become increasingly important in TGD framework. In particular, the recent progress has made clear that these objects might be crucial for the understanding of quantum TGD. The singular objects are associated not only with the induced metric but also with the effective metric defined by the anti-commutators of the modified gamma matrices appearing in the modified Dirac equation and determined by the Kähler action.

The singular objects associated with the induced metric

Consider first the singular objects associated with the induced metric.

(a) At light-like 3-surfaces defined by wormhole throats the signature of the induced metric changes from Euclidian to Minkowskian so that 4-metric is degenerate. These surfaces are carriers of elementary particle quantum numbers and the 4-D induced metric degenerates locally to 3-D one at these surfaces.

(b) Braid strands at light-like 3-surfaces are most naturally light-like curves: this correspond to the boundary condition for open strings. One can assign fermion number to the braid strands. Braid strands allow an identification as curves along which the Euclidian signature of the string world sheet in Euclidian region transforms to Minkowskian one. Number theoretic interpretation would be as a transformation of complex regions to hyper-complex regions meaning that imaginary unit i satisfying $i^2 = -1$ becomes hyper-complex unit e satisfying $e^2 = 1$. The complex coordinates $(z, \overline{z})$ become hyper-complex coordinates $(u = t + ex, v = t - ex)$ giving the standard light-like coordinates when one puts $e = 1$.

The singular objects associated with the effective metric

There are also singular objects assignable to the effective metric. According to the simple arguments already developed, string world sheets and possibly also partonic 2-surfaces are singular objects with respect to the effective metric defined by the anti-commutators of the modified gamma matrices rather than induced gamma matrices. Therefore the effective metric seems to be much more than a mere formal structure.

(a) For instance, quaternionicity of the space-time surface could allow an elegant formulation in terms of the effective metric avoiding the problems due to the Minkowski signature. This is achieved if the effective metric has Euclidian signature $\epsilon \times (1,1,1,1)$, $\epsilon = \pm 1$ or a complex counterpart of the Minkowskian signature $\epsilon(1,1,-1,-1)$.

(b) String word sheets and perhaps also partonic 2-surfaces could be understood as singularities of the effective metric. What happens that the effective metric with Euclidian signature $\epsilon \times (1,1,1,1)$ transforms to the signature $\epsilon(1,1,-1,-1)$ (say) at string world sheet so that one would have the degenerate signature $\epsilon \times (1,1,0,0)$ at the string world sheet.

    What is amazing is that this works also number theoretically. It came as a total surprise to me that the notion of hyper-quaternions as a closed algebraic structure indeed exists. The hyper-quaternionic units would be given by $(1, I, iJ, iK)$, where i is a commuting imaginary unit satisfying $i^2 = -1$. Hyper-quaternionic numbers defined as combinations of these units with real coefficients do form a closed algebraic structure which however fails to be a number field just like hyper-complex numbers do. Note that the hyper-quaternions obtained with real coefficients from the basis $(1, iI, iJ, iK)$ fail to form an algebra since the product is not hyper-quaternion in this sense but belongs to the algebra of complexified quaternions. The same problem is encountered in the case of hyper-octonions defined in this manner. This has been a stone in my shoe since I feel strong disrelish towards Wick rotation as a trick for moving between different signatures.

(c) Could also partonic 2-surfaces correspond to this kind of singular 2-surfaces? In principle, 2-D surfaces of 4-D space intersect at discrete points just as string world sheets and partonic 2-surfaces do so that this might make sense. By complex structure the situation is algebraically equivalent to the analog of plane with non-flat metric allowing all possible signatures (ϵ_1, ϵ_2) in various regions. At light-like curve either ϵ_1 or ϵ_2 changes sign and light-like curves for these two kinds of changes can intersect as one can easily verify by drawing what happens. At the intersection point the metric is completely degenerate and simply vanishes.

(d) Replacing real 2-dimensionality with complex 2-dimensionality, one obtains by the universality of algebraic dimension the same result for partonic 2-surfaces and string world sheets. The braid ends at partonic 2-surfaces representing the intersection points of 2-surfaces of this kind would have completely degenerate effective metric so that the modified gamma matrices would vanish implying that energy momentum tensor vanishes as does also the induced Kähler field.

(e) The effective metric suffers a local conformal scaling in the critical deformations identified in the proposed manner. Since ordinary conformal group acts on Minkowski space and leaves the boundary of light-cone invariant, one has two conformal groups. It is not however clear whether the M^4 conformal transformations can act as symmetries in TGD, where the presence of the induced metric in Kähler action breaks M^4 conformal symmetry. As found, also in TGD framework the Kac-Moody currents assigned to the braid strands generate Yangian: this is expected to be true also for the Kac-Moody counterparts of the conformal algebra associated with quantum criticality. On the other hand, in twistor program one encounters also two conformal groups and the space in which the second conformal group acts remains somewhat mysterious object. The Lie algebras for the two conformal groups generate the conformal Yangian and the integrands of the scattering amplitudes are Yangian invariants. Twistor approach should apply in TGD if zero energy ontology is right. Does this mean a deep connection?

    What is also intriguing that twistor approach in principle works in strict mathematical sense only at signatures $\epsilon \times (1,1,-1-1)$ and the scattering amplitudes in Minkowski signature are obtained by analytic continuation. Could the effective metric give rise to the desired signature? Note that the notion of massless particle does not make sense in the signature $\epsilon \times (1,1,1,1)$.

These arguments provide genuine a support for the notion of quaternionicity and suggest a connection with the twistor approach.

6.4.9 Realization of large $\mathcal{N}$ SUSY in TGD

The generators large $\mathcal{N}$ SUSY algebras are obtained by taking fermionic currents for second quantized fermions and replacing either fermion field or its conjugate with its particular mode. The resulting super currents are conserved and define super charges. By replacing both fermion and its conjugate with modes one obtains c number valued currents. Therefore $\mathcal{N} = \infty$ SUSY - presumably equivalent with super-conformal invariance - or its finite $\mathcal{N}$ cutoff is realized in TGD framework and the challenge is to understand the realization in more detail.

Super-space viz. Grassmann algebra valued fields

Standard SUSY induces super-space extending space-time by adding anti-commuting coordinates as a formal tool. Many mathematicians are not enthusiastic about this approach because of the purely formal nature of anti-commuting coordinates. Also I regard them as a non-sense geometrically and there is actually no need to introduce them as the following little argument shows.

Grassmann parameters (anti-commuting theta parameters) are generators of Grassmann algebra and the natural object replacing super-space is this Grassmann algebra with coefficients of Grassmann algebra basis appearing as ordinary real or complex coordinates. This is just an ordinary space with additional algebraic structure: the mysterious anti-commuting coordinates are not needed. To me this notion is one of the conceptual monsters created by the over-pragmatic thinking of theoreticians.

This allows allows to replace field space with super field space, which is completely well-defined object mathematically, and leave space-time untouched. Linear field space is simply replaced with its Grassmann algebra. For non-linear field space this replacement does not work. This allows to formulate the notion of linear super-field just in the same manner as it is done usually.

The generators of super-symmetries in super-space formulation reduce to super translations , which anti-commute to translations. The super generators Q_α and $\overline{Q}_{\dot{\beta}}$ of super Poincare algebra are Weyl spinors commuting with momenta and anti-commuting to momenta:

$$\{Q_\alpha, \overline{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\ \dot{beta}} P_\mu \ . \tag{6.4.10}$$

One particular representation of super generators acting on super fields is given by

$$\begin{aligned} D_\alpha &= i\frac{\partial}{\partial_{\theta_\alpha}} \ , \\ D_{\dot{\alpha}} &= i\frac{\partial}{\partial_{\overline{\theta}_{\dot{alpha}}}} + \theta^\beta \sigma^\mu_{\beta\dot{\alpha}}\partial_\mu \end{aligned} \tag{6.4.11}$$

Here the index raising for 2-spinors is carried out using antisymmetric 2-tensor $\epsilon^{\alpha\beta}$. Super-space interpretation is not necessary since one can interpret this action as an action on Grassmann algebra valued field mixing components with different fermion numbers.

Chiral superfields are defined as fields annihilated by $D_{\dot{\alpha}}$. Chiral fields are of form $\Psi(x^\mu + i\overline{\theta}\sigma^\mu\theta, \theta)$. The dependence on $\overline{\theta}_{\dot{\alpha}}$ comes only from its presence in the translated Minkowski coordinate annihilated by $D_{\dot{\alpha}}$. Super-space enthusiast would say that by a translation of M^4 coordinates chiral fields reduce to fields, which depend on θ only.

The space of fermionic Fock states at partonic 2-surface as TGD counterpart of chiral super field

As already noticed, another manner to realize SUSY in terms of representations the super algebra of conserved super-charges. In TGD framework these super charges are naturally associated with the modified Dirac equation, and anti-commuting coordinates and super-fields do not appear anywhere. One can however ask whether one could identify a mathematical structure replacing the notion of chiral super field.

In [K19] it was proposed that generalized chiral super-fields could effectively replace induced spinor fields and that second quantized fermionic oscillator operators define the analog of SUSY algebra. One would have $\mathcal{N} = \infty$ if all the conformal excitations of the induced spinor field restricted on 2-surface are present. For right-handed neutrino the modes are labeled by two integers and de-localized to the interior of Euclidian or Minkowskian regions of space-time sheet.

The obvious guess is that chiral super-field generalizes to the field having as its components many-fermions states at partonic 2-surfaces with theta parameters and their conjugates in one-one correspondence with fermionic creation operators and their hermitian conjugates.

(a) Fermionic creation operators - in classical theory corresponding anti-commuting Grassmann parameters - replace theta parameters. Theta parameters and their conjugates are not in one-one correspondence with spinor components but with the fermionic creation operators and their hermitian conjugates. One can say that the super-field in question is defined in the "world of classical worlds" (WCW) rather than in space-time. Fermionic Fock state at the partonic 2-surface is the value of the chiral super field at particular point of WCW.

(b) The matrix defined by the $\sigma^\mu \partial_\mu$ is replaced with a matrix defined by the modified Dirac operator D between spinor modes acting in the solution space of the modified Dirac equation. Since modified Dirac operator annihilates the modes of the induced spinor field, super covariant derivatives reduce to ordinary derivatives with respect the theta parameters labeling the modes. Hence the chiral super field is a field that depends on θ_m or conjugates $\overline{\theta}_m$ only. In second quantization the modes of the chiral super-field are many-fermion states assigned to partonic 2-surfaces and string world sheets. Note that this is the only possibility since the notion of super-coordinate does not make sense now.

(c) It would seem that the notion of super-field does not bring anything new. This is not the case. First of all, the spinor fields are restricted to 2-surfaces. Second point is that one cannot assign to the fermions of the many-fermion states separate non-parallel or even parallel four-momenta. The many-fermion state behaves like elementary particle. This has non-trivial implications for propagators and a simple argument [K19] leads to the proposal that propagator for N-fermion partonic state is proportional to $1/p^N$. This would mean that only the states with fermion number equal to 1 or 2 behave like ordinary elementary particles.

How the fermionic anti-commutation relations are determined?

Understanding the fermionic anti-commutation relations is not trivial since all fermion fields except right-handed neutrino are assumed to be localized at 2-surfaces. Since fermionic conserved currents must give rise to well-defined charges as 3-D integrals the spinor modes must be proportional to a square root of delta function in normal directions. Furthermore, the modified Dirac operator must act only in the directions tangential to the 2-surface in order that the modified Dirac equation can be satisfied.

The square root of delta function can be formally defined by starting from the expansion of delta function in discrete basis for a particle in 1-D box. The product of two functions in x-space is convolution of Fourier transforms and the coefficients of Fourier transform of delta function are apart from a constant multiplier equal to 1: $\delta(x) = K \sum_n exp(inx/2\pi L)$.

Therefore the Fourier transform of square root of delta function is obtained by normalizing the Fourier transform of delta function by $1/\sqrt{N}$, where $N \rightarrow \infty$ is the number of plane waves. In other words: $\sqrt{\delta(x)} = \sqrt{\frac{K}{N}} \sum_n \sum exp(inx/2\pi L)$.

Canonical quantization defines the standard approach to the second quantization of the Dirac equation.

(a) One restricts the consideration to time=constant slices of space-time surface. Now the 3-surfaces at the ends of CD are natural slices. The intersection of string world sheet with these surfaces is 1-D whereas partonic 2-surfaces have 2-D Euclidian intersection with them.

(b) The canonical momentum density is defined by

$$\begin{aligned} \Pi_\alpha &= \frac{\partial L}{\partial_t \overline{\Psi}_\alpha(x)} = \Gamma^t \Psi \ , \\ \Gamma^t &= \frac{\partial L_K}{\partial(\partial_t h^k)} \ . \end{aligned} \tag{6.4.12}$$

L_K denotes Kähler action density: consistency requires $D_\mu \Gamma^\mu = 0$, and this is guaranteed only by using the modified gamma matrices defined by Kähler action. Note that Γ^t contains also the $\sqrt{g_4}$ factor. Induced gamma matrices would require action defined by four-volume. t is time coordinate varying in direction tangential to 2-surface.

(c) The standard equal time canonical anti-commutation relations state

$$\{\Pi_\alpha, \overline{\Psi}_\beta\} = \delta^3(x,y)\delta_{\alpha\beta} \ . \tag{6.4.13}$$

Can these conditions be applied both at string world sheets and partonic 2-surfaces.

(a) Sttring world sheets do not pose problems. The restriction of the modes to string world sheets means that the square root of delta function in the normal direction of string world sheet takes care of the normal dimensions and the dynamical part of anti-commutation relations is 1-dimensional just as in the case of strings.

(b) Partonic 2-surfaces are problematic. The $\sqrt{g_4}$ factor in Γ^t implies that Γ^t approaches zero at partonic 2-surfaces since they belong to light-like wormhole throats at which the signature of the induced metric changes. Energy momentum tensor appearing in Γ^t involves to index raisins by induced metric so that it can grow without limit as one approaches partonic two-surface. Therefore it is quite possible that the limit is finite and the boundary conditions defined by the weak form of electric magnetic duality might imply that the limit is finite. The open question is whether one can apply canonical quantization at partonic 2-surfaces. One can also ask whether one can define induced spinor fields at wormhole throats only at the ends of string world sheets so that partonic 2-surface would be effectively discretized. This cautious conclusion emerged in the earlier study of the modified Dirac equation [K18].

(c) Suppose that one can assume spinor modes at partonic 2-surfaces. 2-D conformal invariance suggests that the situation reduces to effectively one-dimensional also at the partonic two-surfaces. If so, one should pose the anti-commutation relations at some 1-D curves of the partonic 2-surface only. This is the only sensical option. The point is that the action of the modified Dirac operator is tangential so that also the canonical momentum current must be tangential and one can fix anti-commutations only at some set of curves of the partonic 2-surface.

One can of course worry what happens at the limit of vacuum extremals. The problem is that Γ^t vanishes for space-time surfaces reducing to vacuum extremals at the 2-surfaces carrying fermions so that the anti-commutations are inconsistent. Should one require - as done earlier- that the anti-commutation relations make sense at this limit and cannot therefore have the standard form but involve the scalar magnetic flux formed from the induced Kähler form by permuting it with the 2-D permutations symbol? The restriction to preferred extremals, which are always non-vacuum extremals, might allow to avoid this kind of problems automatically.

In the case of right-handed neutrino the situation is genuinely 3-dimensional and in this case non-vacuum extremal property must hold true in the regions where the modes of ν_R are non-vanishing. The same mechanism would save from problems also at the partonic 2-surfaces. The dynamics of induced spinor fields must avoid classical vacuum. Could this relate to color confinement? Could hadrons be surrounded by an insulating layer of Kähler vacuum?

6.4.10 Comparison of TGD and stringy views about super-conformal symmetries

The best manner to represent TGD based view about conformal symmetries is by comparison with the conformal symmetries of super string models.

Basic differences between the realization of super conformal symmetries in TGD and in super-string models

The realization super conformal symmetries in TGD framework differs from that in string models in several fundamental aspects.

(a) In TGD framework super-symmetry generators acting as configuration space gamma matrices carry either lepton or quark number. Majorana condition required by the hermiticity of super generators which is crucial for super string models would be in conflict with the conservation of baryon and lepton numbers and is avoided. This is made possible by the realization of bosonic generators represented as Hamiltonians of X^2-local symplectic transformations rather than vector fields generating them [K10] . This kind of representation applies also in Kac-Moody sector since the local transversal isometries localized in X_l^3 and respecting light-likeness condition can be regarded as X^2 local symplectic transformations, whose Hamiltonians generate also isometries. Localization is not complete: the functions of X^2 coordinates multiplying symplectic and Kac-Moody generators are functions of the symplectic invariant $J = \epsilon^{\mu\nu} J_{\mu\nu}$ so that effective one-dimensionality results but in different sense than in conformal field theories. This realization of super symmetries is what distinguishes between TGD and super string models and leads to a totally different physical interpretation of super-conformal symmetries. The fermionic representations of super-symplectic and super Kac-Moody generators can be identified as Noether charges in standard manner.

(b) A long-standing problem of quantum TGD was that stringy propagator $1/G$ does not make sense if G carries fermion number. The progress in the understanding of second quantization of the modified Dirac operator made it however possible to identify the counterpart of G as a c-number valued operator and interpret it as different representation of G [K12] .

(c) The notion of super-space is not needed at all since Hamiltonians rather than vector fields represent bosonic generators, no super-variant of geometry is needed. The distinction between Ramond and N-S representations important for $N = 1$ super-conformal symmetry and allowing only ground state weight 0 an 1/2 disappears. Indeed, for $N = 2$ super-conformal symmetry it is already possible to generate spectral flow transforming these Ramond and N-S representations to each other (G_n is not Hermitian anymore).

(d) If Kähler action defines the modified Dirac operator, the number of spinor modes could be finite. One must be here somewhat cautious since bound state in the Coulomb potential associated with electric part of induced electro-weak gauge field might give rise to an infinite number of bound states which eigenvalues converging to a fixed eigenvalue (as in the case of hydrogen atom). Finite number of generalized eigenmodes means that the representations of super-conformal algebras reduces to finite-dimensional ones in TGD framework. Also the notion of number theoretic braid indeed implies this. The physical interpretation would be in terms of finite measurement resolution. If Kähler action is complexified to include imaginary part defined by CP breaking instanton term, the number of stringy mass square eigenvalues assignable to the spinor modes becomes infinite since conformal excitations are possible. This means breakdown of exact holography and effective 2-dimensionality of 3-surfaces. It seems that the inclusion of instanton term is necessary for several reasons. The notion of finite measurement resolution forces conformal cutoff also now. There are arguments suggesting that only the modes with vanishing conformal weight contribute to the Dirac determinant defining vacuum functional identified as exponent of Kähler function in turn identified as Kähler action for its preferred extremal.

(e) What makes spinor field mode a generator of gauge super-symmetry is that is c-number and not an eigenmode of $D_K(X^2)$ and thus represents non-dynamical degrees of freedom. If the number of eigen modes of $D_K(X^2)$ is indeed finite means that most of spinor field modes represent super gauge degrees of freedom.

The super generators G are not Hermitian in TGD!

The already noticed important difference between TGD based and the usual Super Virasoro representations is that the Super Virasoro generator G cannot Hermitian in TGD. The reason is that WCW gamma matrices possess a well defined fermion number. The hermiticity of the WCW gamma matrices Γ and of the Super Virasoro current G could be achieved by posing Majorana conditions on the second quantized H-spinors. Majorana conditions can be however realized only for space-time dimension $D\ mod\ 8 = 2$ so that super string type approach does not work in TGD context. This kind of conditions would also lead to the non-conservation of baryon and lepton numbers.

An analogous situation is encountered in super-symmetric quantum mechanics, where the general situation corresponds to super symmetric operators S, $S^\dagger$, whose anti-commutator is Hamiltonian: $\{S, S^\dagger\} = H$. One can define a simpler system by considering a Hermitian operator $S_0 = S+S^\dagger$ satisfying $S_0^2 = H$: this relation is completely analogous to the ordinary Super Virasoro relation $GG = L$. On basis of this observation it is clear that one should replace ordinary Super Virasoro structure $GG = L$ with $GG^\dagger = L$ in TGD context.

It took a long time to realize the trivial fact that $N = 2$ super-symmetry is the standard physics counterpart for TGD super symmetry. $N = 2$ super-symmetry indeed involves the doubling of super generators and super generators carry $U(1)$ charge having an interpretation as fermion number in recent context. The so called short representations of $N = 2$ super-symmetry algebra can be regarded as representations of $N = 1$ super-symmetry algebra.

WCW gamma matrix Γ_n, $n > 0$ corresponds to an operator creating fermion whereas Γ_n, $n < 0$ annihilates anti-fermion. For the Hermitian conjugate $\Gamma_n^\dagger$ the roles of fermion and anti-fermion are interchanged. Only the anti-commutators of gamma matrices and their Hermitian conjugates are non-vanishing. The dynamical Kac Moody type generators are Hermitian and are constructed as bilinears of the gamma matrices and their Hermitian conjugates and, just like conserved currents of the ordinary quantum theory, contain parts proportional to $a^\dagger a$, $b^\dagger b$, $a^\dagger b^\dagger$ and ab (a and b refer to fermionic and anti-fermionic oscillator operators). The commutators between Kac Moody generators and Kac Moody generators and gamma matrices remain as such.

For a given value of m G_n, $n > 0$ creates fermions whereas G_n, $n < 0$ annihilates anti-fermions. Analogous result holds for $G_n^\dagger$. Virasoro generators remain Hermitian and decompose just like Kac Moody generators do. Thus the usual anti-commutation relations for the super Virasoro generators must be replaced with anti-commutations between G_m and $G_n^\dagger$ and one has

$$\begin{aligned} \{G_m, G_n^\dagger\} &= 2L_{m+n} + \frac{c}{3}(m^2 - \frac{1}{4})\delta_{m,-n} \ , \\ \{G_m, G_n\} &= 0 \ , \\ \{G_m^\dagger, G_n^\dagger\} &= 0 \ . \end{aligned} \tag{6.4.14}$$

The commutators of type $[L_m, L_n]$ are not changed. Same applies to the purely kinematical commutators between L_n and $G_m/G_m^\dagger$.

The Super Virasoro conditions satisfied by the physical states are as before in case of L_n whereas the conditions for G_n are doubled to those of G_n, $n<0$ and $G_n^\dagger$, $n>0$.

What could be the counterparts of stringy conformal fields in TGD framework?

The experience with string models would suggest the conformal symmetries associated with the complex coordinates of X^2 as a candidate for conformal super-symmetries. One can imagine two counterparts of the stringy coordinate z in TGD framework.

(a) Super-symplectic and super Kac-Moody symmetries are local with respect to X^2 in the sense that the coefficients of generators depend on the invariant $J=\epsilon^{\alpha\beta}J_{\alpha\beta}\sqrt{g_2}$ rather than being completely free [K10] . Thus the real variable J replaces complex (or hyper-complex) stringy coordinate and effective 1-dimensionality holds true also now but in different sense than for conformal field theories.

(b) The slicing of X^4 by string world sheets Y^2 and partonic 2-surfaces X^2 implied by number theoretical compactification implies string-parton duality and involves the super conformal fermionic gauge symmetries associated with the coordinates u and w in the dual dimensional reductions to stringy and partonic dynamics. These coordinates define the natural analogs of stringy coordinate. The effective reduction of X_l^3 to braid by finite measurement resolution implies the effective reduction of $X^4(X^3)$ to string world sheet. This implies quite strong resemblance with string model. The realization that spinor modes with well- define em charge must be localized at string world sheets makes the connection with strings even more explicit [K69].

    One can understand how Equivalence Principle emerges in TGD framework at space-time level when many-sheeted space-time (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book) is replaced with effective space-time lumping together the space-time sheets to M^4 endowed with effective metric. The quantum counterpart EP has most feasible interpretation in terms of Quantum Classical Correspondence (QCC): the conserved Kähler four-momentum equals to an eigenvalue of conserved Kähler-Dirac four-momentum acting as operator.

(c) The conformal fields of string model would reside at X^2 or Y^2 depending on which description one uses and complex (hyper-complex) string coordinate would be identified accordingly. Y^2 could be fixed as a union of stringy world sheets having the strands of number theoretic braids as its ends. The proposed definition of braids is unique and characterizes finite measurement resolution at space-time level. X^2 could be fixed uniquely as the intersection of X_l^3 (the light-like 3-surface at which induced metric of space-time surface changes its signature) with $\delta M_\pm^4\times CP_2$. Clearly, wormhole throats X_l^3 would take the role of branes and would be connected by string world sheets defined by number theoretic braids.

(d) An alternative identification for TGD parts of conformal fields is inspired by M^8-H duality. Conformal fields would be fields in WCW. The counterpart of z coordinate could be the hyper-octonionic M^8 coordinate m appearing as argument in the Laurent series of WCW Clifford algebra elements. m would characterize the position of the tip of CD and the fractal hierarchy of CDs within CDs would give a hierarchy of Clifford algebras and thus inclusions of hyper-finite factors of type II_1. Reduction to hyper-quaternionic field -that is field in M^4 center of mass degrees of freedom- would be needed to obtained associativity. The arguments m at various level might correspond to arguments of N-point function in quantum field theory.

6.5 Appendix: Hamilton-Jacobi structure

In the following the definition of Hamilton-Jacobi structure is discussed in detail.

6.5.1 Hermitian and hyper-Hermitian structures

The starting point is the observation that besides the complex numbers forming a number field there are hyper-complex numbers. Imaginary unit i is replaced with e satisfying $e^2 = 1$. One obtains an algebra but not a number field since the norm is Minkowskian norm $x^2 - y^2$, which vanishes at light-cone $x = y$ so that light-like hypercomplex numbers $x \pm e$) do not have inverse. One has "almost" number field.

Hyper-complex numbers appear naturally in 2-D Minkowski space since the solutions of a massless field equation can be written as $f = g(u = t-ex)+h(v = t+ex)$ whith $e^2 = 1$ realized by putting $e = 1$. Therefore Wick rotation relates sums of holomorphic and antiholomorphic functions to sums of hyper-holomorphic and anti-hyper-holomorphic functions. Note that u and v are hyper-complex conjugates of each other.

Complex n-dimensional spaces allow Hermitian structure. This means that the metric has in complex coordinates $(z_1, ..., z_n)$ the form in which the matrix elements of metric are non-vanishing only between z_i and complex conjugate of z_j. In 2-D case one obtains just $ds^2 = g_{z\overline{z}}dzd\overline{z}$. Note that in this case metric is conformally flat since line element is proportional to the line element $ds^2 = dzd\overline{z}$ of plane. This form is always possible locally. For complex n-D case one obtains $ds^2 = g_{i\overline{j}}dz^id\overline{z}^j$. $g_{i\overline{j}} = \overline{g_{j\overline{i}}}$ guaranteeing the reality of ds^2. In 2-D case this condition gives $g_{z\overline{z}} = \overline{g_{z\overline{z}}}$.

How could one generalize this line element to hyper-complex n-dimensional case. In 2-D case Minkowski space M^2 one has $ds^2 = g_{uv}dudv$, $g_{uv} = 1$. The obvious generalization would be the replacement $ds^2 = g_{u_iv_j}du^idv^j$. Also now the analogs of reality conditions must hold with respect to $u_i \leftrightarrow v_i$.

6.5.2 Hamilton-Jacobi structure

Consider next the path leading to Hamilton-Jacobi structure.

4-D Minkowski space $M^4 = M^2 \times E^2$ is Cartesian product of hyper-complex M^2 with complex plane E^2, and one has $ds^2 = dudv + dzd\overline{z}$ in standard Minkowski coordinates. One can also consider more general integrable decompositions of M^4 for which the tangent space $TM^4 = M^4$ at each point is decomposed to $M^2(x) \times E^2(x)$. The physical analogy would be a position dependent decomposition of the degrees of freedom of massless particle to longitudinal ones ($M^2(x)$: light-like momentum is in this plane) and transversal ones ($E^2(x)$: polarization vector is in this plane). Cylindrical and spherical variants of Minkowski coordinates define two examples of this kind of coordinates (it is perhaps a good exercise to think what kind of decomposition of tangent space is in question in these examples). An interesting mathematical problem highly relevant for TGD is to identify all possible decompositions of this kind for empty Minkowski space.

The integrability of the decomposition means that the planes $M^2(x)$ are tangent planes for 2-D surfaces of M^4 analogous to Euclidian string world sheet. This gives slicing of M^4 to Minkowskian string world sheets parametrized by euclidian string world sheets. The question is whether the sheets are stringy in a strong sense: that is minimal surfaces. This is not the case: for spherical coordinates the Euclidian string world sheets would be spheres which are not minimal surfaces. For cylindrical and spherical coordinates howeverr $M^2(x)$ integrate to plane M^2, which is minimal surface.

Integrability means in the case of $M^2(x)$ the existence of light-like vector field J whose flow lines define a global coordinate. Its existence implies also the existence of its conjugate and together these vector fields give rise to $M^2(x)$ at each point. This means that one has $J = \Psi\nabla\Phi$: Φ indeed defines the global coordinate along flow lines. In the case of M^2 either the coordinate u or v would be the coordinate in question. This kind of flows are called Beltrami flows. Obviously the same holds for the transversal planes E^2.

One can generalize this metric to the case of general 4-D space with Minkowski signature of metric. At least the elements g_{uv} and $g_{z\overline{z}}$ are non-vanishing and can depend on both u, v

and $z, \overline{z}$. They must satisfy the reality conditions $g_{z\overline{z}} = \overline{g_{z\overline{z}}}$ and $g_{uv} = \overline{g_{vu}}$ where complex conjugation in the argument involves also $u \leftrightarrow v$ besides $z \leftrightarrow \overline{z}$.

The question is whether the components g_{uz}, g_{vz}, and their complex conjugates are non-vanishing if they satisfy some conditions. They can. The direct generalization from complex 2-D space would be that one treats u and v as complex conjugates and therefore requires a direct generalization of the hermiticity condition

$$g_{uz} = \overline{g_{v\overline{z}}} \ , \ g_{vz} = \overline{g_{u\overline{z}}} \ .$$

This would give complete symmetry with the complex 2-D (4-D in real sense) spaces. This would allow the algebraic continuation of hermitian structures to Hamilton-Jacobi structures by just replacing i with e for some complex coordinates.

Chapter 7

Recent View about Kähler Geometry and Spin Structure of "World of Classical Worlds"

7.1 Introduction

The construction of Kähler geometry of WCW ("world of classical worlds") is fundamental to TGD program. I ended up with the idea about physics as WCW geometry around 1985 and made a breakthrough around 1990, when I realized that Kähler function for WCW could correspond to Kähler action for its preferred extremals defining the analogs of Bohr orbits so that classical theory with Bohr rules would become an exact part of quantum theory and path integral would be replaced with genuine integral over WCW. The motivating construction was that for loop spaces leading to a unique Kähler geometry [A37]. The geometry for the space of 3-D objects is even more complex than that for loops and the vision still is that the geometry of WCW is unique from the mere existence of Riemann connection.

The basic idea is that WCW is union of symmetric spaces G/H labelled by zero modes which do not contribute to the WCW metric. There have been many open questions and it seems the details of the ealier approach [?]ust be modified at the level of detailed identifications and interpretations.

(a) A longstanding question has been whether one could assign Equivalence Principle (EP) with the coset representation formed by the super-Virasoro representation assigned to G and H in such a manner that the four-momenta associated with the representations and identified as inertial and gravitational four-momenta would be identical. This does not seem to be the case. The recent view will be that EP reduces to the view that the classical four-momentum associated with Kähler action is equivalent with that assignable to modified Dirac action supersymmetrically related to Kähler action: quantum classical correspondence (QCC) would be in question. Also strong form of general coordinate invariance implying strong form of holography in turn implying that the super-symplectic representations assignable to space-like and light-like 3-surfaces are equivalent could imply EP with gravitational and inertial four-momenta assigned to these two representations.

At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least.

(b) The detailed identification of groups G and H and corresponding algebras has been a longstanding problem. Symplectic algebra associated with$\delta M^4_\pm \times CP2$ ($\delta M^4_\pm$ is light-cone boundary - or more precisely, with the boundary of causal diamond (CD) defined as Cartesian product of CP_2 with intersection of future and past direct light cones of M^4 has Kac-Moody type structure with light-like radial coordinate replacing complex coordinate z. Virasoro algebra would correspond to radial diffeomorphisms. I have also introduced Kac-Moody algebra assigned to the isometries and localized with respect to internal coordinates of the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian and which serve as natural correlates for elementary particles (in very general sense!). This kind of localization by force could be however argued to be rather ad hoc as opposed to the inherent localization of the symplectic algebra containing the symplectic algebra of isometries as sub-algebra. It turns out that one obtains direct sum of representations of symplectic algebra and Kac-Moody algebra of isometries naturally as required by the success of p-adic mass calculations.

(c) The dynamics of Kähler action is not visible in the earlier construction. The construction also expressed WCW Hamiltonians as 2-D integrals over partonic 2-surfaces. Although strong form of general coordinate invariance (GCI) implies strong form of holography meaning that partonic 2-surfaces and their 4-D tangent space data should code for quantum physics, this kind of outcome seems too strong. The progress in the understanding of the solutions of modified Dirac equation led however to the conclusion that spinor modes other than right-handed neutrino are localized at string world sheets with strings connecting different partonic 2-surfaces. This leads to a modification of earlier construction in which WCW super-Hamiltonians are essentially integrals with integrand identified as a Noether super current for the deformations in G Each spinor mode gives rise to super current and the modes of right-handed neutrino and other fermions differ in an essential manner. Right-handed neutrino would correspond to symplectic algebra and other modes to the Kac-Moody algebra and one obtains the crucial 5 tensor factors of Super Virasoro required by p-adic mass calculations.

The matrix elements of WCW metric between Killing vectors are expressible as anti-commutators of super-Hamiltonians identifiable as contractions of WCW gamma matrices with these vectors and give Poisson brackets of corresponding Hamiltonians. The anti-commutation relates of induced spinor fields are dictated by this condition. Everything is 3-dimensional although one expects that symplectic transformations localized within interior of X^3 act as gauge symmetries so that in this sense effective 2-dimensionality is achieved. The components of WCW metric are labelled by standard model quantum numbers so that the connection with physics is extremely intimate.

(d) An open question in the earlier visions was whether finite measurement resolution is realized as discretization at the level of fundamental dynamics. This would mean that only certain string world sheets from the slicing by string world sheets and partonic 2-surfaces are possible. The requirement that anti-commutations are consistent suggests that string world sheets correspond to surfaces for which Kähler magnetic field is constant along string in well defined sense ($J_{\mu\nu}\epsilon^{\mu\nu}g^{1/2}$ remains constant along string). It however turns that by a suitable choice of coordinates of 3-surface one can guarantee that this quantity is constant so that no additional constraint results.

(e) Quantum criticality is one of the basic notions of quantum TGD and its relationship to coset construction has remained unclear. In this chapter the concrete realization of criticality in terms of symmetry breaking hierarchy of Super Virasoro algebra acting on symplectic and Kac-Moody algebras. Also a connection with finite measurement resolution - second key notion of TGD - emerges naturally.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L13]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L14]. The topics relevant to this chapter are given by the following list.

- Hierarchy of Planck constants [L21]

- Hyperfinite factors and TGD [L23]
- Structure of WCW [L33]
- TGD as infinite-dimensional geometry [L37]
- WCW gamma matrices [L41]
- WCW spinor fields [L42]
- Weak form of electric-magnetic duality [L43]
- Zero Energy Ontology (ZEO) [L44]
- 4-D spin glass degeneracy [L15]
- Equivalence Principle [L18]
- Holography [L22]
- Quantum Classical Correspondence [L28]
- Quantum criticality [L29]
- Symmetries of WCW [L34]
- TGD as ATQFT [L36]
- Vacuum functional in TGD [L40]
- KD equation [L25]
- Kaehler-Dirac action [L24]

7.2 WCW as a union of homogenous or symmetric spaces

In the following the vision about WCW as union of coset spaces is discussed in more detail.

7.2.1 Basic vision

The basic view about coset space construction for WCW has not changed.

(a) The idea about WCW as a union of coset spaces G/H labelled by zero modes is extremely attractive. The structure of homogenous space [A9] (`http://en.wikipedia.org/wiki/Homogenous_space`) means at Lie algebra level the decomposition $g = h \oplus t$ to sub-Lie-algebra h and its complement t such that $[h, t] \subset t$ holds true. Homogeneous spaces have G as its isometries. For symmetric space the additional condition $[t, t] \subset h$ holds true and implies the existence of involution changing at the Lie algebra level the sign of elements of t and leaving the elements of h invariant. The assumption about the structure of symmetric space [A24] (`http://en.wikipedia.org/wiki/Symmetric_space`) implying covariantly constant curvature tensor is attractive in infinite-dimensional case since it gives hopes about calculability.

An important source of intuition is the analogy with the construction of CP_2, which is symmetric space A particular choice of h corresponds to Lie-algebra elements realized as Killing vector fields which vanish at particular point of WCW and thus leave 3-surface invariant. A preferred choice for this point is as maximum or minimum of Kähler function. For this 3-surface the Hamiltonians of h should be stationary. If symmetric space property holds true then commutators of $[t, t]$ also vanish at the minimum/maximum. Note that Euclidian signature for the metric of WCW requires that Kähler function can have only maximum or minimum for given zero modes.

(b) The basic objection against TGD is that one cannot use the powerful canonical quantization using the phase space associated with configuration space - now WCW. The reason is the extreme non-linearity of the Kähler action and its huge vacuum degeneracy, which do not allow the construction of Hamiltonian formalism. Symplectic and Kähler structure must be realized at the level of WCW. In particular, Hamiltonians must be represented in completely new manner. The key idea is to construct WCW

Hamiltonians as anti-commutators of super-Hamiltonians defining the contractions of WCW gamma matrices with corresponding Killing vector fields and therefore defining the matrix elements of WCW metric in the tangent vector basis defined by Killing vector fields. Super-symmetry therefre gives hopes about constructing quantum theory in which only induced spinor fields are second quantized and imbedding space coordinates are treated purely classically.

(c) It is important to understand the difference between symmetries and isometries assigned to the Kähler function. Symmetries of Kähler function do not affect it. The symmetries of Kähler action are also symmetries of Kähler action because Kähler function is Kähler action for a preferred extremal (here there have been a lot of confusion). Isometries leave invariant only the quadratic form defined by Kähler metric $g_{M\overline{N}} = \partial_M \partial_{\overline{L}} K$ but not Kähler function in general. For G/H decomposition G represents isometries and H both isometries and symmetries of Kähler function.

CP_2 is familiar example: $SU(3)$ represents isometries and $U(2)$ leaves also Kähler function invariant since it depends on the $U(2)$ invariant radial coordinate r of CP_2. The origin $r=0$ is left invariant by $U(2)$ but for $r>0$ $U(2)$ performs a rotation at $r = constant$ 3-sphere. This simple picture helps to understand what happens at the level of WCW.

How to then distinguish between symmetries and isometries? A natural guess is that one obtains also for the isometries Noether charges but the vanishing of boundary terms at spatial infinity crucial in the argument leading to Noether theorem as $\Delta S = \Delta Q = 0$ does not hold true anymore and one obtains charges which are not conserved anymore. The symmetry breaking contributions would now come from effective boundaries defined by wormhole throats at which the induce metric changes its signature from Minkowskian to Euclidian. A more delicate situation is in which first order contribution to ΔS vanishes and therefore also ΔQ and the contribution to ΔS comes from second variation allowing also to define Noether charge which is not conserved.

(d) The simple picture about CP_2 as symmetric space helps to understand the general vision if one assumes that WCW has the structure of symmetric space. The decomposition $g = h+t$ corresponds to decomposition of symplectic deformations to those which vanish at 3-surface (h) and those which do not (t).

For the symmetric space option, the Poisson brackets for super generators associated with t give Hamiltonians of h identifiable as the matrix elements of WCW metric. They would not vanish although they are stationary at 3-surface meaning that Riemann connection vanishes at 3-surface. The Hamiltonians which vanish at 3-surface X^3 would correspond to t and the Hamiltonians for which Killing vectors vanish and which therefore are stationary at X^3 would correspond to h. Outside X^3 the situation would of course be different. The metric would be obtained by parallel translating the metric from the preferred point of WCW to elsewhere and symplectic transformations would make this parallel translation.

For the homogenous space option the Poisson brackets for super generators of t would still give Hamiltonians identifiable as matrix elements of WCW metric but now they would be necessary those of h. In particular, the Hamiltonians of t do not in general vanish at X^3.

7.2.2 Equivalence Principle and WCW

7.2.3 EP at quantum and classical level

Quite recently I returned to an old question concerning the meaning of Equivalence Principle (EP) in TGD framework.

Heretic would of course ask whether the question about whether EP is true or not is a pseudo problem due to uncritical assumption there really are two different four-momenta which must be identified. If even the identification of these two different momenta is difficult, the pondering of this kind of problem might be waste of time.

At operational level EP means that the scattering amplitudes mediated by graviton exchange are proportional to the product of four-momenta of particles and that the proportionality constant does not depend on any other parameters characterizing particle (except spin). The are excellent reasons to expect that the stringy picture for interactions predicts this.

(a) The old idea is that EP reduces to the coset construction for Super Virasoro algebra using the algebras associated with G and H. The four-momenta assignable to these algebras would be identical from the condition that the differences of the generators annihilate physical states and identifiable as inertial and gravitational momenta. The objection is that for the preferred 3-surface H by definition acts trivially so that time-like translations leading out from the boundary of CD cannot be contained by H unlike G. Hence four-momentum is not associated with the Super-Virasoro representations assignable to H and the idea about assigning EP to coset representations does not look promising.

(b) Another possibility is that EP corresponds to quantum classical correspondence (QCC) stating that the classical momentum assignable to Kähler action is identical with gravitational momentum assignable to Super Virasoro representations. This forced to reconsider the questions about the precise identification of the Kac-Moody algebra and about how to obtain the magic five tensor factors required by p-adic mass calculations [K56].

    A more precise formulation for EP as QCC comes from the observation that one indeed obtains two four-momenta in TGD approach. The classical four-momentum assignable to the Kähler action and that assignable to the modified Dirac action. This four-momentum is an operator and QCC would state that given eigenvalue of this operator must be equal to the value of classical four-momentum for the space-time surfaces assignable to the zero energy state in question. In this form EP would be highly non-trivial. It would be justified by the Abelian character of four-momentum so that all momentum components are well-defined also quantum mechanically. One can also consider the splitting of four-momentum to longitudinal and transversal parts as done in the parton model for hadrons: this kind of splitting would be very natural at the boundary of CD. The objection is that this correspondence is nothing more than QCC.

(c) A further possibility is that duality of light-like 3-surfaces and space-like 3-surfaces holds true. This is the case if the action of symplectic algebra can be defined at light-like 3-surfaces or even for the entire space-time surfaces. This could be achieved by parallel translation of light-cone boundary providing slicing of CD. The four-momenta associated with the two representations of super-symplectic algebra would be naturally identical and the interpretation would be in terms of EP.

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets.

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

(c) Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

(d) The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K78].

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

7.2.4 Criticism of the earlier construction

The earlier detailed realization of super-Hamiltonians and Hamiltonians can be criticized.

(a) Even after these more than twenty years it looks strange that the Hamiltonians should reduce to flux integrals over partonic 2-surfaces. The interpretation has been in terms of effective 2-dimensionality suggested strongly by strong form of general coordinate invariance stating that the descriptions based on light-like orbits of partonic 2-surfaces and space-like three surfaces at the ends of causal diamonds are dual so that only partonic 2-surfaces and 4-D tangent space data at them would matter. Strong form of holography implies effective 2-dimensionality but this should correspond gauge character for the action of symplectic generators in the interior the space-like 3-surfaces at the ends of CDs, which is something much milder.

    One expects that the strings connecting partonic 2-surfaces could bring something new to the earlier simplistic picture. The guess is that imbedding space Hamiltonian assignable to a point of partonic 2-surface should be replaced with that defined as integral over string attached to the point. Therefore the earlier picture would suffer no modification at the level of general formulas.

(b) The fact that the dynamics of Kähler action and modified Dirac action are not directly involved with the earlier construction raises suspicions. I have proposed that Kähler function could allow identification as Dirac determinant [K18] but one would expect more intimate connection. Here the natural question is whether super-Hamiltonians for the modified Dirac action could correspond to Kähler charges constructible using Noether's theorem for corresponding deformations of the space-time surface and would also be identifiable as WCW gamma matrices.

7.2.5 Is WCW homogenous or symmetric space?

A key question is whether WCW can be symmetric space [A24] (`http://en.wikipedia.org/wiki/Riemannian_symmetric_space`) or whether only homogenous structure is needed. The lack of covariant constancy of curvature tensor might produce problems in infinite-dimensional context.

The algebraic conditions for symmetric space are $g = h + t$, $[h, t] \subset t$, $[t, t] \subset h$. The latter condition is the difficult one.

(a) δCD Hamiltonians should induce diffeomorphisms of X^3 indeed leaving it invariant. The symplectic vector fields would be parallel to X^3. A stronger condition is that they induce symplectic transformations for which all points of X^3 remain invariant. Now symplectic vector fields vanish at preferred 3-surface (note that the symplectic transformations are r_M local symplectic transformations of $S^2 \times CP_2$).

(b) For Kac-Moody algebra inclusion $H \subset G$ for the finite-dimensional Lie-algebra induces the structure of symmetric space. If entire algebra is involved this does not look physically very attractive idea unless one believes on symmetry breaking for both $SU(3)$, $U(2)_{ew}$, and $SO(3)$ and E_2 (here complex conjugation corresponds to the involution). If one assumes only Kac-Moody algebra as critical symmetries, the number of tensor factors is 4 instead of five, and it is not clear whether one can obtain consistency with p-adic mass calculations.

Examples of 3-surfaces remaining invariant under $U(2)$ are 3-spheres of CP_2. They could correspond to intersections of deformations of CP_2 type vacuum extremals with the boundary of CD. Also geodesic spheres S^2 of CP_2 are invariant under $U(2)$ subgroup and would relate naturally to cosmic strings. The corresponding 3-surface would be $L \times S^2$, where L is a piece of light-like radial geodesic.

(c) In the case of symplectic algebra one can construct the imbedding space Hamiltonians inducing WCW Hamiltonians as products of elements of the isometry algebra of $S^2 \times CP_2$ for with parity under involution is well-defined. This would give a decomposition of the symplectic algebra satisfying the symmetric space property at the level imbedding space. This decomposition does not however look natural at the level of WCW since the only single point of CP_2 and light-like geodesic of δM^4_+ can be fixed by $SO(2) \times U(2)$ so that the 3-surfaces would reduce to pieces of light rays.

(d) A more promising involution is the inversion $r_M \rightarrow 1/r_M$ of the radial coordinate mapping the radial conformal weights to their negatives. This corresponds to the inversion in Super Virasoro algebra. t would correspond to functions which are odd functions of $u \equiv log(r_M/r_0)$ and h to even function of u. Stationary 3-surfaces would correspond to $u = 1$ surfaces for which $log(u) = 0$ holds true. This would assign criticality with Virasoro algebra as one expects on general grounds.

$r_M = constant$ surface would most naturally correspond to a maximum of Kähler function which could indeed be highly symmetric. The elements with even u-parity should define Hamiltonians, which are stationary at the maximum of Kähler function. For other 3-surfaces obtained by $/r_M$-local) symplectic transformations the situation is different: now H is replaced with its symplectic conjugate hHg^{-1} of H is acceptable and if the conjecture is true one would obtained 3-surfaces assignable to perturbation theory around given maximum as symplectic conjugates of the maximum. The condition that H leaves X^3 invariant in poin-twise manner is certainly too strong and imply that the 3-surface has single point as CP_2 projection.

(e) One can also consider the possibility that critical deformations correspond to h and non-critical ones to t for the preferred 3-surface. Criticality for given h would hold only for a preferred 3-surface so that this picture would be very similar that above. Symplectic conjugates of h would define criticality for other 3-surfaces. WCW would decompose to a union corresponding to different criticalities perhaps assignable to the hierarchy of sub-algebras of conformal algebra labelled by integer whose multiples give the allowed conformal weights. Hierarchy of breakings of conformal symmetries would characterize this hierarchy of sectors of WCW.

For sub-algebras of the conformal algebras (Kac-Moody and symplectic algebra) the condition $[t, t] \subset h$ cannot hold true so that one would obtain only the structure of homogenous space.

7.2.6 Symplectic and Kac-Moody algebras as basic building bricks

The basic building bricks are symplectic algebra of δCD (this includes CP_2 besides light-cone boundary) and Kac-Moody algebra assignable to the isometries of δCD [K10]. It seems however that the longheld view about the role of Kac-Moody algebra must be modified. Also the earlier realization of super-Hamiltonians and Hamiltonians seems too naive.

(a) I have been accustomed to think that Kac-Moody algebra could be regarded as a sub-algebra of symplectic algebra. p-Adic mass calculations however requires five tensor factors for the coset representation of Super Virasoro algebra naturally assigned to the coset structure G/H of a sector of WCW with fixed zero modes. Therefore Kac-Moody algebra cannot be regarded as a sub-algebra of symplectic algebra giving only single tensor factor and thus inconsistent with interpretation of p-adic mass calculations.

(b) The localization of Kac-Moody algebra generators with respect to the internal coordinates of light-like 3-surface taking the role of complex coordinate z in conformal field theory is also questionable: the most economical option relies on localization with respect to light-like radial coordinate of light-cone boundary as in the case of symplectic algebra. Kac-Moody algebra cannot be however sub-algebra of the symplectic algebra assigned with covariantly constant right-handed neutrino in the earlier approach.

(c) Right-handed covariantly constant neutrino as a generator of super symmetries plays a key role in the earlier construction of symplectic super-Hamiltonians. What raises doubts is that other spinor modes - both those of right-handed neutrino and electroweakly charged spinor modes - are absent. All spinor modes should be present and thus provide direct mapping from WCW geometry to WCW spinor fields in accordance with super-symmetry and the general idea that WCW geometry is coded by WCW spinor fields.

   Hence it seems that Kac-Moody algebra must be assigned with the modes of the induced spinor field which carry electroweak quantum numbers. If would be natural that the modes of right-handed neutrino having no weak and color interactions would generate the huge symplectic algebra of symmetries and that the modes of fermions with electroweak charges generate much smaller Kac-Moody algebra.

(d) The dynamics of Kähler action and modified Dirac action action are invisible in the earlier construction. This suggests that the definition of WCW Hamiltonians is too simplistic. The proposal is that the conserved super charges derivable as Noether charges and identifiable as super-Hamiltonians define WCW metric and Hamiltonians as their anti-commutators. Spinor modes would become labels of Hamiltonians and WCW geometry relates directly to the dynamics of elementary particles.

(e) Note that light-cone boundary $\delta M^4_+ = S^2 \times R_+$ allows infinite-dimensional group of isometries consisting of conformal transformation of the sphere S^2 with conformal scaling compensated by an S^2 local scaling or the light-like radial coordinate of R_+. These isometries contain as a subgroup symplectic isometries and could act as gauge symmetries of the theory.

7.3 Preferred extremals of Kähler action, solutions of the modified Dirac operator, and quantum criticality

Perhaps due to my natural laziness I have not bothered to go through the basic construction [K10, K9] although several new ideas have emerged during last years [K69].

(a) The new view about preferred extremals of Kähler action involves the slicing of space-time surface to string world sheets labelled by points of any partonic two-surface or vice versa. I have called this structure Hamilton-Jacobi structure [K5]. A number theoretic interpretation based on the octonionic representation of imbedding space gamma matrices. A gauge theoretic interpretation in terms of two orthogonal 2-D spaces assignable to polarization and momentum of massless field mode is also possible. The slicing suggests duality between string world sheets and conformal field theory at partonic 2-surfaces analogous to AdS/CFT. Strong form of holography implied by strong form of GCI would be behind the duality.

(b) The new view about the solutions of modified Dirac equation involves localization of the modes at string world sheets: this emerges from the condition that electric charge is well defined quantum number for the modes. The effective 2-dimensionality of the space of the modified gamma matrices is crucial for the localization. This leads to a concrete model of elementary particles as string like objects involving two space-time sheets and flux tubes carrying Kähler magnetic monopole flux. Holomorphy and complexification of modified gamma matrices are absolutely essential consequences of the localization and is expected to be crucial also in the construction of WCW geometry. The weakest interpretation is that the general solution of modified Dirac is superposition of these localized modes parametrized by the points of partonic 2-surface and integer labelling the modes themselves as in string theory. One has the same general picture as in ordinary quantum theory.

One can wonder whether finite measurement resolution is realized dynamically in the sense that a discrete set of stringy world sheets are possible. It will be found that quantization of induced spinor fields leads to a concrete proposal realizing this: strings would be identified as curves along which Kähler magnetic field has constant value.

(c) Quantum criticality is central notion in TGD framework: Kähler coupling strength is the only coupling parameter appearing in Kähler action and is analogous to temperature. The idea of quantum criticality is that TGD Universe is quantum critical so that Kähler coupling strength is analogous to critical temperature. The hope is that this could make the theory unique. I have not however been able to really understand it and relate it to the coset space construction of WCW and to coset representations of Super Virasoro.

7.3.1 What criticality is?

The basic technical problem has been characterization of it quantitatively [K18]. Here there is still a lot of fuzzy thinking and unanswered questions. What is the precise definition of criticality and what is its relation to G/H decomposition of WCW? Could H correspond to critical deformations so that it would have purely group theoretical characterization, and one would have nice unification of two approaches to quantum TGD?

1. Does criticality correspond to the failure of classical determinism?

The intuitive guess is that quantum criticality corresponds classically to the criticality of Kähler action implying non-determinism. The preferred extremal associated with given 3-surface at the boundary of CD is not unique. There are several deformations of space-time surface *vanishing* at X^3 and leaving the Kähler action and thus Kähler function invariant.

Some nitpicking before continuing is in order.

(a) The key word is *"vanishing"* in the above definition of criticality relying on classical non-determinism. Could one allow also non-vanishing deformations of X^3 with the property that Kähler function and Kähler action are not changed? This would correspond to the idea that critical directions correspond to flat directions for the potential in quadratic approximation: now it would be Kähler function in quadratic approximation. The flat direction would not contribute to Kähler metric $G_{K\overline{L}} = \partial_K \partial_{\overline{L}}$.

Clearly, the subalgebra h associated with H would satisfy criticality in this sense for all 3-surfaces except the one for which it acts as isotropy group: in this case one would have criticality in the strong sense.

This identification of criticality is consistent with that based on non-determinism *only if* the deformations in H leaving X^3 fixed do not leave $X^4(X^3)$ fixed. This would apply also to h. One would have bundle like structure: 3-surface would represent base point of the bundle and space-time surfaces associated with it would correspond to the points in the fiber permuted by h.

(b) What about zero modes, which appear only in the conformal scaling factor of WCW metric but not in the differentials appearing the line element? Are the critical modes zero modes but only up to second order in functional Taylor expansion?

Returning to the definition of criticality relying on classical non-determinism. One can try to fix $X^4(X^3)$ uniquely by fixing 3-surface at the second end of CD but even this need not be enough? One expects non-uniqueness in smaller scales in accordance with approximate scaling invariance and fractality assignable to criticality.

A possible interpretation would be in terms of dynamical symmetry analogous to gauge symmetry assignable to H and having interpretation in terms of measurement resolution. Increasing the resolution would mean fixing X^3 at upper and lower boundaries in shorter scale. Finite measurement resolution would give rise to dynamical gauge symmetry. This conforms with the idea that TGD Universe is analogous to a Turing machine able to mimic any gauge dynamics. The hierarchy of inclusions for hyper-finite factors of type II_1 supports this view too [K60].

Criticality would be a space-time correlate for quantum non-determinism. I have assigned this nondeterminism to multi-furcations of space-time sheets giving rise to the hierarchy of Planck constants. This involves however something new: namely the idea that several alternative paths are selected in the multi-furcation simultaneously [K17, K62].

2. Further aspects of criticality

(a) Mathematically the situation at criticality of Kähler action for $X^4(X^3)$ (as distinguished from Kähler function for X^3) is analogous to that at the extremum of potential when the Hessian defined by second derivatives has vanishing determinant and there are zero modes. Now one would have an infinite number of deformations leaving Kähler action invariant in second order. What is important that critical deformations leave X^3 invariant so that they cannot correspond to the sub-algebra h except possibly at point for which H acts as an isotropy group.

(b) Criticality would suggest that conserved charges linear in deformation vanish: this because deformation vanishes at X^3. Second variation would give rise to charges to and invariance of the Kähler action in this action would mean that $\Delta S_2 = \Delta Q_2 = 0$ holds true unless effective boundary terms spoil the situation. Second order charges would be quadratic in the variation and it is not at all clear whether there is any hope about having a non-linear analog of Lie-algebra or super algebra structure. I do not know whether mathematicians have considered this kind of possibility. Yangian algebra represent involving besides Lie algebra generators also generators coming as their multilinears have some formal resemblance with this kind of non-linear structure.

(c) Supersymmetry would suggest that criticality for the Kähler action implies criticality for the modified Dirac action. The first order charges for Dirac action involve the partial derivatives of the canonical momentum currents T_k^α with respect to partial derivatives $\partial_\beta h^l$ of imbedding space coordinates just as the second order charges for Kähler action do. First order Noether charges vanish if criticality means that variation vanishes at X^3 but not at $X^4(X^3)$ since they involve linearly δh^k vanishing at X^3. Second order charges for modified Dirac action get second contribution from the modification of the induced spinor field by a term involving spin rotation and from the second variation of the modified gamma matrices. Here it is essential that derivatives of $\partial_k \delta h^l$, which need not vanish, are involved.

    Note: I use the notation ∂_α for space-time partial derivatives and ∂_k for imbedding space partial derivatives).

7.3.2 Do critical deformations correspond to Super Virasoro algebra?

One can try to formulate criticality a in terms of super-conformal algebras and their subalgebras $h_{c,m}$ for which conformal weights are integer multiples of integer m. Now I mean with super-conformal algebra also symplectic and super Kac-Moody algebras. These decompositions - call them just $g_c = t_c \oplus h_c$ need *not* correspond to $g + h$ associated with G/H although it could do so. For instance, if g_c corresponds to Super Virasoro algebra then the decomposition $g_c = t_c \oplus h_c$ does not correspond to $g = t \oplus h$.

(a) There would be a hierarchy of included sub-algebras $h_{c,m}$, which corresponds to hierarchy of conformal algebras assignable to the light-like radial coordinate of the boundary of light-cone and criticalities could form hierarchy in this sense. The algebras form inclusion hierarchies $h_{m_1} \supset h_{m_2} \supset ...$ labelled by sequences consisting of integers such that given integer is divisible by the previous integer in the sequence: $m_n \ mod \ m_{n-1} = 0$.

Critical deformations assignable to $h_{c,m}$ would vanish at preferred X^3 for which H is isotropy group and leave Kähler action invariant and would not therefore contribute to Kähler metric at X^3. They could however affect $X^4(X^3)$.

Non-critical deformation would correspond to the complement of this sub-algebra affecting both $X^4(X^3)$ and X^3. This hierarchy would correspond to an infinite hierarchy of conformal symmetry breakings and would be manifested at the level of WCW geometry. Also a connection with the inclusion hierarchy for hyper-finite factors of type II_1 [K60] having interpretation in terms of finite measurement resolution is suggested by this hierarchy. Super Virasoro generators with conformal weight coming as a multiple of m would annihilate physical states so that effectively the criticality correspond to finite-D Hilbert space. This is something new as compared to the ordinary view about criticality for which all Super Virasoro generators annihilate the states.

(b) A priori $g = t + h$ decomposition need not have anything to do with the decomposition of deformations to non-critical and critical ones. Critical deformations could indeed appear as sub-algebra of $g = t + h$ and be present for both t and h in the same manner: that is as sub-algebras of super- Virasoro algebras: Super Virasoro would represent the non-determinism and criticality and in 2-D conformal theories describing criticality this is indeed the case. In this case the actions of G and H identified as super-symplectic and super Kac-Moody algebras could be unique and non-deterministic aspect would not be present. This corresponds to the physical intuition.

If criticality corresponds to G/H structure, symmetric space property $[t, t] \subset h$ would not hold true as is clear from the additivity of super-conformal weights in the commutators of conformal algebras. The reduction of G/H structure to criticality would be very nice but personally I would give up covariant constancy of curvature tensor in infinite-dimensional context only with heavy heart.

(c) The super-symmetric relation between Kähler action and corresponding modified Dirac action suggests that the criticality of Kähler action implies vanishing conserved charges also for the modified Dirac action (both ordinary and super charges so that one has super-symmetry). The reason is that conserved charge is linear in deformation. Conservation in turn means that Kähler action is not changed: $\Delta S = \Delta Q = 0$. For non-critical deformations the boundary terms at the orbits wormhole throats imply non-conservation so that ΔQ (the difference of charges at space-like ends of space-time surface) is non-vanishing although local conservation law holds strue. This in terms implies that the contribution to the Kähler metric is non-trivial.

At criticality both bosonic and fermionic conserved currents can be assigned to the second variation and are thus quadratic in deformation just like that associated with Kähler action. If effective boundary terms vanish the criticality for Kähler action implies the conservation of second order charges by $\Delta_2 S = \Delta_2 Q = 0$.

7.3.3 Connection with the vanishing of second variation for Kähler action

There are three general conjectures related to modified Dirac equation and the conserved currents associated with the vanishing second variation of Kähler action at critical points analogous to extrema of potential function at which flat directions appear and the determinant defined by second derivatives of the potential function does not have maximal rank.

(a) Quantum criticality has as a correlate the vanishing of the second variation of Kähler action for critical deformations. The conjecture is that the number of these directions is infinite and corresponds to sub-algebras of Super Virasorol algebra corresponding to conformal weights coming as integer multiples of integer. Super Virasoro hypothesis implies that preferred extremals have same algebra of critical deformations at all points.

Noether theorem applied to critical variations gives rise to conserved currents and charges which are quadratic in deformation. For non-critical deformations one obtains linearity in deformation and this charges define the super- conformal algebras.

Super Virasoro algebra indeed has a standard representation in which generators are indeed quadratic in Kac-Moody (and symplectic generators in the recent case). This quadratic character would have interpretation in terms of criticality not allowing linear representation.

(b) Modified Dirac operator is assumed to have a solution spectrum for which both non-critical and critical deformations act as symmetries. The critical currents vanish in the first order. Second variation involving first variation for the modified gamma matrices and first variation for spinors (spinor rotation term) gives and second variation for canonical momentum currents gives conserved current. The general form of the current is very similar to the corresponding current associated with Kähler action.

(c) The currents associated with the modified Dirac action and Kähler action have same origin. In other words: the conservation of Kähler currents implies the conservation of the currents associated with the modes of the modified Dirac operator. A question inspired by quantum classical correspondence is whether the eigen values of the fermionic charges correspond to the values of corresponding classical conserved charges for Kähler action in the Cartan algebra. This would imply that all space-time surfaces in superposition representing momentum eigen state have the same value of classical four-momentum. A stronger statement of QCC would be that classical correlation functions are same as the quantal ones.

7.4 Quantization of the modified Dirac action

The quantization of the modified Dirac action follows standard rules.

(a) The general solution is written as a superposition of modes, which are for other fermions than ν_R localized to string world sheets and parametrized by a point of partonic 2-surface which can be chosen to be the intersection of light-like 3-surface at which induced metric changes signature with the boundary of CD.

(b) The anti-commutations for the induced spinor fields are dictated from the condition that the anti-commutators of the super-Hamiltonians identified as WCW gamma matrices give WCW Hamiltonians as matrix elements of WCW metric. Super Hamiltonians are identified as Noether charges for the modified Dirac action assignable to the symplectic algebra of δCD being labelled also by the quantum numbers labelling the modes of the induced spinor field.

(c) Consistency conditions for the modified Dirac operator require that the modified gamma matrices have vanishing divergence: this is true for the extremals of Kähler action.

(d) The guess for the critical algebra is as sub-algebra of Super Virasoro algebra affecting on the radial light-like coordinate of δCD as diffeomorphisms. The deformations of the modified Dirac operator should annihilate spinor modes. This requires that the deformation corresponds to a gauge transformation for the induced gauge fields. Furthermore, the deformation for the modified gamma matrices determined by the deformation of the canonical momentum densities contracted covariant derivatives should annihilate the spinor modes. The situation is analogous to that for massless Dirac operator: Dirac equation for momentum eigenstate does not imply vanishing of the momentum but only that of mass. The condition that the divergence for the deformation of the modified gamma matrices vanishes as does also the divergence of the modified gamma matrices implies that the second variation of Kähler action vanishes. One obtains classical Kähler charges and Dirac charges: the latter act as operators. The equivalence of the two definitions of of four-momenta would corresponds to EP and QCC.

(e) An interesting question of principle is what the almost topological QFT property meaning that Kähler action reduces to Chern-Simons form integrated over boundary of space-time and over the light-like 3-surfaces means. Could one write the currents in terms of Chern-Simons form alone? Could one use also Chern-Simons analog of modified Dirac action. What looks like problem at the first glance is that only the charges associated with the symplectic group of CP_2 would be non-vanishing. Here the weak form of electric-magnetic duality [K18, K69] however introduce constraint terms to the action implying that all charges can be non-vanishing.

The challenge is to construct explicit representations of super charges and demonstrate that suitably defined anti-commutations for spinor fields reproduce the anti-commutations of the super-symplectic algebra.

7.4.1 Integration measure in the superposition over modes

One can express Ψ as a superposition over modes as usually. Except for ν_R, the modes are localized at string world sheets and can be labelled by a point of X^2, integer characterizing the mode and analogous to conformal weight, and quantum numbers characterizing spin, electroweak quantum numbers, and M^4 handedness. The de-localization of the modes of ν_R decouple from left-handed neutrino if the modified gamma matrices involved only M^4 or CP_2 gamma matrices. It might be possible to choose the string coordinate to be light-like radial coordinate of δCD but this is by no means necessary.

The integration measure $d\mu$ in the superposition of modes has nothing to do with the metric determinants assignable to 3-surface X^3 or with the corresponding space-time surface at X^3. $d\mu$ at partonic 1-surface X^2 must be taken to be such that its square multiplied by transversal delta function resulting in anti-commutation of two modes gives a measure defined by the Kähler form $J_{\mu\nu}$ and given by $d\mu = J_{\mu\nu}dx^\mu dx^\nu = J\sqrt{g_2}dx^1 \wedge dx^2$, $J = J_{\mu\nu}\epsilon^{\mu\nu}$ (note that permutation tensor is inversely proportional $\sqrt{g_2}$). This measure appears in the earlier definition of WCW Hamiltonian as the analog of flux integral $\oint H_A J dx^1 \wedge dx^2$, where H_A is Hamiltonian to be replaced with its integral over string.

There are two manners to get J to the measure for Hamiltonian flux.

- Option I: One uses for super charges has "half integration measure" given by $d\mu_{1/2} = \sqrt{J\sqrt{g_2}}dx^1 \times dx^2$. Note that $\sqrt{J}$ is imaginary for $J < 0$ and also the unique choice of sign of the square root might produce problems.
- Option II: The integration measure is $d\mu = J(x,end)\sqrt{g_2}dx^1 \wedge dx^2$ for the super charge and anti-commutations of Ψ at string are proportional to $1/J(x,end)\sqrt{g_2}$ so that anti-commutator of supercharges would be proportional to $J(x,end)\sqrt{g_2}$ and metric determinant disappears from the integration measure. Note that the vanishing of $J(x,end)$ does not produce any problems in anti-commutators.

  $J(x,end)$ means a non-locality in the anti-commutator. If the string is interpreted as beginning from the partonic surface at its second end, one obtains two different anti-commutation relations unless strings are $J(x,y)\sqrt{g_2} = constant$ curves. This could make sense for flux tubes which are indeed assumed to carry the Kähler flux. Note also that partonic 2-surface decomposes naturally into regions with fixed sign of J forming flux tubes.

  $J(x,y)\sqrt{g_2} = constant$ condition seems actually trivial. The reason is that by a suitable coordinate transformations $(x,y) \rightarrow (f(x,y)$ leaving string coordinate invariant the $\sqrt{g_2}$ gains a factor equal to the Jacobian of the transformation which reduces to 2-D Jacobian for the transformation for the coordinates of partonic 2-surface. By a suitable choice of this transformation $J(x,y)\sqrt{g_2} = constant$ condition is satisfied along string world sheets. This transformation is determined only modulo an area preserving - thus symplectic - transformation for each partonic 2-surface in the slicing. One obtains space-time analog of symplectic invariance as an additional symmetry having identification as a remnant of 3-D GCI. Since also string parameterizations $t \rightarrow f(t)$ are allowed so that 3-D GCI reduces to 1-D Diff and 2-D Sympl. Natural 4-D extension of string reparameterizations would be to the analogs of conformal transformations associated with the effective metric defined by modified gamma matrices so that 4-D Diff would reduce to a product of 2-D conformal and symplectic groups.

The physical state is specified by a finite number of fermion number carrying string world sheets (one can of course have a superposition of these states with different locations of string world sheets). One can ask whether QCC forces the space-time surface to code this state in its geometry in the sense that only these string world sheets are possible. $J(x,y)\sqrt{g_2} = constant$ condition does not force this.

- Option III: If one assumes slicing by partonic 2-surfaces with common coordinates $x = (x^1, x^2)$ and that $J(x,y)\sqrt{g}$ is included to current density at the point of string and that $1/J(x,y)\sqrt{g_2}$ in the anti-commutations is evaluated at the point x of the partonic surface intersecting the string at y, the flux is replaced with the superposition of local fluxes from all points in the slicing by partonic 2-surfaces and $J(x,y)$. For $J\sqrt{g_2}$= constant along strings Options II an III are equivalent.

On basis of physical picture Option II with $J\sqrt{g_2}$= constant achieved by a proper choice of partonic coordinates for the slicing looks very attractive.

7.4.2 Fermionic supra currents as Noether currents

Fermionic supra currents can be taken as Noether currents assignable to the modified Dirac action. Charges are obtained by integrating over string. Here possible technical problems relate to the correct identification of the integration measure. In the normal situation the integration measure is $\sqrt{g_4}$ but now 2-D delta function restricts the charge density for a given mode to the string world sheet and might produce additional factors.

The general form of the super current at given string world sheet corresponding to a given string world sheet is given by

$$\begin{aligned} J^\alpha &= \left[\overline{\Psi_n} O^\alpha_{\beta,k} \delta h^k D_\alpha \Psi + \overline{\Psi_n} \Gamma^\alpha \delta\Psi\right] \sqrt{g_4} \ , \\ O^\alpha_{\beta,k} &= \frac{\partial \Gamma^\alpha}{\partial(\partial_\beta h^k)} \ . \end{aligned} \tag{7.4.1}$$

The covariant divergence of J^α vanishes. Modified gamma matrices appearing in the equation are defined as contractions of the canonical momentum densities T^α_k of Kähler action with imbedding space gamma matrices Γ^k as

$$\begin{aligned} \Gamma^\alpha &= T^\alpha_k \Gamma^k \ , \\ T^\alpha_k &= \frac{L_K}{\partial(\partial_\beta h^k)} \ , \end{aligned} \tag{7.4.2}$$

Ψ_n is the mode of induced spinor field considered. $\delta\Psi$ is the change of Ψ in spin rotation given by

$$\delta\Psi = \partial_l j_k \Sigma^{kl} \ . \tag{7.4.3}$$

The corresponding current is obtained by replacing Ψ_n with Ψ and integrating over the modes.

The current could quite well vanish. The reason is that holography means that one half of modified gamma matrices whose number is effectively 2 annihilates the spinor modes. Also the covariant derivative D_z or $D_{\overline{z}}$ annihilates it. One obtains vanishing result if the quantity $O^z_{\beta,k}$ is proportional to Γ^z. This can be circumvented if it is superposition of gamma matrices which are not parallel to the string world sheet or if is superposition of Γ^z and $\Gamma^{\overline{z}}$: this could have interpretation as breaking of conformal invariance.

For critical deformations vanishing at X^3 δh^k appearing in the formula of current vanishes so that one obtains non-vanishing charge only for second variation.

Note that the quantity $O^\alpha_{\beta,k}$ involves terms $J^{\alpha k}J^l_\beta$ and can be non-vanishing even when J vanishes. The replacement of ordinary γ^0 in fermionic anti-commutation relations with the modified gamma matrix Γ^0 helps here since modified gamma matrices vanish when J vanishes.

Note that for option II favoured by the existing physical picture J is constant along the strings and anti-commutation relations are non-singular for $J \neq 0$.

7.4.3 Anti-commutators of super-charges

The anti-commutators for fermionic fields- or more generally, quantities related to them - should be such that the anti-commutator of fermionic super-Hamiltonians defines WCW Hamiltonian with correct group theoretical properties. To obtain the correct anti-commutator requires that one obtains Poisson bracket of δCD Hamiltonians appearing in the super-Hamiltonians. This is the case if the anti-commutator involved is proportional to iJ_{kl} since this gives the desired Poisson bracket

$$J_{kl}j^k_A j^l_B = \{H_A, H_B\} \ . \tag{7.4.4}$$

This is achieved if one replaces the anti-commutators of Ψ and $\overline{\Psi}$ with anti-commutator of $A_k \equiv O^0_k\Psi$ and $\overline{A_l} \equiv \overline{\Psi}O^0_l$ (O^α_k was defined in Eq. 7.4.1) and assumes

$$\{A_k, \overline{A}_l\} = iJ_{kl}\Gamma^0\delta_2(x_2,y_2)\delta_1(y_1,y_2)\frac{X}{g_4^{1/2}} \ . \tag{7.4.5}$$

Here Γ^0 is modified gamma matrix and δ_2 is delta function assignable to the partonic 2-surface and δ_1 is delta function assignable with the string. Depending on whether one assumes option I, II, or III one has $X = 1$, $X = 1/J_{x,end}$ or $1/J(x_1,x_2,y)$.

The modified anti-commutation relations do not make sense in higher imbedding space dimensions since the number of spinor components exceeds imbedding space dimension. For $D = 8$ the dimension of H and the number of independent spinor components with given H-chirality are indeed same (leptons and quarks have opposite H-chirality). This makes the dimension $D = 8$ unique in TGD framework.

7.4.4 Strong form of General Coordinate Invariance and strong form of holography

Strong form of general coordinate invariance (GCI) suggests a duality between descriptions using light-like 3-surfaces X^3_l at which the signature of the induced metric changes and space-like 3-surface X^3 at the ends of the space-time surface. Also the translates of these surfaces along slicing might define the theory but with a Kähler function to which real part of a holomorphic function defined in WCW is added.

In order to define the formalism for light-like 3-surfaces, one should be able to define the symplectic algebra. This requires the translation of the boundaries of the light-cone along the line connecting the tips of the CD so that the Hamiltonians of δM^4_+ or δM^4_- make sense at X^3_l. Depending on whether the the state function reduction has occurred on upper or lower boundary of CD one must use translates of δM^4_+ or δM^4_-: this would be one particular manifestation for the arrow of time.

7.4.5 Radon, Penrose ja TGD

The construction of the induced spinor field as a superposition of modes restricted to string world sheets to have well-defined em charge (except in the case of right-handed neutrino) brings in mind Radon transform [A21] (http://en.wikipedia.org/wiki/Radon_transform) and Penrose transform [A19] (http://en.wikipedia.org/wiki/Penrose_transform). In Radon transform the function defined in Euclidian space E^n is coded by its integrals over $n-1$ dimensional hyper-planes. All planes are allowed and are characterized by their normal whose direction corresponds to a point of $n-1$-dimensional sphere S^{n-1} and by the orthogonal distance of the plane from the origin. Note that the space of hyper-planes is n-dimensional as it should be if it is to carry same information as the function itself. One can easily demonstrate that n-dimensional Fourier transform is composite of 1-dimensional Fourier transform in the direction normal vector parallel to wave vector obtained integrating over the distance parameter associated with $n-$dimensional Radon transform defined by function multplied by the plane wave.

In the case of Penrose transform [A19] (http://en.wikipedia.org/wiki/Penrose_transform) one has 6-dimensional twistor space CP_3 and the space of complex two - planes- topologically spheres in CP_3 - one for each point of in CP_3 - defines 4-D compactified Minkowski space. A massless field in M^4 has a representation in CP_3 with field value at given point of M^4 represented as an integral over S^3 of holomorphic field in CP_3.

In the recent case the situation resembles very much that for Penrose transform. In the case of space-like 3-surface CP_3 is replaced with the space of strings emanating from the partonic 2-surface and its points are labelled by points of partonic 2-surface and points of string so that dimension is still $D = 3$. The transform describes second quantize spinor field as a collection of "Fourier components" along stringy curves. In 4-D case one has 4-D space-time surface and collection of "Fourier components" along string world sheets. One could say that charge densities assignable to partonic 2-surfaces replace the massless fields in M^4. Now however the decomposition into strings and string world sheets takes place at the level of physics rather than only mathematically.

7.5 About the notion of four-momentum in TGD framework

The starting point of TGD was the energy problem of General Relativity [K56]. The solution of the problem was proposed in terms of sub-manifold gravity and based on the lifting of the isometries of space-time surface to those of $M^4 \times CP_2$ in which space-times are realized as 4-surfaces so that Poincare transformations act on space-time surface as an 4-D analog of rigid body rather than moving points at space-time surface. It however turned out that the situation is not at all so simple.

There are several conceptual hurdles and I have considered several solutions for them. The basic source of problems has been Equivalence Principle (EP): what does EP mean in TGD framework [K56, K78]? A related problem has been the interpretation of gravitational and inertial masses, or more generally the corresponding 4-momenta. In General Relativity based cosmology gravitational mass is not conserved and this seems to be in conflict with the conservation of Noether charges. The resolution is in terms of zero energy ontology (ZEO), which however forces to modify slightly the original view about the action of Poincare transformations.

A further problem has been quantum classical correspondence (QCC): are quantal four-momenta associated with super conformal representations and classical four-momenta associated as Noether charges with Kähler action for preferred extremals identical? Could inertial-gravitational duality - that is EP - be actually equivalent with QCC? Or are EP and QCC independent dualities. A powerful experimental input comes p-adic mass calculations [K74] giving excellent predictions provided the number of tensor factors of super-Virasoro representations is five, and this input together with Occam's razor strongly favors QCC=EP identification.

There is also the question about classical realization of EP and more generally, TGD-GRT correspondence.

Twistor Grassmannian approach has meant a technical revolution in quantum field theory (for attempts to understand and generalize the approach in TGD framework see [K61, K44]. This approach seems to be extremely well suited to TGD and I have considered a generalization of this approach from $\mathcal{N} = 4$ SUSY to TGD framework by replacing point like particles with string world sheets in TGD sense and super-conformal algebra with its TGD version: the fundamental objects are now massless fermions which can be regarded as on mass shell particles also in internal lines (but with unphysical helicity). The approach solves old problems related to the realization of stringy amplitudes in TGD framework, and avoids some problems of twistorial QFT (IR divergences and the problems due to non-planar diagrams). The Yangian variant of 4-D conformal symmetry is crucial for the approach in $\mathcal{N} = 4$ SUSY, and implies the recently introduced notion of amplituhedron [B6]. A Yangian generalization of various super-conformal algebras seems more or less a "must" in TGD framework. As a consequence, four-momentum is expected to have characteristic multilocal contributions identifiable as multipart on contributions now and possibly relevant for the understanding of bound states such as hadrons.

7.5.1 Scale dependent notion of four-momentum in zero energy ontology

Quite generally, General Relativity does not allow to identify four-momentum as Noether charges but in GRT based cosmology one can speak of non-conserved mass [K46], which seems to be in conflict with the conservation of four-momentum in TGD framework. The solution of the problem comes in terms of zero energy ontology (ZEO) [K4, K70], which transforms four-momentum to a scale dependent notion: to each causal diamond (CD) one can assign four-momentum assigned with say positive energy part of the quantum state defined as a quantum superposition of 4-surfaces inside CD.

ZEO is necessary also for the fusion of real and various p-adic physics to single coherent whole. ZEO also allows maximal "free will" in quantum jump since every zero energy state can be created from vacuum and at the same time allows consistency with the conservation laws. ZEO has rather dramatic implications: in particular the arrow of thermodynamical time is predicted to vary so that second law must be generalized. This has especially important implications in living matter, where this kind of variation is observed.

More precisely, this superposition corresponds to a spinor field in the "world of classical worlds" (WCW) [K70]: its components - WCW spinors - correspond to elements of fermionic Fock basis for a given 4-surface - or by holography implied by general coordinate invariance (GCI) - for 3-surface having components at both ends of CD. Strong form of GGI implies strong form of holography (SH) so that partonic 2-surfaces at the ends of space-time surface plus their 4-D tangent space data are enough to fix the quantum state. The classical dynamics in the interior is necessary for the translation of the outcomes of quantum measurements to the language of physics based on classical fields, which in turn is reduced to sub-manifold geometry in the extension of the geometrization program of physics provided by TGD.

Holography is very much reminiscent of QCC suggesting trinity: GCI-holography-QCC. Strong form of holography has strongly stringy flavor: string world sheets connecting the wormhole throats appearing as basic building bricks of particles emerge from the dynamics of induced spinor fields if one requires that the fermionic mode carries well-defined electromagnetic charge [K69].

7.5.2 Are the classical and quantal four-momenta identical?

One key question concerns the classical and quantum counterparts of four-momentum. In TGD framework classical theory is an exact part of quantum theory. Classical four-momentum corresponds to Noether charge for preferred extremals of Kähler action. Quantal four-momentum in turn is assigned with the quantum superposition of space-time sheets assigned with CD - actually WCW spinor field analogous to ordinary spinor field carrying fermionic degrees of freedom as analogs of spin. Quantal four-momentum emerges just as it does in super string models - that is as a parameter associated with the representations of super-conformal algebras. The precise action of translations in the representation remains poorly specified. Note that quantal four-momentum does not emerge as Noether charge: at at least it is not at all obvious that this could be the case.

Are these classical and quantal four-momenta identical as QCC would suggest? If so, the Noether four-momentum should be same for all space-time surfaces in the superposition. QCC suggests that also the classical correlation functions for various general coordinate invariant local quantities are same as corresponding quantal correlation functions and thus same for all 4-surfaces in quantum superposition - this at least in the measurement resolution used. This would be an extremely powerful constraint on the quantum states and to a high extend could determined the U-, M-, and S-matrices.

QCC seems to be more or less equivalent with SH stating that in some respects the descriptions based on classical physics defined by Kähler action in the interior of space-time surface and the quantal description in terms of quantum states assignable to the intersections of space-like 3-surfaces at the boundaries of CD and light-like 3-surfaces at which the signature of induced metric changes. SH means effective 2-dimensionality since the four-dimensional tangent space data at partonic 2-surfaces matters. SH could be interpreted as Kac-Mody and symplectic symmetries meaning that apart from central extension they act almost like gauge symmetries in the interiors of space-like 3-surfaces at the ends of CD and in the interiors of light-like 3-surfaces representing orbits of partonic 2-surfaces. Gauge conditions are replaced with Super Virasoro conditions. The word "almost" is of course extremely important.

7.5.3 What Equivalence Principle (EP) means in quantum TGD?

EP states the equivalence of gravitational and inertial masses in Newtonian theory. A possible generalization would be equivalence of gravitational and inertial four-momenta. In GRT this correspondence cannot be realized in mathematically rigorous manner since these notions are poorly defined and EP reduces to a purely local statement in terms of Einstein's equations.

What about TGD? What could EP mean in TGD framework?

(a) Is EP realized at both quantum and space-time level? This option requires the identification of inertial and gravitational four-momenta at both quantum and classical level. It is now clear that at classical level EP follows from very simple assumption that GRT space-time is obtained by lumping together the space-time sheets of the many-sheeted space-time and by the identification the effective metric as sum of M^4 metric and deviations of the induced metrics of space-time sheets from M^2 metric: the deviations indeed define the gravitational field defined by multiply topologically condensed test particle. Similar description applies to gauge fields. EP as expressed by Einstein's equations would follow from Poincare invariance at microscopic level defined by TGD space-time. The effective fields have as sources the energy momentum tensor and YM currents defined by topological inhomogenities smaller than the resolution scale.

(b) QCC would require the identification of quantal and classical counterparts of both gravitational and inertial four-momenta. This would give three independent equivalences, say

$P_{I,class} = P_{I,quant}$, $P_{gr,class} = P_{gr,quant}$, $P_{gr,class} = P_{I,quant}$, which imply the remaining ones.

Consider the condition $P_{gr,class} = P_{I,class}$. At classical level the condition that the standard energy momentum tensor associated with Kähler action has a vanishing divergence is guaranteed if Einstein's equations with cosmological term are satisfied. If preferred extremals satisfy this condition they are constant curvature spaces for non-vanishing cosmological constant. A more general solution ansatz involves several functions analogous to cosmological constant corresponding to the decomposition of energy momentum tensor to terms proportional to Einstein tensor and several lower-dimensional projection operators [K78]. It must be emphasized that field equations are extremely non-linear and one must also consider preferred extremals (which could be identified in terms of space-time regions having so called Hamilton-Jacobi structure): hence these proposals are guesses motivated by what is known about exact solutions of field equations.

Consider next $P_{gr,class} = P_{I,class}$. At quantum level I have proposed coset representations for the pair of super conformal algebras g and $h \subset g$ which correspond to the coset space decomposition of a given sector of WCW with constant values of zero modes. The coset construction would state that the differences of super-Virasoro generators associated with g resp. h annhilate physical states.

The identification of the algebras g and h is not straightforward. The algebra g could be formed by the direct sum of super-symplectic and super Kac-Moody algebras and its sub-algebra h for which the generators vanish at partonic 2-surface considered. This would correspond to the idea about WCW as a coset space G/H of corresponding groups (consider as a model $CP_2 = SU(3)/U(2)$ with $U(2)$ leaving preferred point invariant). The sub-algebra h in question includes or equals to the algebra of Kac-Moody generators vanishing at the partonic 2-surface. A natural choice for the preferred WCW point would be as maximum of Kähler function in Euclidian regions: positive definiteness of Kähler function allows only single maximum for fixed values of zero modes). Coset construction states that differences of super Virasoro generators associated with g and h annihilate physical states. This implies that corresponding four-momenta are identical that is Equivalence Principle.

(c) Does EP at quantum level reduce to one aspect of QCC? This would require that classical Noether four-momentum identified as inertial momentum equals to the quantal four-momentum assignable to the states of super-conformal representations and identifiable as gravitational four-momentum. There would be only one independent condition: $P_{class} \equiv P_{I,class} = P_{gr,quant} \equiv P_{quant}$.

Holography realized as AdS/CFT correspondence states the equivalence of descriptions in terms of gravitation realized in terms of strings in 10-D space-time and gauge fields at the boundary of AdS. What is disturbing is that this picture is not completely equivalent with the proposed one. In this case the super-conformal algebra would be direct sum of super-symplectic and super Kac-Moody parts.

Which of the options looks more plausible? The success of p-adic mass calculations [K74] have motivated the use of them as a guideline in attempts to understand TGD. The basic outcome was that elementary particle spectrum can be understood if Super Virasoro algebra has five tensor factors. Can one decide the fate of the two approaches to EP using this number as an input?

This is not the case. For both options the number of tensor factors is five as required. Four tensor factors come from Super Kac-Moody and correspond to translational Kac-Moody type degrees of freedom in M^4, to color degrees of freedom and to electroweak degrees of freedom ($SU(2) \times U(1)$). One tensor factor comes from the symplectic degrees of freedom in $\Delta CD \times CP_2$ (note that Hamiltonians include also products of δCD and CP_2 Hamiltonians so that one does not have direct sum!).

The reduction of EP to the coset structure of WCW sectors is extremely beautiful property. But also the reduction of EP to QCC looks very nice and deep. It is of course possible that the two realizations of EP are equivalent and the natural conjecture is that this is the case.

For QCC option the GRT inspired interpretation of Equivalence Principle at space-time level remains to be understood. Is it needed at all? The condition that the energy momentum tensor of Kähler action has a vanishing divergence leads in General Relativity to Einstein equations with cosmological term. In TGD framework preferred extremals satisfying the analogs of Einstein's equations with several cosmological constant like parameters can be considered.

Should one give up this idea, which indeed might be wrong? Could the divergence of of energy momentum tensor vanish only asymptotically as was the original proposal? Or should one try to generalize the interpretation? QCC states that quantum physics has classical correlate at space-time level and implies EP. Could also quantum classical correspondence itself have a correlate at space-time level. If so, space-time surface would able to represent abstractions as statements about statements about.... as the many-sheeted structure and the vision about TGD physics as analog of Turing machine able to mimic any other Turing machine suggest.

7.5.4 TGD-GRT correspondence and Equivalence Principle

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see fig. `http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg` or fig. 11 in the appendix of this book).

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

(c) Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

(d) The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K78].

7.5.5 How translations are represented at the level of WCW?

The four-momentum components appearing in the formulas of super conformal generators correspond to infinitesimal translations. In TGD framework one must be able to identify these infinitesimal translations precisely. As a matter of fact, finite measurement resolution implies that it is probably too much to assume infinitesimal translations. Rather, finite exponentials of translation generators are involved and translations are discretized. This does not have practical signficance since for optimal resolution the discretization step is about CP_2 length scale.

Where and how do these translations act at the level of WCW? ZEO provides a possible answer to this question.

Discrete Lorentz transformations and time translations act in the space of CDs: inertial four-momentum

Quantum state corresponds also to wave function in moduli space of CDs. The moduli space is obtained from given CD by making all boosts for its non-fixed boundary: boosts correspond to a discrete subgroup of Lorentz group and define a lattice-like structure at the hyperboloid for which proper time distance from the second tip of CD is fixed to $T_n = n \times T(CP_2)$. The quantization of cosmic redshift for which there is evidence, could relate to this lattice generalizing ordinary 3-D lattices from Euclidian to hyperbolic space by replacing translations with boosts (velocities).

The additional degree of freedom comes from the fact that the integer $n > 0$ obtains all positive values. One has wave functions in the moduli space defined as a pile of these lattices defined at the hyperboloid with constant value of $T(CP_2)$: one can say that the points of this pile of lattices correspond to Lorentz boosts and scalings of CDs defining sub-WCW:s.

The interpretation in terms of group which is product of the group of shifts $T_n(CP_2) \rightarrow T_{n+m}(CP_2)$ and discrete Lorentz boosts is natural. This group has same Cartesian product structure as Galilean group of Newtonian mechanics. This would give a discrete rest energy and by Lorentz boosts discrete set of four-momenta giving a contribution to the four-momentum appearing in the super-conformal representation.

What is important that each state function reduction would mean localisation of either boundary of CD (that is its tip). This localization is analogous to the localization of particle in position measurement in E^3 but now discrete Lorentz boosts and discrete translations $T_n - - > T_{n+m}$ replace translations. Since the second end of CD is necessary del-ocalized in moduli space, one has kind of flip-flop: localization at second end implies de-localization at the second end. Could the localization of the second end (tip) of CD in moduli space correspond to our experience that momentum and position can be measured simultaneously? This apparent classicality would be an illusion made possible by ZEO.

The flip-flop character of state function reduction process implies also the alternation of the direction of the thermodynamical time: the asymmetry between the two ends of CDs would induce the quantum arrow of time. This picture also allows to understand what the experience growth of geometric time means in terms of CDs.

The action of translations at space-time sheets

The action of imbedding space translations on space-time surfaces possibly becoming trivial at partonic 2-surfaces or reducing to action at δCD induces action on space-time sheet which becomes ordinary translation far enough from end end of space-time surface. The four-momentum in question is very naturally that associated with Kähler action and would therefore correspond to inertial momentum for $P_{I,class} = P_{quant,gr}$ option. Indeed, one cannot assign quantal four-momentum to Kähler action as an operator since canonical quantization badly fails. In finite measurement infinitesimal translations are replaced with their exponentials for $P_{I,class} = P_{quant,gr}$ option.

What looks like a problem is that ordinary translations in the general case lead out from given CD near its boundaries. In the interior one expects that the translation acts like ordinary translation. The Lie-algebra structure of Poincare algebra including sums of translation generators with positive coefficient for time translation is preserved if only time-like superpositions if generators are allowed also the commutators of time-like translation generators with boost generators give time like translations. This defines a Lie-algebraic formulation for the arrow of geometric time. The action of time translation on preferred extremal would be ordinary translation plus continuation of the translated preferred extremal backwards in time to the boundary of CD. The transversal space-like translations could be made Kac-Moody algebra by multiplying them with functions which vanish at δCD.

A possible interpretation would be that $P_{quant,gr}$ corresponds to the momentum assignable to the moduli degrees of freedom and $P_{cl,I}$ to that assignable to the time like translations. $P_{quant,gr} = P_{cl,I}$ would code for QCC. Geometrically quantum classical correspondence would state that time-like translation shift both the interior of space-time surface and second boundary of CD to the geometric future/past while keeping the second boundary of space-time surface and CD fixed.

7.5.6 Yangian and four-momentum

Yangian symmetry implies the marvellous results of twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY culminating in the notion of amplituhedron which promises to give a nice projective geometry interpretation for the scattering amplitudes [B6]. Yangian symmetry is a multilocal generalization of ordinary symmetry based on the notion of co-product and implies that Lie algebra generates receive also multilocal contributions. I have discussed these topics from slightly different point of view in [K61], where also references to the work of pioneers can be found.

Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [K61] . Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant and and concrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [B18] . Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N} = 4$ SUSY). One of the conditions conditions is that the tensor product $R \otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n+1$-local in the sense that they involve $n+1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, it is not much to say. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

(a) The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A12] and Virasoro algebras [A23] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.

(b) The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.

(c) This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context)?

(a) At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.

(b) Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.

(c) The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

(d) Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

Chapter 8

Unified Number Theoretical Vision

8.1 Introduction

Octonions, quaternions, quaternionic space-time surfaces, octonionic spinors and twistors and twistor spaces are highly relevant for quantum TGD. In the following some general observations distilled during years are summarized. This summary involves several corrections to the picture which has been developing for a decade or so.

A brief updated view about M^8-H duality and twistorialization is in order. There is a beautiful pattern present suggesting that M^8-H duality makes sense and that $H=M^4\times CP_2$ is completely unique on number theoretical grounds.

(a) M^8-H duality allows to deduce $M^4\times CP_2$ via number theoretical compactification. For the option with minimal number of conjectures the associativity/co-associativity of the space-time surfaces in M^8 guarantees that the space-time surfaces in M^8 define space-time surfaces in H. The tangent/normal spaces of quaternionic/hyper-quaternionic surfaces in M^8 contain also an integrable distribution of hyper-complex tangent planes $M^2(x)$.

An important correction is that associativity/co-associativity does not make sense at the level of H since the spinor structure of H is already complex quaternionic and reducible to the ordinary one by using matrix representations for quaternions. The associativity condition should however have some counterpart at level of H. One could require that the induced gamma matrices at each point could span a *real*-quaternionic sub-space of complexified quaternions for quaternionicity and a purely imaginary quaternionic sub-space for co-quaternionicity. One might hope that it is consistent with - or even better, implies - preferred extremal property. I have not however found a viable definition of quaternionic "reality". On the other hand, it is possible assigne the tangent space M^8 of H with octonion structure and define associativity as in case of M^8.

M^8-H duality could generalize to $H-H$ duality in the sense that also the image of the space-time surface under duality map is not only preferred extremal but also associative (co-associative) surface. The duality map $H\rightarrow H$ could be iterated and would define the arrow for the category formed by preferred extremals.

(b) M^4 and CP_2 are the unique 4-D spaces allowing twistor space with Kähler structure. M^8 allows twistor space for octonionic spinor structure obtained by direct generalization of the standard construction for M^4. $M^4\times CP_2$ spinors can be regarded as tensor products of quaternionic spinors associated with M^4 and CP_2: this trivial observation forces to challenge the earlier rough vision, which however seems to stand up the challenge.

(c) Octotwistors generalise the twistorial construction from M^4 to M^8 and octonionic gamma matrices make sense also for H with quaternionicity condition reducing 12-D

$T(M^8) = G_2/U(1) \times U(1)$ to the 12-D twistor space $T(H) = CP_3 \times SU^3/U(1) \times U(1)$. The interpretation of the twistor space in the case of M^8 is as the space of choices of quantization axes for the 2-D Cartan algebra of G_2 acting as octonionic automorphisms. For CP_2 one has space for the chocies of quantization axes for the 2-D $SU(3)$ Cartan algebra.

(d) It is also possible that the dualities extend to a sequence $M^8 \to H \to H$... by mapping the associative/co-associative tangent space to CP_2 and M^4 point to M^4 point at each step. One has good reasons to expect that this iteration generates fractal as the limiting space-time surface.

(e) A fascinating structure related to octo-twistors is the non-associated analog of Lie group defined by automorphisms by octonionic imaginary units: this group is topologically 7-sphere. Second analogous structure is the 7-D Lie algebra like structure defined by octonionic analogs of sigma matrices.

The analogy of quaternionicity of M^8 pre-images of preferred extremals and quaternionicity of the tangent space of space-time surfaces in H with the Majorana condition central in super string models is very thought provoking. All this suggests that associativity at the level of M^8 indeed could define basic dynamical principle of TGD.

Number theoretical vision about quantum TGD involves both p-adic number fields and classical number fields and the challenge is to unify these approaches. The challenge is non-trivial since the p-adic variants of quaternions and octonions are not number fields without additional conditions. The key idea is that TGD reduces to the representations of Galois group of algebraic numbers realized in the spaces of octonionic and quaternionic adeles generalizing the ordinary adeles as Cartesian products of all number fields: this picture relates closely to Langlands program. Associativity would force sub-algebras of the octonionic adeles defining 4-D surfaces in the space of octonionic adeles so that 4-D space-time would emerge naturally. $M^8 - H$ correspondence in turn would map the space-time surface in M^8 to $M^4 \times CP_2$.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L13]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L14]. The topics relevant to this chapter are given by the following list.

- Physics as generalized number theory [L27]
- Quantum physics as generalized number theory [L30]
- TGD and classical number fields [L35]
- $M^8 - H$ duality [L26]
- Basic notions behind $M^8 - H$ duality [L16]
- Quaternionic planes of octonions [L32]

8.2 Number theoretic compactification and $M^8 - H$ duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to associativity or co-associativity. Originally $M^8 - H$ duality was introduced as a number theoretic explanation for $H = M^4 \times CP_2$. Much later it turned out that the completely exceptional twistorial properties of M^4 and CP_2 are enough to justify $X^4 \subset H$ hypothesis. Skeptic could therefore criticize the introduction of M^8 (actually its complexification) as an un-necessary mathematical complication producing only unproven conjectures and bundle of new statements to be formulated precisely. However, if quaternionicity can be realized in terms of M_c^8 using O_c-real analytic functions and if quaternionicity is equivalent with preferred extremal property, a huge simplification results and one can say that field equations are exactly solvable.

One can question the feasibility of $M^8 - H$ duality if the dynamics is purely number theoretic at the level of M^8 and determined by Kähler action at the level of H. Situation becomes more democratic if Kähler action defines the dynamics in both M^8 and H: this might mean that associativity could imply field equations for preferred extremals or vice versa or there might be equivalence between two. This means the introduction Kähler structure at the level of M^8, and motivates also the coupling of Kähler gauge potential to M^8 spinors characterized by Kähler charge or em charge. One could call this form of duality strong form of $M^8 - H$ duality.

The strong form $M^8 - H$ duality boils down to the assumption that space-time surfaces can be regarded either as 4-surfaces of H or as surfaces of M^8 or even M^8_c composed of associative and co-associative regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric. They have the same induced metric and Kähler form and WCW associated with H should be essentially the same as that associated with M^8. Associativity corresponds to hyper-quaterniocity at the level of tangent space and co-associativity to co-hyper-quaternionicity - that is associativity/hyper-quaternionicity of the normal space. Both are needed to cope with known extremals. Since in Minkowskian context precise language would force to introduce clumsy terms like hyper-quaternionicity and co-hyper-quaternionicity, it is better to speak just about associativity or co-associativity.

Remark: The original assumption was that space-times could be regarded as surfaces in M^8 rather than in its complexification M^8_c identifiable as complexified octonions. This assumption is un-necessarily strong and if one assumes that octonion-real analytic functions characterize these surfaces M^8_c must be assumed.

For the octonionic spinor fields the octonionic analogs of electroweak couplings reduce to mere K̈hler or electromagnetic coupling and the solutions reduce to those for spinor d'Alembertian in 4-D harmonic potential breaking $SO(4)$ symmetry. Due to the enhanced symmetry of harmonic oscillator, one expects that partial waves are classified by $SU(4)$ and by reduction to $SU(3)\times U(1)$ by em charge and color quantum numbers just as for CP_2 - at least formally.

Harmonic oscillator potential defined by self-dual em field splits M^8 to $M^4 \times E^4$ and implies Gaussian localization of the spinor modes near origin so that E^4 effectively compactifies. The The resulting physics brings strongly in mind low energy physics, where only electromagnetic interaction is visible directly, and one cannot avoid associations with low energy hadron physics. These are some of the reasons for considering $M^8 - H$ duality as something more than a mere mathematical curiosity.

Remark: The Minkowskian signatures of M^8 and M^4 produce technical nuisance. One could overcome them by Wick rotation, which is however somewhat questionable trick. $M^8_c = O_c$ provides the proper formulation.

(a) The proper formulation is in terms of complexified octonions and quaternions involving the introduction of commuting imaginary unit j. If complexified quaternions are used for H, Minkowskian signature requires the introduction of two commuting imaginary units j and i meaning double complexification.

(b) Hyper-quaternions/octonions define as subspace of complexified quaternions/octonions spanned by real unit and jI_k, where I_k are quaternionic units. These spaces are obviously not closed under multiplication. One can however however define the notion of associativity for the sub-space of M^8 by requiring that the products and sums of the tangent space vectors generate complexified quaternions.

(c) Ordinary quaternions Q are expressible as $q = q_0 + q^k I_k$. Hyper-quaternions are expressible as $q = q_0 + jq^k I_k$ and form a subspace of complexified quaternions $Q_c = Q \oplus jQ$. Similar formula applies to octonions and their hyper counterparts which can be regarded as subspaces of complexified octonions $O \oplus jO$. Tangent space vectors of H correspond hyper-quaternions $q_H = q_0 + jq^k I_k + jiq_2$ defining a subspace of doubly complexified quaternions: note the appearance of two imaginary units.

The recent definitions of associativity and M^8 duality has evolved slowly from in-accurate characterizations and there are still open questions.

(a) Kähler form for M^8 non-trivial only in $E^4 \subset M^8$ implies unique decomposition $M^8 = M^4 \times E^4$ needed to define $M^8 - H$ duality uniquely. This applies also to M_c^8. This forces to introduce also Kähler action, induced metric and induced Kähler form. Could strong form of duality meant that the space-time surfaces in M^8 and H have same induced metric and induced Kähler form? Could the WCWs associated with M^8 and H be identical with this assumption so that duality would provide different interpretations for the same physics?

(b) One can formulate associativity in M^8 (or M_c^8) by introducing octonionic structure in tangent spaces or in terms of the octonionic representation for the induced gamma matrices. Does the notion have counterpart at the level of H as one might expect if Kähler action is involved in both cases? The analog of this formulation in H might be as quaternionic "reality" since tangent space of H corresponds to complexified quaternions: I have however found no acceptable definition for this notion.

   The earlier formulation is in terms of octonionic flat space gamma matrices replacing the ordinary gamma matrices so that the formulation reduces to that in M^8 tangent space. This formulation is enough to define what associativity means although one can protest. Somehow H is already complex quaternionic and thus associative. Perhaps this just what is needed since dynamics has two levels: *imbedding space level* and *space-time level*. One must have imbedding space spinor harmonics assignable to the ground states of super-conformal representations and quaternionicity and octonionicity of H tangent space would make sense at the level of space-time surfaces.

(c) Whether the associativity using induced gamma matrices works is not clear for massless extremals (MEs) and vacuum extremals with the dimension of CP_2 projection not larger than 2.

(d) What makes this notion of associativity so fascinating is that it would allow to iterate duality as a sequence $M^8 \to H \to H...$ by mapping the space-time surface to $M^4 \times CP_2$ by the same recipe as in case of M^8. This brings in mind the functional composition of O_c-real analytic functions (O_c denotes complexified octonions: complexification is forced by Minkowskian signature) suggested to produced associative or co-associative surfaces. The associative (co-associative) surfaces in M^8 would correspond to loci for vanishing of imaginary (real) part of octonion-real-analytic function.

It might be possible to define associativity in H also in terms of modified gamma matrices defined by Kähler action (certainly not M^8).

(a) All known extremals are associative or co-associative in H in this sense. This would also give direct correlation with the variational principle. For the known preferred extremals this variant is successful partially because the modified gamma matrices need not span the entire tangent space. The space spanned by the modified gammas is not necessarily tangent space. For instance for CP_2 type vacuum extremals the modified gamma matrices are CP_2 gamma matrices plus an additional light-like component from M^4 gamma matrices.

   If the space spanned by modified gammas has dimension D smaller than 3 co-associativity is automatic. If the dimension of this space is $D = 3$ it can happen that the triplet of gammas spans by multiplication entire octonionic algebra. For $D = 4$ the situation is of course non-trivial.

(b) For modified gamma matrices the notion of co-associativity can produce problems since modified gamma matrices do not in general span the tangent space. What does co-associativity mean now? Should one replace normal space with orthogonal complement of the space spanned by modified gamma matrices? Co-associativity option must be considered for $D = 4$ only. CP_2 type vacuum extremals provide a good example. In this case the modified gamma matrices reduce to sums of ordinary CP_2 gamma matrices and ligt-like M^4 contribution. The orthogonal complement for the modified gamma matrices consists of dual light-like gamma matrix and two gammas orthogonal to it: this space is subspace of M^4 and trivially associative.

8.2.1 Basic idea behind $M^8 - M^4 \times CP_2$ duality

If four-surfaces $X^4 \subset M^8$ under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly, the spontaneous compactification of super string models would correspond in TGD to two different manners to interpret the space-time surface. This correspondence could be called number theoretical compactification or $M^8 - H$ duality.

The hard mathematical facts behind the notion of number theoretical compactification are following.

(a) One must assume that M^8 has unique decomposition $M^8 = M^4 \times E^4$. This decomposition generalizes also to the case of M^8_c. This would be most naturally due to Kähler structure in E^4 defined by a self-dual Kähler form defining parallel constant electric and magnetic fields in Euclidian sense. Besides Kähler form there is vector field coupling to sigma matrix representing the analog of strong isospin: the corresponding octonionic sigma matrix however is imaginary unit times gamma matrix - say ie_1 in M^4 - defining a preferred plane M^2 in M^4. Here it is essential that the gamma matrices of E^4 defined in terms of octonion units commute to gamma matrices in M^4. What is involved becomes clear from the Fano triangle illustrating octonionic multiplication table.

(b) The space of hyper-complex structures of the hyper-octonion space - they correspond to the choices of plane $M^2 \subset M^8$ - is parameterized by 6-sphere $S^6 = G^2/SU(3)$. The subgroup $SU(3)$ of the full automorphism group G_2 respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it e_1. Fixed complex structure therefore corresponds to a point of S^6.

(c) Quaternionic sub-algebras of M^8 (and M^8_c) are parametrized by $G_2/U(2)$. The quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space defined by real and preferred imaginary unit and parametrized by a point of S^6) are parameterized by $SU(3)/U(2) = CP_2$ just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of CP_2, as the automorphism sub-group of octonions, and as color group. Thus the space of quaternionic structures can be parametrized by the 10-dimensional space $G_2/U(2)$ decomposing as $S^6 \times CP_2$ locally.

(d) The basic result behind number theoretic compactification and $M^8 - H$ duality is that associative sub-spaces $M^4 \subset M^8$ containing a fixed commutative sub-space $M^2 \subset M^8$ are parameterized by CP_2. The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of e_1) are labeled by $U(2) \subset SU(3)$. The choice of e_2 and e_3 amounts to fixing $e_2 \pm \sqrt{-1} e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of e_1 and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having e_2 and e_3 components. Hence all possible completions of $1, e_1$ by adding e_2, e_3 doublet are labeled by $SU(3)/U(2) = CP_2$.

Consider now the formulation of $M^8 - H$ duality.

(a) The idea of the standard formulation is that associative manifold $X^4 \subset M^8$ has at its each point associative tangent plane. That is X^4 corresponds to an integrable distribution of $M^2(x) \subset M^8$ parametrized 4-D coordinate x that is map $x \rightarrow S^6$ such that the 4-D tangent plane is hyper-quaternionic for each x.

(b) Since the Kähler structure of M^8 implies unique decomposition $M^8 = M^4 \times E^4$, this surface in turn defines a surface in $M^4 \times CP_2$ obtained by assigning to the point of 4-surface point $(m, s) \in H = M^4 \times CP_2$: $m \in M^4$ is obtained as *projection* $M^8 \rightarrow M^4$ (this is modification to the earlier definition) and $s \in CP_2$ parametrizes the quaternionic tangent plane as point of CP_2. Here the local decomposition $G_2/U(2) = S^6 \times CP_2$ is essential for achieving uniqueness.

(c) One could also map the associative surface in M^8 to surface in 10-dimensional $S^6 \times CP_2$. In this case the metric of the image surface cannot have Minkowskian signature and one cannot assume that the induced metrics are identical. It is not known whether S^6 allows genuine complex structure and Kähler structure which is essential for TGD formulation.

(d) Does duality imply the analog of associativity for $X^4 \subset H$? The tangent space of H can be seen as a sub-space of doubly complexified quaternions. Could one think that quaternionic sub-space is replaced with sub-space analogous to that spanned by real parts of complexified quaternions? The attempts to define this notion do not however look promising. One can however define associativity and co-associativity for the tangent space M^8 of H using octonionization and can formulate it also terms of induced gamma matrices.

(e) The associativity defined in terms of induced gamma matrices in both in M^8 and H has the interesting feature that one can assign to the associative surface in H a new associative surface in H by assigning to each point of the space-time surface its M^4 projection and point of CP_2 characterizing its associative tangent space or co-associative normal space. It seems that one continue this series ad infinitum and generate new solutions of field equations! This brings in mind iteration which is standard manner to generate fractals as limiting sets. This certainly makes the heart of mathematician beat.

(f) Kähler structure in $E^4 \subset M^8$ guarantees natural $M^4 \times E^4$ decomposition. Does associativity imply preferred extremal property or vice versa, or are the two notions equivalent or only consistent with each other for preferred extremals?

A couple of comments are in order.

(a) This definition generalizes to the case of M^8_c: all that matters is that tangent space-is is complexified quaternionic and there is a unique identification $M^4 \subset M^8_c$: this allows to assign the point of 4-surfaces a point of $M^4 \times CP_2$. The generalization is needed if one wants to formulate the hypothesis about O_c real-analyticity as a manner to build quaternionic space-time surfaces properly.

(b) This definition differs from the first proposal for years ago stating that each point of X^4 contains a *fixed* $M^2 \subset M^4$ rather than $M_2(x) \subset M^8$ and also from the proposal assuming integrable distribution of $M^2(x) \subset M^4$. The older proposals are not consistent with the properties of massless extremals and string like objects for which the counterpart of M^2 depends on space-time point and is not restricted to M^4. The earlier definition $M^2(x) \subset M^4$ was problematic in the co-associative case since for the Euclidian signature is is not clear what the counterpart of $M^2(x)$ could be.

(c) The new definition is consistent with the existence of Hamilton-Jacobi structure meaning slicing of space-time surface by string world sheets and partonic 2-surfaces with points of partonic 2-surfaces labeling the string world sheets [K5]. This structure has been proposed to characterize preferred extremals in Minkowskian space-time regions at least.

(d) Co-associative Euclidian 4-surfaces, say CP_2 type vacuum extremal do not contain integrable distribution of $M^2(x)$. It is normal space which contains $M^2(x)$. Does this have some physical meaning? Or does the surface defined by $M^2(x)$ have Euclidian analog?

A possible identification of the analog would be as string world sheet at which W boson field is pure gauge so that the modes of the modified Dirac operator [K18] restricted to the string world sheet have well-defined em charge. This condition appears in the construction of solutions of modified Dirac operator.

For octonionic spinor structure the W coupling is however absent so that the condition does not make sense in M^8. The number theoretic condition would be as commutative or co-commutative surface for which imaginary units in tangent space transform to real and imaginary unit by a multiplication with a fixed imaginary unit! One can also formulate co-associativity as a condition that tangent space becomes associative by a multiplication with a fixed imaginary unit.

There is also another justification for the distribution of Euclidian tangent planes. The idea about associativity as a fundamental dynamical principle can be strengthened to the statement that space-time surface allows slicing by hyper-complex or complex 2-surfaces, which are commutative or co-commutative inside space-time surface. The physical interpretation would be as Minkowskian or Euclidian string world sheets carrying spinor modes. This would give a connection with string model and also with the conjecture about the general structure of preferred extremals.

(e) Minimalist could argue that the minimal definition requires octonionic structure and associativity *only* in M^8. There is no need to introduce the counterpart of Kähler action in M^8 since the dynamics would be based on associativity or co-associativity alone. The objection is that one must assumes the decomposition $M^8 = M^4 \times E^4$ without any justification.

The map of space-time surfaces to those of $H = M^4 \times CP_2$ implies that the space-time surfaces in H are in well-defined sense quaternionic. As a matter of fact, the standard spinor structure of H can be regarded as quaternionic in the sense that gamma matrices are essentially tensor products of quaternionic gamma matrices and reduce in matrix representation for quaternions to ordinary gamma matrices. Therefore the idea that one should introduce octonionic gamma matrices in H is questionable. If all goes as in dreams, the mere associativity or co-associativity would code for the preferred extremal property of Kähler action in H. One could at least hope that associativity/co-associativity in H is consistent with the preferred extremal property.

(f) One can also consider a variant of associativity based on modified gamma matrices - but only in H. This notion does not make sense in M^8 since the very existence of quaternionic tangent plane makes it possible to define $M^8 - H$ duality map. The associativity for modified gamma matrices is however consistent with what is known about extremals of Kähler action. The associativity based on induced gamma matrices would correspond to the use of the space-time volume as action. Note however that gamma matrices are *not* necessary in the definition.

8.2.2 Hyper-octonionic Pauli "matrices" and the definition of associativity

Octonionic Pauli matrices suggest an interesting possibility to define precisely what associativity means at the level of M^8 using gamma matrices (for background see [K59]).

(a) According to the standard definition space-time surface $X^4 \subset M^8$ is associative if the tangent space at each point of X^4 in $X^4 \subset M^8$ picture is associative. The definition can be given also in terms of octonionic gamma matrices whose definition is completely straightforward.

(b) Could/should one define the analog of associativity at the level of H? One can identify the tangent space of H as M^8 and can define octonionic structure in the tangent space and this allows to define associativity locally. One can replace gamma matrices with their octonionic variants and formulate associativity in terms of them locally and this should be enough.

Skeptic however reminds M^4 allows hyper-quaternionic structure and CP_2 quaternionic structure so that complexified quaternionic structure would look more natural for H. The tangent space would decompose as $M^8 = HQ + ijQ$, weher j is commuting imaginary unit and HQ is spanned by real unit and by units iI_k, where i second commutating imaginary unit and I_k denotes quaternionic imaginary units. There is no need to make anything associative.

There is however far from obvious that octonionic spinor structure can be (or need to be!) defined globally. The lift of the CP_2 spinor connection to its octonionic variant has questionable features: in particular vanishing of the charged part and reduction of neutral part to photon. Therefore is is unclear whether associativity condition makes sense for $X^4 \subset M^4 \times CP_2$. What makes it so fascinating is that it would allow to iterate duality as a sequences $M^8 \to H \to H....$ This brings in mind the functional composition of octonion real-analytic functions suggested to produced associative or co-associative surfaces.

I have not been able to settle the situation. What seems the working option is associativity in both M^8 and H and modified gamma matrices defined by appropriate Kähler action and correlation between associativity and preferred extremal property.

8.2.3 Are Kähler and spinor structures necessary in M^8?

If one introduces M^8 as dual of H, one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in H are also extremals of M^8 Kähler action with same value of Kähler action defining Kähler function. As found, this leads to the conclusion that the $M^8 - H$ duality is Kähler isometry. Coupling of spinors to Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in H should have full M^8 dual.

Are also the 4-surfaces in M^8 preferred extremals of Kähler action?

It would be a mathematical miracle if associative and co-associative surfaces in M^8 would be in 1-1 correspondence with preferred extremals of Kähler action. This motivates the question whether Kähler action make sense also in M^8. This does not exclude the possibility that associativity implies or is equivalent with the preferred extremal property.

One expects a close correspondence between preferred extremals: also now vacuum degeneracy is obtained, one obtains massless extremals, string like objects, and counterparts of CP_2 type vacuum extremals. All known extremals would be associative or co-associative if modified gamma matrices define the notion (possible only in the case of H).

The strongest form of duality would be that the space-time surfaces in M^8 and H have same induced metric same induced Kähler form. The basic difference would be that the spinor connection for surfaces in M^8 would be however neutral and have no left handed components and only em gauge potential. A possible interpretation is that M^8 picture defines a theory in the phase in which electroweak symmetry breaking has happened and only photon belongs to the spectrum.

The question is whether one can define WCW also for M^8. Certainly it should be equivalent with WCW for H: otherwise an inflation of poorly defined notions follows. Certainly the general formulation of the WCW geometry generalizes from H to M^8. Since the matrix elements of symplectic super-Hamiltonians defining WCW gamma matrices are well defined as matrix elements involve spinor modes with Gaussian harmonic oscillator behavior, the non-compactness of E^4 does not pose any technical problems.

Spinor connection of M^8

There are strong physical constraints on M^8 dual and they could kill the hypothesis. The basic constraint to the spinor structure of M^8 is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different H-chiralities and parity breaking.

(a) By the flatness of the metric of E^4 its spinor connection is trivial. E^4 however allows full S^2 of covariantly constant Kähler forms so that one can accommodate free independent Abelian gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units. This is possible but perhaps a more natural option is the introduction of just single Kähler form as in the case of CP_2.

(b) One should be able to distinguish between quarks and leptons also in M^8, which suggests that one introduce spinor structure and Kähler structure in E^4. The Kähler structure of E^4 is unique apart form $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of S^2 representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of H.

(c) Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and Z^0 contains both axial and vector parts. The naive replacement of sigma matrices appearing in the coupling of electroweak gauge fields takes the left handed parts of these fields to zero so that only neutral part remains. Further, gauge fields correspond to curvature of CP_2 which vanishes for E^4 so that only Kähler form form remains. Kähler form couples to 3L and q so that the basic asymmetry between leptons and quarks remains. The resulting field could be seen as analog of photon.

(d) The absence of weak parts of classical electro-weak gauge fields would conform with the standard thinking that classical weak fields are not important in long scales. A further prediction is that this distinction becomes visible only in situations, where H picture is necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of E^4 partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

Dirac equation for leptons and quarks in M^8

Kähler gauge potential would also couple to octonionic spinors and explain the distinction between quarks and leptons.

(a) The complexified octonions representing H spinors decompose to $1+1+3+\overline{3}$ under SU(3) representing color automorphisms but the interpretation in terms of QCD color does not make sense. Rather, the triplet and single combine to two weak isospin doublets and quarks and leptons corresponds to to "spin" states of octonion valued 2-spinor. The conservation of quark and lepton numbers follows from the absence of coupling between these states.

(b) One could modify the coupling so that coupling is on electric charge by coupling it to electromagnetic charge which as a combination of unit matrix and sigma matrix is proportional to $1+kI_1$, where I_1 is octonionic imaginary unit in $M^2 \subset M^4$. The complexified octonionic units can be chosen to be eigenstates of Q_{em} so that Laplace equation reduces to ordinary scalar Laplacian with coupling to self-dual em field.

(c) One expects harmonic oscillator like behavior for the modes of the Dirac operator of M^8 since the gauge potential is linear in E^4 coordinates. One possibility is Cartesian coordinates is $A(A_x, A_y, A_z, A_t) = k(-y, x, t, -z)$. Thhe coupling would make E^4 effectively a compact space.

(d) The square of Dirac operator gives potential term proportional to $r^2 = x^2+y^2+z^2+t^2$ so that the spectrum of 4-D harmonic oscillator operator and $SO(4)$ harmonics localized near origin are expected. For harmonic oscillator the symmetry enhances to $SU(4)$.

If one replaces Kähler coupling with em charge symmetry breaking of $SO(4)$ to vectorial $SO(3)$ is expected since the coupling is proportional to $1+ike_1$ defining electromagnetic charge. Since the basis of complexified quaternions can be chosen to be eigenstates of e_1 under multiplication, octonionic spinors are eigenstates of em charge and one obtains two color singles $1 \pm e_1$ and color triplet and antitriplet. The color triplets cannot be however interpreted in terms of quark color.

Harmonic oscillator potential is expected to enhance $SO(3)$ to SU(3). This suggests the reduction of the symmetry to $SU(3) \times U(1)$ corresponding to color symmetry and em charge so that one would have same basic quantum numbers as tof CP_2 harmonics. An interesting question is how the spectrum and mass squared eigenvalues of harmonics differ from those for CP_2.

(e) In the square of Dirac equation $J^{kl}\Sigma_{kl}$ term distinguishes between different em charges ($\Sigma_k l$ reduces by self duality and by special properties of octonionic sigma matrices to a term proportional to iI_1 and complexified octonionic units can be chosen to be its eigenstates with eigen value ± 1. The vacuum mass squared analogous to the vacuum energy of harmonic oscillator is also present and this contribution are expected to cancel themselves for neutrinos so that they are massless whereas charged leptons and quarks are massive. It remains to be checked that quarks and leptons can be classified to triality $T = \pm 1$ and $t = 0$ representations of dynamical $SU(3)$ respectively.

What about the analog of Kähler Dirac equation

Only the octonionic structure in $T(M^8)$ is needed to formulate quaternionicity of space-time surfaces: the reduction to O_c-real-analyticity would be extremely nice but not necessary (O_c denotes complexified octonions needed to cope with Minkowskian signature). Most importantly, there might be no need to introduce Kähler action (and Kähler form) in M^8. Even the octonionic representation of gamma matrices is un-necessary. Neither there is any absolute need to define octonionic Dirac equation and octonionic Kähler Dirac equation nor octonionic analog of its solutions nor the octonionic variants of imbedding space harmonics.

It would be of course nice if the general formulas for solutions of the Kähler Dirac equation in H could have counterparts for octonionic spinors satisfying quaternionicity condition. One can indeed wonder whether the restriction of the modes of induced spinor field to string world sheets defined by integrable distributions of hyper-complex spaces $M^2(x)$ could be interpretated in terms of commutativity of fermionic physics in M^8. $M^8 - H$ correspondence could map the octonionic spinor fields at string world sheets to their quaternionic counterparts in H. The fact that only holomorphy is involved with the definition of modes could make this map possible.

8.2.4 How could one solve associativity/co-associativity conditions?

The natural question is whether and how one could solve the associativity/-co-associativity conditions explicitly. One can imagine two approaches besides $M^8 \rightarrow H \rightarrow H...$ iteration generating new solutions from existing ones.

Could octonion-real analyticity be equivalent with associativity/co-associativity?

Analytic functions provide solutions to 2-D Laplace equations and one might hope that also the field equations could be solved in terms of octonion-real-analyticity at the level of M^8 perhaps also at the level of H. Signature however causes problems - at least technical. Also the compactness of CP_2 causes technical difficulties but they need not be insurmountable.

For E^8 the tangent space would be genuinely octonionic and one can define the notion octonion-real analytic map as a generalization of real-analytic function of complex variables (the coefficients of Laurent series are real to guarantee associativity of the series). The argument is complexified octonion in $O \oplus iO$ forming an algebra but not a field. The norm square is Minkowskian as difference of two Euclidian octonionic norms: $N(o_1+io_2) = N(o_1) - N(o_2)$ and vanishes at 15-D light cone boundary. Obviously, differential calculus is possible outside the light-cone boundary. Rational analytic functions have however poles at the light-cone boundary. One can wonder whether the poles at M^4 light-cone boundary, which is subset of 15-D light-cone boundary could have physical significance and relevant for the role of causal diamonds in ZEO.

The candidates for associative surfaces defined by O_c-real-analytic functions (I use O_c for complexified octonions) have Minkowskian signature of metric and are 4-surfaces at which the projection of $f(o_1+io_2)$ to $Im(O_1)$, $iIm(O_2)$, and $iRe(Q_2)\oplus Im(Q_1)$ vanish so that only the projection to hyper-quaternionic Minkowskian sub-space $M^4 = Re(Q_1) + iIm(Q_2)$ with signature $(1,-1,-,1-,1)$ is non-vanishing. The inverse image need not belong to M^8 and in general it belongs to M^8_c but this is not a problem: all that is needed that the tangent space of inverse image is complexified quaternionic. If this is the case then $M^8 - H$ duality maps the tangent space of the inverse image to CP_2 point and image itself defines the point of M^4 so that a point of H is obtained. Co-associative surfaces would be surfaces for which the projections of image to $Re(O_1)$, $iRe(O_2)$, and to $Im(O_1)$ vanish so that only the projection to $iIm(O_2)$ with signature $(-1,-1,-1,-1)$ is non-vanishing.

The inverse images as 4-D sub-manifolds of M^8_c (not M^8!) are excellent candidates for associative and co-associative 4-surfaces since $M^8 - H$ duality assignes to them a 4-surface in $M^4\times CP_2$ if the tangent space at given point is complexified quaternionic. This is true if one believes on the analytic continuation of the intuition from complex analysis (the image of real axes under the map defined by O_c-real-analytic function is real axes in the new coordinates defined by the map: the intuition results by replacing "real" by "complexified quaternionic"). The possibility to solve field equations in this manner would be of enormous significance since besides basic arithmetic operations also the functional decomposition of O_c-real-analytic functions produces similar functions. One could speak of the algebra of space-time surfaces.

What is remarkable that the complexified octonion real analytic functions are obtained by analytic continuation from single real valued function of real argument. The real functions form naturally a hierarchy of polynomials (maybe also rational functions) and number theoretic vision suggests that there coefficients are rationals or algebraic numbers. Already for rational coefficients hierarchy of algebraic extensions of rationals results as one solves the vanishing conditions. There is a temptation to regard this hierarchy coding for space-time sheets as an analog of DNA.

Note that in the recent formulation there is no need to pose separately the condition about integrable distribution of $M^2(x)\subset M^4$.

Quaternionicity condition for space-time surfaces

Quaternionicity actually has a surprisingly simple formulation at the level of space-time surfaces. The following discussion applies to both M^8 and H with minor modifications if one accepts that also H can allow octonionic tangent space structure, which does not require gamma matrices.

(a) Quaternionicity is equivalent with associativity guaranteed by the vanishing of the associator $A(a,b,c) = a(bc) - (ab)c$ for any triplet of imaginary tangent vectors in the tangent space of the space-time surface. The condition must hold true for purely imaginary combinations of tangent vectors.

(b) If one is able to choose the coordinates in such a manner that one of the tangent vectors corresponds to real unit (in the imbedding map imbedding space M^4 coordinate depends only on the time coordinate of space-time surface), the condition reduces to the vanishing of the octonionic product of remaining three induced gamma matrices interpreted as octonionic gamma matrices. This condition looks very simple - perhaps too simple!- since it involves only first derivatives of the imbedding space vectors.

    One can of course whether quaternionicity conditions replace field equations or only select preferred extremals. In the latter case, one should be able to prove that quaternionicity conditions are consistent with the field equations.

(c) Field equations would reduce to tri-linear equations in in the gradients of imbedding space coordinates (rather than involving imbedding space coordinates quadratically). Sum of analogs of 3×3 determinants deriving from $a\times(b\times b)$ for different octonion units is involved.

(d) Written explicitly field equations give in terms of vielbein projections e^A_α, vielbein vectors e^A_k, coordinate gradients $\partial_\alpha h^k$ and octonionic structure constants f_{ABC} the following conditions stating that the projections of the octonionic associator tensor to the space-time surface vanishes:

$$\begin{aligned} e^A_\alpha e^B_\beta e^C_\gamma A^E_{ABC} &= 0 \ , \\ A^E_{ABC} &= f_{AD}{}^E f_{BC}{}^D - f_{AB}{}^D f_{DC}{}^E \ , \\ e^A_\alpha &= \partial_\alpha h^k e^A_k \ , \\ \Gamma_k &= e^A_k \gamma_A \ . \end{aligned} \tag{8.2.1}$$

The very naive idea would be that the field equations are indeed integrable in the sense that they reduce to these tri-linear equations. Tri-linearity in derivatives is highly non-trivial outcome simplifying the situation further. These equations can be formulated as the as purely algebraic equations written above plus integrability conditions

$$F^A_{\alpha\beta} = D_\alpha e^A_\beta - D_\beta e^A_\alpha = 0 \ . \tag{8.2.2}$$

One could say that vielbein projections define an analog of a trivial gauge potential. Note however that the covariant derivative is defined by spinor connection rather than this effective gauge potential which reduces to that in SU(2). Similar formulation holds true for field equations and one should be able to see whether the field equations formulated in terms of derivatives of vielbein projections commute with the associatitivity conditions.

(e) The quaternionicity conditions can be formulated as vanishing of generalization of Cayley's hyperdeterminant for "hypermatrix" a_{ijk} with 2-valued indiced (see `http://en.wikipedia.org/wiki/Hyperdeterminant`). Now one has 8 hyper-matrices with 3 8-valued indices associated with the vanishing $A^E_{BCD}x^B y^C z^D = 0$ of trilinear forms defined by the associators. The conditions say somethig only about the octonioni structure constants and since octonionic space allow quaternionic sub-spaces these conditions must be satisfied.

The inspection of the Fano triangle [A28] expressing the multiplication table for octonionic imaginary units reveals that give any two imaginary octonion units e_1 and e_2 their product e_1e_2 (or equivalently commutator) is imaginary octonion unit (2 times octonion unit) and the three units span together with real unit quaternionic sub-algebra. There it seems that one can generate local quaternionic sub-space from two imaginary units plus real unit. This generalizes to the vielbein components of tangent vectors of space-time surface and one can build the solutions to the quaternionicity conditions from vielbein projections e_1, e_2, their product $e_3 = k(x)e_1e_2$ and real fourth "time-like" vielbein component which must be expressible as a combination of real unit and imaginary units:

$$e_0 = a \times 1 + b^i e_i$$

For static solutions this condition is trivial. Here summation over i is understood in the latter term. Besides these conditions one has integrability conditions and field equations for Kähler action. This formulation suggests that quaternionicity is additional - perhaps defining - property of preferred extremals.

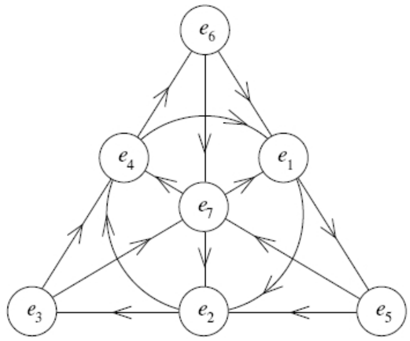

Figure 8.1: Octonionic triangle: the six lines and one circle containing three vertices define the seven associative triplets for which the multiplication rules of the ordinary quaternion imaginary units hold true. The arrow defines the orientation for each associative triplet. Note that the product for the units of each associative triplets equals to real unit apart from sign factor.

8.2.5 Quaternionicity at the level of imbedding space quantum numbers

From the multiplication table of octonions as illustrated by Fano triangle [A28] one finds that all edges of the triangle, the middle circle and the three the lines connecting vertices to the midpoints of opposite side define triplets of quaternionic units. This means that by taking real unit and any imaginary unit in quaternionic M^4 algebra spanning $M^2 \subset M^4$ and two imaginary units in the complement representing CP_2 tangent space one obtains quaternionic algebra. This suggests an explanation for the preferred M^2 contained in tangent space of space-time surface (the M^2:s could form an integrable distribution). Four-momentum restricted to M^2 and I_3 and Y interpreted as tangent vectors in CP_2 tangent space defined quaterionic sub-algebra. This could give content for the idea that quantum numbers are quaternionic.

I have indeed proposed that the four-momentum belongs to M^2. If $M^2(x)$ form a distribution as the proposal for the preferred extremals suggests this could reflect momentum exchanges between different points of the space-time surface such that total momentum is conserved or momentum exchange between two sheets connected by wormhole contacts.

8.2.6 Questions

In following some questions related to $M^8 - H$ duality are represented.

Could associativity condition be formulated using modified gamma matrices?

Skeptic can criticize the minimal form of $M^8 - H$ duality involving no Kähler action in M^8 is unrealistic. Why just Kähler action? What makes it so special? The only defense that I can imagine is that Kähler action is in many respects unique choice.

An alternative approach would replace induced gamma matrices with the modified ones to get the correlation In the case of M^8 this option cannot work. One cannot exclude it for H.

(a) For Kähler action the modified gamma matrices $\Gamma^\alpha = \frac{\partial L_K}{\partial h^k_\alpha}\Gamma^k$, $\Gamma_k = e^A_k \gamma_A$, assign to a given point of X^4 a 4-D space which need not be tangent space anymore or even its sub-space.

The reason is that canonical momentum current contains besides the gravitational contribution coming from the induced metric also the "Maxwell contribution" from the induced Kähler form not parallel to space-time surface. In the case of M^8 the duality map to H is therefore lost.

(b) The space spanned by the modified gamma matrices need not be 4-dimensional. For vacuum extremals with at most 2-D CP_2 projection modified gamma matrices vanish identically. For massless extremals they span 1- D light-like subspace. For CP_2 vacuum extremals the modified gamma matrices reduces to ordinary gamma matrices for CP_2 and the situation reduces to the quaternionicity of CP_2. Also for string like objects the conditions are satisfied since the gamma matrices define associative sub-space as tangent space of $M^2 \times S^2 \subset M^4 \times CP_2$. It seems that associativity is satisfied by all known extremals. Hence modified gamma matrices are flexible enough to realize associativity in H.

(c) Modified gamma matrices in Dirac equation are required by super conformal symmetry for the extremals of action and they also guarantee that vacuum extremals defined by surfaces in $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2, are trivially hyper-quaternionic surfaces. The modified definition of associativity in H does not affect in any manner $M^8 - H$ duality necessarily based on induced gamma matrices in M^8 allowing purely number theoretic interpretation of standard model symmetries. One can however argue that the most natural definition of associativity is in terms of induced gamma matrices in both M^8 and H.

Remark: A side comment not strictly related to associativity is in order. The anti-commutators of the modified gamma matrices define an effective Riemann metric and one can assign to it the counterparts of Riemann connection, curvature tensor, geodesic line, volume, etc... One would have two different metrics associated with the space-time surface. Only if the action defining space-time surface is identified as the volume in the ordinary metric, these metrics are equivalent. The index raising for the effective metric could be defined also by the induced metric and it is not clear whether one can define Riemann connection also in this case. Could this effective metric have concrete physical significance and play a deeper role in quantum TGD? For instance, AdS-CFT duality leads to ask whether interactions be coded in terms of the gravitation associated with the effective metric.

Now skeptic can ask why should one demand $M^8 - H$ correspondence if one in any case is forced to introduced Kähler also at the level of M^8? Does $M^8 - H$ correspondence help to construct preferred extremals or does it only bring in a long list of conjectures? I can repeat the questions of the skeptic.

Minkowskian-Euclidian $\leftrightarrow$ associative–co-associative?

The 8-dimensionality of M^8 allows to consider both associativity of the tangent space and associativity of the normal space- let us call this co-associativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \to k$ duality.

Can $M^8 - H$ duality be useful?

Skeptic could of course argue that $M^8 - H$ duality generates only an inflation of unproven conjectures. This might be the case. In the following I will however try to defend the conjecture. One can however find good motivations for $M^8 - H$ duality: both theoretical and physical.

(a) If $M^8 - H$ duality makes sense for induced gamma matrices also in H, one obtains infinite sequence if dualities allowing to construct preferred extremals iteratively. This might relate to octonionic real-analyticity and composition of octonion-real-analytic functions.

(b) $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action as hyper-quaternionic surfaces. Unfortunately, it is not clear whether one should introduce the counterpart of Kähler action in M^8 and the coupling of M^8 spinors to Kähler form. Note that the Kähler form in E^4 would be self dual and have constant components: essentially parallel electric and magnetic field of same constant magnitude.

(c) $M^8 - H$ duality provides insights to low energy physics, in particular low energy hadron physics. M^8 description might work when H-description fails. For instance, perturbative QCD which corresponds to H-description fails at low energies whereas M^8 description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong $SU(2)_L \times SU(2)_R$ and by flatness of E^4 spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in CP_2. One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin.

    This argument does not seem to be consistent with $SU(3) \times U(1) \subset SU(4)$ symmetry for Mx Dirac equation. One can however argue that $SU(4)$ symmetry combines $SO(4)$ multiplets together. Furthermore, $SO(4)$ represents the isometries leaving Kähler form invariant.

$M^8 - H$ duality in low energy physics and low energy hadron physics

$M^8 - H$ can be applied to gain a view about color confinement. The basic idea would be that $SO(4)$ and $SU(3)$ provide provide dual descriptions of quarks using E^4 and CP_2 partial waves and low energy hadron physics corresponds to a situation in which M^8 picture provides the perturbative approach whereas H picture works at high energies.

A possible interpretation is that the space-time surfaces vary so slowly in CP_2 degrees of freedom that can approximate CP_2 with a small region of its tangent space E^4. One could also say that color interactions mask completely electroweak interactions so that the spinor connection of CP_2 can be neglected and one has effectively E^4. The basic prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics and this is indeed the case.

Consider color confinement at the long length scale limit in terms of $M^8 - H$ duality.

(a) At high energy limit only lowest color triplet color partial waves for quarks dominate so that QCD description becomes appropriate whereas very higher color partial waves for quarks and gluons are expected to appear at the confinement limit. Since WCW degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD.

(b) The success of $SO(4)$ sigma model in the description of low lying hadrons would directly relate to the fact that this group labels also the E^4 Hamiltonians in M^8 picture. Strong $SO(4)$ quantum numbers can be identified as orbital counterparts of right and left handed electro-weak isospin coinciding with strong isospin for lowest quarks. In sigma model pion and sigma boson form the components of E^4 valued vector field or equivalently collection of four E^4 Hamiltonians corresponding to spherical E^4 coordinates. Pion corresponds to S^3 valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the E^4 radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted.

(c) The generalization of sigma model would assign to quarks E^4 partial waves belonging to the representations of $SO(4)$. The model would involve also 6 $SO(4)$ gluons and their $SO(4)$ partial waves. At the low energy limit only lowest representations would be be important whereas at higher energies higher partial waves would be excited and the description based on CP_2 partial waves would become more appropriate.

(d) The low energy quark model would rely on quarks moving $SO(4)$ color partial waves. Left *resp.* right handed quarks could correspond to $SU(2)_L$ *resp.* $SU(2)_R$ triplets so that spin statistics problem would be solved in the same manner as in the standard quark model.

(e) Family replication phenomenon is described in TGD framework the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [K32] .

To my opinion these observations are intriguing enough to motivate a concrete attempt to construct low energy hadron physics in terms of $SO(4)$ gauge theory.

8.2.7 Summary

The overall conclusion is that the most convincing scenario relies on the associativity/co-associativity of space-time surfaces define by induced gamma matrices and applying both for M^8 and H. The fact that the duality can be continued to an iterated sequence of duality maps $M^8 \to H \to H...$ is what makes the proposal so fascinating and suggests connection with fractality.

The introduction of Kähler action and coupling of spinors to Kähler gauge potentials is highly natural. One can also consider the idea that the space-time surfaces in M^8 and H have same induced metric and Kähler form: for iterated duality map this would mean that the steps in the map produce space-time surfaces which identical metric and Kähler form so that the sequence might stop. M^8_H duality might provide two descriptions of same underlying dynamics: M^8 description would apply in long length scales and H description in short length scales.

8.3 Octo-twistors and twistor space

The basic problem of the twistor approach is that one cannot represent massive momenta in terms of twistors in an elegant manner. One can also consider generalization of the notion of spinor and twistor. I have proposed a possible representation of massive states based on the existence of preferred plane of M^2 in the basic definition of theory allowing to express four-momentum as one of two light-like momenta allowing twistor description. One could however ask whether some more elegant representation of massive M^4 momenta might be possible by generalizing the notion of twistor -perhaps by starting from the number theoretic vision.

The basic idea is obvious: in quantum TGD massive states in M^4 can be regarded as massless states in M^8 and $M^4 \times CP_2$ (recall $M^8 - H$ duality). One can therefore map any massive M^4 momentum to a light-like M^8 momentum and hope that this association could be made in a unique manner. One should assign to a massless 8-momentum an 8-dimensional spinor of fixed chirality. The spinor assigned with the light-like four-momentum is not unique without additional conditions. The existence of covariantly constant right-handed neutrino in CP_2 degrees generating the super-conformal symmetries could allow to eliminate the non-uniqueness. 8-dimensional twistor in M^8 would be a pair of this kind of spinors fixing the momentum of massless particle and the point through which the corresponding light-geodesic goes through: the set of these points forms 8-D light-cone and one can assign to each point a spinor. In $M^4 \times CP_2$ definitions makes also in the case of $M^4 \times CP_2$ and twistor space would also now be a lifting of the space of light-like geodesics.

The possibility to interpret M^8 as hyperoctonionic space suggests also the possibility to define the 8-D counterparts of sigma matrices to hyperoctonions to obtain a representation of sigma matrix algebra which is not a matrix representation. The mapping of gamma matrices to this representation allows to define a notion of hyper-quaternionicity in terms of the modified gamma matrices both in M^8 and H.

The basic challenge is to achieve twistorial description of four-momenta or even $M^4 \times CP_2$ quantum numbers: this applies both to the momenta of fundamental fermions at the lines of generalized Feynman diagrams and to the massive incoming and outcoming states identified as their composites.

(a) A rather attractive way to overcome the problem at the level of fermions propagating along the braid strands at the light-like orbits of partonic 2-surfaces relies on the assumption that generalized Feynman diagrammatics effectively reduces to a form in which all fermions in the propagator lines are massless although they can have non-physical helicity [K44]. One can use ordinary M^4 twistors. This is consistent with the idea that space-time surfaces are quaternionic sub-manifolds of octonionic imbedding space.

(b) Incoming and outgoing states are composites of massless fermions and not massless. They are however massless in 8-D sense. This suggests that they could be described using generalization of twistor formalism from M^4 to M^8 and even betterm to $M^4 \times CP_2$.

In the following two possible twistorializations are considered.

8.3.1 Two manners to twistorialize imbedding space

In the following the generalization of twistor formalism for M^8 or $M^4 \times CP_2$ will be considered in more detail. There are two options to consider.

(a) For the first option one assigns to $M^4 \times CP_2$ twistor space as a product of corresponding twistor spaces $T(M_4) = CP_3$ and the flag-manifold $T(CP_2) = SU(3)/U(1) \times U(1)$ parameterizing the choices of quantization axes for $SU(3)$: $T_H = T(M^4) \times T(CP_2)$. Quite remarkably, M^4 and CP_2 are the only 4-D manifolds allowing twistor space with Kähler structure. The twistor space is 12-dimensional. The choice of quantization axis is certainly a physically well-define operation so that $T(CP_2)$ has physical interpretation. If all observable physical states are color singlets situation becomes more complex. If one assumes QCC for color quantum numbers Y and I_3, then also the choice of color quantization axis is fixed at the level of Kähler action from the condition that Y and I_3 have classically their quantal values.

(b) For the second option one generalizes the usual construction for M^8 regarded as tangent space of $M^4 \times CP_2$ (unless one takes $M^8 - H$ duality seriously).

The tangent space option looks like follows.

(a) One can map the points of M^8 to octonions. One can consider 2-component spinors with octonionic components and map points of M^8 light-cone to linear combinations of 2×2 Pauli sigma matrices but with octonionic components. By the same arguments as in the deduction of ordinary twistor space one finds that 7-D light-cone boundary is mapped to 7+8 D space since the octonionic 2-spinor/its conjugate can be multiplied/divided by arbitrary octonion without changing the light-like point. By standard argument this space extends to 8+8-D space. The points of M^8 can be identified as 8-D octonionic planes (analogs of complex sphere CP_1 in this space. An attractive identification is as octonionic projective space OP_2. Remarkably, octonions do not allow higher dimensional projective spaces.

(b) If one assumes that the spinors are quaternionic the twistor space should have dimension 7+4+1=12. This dimension is same as for $M^4 \times CP_2$. Does this mean that quaternionicity assumption reduces $T(M^8) = OP_2$ to $T(H) = CP_3 \times SU(3)/U(1) \times U(1)$? Or does it yield 12-D space $G_2/U(1) \times U(1)$, which is also natural since G_2 has 2-D Cartan algebra? Number theoretical compactification would transform $T(M^8) = G_2/U(1) \times U(1)$ to $T(H) = CP_3 \times SU(3)/U(1) \times U(1)$. This would not be surprising since in $M^8 - H$-duality CP_2 parametrizes (hyper)quaternionic planes containing preferred plane M^2.

Quaternionicity is certainly very natural in TGD framework. Quaternionicity for 8-momenta does not in general imply that they reduce to the observed M^4-momenta unless one identifies M^4 as one particular subspace of M^8. In $M^8 - H$ duality one in principle allows all choices of M^4: it is of course unclear whether this makes any physical difference. Color confinement could be interpreted as a reduction of M^8 momenta to M^4 momenta and would also allow the interpretational problems caused by the fact that CP_2 momenta are not possible.

(c) Since octonions can be regarded as complexified quaternions with non-commuting imaginary unit, one can say that quaternionic spinors in M^8 are "real" and thus analogous to Majorana spinors. Similar interpretation applies at the level of H. Could one can interpret the quaternionicity condition for space-time surfaces and imbedding space spinors as TGD analog of Majorana condition crucial in super string models? This would also be crucial for understanding supersymmetry in TGD sense.

8.3.2 Octotwistorialization of M^8

Consider first the twistorialization in 4-D case. In M^4 one can map light-like momoment to spinors satisfying massless Dirac equation. General point m of M^4 can be mapped to a pair of massless spinors related by incidence relation defining the point m. The essential element of this association is that mass squared can be defined as determinant of the 2×2 matrix resulting in the assignment. Light-likeness is coded to the vanishing of the determinant implying that the spinors defining its rows are linearly independent. The reduction of M^4 inner product to determinant occurs because the 2×2 matrix can be regarded as a matrix representation of complexified quaternion. Massless means that the norm of a complexified quaternion defined as the product of q and its conjugate vanishes. Incidence relation $s_1 = xs_2$ relating point of M^4 and pair of spinors defining the corresponding twistor, can be interpreted in terms of product for complexified quaternions.

The generalization to the 8-D situation is straightforward: replace quaternions with octonions.

(a) The transition to M^8 means the replacement of quaternions with octonions. Masslessness corresponds to the vanishing norm for complexified octonion (hyper-octonion).

(b) One should assign to a massless 8-momentum an 8-dimensional spinor identifiable as octonion - or more precisely as hyper-octonion obtained by multiplying the imaginary part of ordinary octonion with commuting imaginary unit j and defining conjugation as a change of sign of j or that of octonionic imaginar units.

(c) This leads to a generalization of the notion of twistor consisting of pair of massless octonion valued spinors (octonions) related by the incidence relation fixing the point of

M^8. The incidence relation for Euclidian octonions says $s_1 = xs_2$ and can be interpreted in terms of triality for $SO(8)$ relating conjugate spinor octet to the product of vector octed and spinor octet. For Minkowskian subspace of complexified octonions light-like vectors and s_1 and s_2 can be taken light-like as octonions. Light like x can annihilate s_2.

The possibility to interpret M^8 as hyperoctonionic space suggests also the possibility to define the 8-D counterparts of sigma matrices to hyperoctonions to obtain a representation of sigma matrix algebra which is not a matrix representation. The mapping of gamma matrices to this representation allows to define a notion of hyper-quaternionicity in terms of the modified gamma matrices both in M^8 and H.

8.3.3 Octonionicity, $SO(1,7)$, G_2, and non-associative Malcev group

The symmetries assignable with octonions are rather intricate. First of all, octonions (their hyper-variants defining M^8) have SO(8) (SO(1,7)) as isometries. $G_2 \subset SO(7)$ acts as automorphisms of octonions and $SO(1,7) \to G_2$ clearly means breaking of Lorentz invariance.

John Baez has described in a lucid manner G_2 geometrically (`http://math.ucr.edu/home/baez/octonions/node14.html`). The basic observation is that that quaternionic sub-space is generated by two linearly independent imaginary units and by their product. By adding a fourth linearly independent imaginary unit, one can generated all octonions. From this and the fact that G_2 represents subgroup of $SO(7)$, one easily deduces that G_2 is 14-dimensional. The Lie algebra of G_2 corresponds to derivations of octonionic algebra as follows infinitesimally from the condition that the image of product is the product of images. The entire algebra $SO(8)$ is direct sum of G_2 and linear transformations generated by right and left multiplication by imaginary octonion: this gives $14 + 14 = 28 = D(SO(8))$. The subgroup $SO(7)$ acting on imaginary octonsions corresponds to the direct sum of derivations and adjoint transformations defined by commutation with imaginary octonions, and has indeed dimension $14 + 7 = 21$.

One can identify also a non-associative group-like structure.

(a) In the case of octonionic spinors this group like structure is defined by the analog of phase multiplication of spinor generalizing to a multiplication with octonionic unit expressible as linear combinations of 8 octonionic imaginary units and defining 7-sphere plays appear as analog of automorphisms $o \to uou^{-1} = uou^*$.
One can associate with these transformations a non-associative Lie group and Lie algebra like structures by defining the commutators just as in the case of matrices that is as $[a, b] = ab - ba$. One 7-D non-associative Lie group like structure with topology of 7-sphere S^7 whereas G_2 is 14-dimensional exceptional Lie group (having S^6 as coset space $S^6 = G_2/SU(3)$). This group like object might be useful in the treatment of octonionic twistors. In the case of quaternions one has genuine group acting as SO(3) rotations.

(b) Octonionic gamma matrices allow to define as their commutators octonionic sigma matrices:

$$\Sigma_{kl} = \frac{i}{2}[\gamma_k, \gamma_l] \quad . \tag{8.3.1}$$

This algebra is 14-dimensional thanks to the fact that octonionic gamma matrices are of form $\gamma_0 = \sigma_1 \otimes 1$, $\gamma_i = \sigma_2 \otimes e_i$. Due to the non-associativity of octonions this algebra does not satisfy Jacobi identity - as is easy to verify using Fano triangle - and is therefore not a genuine Lie-algebra. Therefore these sigma matrices do not define a representation of G_2 as I thought first.
This algebra has decomposition $g = h + t$, $[h, t] \subset t$, $[t, t] \subset h$ characterizing for symmetric spaces. h is the 7-D algebra generated by Σ_{ij} and identical with the non-associative Malcev algebra generated by the commutators of octonionic units. The complement t corresponds to the generators Σ_{0i}. The algebra is clearly an octonionic non-associative analog fo $SO(1,7)$.

8.3.4 Octonionic spinors in M^8 and real complexified-quaternionic spinors in H?

This above observations about the octonionic sigma matrices raise the problem about the octonionic representation of spinor connection. In $M^8 = M^4 \times E^4$ the spinor connection is trivial but for $M^4 \times CP_2$ not. There are two options.

(a) Assume that octonionic spinor structure makes sense for M^8 only and spinor connection is trivial.

(b) An alternative option is to identify M^8 as tangent space of $M^4 \times CP_2$ possessing quaternionic structure defined in terms of octonionic variants of gamma matrices. Should one replace sigma matrices appearing in spinor connection with their octonionic analogs to get a sigma matrix algebra which is pseudo Lie algebra. Or should one map the holonomy algebra of CP_2 spinor connection to a sub-algebra of $G_2 \subset SO(7)$ and define the action of the sigma matrices as ordinary matrix multiplication of octonions rather than octonionic multiplication? This seems to be possible formally.

    The replacement of sigma matrices with their octonionic counterparts seems to lead to weird looking results. Octonionic multiplication table implies that the electroweak sigma matrices associated with CP_2 tangent space reduce to M^4 sigma matrices so that the spinor connection is quaternionic. Furthermore, left-handed sigma matrices are mapped to zero so that only the neutral part of spinor connection is non-vanishing. This supports the view that only M^8 gamma matrices make sense and that Dirac equation in M^8 is just free massless Dirac equation leading naturally also to the octonionic twistorialization.

    One might think that distinction between different H-chiralities is difficult to make but it turns out that quarks and leptons can be identified as different components of 2-component complexified octonionic spinors.

The natural question is what associativization of octonions gives. This amounts to a condition putting the associator $a(bc)-(ab)c$ to zero. It is enough to consider octonionic imaginary units which are different. By using the decomposition of the octonionic algebra to quaternionic sub-algebra and its complement and general structure of structure constants, one finds that quaternionic sub-algebra remains as such but the products of all imaginary units in the complement with different imaginary units vanish. This means that the complement behaves effectively as 4-D flat space-gamma matrix algebra annihilated by the quaternionic sub-algebra whose imaginary part acts like Lie algebra of $SO(3)$.

8.3.5 What the replacement of $SO(7,1)$ sigma matrices with octonionic sigma matrices could mean?

The basic implication of octonionization is the replacement of $SO(7,1)$ sigma matrices with octonionic sigma matrices. For M^8 this has no consequences since since spinor connection is trivial.

For $M^4 \times CP_2$ situation would be different since CP_2 spinor connection would be replaced with its octonionic variant. This has some rather unexpected consequences and suggests that one should not try to octonionize at the level of $M^4 \times CP_2$ but interepret gamma matrices as tensor products of quaternionic gamma matrices, which can be replaced with their matrix representations. There are however some rather intriguing observations which force to keep mind open.

Octonionic representation of 8-D gamma matrices

Consider first the representation of 8-D gamma matrices in terms of tensor products of 7-D gamma matrices and 2-D Pauli sigma matrices.

(a) The gamma matrices are given by

$$\gamma^0 = 1 \times \sigma_1 \ , \ \gamma^i = \gamma^i \otimes \sigma_2 \ , \ i = 1,..,7 \ . \tag{8.3.2}$$

7-D gamma matrices in turn can be expressed in terms of 6-D gamma matrices by expressing γ^7 as

$$\gamma^{7)}_{i+1} = \gamma^{6)}_i , i = 1,...,6 \ , \ \gamma^{7)}_1 = \gamma^{6)}_7 = \prod_{i=1}^{6} \gamma^{6)}_i \ . \tag{8.3.3}$$

(b) The octonionic representation is obtained as

$$\gamma_0 = 1 \otimes \sigma_1 \ , \ \gamma_i = e_i \otimes \sigma_2 \ . \tag{8.3.4}$$

where e_i are the octonionic units. $e_i^2 = -1$ guarantees that the M^4 signature of the metric comes out correctly. Note that $\gamma_7 = \prod \gamma_i$ is the counterpart for choosing the preferred octonionic unit and plane M^2.

(c) The octonionic sigma matrices are obtained as commutators of gamma matrices:

$$\Sigma_{0i} = je_i \times \sigma_3 \ , \ \Sigma_{ij} = jf_{ij}{}^{k} e_k \otimes 1 \ . \tag{8.3.5}$$

Here j is commuting imaginary unit. These matrices span G_2 algebra having dimension 14 and rank 2 and having imaginary octonion units and their conjugates as the fundamental representation and its conjugate. The Cartan algebra for the sigma matrices can be chosen to be Σ_{01} and Σ_{23} and belong to a quaternionic sub-algebra.

(d) The lower dimension $D = 14$ of the non-associative version of sigma matrix algebra algebra means that some combinations of sigma matrices vanish. All left or right handed generators of the algebra are mapped to zero: this explains why the dimension is halved from 28 to 14. From the octonionic triangle expressing the multiplication rules for octonion units [A17] one finds $e_4e_5 = e_1$ and $e_6e_7 = -e_1$ and analogous expressions for the cyclic permutations of e_4, e_5, e_6, e_7. From the expression of the left handed sigma matrix $I^3_L = \sigma_{23} + \sigma^{30}$ representing left handed weak isospin (see the Appendix about the geometry of CP_2 [L1]) one can conclude that this particular sigma matrix and left handed sigma matrices in general are mapped to zero. The quaternionic sub-algebra $SU(2)_L \times SU(2)_R$ is mapped to that for the rotation group $SO(3)$ since in the case of Lorentz group one cannot speak of a decomposition to left and right handed subgroups. The elements of the complement of the quaternionic sub-algebra are expressible in terms of Σ_{ij} in the quaternionic sub-algebra.

Some physical implications of the reduction of $SO(7,1)$ to its octonionic counterpart

The octonization of spinor connection of CP_2 has some weird physical implications forcing to keep mind to the possibility that the octonionic description even at the level of H might have something to do with reality.

(a) If $SU(2)_L$ is mapped to zero only the right-handed parts of electro-weak gauge field survive octonionization. The right handed part is neutral containing only photon and Z^0 so that the gauge field becomes Abelian. Z^0 and photon fields become proportional to each other ($Z^0 \rightarrow sin^2(\theta_W)\gamma$) so that classical Z^0 field disappears from the dynamics, and one would obtain just electrodynamics.

(b) The gauge potentials and gauge fields defined by CP_2 spinor connection are mapped to fields in $SO(2) \subset SU(2) \times U(1)$ in quaternionic sub-algebra which in a well-defined sense corresponds to M^4 degrees of freedom and gauge group becomes $SO(2)$ subgroup of rotation group of $E^3 \subset M^4$. This looks like catastrophe. One might say that electroweak interactions are transformed to gravimagnetic interactions.

(c) In very optimistic frame of mind one might ask whether this might be a deeper reason for why electrodynamics is an excellent description of low energy physics and of classical physics. This is consistent with the fact that CP_2 coordinates define 4 field degrees of freedom so that single Abelian gauge field should be enough to describe classical physics. This would remove also the interpretational problems caused by the transitions changing the charge state of fermion induced by the classical W boson fields.

(d) Interestingly, the condition that electromagnetic charge is well-defined quantum number for the modes of the induced spinor field for $X^4 \subset H$ leads to the proposal that the solutions of the modified Dirac equation are localized to string world sheets in Minkowskian regions of space-time surface at least. For CP_2 type vacuum extremals one has massless Dirac and this allows only covariantly constant right-handed neutrino as solution. One has however only a piece of CP_2 (wormhole contact) so that holomorphic solutions annihilated by two complexified gamma matrices are possible in accordance with the conformal symmetries.

Can one assume non-trivial spinor connection in M^8

(a) The simplest option encouraged by the requirement of maximal symmetries is that it is absent. Massless 8-momenta would characterize spinor modes in M^8 and this would give physical justification for the octotwistors.

(b) If spinor connection is present at all, it reduces essentially to Kähler connection having different couplings to quarks and leptons identifiable as components of octonionic 2-spinors. It should be $SO(4)$ symmetric and since CP_2 is instant one might argue that now one has also instanton that is self-dual U(1) gauge field in $E^4 \subset M^4 \times E^4$ defining Kähler form. One can loosely say that that one has of constant electric and magnetic fields which are parallel to each other. The rotational symmetry in E^4 would break down to $SO(2)$.

(c) Without spinor connection quarks and leptons are in completely symmetric position at the level of M^8: this is somewhat disturbing. The difference between quarks and leptons in H is made possible by the fact that CP_2 does not allow standard spinor structure. Now this problem is absent. I have also consider the possibility that only leptonic spinor chirality is allowed and quarks result via a kind of anyonization process allowing them to have fractional em charges (see `http://www.tgdtheory.fi/public_html/articles/genesis.pdf`).

(d) If the solutions of the Kähler Dirac equation in Minkowskian regions are localized to two surfaces identifiable as integrable distributions of planes $M^2(x)$ and characterized by a local light-like direction defining the direction of massless momentum, they are holomorphic (in the sense of hyper-complex numbers) such that the second complexified modified gamma matrix annihilates the solution. Same condition makes sense also at the level of M^8 for solutions restricted to string world sheets and the presence or absence of spinor connection does not affect the situation.

Does this mean that the difference between quarks and leptons becomes visible only at the imbedding space level where ground states of super-conformal representations correspond to to imbedding space spinor harmonics which in CP_2 cm degrees are different for quarks and leptons?

Octo-spinors and their relation to ordinary imbedding space spinors

Octo-spinors are identified as octonion valued 2-spinors with basis

$$\begin{aligned} \Psi_{L,i} &= e_i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \\ \Psi_{q,i} &= e_i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad . \end{aligned} \tag{8.3.6}$$

One obtains quark and lepton spinors and conjugation for the spinors transforms quarks to leptons. Note that octospinors can be seen as 2-dimensional spinors with components which have values in the space of complexified octonions.

The leptonic spinor corresponding to real unit and preferred imaginary unit e_1 corresponds naturally to the two spin states of the right handed neutrino. In quark sector this would mean that right handed U quark corresponds to the real unit. The octonions decompose as $1+1+3+\overline{3}$ as representations of $SU(3) \subset G_2$. The concrete representations are given by

$$\begin{array}{ll} \{1 \pm ie_1\} \ , & e_R \text{ and } \nu_R \text{ with spin } 1/2 \ , \\ \{e_2 \pm ie_3\} \ , & e_R \text{ and } \nu_L \text{ with spin -}1/2 \ , \\ \{e_4 \pm ie_5\} & e_L \text{ and } \nu_L \text{ with spin } 1/2 \ , \\ \{e_6 \pm ie_7\} & e_L \text{ and } \nu_L \text{ with spin } 1/2 \ . \end{array} \tag{8.3.7}$$

Instead of spin one could consider helicity. All these spinors are eigenstates of e_1 (and thus of the corresponding sigma matrix) with opposite values for the sign factor $\epsilon = \pm$. The interpretation is in terms of vectorial isospin. States with $\epsilon = 1$ can be interpreted as charged leptons and D type quarks and those with $\epsilon = -1$ as neutrinos and U type quarks. The interpretation would be that the states with vanishing color isospin correspond to right handed fermions and the states with non-vanishing SU(3) isospin (to be not confused with QCD color isospin) and those with non-vanishing SU(3) isospin to left handed fermions.

The importance of this identification is that it allows a unique map of the candidates for the solutions of the octonionic modified Dirac equation to those of ordinary one. There are some delicacies involved due to the possibility to chose the preferred unit e_1 so that the preferred subspace M^2 can corresponds to a sub-manifold $M^2 \subset M^4$.

8.4 Abelian class field theory and TGD

The context leading to the discovery of adeles (`http://en.wikipedia.org/wiki/Adele_ring`) was so called Abelian class field theory. Typically the extension of rationals means that the ordinary primes decompose to the primes of the extension just like ordinary integers decompose to ordinary primes. Some primes can appear several times in the decomposition of ordinary non-square-free integers and similar phenomenon takes place for the integers of extension. If this takes place one says that the original prime is ramified. The simplest example is provided Gaussian integers $Q(i)$. All odd primes are unramified and primes $p \ mod \ 4 = 1$ they decompose as $p = (a+ib)(a-ib)$ whereas primes $p \ mos \ 4 = 3$ do not decompose at all. For $p = 2$ the decomposition is $2 = (1+i)(1-i) = -i(1+i)^2 = i(1-i)^2$ and is not unique $\{\pm 1, \pm i\}$ are the units of the extension. Hence $p = 2$ is ramified.

There goal of Abelian class field theory (`http://en.wikipedia.org/wiki/Class_field_theory`) is to understand the complexities related to the factorization of primes of the original field. The existence of the isomorphism between ideles modulo rationals - briefly ideles - and maximal Abelian Galois Group of rationals (MAGG) is one of the great discoveries of Abelian class field theory. Also the maximal - necessarily Abelian - extension of finite field G_p has Galois group isomorphic to the ideles. The Galois group of $G_p(n)$ with p^n elements is actually the cyclic group Z_n. The isomorphism opens up the way to study the representations of Abelian Galois group and also those of the AGG. One can indeed see these representations as special kind of representations for which the commutator group of AGG is represented trivially playing a role analogous to that of gauge group.

This framework is extremely general. One can replace rationals with any algebraic extension of rationals and study the maximal Abelian extension or algebraic numbers as its extension. One can consider the maximal algebraic extension of finite fields consisting of union of all all finite fields associated with given prime and corresponding adele. One can study function fields defined by the rational functions on algebraic curve defined in finite field and its maximal extension to include Taylor series. The isomorphisms applies in al these cases. One ends up with the idea that one can represent maximal Abelian Galois group in function space of complex valued functions in $GL_e(A)$ right invariant under the action of $GL_e(Q)$. A denotes here adeles.

In the following I will introduce basic facts about adeles and ideles and then consider a possible realization of the number theoretical vision about quantum TGD as a Galois theory for the algebraic extensions of classical number fields with associativity defining the dynamics. This picture leads automatically to the adele defined by p-adic variants of quaternions and octonions, which can be defined by posing a suitable restriction consistent with the basic physical picture provide by TGD.

8.4.1 Adeles and ideles

Adeles and ideles are structures obtained as products of real and p-adic number fields. The formula expressing the real norm of rational numbers as the product of inverses of its p-adic norms inspires the idea about a structure defined as produc of reals and various p-adic number fields.

Class field theory (`http://en.wikipedia.org/wiki/Class_field_theory`) studies Abelian extensions of global fields (classical number fields or functions on curves over finite fields), which by definition have Abelian Galois group acting as automorphisms. The basic result of class field theory is one-one correspondence between Abelian extensions and appropriate classes of ideals of the global field or open subgroups of the ideal class group of the field. For instance, Hilbert class field, which is maximal unramied extension of global field corresponds to a unique class of ideals of the number field. More precisely, reciprocity homomorphism generalizes the quadratic resiprocity for quadratic extensions of rationals. It maps the idele class group of the global field defined as the quotient of the ideles by the multiplicative group of the field - to the Galois group of the maximal Abelian extension of the global field. Each open subgroup of the idele class group of a global field is the image with respect to the norm map from the corresponding class field extension down to the global field.

The idea of number theoretic Langlands correspondence, [A15, A39, A38]. is that n-dimensional representations of Absolute Galois group correspond to infinite-D unitary representations of group $Gl_n(A)$. Obviously this correspondence is extremely general but might be highly relevant for TGD, where imbedding space is replaced with Cartesian product of real imbedding space and its p-adic variants - something which might be related to octonionic and quaternionic variants of adeles. It seems however that the TGD analogs for finite-D matrix groups are analogs of local gauge groups or Kac-Moody groups (in particular symplectic group of $\delta M^4_+ \times CP_2$) so that quite heavy generalization of already extremely abstract formalism is expected.

The following gives some more precise definitions for the basic notions.

(a) Prime ideals of global field, say that of rationals, are defined as ideals which do not decompose to a product of ideals: this notion generalizes the notion of prime. For instance, for p-adic numbers integers vanishing mod p^n define an ideal and ideals can be multiplied. For Abelian extensions of a global field the prime ideals in general decompose to prime ideals of the extension, and the decompostion need not be unique: one speaks of ramification. One of the challenges of tjhe class field theory is to provide information about the ramification. Hilbert class field is define as the maximal unramified extension of global field.

(b) The ring of integral adeles (see http://en.wikipedia.org/wiki/Adele_ring) is defined as $A_Z = R \times \hat{Z}$, where $\hat{Z} = \prod_p Z_p$ is Cartesian product of rings of p-adic integers for all primes (prime ideals) p of assignable to the global field. Multiplication of element of A_Z by integer means multiplication in all factors so that the structure is like direct sum from the point of view of physicist.

(c) The ring of rational adeles can be defined as the tensor product $A_Q = Q \otimes_Z A_Z$. $_Z$ means that in the multiplication by element of Z the factors of the integer can be distributed freely among the factors $\hat{Z}$. Using quantum physics language, the tensor product makes possible entanglement between Q and A_Z.

(d) Another definition for rational adeles is as $R \times \prod'_p Q_p$: the rationals in tensor factor Q have been absorbed to p-adic number fields: given prime power in Q has been absorbed to corresponding Q_p. Here all but finite number of Q_p elements ar p-adic integers. Note that one can take out negative powers of p_i and if their number is not finite the resulting number vanishes.The multiplication by integer makes sense but the multiplication by a rational does not smake sense since all factors Q_p would be multiplied.

(e) Ideles are defined as invertible adeles (http://en.wikipedia.org/wiki/Idele_class_groupIdele class group). The basic result of the class field theory is that the quotient of the multiplicative group of ideles by number field is homomorphic to the maximal Abelian Galois group!

8.4.2 Questions about adeles, ideles and quantum TGD

The intriguing general result of class field theory (http://en.wikipedia.org/wiki/Class_field_theory) is that the the maximal Abelian extension for rationals is homomorphic with the multiplicative group of ideles. This correspondence plays a key role in Langlands correspondence.

Does this mean that it is not absolutely necessary to introduce p-adic numbers? This is actually not so. The Galois group of the maximal abelian extension is rather complex objects (absolute Galois group, AGG, defines as the Galois group of algebraic numbers is even more complex!). The ring $\hat{Z}$ of adeles defining the group of ideles as its invertible elements homeomorphic to the Galois group of maximal Abelian extension is profinite group (http://en.wikipedia.org/wiki/Profinite_group). This means that it is totally disconnected space as also p-adic integers and numbers are. What is intriguing that p-dic integers are however a continuous structure in the sense that differential calculus is possible. A concrete example is provided by 2-adic units consisting of bit sequences which can have literally infinite non-vanishing bits. This space is formally discrete but one can construct differential calculus since the situation is not democratic. The higher the pinary digit in the expansion is, the less significant it is, and p-adic norm approaching to zero expresses the reduction of the insignificance.

1. Could TGD based physics reduce to a representation theory for the Galois groups of quaternions and octonions?

Number theoretical vision about TGD raises questions about whether adeles and ideles could be helpful in the formulation of TGD. I have already earlier considered the idea that quantum TGD could reduce to a representation theory of appropriate Galois groups. I proceed to make questions.

(a) Could real physics and various p-adic physics on one hand, and number theoretic physics based on maximal Abelian extension of rational octonions and quaternions on one hand, define equivalent formulations of physics?

(b) Besides various p-adic physics all classical number fields (reals, complex numbers, quaternions, and octonions) are central in the number theoretical vision about TGD. The technical problem is that p-adic quaternions and octonions exist only as a ring unless one poses some additional conditions. Is it possible to pose such conditions so that one could define what might be called quaternionic and octonionic adeles and ideles?

It will be found that this is the case: p-adic quaternions/octonions would be products of rational quaternions/octonions with a p-adic unit. This definition applies also to algebraic extensions of rationals and makes it possible to define the notion of derivative for corresponding adeles. Furthermore, the rational quaternions define non-commutative automorphisms of quaternions and rational octonions at least formally define a non-associative analog of group of octonionic automorphisms [K77].

(c) I have already earlier considered the idea about Galois group as the ultimate symmetry group of physics. The representations of Galois group of maximal Abelian extension (or even that for algebraic numbers) would define the quantum states. The representation space could be group algebra of the Galois group and in Abelian case equivalently the group algebra of ideles or adeles. One would have wave functions in the space of ideles.

The Galois group of maximal Abelian extension would be the Cartan subgroup of the absolute Galois group of algebraic numbers associated with given extension of rationals and it would be natural to classify the quantum states by the corresponding quantum numbers (number theoretic observables).

If octonionic and quaternionic (associative) adeles make sense, the associativity condition would reduce the analogs of wave functions to those at 4-dimensional associative sub-manifolds of octonionic adeles identifiable as space-time surfaces so that also space-time physics in various number fields would result as representations of Galois group in the maximal Abelian Galois group of rational octonions/quaternions. TGD would reduce to classical number theory! One can hope that WCW spinor fields assignable to the associative and co-associative space-time surfaces provide the adelic representations for super-conformal algebras replacing symmetries for point like objects.

This of course involves huge challenges: one should find an adelic formulation for WCWin terms octonionic and quaternionic adeles, similar formulation for WCW spinor fields in terms of adelic induced spinor fields or their octonionic variants is needed. Also zero energy ontology, causal diamonds, light-like 3-surfaces at which the signature of the induced metric changes, space-like 3-surfaces and partonic 2-surfaces at the boundaries of CDs, $M^8 - H$ duality, possible representation of space-time surfaces in terms of of O_c-real analytic functions (O_c denotes for complexified octonions), etc. should be generalized to adelic framework.

(d) Absolute Galois group is the Galois group of the maximal algebraic extension and as such a poorly defined concept. One can however consider the hierarchy of all finite-dimensional algebraic extensions (including non-Abelian ones) and maximal Abelian extensions associated with these and obtain in this manner a hierarchy of physics defined as representations of these Galois groups homomorphic with the corresponding idele groups.

(e) In this approach the symmetries of the theory would have automatically adelic representations and one might hope about connection with Langlands program [K24], [A15, A39, A38].

2. Adelic variant of space-time dynamics and spinorial dynamics?

As an innocent novice I can continue to pose stupid questions. Now about adelic variant of the space-time dynamics based on the generalization of Kähler action discussed already earlier but without mentioning adeles ([K79]).

(a) Could one think that adeles or ideles could extend reals in the formulation of the theory: note that reals are included as Cartesian factor to adeles. Could one speak about adelic space-time surfaces endowed with adelic coordinates? Could one formulate variational principle in terms of adeles so that exponent of action would be product of actions exponents associated with various factors with Neper number replaced by p for Z_p. The minimal interpretation would be that in adelic picture one collects under the same umbrella real physics and various p-adic physics.

(b) Number theoretic vision suggests that 4:th/8:th Cartesian powers of adeles have interpretation as adelic variants of quaternions/ octonions. If so, one can ask whether adelic quaternions and octonions could have some number theoretical meaning. Adelic quaternions and octonions are not number fields without additional assumptions since the moduli squared for a p-adic analog of quaternion and octonion can vanish so that the inverse fails to exist at the light-cone boundary which is 17-dimensional for complexified octonions and 7-dimensional for complexified quaternions. The reason is that norm squared is difference $N(o_1) - N(o_2)$ for $o_1 \oplus io_2$. This allows to define differential calculus for Taylor series and one can consider even rational functions. Hence the restriction is not fatal.

If one can pose a condition guaranteeing the existence of inverse for octonionic adel, one could define the multiplicative group of ideles for quaternions. For octonions one would obtain non-associative analog of the multiplicative group. If this kind of structures exist then four-dimensional associative/co-associative sub-manifolds in the space of non-associative ideles define associative/co-associative adeles in which ideles act. It is easy to find that octonionic ideles form 1-dimensional objects so that one must accept octonions with arbitrary real or p-adic components.

(c) What about equations for space-time surfaces. Do field equations reduce to separate field equations for each factor? Can one pose as an additional condition the constraint that p-adic surfaces provide in some sense cognitive representations of real space-time surfaces: this idea is formulated more precisely in terms of p-adic manifold concept [K79] (see the appendix of the book). Or is this correspondence an outcome of evolution?

Physical intuition would suggest that in most p-adic factors space-time surface corresponds to a point, or at least to a vacuum extremal. One can consider also the possibility that same algebraic equation describes the surface in various factors of the adele. Could this hold true in the intersection of real and p-adic worlds for which rationals appear in the polynomials defining the preferred extremals.

(d) To define field equations one must have the notion of derivative. Derivative is an operation involving division and can be tricky since adeles are not number field. The above argument suggests this is not actually a problem. Of course, if one can guarantee that the p-adic variants of octonions and quaternions are number fields, there are good hopes about well-defined derivative. Derivative as limiting value $df/dx = \lim(f(x+dx) - f(x))/dx$ for a function decomposing to Cartesian product of real function $f(x)$ and p-adic valued functions $f_p(x_p)$ would require that $f_p(x)$ is non-constant only for a finite number of primes: this is in accordance with the physical picture that only finite number of p-adic primes are active and define "cognitive representations" of real space-time surface. The second condition is that dx is proportional to product $dx \times \prod dx_p$ of differentials dx and dx_p, which are rational numbers. dx goes to xero as a real number but not p-adically for any of the primes involved. dx_p in turn goes to zero p-adically only for Q_p.

(e) The idea about rationals as points common to all number fields is central in number theoretical vision. This vision is realized for adeles in the minimal sense that the action of rationals is well-defined in all Cartesian factors of the adeles. Number theoretical vision allows also to talk about common rational points of real and various p-adic space-time surfaces in preferred coordinate choices made possible by symmetries of the imbedding space, and one ends up to the vision about life as something residing in the intersection of real and p-adic number fields. It is not clear whether and how adeles could allow to formulate this idea.

(f) For adelic variants of imbedding space spinors Cartesian product of real and p-adc variants of imbedding spaces is mapped to their tensor product. This gives justification for the physical vision that various p-adic physics appear as tensor factors. Does this mean that the generalized induced spinors are infinite tensor products of real and various p-adic spinors and Clifford algebra generated by induced gamma matrices is obtained by tensor product construction? Does the generalization of massless Dirac equation reduce to a sum of d'Alembertians for the factors? Does each of them annihilate the appropriate spinor? If only finite number of Cartesian factors corresponds to a space-time surface which is not vacuum extremal vanishing induced Kähler form, Kähler Dirac equation is non-trivial only in finite number of adelic factors.

3. Objections leading to the identification of octonionic adeles and ideles

The basic idea is that appropriately defined invertible quaternionic/octonionic adeles can be regarded as elements of Galois group assignable to quaternions/octonions. The best manner to proceed is to invent objections against this idea.

(a) The first objection is that p-adic quaternions and octonions do not make sense since p-adic variants of quaternions and octonions do not exist in general. The reason is that the p-adic norm squared $\sum x_i^2$ for p-adic variant of quaternion, octonion, or even complex number can vanish so that its inverse does not exist.

(b) Second objection is that automorphisms of the ring of quaternions (octonions) in the maximal Abelian extension are products of transformations of the subgroup of $SO(3)$ (G_2) represented by matrices with elements in the extension and in the Galois group of the extension itself. Ideles separate out as 1-dimensional Cartesian factor from this group so that one does not obtain 4-field (8-fold) Cartesian power of this Galois group.

One can define quaternionic/octonionic ideles in terms of rational quaternions/octonions multiplied by p-adic number. For adeles this condition produces non-sensical results.

(a) This condition indeed allows to construct the inverse of p-adic quaternion/octonion as a product of inverses for rational quaternion/octonion and p-adic number. The reason is that the solutions to $\sum x_i^2 = 0$ involve always p-adic numbers with an infinite number of pinary digits - at least one and the identification excludes this possibility. The ideles form also a group as required.

(b) One can interpret also the quaternionicity/octonionicity in terms of Galois group. The 7-dimensional non-associative counterparts for octonionic automorphisms act as transformations $x \to gxg^{-1}$. Therefore octonions represent this group like structure and the p-adic octonions would have interpretation as combination of octonionic automorphisms with those of rationals.

(c) One cannot assign to ideles 4-D idelic surfaces. The reason is that the non-constant part of all 8-coordinates is proportional to the same p-adic valued function of space-time point so that space-time surface would be a disjoint union of effectively 1-dimensional structures labelled by a subset of rational points of M^8. Induced metric would be 1-dimensional and induced Kähler and spinor curvature would vanish identically.

(d) One must allow p-adic octonions to have arbitrary p-adic components. The action of ideles representing Galois group on these surfaces is well-defined. Number field property is lost but this feature comes in play as poles only when one considers rational functions. Already the Minkowskian signature forces to consider complexified octonions and quaternions leading to the loss of field property. It would not be surprising if p-adic poles would be associated with the light-like orbits of partonic 2-surfaces. Both p-adic and Minkowskian poles might therefore be highly relevant physically and analogous to the poles of ordinary analytic functions. For instance, n-point functions could have poles at the light-like boundaries of causal diamonds and at light-like partonic orbits and explain their special physical role.

   The action of ideles in the quaternionic tangent space of space-time surface would be analogous to the action of of adelic linear group $Gl_n(A)$ in n-dimensional space.

(e) Adelic variants of octonions would be Cartesian products of ordinary and various p-adic octonions and would define a ring. Quaternionic 4-surfaces would define associative local sub-rings of octonion-adelic ring.

Chapter 9

Knots and TGD

9.1 Introduction

Witten has highly inspiring popular lecture about knots and quantum physics [A26] mentioning also his recent work with knots related to an attempt to understand Khovanov homology. Witten manages to explain in rather comprehensible manner both the construction recipe of Jones polynomial and the idea about how Jones polynomial emerges from topological quantum field theory as a vacuum expectation of so called Wilson loop defined by path integral with weighting coming from Chern-Simons action [A62] . Witten also tells that during the last year he has been working with an attempt to understand in terms of quantum theory the so called Khovanov polynomial associated with a much more abstract link invariant whose interpretation and real understanding remains still open. In particular, he mentions the approach of Gukov, Schwartz, and Vafa [A55, A55] as an attempt to understand Khovanov polynomial.

This kind of talks are extremely inspiring and lead to a series of questions unavoidably culminating to the frustrating "Why I do not have the brain of Witten making perhaps possible to answer these questions?". This one must just accept. In the following I summarize some thoughts inspired by the associations of the talk of Witten with quantum TGD and with the model of DNA as topological quantum computer. In my own childish manner I dare believe that these associations are interesting and dare also hope that some more brainy individual might take them seriously.

An idea inspired by TGD approach which also main streamer might find interesting is that the Jones invariant defined as vacuum expectation for a Wilson loop in 2+1-D space-time generalizes to a vacuum expectation for a collection of Wilson loops in 2+2-D space-time and could define an invariant for 2-D knots and for cobordisms of braids analogous to Jones polynomial. As a matter fact, it turns out that a generalization of gauge field known as gerbe is needed and that in TGD framework classical color gauge fields defined the gauge potentials of this field. Also topological string theory in 4-D space-time could define this kind of invariants. Of course, it might well be that this kind of ideas have been already discussed in literature.

Khovanov homology generalizes the Jones polynomial as knot invariant. The challenge is to find a quantum physical construction of Khovanov homology analous to the topological QFT defined by Chern-Simons action allowing to interpret Jones polynomial as vacuum expectation value of Wilson loop in non-Abelian gauge theory.

Witten's approach to Khovanov homology relies on fivebranes as is natural if one tries to define 2-knot invariants in terms of their cobordisms involving violent un-knottings. Despite the difference in approaches it is very useful to try to find the counterparts of this approach in quantum TGD since this would allow to gain new insights to quantum TGD itself as almost topological QFT identified as symplectic theory for 2-knots, braids and braid cobordisms. This comparison turns out to be extremely useful from TGD point of view.

(a) A highly unique identification of string world sheets and therefore also of the braids whose ends carry quantum numbers of many particle states at partonic 2-surfaces emerges if one identifies the string word sheets as singular surfaces in the same manner as is done in Witten's approach.

This identification need not of course be correct and in TGD framework the localization of the modes of the induced spinor fields at 2-D surfaces carrying vanishing induced W boson fields guaranteeing that the em charge of spinor modes is well-defined for a generic preferred extremal is natural. Besides string world sheets partonic 2-surfaces are good candidates for this kind of surfaces. It is not clear whether one can have continuous slicing of this kind by string world sheets and partonic 2-surfaces orthogonal to them or whether only discrete set of these surfaces is possible.

(b) Also a physical interpretation of the operators Q, F, and P of Khovanov homology emerges. P would correspond to instanton number and F to the fermion number assignable to right handed neutrinos. The breaking of M^4 chiral invariance makes possible to realize Q physically. The finding that the generalizations of Wilson loops can be identified in terms of the gerbe fluxes $\int H_A J$ supports the conjecture that TGD as almost topological QFT corresponds essentially to a symplectic theory for braids and 2-knots.

The basic challenge of quantum TGD is to give a precise content to the notion of generalization Feynman diagram and the reduction to braids of some kind is very attractive possibility inspired by zero energy ontology. The point is that no $n > 2$-vertices at the level of braid strands are needed if bosonic emergence holds true.

(a) For this purpose the notion of algebraic knot is introduced and the possibility that it could be applied to generalized Feynman diagrams is discussed. The algebraic structrures kei, quandle, rack, and biquandle and their algebraic modifications as such are not enough. The lines of Feynman graphs are replaced by braids and in vertices braid strands redistribute. This poses several challenges: the crossing associated with braiding and crossing occurring in non-planar Feynman diagrams should be integrated to a more general notion; braids are replaced with sub-manifold braids; braids of braids ....of braids are possible; the redistribution of braid strands in vertices should be algebraized. In the following I try to abstract the basic operations which should be algebraized in the case of generalized Feynman diagrams.

(b) One should be also able to concretely identify braids and 2-braids (string world sheets) as well as partonic 2-surfaces and I have discussed several identifications during last years. Legendrian braids turn out to be very natural candidates for braids and their duals for the partonic 2-surfaces. String world sheets in turn could correspond to the analogs of Lagrangian sub-manifolds or two minimal surfaces of space-time surface satisfying the weak form of electric-magnetic duality. The latter option turns out to be more plausible. This identification - if correct - would solve quantum TGD explicitly at string world sheet level which corresponds to finite measurement resolution.

(c) Also a brief summary of generalized Feynman rules in zero energy ontology is proposed. This requires the identification of vertices, propagators, and prescription for integrating over al 3-surfaces. It turns out that the basic building blocks of generalized Feynman diagrams are well-defined.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L13]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L14].

9.2 Some TGD background

What makes quantum TGD [L4, L5, L8, L9, L6, L3, L7, L10] interesting concerning the description of braids and braid cobordisms is that braids and braid cobordisms emerge both at the level of generalized Feynman diagrams and in the model of DNA as a topological quantum computer [K16] .

9.2.1 Time-like and space-like braidings for generalized Feynman diagrams

(a) In TGD framework space-times are 4-D surfaces in 8-D imbedding space. Basic objects are partonic 2-surfaces at the two ends of causal diamonds CD (intersections of future and past directed light-cones of 4-D Minkowski space with each point replaced with CP_2). The light-like orbits of partonic 2-surfaces define 3-D light-like 3-surfaces identifiable as lines of generalized Feynman diagrams. At the vertices of generalized Feynman diagrams incoming and outgoing light-like 3-surfaces meet. These diagrams are not direct generalizations of string diagrams since they are singular as 4-D manifolds just like the ordinary Feynman diagrams.

By strong form of holography one can assign to the partonic 2-surfaces and their tangent space data space-time surfaces as preferred extremals of Kähler action. This guarantees also general coordinate invariance and allows to interpret the extremals as generalized Bohr orbits.

(b) One can assign to the partonic 2-surfaces discrete sets of points carrying quantum numbers. These sets of points emerge from the solutions of of the Kähler-Dirac equation, which are localized at 2-D surfaces - string world sheets and possibly also partonic 2-surfaces - carrying vanishing induced W fields and also Z^0 fields above weak scale. These points and their orbits identifiable as boundaries of string world sheets define braid strands at the light-like orbits of partonic 2-surfaces. In the generic case the strands get tangled in time direction and one has linking and knotting giving rise to a time-like braiding. String world sheets and also partonic surfaces define 2-braids and 2-knots at 4-D space-time surface so that knot theory generalizes.

(c) Also space-like braidings are possible. One can imagine that the partonic 2-surfaces are connected by space-like curves defining TGD counterparts for strings and that in the initial state these curves define space-like braids whose ends belong to different partonic 2-surfaces. Quite generally, the basic conjecture is that the preferred extremals define orbits of string-like objects with their ends at the partonic 2-surfaces. One would have slicing of space-time surfaces by string world sheets one one hand and by partonic 2-surface on one hand. This string model is very special due to the fact that the string orbits define what could be called braid cobordisms representing which could represent unknotting of braids. String orbits in higher dimensional space-times do not allow this topological interpretation.

9.2.2 Dance metaphor

Time like braidings induces space-like braidings and one can speak of time-like or dynamical braiding and even duality of time-like and space-like braiding. What happens can be understood in terms of dance metaphor.

(a) One can imagine that the points carrying quantum numbers are like dancers at parquettes defined by partonic 2-surfaces. These parquettes are somewhat special in that it is moving and changing its shape.

(b) Space-like braidings means that the feet of the dancers at different parquettes are connected by threads. As the dance continues, the threads connecting the feet of different dancers at different parquettes get tangled so that the dance is coded to the braiding of the threads. Time-like braiding induce space-like braiding. One has what might be called a cobordism for space-like braiding transforming it to a new one.

9.2.3 DNA as topological quantum computer

The model for topological quantum computation is based on the idea that time-like braidings defining topological quantum computer programs. These programs are robust since the topology of braiding is not affected by small deformations.

(a) The first key idea in the model of DNA as topological quantum computer is based on the observation that the lipids of cell membrane form a 2-D liquid whose flow defines the dance in which dancers are lipids which define a flow pattern defining a topological quantum computation. Lipid layers assignable to cellular and nuclear membranes are the parquettes. This 2-D flow pattern can be induced by the liquid flow near the cell membrane or in case of nerve pulse transmission by the nerve pulses flowing along the axon. This alone defines topological quantum computation.

(b) In DNA as topological quantum computer model one however makes a stronger assumption motivated by the vision that DNA is the brain of cell and that information must be communicated to DNA level wherefrom it is communicated to what I call magnetic body. It is assumed that the lipids of the cell membrane are connected to DNA nucleotides by magnetic flux tubes defining a space-like braiding. It is also possible to connect lipids of cell membrane to the lipids of other cell membranes, to the tubulins at the surfaces of microtubules, and also to the aminoadics of proteins. The spectrum of possibilities is really wide.

    The space-like braid strands would correspond to magnetic flux tubes connecting DNA nucleotides to lipids of nuclear or cell membrane. The running of the topological quantum computer program defined by the time-like braiding induced by the lipid flow would be coded to a space-like braiding of the magnetic flux tubes. The braiding of the flux tubes would define a universal memory storage mechanism and combined with 4-D view about memory provides a very simple view about how memories are stored and how they are recalled.

9.3 Could braid cobordisms define more general braid invariants?

Witten says that one should somehow generalize the notion of knot invariant. The above described framework indeed suggests a very natural generalization of braid invariants to those of braid cobordisms reducing to braid invariants when the braid at the other end is trivial. This description is especially natural in TGD but allows a generalization in which Wilson loops in 4-D sense describe invariants of braid cobordisms.

9.3.1 Difference between knotting and linking

Before my modest proposal of a more general invariant some comments about knotting and linking are in order.

(a) One must distinguish between internal knotting of each braid strand and linking of 2 strands. They look the same in the 3-D case but in higher dimensions knotting and linking are not the same thing. Codimension 2 surfaces get knotted in the generic case, in particular the 2-D orbits of the braid strands can get knotted so that this gives additional topological flavor to the theory of strings in 4-D space-time. Linking occurs for two surfaces whose dimension d_1 and d_2 satisfying $d_1 + d_2 = D - 1$, where D is the dimension of the imbedding space.

(b) 2-D orbits of strings do not link in 4-D space-time but do something more radical since the sum of their dimensions is $D = 4$ rather than only $D - 1 = 3$. They intersect and it is impossible to eliminate the intersection without a change of topology of the stringy 2-surfaces: a hole is generated in either string world sheet. With a slight deformation intersection can be made to occur generically at discrete points.

9.3.2 Topological strings in 4-D space-time define knot cobordisms

What makes the 4-D braid cobordisms interesting is following.

(a) The opening of knot by using brute force by forcing the strands to go through each other induces this kind of intersection point for the corresponding 2-surfaces. From 3-D perspective this looks like a temporary cutting of second string, drawing the string ends to some distance and bringing them back and gluing together as one approaches the moment when the strings would go through each other. This surgical operation for either string produces a pair of non-intersecting 2-surfaces with the price that the second string world sheet becomes topologically non-trivial carrying a hole in the region were intersection would occur. This operation relates a given crossing of braid strands to its dual crossing in the construction of Jones polynomial in given step (string 1 above string 2 is transformed to string 2 above string 1).

(b) One can also cut both strings temporarily and glue them back together in such a manner that end a/b of string 1 is glued to the end c/d of string 2. This gives two possibilities corresponding to two kinds of reconnections. Reconnections appears as the second operation in the construction of Jones invariant besides the operation putting the string above the second one below it or vice versa. Jones polynomial relates in a simple manner to Kauffman bracket allowing a recursive construction. At a given step a crossing is replaced with a weighted sum of the two reconnected terms [A1, A11] . Reconnection represents the analog of trouser vertex for closed strings replaced with braid strands.

(c) These observations suggest that stringy diagrams describe the braid cobordisms and a kind of topological open string model in 4-D space-time could be used to construct invariants of braid cobordisms. The dynamics of strand ends at the partonic 2-surfaces would partially induce the dynamics of the space-like braiding. This dynamics need not induce the un-knotting of space-like braids and simple string diagrams for open strings are enough to define a cobordism leading to un-knotting. The holes needed to realize the crossover for braid strands would contribute to the Wilson loop an additional factor corresponding to the rotation of the gauge potential around the boundary of the hole (non-integrable phase factor). In abelian case this gives simple commuting phase factor.

Note that braids are actually much more closer to the real world than knots since a useful strand of knotted structure must end somewhere. The abstract closed loops of mathematician floating in empty space are not very useful in real life albeit mathematically very convenient as Witten notices. Also the braid cobordisms with ends of a collection of space-like braids at the ends of causal diamond are more practical than 2-knots in 4-D space. Mathematician would see these objects as analogous to surfaces in relative homology allowed to have boundaries if they located at fixed sub-manifolds. Homology for curves with ends fixed to be on some surfaces is a good example of this. Now these fixed sub-manifolds would correspond to space-like 3-surfaces at the ends CDs and light-like wormhole throats at which the signature of the induced metric changes and which are carriers of elementary particle quantum numbers.

9.4 Invariants 2-knots as vacuum expectations of Wilson loops in 4-D space-time?

The interpretation of string world sheets in terms of Wilson loops in 4-dimensional space-time is very natural. This raises the question whether Witten's a original identification of the Jones polynomial as vacuum expectation for a Wilson loop in 2+1-D space might be replaced with a vacuum expectation for a collection of Wilson loops in 3+1-D space-time and would characterize in the general case (multi-)braid cobordism rather than braid. If the braid at the lower or upped boundary is trivial, braid invariant is obtained. The intersections of the Wilson loops would correspond to the violent un-knotting operations and the boundaries of the resulting holes give an additional Wilson loop. An alternative interpretation would be as the analog of Jones polynomial for 2-D knots in 4-D space-time generalizing Witten's theory. This description looks completely general and does not require TGD at all.

The following considerations suggest that Wilson loops are not enough for the description of general 2-knots and that that Wilson loops must be replaced with 2-D fluxes. This requires a generalization of gauge field concept so that it corresponds to a 3-form instead of 2-form is needed. In TGD framework this kind of generalized gauge fields exist and their gauge potentials correspond to classical color gauge fields.

9.4.1 What 2-knottedness means concretely?

It is easy to imagine what ordinary knottedness means. One has circle imbedded in 3-space. One projects it in some plane and looks for crossings. If there are no crossings one knows that un-knot is in question. One can modify a given crossing by forcing the strands to go through each other and this either generates or removes knottedness. One can also destroy crossing by reconnection and this always reduces knottedness. Since knotting reduces to linking in 3-D case, one can find a simple interpretation for knottedness in terms of linking of two circles. For 2-knots linking is not what gives rise to knotting.

One might hope to find something similar in the case of 2-knots. Can one imagine some simple local operations which either increase of reduce 2-knottedness?

(a) To proceed let us consider as simple situation as possible. Put sphere in 3-D time= constant section E^3 of 4-space. Add a another sphere to the same section E^3 such that the corresponding balls do not intersect. How could one build from these two spheres a knotted 2-sphere?

(b) From two spheres one can build a single sphere in topological sense by connecting them with a small cylindrical tube connecting the boundaries of disks (circles) removed from the two spheres. If this is done in E^3, a trivial 2-knot results. One can however do the gluing of the cylinder in a more exotic manner by going temporarily to "hyper-space", in other words making a time travel. Let the cylinder leave the second sphere from the outer surface, let it go to future or past and return back to recent but through the interior. This is a good candidate for a knotted sphere since the attempts to deform it to self-non-intersecting sphere in E^3 are expected to fail since the cylinder starting from interior necessarily goes through the surface of sphere if wants to the exterior of the sphere.

(c) One has actually 2×2 manners to perform the connected sum of 2-spheres depending on whether the cylinders leave the spheres through exterior or interior. At least one of them (exterior-exterior) gives an un-knotted sphere and intuition suggests that all the three remaining options requiring getting out from the interior of sphere give a knotted 2-sphere. One can add to the resulting knotted sphere new spheres in the same manner and obtain an infinite number of them. As a matter fact, the proposed 1+3 possibilities correspond to different versions of connected sum and one could speak of knotting and non-knotting connected sums. If the addition of knotted spheres is performed by non-knotting connected sum, one obtains composites of already existing 2-knots. Connected sum composition is analogous to the composition of integer to a product of primes. One indeed speaks of prime knots and the number of prime knots is infinite. Of course, it is far from clear whether the connected sum operation is enough to build all knots. For instance it might well be that cobordisms of 1-braids produces knots not producible in this manner. In particular, the effects of time-like braiding induce braiding of space-like strands and this looks totally different from local knotting.

9.4.2 Are all possible 2-knots possible for stringy world sheets?

Whether all possible 2-knots are allowed for stringy world sheets, is not clear. In particular, if they are dynamically determined it might happen that many possibilities are not realized.

For instance, the condition that the signature of the induced metric is Minkowskian could be an effective killer of 2-knottedness not reducing to braid cobordism.

(a) One must start from string world sheets with Minkowskian signature of the induced metric. In other words, in the previous construction one must E^3 with 3-dimensional Minkowski space M^3 with metric signature 1+2 containing the spheres used in the construction. Time travel is replaced with a travel in space-like hyper dimension. This is not a problem as such. The spheres however have at least one two special points corresponding to extrema at which the time coordinate has a local minimum or maximum. At these points the induced metric is necessarily degenerate meaning that its determinant vanishes. If one allows this kind of singular points one can have elementary knotted spheres. This liberal attitude is encouraged by the fact that the light-like 3-surfaces defining the basic dynamical objects of quantum TGD correspond to surfaces at which 4-D induced metric is degenerate. Otherwise 2-knotting reduces to that induced by cobordisms of 1-braids. If one allows only the 2-knots assignable to the slicings of the space-time surface by string world sheets and even restricts the consideration to those suggested by the duality of 2-D generalization of Wilson loops for string world sheets and partonic 2-surfaces, it could happen that the string world sheets reduce to braidings.

(b) The time=constant intersections define a representation of 2-knots as a continuous sequence of 1-braids. For critical times the character of the 1-braids changes. In the case of braiding this corresponds to the basic operations for 1-knots having interpretation as string diagrams (reconnection and analog of trouser vertex). The possibility of genuine 2-knottedness brings in also the possibility that strings pop up from vacuum as points, expand to closed strings, are fused to stringy words sheet temporarily by the analog of trouser vertex, and eventually return to the vacuum. Essentially trouser diagram but second string open and second string closed and beginning from vacuum and ending to it is in question. Vacuum bubble interacting with open string would be in question. The believer in string model might be eager to accept this picture but one must be cautious.

9.4.3 Are Wilson loops enough for 2-knots?

Suppose that the space-like braid strands connecting partonic 2-surfaces at given boundary of CD and light-like braids connecting partonic 2-surfaces belonging to opposite boundaries of CD form connected closed strands. The collection of closed loops can be identified as boundaries of Wilson loops and the expectation value is defined as the product of traces assignable to the loops. The definition is exactly the same as in 2+1-D case [A62] .

Is this generalization of Wilson loops enough to describe 2-knots? In the spirit of the proposed philosophy one could ask whether there exist two-knots not reducible to cobordisms of 1-knots whose knot invariants require cobordisms of 2-knots and therefore 2-braids in 5-D space-time. Could it be that dimension $D = 4$ is somehow very special so that there is no need to go to $D = 5$? This might be the case since for ordinary knots Jones polynomial is very faithful invariant.

Innocent novice could try to answer the question in the following manner. Let us study what happens locally as the 2-D closed surface in 4-D space gets knotted.

(a) In 1-D case knotting reduces to linking and means that the first homotopy group of the knot complement is changed so that the imbedding of first circle implies that the there exists imbedding of the second circle that cannot be transformed to each other without cutting the first circle temporarily. This phenomenon occurs also for single circle as the connected sum operation for two linked circles producing single knotted circle demonstrates.

(b) In 2-D case the complement of knotted 2-sphere has a non-trivial second homotopy group so that 2-balls have homotopically non- equivalent imbeddings, which cannot be transformed to each other without intersection of the 2-balls taking place during the process. Therefore the description of 2-knotting in the proposed manner would require cobordisms of 2-knots and thus 5-D space-time surfaces. However, since 3-D description for ordinary knots works so well, one could hope that the generalization the notion of Wilson loop could allow to avoid 5-D description altogether. The generalized Wilson loops would be assigned to 2-D surfaces and gauge potential A would be replaced with 2-gauge potential B defining a three-form $F = dB$ as the analog of gauge field.

(c) This generalization of bundle structure known as gerbe structure has been introduced in algebraic geometry [A7, A44] and studied also in theoretical physics [A50] . 3-forms appear as analogs of gauge fields also in the QFT limit of string model. Algebraic geometer would see gerbe as a generalization of bundle structure in which gauge group is replaced with a gauge groupoid. Essentially a structure of structures seems to be in question. For instance, the principal bundles with given structure group for given space defines a gerbe. In the recent case the space of gauge fields in space-time could be seen as a gerbe. Gerbes have been also assigned to loop spaces and WCW can be seen as a generalization of loop space. Lie groups define a much more mundane example about gerbe. The 3-form F is given by $F(X,Y,Z) = B(X,[Y,Z])$, where B is Killing form and for $U(n)$ reduces to $(g^{-1}dg)^3$. It will be found that classical color gauge fields define gerbe gauge potentials in TGD framework in a natural manner.

9.5 TGD inspired theory of braid cobordisms and 2-knots

In the sequel the considerations are restricted to TGD and to a comparison of Witten's ideas with those emerging in TGD framework.

9.5.1 Weak form of electric-magnetic duality and duality of space-like and time-like braidings

Witten notices that much of his work in physics relies on the assumption that magnetic charges exist and that rather frustratingly, cosmic inflation implies that all traces of them disappear. In TGD Universe the non-trivial topology of CP_2 makes possible Kähler magnetic charge and inflation is replaced with quantum criticality. The recent view about elementary particles is that they correspond to string like objects with length of order electro-weak scale with Kähler magnetically charged wormhole throats at their ends. Therefore magnetic charges would be there and LHC might be able to detect their signatures if LHC would get the idea of trying to do this.

Witten mentions also electric-magnetic duality. If I understood correctly, Witten believes that it might provide interesting new insights to the knot invariants. In TGD framework one speaks about weak form of electric magnetic duality. This duality states that Kähler electric fluxes at space-like ends of the space-time sheets inside CDs and at wormhole throats are proportional to Kähler magneic fluxes so that the quantization of Kähler electric charge quantization reduces to purely homological quantization of Kähler magnetic charge.

The weak form of electric-magnetic duality fixes the boundary conditions of field equations at the light-like and space-like 3-surfaces. Together with the conjecture that the Kähler current is proportional to the corresponding instanton current this implies that Kähler action for the preferred extremal sof Kähler action reduces to 3-D Chern-Simons term so that TGD reduces to almost topological QFT. This means an enormous mathematical simplification of the theory and gives hopes about the solvability of the theory. Since knot invariants are defined in terms of Abelian Chern-Simons action for induced Kähler gauge potential, one might hope that TGD could as a by-product define invariants of braid cobordisms in terms of the unitary U-matrix of the theory between zero energy states and having as its rows the non-unitary M-matrices analogous to thermal S-matrices.

Electric magnetic duality is 4-D phenomenon as is also the duality between space-like and time like braidings essential also for the model of topological quantum computation. Also this suggests that some kind of topological string theory for the space-time sheets inside CDs could allow to define the braid cobordism invariants.

9.5.2 Could Kähler magnetic fluxes define invariants of braid cobordisms?

Can one imagine of defining knot invariants or more generally, invariants of knot cobordism in this framework? As a matter fact, also Jones polynomial describes the process of unknotting and the replacement of unknotting with a general cobordism would define a more general invariant. Whether the Khovanov invariants might be understood in this more general framework is an interesting question.

(a) One can assign to the 2-dimensional stringy surfaces defined by the orbits of space-like braid strands Kähler magnetic fluxes as flux integrals over these surfaces and these integrals depend only on the end points of the space-like strands so that one deform the space-like strands in an arbitrarily manner. One can in fact assign these kind of invariants to pairs of knots and these invariants define the dancing operation transforming these knots to each other. In the special case that the second knot is un-knot one obtains a knot-invariant (or link- or braid- invariant).

(b) The objection is that these invariants depend on the orbits of the end points of the space-like braid strands. Does this mean that one should perform an averaging over the ends with the condition that space-like braid is not affected topologically by the allowed deformations for the positions of the end points?

(c) Under what conditions on deformation the magnetic fluxes are not affect in the deformation of the braid strands at 3-D surfaces? The change of the Kähler magnetic flux is magnetic flux over the closed 2-surface defined by initial non-deformed and deformed stringy two-surfaces minus flux over the 2-surfaces defined by the original time-like and space-like braid strands connected by a thin 2-surface to their small deformations. This is the case if the deformation corresponds to a U(1) gauge transformation for a Kähler flux. That is diffeomorphism of M^4 and symplectic transformation of CP_2 inducing the U(1) gauge transformation.

    Hence a natural equivalence for braids is defined by these transformations. This is quite not a topological equivalence but quite a general one. Symplectic transformations of CP_2 at light-like and space-like 3-surfaces define isometries of the world of classical worlds so that also in this sense the equivalence is natural. Note that the deformations of space-time surfaces correspond to this kind of transformations only at space-like 3-surfaces at the ends of CDs and at the light-like wormhole throats where the signature of the induced metric changes. In fact, in quantum TGD the sub-spaces of world of classical worlds with constant values of zero modes (non-quantum fluctuating degrees of freedom) correspond to orbits of 3-surfaces under symplectic transformations so that the symplectic restriction looks rather natural also from the point of view of quantum dynamics and the vacuum expectation defined by Kähler function be defined for physical states.

(d) A further possibility is that the light-like and space-like 3-surfaces carry vanishing induced Kähler fields and represent surfaces in $M^4 \times Y^2$, where Y^2 is Lagrangian submanifold of CP_2 carrying vanishing Kähler form. The interior of space-time surface could in principle carry a non-vanishing Kähler form. In this case weak form of self-duality cannot hold true. This however implies that the Kähler magnetic fluxes vanish identically as circulations of Kähler gauge potential. The non-integrable phase factors defined by electroweak gauge potentials would however define non-trivial classical Wilson loops. Also electromagnetic field would do so. It would be therefore possible to imagine vacuum expectation value of Wilson loop for given quantum state. Exponent of Kähler action would define for non-vacuum extremals the weighting. For 4-D vacuum extremals this exponent is trivial and one might imagine of using imaginary exponent of electroweak Chern-Simons action. Whether the restriction to vacuum extremals in the definition of vacuum expectations of electroweak Wilson loops could define general enough invariants for braid cobordisms remains an open question.

(e) The quantum expectation values for Wilson loops are non-Abelian generalizations of exponentials for the expectation values of Kähler magnetic fluxes. The classical color field is proportional to the induced Kähler form and its holonomy is Abelian which raises the question whether the non-Abelian Wilson loops for classical color gauge field could be expressible in terms of Kähler magnetic fluxes.

9.5.3 Classical color gauge fields and their generalizations define gerbe gauge potentials allowing to replace Wilson loops with Wilson sheets

As already noticed, the description of 2-knots seems to necessitate the generalization of gauge field to 3-form and the introduction of a gerbe structure. This seems to be possible in TGD framework.

(a) Classical color gauge fields are proportional to the products $B_A = H_A J$ of the Hamiltonians of color isometries and of Kähler form and the closed 3-form $F_A = dB_A = dH_A \wedge J$ could serve as a colored 3-form defining the analog of U(1) gauge field. What would be interesting that color would make F non-vanishing. The "circulation" $h_A = \oint H_A J$ over a closed partonic 2-surface transforms covariantly under symplectic transformations of CP_2, whose Hamiltonians can be assigned to irreps of SU(3): just the commutator of Hamiltonians defined by Poisson bracket appears in the infinitesimal transformation. One could hope that the expectation values for the exponents of the fluxes of B_A over 2-knots could define the covariants able to catch 2-knotted-ness in TGD framework. The exponent defining Wilson loop would be replaced with $exp(iQ^A h_A)$, where Q^A denote color charges acting as operators on particles involved.

(b) Since the symplectic group acting on partonic 2-surfaces at the boundary of CD replaces color group as a gauge group in TGD, one can ask whether symplectic SU(3) should be actually replaced with the entire symplectic group of $\cup_{\pm}\delta M^4_{\pm} \times CP_2$ with Hamiltonians carrying both spin and color quantum numbers. The symplectic fluxes $\oint H_A J$ are indeed used in the construction of both quantum states and of WCW geometry. This generalization is indeed possible for the gauge potentials $B_A J$ so that one would have infinite number of classical gauge fields having also interpretation as gerbe gauge potentials.

(c) The objection is that symplectic transformations are not symmetries of Kähler action. Therefore the action of symplectic transformation induced on the space-time surface reduces to a symplectic transformation only at the partonic 2-surfaces. This spoils the covariant transformation law for the 2-fluxes over stringy world sheets unless there exist preferred stringy world sheets for which the action is covariant. The proposed duality between the descriptions based on partonic 2-surfaces and stringy world sheets realized in terms of slicings of space-time surface by string world sheets and partonic 2-surfaces suggests that this might be the case.

   This would mean that one can attach to a given partonic 2-surface a unique collection string world sheets. The duality suggests even stronger condition stating that the total exponents $exp(iQ^A h_A)$ of fluxes are the same irrespective whether h_A evaluated for partonic 2-surfaces or for string world sheets defining the analog of 2-knot. This would mean an immense calculational simplification! This duality would correspond very closely to the weak form of electric magnetic duality whose various forms I have pondered as a must for the geometry of WCW. Partonic 2-surfaces indeed correspond to magnetic monopoles at least for elementary particles and stringy world sheets to surfaces carrying electric flux (note that in the exponent magnetic charges do not make themselves visible so that the identity can make sense also for $H_A = 1$).

(d) Quantum expectation means in TGD framework a functional integral over the symplectic orbits of partonic 2-surfaces plus 4-D tangent space data assigned to the upper and lower boundaries of CD. Suppose that holography fixes the space-like 3-surfaces at the ends of CD and light-like orbits of partonic 2-surfaces. In completely general case the braids and the stringy space-time sheets could be fixed using a representation in terms of space-time coordinates so that the representation would be always the same but the imbedding varies as also the values of the exponent of Kähler function, of the Wilson loop, and of its 2-D generalization. The functional integral over symplectic transforms of 3-surfaces implies that Wilson loop and its 2-D generalization varies.

The proposed duality however suggests that both Wilson loop and its 2-D generalization are actually fixed by the dynamics of quantum TGD. One can ask whether the presence of 2-D analog of Wilson loop has a direct physical meaning bringing into almost topological stringy dynamics associated with color quantum numbers and coding explicit information about space-time interior and topology of field lines so that color dynamics would also have interpretation as a theory of 2-knots. If the proposed duality suggested by holography holds true, only the data at partonic 2-surfaces would be needed to calculate the generalized Wilson loops.

In TGD framework the localization of the modes of the induced spinor fields at 2-D surfaces carrying vanishing induced W boson fields guaranteeing that the em charge of spinor modes is well-defined for a generic preferred extremal is natural [K69]. Besides string world sheets partonic 2-surfaces are good candidates for this kind of surfaces. It is not clear whether one can have a continuous slicing of this kind by string world sheets and partonic 2-surfaces orthogonal to them or whether only discrete set of these surfaces is possible.

This picture is very speculative and sounds too good to be true but follows if one consistently applies holography.

9.5.4 Summing sup the basic ideas

Let us summarize the ideas discussed above.

(a) Instead of knots, links, and braids one could study knot and link cobordisms, that is their dynamical evolutions concretizable in terms of dance metaphor and in terms of interacting string world sheets. Each space-like braid strand can have purely internal knotting and braid strands can be linked. TGD could allow to identify uniquely both space-like and time-like braid strands and thus also the stringy world sheets more or less uniquely and it could be that the dynamics induces automatically the temporary cutting of braid strands when knot is opened violently so that a hole is generated. Gerbe gauge potentials defined by classical color gauge fields could make also possible to characterize 2-knottedness in symplectic invariant manner in terms of color gauge fluxes over 2-surfaces.

The weak form of electric-magnetic duality would reduce the situation to almost topological QFT in general case with topological invariance replaced with symplectic one which corresponds to the fixing of the values of non-quantum fluctuating zero modes in quantum TGD. In the vacuum sector it would be possible to have the counterparts of Wilson loops weighted by 3-D electroweak Chern-Simons action defined by the induced spinor connection.

(b) One could also leave TGD framework and define invariants of braid cobordisms and 2-D analogs of braids as vacuum expectations of Wilson loops using Chern-Simons action assigned to 3-surfaces at which space-like and time-like braid strands end. The presence of light-like and space-like 3-surfaces assignable to causal diamonds could be assumed also now.

I checked whether the article of Gukov, Scwhartz, and Vafa entitled "Khovanov-Rozansky Homology and Topological Strings" [A55, A55] relies on the primitive topological observations made above. This does not seem to be the case. The topological strings in this case are strings in 6-D space rather than 4-D space-time.

There is also an article by Dror Bar-Natan with title "Khovanov's homology for tangles and cobordisms" [A29] . The article states that the Khovanov homology theory for knots and links generalizes to tangles, cobordisms and 2-knots. The article does not say anything explicit about Wilson loops but talks about topological QFTs.

An article of Witten about his physical approach to Khovanov homology has appeared in arXiv [A63] . The article is more or less abracadabra for anyone not working with M-theory but the basic idea is simple. Witten reformulates 3-D Chern-Simons theory as a path integral for $\mathcal{N} = 4$ SYM in the 4-D half space $W\times; R$. This allows him to use dualities and bring in the machinery of M-theory and 6-branes. The basic structure of TGD forces a highly analogous approach: replace 3-surfaces with 4-surfaces, consider knot cobordisms and also 2-knots, introduce gerbes, and be happy with symplectic instead of topological QFT, which might more or less be synonymous with TGD as almost topological QFT. Symplectic QFT would obviously make possible much more refined description of knots.

9.6 Witten's approach to Khovanov homology from TGD point of view

Witten's approach to Khovanov comohology [A63] relies on fivebranes as is natural if one tries to define 2-knot invariants in terms of their cobordisms involving violent un-knottings. Despite the difference in approaches it is very useful to try to find the counterparts of this approach in quantum TGD since this would allow to gain new insights to quantum TGD itself as almost topological QFT identified as symplectic theory for 2-knots, braids and braid cobordisms.

An essentially unique identification of string world sheets and therefore also of the braids whose ends carry quantum numbers of many particle states at partonic 2-surfaces emerges if one identifies the string word sheets as singular surfaces in the same manner as is done in Witten's approach [A63] .

Also a physical interpretation of the operators Q, F, and P of Khovanov homology emerges. P would correspond to instanton number and F to the fermion number assignable to right handed neutrinos. The breaking of M^4 chiral invariance makes possible to realize Q physically. The finding that the generalizations of Wilson loops can be identified in terms of the gerbe fluxes $\int H_A J$ supports the conjecture that TGD as almost topological QFT corresponds essentially to a symplectic theory for braids and 2-knots.

9.6.1 Intersection form and space-time topology

The violent unknotting corresponds to a sequence of steps in which braid or knot becomes trivial and this very process defines braid invariants in TGD approach in nice concordance with the basic recipe for the construction of Jones polynomial. The topological invariant characterizing this process as a dynamics of 2-D string like objects defined by braid strands becomes knot invariant or more generally, invariant depending on the initial and final braids.

The process is describable in terms of string interaction vertices and also involves crossings of braid strands identifiable as self-intersections of the string world sheet. Hence the intersection form for the 2-surfaces defining braid strand orbits becomes a braid invariant. This intersection form is also a central invariant of 4-D manifolds and Donaldson's theorem [A5] says that for this invariant characterizes simply connected smooth 4-manifold completely. Rank, signature, and parity of this form in the basis defined by the generators of 2-homology (excluding torsion elements) characterize smooth closed and orientable 4-manifold. It is possible to diagonalize this form for smoothable 4-surfaces. Although the situation in the recent case differs from that in Donaldson theory in that the 4-surfaces have boundary and even fail to be manifolds, there are reasons to believe that the theory of braid cobordisms and 2-knots becomes part of the theory of topological invariants of 4-surfaces just as knot theory becomes part of the theory of 3-manifolds. The representation of 4-manifolds as space-time surfaces might also bring in physical insights.

This picture leads to ideas about string theory in 4-D space-time as a topological QFT. The string world sheets define the generators of second relative homology group. "Relative" means that closed surfaces are replaced with surfaces with boundaries at wormhole throats and ends of CD. These string world sheets, if one can fix them uniquely, would define a natural basis for homology group defining the intersection form in terms of violent unbraiding operations (note that also reconnections are involved).

Quantum classical correspondence encourages to ask whether also physical states must be restricted in such a manner that only this minimum number of strings carrying quantum numbers at their ends ending to wormhole throats should be allowed. One might hope that there exists a unique identification of the topological strings implying the same for braids and allowing to identify various symplectic invariants as Hamiltonian fluxes for the string world sheets.

9.6.2 Framing anomaly

In 3-D approach to knot theory the framing of links and knots represents an unavoidable technical problem [A63] . Framing means a slight shift of the link so that one can define self-linking number as a linking number for the link and its shift. The problem is that this framing of the link - or trivialization of its normal bundle in more technical terms- is not topological invariant and one obtains a large number of framings. For links in S^3 the framing giving vanishing self-linking number is the unique option and Atyiah has shown that also in more general case it is possible to identify a unique framing.

For 2-D surfaces self-linking is replaced with self-intersection. This is well-defined notion even without framing and indeed a key invariant. One might hope that framing is not needed also for string world sheets. If needed, this framing would induce the framing at the space-like and light-like 3-surfaces. The restriction of the section of the normal bundle of string world sheet to the 3-surfaces must lie in the tangent space of 3-surfaces. It is not clear whether this is enough to resolve the non-uniqueness problem.

9.6.3 Khovanov homology briefly

Khovanov homology involves three charges Q, F, and P. Q is analogous to super charge and satisfies $Q^2 = 0$ for the elements of homology. The basic commutation relations between the charges are $[F,Q] = Q$ and $[P,Q] = 0$. One can show that the Khovanov homology $\kappa(L)$ for link can be expressed as a bi-graded direct sum of the eigen-spaces $V_{m,n}$ of F and P, which have integer valued spectra. Obviously Q increases the eigenvalue of F and maps $V_{m,n}$ to $V_{m+1,n}$ just as exterior derivative in de-Rham comology increases the degree of differential form. P acts as a symmetry allowing to label the elements of the homology by an integer valued charge n.

Jones polynomial can be expressed as an index assignable to Khovanov homology:

$$\mathcal{J}(q|L) = Tr((-1)^F q^P \quad . \tag{9.6.1}$$

Here q defining the argument of Jones polynomial is root of unity in Chern-Simons theory but can be extended to complex numbers by extending the positive integer valued Chern-Simons coupling k to a complex number. The coefficients of the resulting Laurent polynomial are integers: this result does not follow from Chern-Simons approach alone. Jones polynomial depends on the spectrum of F only modulo 2 so that a lot of information is lost as the homology is replaced with the polynomial.

Both the need to have a more detailed characterization of links and the need to understand why the Wilson loop expectation is Laurent polynomial with integer coefficients serve as motivations of Witten for searching a physical approach to Khovanov polynomial.

The replacement of $D = 2$ in braid group approach to Jones polynomial with $D = 3$ for Chern-Simons approach replaced by something new in $D = 4$ would naturally correspond to the dimensional hierarchy of TGD in which partonic 2-surfaces plus their 2-D tangent space data fix the physics. One cannot quite do with partonic 2-surfaces and the inclusion of 2-D tangent space-data leads to holography and unique space time surfaces and perhaps also unique string world sheets serving as duals for partonic 2-surfaces. This would realize the weak form of electric magnetic duality at the level of homology much like Poincare duality relates cohomology and homology.

9.6.4 Surface operators and the choice of the preferred 2-surfaces

The choice of preferred 2-surfaces and the identification of surface operators in $\mathcal{N} = 4$ YM theory is discussed in [A52]. The intuitive picture is that preferred 2-surfaces- now string world sheets defining braid cobordisms and 2-knots- correspond to singularities of classical gauge fields. Surface operator can be said to create this singularity. In functional integral this means the presence of the exponent defining the analog of Wilson loop.

(a) In [A52] the 2-D singular surfaces are identified as poles for the magnitude r of the Higgs field. One can assign to the 2-surface fractional magnetic charges defined for the Cartan algebra part A_C of the gauge connection as circulations $\oint A_C$ around a small circle around the axis of singularity at $r = \infty$. What happens that 3-D $r = constant$ surface reduces to a 2-D surface at $r = \infty$ whereas A_C and entire gauge potential behaves as $A = A_C = \alpha d\phi$ near singularity. Here ϕ is coordinate analogous to angle of cylindrical coordinates when t-z plane represents the singular 2-surface. α is a linear combination of Cartan algebra generators.

(b) The phase factor assignable to the circulation is essentially $exp(i2\pi\alpha)$ and for non-fractional magnetic charges it differs from unity. One might perhaps say that string word sheets correspond to singularities for the slicing of space-time surface with 3-surfaces at which 3-surfaces reduce to 2-surfaces.

Consider now the situation in TGD framwork.

(a) The gauge group is color gauge group and gauge color gauge potentials correspond to the quantities H_AJ. One can also consider a generalization by allowing all Hamiltonians generating symplectic transformations of CP_2. Kähler gauge potential is in essential role since color gauge field is proportional to Kähler form.

(b) The singularities of color gauge fields can be identified by studing the theory locally as a field theory from CP_2 to M^4. It is quite possible to have space-time surfaces for which M^4 coordinates are many-valued functions of CP_2 coordinates so that one has a covering of CP_2 locally. For singular 2-surfaces this covering becomes singular in the sense that separate sheets coincide. These coverings do not seem to correspond to those assignable to the hierarchy of Planck constants implied by the many-valuedness of the time derivatives of the imbedding space coordinates as functions of canonical momentum densities but one must be very cautious in making too strong conclusions here.

(c) To proceed introduce the Eguchi-Hanson coordinates

$$(\xi^1, \xi^2) = [rcos(\theta/2)exp(i(\Psi + \Phi)/2), rsin(\theta/2)exp(i(-\Psi + \Phi)/2]$$

for CP_2 with the defining property that the coordinates transform linearly under $U(2) \subset SU(3)$. In QFT context these coordinates would be identified as Higgs fields. The choice of these coordinates is unique apart from the choice of the $U(2)$ subgroup and rotation by element of $U(2)$ once this choice has been made. In TGD framework the definition of CD involves the fixing of these coordinates and the interpretation is in terms of quantum classical correspondence realizing the choice of quantization axes of color at the level of the WCW geometry.

r has a natural identification as the magnitude of Higgs field invariant under $U(2) \subset SU(3)$. The $SU(2) \times U(1)$ invariant 3-sphere reduces to a homologically non-trivial geodesic 2-sphere at $r = \infty$ so that for this choice of coordinates this surface defines in very natural manner the counterpart of singular 2-surface in CP_2 geometry. At this sphere the second phase associated with CP_2 coordinates- Φ - becomes a redundant coordinate just like the angle Φ at the poles of sphere. There are two other similar spheres and these three spheres are completely analogous to North and South poles of 2-sphere.

(d) One possibility is that the singular surfaces correspond to the inverse images for the projection of the imbedding map to $r = \infty$ geodesic sphere of CP_2 for a CD corresponding to a given choice of quantization axes. Also the inverse images of all homological non-trivial geodesic spheres defining the three poles of CP_2 can be considered. The inverse images of this geodesic 2-sphere under the imbedding-projection map would naturally correspond to 2-D string world sheets for the preferred extremals for a generic space-time surface. For cosmic strings and massless extremals the inverse image would be 4-dimensional but this problem can be circumvented easily. The identification turned out to be somewhat ad hoc and later a much more convincing unique identification of string world sheets emerged and will be discussed in the sequel. Despite this the general aspects of the proposal deserves a discussion.

(e) The existence of dual slicings of space-time surface by 3-surfaces and lines on one hand and by string world sheets Y^2 and 2-surfaces X^2 with Euclidian signature of metric on one hand, is one of the basic conjectures about the properties of preferred extremals of Kähler action. A stronger conjecture is that partonic 2-surfaces represent particular instances of X^2. The proposed picture suggests an amazingly simple and physically attractive identification of these slicings.

   i. The slicing induced by the slicing of CP_2 by $r = constant$ surfaces defines an excellent candidate for the slicing by 3-surfaces. Physical the slices would correspond to equivalence classes of choices of the quantization axes for color group related by $U(2)$. In gauge theory context they would correspond to different breakings of $SU(3)$ symmetry labelled by the vacuum expectation of the Higgs field r which would be dynamical for CP_2 projections and play the role of time coordinate.

   ii. The slicing by string world sheets would naturally correspond to the slicing induced by the 2-D space of homologically non-trivial geodesic spheres (or triplets of them) and could be called "CP_2/S^2". One has clearly bundle structure with S^2 as base space and "CP_2/S^2" as fiber. Partonic 2-surfaces could be seen locally as sections of this bundle like structure assigning a point of "CP_2/S^2" to each point of S^2. Globally this does not make sense for partonic 2-surfaces with genus larger than $g = 0$.

(f) In TGD framework the Cartan algebra of color gauge group is the natural identification for the Cartan algebra involved and the fluxes defining surface operators would be the classical fluxes $\int H_A J$ over the 2-surfaces in question restricted to Cartan algebra. What would be new is the interpretation as gerbe gauge potentials so that flux becomes completely analogous to Abelian circulation.

If one accepts the extension of the gauge algebra to a symplectic algebra, one would have the Cartan algebra of the symplectic algebra. This algebra is defined by generators which depend on the second half P_i or Q_i of Darboux coordinates. If P_i are chosen to be functions of the coordinates (r, θ) of CP_2 coordinates whose Poisson brackets with color isospin and hyper charge generators inducing rotations of phases (Ψ, Φ) of CP_2 complex coordinates vanish, the symplectic Cartan algebra would correspond to color neutral Hamiltonians. The spherical harmonics with non-vanishing angular momentum vanish at poles and one expects that same happens for CP_2 spherical harmonics at the three poles of CP_2. Therefore Cartan algebra is selected automatically for gauge fluxes. This subgroup leaves the ends of the points of braids at partonic 2-surfaces invariant so that symplectic transformations do not induce braiding.

If this picture -resulting as a rather straightforward translation of the picture applied in QFT context- is correct, TGD would predict uniquely the preferred 2-surfaces and therefore also the braids as inverse images of CP_2 geodesic sphere for the imbedding of space-time surface to $CD \times CP_2$. Also the conjecture slicings by 3-surfaces and string world sheets could be identified. The identification of braids and slicings has been indeed one of the basic challenges in quantum TGD since in quantum theory one does not have anymore the luxury of topological invariance and I have proposed several identifications. If one accepts only these space-time sheets then the stringy content for a given space-time surface would be uniquely fixed.

The assignment of singularities to the homologically non-trivial geodesic sphere suggests that the homologically non-trivial space-time sheets could be seen as 1-dimensional idealizations of magnetic flux tubes carrying Kähler magnetic flux playing key role also in applications of TGD, in particular biological applications such as DNA as topological quantum computer and bio-control and catalysis.

9.6.5 The identification of charges Q, P and F of Khovanov homology

The challenge is to identify physically the three operators Q, F, and P appearing in Khovanov homology. Taking seriously the proposal of Witten [A63] and looking for its direct counterpart in TGD leads to the identification and physical interpretation of these charges in TGD framework.

(a) In Witten's approach P corresponds to instanton number assignable to the classical gauge field configuration in space-time. In TGD framework the instanton number would naturally correspond to that assignable to CP_2 Kähler form. One could consider the possibility of assigning this charge to the deformed CP_2 type vacuum extremals assigned to the space-like regions of space-time representing the lines of generalized Feynman diagrams having elementary particle interpretation. P would be or at least contain the sum of unit instanton numbers assignable to the lines of generalized Feynman diagrams with sign of the instanton number depending on the orientation of CP_2 type vacuum extremal and perhaps telling whether the line corresponds to positive or negative energy state. Note that only pieces of vacuum extremals defined by the wormhole contacts are in question and it is somewhat questionable whether the rest of them in Minkowskian regions is included.

(b) F corresponds to $U(1)$ charge assignable to R-symmetry of $N = 4$ SUSY in Witten's theory. The proposed generalization of twistorial approach in TGD framework suggests strongly that this identification generalizes to TGD. In TGD framework all solutions of modified Dirac equation at wormhole throats define super-symmetry generators but the supersymmetry is badly broken. The covariantly constant right handed neutrino defines the minimally broken supersymmetry since there are no direct couplings to gauge fields. This symmetry is however broken by the mixing of right and left handed M^4 chiralities present for both Dirac actions for induced gamma matrices and for modified Dirac equations defined by Kähler action and Chern-Simons action at parton orbits. It is caused by the fact that both the induced and modified gamma matrices are combinations of M^4 and CP_2 gamma matrices. F would therefore correspond to the net fermion number assignable to right handed neutrinos and antineutrinos. F is not conserved because of the chirality mixing and electroweak interactions respecting only the conservation of lepton number.

Note that the mixing of M^4 chiralities in sub-manifold geometry is a phenomenon characteristic for TGD and also a direct signature of particle massivation and SUSY breaking. It would be nice if it would allow the physical realization of Q operator of Khovanov homology.

(c) Witten proposes an explicit formula for Q in terms of 5-dimensional time evolutions interpolating between two 4-D instantons and involving sum of sign factors assignable to

Dirac determinants. In TGD framework the operator Q should increase the right handed neutrino number by one unit and therefore transform one right-handed neutrino to a left handed one in the minimal situation. In zero energy ontology Q should relate to a time evolution either between ends of CD or between the ends of single line of generalized Feynman diagram. If instanton number can be assigned solely to the wormhole contacts, this evolution should increase the number of CP_2 type extremals by one unit. 3-particle vertex in which right handed neutrino assignable to a partonic 2-surface transforms to a left handed one is thus a natural candidate for defining the action of Q.

(d) Note that the almost topological QFT property of TGD together with the weak form of electric-magnetic duality implies that Kähler action reduces to Abelian Chern-Simons term. Ordinary Chern-Simons theory involves imaginary exponent of this term but in TGD the exponent would be real. Should one replace the real exponent of Kähler function with imaginary exponent? If so, TGD would be very near to topological QFT also in this respect. This would also force the quantization of the coupling parameter k in Chern-Simons action. On the other hand, the Chern-Simons theory makes sense also for purely imaginary k [A63] .

9.6.6 What does the replacement of topological invariance with symplectic invariance mean?

One interpretation for the symplectic invariance is as an analog of diffeo-invariance. This would imply color confinement. Another interpretation would be based on the identification of symplectic group as a color group. Maybe the first interpretation is the proper restriction when one calculates invariants of braids and 2-knots.

The replacement of topological symmetry with symplectic invariance means that TGD based invariants for braids carry much more refined information than topological invariants. In TGD approach M^4 diffeomorphisms act freely on partonic 2-surfaces and 4-D tangent space data but the action in CP_2 degrees of freedom reduces to symplectic transformations. One could of course consider also the restriction to symplectic transformations of the light-cone boundary and this would give additional refinements.

It is is easy to see what symplectic invariance means by looking what it means for the ends of braids at a given partonic 2-surface.

(a) Symplectic transformations respect the Kähler magnetic fluxes assignable to the triangles defined by the finite number of braid points so that these fluxes defining symplectic areas define some minimum number of coordinates parametrizing the moduli space in question. For topological invariance all n-point configurations obtained by continuous or smooth transformations are equivalent braid end configurations. These finite-dimensional moduli spaces would be contracted with point in topological QFT.

(b) This picture led to a proposal of what I call symplectic QFT [K8] in which the associativity condition for symplectic fusion rules leads the hierarchy of algebras assigned with symplectic triangulations and forming a structures known as operad in category theory. The ends of braids at partonic 2-surfaces would would define unique triangulation of this kind if one accepts the identification of string like 2-surfaces as inverse images of homologically non-trivial geodesic sphere.

Note that both diffeomorphisms and symplectic transformations can in principle induce braiding of the braid strands connecting two partonic 2-surfaces. Should one consider the possibility that the allow transformations are restricted so that they do not induce braiding?

(a) These transformations induce a transformation of the space-time surface which however is not a symplectic transformation in the interior in general. An attractive conjecture is that for the preferred extremals this is the case at the inverse images of the homologically non-trivial geodesic sphere. This would conform with the proposed duality between partonic 2-surfaces and string world sheets inspired by holography and also with quantum classical correspondence suggesting that at string world sheets the transformations induced by symplectic transformations at partonic 2-surfaces act like symplectic transformations.

(b) If one allows only the symplectic transformations in Cartan algebra leaving the homologically non-trivial geodesic sphere invariant, the infinitesimal symplectic transformations would affect neither the string word sheets nor braidings but would modify the partonic 2-surfaces at all points except at the intersections with string world sheets.

9.7 Algebraic braids, sub-manifold braid theory, and generalized Feynman diagrams

Ulla send me a link to an article by Sam Nelson about very interesting new-to-me notion known as algebraic knots [A53, A34], which has initiated a revolution in knot theory. This notion was introduced 1996 by Louis Kauffmann [A48] so that it is already 15 year old concept. While reading the article I realized that this notion fits perfectly the needs of TGD and leads to a progress in attempts to articulate more precisely what generalized Feynman diagrams are.

In the following I will summarize briefly the vision about generalized Feynman diagrams, introduce the notion of algebraic knot, and after than discuss in more detail how the notion of algebraic knot could be applied to generalized Feynman diagrams. The algebraic structrures kei, quandle, rack, and biquandle and their algebraic modifications as such are not enough. The lines of Feynman graphs are replaced by braids and in vertices braid strands redistribute. This poses several challenges: the crossing associated with braiding and crossing occurring in non-planar Feynman diagrams should be integrated to a more general notion; braids are replaced with sub-manifold braids; braids of braids ....of braids are possible; the redistribution of braid strands in vertices should be algebraized. In the following I try to abstract the basic operations which should be algebraized in the case of generalized Feynman diagrams.

One should be also able to concretely identify braids and 2-braids (string world sheets) as well as partonic 2-surfaces and I have discussed several identifications during last years. Legendrian braids turn out to be very natural candidates for braids and their duals for the partonic 2-surfaces. String world sheets in turn could correspond to the analogs of Lagrangian sub-manifolds or to minimal surfaces of space-time surface satisfying the weak form of electric-magnetic duality. The latter option turns out to be more plausible. Finite measurement resolution would be realized as symplectic invariance with respect to the subgroup of the symplectic group leaving the end points of braid strands invariant. In accordance with the general vision TGD as almost topological QFT would mean symplectic QFT. The identification of braids, partonic 2-surfaces and string world sheets - if correct - would solve quantum TGD explicitly at string world sheet level in other words in finite measurement resolution.

Irrespective of whether the algebraic knots are needed, the natural question is what generalized Feynman diagrams are. It seems that the basic building bricks can be identified so that one can write rather explicit Feynman rules already now. Of course, the rules are still far from something to be burned into the spine of the first year graduate student.

9.7.1 Generalized Feynman diagrams, Feynman diagrams, and braid diagrams

How knots and braids a la TGD differ from standard knots and braids?

TGD approach to knots and braids differs from the knot and braid theories in given abstract 3-manifold (4-manifold in case of 2-knots and 2-braids) is that space-time is in TGD framework identified as 4-D surface in $M^4 \times CP_2$ and preferred 3-surfaces correspond to light-like 3-surfaces defined by wormhole throats and space-like 3-surfaces defined by the ends of space-time sheets at the two light-like boundaries of causal diamond CD.

The notion of finite measurement resolution effectively replaces 3-surfaces of both kinds with braids and space-time surface with string world sheets having braids strands as their ends. The 4-dimensionality of space-time implies that string world sheets can be knotted and intersect at discrete points (counterpart of linking for ordinary knots). Also space-time surface can have self-intersections consisting of discrete points.

The ordinary knot theory in E^3 involves projection to a preferred 2-plane E^2 and one assigns to the crossing points of the projection an index distinguishing between two cases which are transformed to each other by violently taking the first piece of strand through another piece of strand. In TGD one must identify some physically preferred 2-dimensional manifold in imbedding space to which the braid strands are projected. There are many possibilities even when one requires maximal symmetries. An obvious requirement is however that this 2-manifold is large enough.

(a) For the braids at the ends of space-time surface the 2-manifold could be large enough sphere S^2 of light-cone boundary in coordinates in which the line connecting the tips of CD defines a preferred time direction and therefore unique light-like radial coordinate. In very small knots it could be also the geodesic sphere of CP_2 (apart from the action of isometries there are two geodesic spheres in CP_2).

(b) For light-like braids the preferred plane would be naturally M^2 for which time direction corresponds to the line connecting the tips of CD and spatial direction to the quantization axis of spin. Note that these axes are fixed uniquely and the choices of M^2 are labelled by the points of projective sphere P^2 telling the direction of space-like axis. Preferred plane M^2 emerges naturally also from number theoretic vision and corresponds in octonionic pictures to hyper-complex plane of hyper-octonions. It is also forced by the condition that the choice of quantization axes has a geometric correlate both at the level of imbedding space geometry and the geometry of the "world of classical worlds".

The braid theory in TGD framework could be called sub-manifold braid theory and certainly differs from the standard one.

(a) If the first homology group of the 3-surface is non-trivial as it when the light-like 3-surfaces represents an orbit of partonic 2-surface with genus larger than zero, the winding of the braid strand (wrapping of branes in M-theory) meaning that it represents a homologically non-trivial curve brings in new effects not described by the ordinary knot theory. A typical new situation is the one in which 3-surface is locally a product of higher genus 2-surface and line segment so that knot strand can wind around the 2-surface. This gives rise to what are called non-planar braid diagrams for which the projection to plane produces non-standard crossings.

(b) In the case of 2-knots similar exotic effects could be due to the non-trivial 2-homology of space-time surface. Wormhole throats assigned with elementary particle wormhole throats are homologically non-trivial 2-surfaces and might make this kind of effects possible for 2-knots if they are possible.

The challenge is to find a generalization of the usual knot and braid theories so that they apply in the case of braids (2-braids) imbedded in 3-D (4-D) surfaces with preferred highly symmetry sub-manifold of $M^4 \times CP_2$ defining the analog of plane to which the knots are projected. A proper description of exotic crossings due to non-trivial homology of 3-surface (4-surface) is needed.

Basic questions

The questions are following.

(a) How the mathematical framework of standard knot theory should be modified in order to cope with the situation encountered in TGD? To my surprise I found that this kind of mathematical framework exists: so called algebraic knots [A53, A34] define a generalization of knot theory very probably able to cope with this kind of situation.

(b) Second question is whether the generalized Feynman diagrams could be regarded as braid diagrams in generalized sense. Generalized Feynman diagrams are generalizations of ordinary Feynman diagrams. The lines of generalized Feynman diagrams correspond to the orbits of wormhole throats and of wormhole contacts with throats carrying elementary particle quantum numbers.

The lines meet at vertices which are partonic 2-surfaces. Single wormhole throat can describe fermion whereas bosons have wormhole contacts with fermion and anti-fermion at the opposite throats as building bricks. It seems however that all fermions carry Kähler magnetic charge so that physical particles are string like objects with magnetic charges at their ends.

The short range of weak interactions results from the screening of the axial isospin by neutrinos at the other end of string like object and also color confinement could be understood in this manner. One cannot exclude the possibility that the length of magnetic flux tube is of order Compton length.

(c) Vertices of the generalized Feynman diagrams correspond to the partonic 2-surfaces along which light-like 3-surfaces meet and this is certainly a challenge for the required generalization of braid theory. The basic objection against the reduction to algebraic braid diagrams is that reaction vertices for particles cannot be described by ordinary braid theory: the splitting of braid strands is needed.

The notion of bosonic emergence [K36] however suggests that 3-vertex and possible higher vertices correspond to the splitting of braids rather than braid strands. By allowing braids which come from both past and future and identifying free fermions as wormhole throats and bosons as wormhole contacts consisting of a pair of wormhole throats carrying fermion and anti-fermion number, one can understand boson excanges as recombinations without anyneed to have splitting of braid strands. Strictly and technically speaking, one would have tangles like objects instead of braids. This would be an enormous simplification since $n > 2$-vertices which are the source of divergences in QFT:s would be absent.

(d) Non-planar Feynman diagrams are the curse of the twistor approach and I have already earlier proposed that the generalized Feynman amplitudes and perhaps even twistorial amplitudes could be constructed as analogs of knot invariants by recursively transforming non-planar Feynman diagrams to planar ones for which one can write twistor amplitudes. This forces to answer two questions.

i. Does the non-nonplanarity of Feynman diagrams - completely combinatorial objects identified as diagrams in plane - have anything to do with the non-planarity of algebraic knot diagrams and with the non-planarity of generalized Feynman diagrams which are purely geometric objects?

ii. Could these two kind of non-planarities be fused to together by identifying the projection 2-plane as preferred $M^2 \subset M^4$. This would mean that non-planarity in QFT sense is defined for entire braids: braid A can have virtual crossing with B. Non-planarity in the sense of knot theory would be defined for braid strands inside the braids. At vertices braid strands are redistributed between incoming lines and the analog of virtual crossing be identifiable as an exchange of braid strand between braids. Several kinds of non-planarities would be present and the idea about gradual unknotting of a non-planar diagram so that a planar diagram results as the final outcome might make sense and allow to generalize the recursion recipe for the twistorial amplitudes.

iii. One might consider the possibility that inside orbits of wormhole throats defining the lines of Feynman diagrams the R-matrix for integrable QFT in M^2 (only permutations of momenta are allowed) describes the dynamics so that one obtains just a permutation of momenta assigned to the braid strands. Ordinary braiding would be described by existing braid theories. The core problem would be the representation of the exchange of a strand between braids algebraically.

It has become clear that there is different and much simpler general approach to the non-planarity problem. In twistor Grassmannian approach [K44] generalized Feynman diagrams correspond to TGD variants of stringy diagrams. In stringy approach one gets rid of non-planarity problem altogether.

9.7.2 Brief summary of algebraic knot theory

Basic ideas of algebraic knot theory

In ordinary knot theory one takes as a starting point the representation of knots of E^3 by their plane plane projections to which one attach a "color" to each crossing telling whether the strand goes over or under the strand it crosses in planar projection. These numbers are fixed uniquely as one traverses through the entire knot in given direction.

The so called Reidermeister moves are the fundamental modifications of knot leaving its isotopy equivalence class unchanged and correspond to continuous deformations of the knot. Any algebraic invariant assignable to the knot must remain unaffected under these moves. Reidermeister moves as such look completely trivial and the non-trivial point is that they represent the minimum number of independent moves which are represented algebraically.

In algebraic knot theory topological knots are replaced by typographical knots resulting as planar projections. This is a mapping of topology to algebra. It turns out that the existing knot invariants generalize and ordinary knot theory can be seen as a special case of the algebraic knot theory. In a loose sense one can say that the algebraic knots are to the classical knot theory what algebraic numbers are to rational numbers.

Virtual crossing is the key notion of the algebraic knot theory. Virtual crossing and their rules of interaction were introduced 1996 by Louis Kauffman as basic notions [A1]. For instance, a strand with only virtual crossings should be replaceable by any strand with the same number of virtual crossings and same end points. Reidermeister moves generalize to virtual moves. One can say that in this case crossing is self-intersection rather than going under or above. I cannot be eliminated by a small deformation of the knot. There are actually several kinds of non-standard crossings: examples listed in figure 7 of [A53]) are virtual, flat, singular, and twist bar crossings.

Algebraic knots have a concrete geometric interpretation.

i. Virtual knots are obtained if one replaces E^3 as imbedding space with a space which has non-trivial first homology group. This implies that knot can represent a homologically non-trivial curve giving an additional flavor to the unknottedness since homologically non-trivial curve cannot be transformed to a curve which is homologically non-trivial by any continuous deformation.

ii. The violent projection to plane leads to the emergence of virtual crossings. The product $(S^1 \times S^1) \times D$, where $(S^1 \times S^1)$ is torus D is finite line segment, provides the simplest example. Torus can be identified as a rectangle with opposite sides identified and homologically non-trivial knots correspond to curves winding n_1 times around the first S^1 and n_2 times around the second S^1. These curves are not continuous in the representation where $S^1 \times S^1$ is rectangle in plane.

iii. A simple geometric visualization of virtual crossing is obtained by adding to the plane a handle along which the second strand traverses and in this manner avoids intersection. This visualization allows to understand the geometric motivation for the the virtual moves.

This geometric interpretation is natural in TGD framework where the plane to which the projection occurs corresponds to $M^2 \subset M^4$ or is replaced with the sphere at the boundary of S^2 and 3-surfaces can have arbitrary topology and partonic 2-surfaces defining as their orbits light-like 3-surfaces can have arbitrary genus.

In TGD framework the situation is however more general than represented by sub-manifold braid theory. Single braid represents the line of generalized Feynman diagram. Vertices represent something new: in the vertex the lines meet and the braid strands are redistributed but do not disappear or pop up from anywhere. That the braid strands can come both from the future and past is also an important generalization.

There are physical argments suggesting that there are only 3-vertices for braids but not higher ones [K11]. The challenge is to represent algebraically the vertices of generalized Feynman diagrams.

Algebraic knots

The basic idea in the algebraization of knots is rather simple. If x and y are the crossing portions of knot, the basic algebraic operation is binary operation giving "the result of x going under y", call it $x \triangleright y$ telling what happens to x. "Portion of knot" means the piece of knot between two crossings and $x \triangleright y$ denotes the portion of knot next to x. The definition is asymmetrical in x and y and the dual of the operation would be $y \triangleleft x$ would be "the result of y going above x". One can of course ask, why not to define the outcome of the operation as a pair $(x \triangleleft y, y \triangleright x)$. This operation would be bi-local in a well-defined sense. One can of course do this: in this case one has binary operation from $X \times X \rightarrow X \times X$ mapping pairs of portions to pairs of portions. In the first case one has binary operation $X \times X \rightarrow X$.

The idea is to abstract this basic idea and replace X with a set endowed with operation $\triangleright$ or $\triangleleft$ or both and formukate the Reidermeister conditions given as conditions satisfied by the algebra. One ends up to four basic algebraic structures kei, quandle, rack, and biquandle.

i. In the case of non-oriented knots the kei is the algebraic structure. Kei - or invontary quandle-is a set X with a map $X \times X \rightarrow X$ satisfying the conditions
   A. $x \triangleright x = x$ (idenpotency, one of the Reidemeister moves)
   B. $(x \triangleright y) \triangleright y$ =x (operation is its own right inverse having also interpretation as Reidemeister move)
   C. $(x \triangleright y) \triangleright z = (x \triangleright z) \triangleright (y \triangleright z)$ (self-distributivity)

   $Z([t])/(t^2)$ module with $x \triangleright y = tx + (1-t)y$ is a kei.

ii. For orientable knot diagram there is preferred direction of travel along knot and one can distinguish between $\triangleright$ and its right inverse $\triangleright^{-1}$. This gives quandle satisfying the axios
   A. $x \triangleright x = x$
   B. $(x \triangleright y) \triangleright^{-1} y = (x \triangleright^{-1} y) \triangleright y = x$
   C. $(x \triangleright y) \triangleright z = (x \triangleright z) \triangleright (y \triangleright z)$

   $Z[t^{\pm 1}]$ nodule with $x \triangleright y = tx + (1-t)y$ is a quandle.

iii. One can also introduce framed knots: intuitively one attaches to a knot very near to it. More precise formulation in terms of a section of normal bundle of the knot. This makes possible to speak about self-linking. Reidermeister moves must be modified appropriately. In this case rack is the appropriate structure. It satisfied the axioms of quandle except the first axiom since corresponding operation is not a move anymore. Rack axioms are eqivalent with the requirement that functions $f_y : X \rightarrow X$ defined by $f_y(x)x \triangleright y)$ are automorphisms of the structure. Therefore the elements of rack represent its morphisms. The modules over $Z[t^{\pm 1}, s]/s(t+s-1)$ are racks. Coxeter racks are inner product spaces with $x \triangleright y$ obtained by reflecting x across y.

iv. Biquandle consists of arcs connecting the subsequent crossings (both under- and over-) of oriented knot diagram. Biquandle operation is a map $B : X \times X \rightarrow X \times X$ of order pairs satisfying certain invertibility conditions together with set theoretic Yang-Baxter equation:

$$(B \times I)(I \times B)(B \times I) = (I \times B)(B \times I)(I \times B) \ .$$

   Here $I : X \rightarrow X$ is the identity map. The three conditions to which Yang-Baxter equation decomposes gives the counterparts of the above discussed axioms. Alexander biquandle is the module $Z(t^{\pm 1}, s^{\pm}1$ with $B(x, y) = (ty + (1-ts)x, sx)$ where one has $s \neq 1$. If one includes virtual, flat and singular crossings one obtains virtual/singular aundles and semiquandles.

9.7.3 Generalized Feynman diagrams as generalized braid diagrams?

Zero energy ontology suggests the interpretation of the generalized Feynman diagrams as generalized braid diagrams so that there would be no need for vertices at the fundamental braid strand level. The notion of algebraic braid (or tangle) might allow to formulate this idea more precisely.

Could one fuse the notions of braid diagram and Feynman diagram?

The challenge is to fuse the notions of braid diagram and Feynman diagram having quite different origin.

i. All generalized Feynman diagrams are reduced to sub-manifold braid diagrams at microscopic level by bosonic emergence (bosons as pairs of fermionic wormhole throats). Three-vertices appear only for entire braids and are purely topological whereas braid strands carrying quantum numbers are just re-distributed in vertices. No 3-vertices at the really microscopic level! This is an additional nail to the coffin of divergences in TGD Universe.

ii. By projecting the braid strands of generalized Feynman diagrams to preferred plane $M^2 \subset M^4$ (or rather 2-D causal diamond), one could achieve a unified description of non-planar Feynman diagrams and braid diagrams. For Feynman diagrams the intersections have a purely combinatorial origin coming from representations as 2-D diagrams.
For braid diagrams the intersections have different origin and non-planarity has different meaning. The crossings of entire braids analogous to those appearing in non-planar Feynman diagrams should define one particular exotic crossing besides virtual crossings of braid strands due to non-trivial first homology of 3-surfaces.

iii. The necessity to choose preferred plane M^2 looks strange from QFT point of view. In TGD framework it is forced by the number theoretic vision in which M^2 represents hyper-complex plane of sub-space of hyper-octonions which is subspace of complexified octonions. The choice of M^2 is also forced by the condition that the choice of quantization axes has a geometric correlate both at the level of imbedding space geometry and the geometry of the "world of classical worlds".

iv. Also 2-braid diagrams defined as projections of string world sheets are suggestive and would be defined by a projections to the 3-D boundary of CD or to $M^3 \subset M^4$. They would provide a more concrete stringy illustration about generalized Feynman diagram as analog of string diagram. Another attractive illustration is in terms of dance metaphor with the boundary of CD defining the 3-D space-like parquette. The duality between space-like and light-like braids is expected to be of importance.

The obvious conjecture is that Feynman amplitudes are a analogous to knot invariants constructible by gradually reducing non-planar Feynman diagrams to planar ones after which the already existing twistor theoretical machinery of $\mathcal{N} = 4$ SYMs would apply [K59].

Does 2-D integrable QFT dictate the scattering inside the lines of generalized Feynman diagrams

The preferred plane M^2 (more precisely, 2-D causal diamond having also interpretation as Penrose diagram) plays a key role as also the preferred sphere S^2 at the boundary of CD. It is perhaps not accident that a generalization of braiding was discovered in integrable quantum field theories in M^2. The S-matrix of this theory is rather trivial looking: particle moving with different velocities cross each other and suffer a phase lag and permutation of 2-momenta which has physical effects only in the case of non-identical particles. The R-matrix describing this process reduces to the R-matrix describing the basic braiding operation in braid theories at the static limit.

I have already earlier conjectured that this kind of integrable QFT is part of quantum TGD [K13]. The natural guess is that it describes what happens for the projections of 4-momenta in M^2 in scattering process inside lines of generalized Feynman diagrams. If integrable theories in M^2 control this scattering, it would cause only phase changes and permutation of the M^2 projections of the 4-momenta. The most plausible guess is that M^2 QFT characterized by R-matrix describes what happens to the braid momenta during the free propagation and the remaining challenge would be to understand what happens in the vertices defined by 2-D partonic surfaces at which re-distribution of braid strands takes place.

How quantum TGD as almost topological QFT differs from topological QFT for braids and 3-manifolds

One must distinguish between two topological QFTs. These correspond to topological QFT defining braid invariants and invariants of 3-manifolds respectively. The reason is that knots are an essential element in the procedure yielding 3-manifolds. Both 3-manifold invariants and knot invariants would be defined as Wilson loops involving path integral over gauge connections for a given 3-manifold with exponent o non-Abelkian f Chern-Simons action defining the weight.

i. In TGD framework the topological QFT producing braid invariants for a given 3-manifold is replaced with sub-manifold braid theory. Kähler action reduces Chern-Simons terms for preferred extremals and only these contribute to the functional integral. What is the counterpart of topological invariance in this framework? Are general isotopies allowed or should one allow only sub-group of symplectic group of CD boundary leaving the end points of braids invariant? For this option Reidermeister moves are undetectable in the finite measurement resolution defined by the subgroup of the symplectic group. Symplectic transformations would not affect 3-surfaces as the analogs of abstract contact manifold since induced Kähler form would not be affected and only the imbedding would be changed.

   In the approach based on inclusions of HFFs gauge invariance or its generalizations would represent finite measurement resolution (the action of included algebra would generate states not distiguishable from the original one).

ii. There is also ordinary topological QFT allowing to construct topological invariants for 3-manifold. In TGD framework the analog of topological QFT is defined by Chern-Simons-Kähler action in the space of preferred 3-surfaces. Now one sums over small deformations of 3-surface instead of gauge potentials. If extremals of Chern-Simons-Kähler action are in question, symplectic invariance is the most that one can hope for and this might be the situation quite generally. If all light-like 3-surfaces are allowed so that only weak form of electric-magnetic duality at them would bring metric into the theory, it might be possible to have topological invariance at 3-D level but not at 4-D level. It however seems that symplectic invariance with respect to subgroup leaving end points of braids invariant is the realistic expectation.

Could the allowed braids define Legendrian sub-manifolds of contact manifolds?

The basic questions concern the identification of braids and 2-braids. In quantum TGD they cannot be arbitrary but determined by dynamics providing space-time correlates for quantum dynamics. The deformations of braids should mean also deformations of 3-surfaces which as topological manifolds would however remain as such. Therefore topological QFT for given 3-manifold with path integral over gauge connections would in TGD correspond to functional integral of 3-surfaces corresponding to same topology even symplectic structure. The quantum fluctuating degrees of freedom indeed correspond to symplectic group divided by its subgroup defining measurement resolution.

What is the dynamics defining the braids strands? What selects them? I have considered this problem several times. Just two examples is enough here.

i. Could they be some special light-like curves? Could the condition that the end points of the curves correspond to rational points in some preferred coordinates allow to select these light-like curves? But what about light-like curves associated with the ends of the space-time surface?
ii. The solutions of modified Dirac equation [K18] are localized to curves by using the analog of periodic boundary conditions: the length of the curve is quantized in the effective metric defined by the modified gamma matrices. Here one however introcuced a coordinate along light-like 3-surface and it is not clear how one should fix this preferred coordinate.

1. Legendrian and Lagrangian sub-manifolds

A hint about what is missing comes from the observation that a non-vanishing Chern-Simons-Kähler form A defines a contact structure [A4] at light-like 3-surfaces if one has $A \wedge dA \neq 0$. This condition states complete non-intebrability of the distribution of 2-planes defined by the condition $A_\mu t^\mu = 0$, where t is tangent vector in the tangent bundle of light-like 3-surface. It also states that the flow lines of A do *not* define global coordinate varying along them.

i. It is however possible to have 1-dimensional curves for which $A_\mu t^\mu = 0$ holds true at each point. These curves are known as Legendrian sub-manifolds to be distinguished from Lagrangian manifolds for which the projection of symplectic form expressible locally as $J = dA$ vanishes. The set of this curves is discrete so that one obtains braids. Legendrian knots are the simplest example of Legendrian sub-manifolds and the question is whether braid strands could be identified as Legendrian knots. For Legendrian braids symplectic invariance replaces topological invariance and Legendrian knots and braids can be trivial in topological sense. In some situations the property of being Legendrian implies un-knottedness.
ii. For Legendrian braid strands the Kähler gauge potential vanishes. Since the solutions of the modified Dirac equation are localized to braid strands, this means that the coupling to Kähler gauge potential vanishes. From physics point of view a generalization of Legendre braid strand by allowing gauge transformations $A \rightarrow A + d\Phi$ looks natural since it means that the coupling of induced spinors is pure gauge terms and can be eliminated by a gauge transformation.

2. 2-D duals of Legendrian sub-manifolds

One can consider also what might be called 2-dimensional duals of Legendrian sub-manifolds.

i. Also the one-form obtained from the dual of Kähler magnetic field defined as $B^\mu = \epsilon^{\mu\nu\gamma} J_{\nu\nu}$ defines a distribution of 2-planes. This vector field is ill-defined for light-like surfaces since contravariant metric is ill-defined. One can however multiply B with the square root of metric determining formally so that metric would disappear completely just as it disappears from Chern-Simons action. This looks however somewhat tricky mathematically. At the 3-D space-like ends of space-time sheets at boundaries of CD B^μ is however well-defined as such.
ii. The distribution of 2-planes is integrable if one has $B \wedge dB = 0$ stating that one has Beltrami field: physically the conditions states that the current dB feels no Lorentz force. The geometric content is that B defines a global coordinate varying along its flow lines. For the preferred extremals of Kähler action Beltrami condition is satisfied by isometry currents and Kähler current in the interior of space-time sheets. If this condition holds at 3-surfaces, one would have an global time coordinate and integrable distribution of 2-planes defining a slicing of the 2-surface. This would realize the conjecture that space-time surface has a slicing by partonic 2-surfaces. One could say that the 2-surfaces defined by the distribution are orthogonal to B. This need not however mean that the projection of J to these 2-surfaces vanishes. The condition $B \wedge dB = 0$ on the space-like 3-surfaces could be interpreted in terms of effective 2-dimensionality. The simplest option posing no additional conditions

would allow two types of braids at space-like 3-surfaces and only Legendrian braids at light-like 3-surfaces.

These observations inspire a question. Could it be that the conjectured dual slicings of space-time sheets by space-like partonic 2-surfaces and by string world sheets are defined by A_μ and B^μ respectively associated with slicings by light-like 3-surfaces and space-like 3-surfaces? Could partonic 2-surfaces be identified as 2-D duals of 1-D Legendrian sub-manifolds?

The identification of braids as Legendrian braids for light-like 3-surfaces and with Legendrian braids or their duals for space-like 3-surfaces would in turn imply that topological braid theory is replaced with a symplectic braid theory in accordance with the view about TGD as almost topological QFT. If finite measurement resolution corresponds to the replacement of symplectic group with the coset space obtained by dividing by a subgroup, symplectic subgroup would take the role of isotopies in knot theory. This symplectic subgroup could be simply the symplectic group leaving the end points of braids invariant.

An attempt to identify the constraints on the braid algebra

The basic problems in understanding of quantum TGD are conceptual. One must proceed by trying to define various concepts precisely to remove the many possible sources of confusion. With this in mind I try collect essential points about generalized Feynman diagrams and their relation to braid diagrams and Feynman diagrams and discuss also the most obvious constraints on algebraization.

Let us first summarize what generalized Feynman diagrams are.

i. Generalized Feynman diagrams are 3-D (or 4-D, depends on taste) objects inside $CD\times CP_2$. Ordinary Feynman diagrams are in plane. If finite measurement resolution has as a space-time correlate discretization at the level of partonic 2-surfaces, both space-like and light-like 3-surfaces reduce to braids and the lines of generalized Feynman diagrams correspond to braids. It is possible to obtain the analogs of ordinary Feynman diagrams by projection to $M^2 \subset M^4$ defined uniquely for given CD. The resulting apparent intersections would represent ne particular kind of exotic intersection.

ii. Light-like 3-surfaces define the lines of generalized Feynman diagrams and the braiding results naturally. Non-trivial first homology for the orbits of partonic 2-surfaces with genus $g > 0$ could be called homological virtual intersections.

iii. It zero energy ontology braids must be characterized by time orientation. Also it seems that one must distinguish in zero energy ontology between on mass shell braids and off mass shell braid pairs which decompose to pairs of braids with positive and negative energy massless on mass shell states. In order to avoid confusion one should perhaps speak about tangles insie CD rather than braids. The operations of the algebra are same except that the braids can end either to the upper or lower light-like boundary of CD. The projection to M^2 effectively reduces the CD to a 2-dimensional causal diamond.

iv. The vertices of generalized Feynman diagrams are partonic 2-surfaces at which the light-like 3-surfaces meet. This is a new element. If the notion of bosonic emergence is accepted no $n > 2$-vertices are needed so that braid strands are redistributed in the reaction vertices. The redistribution of braid strands in vertices must be introduced as an additional operation somewhat analogous to $\triangleright$ and the challenge is to reduce this operation to something simple. Perhaps the basic operation reduces to an exchange of braid strand between braids. The process can be seen as a decay of of braid with the conservation of braid strands with strands from future and past having opposite strand numbers. Also for this operation the analogs of Reidermeister moves should be identified. In dance metaphor this operation corresponds to a situation in which the dancer leaves the group to which it belongs and goes to a new one.

v. A fusion of Feynman diagrammatic non-planarity and braid theoretic non-planarity is needed and the projection to M^2 could provide this fusion when at least two kinds of virtual crossings are allowed. The choice of M^2 could be global. An open question is whether the choice of M^2 could characterize separately each line of generalized Feynman diagram characterized by the four-momentum associated with it in the rest system defined by the tips of CD. Somehow the theory should be able to fuse the braiding matrix for integrable QFT in M^2 applying to entire braids with the braiding matrix for braid theory applying at the level of single braid.

Both integral QFTs in M^2 and braid theories suggest that biquandle structure is the structure that one should try to generalized.

i. The representations of resulting bi-quandle like structure could allow abstract interesting information about generalized Feynman diagrams themselves but the dream is to construct generalized Feynman diagrams as analogs of knot invariants by a recursive procedure analogous to un-knotting of a knot.

ii. The analog of bi-quandle algebra should have a hierarchical structure containing braid strands at the lowest level, braids at next level, and braids of braids...of braids at higher levels. The notion of operad would be ideal for formulating this hierarchy and I have already proposed that this notion must be essential for the generalized Feynman diagrammatics. An essential element is the vanishing of total strand number in the vertex (completely analogous to conserved charged such as fermion number). Again a convenient visualization is in terms of dancers forming dynamical groups, forming groups of groups forming .....
I have already earlier suggested [K13] that the notion of operad [A18] relying on permutation group and its subgroups acting in tensor products of linear spaces is central for understanding generalized Feynman diagrams. $n \rightarrow n_1 + n_2$ decay vertex for n-braid would correspond to "symmetry breaking" $S_n \rightarrow S_{n_1} \times S_{n_2}$. Braid group represents the covering of permutation group so that braid group and its subgroups permuting braids would suggest itself as the basic group theoretical notion. One could assign to each strand of n-braid decaying to n_1 and n_2 braids a two-valued color telling whether it becomes a strand of n_1-braid or n_2-braid. Could also this "color" be interpreted as a particular kind of exotic crossing?

iii. What could be the analogs of Reidermaster moves for braid strands?

   A. If the braid strands are dynamically determined, arbitrary deformations are not possible. If however all isotopy classes are allowed, the interpretation would be that a kind of gauge choice selecting one preferred representation of strand among all possible ones obtained by continuous deformations is in question.

   B. Second option is that braid strands are dynamically determined within finite measurement resolution so that one would have braid theory in given length scale resolution.

   C. Third option is that topological QFT is replaced with symplectic QFT: this option is suggested by the possibility to identify braid strands as Legendrian knots or their duals. Subgroup of the symplectic group leaving the end points of braids invariant would act as the analog of continous transformations and play also the role of gauge group. The new element is that symplectic transformations affect partonic 2-surfaces and space-time surfaces except at the end points of braid.

iv. Also 2-braids and perhaps also 2-knots could be useful and would provide string theory like approach to TGD. In this case the projections could be performed to the ends of CD or to M^3, which can be identified uniquely for a given CD.

v. There are of course many additional subtleties involved. One should not forget loop corrections, which naturally correspond to sub-CDs. The hierarchy of Planck constants and number theoretical universality bring in additional complexities.

All this looks perhaps hopelessly complex but the Universe around is complex even if the basic principles could be very simple.

9.7.4 About string world sheets, partonic 2-surfaces, and two-knots

String world sheets and partonic 2-surfaces provide a beatiful visualization of generalized Feynman diagrams as braids and also support for the duality of string world sheets and partonic 2-surfaces as duality of light-like and space-like braids. Dance metaphor is very helpful here.

i. The projection of string world sheets and partonic 2-surfaces to 3-D space replaces knot projection. In TGD context this 3-D of space could correspond to the 3-D light-like boundary of CD and 2-knot projection would correspond to the projection of the braids associated with the lines of generalized Feynman diagram. Another identification would be as $M^1 \times E^2$, where M^1 is the line connecting the tips of CD and E^2 the orthogonal complement of M^2.

ii. Using dance metaphor for light-like braiding, braids assignable to the lines of generalized Feynman diagrams would correspond to groups of dancers. At vertices the dancing groups would exchange members and completely new groups would be formed by the dancers . The number of dancers (negative for those dancing in the reverse time direction) would be conserved. Dancers would be connected by threads representing strings having braid points at their ends. During the dance the light-like braiding would induce space-like braiding as the threads connecting the dancers would get entangled. This would suggest that the light-like braids and space-like braidings are equivalent in accordance with the conjectured duality between string-world sheets and partonic 2-surfaces. The presence of genuine 2-knottedness could spoil this equivalence unless it is completely local.

Can string world sheets and partonic 2-surfaces get knotted?

i. Since partonic 2-surfaces (wormhole throats) are imbedded in light-cone boundary, the preferred 3-D manifolds to which one can project them is light-cone boundary (boundary of CD). Since the projection reduces to inclusion these surfaces cannot get knotted. Only if the partonic 2-surfaces contains in its interior the tip of the light-cone something non-trivial identifiable as virtual 2-knottedness is obtained.

ii. One might argue that the conjectured duality between the descriptions provided by partonic 2-surfaces and string world sheets requires that also string world sheets represent trivial 2-braids. I have shown earlier that nontrivial local knots glued to the string world sheet require that M^4 time coordinate has a local maximum. Does this mean that 2-knots are excluded? This is not obvious: TGD allows also regions of space-time surface with Euclidian signature and generalized Feynman graphs as 4-D space-time regions are indeed Euclidian. In these regions string world sheets could get knotted.

What happens for knot diagrams when the dimension of knot is increased to two? According to the articles of Nelson [A53] and Carter [A34] the crossings for the projections of braid strands are replaced with more complex singularities for the projections of 2-knots. One can decompose the 2-knots to regions surrounded by boxes. Box can contain just single piece of 2-D surface; it can contain two intersection pieces of 2-surfaces as the counterpart of intersecting knot strands and one can tell which of them is above which; the box can contain also a discrete point in the intersection of projections of three disjoint regions of knot which consists of discrete points; and there is also a box containing so called cone point. Unfortunately, I failed to understand the meaning of the cone point.

For 2-knots Reidemeister moves are replaced with Roseman moves. The generalization would allow virtual self intersections for the projection and induced by the non-trivial second homology of 4-D imbedding space. In TGD framework elementary particles have homologically non-trivial partonic 2-surfaces (magnetic monpoles) as their building bricks so that even if 2-knotting in standard sense might be not allowed, virtual 2-knotting would be possible. In TGD framework one works with a subgroup of symplectic transformations defining measurement resolution instead of isotopies and this might reduce the number of allowed mov

The dynamics of string world sheets and the expression for Kähler action

The dynamics of string world sheets is an open question. Effective 2-dimensionality suggests that Kähler action for the preferred extremal should be expressible using 2-D data but there are several guesses for what the explicit expression could be, and one can only make only guesses at this moment and apply internal consistency conditions in attempts to kill various options.

1. Could weak form of electric-magnetic duality hold true for string world sheets?

If one believes on duality between string world sheets and partonic 2-surfaces, one can argue that string world sheets are most naturally 2-surfaces at which the weak form of electric magnetic duality holds true. One can even consider the possibility that the weak form of electric-magnetic duality holds true only at the the string world sheets and partonic 2-surfaces but not at the preferred 3-surfaces.

i. The weak form of electric magnetic duality would mean that induced Kähler form is non-vanishing at them and Kähler magnetic flux over string world sheet is proportional to Kähler electric flux.
ii. The flux of the induced Kähler form of CP_2 over string world sheet would define a dimensionless "area". Could Kähler action for preferred extremals reduces to this flux apart from a proportionality constant. This "area" would have trivially extremum with respect to symplectic variations if the braid strands are Legendrian sub-manifolds since in this case the projection of Kähler gauge potential on them vanishes. This is a highly non-trivial point and favors weak form of electric-magnetic duality and the identification of Kähler action as Kähler magnetic flux. This option is also in spirit with the vision about TGD as almost topological QFT meaning that induced metric appears in the theory only via electric-magnetic duality.
iii. Kähler magnetic flux over string world sheet has a continuous spectrum so that the identification as Kähler action could make sense. For partonic 2-surfaces the magnetic flux would be quantized and give constant term to the action perhaps identifiable as the contribution of CP_2 type vacuum extremals giving this kind of contribution.

The change of space-time orientation by changing the sign of permutation symbol would change the sign in electric-magnetic duality condition and would not be a symmetry. For a given magnetic charge the sign of electric charge changes when orientation is changed. The value of Kähler action does not depend on space-time orientation but weak form of electric-magnetic duality as boundary condition implies dependence of the Kähler action on space-time orientation. The change of the sign of Kähler electric charge suggests the interpretation of orientation change as one aspect of charge conjugation. Could this orientation dependence be responsible for matter antimatter asymmetry?

2. Could string world sheets be Lagrangian sub-manifolds in generalized sense?

Legendrian sub-manifolds can be lifted to Lagrangian sub-manifolds [A4] Could one generalize this by replacing Lagrangian sub-manifold with 2-D sub-manifold of space-times surface for which the projection of the induced Kähler form vanishes? Could string world sheets be Lagrangian sub-manifolds?

I have also proposed that the inverse image of homologically non-trivial sphere of CP_2 under imbedding map could define counterparts of string world sheets or partonic 2-surfaces. This conjecture does not work as such for cosmic strings, massless extremals having 2-D projection since the inverse image is in this case 4-dimensional. The option based on homologically non-trivial geodesic sphere is not consistent with the identification as analog of Lagrangian manifold but the identification as the inverse image of homologically trivial geodesic sphere is.

The most general option suggested is that string world sheet is mapped to 2-D Lagrangian sub-manifold of CP_2 in the imbedding map. This would mean that theory is exactly solvable at string world sheet level. Vacuum extremals with a vanishing induced

Kähler form would be exceptional in this framework since they would be mapped as a whole to Lagrangian sub-manifolds of CP_2. The boundary condition would be that the boundaries of string world sheets defined by braids at preferred 3-surfaces are Legendrian sub-manifolds. The generalization would mean that Legendrian braid strands could be continued to Lagrangian string world sheets for which induced Kähler form vanishes. The physical interpretation would be that if particle moves along this kind of string world sheet, it feels no covariant Lorentz-Kähler force and contra variant Lorentz forces is orthogonal to the string world sheet.

There are however serious objections.

i. This proposal does not respect the proposed duality between string world sheets and partonic 2-surfaces which as carries of Kähler magnetic charges cannot be Lagrangian 2-manifolds.

ii. One loses the elegant identification of Kähler action as Kähler magnetic flux since Kähler magnetic flux vanishes. Apart from proportionality constant Kähler electric flux

$$\int_{Y^2} *J$$

is as a dimensionless scaling invariant a natural candidate for Kähler action but need not be extremum if braids are Legendrian sub-manifolds whereas for Kähler magnetic flux this is the case. There is however an explicit dependence on metric which does not conform with the idea that almost topological QFT is symplectic QFT.

iii. The sign factor of the dual flux which depends on the orientation of the string world sheet and thus changes sign when the orientation of space-time sheet is changed by changing that of the string world sheet. This is in conflict with the independence of Kähler action on orientation. One can however argue that the orientation makes itself actually physically visible via the weak form of electric-magnetic duality. If the above discussed duality holds true, the net contribution to Kähler action would vanish as the total Kähler magnetic flux for partonic 2-surfaces. Therefore the duality cannot hold true if Kähler action reduces to dual flux.

iv. There is also a purely formal counter argument. The inverse images of Lagrangian sub-manifolds of CP_2 can be 4-dimensional (cosmic strings and massless extremals) whereas string world sheets are 2-dimensonal.

String world sheets as minimal surfaces

Effective 2-dimensionality suggests a reduction of Kähler action to Chern-Simons terms to the area of minimal surfaces defined by string world sheets holds true [K22]. Skeptic could argue that the expressibility of Kähler action involving no dimensional parameters except CP_2 scaled does not favor this proposal. The connection of minimal surface property with holomorphy and conformal invariance however forces to take the proposal seriously and it is easy to imagine how string tension emerges since the size scale of CP_2 appears in the induced metric [K22].

One can ask whether the mimimal surface property conforms with the proposal that string worlds sheets obey the weak form of electric-magnetic duality and with the proposal that they are generalized Lagrangian sub-manifolds.

i. The basic answer is simple: minimal surface property and possible additional conditions (Lagrangian sub-manifold property or the weak form of electric magnetic duality) poses only additional conditions forcing the space-time sheet to be such that the imbedded string world sheet is a minimal surface of space-time surface: minimal surface property is a condition on space-time sheet rather than string world sheet. The weak form of electric-magnetic duality is favored because it poses conditions on the first derivatives in the normal direction unlike Lagrangian sub-manifold property.

ii. Any proposal for 2-D expression of Kähler action should be consistent with the proposed real-octonion analytic solution ansatz for the preferred extremals [K5]. The ansatz is based on real-octonion analytic map of imbedding space to itself obtained by algebraically continuing real-complex analytic map of 2-D sub-manifold of imbedding space to another such 2-D sub-manifold. Space-time surface is obtained by requiring that the "imaginary" part of the map vanishes so that image point is hyper-quaternion valued. Wick rotation allows to formulate the conditions using octonions and quaternions. Minimal surfaces (of space-time surface) are indeed objects for which the imbedding maps are holomorphic and the real-octonion analyticity could be perhaps seen as algebraic continuation of this property.

iii. Does Kähler action for the preferred exremals reduce to the area of the string world sheet or to Kähler magnetic flux or are the representations equivalent so that the induced Kähler form would effectively define area form? If the Kähler form form associated with the induced metric on string world sheet is proportional to the induced Kähler form the Kähler magnetic flux is proportional to the area and Kähler action reduces to genuine area. Could one pose this condition as an additional constraint on string world sheets? For Lagrangian sub-manifolds Kähler electric field should be proportional to the area form and the condition involves information about space-time surface and is therefore more complex and does not look plausible.

Explicit conditions expressing the minimal surface property of the string world sheet

It is instructive to write explicitly the condition for the minimal surface property of the string world sheet and for the reduction of the area Kähler form to the induced Kähler form. For string world sheets with Minkowskian signature of the induced metric Kähler structure must be replaced by its hyper-complex analog involving hyper-complex unit e satisfying $e^2 = 1$ but replaced with real unit at the level hyper-complex coordinates. e can be represented as antisymmetric Kähler form J_g associated with the induced metric but now one has $J_g^2 = g$ instead of $J_g^2 = -g$. The condition that the signed area reduces to Kähler electric flux means that J_g must be proportional to the induced Kähler form: $J_g = kJ$, $k = constant$ in a given space-time region.
One should make an educated guess for the imbedding of the string world sheet into a preferred extremal of Kähler action. To achieve this it is natural to interpret the minimal surface property as a condition for the preferred Kähler extremal in the vicinity of the string world sheet guaranteeing that the sheet is a minimal surface satisfying $J_g = kJ$. By the weak form of electric-magnetic duality partonic 2-surfaces represent both electric and magnetic monopoles. The weak form of electric-magnetic duality requires for string world sheets that the Kähler magnetic field at string world sheet is proportional to the component of the Kähler electric field parallel to the string world sheet. Kähler electric field is assumed to have component only in the direction of string world sheet.

1. Minkowskian string world sheets

Let us try to formulate explicitly the conditions for the reduction of the signed area to Kähler electric flux in the case of Minkowskian string world sheets.

i. Let us assume that the space-time surface in Minkowskian regions has coordinates coordinates $(u, v, w, \overline{w})$ [K5]. The pair (u, v) defines light-like coordinates at the string world sheet having identification as hyper-complex coordinates with hyper-complex unit satisfying $e = 1$. u and v need not - nor cannot as it turns out - be light-like with respect to the metric of the space-time surface. One can use (u, v) as coordinates for string world sheet and assume that $w = x^1 + ix^2$ and $\overline{w}$ are constant for the string world sheet. Without a loss of generality one can assume $w = \overline{w} = 0$ at string world sheet.

ii. The induced Kähler structure must be consistent with the metric. This implies that the induced metric satisfies the conditions

$$g_{uu} = g_{vv} = 0 \ . \tag{9.7.1}$$

The analogs of these conditions in regions with Euclidian signature would be $g_{zz} = g_{\overline{z}\overline{z}} = 0$.

iii. Assume that the imbedding map for space-time surface has the form

$$s^m = s^m(u,v) + f^m(u,v,x^m)_{kl}x^kx^l \ , \tag{9.7.2}$$

so that the conditions

$$\partial_l k s^m = 0 \ , \ \partial_k\partial_u s^m = 0, \ \partial_k\partial_v s^m = 0 \tag{9.7.3}$$

are satisfies at string world sheet. These conditions imply that the only non-vanishing components of the induced CP_2 Kähler form at string world sheet are J_{uv} and $J_{w\overline{w}}$. Same applies to the induced metric if the metric of M^4 satisfies these conditions (no non-vanishing components of form m_{uk} or m_{vk}).

iv. Also the following conditions hold true for the induced metric of the space-time surface

$$\partial_k g_{uv} = 0 \ , \ \partial_u g_{kv} = 0 \ , \ \partial_v g_{ku} = 0 \ . \tag{9.7.4}$$

at string world sheet as is easy to see by using the ansatz.

Consider now the minimal surface conditions stating that the trace of the four components of the second fundamental form whose components are labelled by the coordinates $\{x^\alpha\} \equiv (u,v,w,\overline{w})$ vanish for string world sheet.

i. Since only g_{uv} is non-vanishing, only the components H^k_{uv} of the second fundamental form appear in the minimal surface equations. They are given by the general formula

$$\begin{aligned} H^\alpha_{uv} &= H^\gamma P^\alpha_\gamma \ , \\ H^\alpha &= \left(\partial_u\partial_v x^\alpha + \left({}_\beta{}^\alpha{}_\gamma\right)\partial_u x^\beta \partial_v x^\gamma\right) \ . \end{aligned} \tag{9.7.5}$$

Here P^α_γ is the projector to the normal space of the string world sheet. Formula contains also Christoffel symbols $\left({}_\beta{}^\alpha{}_\gamma\right)$.

ii. Since the imbedding map is simply $(u,v) \to (u,v,0,0)$ all second derivatives in the formula vanish. Also $H^k = 0, k \in \{w,\overline{w}\}$ holds true. One has also $\partial_u x^\alpha = \delta^\alpha_u$ and $\partial_v x^\beta = \delta^\beta_v$. This gives

$$H^\alpha = \left({}_u{}^\alpha{}_v\right) \ . \tag{9.7.6}$$

All these Christoffel symbols however vanish if the assumption $g_{uu} = g_{vv} = 0$ and the assumptions about imbedding ansatz hold true. Hence a minimal surface is in question.

Consider now the conditions on the induced metric of the string world sheet

i. The conditions reduce to

$$g_{uu} = g_{vv} = 0 \ . \tag{9.7.7}$$

The conditions on the diagonal components of the metric are the analogs of Virasoro conditions fixing the coordinate choices in string models. The conditions state that the coordinate lines for u and v are light-like curves in the induced metric.

ii. The conditions can be expressed directly in terms of the induced metric and read

$$\begin{aligned} m_{uu} + s_{kl}\partial_u s^k \partial_u s^l &= 0 \ , \\ m_{vv} + s_{kl}\partial_v s^k \partial_v s^l &= 0 \ . \end{aligned} \tag{9.7.8}$$

The CP_2 contribution is negative for both equations. The conditions make sense only for $(m_{uu} > 0, m_{vv} > 0)$. Note that the determinant condition $m_{uu}m_{vv} - m_{uv}m_{vu} < 0$ expresses the Minkowskian signature of the (u,v) coordinate plane in M^4.

The additional condition states

$$J^g_{uv} = kJ_{uv} . \quad (9.7.9)$$

It reduces signed area to Kähler electric flux. If the weak form of electric-magnetic duality holds true one can interpret the area as magnetic flux defined as the flux of the dual of induced Kähler form over space-like surface and defining electric charge. A further condition is that the boundary of string world sheet is Legendrean manifold so that the flux and thus area is extremized also at the boundaries.

2.Conditions for the Euclidian string world sheets

One can do the same calculation for string world sheet with Euclidian signature. The only difference is that (u, v) is replaced with $(z, \overline{z})$. The imbedding map has the same form assuming that space-time sheet with Euclidian signature allows coordinates $(z, \overline{z}, w, \overline{w})$ and the local conditions on the imbedding are a direct generalization of the above described conditions. In this case the vanishing for the diagonal components of the string world sheet metric reads as

$$\begin{aligned} h_{kl}\partial_z s^k \partial_z s^l &= 0 \ , \\ h_{kl}\partial_{\overline{z}} s^k \partial_{\overline{z}} s^l &= 0 \ . \end{aligned} \quad (9.7.10)$$

The natural ansatz is that complex CP_2 coordinates are holomorphic functions of the complex coordinates of the space-time sheet.

3. Wick rotation for Minkowskian string world sheets leads to a more detailed solution ansatz

Wick rotation is a standard trick used in string models to map Minkowskian string world sheets to Euclidian ones. Wick rotation indeed allows to define what one means with real-octonion analyticity. Could one identify string world sheets in Minkowskian regions by using Wick rotation and does this give the same result as the direct approach?

Wick rotation transforms space-time surfaces in $M^4 \times CP_2$ to those in $E^4 \times CP_2$. In $E^4 \times CP_2$ octonion real-analyticity is a well-defined notion and one can identify the space-time surfaces surfaces at which the imaginary part of of octonion real-analytic function vanishes: imaginary part is defined via the decomposition of octonion to two quaternions as $o = q_1 + Iq_2$ where I is a preferred octonion unit. The reverse of the Wick rotation maps the quaternionic surfaces to what might be called hyper-quaternionic surfaces in $M^4 \times CP_2$.

In this picture string world sheets would be hyper-complex surfaces defined as inverse imagines of complex surfaces of quaternionic space-time surface obtained by the inverse of Wick rotation. For this approach to be equivalent with the above one it seems necessary to require that the the treatment of the conditions on metric should be equivalent to that for which hyper-complex unit e is not put equal to 1. This would mean that the conditions reduce to independent conditions for the real and imaginary parts of the real number formally represented as hyper-complex number with $e = 1$.

Wick rotation allows to guess the form of the ansatz for CP_2 coordinates as functions of space-time coordinates In Euclidian context holomorphich functions of space-time coordinates are the natural ansatz. Therefore the natural guess is that one can map the hypercomplex number $t \pm ez$ to complex coordinate $t \pm iz$ by the analog of Wick rotation and assume that CP_2 complex coordinates are analytic functions of the complex space-time coordinates obtained in this manner.

The resulting induced metric could be obtained directly using real coordinates (t, z) for string world sheet or by calculating the induced metric in complex coordinates $t \pm iz$ and by mapping the expressions to hyper-complex numbers by Wick rotation (by replacing i with $e = 1$). If the diagonal components of the induced metric vanish for $t \pm iz$ they vanish also for hyper-complex coordinates so that this approach seem to make sense.

Electric-magnetic duality for flux Hamiltonians and the existence of Wilson sheets

One must distinguish between two conjectured dualities. The weak form of electric-magnetic duality and the duality between string world sheets and partonic 2-surfaces. Could the first duality imply equivalence of not only electric and magnetic flux Hamiltonians but also electric and magnetic Wilson sheets? Could the latter duality allow two different representations of flux Hamiltonians?

i. For electric-magnetic duality holding true at string world sheets one would have non-vanishing Kähler form and the fluxes would be non-vanishing. The Hamiltonian fluxes

$$Q_{m,A} = \int_{X^2} JH_A dx^1 dx^2 = \int_{X^2} H_A J_{\alpha\beta} dx^\alpha \wedge dx^\beta \tag{9.7.11}$$

for partonic 2-surfaces X^2 define WCW Hamiltonians playing a key role in the definition of WCW Kähler geometry. They have also interpretation as a generalization of Wilson loops to Wilson 2-surfaces.

ii. Weak form of electric magnetic duality would imply both at partonic 2-surfaces and string world sheets the proportionality

$$Q_{m,A} = \int_{X^2} JH_A dx^1 \wedge dx^2 \propto Q^*_{m,A} = \int_{X^2} H_A * J_{\alpha\beta} dx^\alpha \wedge dx^\beta \ . \tag{9.7.12}$$

Therefore the electric-magnetic duality would have a concrete meaning also at the level of WCW geometry.

iii. If string world sheets are Lagrangian sub-manifolds Hamiltonian fluxes would vanish identically so that the identification as Wilson sheets does not make sense. One would lose electric-magnetic duality for flux sheets. The dual fluxes

$$*Q_A = \int_{Y^2} *JH_A dx^1 \wedge dx^2 = \int_{Y^2} \epsilon_{\alpha\beta}{}^{\gamma\delta} J_{\gamma\delta} = \int_{Y^2} \frac{\sqrt{det(g_4)}}{det(g_2^\perp)} J_{34}^\perp dx^1 \wedge dx^2$$

for string world sheets Y^2 are however non-vanishing. Unlike fluxes, the dual fluxes depend on the induced metric although they are scaling invariant.

Under what conditions the conjectured duality between partonic 2-surface and string world sheets hold true at the level of WCW Hamiltonians?

i. For the weak form of electric-magnetic duality at string world sheets the duality would mean that the sum of the fluxes for partonic 2-surfaces and sum of the fluxes for string world sheets are identical apart from a proportionality constant:

$$\sum_i Q_A(X_i^2) \propto \sum_i Q_A(Y_i^2) \ . \tag{9.7.13}$$

Note that in zero ontology it seems necessary to sum over all the partonic surfaces (at both ends of the space-time sheet) and over all string world sheets.

ii. For Lagrangian sub-manifold option the duality can hold true only in the form

$$\sum_i Q_A(X_i^2) \propto \sum_i Q_A^*(Y_i^2) \ . \tag{9.7.14}$$

Obviously this option is less symmetric and elegant.

Summary

There are several arguments favoring weak form of electric-magnetic duality for both string world sheets and partonic 2-surfaces. Legendrian sub-manifold property for braid strands follows from the assumption that Kähler action for preferred extremals is proportional to the Kähler magnetic flux associated with preferred 2-surfaces and is stationary with respect to the variations of the boundary. What is especially nice is that Legendrian sub-manifold property implies automatically unique braids. The minimal option favored by the idea that 3-surfaces are basic dynamical objects is the one for which weak form of electric-magnetic duality holds true only at partonic 2-surfaces and string world sheets. A stronger option assumes it at preferred 3-surfaces. Duality between string world sheets and partonic 2-surfaces suggests that WCW Hamiltonians can be defined as sums of Kähler magnetic fluxes for either partonic 2-surfaces or string world sheets.

9.7.5 What generalized Feynman rules could be?

After all these explanations the skeptic reader might ask whether this lengthy discussion gives any idea about what the generalized Feynman rules might look like. The attempt to answer this question is a good manner to make a map about what is understood and what is not understood. The basic questions are simple. What constraints does zero energy ontology (ZEO) pose? What does the necessity to projecti the four-momenta to a preferred plane M^2 mean? What mathematical expressions one should assign to the propagator lines and vertices? How does one perform the functional integral over 3-surfaces in finite measurement resolution? The following represents tentatative answers to these questions but does not say much about exact role of algebraic knots.

Zero energy ontology

Zero energy ontology (ZEO) poses very powerful constraints on generalized Feynman diagrams and gives hopes that both UV and IR divergences cancel.

i. ZEO predicts that the fermions assigned with braid strands associated with the virtual particles are on mass shell massless particles for which the sign of energy can be also negative: in the case of wormhole throats this can give rise to a tachyonic exchange.

ii. The on mass shell conditions for each wormhole throat in the diagram involving loops are very stringent and expected to eliminate very large classes of diagrams. If however given diagonal diagram leading from n-particle state to the same n-particle state -completely analogous to self energy diagram- is possible then the ladders form by these diagrams are also possible and one one obtains infinite of this kind of diagrams as generalized self energy correction and is excellent hopes that geometric series gives a closed algebraic function.

iii. IR divergences plaguing massless theories are cancelled if the incoming and outgoing particles are massive bound states of massless on mass shell particles. In the simplest manner this is achieved when the 3-momenta are in opposite direction. For internal lines the massive on-mass shell-condition is not needed at all. Therefore there is an almost complete separation of the problem how bound state masses are determined from the problem of constructing the scattering amplitudes.

iv. What looks like a problematic aspect ZEO is that the massless on-mass-shell propagators would diverge for wormhole throats. The solution comes from the projection of 4-momenta to M^2. In the generic the projection is time-like and one avoids the singularity. The study of solutions of the modified Dirac equation [K18] and number theoretic vision [K50] indeed suggests that the four-momenta are obtained by rotating massless M^2 momenta and their projections to M^2 are in general integer multiples of hyper-complex primes or light-like. The light-like momenta would be treated like in the case of ordinary Feynman diagrams using $i\epsilon$-prescription of the propagator and would also give a finite contributions corresponding to integral over physical on mass shell states. This guarantees also the vanishing of the possible IR divergences coming from the summation over different M^2 momenta.
There is a strong temptation to identify - or at least relate - the M^2 momenta labeling the solutions of the modified Dirac equation with the region momenta of twistor approach [K61]. The reduction of the region momenta to M^2 momenta could dramatically simplify the twistorial description. It does not seem however plausible

that $\mathcal{N} = 4$ super-symmetric gauge theory could allow the identification of M^2 projections of 4-momenta as region momenta. On the other hand, there is no reason to expect the reduction of TGD certainly to a gauge theory containing QCD as part. For instance, color magnetic flux tubes in many-sheeted space-time are central for understanding jets, quark gluon plasma, hadronization and fragmentation [L11] but cannot be deduced from QCD. Note also that the splitting of parton momenta to their M^2 projections and transversal parts is an ad hoc assumption motivated by parton model rather than first principle implication of QCD: in TGD framework this splitting would emerge from first principles.

v. ZEO strongly suggests that all particles (including photons, gluons, and gravitons) have mass which can be arbitrarily small and could be perhaps seen as being due to the fact that particle "eats" Higgs like states giving it the otherwise lacking polarization states. This would mean a generalization of the notion of Higgs particle to a Higgs like particle with spin. It would also mean rearrangmenet of massless states at wormhole throat level to massives physical states. The slight massication of photon by p-adic thermodynamics does not however mean disappearance of Higgs from spectrum, and one can indeed construct a model for Higgs like states [K65].
The projection of the momenta to M^2 is consistent with this vision. The natural generalization of the gauge condition $p \cdot \epsilon = 0$ is obtained by replacing p with the projection of the total momentum of the boson to M^2 and ϵ with its polarization so that one has $p_{||} \cdot \epsilon$. If the projection to M^2 is light-like, three polarization states are possible in the generic case, so that massivation is required by internal consistency. Note that if intermediate states in the unitary condition were states with light-like M^2-momentum one could have a problematic situation.

vi. A further assumption vulnerable to criticism is that the M^2 projections of all momenta assignable to braid strands are parallel. Only the projections of the momenta to the orthogonal complement E^2 of M^2 can be non-parallel and for massive wormhole throats they must be non-parallel. This assumption does not break Lorentz invariance since in the full amplitude one must integrate over possible choices of M^2. It also interpret the gauge conditions either at the level of braid strands or of partons. Quantum classical correspondence in strong form would actually suggests that quantum 4-momenta should co-incide with the classical ones. The restriction to M^2 projections is however necessary and seems also natural. For instance, for massless extremals only M^2 projection of wave-vector can be well-defined: in transversal degrees of freedom there is a superposition over Fourier components with diffrent transversal wave-vectors. Also the partonic description of hadrons gives for the M^2 projections of the parton momenta a preferred role. It is highly encouraging that this picture emerged first from the modified Dirac equation and purely number theoretic vision based on the identification of M^2 momenta in terms of hyper-complex primes.
The number theoretical approach also suggests a number theoretical quantization of the transversal parts of the momenta [K50]: four-momenta would be obtained by rotating massless M^2 momenta in M^4 in such a manner that the components of the resulting 3-momenta are integer valued. This leads to a classical problem of number theory which is to deduce the number of 3-vectors of fixed length with integer valued components. One encounters the n-dimensional generalization of this problem in the construction of discrete analogs of quantum groups (these "classical" groups are analogous to Bohr orbits) and emerge in quantum arithmetics [K64], which is a deformation of ordinary arithmetics characterized by p-adic prime and giving rigorous justification for the notion of canonical identification mapping p-adic numbers to reals.

vii. The real beauty of Feynman rules is that they guarantee unitarity automatically. In fact, unitarity reduces to Cutkosky rules which can be formulated in terms of cut obtained by putting certain subset of interal lines on mass shell so that it represents on mass shell state. Cut analyticity implies the usual $iDisc(T) = TT^{\dagger}$. In the recent context the cutting of the internal lines by putting them on-mass-shell requires a generalization.

A. The first guess is that on mass shell property means that M^2 projection for the momenta is light-like. This would mean that also these momenta contribute to the amplitude but the contribution is finite just like in the usual case. In this formulation the real particles would be the massless wormhole throats.

B. Second possibility is that the internal lines on on mass shell states corresponding to massive on mass-shell-particles. This would correspond to the experimental meaning of the unitary conditions if real particles are the massive on mass shell particles. Mathematically it seems possible to pick up from the amplitude the states which correspond to massive on mass shell states but one should understand why the discontinuity should be associated with physical net masses for wormhole contacts or many-particle states formed by them. General connection with unitarity and analyticity might allow to understand this.

viii. CDs are labelled by various moduli and one must integrate over them. Once the tips of the CD and therefore a preferred M^1 is selected, the choice of angular momentum quantization axis orthogonal to M^1 remains: this choice means fixing M^2. These choices are parameterized by sphere S^2. It seems that an integration over different choices of M^2 is needed to achieve Poincare invariance.

How the propagators are determined?

In accordance with previous sections it will be assumed that the braid are Legendrian braids and therefore completely well-defined. One should assign propagator to the braid. A good guess is that the propagator reduces to a product of three terms.

i. A multi-particle propagator which is a product of collinear massless propagators for braid strands with fermionin number $F = 0, 1 - 1$. The constraint on the momenta is $p_i = \lambda_i p$ with $\sum_i \lambda_i = 1$. So that the fermionic propagator is $\frac{1}{\prod_i \lambda_i} p^k \gamma_k$. If one gas $p = nP$, where P is hyper-complex prime, one must sum over combinations of $\lambda_i = n_i$ satisfying $\sum_i n_i = n$.

ii. A unitary S-matrix for integrable QFT in M^2 in which the velocities of particles assignable to braid strands appear for which fixed by R-matrix defines the basic 2-vertex representing the process in which a particle passes through another one. For this S-matrix braids are the basic units. To each crossing appearing in non-planar Feynman diagram one would have an R-matrix representing the effect of a reconnection the ends of the lines coming to the crossing point. In this manner one could gradually transform the non-planar diagram to a planar diagram. One can ask whether a formulation in terms of a suitable R-matrix could allow to generalize twistor program to apply in the case of non-planar diagrams.

iii. An S-matrix predicted by topological QFT for a given braid. This S-matrix should be constructible in terms of Chern-Simons term defining a sympletic QFT.

There are several questions about quantum numbers assignable to the braid strands.

i. Can braid strands be only fermionic or can they also carry purely bosonic quantum numbers corresponding to WCW Hamiltonians and therefore to Hamiltonians of $\delta M^4_\pm \times CP_2$? Nothing is lost if one assumes that both purely bosonic and purely fermionic lines are possible and looks whether this leads to inconsistencies. If virtual fermions correspond to single wormhole throat they can have only time-like M^2-momenta. If virtual fermions correspond to pairs of wormhole throats with second throat carrying purely bosonic quantum numbers, also fermionic can have space-like net momenta. The interpretation would be in terms of topological condensation. This is however not possible if all strands are fermionic. Situation changes if one identifies physical fermions wormhole throats at the ends of Kähler magnetic flux tube as one indeed does: in this case virtual net momentum can be space-like if the sign of energy is opposite for the ends of the flux tube.

ii. Are the 3-momenta associated with the wormholes of wormhole contact parallel so that only the sign of energy could distinguish between them for space-like total momentum and M^2 mass squared would be the same? This assumption simplifies the situation but is not absolutely necessary.

iii. What about the momentum components orthogonal to M^2? Are they restricted only by the massless mass shell conditions on internal lines and quantization of the M^2 projection of 4-momentum?

iv. What kind of braids do elementary particles correspond? The braids assigned to the wormhole throat lines can have arbitrary number n of strands and for $n = 1, 2$ the treatment of braiding is almost trivial. A natural assumption is that propagator is simply a product of massless collinear propagators for M^2 projection of momentum [K19]. Collinearity means that propagator is product of a multifermion propagator $\frac{1}{\lambda_i p_k \gamma_k}$, znd multiboson propagator $\frac{1}{\mu_i p_k \gamma_k}$, $\sum \lambda_i + \sum_i \mu_i = 1$. There are also quantization conditions on M^2 projections of momenta from modified Dirac equation implying that multiplies of hyper-complex prime are in question in suitable units. Note however that it is not clear whether purely bosonic strands are present.

v. For ordinary elementary particles with propagators behaving like $\prod_i \lambda_i^{-1} 1 p^{-n}$, only $n \leq 2$ is possible. The topologically really interesting states with more than two braid strands are something else than what we have used to call elementary particles. The proposed interpretation is in terms of anyonic states [K37]. One important implication is that $\mathcal{N} = 1$ SUSY generated by right-handed neutrino or its antineutrino is SUSY for which all members of the multiplet assigned to a wormhole throat have braid number smaller than 3. For $\mathcal{N} = 2$ SUSY generated by right-handed neutrino and its antiparticle the states containing fermion and neutrino-antineutrino pair have three braid strands and SUSY breaking is expected to be strong.

Vertices

Conformal invariance raises the hope that vertices can be deduced from super-conformal invariance as n-point functions. Therefore lines would come from integrable QFT in M^2 and topological braid theory and vertices from confofmal field theory: both theories are integrable.

The basic questions is how the vertices are defined by the 2-D partonic surfaces at which the ends of lines meet. Finite measurement resolution reduces the lines to braids so that the vertices reduces to the intersection of braid strands with the partonic 2-surface.

i. Conformal invariance is the basic symmetry of quantum TGD. Does this mean that the vertices can be identified as n-point functions for points of the partonic 2-surface defined by the incoming and outgoing braid strands? How strong constraints can one pose on this conformal field theory? Is this field theory free and fixed by anti-commutation relations of induced spinor fields so that correlation function would reduce to product of fermionic two points functions with standard operator in the vertices represented by strand ends. If purely bosonic vertices are present, their correlation functions must result from the functional integral over WCW.

ii. For the fermionic fields associated with each incoming braid the anti-commutators of fermions and anti-fermions are trivial just as the usual equal time anti-commutation relations. This means that the vertex reduces to sum of products of fermionic correlation functions with arguments belonging to different incoming and outgoing lines. How can one calculate the correlators?

    A. Should one perform standard second quantization of fermions at light-like 3-surface allowing infinite number of spinor modes, apply a finite measurement resolution to obtain braids, for each partonic 2-surface, and use the full fermion fields to calculate the correlators? In this case braid strands would be discontinuous in vertices. A possible problem might be that the cutoff in spinor modes seems to come from the theory itself: finite measurement resolution is a property of quantum state itself.

B. Could finite measurement resolution allow to approximate the braid strands with continuous ones so that the correlators between strands belonging to different lines are given by anti-commutation relations? This would simplify enormously the situation and would conform with the idea of finite measurement resolution and the vision that interaction vertices reduce to braids. This vision is encouraged by the previous considerations and would mean that replication of braid strands analogous to replication of DNA strands can be seen as a fundamental process of Nature. This of course represents an important deviation from the standard picture.

iii. Suppose that one accepts the latter option. What can happen in the vertex, where line goes from one braid to another one?

A. Can the direction of momentum changed as visual intuition suggests? Is the total braid momentum conservation the only constraint so that the velocities assignable braid strands in each line would be constrained by the total momentum of the line.

B. What kind of operators appear in the vertex? To get some idea about this one can look for the simplest possible vertex, namely FFB vertex which could in fact be the only fundamental vertex as the arguments of [K11] suggest. The propagator of spin one boson decomposes to product of a projection operator to the polarization states divited by p^2 factor. The projection operator sum over products $\epsilon_i^k\gamma_k$ at both ends where γ_k acts in the spinor space defined by fermions. Also fermion lines have spinor and its conjugate at their ends. This gives rise to $p^k\gamma_k/p^2$. $p^k\gamma_k$ is the analog of the bosonic polarization tensor factorizing into a sum over products of fermionic spinors and their conjugates. This gives the BFF vertex $\epsilon_i^k\gamma_k$ slashed between the fermionic propagators which are effectively 2-dimensional.

C. Note that if H-chiralities are same at the throats of the wormhole contact, only spin one states are possible. Scalars would be leptoquarks in accordance with general view about lepton and quark number conservation. One particular implication is that Higgs in the standard sense is not possible in TGD framework. It can appear only as a state with a polarization which is in CP_2 direction. In any case, Higgs like states would be eaten by massless state so that all particles would have at least a small mass.

Functional integral over 3-surfaces

The basic question is how one can functionally integrate over light-like 3-surfaces or space-like 3-surfaces.

i. Does effective 2-dimensionality allow to reduce the functional integration to that over partonic 2-surfaces assigned with space-time sheet inside CD plus radiative corrections from the hierarchy of sub-CDs?

ii. Does finite measurement resolution reduce the functional integral to a ordinary integral over the positions of the end points of braids and could this integral reduce to a sum? Symplectic group of $\delta M^4_\pm \times CP_2$ basically parametrizes the quantum fluctuating degrees of freedom in WCW. Could finite measurement resolution reduce the symplectic group of $\delta M^4_\pm \times CP_2$ to a coset space obtained by dividing with symplectic transformations leaving the end points invariant and could the outcome be a discrete group as proposed? Functional integral would reduce to sum.

iii. If Kähler action reduces to Chern-Simons-Kähler terms to surface area terms in the proposed manner, the integration over WCW would be very much analogous to a functional integral over string world sheets and the wisdom gained in string models might be of considerable help.

Summary

What can one conclude from these argument? To my view the situation gives rise to a considerable optimism. I believe that on basis of the proposed picture it should be possible to build a concrete mathematical models for the generalized Feynman graphics and the idea about reduction to generalized braid diagrams having algebraic representations could pose additional powerful constraints on the construction. Braid invariants could also be building bricks of the generalized Feynman diagrams. In particular, the treatment of the non-planarity of Feynman diagrams in terms of M^2 braiding matrix would be something new and therefore can be questioned.

Few years after writing these lines a view about generalized Feynman diagrams as a stringy generalization of twistor Grassmannian diagrams has emerged [K44]. This approach relies heavily on the localization of spinor modes on 2-D string world sheets (covariantly constant right-handed neutrino is an exception) [K69]. This approach can be regarded as an effective QFT (or rather, effective string theory) approach: all information about the microscopic character of the fundamental particle like entities has been integrated out so that a string model type description at the level of imbdding space emerges. The presence of gigantic symmetries, in particular, the Yangian generalization of super-conformal symmetries, raises hopes that this approach could work. The approach to generalized Feynman diagrams considered above is obviously microscopic.

9.8 Electron as a trefoil or something more general?

The possibility that electron, and also other elementary particles could correspond to knot is very interesting. The video model [B14] was so fascinating (I admire the skills of the programmers) that I started to question my belief that all related to knots and braids represents new physics (say anyons), [K37] and that it is hopeless to try to reduce standard model quantum numbers with purely group theoretical explanation (except family replication) to topological quantum numbers.

Electroweak and color quantum numbers should by quantum classical correspondence have geometric correlates in space-time geometry. Could these correlates be topological? As a matter of fact, the correlates existing if the present understanding of the situation is correct but they are not topological.

Despite this, I played with various options and found that in TGD Universe knot invariants do not provide plausible space-time correlates for electroweak quantum numbers. The knot invariants and many other topological invariants are however present and mean new physics. As following arguments try to show, elementary particles in TGD Universe are characterized by extremely rich spectrum of topological quantum numbers, in particular those associated with knotting and linking: this is basically due to the 3-dimensionality of 3-space.

For a representation of trefoil knot by R.W. Gray see
http://www.rwgrayprojects.com/Lynn/Presentation20070926/p008.html. The homepage of Louis Kauffman [A8] is a treasure trove for anyone interested in ideas related to possible applications of knots to physics. One particular knotty idea is discussed in the article Emergent Braided Matter of Quantum Geometry by Bilson-Thompson, Hackett, and Kauffman [B26].

9.8.1 Space-time as 4-surface and the basic argument

Space-time as a 4-surface in $M^4 \times CP_2$ is the key postulate. The dynamics of space-time surfaces is determined by so called Kähler action - essentially Maxwell action for the Kähler form of CP_2 induced to X^4 in induced metric. Only so called preferred extremals are accepted and one can in very loose sense say that general coordinate invariance is realized by assigning to a given 3-surface a unique 4-surface as a preferred extremal analogous to Bohr orbit for a particle identified as 3-D surface rather than point-like object.

One ends up with a radical generalization of space-time concept to what I call many-sheeted space-time. The sheets of many-sheeted space-time are at distance of CP_2 size scale (10^4 Planck lengths as it turns out) and can touch each other which means formation of wormhole contact with wormhole throats as its ends. At throats the signature of the induced metric changes from Minkowskian to Euclidian. Euclidian regions are identified as 4-D analogs of lines of generalized Feynman diagrams and the M^4 projection of wormhole contact can be arbitrarily large: macroscopic, even astrophysical. Macroscopic object as particle like entity means that it is accompanied by Euclidian region of its size.

Elementary particles are identified as wormhole contacts. The wormhole contacts born in mere touching are not expected to be stable. The situation changes if there is a monopole magnetic flux (CP_2 carries self dual purely homological monopole Kähler form defining Maxwell field, this is not Dirac monopole) since one cannot split the contact. The lines of the Kähler magnetic field must be closed, and this requires that there is another wormhole contact nearby. The magnetic flux from the upper throat of contact A travels to the upper throat of contact B along "upper space-time sheet", goes to "lower" space-time sheet along contact B and returns back to the wormhole contact A so that closed loop results.

In principle, wormhole throat can have arbitrary orientable topology characterized by the number g of handles attached to sphere and known as genus. The closed flux tube corresponds to topology $X^2_g \times S^1$, g=0,1,2,... Genus-generation correspondence [K11] states that electron, muon, and tau lepton and similarly quark generations correspond to $g = 0, 1, 2$ in TGD Universe and CKM mixing is induced by topological mixing.

Suppose that one can assign to this flux tube a closed string: this is indeed possible but I will not bother reader with details yet. What one can say about the topology of this string?

i. X^2_g has homology Z^{2g} and S^1 homology S^1. The entire homology is Z^{2g+1} so that there are $2g + 1$ additional integer valued topological quantum numbers besides genus. Z^{2g+1} obviously breaks topologically universality stating that fermion generations are exact copies of each other apart from mass. This would be new physics. If the size of the flux loop is of order Compton length, the topological excitations need not be too heavy. One should however know how to excite them.

ii. The circle S^1 is imbedded in 3-surface and can get knotted. This means that all possible knots characterize the topological states of the the fermion. Also this means extremely rich spectrum of new physics.

9.8.2 What is the origin of strings going around the magnetic flux tube?

What is then the origin of these knotted strings? The study of the modified Dirac equation [K69] determining the dynamics of induced spinor fields at space-time surface led to a considerable insight here. This requires however additional notions such as zero energy ontology (ZEO), and causal diamond (CD) defined as intersection of future and past directed light-cones (double 4-pyramid is the M^4 projection. Note that CD has CP_2 as Cartesian factor and is analogous to Penrose diagram.

i. ZEO means the assumption that space-time surfaces for a particular sub-WCW ("world of classical worlds") are contained inside given CD identifiable as a the correlate for the "spotlight of consciousness" in TGD inspired theory of consciousness. The space-time surface has ends at the upper and lower light-like boundaries of CD. The 3-surfaces at the the ends define space-time correlates for the initial and final states in positive energy ordinary ontology. In ZEO they carry opposite total quantum numbers.

ii. General coordinate invariance (GCI) requires that once the 3-D ends are known, space-time surface connecting the ends is fixed (there is not path integral since it simply fails). This reduces ordinary holography to GCI and makes classical physics

defined by preferred extremals an exact part of quantum theory, actually a key element in the definition of Kähler geometry of WCW.
Strong form of GCI is also possible. One can require that 3-D light-like orbits of wormhole throats at which the induced metric changes its signature, and space-like 3-surfaces at the ends of CD give equivalent descriptions. This implies that quantum physics is coded by the their intersections which I call partonic 2-surfaces - wormhole throats - plus the 4-D tangent spaces of X^4 associated with them. One has strong form of holography. Physics is almost 2-D but not quite: 4-D tangent space data is needed.

iii. The study of the modified Dirac equation [K69] leads to further results. The mere conservation of electromagnetic charge defined group theoretically for the induced spinors of $M^4 \times CP_2$ carrying spin and electroweak quantum numbers implies that for all other fermion states except right handed neutrino (, which does not couple at all all to electroweak fields), are localized at 2-D string world sheets and partonic 2-surfaces.
String world sheets intersect the light-like orbits of wormhole throats along 1-D curves having interpretation as time-like braid strands (a convenient metaphor: braiding in time direction si created by dancers in the parquette).
One can say that dynamics automatically implies effective discretization: the ends of time like braid strands at partonic 2-surfaces at the ends of CD define a collection of discrete points to each of which one can assign fermionic quantum numbers.

iv. Both throats of the wormhole contact can carry many fermion state and known fermions correspond to states for which either throat carries single braid strand. Known bosons correspond to states for which throats carry fermion and anti-fermion number.

v. Partonic 2-surface is replaced with discrete set of points effectively. The interpretation is in terms of a space-time correlate for finite measurement resolution. Quantum correlate would be the inclusion of hyperfinite factors of type II_1.

This interpretation brings in even more topology!

i. String world sheets - present both in Euclidian and Minkowskian regions - intersect the 3-surfaces at the ends of CD along curves - one could speak of strings. These strings give rise to the closed curves that I discussed above. These strings can be homologically non-trivial - in string models this corresponds to wrapping of branes.

ii. For known bosons one has two closed loop but these loops could fuse to single. Space-like 2-braiding (including linking) becomes possible besides knotting.

iii. When the partonic 2-surface contains several fermionic braid ends one obtains even more complex situation than above when one has only single braid end. The loops associated with the braid ends and going around the monopole flux tube can form space-like N-braids. The states containing several braid ends at either throat correspond to exotic particles not identifiable as ordinary elementary particles.

9.8.3 How elementary particles interact as knots?

Elementary particles could reveal their knotted and even braided character via the topological interactions of knots. There are two basic interactions.

i. The basic interaction for single string is by self-touching and this can give to a local connected sum or a reconnection. In both cases the knot invariants can change and it is possible to achieve knotting or unknotting of the string by this mechanism. String can also split into two pieces but this might well be excluded in the recent case.
The space-time dynamics for these interactions is that of closed string model with 4-D target space. The first guess would be topological string model describing only the dynamics of knots. Note that string world sheets define 2-knots and braids.

ii. The basic interaction vertex for generalized Feynman diagrams (lines are 4-D space-time regions with Euclidian signature) is join along 3-D boundaries for the three particles involved: this is just like ordinary 3-vertex for Feynman diagrams and is not encountered in string models. The ends of lines must have same genus g. In this interaction vertex the homology charges in Z^{2g+1} is conserved so that these charges are analogous to U(1) gauge charges. The strings associated with the two particles can touch each other and connected sum or reconnection is the outcome.

Consider now in more detail connected sum and reconnection vertices responsible for knotting and un-knotting.

i. The first interaction is connected sum of knots [A3]. A little mental exercise demonstrates that a local connected sum for the pieces of knot for which planar projections cross, can lead to a change in knotted-ness. Local connected sum is actually used to un-knot the knot in the construction of knot invariants.
In dimension 3 knots form a module with respect to the connected sum. One can identify unique prime knots and construct all knots as products of prime knots with product defined as a connected sum of knots. In particular, one cannot have a situation on which a product of two non-trivial knots is un-knot so that one could speak about the inverse of a knot (indeed, the inverse of ordinary prime is not an integer!). For higher-dimensional knots the situation changes (string world sheets at space-time surface could form 2-knots but instead of linking they intersect at discrete points).
Connected sum in the vertex of generalized Feynman graph (as described above) can lead to a decay of particle to two particles, which correspond to the summands in the connected sum as knots. Could one consider a situation in which un-knotted particle decomposes via the time inverse of the connected sum to a pair of knotted particles such that the knots are inverses of each other? This is not possible since knots do not have inverse.

ii. Touching knots can also reconnect. For braids the strands $A \to B$ and $C \to D$ touch and one obtains strands $A \to D$ and $C \to B$. If this reaction takes place for strands whose planar projections cross, it can also change the character of the knot. One one can transform knot to un-knot by repeatedly applying connected sum and reconnection for crossing strands (the Alexandrian way).

iii. In the evolution of knots as string world sheets these two vertices corresponds to closed string vertices. These vertices can lead to topological mixing of knots leading to a quantum superposition of different knots for a given elementary particle. This mixing would be analogous to CKM mixing understood to result from the topological mixing of fermion genera in TGD framework. It could also imply that knotted particles decay rapidly to un-knots and make the un-knot the only long-lived state.
A naive application of Uncertainty Principle suggests that the size scale of string determines the life time of particular knot configuration. The dependence on the length scale would however suggest that purely topological string theory cannot be in question. Zero energy ontology suggests that the size scale of the causal diamond assignable to elementary particle determines the time scale for the rates as secondary p-adic time scale: in the case of electron the time scale would be .1 seconds corresponding to Mersenne prime $M_{127} = 2^{127} - 1$ so that knotting and unknotting would be very slow processes. For electron the estimate for the scale of mass differences between different knotted states would be about $10^{-19} m_e$: electron mass is known for certain for 9 decimals so that there is no hope of detecting these mass differences. The pessimistic estimate generalizes to all other elementary particles: for weak bosons characterized by M_{89} the mass difference would be of order $10^{-13} m_W$.

iv. A natural guess is that p-adic thermodynamics can be applied to the knotting. In p-adic thermodynamics Boltzmann weights in are of form $p^{H/T}$ (p-adic number) and the allowed values of the Hamiltonian H are non-negative integer powers of p. Clearly, H representing a contribution to p-adic valued mass squared must be a non-negative integer valued invariant additive under connected sum. This guarantees extremely rapid convergence of the partition function and mass squared expectation value as the number of prime knots in the decomposition increases.

An example of an knot invariant [A14] additive under connected sum is knot genus [A13] defined as the minimal genus of 2-surface having the knot as boundary (Seifert surface). For trefoil and figure eight knot one has $g = 1$. For torus knot $(p, q) \equiv (q, p)$ one has $g = (p-1)(q-1)/2$. Genus vanishes for un-knot so that it gives the dominating contribution to the partition function but a vanishing contribution to the p-adic mass squared.

p-Adic mass scale could be assumed to correspond to the primary p-adic mass scale just as in the ordinary p-adic mass calculations. If the p-adic temperature is $T = 1$ in natural units (highest possible), and if one has $H = 2g$, the lowest order contribution corresponds to the value $H = 2$ of the knot Hamiltonian, and is obtained for trefoil and figure eight knot so that the lowest order contribution to the mass would indeed be about $10^{-19} m_e$ for electron. An equivalent interpretation is that $H = g$ and $T = 1/2$ as assumed for gauge bosons in p-adic mass calculations.

There is a slight technical complication involved. When the string has a non-trivial homology in $X^2_g \times S^1$ (it always has by construction), it does not allow Seifert surface in the ordinary sense. One can however modify the definition of Seifert surface so that it isolates knottedness from homology. One can express the string as connected sum of homologically non-trivial un-knot carrying all the homology and of homologically trivial knot carrying all knottedness and in accordance with the additivity of genus define the genus of the original knot as that for the homologically trivial knot.

v. If the knots assigned with the elementary particles have large enough size, both connected sum and reconnection could take place for the knots associated with different elementary particles and make the many particle system a single connected structure. TGD based model for quantum biology is indeed based on this kind of picture. In this case the braid strands are magnetic flux tubes and connect bio-molecules to single coherent whole. Could electrons form this kind of stable connected structures in condensed matter systems? Could this relate to superconductivity and Cooper pairs somehow? If one takes p-adic thermodynamics for knots seriously then knotted and braided magnetic flux tubes are more attractive alternative in this respect.

What if the thermalization of knot degrees of freedom does not take place? One can also consider the possibility that knotting contributes only to the vacuum conformal weight and thus to the mass squared but that no thermalization of ground states takes place. If the increment Δm of inertial mass squared associated with knotting is of from kgp^2, where k is positive integer and g the above described knot genus, one would have $\Delta m/m \simeq 1/p$. This is of order $M_{127}^{-1} \simeq 10^{-38}$ for electron.

Could the knotting and linking of elementary particles allow topological quantum computation at elementary particle level? The huge number of different knottings would give electron a huge ground state degeneracy making possible negentropic entanglement. For negentropic entanglement probabilities must belong to an algebraic extension of rationals: this would be the case in the intersection of p-adic and real worlds and there is a temptation to assign living matter to this intersection. Negentropy Maximization Principle could stabilize negentropic entanglement and therefore allow to circumvent the problems due to the fact that the energies involved are extremely tiny and far below thus thermal energy. In this situation bit would generalize to "nit" corresponding to N different ground states of particle differing by knotting.

A very naive dimensional analysis using Uncertainty Principle would suggest that the number changes of electron state identifiable as quantum computation acting on q-nits is of order $1/\Delta t = \Delta m/hbar$. More concretely, the minimum duration of the quantum computation would be of order $\Delta t = \hbar/\Delta m$. Single quantum computation would take an immense amount time: for electron single operation would take time of order 10^{17} s, which is of the order of the recent age of the Universe. Therefore this quantum computation would be of rather limited practical value!

Chapter 10

Ideas Emerging from TGD

10.1 Introduction

I have gathered to this chapter those ideas related to quantum TGD which are not absolutely central and whose status is not clear in the long run. I have represented earlier these ideas in chapters and the outcome was a total chaos and reader did not have a slightest idea what is they real message. I hope that this organization of material makes it easier for the reader to grasp the topology of TGD correctly. The representation includes various ideas and notions such as $M^8 - H$ duality, hierarchy of Planck constants, and the notion of number theoretic braid. Sections about twistor approach and octonionic spinors are included as well as considerations related to WCW integration and about possible topological invariances defined by geometric invariants for preferred extremals of Kähler action.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L13]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L14]. The topics relevant to this chapter are given by the following list.

- Quantum theory [L31]
- Emergent ideas and notions [L17]
- Weak form of electric-magnetic duality [L43]
- $M^8 - H$ duality [L26]
- Hierarchy of Planck constants [L21]
- Hyperfinite factors and TGD [L23]
- The unique role of twistors in TGD [L38]
- Twistors and TGD [L39]

10.2 Hierarchy of Planck constants and the generalization of the notion of imbedding space

In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is summarized. The question is whether it might be possible in some sense to replace H or its Cartesian factors by their necessarily singular multiple coverings and factor spaces. One can consider two options: either M^4 or the causal diamond CD. The latter one is the more plausible option from the point of view of WCW geometry.

10.2.1 The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

i. The starting point was the proposal of Nottale [E2] that the orbits of inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [K45, K35] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.

ii. Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [K46] . TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or over-critical mass density and the "pressure" associated with these cosmologies is negative.

iii. The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the imbedding space obtained by gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.

iv. Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K54] .

v. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase. If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E2] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr}G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

vi. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L2, K54] , [L2] .

10.2.2 The most general option for the generalized imbedding space

Simple physical arguments pose constraints on the choice of the most general form of the imbedding space.

i. The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

ii. CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is un-acceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the singular coverings and factor spaces of CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.

   A. The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the modified gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.

   B. The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

iii. For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD}\hat{\times}G_a) \times (\hat{CP}_2\hat{\times}G_b)$, $(\hat{CD}\hat{\times}G_a) \times \hat{CP}_2/G_b$, $\hat{CD}/G_a \times (\hat{CP}_2\hat{\times}G_b)$, and $\hat{CD}/G_a \times \hat{CP}_2/G_b$.

iv. The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tetrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

10.2.3 About the phase transitions changing Planck constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

i. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.

ii. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

iii. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

10.2.4 How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

i. One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

ii. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.

iii. The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to n_an_b sheets and multiplies YM action action by n_an_b which is equivalent with the $\hbar = n_an_b\hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_an_b\hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.
This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	n_an_b	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_an_b}$

10.2.5 Preferred values of Planck constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k\prod_s F_s$, where $F_s = 2^{2^s}+1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0,1,2,3,4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} seem to be especially favored as values of n_a in living matter [K14] .

10.2.6 How Planck constants are visible in Kähler action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anti-commutation relations of various superconformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck constants. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

10.2.7 Could the dynamics of Kähler action predict the hierarchy of Planck constants?

The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. The formula for the Planck constant involves heuristic guess work and physical plausibility arguments. There are good arguments in favor of the hypothesis that only coverings are possible. Only a finite number of pages of the Big Book correspond to a given value of Planck constant, biological evolution corresponds to a gradual dispersion to the pages of the Big Book with larger Planck constant, and a connection with the hierarchy of infinite primes and p-adicization program based on the mathematical realization of finite measurement resolution emerges.

One can however ask whether this hierarchy could emerge directly from the basic quantum TGD rather than as a separate hypothesis. The following arguments suggest that this might be possible. One finds also a precise geometric interpretation of preferred extremal property interpreted as criticality in zero energy ontology.

1-1 correspondence between canonical momentum densities and time derivatives fails for Kähler action

The basic motivation for the geometrization program was the observation that canonical quantization for TGD fails. To see what is involved let us try to perform a canonical quantization in zero energy ontology at the 3-D surfaces located at the light-like boundaries of $CD \times CP_2$.

i. In canonical quantization canonical momentum densities $\pi^0_k \equiv \pi_k = \partial L_K/\partial(\partial_0 h^k)$, where $\partial_0 h^k$ denotes the time derivative of imbedding space coordinate, are the physically natural quantities in terms of which to fix the initial values: once their value distribution is fixed also conserved charges are fixed. Also the weak form of electric-magnetic duality given by $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and a mild generalization of this condition to be discussed below can be interpreted as a manner to fix the values of conserved gauge charges (not Noether charges) to their quantized values since Kähler magnetic flux equals to the integer giving the homology class of the (wormhole) throat. This condition alone need not characterize criticality, which requires an infinite number of deformations of X^4 for which the second variation of the Kähler action vanishes and implies infinite number conserved charges. This in fact gives hopes of replacing π_k with these conserved Noether charges.

ii. Canonical quantization requires that $\partial_0 h^k$ in the energy is expressed in terms of π_k. The equation defining π_k in terms of $\partial_0 h^k$ is however highly non-linear although algebraic. By taking squares the equations reduces to equations for rational functions of $\partial_0 h^k$. $\partial_0 h^k$ appears in contravariant and covariant metric at most quadratically and in the induced Kähler electric field linearly and by multiplying the equations by $det(g_4)^3$ one can transform the equations to a polynomial form so that in principle $\partial_0 h^k$ can obtained as a solution of polynomial equations.

iii. One can always eliminate one half of the coordinates by choosing 4 imbedding space coordinates as the coordinates of the space-time surface so that the initial value conditions reduce to those for the canonical momentum densities associated with the remaining four coordinates. For instance, for space-time surfaces representable as map $M^4 \to CP_2$ M^4 coordinates are natural and the time derivatives $\partial_0 s^k$ of CP_2 coordinates are multi-valued. One would obtain four polynomial equations with $\partial_0 s^k$ as unknowns. In regions where CP_2 projection is 4-dimensional -in particular for the deformations of CP_2 vacuum extremals the natural coordinates are CP_2 coordinates and one can regard $\partial_0 m^k$ as unknowns. For the deformations of cosmic strings, which are of form $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, one can use coordinates of $M^2 \times S^2$, where S^2 is geodesic sphere as natural coordinates and regard as unknowns E^2 coordinates and remaining CP_2 coordinates.

iv. One can imagine solving one of the four polynomials equations for time derivaties in terms of other obtaining N roots. Then one would substitute these roots to the remaining 3 conditions to obtain algebraic equations from which one solves then second variable. Obviously situation is very complex without additional symmetries. The criticality of the preferred extremals might however give additional conditions allowing simplifications. The reasons for giving up the canonical quantization program was following. For the vacuum extremals of Kähler action π_k are however identically vanishing and this means that there is an infinite number of value distributions for $\partial_0 h^k$. For small deformations of vacuum extremals one might however hope a finite number of solutions to the conditions and thus finite number of space-time surfaces carrying same conserved charges.

If one assumes that physics is characterized by the values of the conserved charges one must treat the the many-valuedness of $\partial_0 h^k$. The most obvious guess is that one should replace the space of space-like 4-surfaces corresponding to different roots $\partial_0 h^k = F^k(\pi_l)$ with four-surfaces in the covering space of $CD \times CP_2$ corresponding to different branches of the many-valued function $\partial_0 h^k = F(\pi_l)$ co-inciding at the ends of CD.

Do the coverings forces by the many-valuedness of $\partial_0 h^k$ correspond to the coverings associated with the hierarchy of Planck constants?

The obvious question is whether this covering space actually corresponds to the covering spaces associated with the hierarchy of Planck constants. This would conform with quantum classical correspondence. The hierarchy of Planck constants and hierarchy of covering spaces was introduced to cure the failure of the perturbation theory at quantum level. At classical level the multi-valuedness of $\partial_0 h^k$ means a failure of perturbative canonical quantization and forces the introduction of the covering spaces. The interpretation would be that when the density of matter becomes critical the space-time surface splits to several branches so that the density at each branches is sub-critical. It is of course not at all obvious whether the proposed structure of the Big Book is really consistent with this hypothesis and one also consider modifications of this structure if necessary. The manner to proceed is by making questions.

i. The proposed picture would give only single integer characterizing the covering. Two integers assignable to CD and CP_2 degrees of freedom are however needed. How these two coverings could emerge?

   A. One should fix also the values of $\pi^n_k = \partial L_K/\partial h^k_n$, where n refers to space-like normal coordinate at the wormhole throats. If one requires that charges do not flow between regions with different signatures of the metric the natural condition is $\pi^n_k = 0$ and allows also multi-valued solution. Since wormhole throats carry magnetic charge and since weak form of electric-magnetic duality is assumed, one can assume that CP_2 projection is four-dimensional so that one can use CP_2 coordinates and regard $\partial_0 m^k$ as un-knows. The basic idea about topological condensation in turn suggests that M^4 projection can be assumed to be 4-D inside space-like 3-surfaces so that here $\partial_0 s^k$ are the unknowns. At partonic 2-surfaces one would have conditions for both π^0_k and π^n_k. One might hope that the numbers of solutions are finite for preferred extremals because of their symmetries and given by n_a for $\partial_0 m^k$ and by n_b for $\partial_0 s^k$. The optimistic guess is that n_a and n_b corresponds to the numbers of sheets for singular coverings of CD and CP_2. The covering could be visualized as replacement of space-time surfaces with space-time surfaces which have $n_a n_b$ branches. n_b branches would degenerate to single branch at the ends of diagrams of the generaled Feynman graph and n_a branches would degenerate to single one at wormhole throats.

   B. This picture is not quite correct yet. The fixing of π^0_k and π^n_k should relate closely to the effective 2-dimensionality as an additional condition perhaps crucial for criticality. One could argue that both π^0_k and π^n_k must be fixed at X^3 and X^3_l in order to effectively bring in dynamics in two directions so that X^3 could be interpreted as a an orbit of partonic 2-surface in space-like direction and X^3_l as its orbit in light-like direction. The additional conditions could be seen as gauge conditions made possible by symplectic and Kac-Moody type conformal symmetries. The conditions for π^k_0 would give n_b branches in CP_2 degrees of freedom and the conditions for π^n_k would split each of these branches to n_a branches.

   C. The existence of these two kinds of conserved charges (possibly vanishing for π^n_k) could relate also very closely to the slicing of the space-time sheets by string world sheets and partonic 2-surfaces.

ii. Should one then treat these branches as separate space-time surfaces or as a single space-time surface? The treatment as a single surface seems to be the correct thing to do. Classically the conserved changes would be $n_a n_b$ times larger than for single branch. Kähler action need not (but could!) be same for different branches but the total action is n_an_b times the average action and this effectively corresponds to the replacement of the $\hbar_0/g_K^2$ factor of the action with $\hbar/g_K^2$, $r \equiv \hbar/\hbar_0 = n_an_b$. Since the conserved quantum charges are proportional to $\hbar$ one could argue that $r = n_an_b$ tells only that the charge conserved charge is n_an_b times larger than without multi-valuedness. $\hbar$ would be only effectively n_an_b fold. This is of course poor man's argument but might catch something essential about the situation.

iii. How could one interpret the condition $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and its generalization to be discussed below in this framework? The first observation is that the total Kähler electric charge is by $\alpha_K \propto 1/(n_an_b)$ same always. The interpretation would be in terms of charge fractionization meaning that each branch would carry Kähler electric charge $Q_K = ng_K/n_an_b$. I have indeed suggested explanation of charge fractionization and quantum Hall effect based on this picture.

iv. The vision about the hierarchy of Planck constants involves also assumptions about imbedding space metric. The assumption that the M^4 covariant metric is proportional to $\hbar^2$ follows from the physical idea about $\hbar$ scaling of quantum lengths as what Compton length is. One can always introduce scaled M^4 coordinates bringing M^4 metric into the standard form by scaling up the M^4 size of CD. It is not clear whether the scaling up of CD size follows automatically from the proposed scenario. The basic question is why the M^4 size scale of the critical extremals must scale like n_an_b? This should somehow relate to the weak self-duality conditions implying that Kähler field at each branch is reduced by a factor $1/r$ at each branch. Field equations should posses a dynamical symmetry involving the scaling of CD by integer k and $J^{0\beta}\sqrt{g_4}$ and $J^{n\beta}\sqrt{g_4}$ by $1/k$. The scaling of CD should be due to the scaling up of the M^4 time interval during which the branched light-like 3-surface returns back to a non-branched one.

v. The proposed view about hierarchy of Planck constants is that the singular coverings reduce to single-sheeted coverings at $M^2 \subset M^4$ for CD and to $S^2 \subset CP_2$ for CP_2. Here S^2 is any homologically trivial geodesic sphere of CP_2 and has vanishing Kähler form. Weak self-duality condition is indeed consistent with any value of $\hbar$ and impies that the vacuum property for the partonic 2-surface implies vacuum property for the entire space-time sheet as holography indeed requires. This condition however generalizes. In weak self-duality conditions the value of $\hbar$ is free for any 2-D Lagrangian sub-manifold of CP_2.
The branching along M^2 would mean that the branches of preferred extremals always collapse to single branch when their M^4 projection belongs to M^2. Magnetically charged light-light-like throats cannot have M^4 projection in M^2 so that self-duality conditions for different values of $\hbar$ do not lead to inconsistencies. For space-like 3-surfaces at the boundaries of CD the condition would mean that the M^4 projection becomes light-like geodesic. Straight cosmic strings would have M^2 as M^4 projection. Also CP_2 type vacuum extremals for which the random light-like projection in M^4 belongs to M^2 would represent this of situation. One can ask whether the degeneration of branches actually takes place along any string like object $X^2 \times Y^2$, where X^2 defines a minimal surface in M^4. For these the weak self-duality condition would imply $\hbar = \infty$ at the ends of the string. It is very plausible that string like objects feed their magnetic fluxes to larger space-times sheets through wormhole contacts so that these conditions are not encountered.

Connection with the criticality of preferred extremals

Also a connection with quantum criticality and the criticality of the preferred extremals suggests itself. Criticality for the preferred extremals must be a property of space-like 3-surfaces and light-like 3-surfaces with degenerate 4-metric and the degeneration of the n_an_b branches of the space-time surface at the its ends and at wormhole throats is exactly what happens at criticality. For instance, in catastrophe theory roots of the polynomial equation giving extrema of a potential as function of control parameters co-incide at criticality. If this picture is correct the hierarchy of Planck constants would be an outcome of criticality and of preferred extremal property and preferred extremals would be just those multi-branched space-time surfaces for which branches co-incide at the the boundaries of $CD \times CP_2$ and at the throats.

10.2.8 Updated view about the hierarchy of Planck constants

The original hypothesis was that the hierarchy of Planck constants is real. In this formulation the imbedding space was replaced with its covering space assumed to decompose to a Cartesian product of singular finite-sheeted coverings of M^4 and CP_2.

Few years ago came the realization that it could be only effective but have same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $\hbar_{eff} = n\hbar$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. In this formulation the singular covering of the imbedding space became only a convenient auxiliary tool. It is no more necessary to assume that the covering reduces to a Cartesian product of singular coverings of M^4 and CP_2 but for some reason I kept this assumption.

The formulation based on multi-furcations of space-time surfaces to N branches. For some reason I assumed that they are simultaneously present. This is too restrictive an assumption. The N branches are very much analogous to single particle states and second quantization allowing all $0 < n \leq N$-particle states for given N rather than only N-particle states looks very natural. As a matter fact, this interpretation was the original one, and led to the very speculative and fuzzy notion of N-atom, which I later more or less gave up. Quantum multi-furcation could be the root concept implying the effective hierarchy of Planck constants, anyons and fractional charges, and related notions- even the notions of N-nuclei, N-atoms, and N-molecules.

Basic physical ideas

The basic phenomenological rules are simple and there is no need to modify them.

i. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K55].

ii. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) [K37] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

iii. In astrophysics and cosmology the implications are even more dramatic if one believes that also $\hbar_{gr}$ corresponds to effective Planck constant interpreted as number of sheets of multi-furcation. It was Nottale [E2] who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This would suggest that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation. It must be however emphasized that the interpretation of $\hbar_{gr}$ could be different, and it will be found that one can develop an argument demonstrating how $\hbar_{gr}$ with a correct order of magnitude emerges from the effective space-time metric defined by the anti-commutators appearing in the modified Dirac equation. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents $\partial L_K/\partial(\partial_\alpha h^k)$ defining the modified gamma matrices [K69] and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of imbedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system. What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches co-incide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis.

In light of this the working hypothesis adopted during last years has been too limited: for some reason I ended up to propose that only N-sheeted covering corresponding to a situation in which all N branches are present is possible. Before that I quite correctly considered more general option based on intuition that one has many-particle states in the multi-sheeted space. The erratic form of the working hypothesis has not been used in applications.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

Basic phenomenological rules of thumb in the new framework

It is important to check whether or not the refreshed view about dark matter is consistent with existent rules of thumb.

i. The interpretation of quantized multi-furcations as WCW anyons explains also why the effective hierarchy of Planck constants defines a hierarchy of phases which are dark relative to each other. This is trivially true since the phases with different number of branches in multi-furcation correspond to disjoint regions of WCW so that the particles with different effective value of Planck constant cannot appear in the same vertex.
ii. The phase transitions changing the value of Planck constant are just the multi-furcations and can be induced by changing the values of the external parameters controlling the properties of preferred extremals. Situation is very much the same as in any non-linear system.
iii. In the case of massless particles the scaling of wavelength in the effective scaling of $\hbar$ can be understood if dark n-photons consist of n photons with energy E/n and wavelength $n\lambda$.
iv. For massive particle it has been assumed that masses for particles and they dark counterparts are same and Compton wavelength is scaled up. In the new picture this need not be true. Rather, it would seem that wave length are same as for ordinary electron.

   On the other hand, p-adic thermodynamics predicts that massive elemenetary particles are massless most of the time. ZEO predicts that even virtual wormhole throats are massless. Could this mean that the picture applying on massless particle should apply to them at least at relativistic limit at which mass is negligible. This might be the case for bosons but for fermions also fermion number should be fractionalized and this is not possible in the recent picture. If one assumes that the

n-electron has same mass as electron, the mass for dark single electron state would be scaled down by $1/n$. This does not look sensible unless the p-adic length defined by prime is scaled down by this fact in good approximation.
This suggests that for fermions the basic scaling rule does not hold true for Compton length $\lambda_c = \hbar_m$. Could it however hold for de-Broglie lengths $\lambda = \hbar/p$ defined in terms of 3-momentum? The basic overlap rule for the formation of macroscopic quantum states is indeed formulated for de Broglie wave length. One could argue that an $1/N$-fold reduction of density that takes place in the de-localization of the single particle states to the N branches of the cover, implies that the volume per particle increases by a factor N and single particle wave function is de-localized in a larger region of 3-space. If the particles reside at effectively one-dimensional 3-surfaces - say magnetic flux tubes - this would increase their de Broglie wave length in the direction of the flux tube and also the length of the flux tube. This seems to be enough for various applications.

One important notion in TGD inspired quantum biology is dark cyclotron state.

i. The scaling $\hbar \rightarrow k\hbar$ in the formula $E_n = (n + 1/2)\hbar eB/m$ implies that cyclotron energies are scaled up for dark cyclotron states. What this means microscopically has not been obvious but the recent picture gives a rather clearcut answer. One would have k-particle state formed from cyclotron states in N-fold branched cover of space-time surface. Each branch would carry magnetic field B and ion or electron. This would give a total cyclotron energy equal to kE_n. These cyclotron states would be excited by k-photons with total energy $E = khf$ and for large enough value of k the energies involved would be above thermal threshold. In the case of Ca^{++} one has $f = 15$ Hz in the field $B_{end} = .2$ Gauss. This means that the value of $\hbar$ is at least the ratio of thermal energy at room temperature to $E = hf$. The thermal frequency is of order 10^{12} Hz so that one would have $k \simeq 10^{11}$. The number branches would be therefore rather high.
ii. It seems that this kinds of states which I have called cyclotron Bose-Einstein condensates could make sense also for fermions. The dark photons involved would be Bose-Einstein condensates of k photons and wall of them would be simultaneously absorbed. The biological meaning of this would be that a simultaneous excitation of large number of atoms or molecules can take place if they are localized at the branches of N-furcation. This would make possible coherent macroscopic changes. Note that also Cooper pairs of electrons could be $n = 2$-particle states associated with N-furcation.

There are experimental findings suggesting that photosynthesis involves de-localized excitations of electrons and it is interesting so see whether this could be understood in this framework.

i. The TGD based model relies on the assumption that cyclotron states are involved and that dark photons with the energy of visible photons but with much longer wavelength are involved. Single electron excitations (or single particle excitations of Cooper pairs) would generate negentropic entanglement automatically.
ii. If cyclotron excitations are the primary ones, it would seem that they could be induced by dark n-photons exciting all n electrons simultaneously. n-photon should have energy of a visible photon. The number of cyclotron excited electrons should be rather large if the total excitation energy is to be above thermal threshold. In this case one could not speak about cyclotron excitation however. This would require that solar photons are transformed to n-photons in N-furcation in biosphere.
iii. Second - more realistic looking - possibility is that the incoming photons have energy of visible photon and are therefore $n = 1$ dark photons de-localized to the branches of the N-furcation. They would induce de-localized single electron excitation in WCW rather than 3-space.

Charge fractionalization and anyons

It is easy to see how the effective value of Planck constant as an integer multiple of its standard value emerges for multi-sheeted states in second quantization. At the level of Kähler action one can assume that in the first approximation the value of Kähler action for each branch is same so that the total Kähler action is multiplied by n. This corresponds effectively to the scaling $\alpha_K \rightarrow \alpha_K/n$ induced by the scaling $\hbar_0 \rightarrow n\hbar_0$.

Also effective charge fractionalization and anyons emerge naturally in this framework.

i. In the ordinary charge fractionalization the wave function decomposes into sharply localized pieces around different points of 3-space carrying fractional charges summing up to integer charge. Now the same happens at at the level of WCW ("world of classical worlds") rather than 3-space meaning that wave functions in E^3 are replaced with wave functions in the space-time of 3-surfaces (4-surfaces by holography implied by General Coordinate Invariance) replacing point-like particles. Single particle wave function in WCW is a sum of N sharply localized contributions: localization takes place around one particular branch of the multi-sheeted space time surface. Each branch carries a fractional charge q/N for teh analogs of plane waves. Therefore all quantum numbers are additive and fractionalization is only effective and observable in a localization of wave function to single branch occurring with probability $p = 1/N$ from which one can deduce that charge is q/N.

ii. The is consistent with the proposed interpretation of dark photons/gravitons since they could carry large spin and this kind of situation could decay to bunches of ordinary photons/gravitons. It is also consistent with electromagnetic charge fractionalization and fractionalization of spin.

iii. The original - and it seems wrong - argument suggested what might be interpreted as a genuine fractionalization for orbital angular momentum and also of color quantum numbers, which are analogous to orbital angular momentum in TGD framework. The observation was that a rotation through 2π at space-time level moving the point along space-time surface leads to a new branch of multi-furcation and $N + 1$:th branch corresponds to the original one. This suggests that angular momentum fractionalization should take place for M^4 angle coordinate ϕ because for it 2π rotation could lead to a different sheet of the effective covering.

   The orbital angular momentum eigenstates would correspond to waves $exp(i\phi m/N)$, $m = 0, 2, ..., N-1$ and the maximum orbital angular momentum would correspond the sum $\sum_{m=0}^{N-1} m/N = (N-1)/2$. The sum of spin and orbital angular momentum be therefore fractional.

   The different prediction is due to the fact that rotations are now interpreted as flows rotating the points of 3-surface along 3-surface rather than rotations of the entire partonic surface in imbedding space. In the latter interpretation the rotation by 2π does nothing for the 3-surface. Hence fractionalization for the total charge of the single particle states does not take place unless one adopts the flow interpretation. This view about fractionalization however leads to problems with fractionalization of electromagnetic charge and spin for which there is evidence from fractional quantum Hall effect.

What about the relationship of gravitational Planck constant to ordinary Planck constant?

Gravitational Planck constant is given by the expression $\hbar_{gr} = GMm/v_0$, where $v_0 < 1$ has interpretation as velocity parameter in the units $c = 1$. Can one interpret also $\hbar_{gr}$ as effective value of Planck constant so that its values would correspond to multi-furcation with a gigantic number of sheets. This does not look reasonable.

Could one imagine any other interpretation for $\hbar_{gr}$? Could the two Planck constants correspond to inertial and gravitational dichotomy for four-momenta making sense also for angular momentum identified as a four-vector? Could gravitational angular momentum and the momentum associated with the flux tubes mediating gravitational interaction be quantized in units of $\hbar_{gr}$ naturally?

i. Gravitational four-momentum can be defined as a projection of the M^4-four-momentum to space-time surface. It's length can be naturally defined by the effective metric $g^{\alpha\beta}_{eff}$ defined by the anti-commutators of the modified gamma matrices. Gravitational four-momentum appears as a measurement interaction term in the modified Dirac action and can be restricted to the space-like boundaries of the space-time surface at the ends of CD and to the light-like orbits of the wormhole throats and which induced 4- metric is effectively 3-dimensional.
ii. At the string world sheets and partonic 2-surfaces the effective metric degenerates to 2-D one. At the ends of braid strands representing their intersection, the metric is effectively 4-D. Just for definiteness assume that the effective metric is proportional to the M^4 metric or rather - to its M^2 projection: $g^{kl}_{eff} = K^2 m^{kl}$.
One can express the length squared for momentum at the flux tubes mediating the gravitational interaction between massive objects with masses M and m as

$$g^{\alpha\beta}_{eff} p_\alpha p_\beta = g^{\alpha\beta}_{eff} \partial_\alpha h^k \partial_\beta h^l p_k p_l \equiv g^{kl}_{eff} p_k p_l = n^2 \frac{\hbar^2}{L^2} \quad . \tag{10.2.1}$$

Here L would correspond to the length of the flux tube mediating gravitational interaction and p_k would be the momentum flowing in that flux tube. $g^{kl}_{eff} = K^2 m^{kl}$ would give

$$p^2 = \frac{n^2\hbar^2}{K^2L^2} \quad .$$

$\hbar_{gr}$ could be identifed in this simplified situation as $\hbar_{gr} = \hbar/K$.
iii. Nottale's proposal requires $K = GMm/v_0$ for the space-time sheets mediating gravitational interacting between massive objects with masses M and m. This gives the estimate

$$p_{gr} = \frac{GMm}{v_0}\frac{1}{L} \quad . \tag{10.2.2}$$

For $v_0 = 1$ this is of the same order of magnitude as the exchanged momentum if gravitational potential gives estimate for its magnitude. v_0 is of same order of magnitude as the rotation velocity of planet around Sun so that the reduction of v_0 to $v_0 \simeq 2^{-11}$ in the case of inner planets does not mean that the propagation velocity of gravitons is reduced.
iv. Nottale's formula requires that the order of magnitude for the components of the energy momentum tensor at the ends of braid strands at partonic 2-surface should have value GMm/v_0. Einstein's equations $T = \kappa G + \Lambda g$ give a further constraint. For the vacuum solutions of Einstein's equations with a vanishing cosmological constant the value of h_{gr} approaches infinity. At the flux tubes mediating gravitational interaction one expects T to be proportional to the factor GMm simply because they mediate the gravitational interaction.
v. One can consider similar equation for gravitational angular momentum:

$$g^{\alpha\beta}_{eff} L_\alpha L_\beta = g^{kl}_{eff} L_k L_l = l(l+1)\hbar^2 \quad . \tag{10.2.3}$$

This would give under the same simplifying assumptions

$$L^2 = l(l+1)\frac{\hbar^2}{K^2} \quad . \tag{10.2.4}$$

This would justify the Bohr quantization rule for the angular momentum used in the Bohr quantization of planetary orbits.

Maybe the proposed connection might make sense in some more refined formulation. In particular the proportionality between $m^{kl}_{eff} = Km^{kl}$ could make sense as a quantum average. Also the fact, that the constant v_0 varies, could be understood from the dynamical character of m^{kl}_{eff}.

Could $h_{gr} = h_{eff}$ hold true?

The obvious question is whether the gravitational Planck constant deduced from the Nottale's considerations and the effective Planck constant $h_{eff} = nh$ deduced from ELF effects on vertebrate brain and explained in terms of non-determinism of Kähler action could be identical. At first this seems to be non-sensical idea since $h_{gr} = GMm/v_0$ has gigantic value.

It is however essential to realize that by Equivalence Principle one describe gravitational interaction by reducing it to elementary particle level. For instance, gravitational Compton lengths do not depend at all on the masses of particles. Also the radii of the planetary orbits are independent of the mass of particle mass in accordance with Equivalence Principle. For elementary particles the values of h_{gr} are in the same range as in quantum biological applications. Typically 10 Hz ELF radiation should correspond to energy $E = h_{eff}f$ of UV photon if one assumes that dark ELF photons have energies of biophotons and transform to them. The order of magnitude for n would be therefore $n \simeq 10^{14}$.

The experiments of M. Tajmar et al [E1, E3] discussed in [K76] provide a support for this picture. The value of gravimagnetic field needed to explain the findings is 28 orders of magnitude higher than theoretical value if one extrapolates the model of Meissner effect to gravimagnetic context. The amazing finding is that if one replaces Planck constant in the formula of gravimagnetic field with h_{gr} associated with Earth-Cooper pair system and assumes that the velocity parameter v_0 appearing in it corresponds to the Earth's rotation velocity around its axis, one obtains correct order of magnitude for the effect requiring $r \simeq 3.6 \times 10^{14}$.

The most important implications are in quantum biology and Penrose's vision about importance of quantum gravitation in biology might be correct.

i. This result allows by Equivalence Principle the identification $h_{gr} = h_{eff}$ at elementary particle level at least so that the two views about hierarchy of Planck constants would be equivalent. If the identification holds true for larger units it requires that space-time sheet identifiable as quantum correlates for physical systems are macroscopically quantum coherent and gravitation causes this. If the values of Planck constant are really additive, the number of parallel space-time sheets corresponding to non-determinism evolution for the flux tube connecting systems with masses M and m is proportional to the masses M and m using Planck mass as unit. Information theoretic interpretation is suggestive since hierarchy of Planck constants is assumed to relate to negentropic entanglement very closely in turn providing physical correlate for the notions of rule and concept.

ii. That gravity would be fundamental for macroscopic quantum coherence would not be surprising since by EP all particles experience same acceleration in constant gravitational field, which therefore has tendency to create coherence unlike other basic interactions. This in principle allows to consider hierarchy in which the integers $h_{gr,i}$ are additive but give rise to the same universal dark Compton length.

iii. The model for quantum biology relying on the notions of magnetic body and dark matter as hierarchy of phases with $h_{eff} = nh$, and biophotons [K72, K71] identified as decay produces of dark photons. The assumption $h_{gr} \propto m$ becomes highly predictable since cyclotron frequencies would be independent of the mass of the ion.

    A. If dark photons with cyclotron frequencies decay to biophotons, one can conclude that biophoton spectrum reflects the spectrum of endogenous magnetic field strengths. In the model of EEG [K14] it has been indeed assumed that this kind spectrum is there: the inspiration came from music metaphors suggesting that musical scales are realized in terms of values of magnetic field strength. The new quantum physics associated with gravitation would also become key part of quantum biophysics in TGD Universe.

    B. For the proposed value of h_{gr} 1 Hz cyclotron frequency associated to DNA sequences would correspond to ordinary photon frequency $f = 3.6 \times 10^{14}$ Hz and energy 1.2 eV just at the lower limit of visible frequencies. For 10 Hz alpha band the energy would be 12 eV in UV. This plus the fact that molecular energies are in eV range suggests very simple realization of biochemical control by magnetic body. Each ion has its own cyclotron frequency but same energy for the corresponding biophoton.

C. Biophoton with a given energy would activate transitions in specific bio-molecules or atoms: ionization energies for atoms except hydrogen have lower bound about 5 eV (`http://en.wikipedia.org/wiki/Ionization_energy`). The energies of molecular bonds are in the range 2-10 eV (`http://en.wikipedia.org/wiki/Bond-dissociation_energy`). If one replaces v_0 with $2v_0$ in the estimate, DNA corresponds to .62 eV photon with energy of order metabolic energy currency and alpha band corresponds to 6 eV energy in the molecular region and also in the region of ionization energies.
   Each ion at its specific magnetic flux tubes with characteristic palette of magnetic field strengths would resonantly excite some set of biomolecules.This conforms with the earlier vision about dark photon frequencies as passwords.
   It could be also that biologically important ions take care of their ionization self. This would be achieved if the magnetic field strength associated with their flux tubes is such that dark cyclotron energy equals to ionization energy. EEG bands labelled by magnetic field strengths could reflect ionization energies for these ions.

D. The hypothesis means that the scale of energy spectrum of biophotons depends on the ratio M/v_0 of the planet and on the strength of the endogenous magnetic field, which is .2 Gauss for Earth (2/5 of the nominal value of the Earth's magnetic field). Therefore the astrophysical characteristics of planets should be tuned for molecular life. Taking v_0 to be rotational velocity one obtains for the ratio $M(planet)/v_0(planet)$ using the ratio for Earth as unit the following numbers for the planets (Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptune): $M/v_0 = (8.5, 209, 1, .214223, 1613, 6149, 9359)$. If the energy scale of biophotons is required to be the same, the scale of endogenous magnetic field should be divided by this ratio in order to obtain the same situation as in Earth. For instance, in Mars the magnetic field should be roughly 5 times stronger: in reality the magnetic field of Mars is much weaker. Just for fun one can notice that for Sun the ratio is 1.4×10^6 so that magnetic field should be by the inverse of this factor weaker.

iv. An interesting question is how large systems can behave as coherent units with $h_{gr} = GMm/v_0$. In living matter one might consider the possibility that entire organism might be this kind of system. Interestingly, for larger masses the gravitational quantum coherence would be easier. For particle with mass m $h_{gr}/h > 1$ requires larger mass to satisfy $M > M_P^2/m_e$. The first guess that life has evolved from long to shorter scales and reached elementary particle last. Planck mass is the critical mass corresponds to the mass of water blog with volume of size scale of 10^{-4} m (big neuron) is the limit.

v. The Universal gravitational Compton wave length of $GM/v_0 \simeq 864$ meters gives an idea about largest possible living matter system if Earth is the second body. Of course, also other large bodies are possible.In the case of solar system this length is 3×10^3 km. The radius of Earth is 6.37×10^3 km - roughly twice the Compton length. The radii of Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptunus are (.38,.99, .533, 1, 10.6, 8.6, 4.0, 3.9) using Earth radius as unit the value of h_{gr} is by factor 5 larger than for three inner planets so that the values are reasonably near to gravitational Compton length or twice it. Does this mean that dark matter associated with Earth and maybe also other planets is in macroscopic quantum state at some level of the hierarchy of space-time sheets? Does this mean that Mother Gaia as conscious entity might make sense. One can of course make same question in the case of Sun. The universal gravitational Compton length in Sun would be 18 per cent of the radius of Sun if v_0 is taken to be the rotational velocity at the surface of Sun. The radius of solar core, where fusion takes place, is 20-25 per cent of solar radius.

vi. There are further interesting numerical co-incidences. One can for a moment forget the standard hostility of scientist towards horoscopes and ask whether Sun and Moon could have somehow affect our life via astrocopic quantum coherence. The gravitational Compton length for particle-Moon or particle-Sun system multiplied by the natural value of magnetic field is the relevant parameter. For Sun the parameters in question are mass of Sun, and rotational velocity of Earth with respect to Sun, plus magnetic fields of Sun at flux tubes associated with solar magnetic field measured to be about 5 nT at the position of Earth and 100 times stronger than expected from dipole field behavior. This gives that the range of biophoton energies is scaled down with factor of 1/4 in good approximation so that Father Sun might affect terrestrial biology! If one uses for the rotational velocity of particle at surface of Moon as parameter v_0 (particle would be at Moon), biophoton energy scaled scaled up by factor 1.2.

The general proposal discussed above is testable. In particular, a detailed study of molecular energies with those associated with resonances of EEG could be highly rewarding and reveal the speculated spectroscopy of consciousness.

Summary

The hierarchy of Planck constants reduces to second quantization of multi-furcations in TGD framework and the hierarchy is only effective. Anyonic physics and effective charge fractionalization are consequences of second quantized multi-furcations. This framework also provides quantum version for the transition to chaos via quantum multi-furcations and living matter represents the basic application. The key element of dynamics of TGD is vacuum degeneracy of Kähler action making possible quantum criticality having the hierarchy of multi-furcations as basic aspect. The potential problems relate to the question whether the effective scaling of Planck constant involves scaling of ordinary wavelength or not. For particles confined inside linear structures such as magnetic flux tubes this seems to be the case.

There is also an intriguing connection with the vision about physics as generalized number theory. The conjecture that the preferred extremals of Kähler action consist of quaternionic or co-quaternionic regions led to a construction of them using iteration and also led to the hierarchy of multi-furcations [K69]. Therefore it seems that the dynamics of preferred extremals might indeed reduce to associativity/co-associativity condition at space-time level , to commutativity/co-commutativity condition at the level of string world sheets and partonic 2-surfaces, and to reality at the level of stringy curves (conformal invariance makes stringy curves causal determinants [K61] so that conformal dynamics represents conformal evolution) [K52].

10.3 Number theoretic braids and global view about anti-commutations of induced spinor fields

The anti-commutations of the induced spinor fields are reasonably well understood locally. The basic objects are 3-dimensional light-like 3-surfaces. These surfaces can be however seen as random light-like orbits of partonic 2-D partonic surface and the effective 2-dimensionality means that partonic 2-surfaces plus there 4-D tangent space take the role of fundamental dynamical objects. This is expressed concretely by the condition that the ends of the space-time surface and wormhole throats are extremals of Chern-Simons action. Conformal invariance would in turn make the 2-D partons 1-D objects (analogous to Euclidian strings) and braids, which can be regarded as the ends of string world sheets with Minkowskian signature, in turn would discretize these Euclidian strings. It must be however noticed that the status of Euclidian strings is uncertain.

Somehow these views should be unifiable into a more global view about the situation allowing to understand the reduction of effective dimension of the system as one goes to short scales.

i. The notions of measurement resolution and braid concept indeed provides the needed physical insights in this respect. The precise definition of the notion of braid and its number theoretic counterpart has however remained open and I have considered several alternatives. The topological character of braid indeed allows flexibility in its definition but it would be nice to have some canonical definition with a clear physical meaning.

ii. It turned out that the braid concept emerges automatically from the localization of the modes of Kähler-Dirac action to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces - with vanishing induced W fields and above weak scale also induced Z^0 fields. The boundaries of string world sheets can be identified as braids and string world sheets as 2-braids. Hence the identification of braids is unique although their topological character does not necessitate this. The attribute "number theoretic" would mean that the intersections of braids with partonic 2-surfaces corresponds to points with preferred imbedding space coordinates having values which are algebraic numbers in some extension of rational numbers. This selects preferred extremals among all extremals and they could perhaps be said to belong to the intersection of real and p-adic space-time sheets.

10.3.1 Quantization of the modified Dirac action and configuration space geometry

The quantization of the modified Dirac action involves a fusion of various number theoretical ideas. The naive approach would be based on standard canonical quantization of induced spinor fields by posing anti-commutation relations between Ψ and canonical momentum density $\partial L/\partial(\partial_t\Psi)$.

One can imagine two alternative forms of the anti-commutation relations.

i. The standard canonical anti-commutation relations for the induced the spinor fields would be given by

$$\{\overline{\Psi}\hat{\Gamma}^0(x),\Psi(y)\} = \delta^2_{x,y} \ . \tag{10.3.1}$$

The factor that $\hat{\Gamma}^0(x)$ corresponds to the canonical momentum density associated with Kähler action. The discrete variant of the anti-commutation relations applying in the case of non-stringy space-time sheets is

$$\{\overline{\Psi}\hat{\Gamma}^0(x_i),\Psi(x_j)\} = \delta_{i,j} \ . \tag{10.3.2}$$

where x_i and x_j label the points of the number theoretic braid. These anti-commutations are are inconsistent at the limit of vacuum extremal and also extremely non-linear in the imbedding space coordinates.

ii. The construction of WCW gamma matrices leads to a nonsingular form of anti-commutation relations given by

$$\{\overline{\Psi}(x)\gamma^0,\Psi(x)\} = (1+K)J\delta_{x,y} \ . \tag{10.3.3}$$

Here J denotes the Kähler magnetic flux J_m and Kähler electric flux relates to via the formula $J_e = KJ_m$, where K is symplectic invariant. What is nice that at the limit of vacuum extremals the right hand side vanishes so that spinor fields become non-dynamical. Therefore this option- actually the original one - seems to be the only reasonable choice.

For the latter option the super counterparts of local flux Hamiltonians can be written in the form

$$
\begin{aligned}
H_{A,+,n} &= H_{A,+,q,n} + H_{A,+,L,n} \ , \ H_{A,-,n} = H_{A,-,q,n} + H_{A,-,L,n} \ , \\
H_{A,+,q,n} &= \oint \overline{\Psi} J_+^A q_n d^2x \ , \\
H_{A,-,q,n} &= \oint \overline{q}_n J_-^A \Psi d^2x \ , \\
H_{A,-,L,n} &= \oint \overline{\Psi} J_+^A L_n d^2x \ , \\
H_{A,+,L,n} &= \oint \overline{L}_n J_-^A \Psi d^2x \ , \\
J_+^A &= j^{Ak}\Gamma_k \ , \ J_-^A = j^{A\overline{k}}\Gamma_{\overline{k}} \ .
\end{aligned} \tag{10.3.4}
$$

Suppose that there is a one-one correspondence between quark modes and leptonic modes is satisfied and the label n decomposes as $n = (m, i)$, where n labels a scalar function basis and i labels spinor components. This would give

$$
\begin{aligned}
q_n = q_{m,i} &= \Phi_m q_i \ , \\
L_n = L_{m,i} &= \Phi_m L_i \ , \\
\overline{q}_i \gamma^0 q_j &= \overline{L}_i \gamma^0 L_j = g_{ij} \ .
\end{aligned} \tag{10.3.5}
$$

Suppose that the inner products g_{ij} are constant. The simplest possibility is $g_{ij} = \delta_{ij}$ Under these assumptions the anti-commutators of the super-symmetric flux Hamiltonians give flux Hamiltonians.

$$
\{H_{A,+,n}, H_{A,-,n}\} = g_{ij} \oint (1+K)\overline{\Phi}_m \Phi_n H_A J d^2x \ . \tag{10.3.6}
$$

The product of scalar functions can be expressed as

$$
\overline{\Phi}_m \Phi_n = c_{mn}^{\ \ k} \Phi_k \ . \tag{10.3.7}
$$

Note that the notion of symplectic QFT led to a scalar function algebra of similar kind consisting of phase factors and there excellent reasons to consider the possibility that there is a deep connection with this approach.

One expects that the symplectic algebra is restricted to a direct sum of symplectic algebras localized to the regions where the induced Kähler form is non-vanishing implying that the algebras associated with different region form to a direct sum. Also the contributions to WCW metric are direct sums. The symplectic algebras associated with different region can be truncated to finite-dimensional spaces of symplectic algebras $S^2 \times S$ associated with the regions in question. As far as coordinatization of the reduced configuration space is considered, these symplectic sub-spaces are enough. These truncated algebras naturally correspond to the hyper-finite factor property of the Clifford algebra of WCW.

10.3.2 Expressions for WCW super-symplectic generators in finite measurement resolution

The expressions of WCW Hamiltonians and their super counterparts just discussed were based on 2-dimensional integrals. This is problematic for several reasons.

i. In p-adic context integrals do not makes sense so that this representation fails in p-adic context. Sums would be more appropriate if one wants number theoretic universality at the level of basic formulas.

ii. The use of sums would also conform with the notion of finite measurement resolution having discretization in terms of intersections of X^2 with number theoretic braids as a space-time correlate.

iii. Number theoretic duality suggests a unique realization of the discretization in the sense that only the points of partonic 2-surface X^2 whose $\delta M^4_\pm$ projections commute in hyper-octonionic sense and thus belong to the intersections of the projection $P_{M^4}(X^2)$ with radial light-like geodesics $M_\pm$ representing intersections of $M^2 \subset M^4 \subset M^8$ with $\delta M^4_\pm \times CP_2$ contribute to WCW Hamiltonians and super Hamiltonians and therefore to the WCW metric.

Clearly, finite measurement resolution seems to be an unavoidable aspect of the geometrization of WCW as one can expect on basis of the fact that WCW Clifford algebra provides representation for hyper-finite factors of type II_1 whose inclusions provide a representation for the finite measurement resolution. This means that the infinite-dimensional WCW can be represented as a finite-dimensional space in arbitrary precise approximation so that also also configuration Clifford algebra and WCW spinor fields becomes finite-dimensional.

The modification of anti-commutation relations to this case is

$$\{\overline{\Psi}(x_m)\gamma^0, \Psi(x_n)\} \quad = \quad (1+K)J\delta_{x_m,x_n} \ . \tag{10.3.8}$$

Note that the constancy of γ^0 implies a complete symmetry between the two points. The number of points must be the maximal one consistent with the Kronecker delta type anti-commutation relations so that information is not lost.

The question arises about the choice of the points x_m. This choice should ge general coordinate invariant. As already described, the localization of the modes of the Kähler-Dirac action to 2-D surfaces resolves this problem: the points x_m correspond to points of imbedding space which in preferred imbedding space coordinates have values in some algebraic extension of rationals.

10.3.3 QFT description of particle reactions at the level of braids

The overall view conforms with zero energy ontology in which hierarchy of causal diamonds (CDs) within CDs gives rise to a hierarchy of generalized Feynman diagrams and geometric description of the radiative corrections. Each sub-CD gives also rise to to zero energy states and thus particle reactions in its own time scale so that improvement of the time resolution brings in also new physics as it does also in reality.

The natural question is what happens to the braids at vertices.

i. The vision based on infinite primes led to the conclusion that the selection rules of arithmetic quantum field theory based on the conservation of the total number theoretic momentum $P = \sum n_i log(p_i)$ dictate the selection rules at the vertices. For given p_i the momentum $n_i log(p_i)$ can be shared between the outgoing lines and this allows several combinations of infinite primes in outgoing lines having interpretations in terms of singular coverings of CD and CP_2.

ii. What happens then to the braid strands? If the bosons and fermions with given p_i are shared between several outgoing particles, does this require that the braid strands replicate? Or is their number preserved if one regards each braid strand as having n_a *resp.* n_b copies at the sheets of the corresponding coverings? This is required by the conservation of number theoretic momentum if one accepts the connection between the hierarchy of Planck constants and infinite primes.

iii. The question raised already earlier is whether DNA replication could have a counterpart at the level of fundamental physics. The interpretation of the incoming lines of generalized Feynman diagram as representations of topological quantum computations and the virtual particle lines as representations of quantum communications would support this picture. The no-cloning theorem [B3] would hold true since exact copies of quantum states would not be possible by the conservation of the number theoretical momentum. One could however say that the bosonic occupation number n_i means the presence of n_i-fold copy of same piece of information so that the sharing of information by sharing the pages of the singular covering associated with n_i would be possible in the limits posed by the values of n_i. Note again that the identification $n_i = n_a$ or $n_i = n_b$ (two infinite primes characterize the quantum state) makes sense only if only one of the p-adic primes associated with the 3-surface is realized as a physical state since the identification forces the selection of the covering. The quantum model for DNA based on hierarchy of Planck constants [K57] inspires the question whether DNA replication could be actually accompanied by its proposed counterpart at the fundamental level defining the fundamental information transfer process.

iv. The localization of the quantum numbers to braid strands suggests that braid ends of a given braid continue to one particular line or more generally, are shared between several lines. This condition is quite strong since without additional quantization conditions the ends of the braids of outgoing particles do not co-incide with the ends of the incoming braid. These kind of quantization conditions would conform with the generalized Bohr orbit property of light-like 3-surfaces.

v. Without these quantization conditions one meets the challenge of calculating the anti-commutators of fermionic oscillator operators associated with non-co-inciding points of the incoming and outgoing braids. This raises the question whether one should regard the quantizations of induced spinor fields based on the L_{min} as one possible gauge only and allow the variation of L_{min} in some limits. If these quantizations are equivalent, the fermionic oscillator operators would be unitarily related. How to deduce this unitary transformation would be the non-trivial problem and it seems that the simpler picture is much more attractive.

This picture means that particle reactions occur at several levels which brings in mind a kind of universal mimicry inspired by Universe as a Universal Computer hypothesis. Particle reactions in QFT sense correspond to the reactions for the number theoretic braids inside partons. This level seems to be the simplest one to describe mathematically. At parton level particle reactions correspond to generalized Feynman diagrams obtained by gluing partonic 3-surfaces along their ends at vertices. Particle reactions are realized also at the level of 4-D space-time surfaces. One might hope that this multiple realization could code the dynamics already at the simple level of single partonic 3-surface.

10.3.4 How do generalized braid diagrams relate to the perturbation theory?

The association of generalized braid diagrams characterized by infinite primes to the incoming and outgoing partonic legs and internal lines of the generalized Feynman diagrams forces to ask whether the generalized braid diagrams could give rise to a counterpart of perturbation theoretical formalism via the functional integral over configuration space degrees of freedom.

The basic question is how the functional integral over configuration space degrees of freedom relates to the generalized braid diagrams.

i. If one believes in perturbation theoretic approach, the basic conjecture motivated also number theoretically is that radiative corrections in this sense sum up to zero for critical values of Kähler coupling strength and Kähler function codes radiative corrections to classical physics via the dependence of the scale of M^4 metric on Planck constant. Cancelation could occur only for critical values of Kähler coupling strength α_K: for general values of α_K the cancellation would require separate vanishing of each term in the sum and does not occur.

In perturbative approach the expression of Kähler function as Chern-Simons action could be used and propagator would correspond to the inverse of the 1-1 part of the second variation of the Chern-Simons action with respect to complex WCW coordinates evaluated allowing only the extrema of Chern-Simons action for the ends of space-time surface and for wormhole throats. One would have perturbation theory for a sum over maxima of Kähler function. From the expression of the Kähler function as Dirac determinant the maxima would correspond to the local minima of $L_p = \sqrt{p}L_{min}$ for a given infinite prime. The connection between Chern-Simons representation and Dirac determinant representation of Kähler function would be obviously highly desirable.

ii. The possibility to define WCW functional integral in terms of harmonic analysis for infinite-dimensional spaces leads to a non-perturbative approach to functional integration allowing also a generalization the p-adic context [K52] . In this approach there is no need to make additional assumptions.

For both cases the assignment of the collection of braids characterized by pairs of infinite primes allows to organize the generalized Feynman diagrams into a sum of generalized Feynman diagrams and for each diagram type the exponent of Kähler function - if given by the Dirac determinant- would be simply the product $\prod_i L_{p_i}^{-1}$, $L_p = \sqrt{p}L_{min}$. One should perform a sum over different infinite primes in the internal lines subject to the conservation of the total number theoretic momenta. The conservation of the incoming number theoretic momentum would allow only a finite number of configurations for the intermediate lines. For the approach based on harmonic analysis the expression of the Kähler function in terms of the Dirac determinant would be optimal since it is manifestly algebraic function.

Both approaches involve a perturbative summation in the sense of introducing sub-CDs with time scales coming as 2^{-n} powers of the time scale of CD defining the infrared cutoff.

i. The addition of zero energy insertions corresponding to sub-CDs as radiative corrections allows to improve measurement resolution. Hence a connection with QFT type Feyman diagram expansion would be obtained and Connes tensor product would have a practical computational realization.

ii. The time scale resolution defined by the temporal distance between the tips of the causal diamond defined by the future and past light-cones applies to the addition of zero energy sub-states and one obtains a direct connection with p-adic length scale evolution of coupling constants since the time scales in question naturally come as negative powers of two. More precisely, p-adic primes near power of two are very natural since the coupling constant evolution comes in powers of two of fundamental 2-adic length scale.

10.3.5 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

The condition $T_n = 2^n T_0$ would assign to the hierarchy of CDs as hierarchy of time scales coming as octaves. A weaker condition would be $T_p = pT_0$, p prime, and would assign all secondary p-adic time scales to the size scale hierarchy of CDs.

One can wonder how this picture relates to the earlier hypothesis that p-adic length coupling constant evolution. Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks like an attractive idea but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

i. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

ii. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1.\times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

iii. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3. For $T_p = pT_0$ the above argument is not enough for p-adic length scale hypothesis and p-adic length scale hypothesis might be seen as an outcome of a process analogous to natural selection. Resonance like effect favoring octaves of a fundamental frequency might be in question. In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of WCW.

10.4 Twistor revolution and TGD

Lubos Motl wrote a nice summary about the talk of Nima Arkani Hamed about twistor revolution in Strings 2012 and gave also a link to the talk [B7]. It seems that Nima and collaborators are ending to a picture about scattering amplitudes which strongly resembles that provided by generalized Feynman diagrammatics in TGD framework

TGD framework is much more general than $\mathcal{N} = 4$ SYM and is to it same as general relativity for special relativity whereas the latter is completely explicit. Of course, I cannot hope that TGD view could be taken seriously - at least publicly. One might hope that these approaches could be combined some day: both have a lot to give for each other. Below I compare these approaches.

The recent approach below emerges from the study of preferred extremals of Kähler and solutions of the modified Dirac equations so that it begins directly from basic TGD whereas the approaches hitherto have been based on general arguments and the precise role of right-handed neutrino has remained enigmatic. Chapters "Construction of quantum TGD: Symmetries" [K13] and "The recent vision about preferred extremals and solutions of the modified Dirac equation" [K69] contain section explaining how super-conformal and Yangian algebras crucial for the Grassmannian approach emerge from the basic TGD.

10.4.1 The origin of twistor diagrammatics

In TGD framework zero energy ontology forces to replace the idea about continuous unitary evolution in Minkowski space with something more general assignable to causal diamonds (CDs), and S-matrix is replaced with a square root of density matrix equal to a hermitian l square root of density matrix multiplied by unitary S-matrix. Also in twistor approach unitarity has ceased to be a star actor. In p-Adic context continuous unitary time evolution fails to make sense also mathematically.

Twistor diagrammatics involves only massless on mass shell particles on both external and internal lines. Zero energy ontology (ZEO) requires same in TGD: wormhole lines carry parallelly moving massless fermions and anti-fermions. The mass shell conditions at vertices are enormously powerful and imply UV finiteness. Also IR finiteness follows if external particles are massive.

What one means with mass is however a delicate matter. What does one mean with mass? I have pondered 35 years this question and the recent view is inspired by p-adic mass calculations and ZEO, and states that observed mass is in a well-defined sense expectation value of longitudinal mass squared for all possible choices of $M^2 \subset M^4$ characterizing the choices of quantization axis for energy and spin at the level of "world of classical worlds" (WCW) assignable with given causal diamond CD.

The choice of quantization axis thus becomes part of the geometry of WCW. All wormhole throats are massless but develop non-vanishing longitudinal mass squared. Gauge bosons correspond to wormhole contacts and thus consist of pairs of massless wormhole throats. Gauge bosons could develop 4-D mass squared but also remain massless in 4-D sense if the throats have parallel massless momenta. Longitudinal mass squared is however non-vanishing and p-adic thermodynamics predicts it.

10.4.2 The emergence of 2-D sub-dynamics at space-time level

Nima et al introduce ordering of the vertices in 4-D case. Ordering and related braiding are however essentially 2-D notions. Somehow 2-D theory must be a part of the 4-D theory also at space-time level, and I understood that understanding this is the challenge of the twistor approach at this moment.

The twistor amplitude can be represented as sum over the permutations of n external gluons and all diagrams corresponding to the same permutation are equivalent. Permutations are more like braidings since they carry information about how the permutation proceeded as a homotopy. Yang-Baxter equation emerges and states associativity of the braid group. The allowed braidings are minimal braidings in the sense that the repetitions of permutations of two adjacent vertices are not considered to be separate. Minimal braidings reduce to ordinary permutations. Nima also talks about affine braidings which I interpret as analogs of Kac-Moody algebras meaning that one uses projective representations which for Kac-Moody algebra mean non-trivial central extension. Perhaps the condition is that the square of a permutation permuting only two vertices which each other gives only a non-trivial phase factor. Lubos suggests an alternative interpretation which would select only special permutations and cannot be therefore correct.

There are rules of identifying the permutation associated with a given diagram involving only basic 3-gluon vertex with white circle and its conjugate. Lubos explains this "Mickey Mouse in maze" rule in his posting in detail: to determine the image $p(n)$ of vertex n in the permutation put a mouse in the maze defined by the diagram and let it run around obeying single rule: if the vertex is black turn to the right and if the vertex is white turn to the left. The mouse cannot remain in a loop: if it would do so, the rule would force it to run back to n after single full loop and one would have a fixed point: $p(n) = n$. The reduction in the number of diagrams is enormous: the infinity of different diagrams reduces to $n!$ diagrams!

What happens in TGD framework?

i. In TGD framework string world sheets and partonic 2-surfaces (or either or these if they are dual notions as conjectured) at space-time surface would define the sought for 2-D theory, and one obtains indeed perturbative expansion with fermionic propagator defined by the inverse of the modified Dirac operator and bosonic propagator defined by the correlation function for small deformations of the string world sheet. The vertices of twistor diagrams emerge as braid ends defining the intersections of string world sheets and partonic 2-surfaces.

   String model like description becomes part of TGD and the role of string world sheets in X^4 is highly analogous to that of string world sheets connecting branes in $AdS^5 \times S^5$ of $\mathcal{N} = 4$ SYM. In TGD framework 10-D $AdS^5 \times S^5$ is replaced with 4-D space-time surface in $M^4 \times CP_2$. The meaning of the analog of AdS^5 duality in TGD framework should be understood. In particular, it could it be that the descriptions involving string world sheets on one hand and partonic 2-surfaces - or

3-D orbits of wormhole throats defining the generalized Feynman diagram- on the other hand are dual to each other. I have conjectured something like this earlier but it takes some time for this kind of issues to find their natural answer.

ii. As described in the article, string world sheets and partonic 2-surfaces emerge directly from the construction of the solutions of the modified Dirac equation by requiring conservation of em charge. This result has been conjectured already earlier but using other less direct arguments. 2-D "string world sheets" as submanifolds of the space-time surface make the ordering possible, and guarantee the finiteness of the perturbation theory involving n-point functions of a conformal QFT for fermions at wormhole throats and n-point functions for the deformations of the space-time surface. Conformal invariance should dictate these n-point functions to a high degree. In TGD framework the fundamental 3-vertex corresponds to joining of light-like orbits of three wormhole contacts along their 2-D ends (partonic 2-surfaces).

10.4.3 The emergence of Yangian symmetry

Yangian symmetry associated with the conformal transformations of M^4 is a key symmetry of Grassmannian approach. Is it possible to derive it in TGD framework?

i. TGD indeed leads to a concrete representation of Yangian algebra as generalization of color and electroweak gauge Kac-Moody algebra using general formula discussed in Witten's article about Yangian algebras (see the article).

ii. Article discusses also a conjecture about 2-D Hodge duality of quantized YM gauge potentials assignable to string world sheets with Kac-Moody currents. Quantum gauge potentials are defined only where they are needed - at string world sheets rather than entire 4-D space-time.

iii. Conformal scalings of the effective metric defined by the anti-commutators of the modified gamma matrices emerge as realization of quantum criticality. They are induced by critical deformations (second variations not changing Kähler action) of the space-time surface. This algebra can be generalized to Yangian using the formulas in Witten's article (see the article).

iv. Critical deformations induce also electroweak gauge transformations and even more general symmetries for which infinitesimal generators are products of U(n) generators permuting n modes of the modified Dirac operator and infinitesimal generators of local electro-weak gauge transformations. These symmetries would relate in a natural manner to finite measurement resolution realized in terms of inclusions of hyperfinite factors with included algebra taking the role of gauge group transforming to each other states not distinguishable from each other.

v. How to end up with Grassmannian picture in TGD framework? This has inspired some speculations in the past. From Nima's lecture one however learns that Grassmannian picture emerges as a convenient parameterization. One starts from the basic 3-gluon vertex or its conjugate expressed in terms of twistors. Momentum conservation implies that with the three twistors λ_i or their conjugates are proportional to each other (depending on which is the case one assigns white or black dot with the vertex). This constraint can be expressed as a delta function constraint by introducing additional integration variables and these integration variables lead to the emergence of the Grassmannian $G_{n,k}$ where n is the number of gluons, and k the number of positive helicity gluons.
Since only momentum conservation is involved, and since twistorial description works because only massless on mass shell virtual particles are involved, one is bound to end up with the Grassmannian description also in TGD.

10.4.4 The analog of AdS^5 duality in TGD framework

The generalization of AdS^5 duality of $\mathcal{N} = 4$ SYMs to TGD framework is highly suggestive and states that string world sheets and partonic 2-surfaces play a dual role in the construction of M-matrices. Some terminology first.

i. Let us agree that string world sheets and partonic 2-surfaces refer to 2-surfaces in the slicing of space-time region defined by Hermitian structure or Hamilton-Jacobi structure.
ii. Let us also agree that *singular* string world sheets and partonic 2-surfaces are surfaces at which the *effective* metric defined by the anti-commutators of the modified gamma matrices degenerates to effectively 2-D one.
iii. Braid strands at wormhole throats in turn would be loci at which the *induced* metric of the string world sheet transforms from Euclidian to Minkowskian as the signature of induced metric changes from Euclidian to Minkowskian.

AdS^5 duality suggest that string world sheets are in the same role as string world sheets of 10-D space connecting branes in AdS^5 duality for $\mathcal{N}=4$ SYM. What is important is that there should exist a duality meaning two manners to calculate the amplitudes. What the duality could mean now?

i. Also in TGD framework the first manner would be string model like description using string world sheets. The second one would be a generalization of conformal QFT at light-like 3-surfaces (allowing generalized conformal symmetry) defining the lines of generalized Feynman diagram. The correlation functions to be calculated would have points at the intersections of partonic 2-surfaces and string world sheets and would represent braid ends.
ii. General Coordinate Invariance (GCI) implies that physics should be codable by 3-surfaces. Light-like 3-surfaces define 3-surfaces of this kind and same applies to space-like 3-surfaces. There are also preferred 3-surfaces of this kind. The orbits of 2-D wormhole throats at which 4-metric degenerates to 3-dimensional one define preferred light-like 3-surfaces. Also the space-like 3-surfaces at the ends of space-time surface at light-like boundaries of causal diamonds (CDs) define preferred space-like 3-surfaces. Both light-like and space-like 3-surfaces should code for the same physics and therefore their intersections defining partonic 2-surfaces plus the 4-D tangent space data at them should be enough to code for physics. This is strong form of GCI implying effective 2-dimensionality. As a special case one obtains singular string world sheets at which the effective metric reduces to 2-dimensional and singular partonic 2-surfaces defining the wormhole throats. For these 2-surfaces situation could be especially simple mathematically.
iii. The guess inspired by strong GCI is that string world sheet -partonic 2-surface duality holds true. The functional integrals over the deformations of 2 kinds of 2-surfaces should give the same result so that functional integration over either kinds of 2-surfaces should be enough. Note that the members of a given pair in the slicing intersect at discrete set of points and these points define braid ends carrying fermion number. Discretization and braid picture follow automatically.
iv. Scattering amplitudes in the twistorial approach could be thus calculated by using *any* pair in the slicing - or only either member of the pair if the analog of AdS^5 duality holds true as argued. The possibility to choose any pair in the slicing means general coordinate invariance as a symmetry of the Kähler metric of WCW and of the entire theory suggested already early: Kähler functions for difference choices in the slicing would differ by a real part of holomorphic function and give rise to same Kähler metric of "world of classical worlds" (WCW). For a general pair one obtains functional integral over deformations of space-time surface inducing deformations of 2-surfaces with only other kind 2-surface contributing to amplitude. This means the analog of stringy QFT: Minkowskian or Euclidian string theory depending on choice.
v. For singular string world sheets and partonic 2-surfaces an enormous simplification results. The propagators for fermions and correlation functions for deformations reduce to 1-D instead of being 2-D: the propagation takes place only along the light-like lines at which the string world sheets with Euclidian signature (inside CP_2 like regions) change to those with Minkowskian signature of induced metric.

The local reduction of space-time dimension would be very real for particles moving along sub-manifolds at which higher dimensional space-time has reduced metric dimension: they cannot get out from lower-D sub-manifold. This is like ending down to 1-D black hole interior and one would obtain the analog of ordinary Feynman diagrammatics. This kind of Feynman diagrammatics involving only braid strands is what I have indeed ended up earlier so that it seems that I can trust good intuition combined with a sloppy mathematics sometimes works;-).

These singular lines represent orbits of point like particles carrying fermion number at the orbits of wormhole throats. Furthermore, in this representation the expansions coming from string world sheets and partonic 2-surfaces are identical automatically. This follows from the fact that only the light-like lines connecting points common to singular string world sheets and singular partonic 2-surfaces appear as propagator lines!

vi. The TGD analog of AdS^5 duality of $\mathcal{N} = 4$ SUSYs would be trivially true as an identity in this special case, and the good guess is that it is true also generally. One could indeed use integral over either string world sheets or partonic 2-sheets to deduce the amplitudes.

What is important to notice that singularities of Feynman diagrams crucial for the Grassmannian approach of Nima and others would correspond at space-time level 2-D singularities of the effective metric defined by the modified gamma matrices defined as contractions of canonical momentum currents for Kähler action with ordinary gamma matrices of the imbedding space and therefore directly reflecting classical dynamics.

10.4.5 Problems of the twistor approach from TGD point of view

Twistor approach has also its problems and here TGD suggests how to proceed. Signature problem is the first problem.

i. Twistor diagrammatics works in a strict mathematical sense only for $M^{2,2}$ with metric signature (1,1,-1,-1) rather than M^4 with metric signature (1,-1,-1,-1). Metric signature is wrong in the physical case. This is a real problem which must be solved eventually.

ii. Effective metric defined by anti-commutators of the modified gamma matrices (to be distinguished from the induced gamma matrices) could solve that problem since it would have the correct signature in TGD framework (see the article). String world sheets and partonic 2-surfaces would correspond to the 2-D singularities of this effective metric at which the even-even signature (1,1,1,1) changes to even-even signature (1,1,-1,-1). Space-time at string world sheet would become locally 2-D with respect to effective metric just as space-time becomes locally 3-D with respect to the induced metric at the light-like orbits of wormhole throats. String world sheets become also locally 1-D at light-like curves at which Euclidian signature of world sheet in induced metric transforms to Minkowskian.

iii. Twistor amplitudes are indeed singularities and string world sheets implied in TGD framework by conservation of em charge would represent these singularities at space-time level. At the end of the talk Nima conjectured about lower-dimensional manifolds of space-time as representation of space-time singularities. Note that string world sheets and partonic 2-surfaces have been part of TGD for years. TGD is of course to $\mathcal{N} = 4$ SYM what general relativity is for the special relativity. Space-time surface is dynamical and possesses induced and effective metrics rather than being flat.

Second limitation is that twistor diagrammatics works only for planar diagrams. This is a problem which must be also fixed sooner or later.

i. This perhaps dangerous and blasphemous statement that I will regret it some day but I will make it;-). Nima and others have not yet discovered that $M^2 \subset M^4$ must be there but will discover it when they begin to generalize the results to non-planar diagrams and realize that Feynman diagrams are analogous to knot diagrams in 2-D plane (with crossings allowed) and that this 2-D plane must correspond to $M^2 \subset M^4$. The different choices of causal diamond CD correspond to different choices of M^2 representing choice of quantization axes 4-momentum and spin. The integral over these choices guarantees Lorentz invariance. Gauge conditions are modified: longitudinal M^2 projection of massless four-momentum is orthogonal to polarization so that three polarizations are possible: states are massive in longitudinal sense.

ii. In TGD framework one replaces the lines of Feynman diagrams with the light-like 3-surfaces defining orbits of wormhole throats. These lines carry many fermion states defining braid strands at light-like 3-surfaces. There is internal braiding associated with these braid strands. String world sheets connect fermions at different wormhole throats with space-like braid strands. The M^2 projections of generalized Feynman diagrams with 4-D "lines" replaced with genuine lines define the ordinary Feynman diagram as the analog of braid diagram. The conjecture is that one can reduce non-planar diagrams to planar diagrams using a procedure analogous to the construction of knot invariants by un-knotting the knot in Alexandrian manner by allowing it to be cut temporarily.

iii. The permutations of string vertices emerge naturally as one constructs diagrams by adding to the interior of polygon sub-polygons connected to the external vertices. This corresponds to the addition of internal partonic two-surfaces. There are very many equivalent diagrams of this kind. Only permutations matter and the permutation associated with a given diagram of this kind can be deduced by the Mickey-Mouse rule described explicitly by Lubos. A connection with planar operads is highly suggestive and also conjecture already earlier in TGD framework.

10.4.6 Could $\mathcal{N}=2$ or $\mathcal{N}=4$ SYM be a part of TGD after all?

Whether right-handed neutrinos generate a supersymmetry in TGD has been a long standing open question. $\mathcal{N}=1$ SUSY is certainly excluded by fermion number conservation but already $\mathcal{N}=2$ defining a "complexification" of $\mathcal{N}=1$ SUSY is possible and could generate right-handed neutrino and its antiparticle. These states should however possess a non-vanishing light-like momentum since the fully covariantly constant right-handed neutrino generates zero norm states. So called massless extremals (MEs) allow massless solutions of the modified Dirac equation for right-handed neutrino in the interior of space-time surface, and this seems to be case quite generally in Minkowskian signature for preferred extremals. This suggests that particle represented as magnetic flux tube structure with two wormhole contacts sliced between two MEs could serve as a starting point in attempts to understand the role of right handed neutrinos and how $\mathcal{N}=2$ or $\mathcal{N}=4$ SYM emerges at the level of space-time geometry. The following arguments inspired by the article of Nima Arkani-Hamed et al [B13] about twistorial scattering amplitudes suggest a more detailed physical interpretation of the possible SUSY associated with the right-handed neutrinos.

The fact that right handed neutrinos have only gravitational interaction suggests a radical re-interpretation of SUSY: no SUSY breaking is needed since it is very difficult to distinguish between mass degenerate spartners of ordinary particles. In order to distinguish between different spartners one must be able to compare the gravitomagnetic energies of spartners in slowly varying external gravimagnetic field: this effect is extremely small.

Scattering amplitudes and the positive Grassmannian

The work of Nima Arkani-Hamed and others represents something which makes me very optimistic and I would be happy if I could understand the horrible technicalities of their work. The article Scattering Amplitudes and the Positive Grassmannian by Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, and Trnka [B13] summarizes the recent situation in a form, which should be accessible to ordinary physicist. Lubos has already discussed the article. The following considerations do not relate much to the main message of the article (positive Grassmannians) but more to the question how this approach could be applied in TGD framework.

1. All scattering amplitudes have on shell amplitudes for massless particles as building bricks

The key idea is that all planar amplitudes can be constructed from on shell amplitudes: all virtual particles are actually real. In zero energy ontology I ended up with the representation of TGD analogs of Feynman diagrams using only mass shell massless states with both positive and negative energies. The enormous number of kinematic constraints eliminates UV and IR divergences and also the description of massive particles as bound states of massless ones becomes possible.

In TGD framework quantum classical correspondence requires a space-time correlate for the on mass shell property and it indeed exists. The mathematically ill-defined path integral over all 4-surfaces is replaced with a superposition of preferred extremals of Kähler action analogous to Bohr orbits, and one has only a functional integral over the 3-D ends at the light-like boundaries of causal diamond (Euclidian/Minkowskian space-time regions give real/imaginary Chern-Simons exponent to the vacuum functional). This would be obviously the deeper principle behind on mass shell representation of scattering amplitudes that Nima and others are certainly trying to identify. This principle in turn reduces to general coordinate invariance at the level of the world of classical worlds.

Quantum classical correspondence and quantum ergodicity would imply even stronger condition: the quantal correlation functions should be identical with classical correlation functions for any preferred extremal in the superposition: all preferred extremals in the superposition would be statistically equivalent [K69]. 4-D spin glass degeneracy of Kähler action however suggests that this is is probably too strong a condition applying only to building bricks of the superposition.

Minimal surface property is the geometric counterpart for masslessness and the preferred extremals are also minimal surfaces: this property reduces to the generalization of complex structure at space-time surfaces, which I call Hamilton-Jacobi structure for the Minkowskian signature of the induced metric. Einstein Maxwell equations with cosmological term are also satisfied.

2. Massless extremals and twistor approach

The decomposition $M^4 = M^2 \times E^2$ is fundamental in the formulation of quantum TGD, in the number theoretical vision about TGD, in the construction of preferred extremals, and for the vision about generalized Feynman diagrams. It is also fundamental in the decomposition of the degrees of string to longitudinal and transversal ones. An additional item to the list is that also the states appearing in thermodynamical ensemble in p-adic thermodynamics correspond to four-momenta in M^2 fixed by the direction of the Lorentz boost. In twistor approach to TGD the possibility to decompose also internal lines to massless states at parallel space-time sheets is crucial.

Can one find a concrete identification for $M^2 \times E^2$ decomposition at the level of preferred extremals? Could these preferred extremals be interpreted as the internal lines of generalized Feynman diagrams carrying massless momenta? Could one identify the mass of particle predicted by p-adic thermodynamics with the sum of massless classical momenta assignable to two preferred extremals of this kind connected by wormhole contacts defining the elementary particle?

Candidates for this kind of preferred extremals indeed exist. Local $M^2 \times E^2$ decomposition and light-like longitudinal massless momentum assignable to M^2 characterizes "massless extremals" (MEs, "topological light rays"). The simplest MEs correspond to single space-time sheet carrying a conserved light-like M^2 momentum. For several MEs connected by wormhole contacts the longitudinal massless momenta are not conserved anymore but their sum defines a time-like conserved four-momentum: one has a bound states of massless MEs. The stable wormhole contacts binding MEs together possess

Kähler magnetic charge and serve as building bricks of elementary particles. Particles are necessary closed magnetic flux tubes having two wormhole contacts at their ends and connecting the two MEs.

The sum of the classical massless momenta assignable to the pair of MEs is conserved even when they exchange momentum. Quantum classical correspondence requires that the conserved classical rest energy of the particle equals to the prediction of p-adic mass calculations. The massless momenta assignable to MEs would naturally correspond to the massless momenta propagating along the internal lines of generalized Feynman diagrams assumed in zero energy ontology. Masslessness of virtual particles makes also possible twistor approach. This supports the view that MEs are fundamental for the twistor approach in TGD framework.

3. Scattering amplitudes as representations for braids whose threads can fuse at 3-vertices

Just a little comment about the content of the article. The main message of the article is that non-equivalent contributions to a given scattering amplitude in $\mathcal{N}=4$ SYM represent elements of the group of permutations of external lines - or to be more precise - decorated permutations which replace permutation group S_n with $n!$ elements with its decorated version containing $2^n n!$ elements. Besides 3-vertex the basic dynamical process is permutation having the exchange of neighboring lines as a generating permutation completely analogous to fundamental braiding. BFCW bridge has interpretation as a representations for the basic braiding operation.

This supports the TGD inspired proposal (TGD as almost topological QFT) that generalized Feynman diagrams are in some sense also knot or braid diagrams allowing besides braiding operation also two 3-vertices [K23]. The first 3-vertex generalizes the standard stringy 3-vertex but with totally different interpretation having nothing to do with particle decay: rather particle travels along two paths simultaneously after $1 \rightarrow 2$ decay. Second 3-vertex generalizes the 3-vertex of ordinary Feynman diagram (three 4-D lines of generalized Feynman diagram identified as Euclidian space-time regions meet at this vertex). The main idea is that in TGD framework knotting and braiding emerges at two levels.

i. At the level of space-time surface string world sheets at which the induced spinor fields (except right-handed neutrino [K69]) are localized due to the conservation of electric charge can form 2-knots and can intersect at discrete points in the generic case. The boundaries of strings world sheets at light-like wormhole throat orbits and at space-like 3-surfaces defining the ends of the space-time at light-like boundaries of causal diamonds can form ordinary 1-knots, and get linked and braided. Elementary particles themselves correspond to closed loops at the ends of space-time surface and can also get knotted (possible effects are discussed in [K23]).
ii. One can assign to the lines of generalized Feynman diagrams lines in M^2 characterizing given causal diamond. Therefore the 2-D representation of Feynman diagrams has concrete physical interpretation in TGD. These lines can intersect and what suggests itself is a description of non-planar diagrams (having this kind of intersections) in terms of an algebraic knot theory. A natural guess is that it is this knot theoretic operation which allows to describe also non-planar diagrams by reducing them to planar ones as one does when one constructs knot invariant by reducing the knot to a trivial one. Scattering amplitudes would be basically knot invariants.

"Almost topological" has also a meaning usually not assigned with it. Thurston's geometrization conjecture stating that geometric invariants of canonical representation of manifold as Riemann geometry, defined topological invariants, could generalize somehow. For instance, the geometric invariants of preferred extremals could be seen as topological or more refined invariants (symplectic, conformal in the sense of 4-D generalization of conformal structure). If quantum ergodicity holds true, the statistical geometric invariants defined by the classical correlation functions of various induced classical gauge fields for preferred extremals could be regarded as this kind of invariants for sub-manifolds. What would distinguish TGD from standard topological QFT would be that the invariants in question would involve length scale and thus have a physical content in the usual sense of the word!

Could $\mathcal{N}$ =2 or $\mathcal{N}$ = 4 SUSY have something to do with TGD?

$\mathcal{N} = 4$ SYM has been the theoretical laboratory of Nima and others. $\mathcal{N} = 4$ SYM is definitely a completely exceptional theory, and one cannot avoid the question whether it could in some sense be part of fundamental physics. In TGD framework right handed neutrinos have remained a mystery: whether one should assign space-time SUSY to them or not. Could they give rise to something resembling $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SUSY with fermion number conservation?

1. Earlier results

My latest view is that *fully* covariantly constant right-handed neutrinos decouple from the dynamics completely. I will repeat first the earlier arguments which consider only fully covariantly constant right-handed neutrinos.

i. $\mathcal{N} = 1$ SUSY is certainly excluded since it would require Majorana property not possible in TGD framework since it would require superposition of left and right handed neutrinos and lead to a breaking of lepton number conservation. Could one imagine SUSY in which both MEs between which particle wormhole contacts reside have $\mathcal{N} = 2$ SUSY which combine to form an $\mathcal{N} = 4$ SUSY?

ii. Right-handed neutrinos which are covariantly constant right-handed neutrinos in both M^4 degrees of freedom cannot define a non-trivial theory as shown already earlier. They have no electroweak nor gravitational couplings and carry no momentum, only spin.
The fully covariantly constant right-handed neutrinos with two possible helicities at given ME would define representation of SUSY at the limit of vanishing light-like momentum. At this limit the creation and annihilation operators creating the states would have vanishing anti-commutator so that the oscillator operators would generate Grassmann algebra. Since creation and annihilation operators are hermitian conjugates, the states would have zero norm and the states generated by oscillator operators would be pure gauge and decouple from physics. This is the core of the earlier argument demonstrating that $\mathcal{N} = 1$ SUSY is not possible in TGD framework: LHC has given convincing experimental support for this belief.

2. Could massless right-handed neutrinos covariantly constant in CP_2 degrees of freedom define $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SUSY?

Consider next right-handed neutrinos, which are covariantly constant in CP_2 degrees of freedom but have a light-like four-momentum. In this case fermion number is conserved but this is consistent with $\mathcal{N} = 2$ SUSY at both MEs with fermion number conservation. $\mathcal{N} = 2$ SUSYs could emerge from $\mathcal{N} = 4$ SUSY when one half of SUSY generators annihilate the states, which is a basic phenomenon in supersymmetric theories.

i. At space-time level right-handed neutrinos couple to the space-time geometry - gravitation - although weak and color interactions are absent. One can say that this coupling forces them to move with light-like momentum parallel to that of ME. At the level of space-time surface right-handed neutrinos have a spectrum of excitations of four-dimensional analogs of conformal spinors at string world sheet (Hamilton-Jacobi structure).
For MEs one indeed obtains massless solutions depending on longitudinal M^2 coordinates only since the induced metric in M^2 differs from the light-like metric only by a contribution which is light-like and contracts to zero with light-like momentum in the same direction. These solutions are analogs of (say) left movers of string theory. The dependence on E^2 degrees of freedom is holomorphic. That left movers are only possible would suggest that one has only single helicity and conservation of

fermion number at given space-time sheet rather than 2 helicities and non-conserved fermion number: two real Majorana spinors combine to single complex Weyl spinor.

ii. At imbedding space level one obtains a tensor product of ordinary representations of $\mathcal{N}=2$ SUSY consisting of Weyl spinors with opposite helicities assigned with the ME. The state content is same as for a reduced $\mathcal{N}=4$ SUSY with four $\mathcal{N}=1$ Majorana spinors replaced by two complex $\mathcal{N}=2$ spinors with fermion number conservation. This gives 4 states at both space-time sheets constructed from ν_R and its antiparticle. Altogether the two MEs give 8 states, which is one half of the 16 states of $\mathcal{N}=4$ SUSY so that a degeneration of this symmetry forced by non-Majorana property is in question.

3. Is the dynamics of $\mathcal{N}=2$ or $\mathcal{N}=4$ SYM possible in right-handed neutrino sector?

Could $\mathcal{N}=2$ or $\mathcal{N}=4$ SYM be a part of quantum TGD? Could TGD be seen a fusion of a degenerate $\mathcal{N}=4$ SYM describing the right-handed neutrino sector and string theory like theory describing the contribution of string world sheets carrying other leptonic and quark spinors? Or could one imagine even something simpler?

What is interesting that the net momenta assigned to the right handed neutrinos associated with a pair of MEs would correspond to the momenta assignable to the particles and obtained by p-adic mass calculations. It would seem that right-handed neutrinos provide a representation of the momenta of the elementary particles represented by wormhole contact structures. Does this mimicry generalize to a full duality so that all quantum numbers and even microscopic dynamics of defined by generalized Feynman diagrams (Euclidian space-time regions) would be represented by right-handed neutrinos and MEs? Could a generalization of $\mathcal{N}=4$ SYM with non-trivial gauge group with proper choices of the ground states helicities allow to represent the entire microscopic dynamics?

Irrespective of the answer to this question one can compare the TGD based view about supersymmetric dynamics with what I have understood about $\mathcal{N}=4$ SYM.

i. In the scattering of MEs induced by the dynamics of Kähler action the right-handed neutrinos play a passive role. Modified Dirac equation forces them to adopt the same direction of four-momentum as the MEs so that the scattering reduces to the geometric scattering for MEs as one indeed expects on basic of quantum classical correspondence. In ν_R sector the basic scattering vertex involves four MEs and could be a re-sharing of the right-handed neutrino content of the incoming two MEs between outgoing two MEs respecting fermion number conservation. Therefore $\mathcal{N}=4$ SYM with fermion number conservation would represent the scattering of MEs at quantum level.

ii. $\mathcal{N}=4$ SUSY would suggest that also in the degenerate case one obtains the full scattering amplitude as a sum of permutations of external particles followed by projections to the directions of light-like momenta and that BCFW bridge represents the analog of fundamental braiding operation. The decoration of permutations means that each external line is effectively doubled. Could the scattering of MEs can be interpreted in terms of these decorated permutations? Could the doubling of permutations by decoration relate to the occurrence of pairs of MEs?

One can also revert these questions. Could one construct massive states in $\mathcal{N}=4$ SYM using pairs of momenta associated with particle with integer label k and its decorated copy with label $k+n$? Massive external particles obtained in this manner as bound states of massless ones could solve the IR divergence problem of $\mathcal{N}=4$ SYM.

iii. The description of amplitudes in terms of leading singularities means picking up of the singular contribution by putting the fermionic propagators on mass shell. In the recent case it would give the inverse of massless Dirac propagator acting on the spinor at the end of the internal line annihilating it if it is a solution of Dirac equation.

The only way out is a kind of cohomology theory in which solutions of Dirac equation represent exact forms. Dirac operator defines the exterior derivative d and virtual lines correspond to non-physical helicities with $d\Psi \neq 0$. Virtual fermions would be on mass-shell fermions with non-physical polarization satisfying $d^2\Psi = 0$. External particles would be those with physical polarization satisfying $d\Psi = 0$, and one can say that the Feynman diagrams containing physical helicities split into products of Feynman diagrams containing only non-physical helicities in internal lines.

iv. The fermionic states at wormhole contacts should define the ground states of SUSY representation with helicity +1/2 and -1/2 rather than spin 1 or -1 as in standard realization of $\mathcal{N} = 4$ SYM used in the article. This would modify the theory but the twistorial and Grassmannian description would remain more or less as such since it depends on light-likeness and momentum conservation only.

4. 3-vertices for sparticles are replaced with 4-vertices for MEs

In $\mathcal{N} = 4$ SYM the basic vertex is on mass-shell 3-vertex which requires that for real light-like momenta all 3 states are parallel. One must allow complex momenta in order to satisfy energy conservation and light-likeness conditions. This is strange from the point of view of physics although number theoretically oriented person might argue that the extensions of rationals involving also imaginary unit are rather natural.

The complex momenta can be expressed in terms of two light-like momenta in 3-vertex with one real momentum. For instance, the three light-like momenta can be taken to be p, k, and $p - ka$ with $k = ap_R$. Here p (incoming momentum) and p_R are real light-like momenta satisfying $p \cdot p_R = 0$ but with opposite sign of energy, and a is complex number. What is remarkable that also the negative sign of energy is necessary also now.

Should one allow complex light-like momenta in TGD framework? One can imagine two options.

i. Option I: no complex momenta. In zero energy ontology the situation is different due to the presence of a pair of MEs meaning replaced of 3-vertices with 4-vertices or 6-vertices, the allowance of negative energies in internal lines, and the fact that scattering is of sparticles is induced by that of MEs. In the simplest vertex a massive external particle with non-parallel MEs carrying non-parallel light-like momenta can decay to a pair of MEs with light-like momenta. This can be interpreted as 4-ME-vertex rather than 3-vertex (say) BFF so that complex momenta are not needed. For an incoming boson identified as wormhole contact the vertex can be seen as BFF vertex.

   To obtain space-like momentum exchanges one must allow negative sign of energy and one has strong conditions coming from momentum conservation and light-likeness which allow non-trivial solutions (real momenta in the vertex are not parallel) since basically the vertices are 4-vertices. This reduces dramatically the number of graphs. Note that one can also consider vertices in which three pairs of MEs join along their ends so that 6 MEs (analog of 3-boson vertex) would be involved.

ii. Option II: complex momenta are allowed. Proceeding just formally, the $\sqrt{g_4}$ factor in Kähler action density is imaginary in Minkowskian and real in Euclidian regions. It is now clear that the formal approach is correct: Euclidian regions give rise to Kähler function and Minkowskian regions to the analog of Morse function. TGD as almost topological QFT inspires the conjecture about the reduction of Kähler action to boundary terms proportional to Chern-Simons term. This is guaranteed if the condition $j^{\mu}_K A_{\mu} = 0$ holds true: for the known extremals this is the case since Kähler current j_K is light-like or vanishing for them. This would seem that Minkowskian and Euclidian regions provide dual descriptions of physics. If so, it would not be surprising if the real and complex parts of the four-momentum were parallel and in constant proportion to each other.

   This argument suggests that also the conserved quantities implied by the Noether theorem have the same structure so that charges would receive an imaginary contribution from Minkowskian regions and a real contribution from Euclidian regions (or vice versa). Four-momentum would be complex number of form $P = P_M + iP_E$. Generalized light-likeness condition would give $P_M^2 = P_E^2$ and $P_M \cdot P_E = 0$. Complexified momentum would have 6 free components. A stronger condition would be

$P_M^2 = 0 = P_E^2$ so that one would have two light-like momenta "orthogonal" to each other. For both relative signs energy P_M and P_E would be actually parallel: parameterization would be in terms of light-like momentum and scaling factor. This would suggest that complex momenta do not bring in anything new and Option II reduces effectively to Option I. If one wants a complete analogy with the usual twistor approach then $P_M^2 = P_E^2 \neq 0$ must be allowed.

5. Is SUSY breaking possible or needed?

It is difficult to imagine the breaking of the proposed kind of SUSY in TGD framework, and the first guess is that all these 4 super-partners of particle have identical masses. p-Adic thermodynamics does not distinguish between these states and the only possibility is that the p-adic primes differ for the spartners. But is the breaking of SUSY really necessary? Can one really distinguish between the 8 different states of a given elementary particle using the recent day experimental methods?

i. In electroweak and color interactions the spartners behave in an identical manner classically. The coupling of right-handed neutrinos to space-time geometry however forces the right-handed neutrinos to adopt the same direction of four-momentum as MEs has. Could some gravitational effect allow to distinguish between spartners? This would be trivially the case if the p-adic mass scales of spartners would be different. Why this should be the case remains however an open question.

ii. In the case of unbroken SUSY only spin distinguishes between spartners. Spin determines statistics and the first naive guess would be that bosonic spartners obey totally different atomic physics allowing condensation of selectrons to the ground state. Very probably this is not true: the right-handed neutrinos are de-localized to 4-D MEs and other fermions correspond to wormhole contact structures and 2-D string world sheets.
The coupling of the spin to the space-time geometry seems to provide the only possible manner to distinguish between spartners. Could one imagine a gravimagnetic effect with energy splitting proportional to the product of gravimagnetic moment and external gravimagnetic field B? If gravimagnetic moment is proportional to spin projection in the direction of B, a non-trivial effect would be possible. Needless to say this kind of effect is extremely small so that the unbroken SUSY might remain undetected.

iii. If the spin of sparticle be seen in the classical angular momentum of ME as quantum classical correspondence would suggest then the value of the angular momentum might allow to distinguish between spartners. Also now the effect is extremely small.

6. What can one say about scattering amplitudes?

One expect that scattering amplitudes factorize with the only correlation between right-handed neutrino scattering and ordinary particle scattering coming from the condition that the four-momentum of the right-handed neutrino is parallel to that of massless extremal of more general preferred extremal having interpretation as a geometric counterpart of radiation quantum. This momentum is in turn equal to the massless four-momentum associated with the space-time sheet in question such that the sum of classical four-momenta associated with the space-time sheets equals to that for all wormhole throats involved. The right-handed neutrino amplitude itself would be simply constant. This certainly satisfies the SUSY constraint and it is actually difficult to find other candidates for the amplitude. The dynamics of right-handed neutrinos would be therefore that of spectator following the leader.

Right-handed neutrino as inert neutrino?

10.4.7 Right-handed neutrino as inert neutrino?

There is a very interesting posting by Jester in Resonaances with title How many neutrinos in the sky? [C1]. Jester tells about the recent 9 years WMAP data [C4] and compares it with earlier 7 years data. In the earlier data the effective number of neutrino types was $N_{eff} = 4.34 \pm 0.87$ and in the recent data it is $N_{eff} = 3.26 \pm 0.35$. WMAP alone would give $N_{eff} = 3.89 \pm 0.67$ also in the recent data but also other data are used to pose constraints on N_{eff}.

To be precise, N_{eff} could include instead of fourth neutrino species also some other weakly interacting particle. The only criterion for contributing to N_{eff} is that the particle is in thermal equilibrium with other massless particles and thus contributes to the density of matter considerably during the radiation dominated epoch.

Jester also refers to the constraints on N_{eff} from nucleosynthesis, which show that $N_{eff} \sim 4$ us slightly favored although the entire range $[3, 5]$ is consistent with data.

It seems that the effective number of neutrinos could be 4 instead of 3 although latest WMAP data combined with some other measurements favor 3. Later a corrected version of the eprint appeared [C4] telling that the original estimate of N_{eff} contained a mistake and the correct estimate is $N_{eff} = 3.84 \pm 0.40$.

An interesting question is what $N_{eff} = 4$ could mean in TGD framework?

i. One poses to the modes of the modified Dirac equation the following condition: electric charge is conserved in the sense that the time evolution by modified Dirac equation does not mix a mode with a well-defined em charge with those with different em charge. The implication is that all modes except pure right handed neutrino are restricted at string world sheets. The first guess is that string world sheets are minimal surfaces of space-time surface (rather than those of imbedding space). One can also consider minimal surfaces of imbedding space but with effective metric defined by the anti-commutators of the modified gamma matrices. This would give a direct physical meaning for this somewhat mysterious effective metric.

   For the neutrino modes localized at string world sheets mixing of left and right handed modes takes place and they become massive. If only 3 lowest genera for partonic 2-surfaces are light, one has 3 neutrinos of this kind. The same applies to all other fermion species. The argument for why this could be the case relies on simple observation [K11]: the genera g=0,1,2 have the property that they allow for all values of conformal moduli Z_2 as a conformal symmetry (hyper-ellipticity). For $g > 2$ this is not the case. The guess is that this additional conformal symmetry is the reason for lightness of the three lowest genera.

ii. Only purely right-handed neutrino is completely de-localized in 4-volume so that one cannot assign to it genus of the partonic 2-surfaces as a topological quantum number and it effectively gives rise to a fourth neutrino very much analogous to what is called sterile neutrino. De-localized right-handed neutrinos couple only to gravitation and in case of massless extremals this forces them to have four-momentum parallel to that of ME: only massless modes are possible. Very probably this holds true for all preferred extremals to which one can assign massless longitudinal momentum direction which can vary with spatial position.

iii. The coupling of ν_R is to gravitation alone and all electroweak and color couplings are absent. According to standard wisdom de-localized right-handed neutrinos cannot be in thermal equilibrium with other particles. This according to standard wisdom. But what about TGD?

   One should be very careful here: de-localized right-handed neutrinos is proposed to give rise to SUSY (not $\mathcal{N} = 1$ requiring Majorana fermions) and their dynamics is that of passive spectator who follows the leader. The simplest guess is that the dynamics of right handed neutrinos at the level of amplitudes is completely trivial and thus trivially supersymmetric. There are however correlations between four-momenta.

A. The four-momentum of ν_R is parallel to the light-like momentum direction assignable to the massless extremal (or more general preferred extremal). This direct coupling to the geometry is a special feature of the modified Dirac operator and thus of sub-manifold gravity.

B. On the other hand, the sum of massless four-momenta of two parallel pieces of preferred extremals is the - in general massive - four-momentum of the elementary particle defined by the wormhole contact structure connecting the space-time sheets (which are glued along their boundaries together since this is seems to be the only manner to get rid of boundary conditions requiring vacuum extremal property near the boundary). Could this direct coupling of the four-momentum direction of right-handed neutrino to geometry and four-momentum directions of other fermions be enough for the right handed neutrinos to be counted as a fourth neutrino species in thermal equilibrium? This might be the case!

One cannot of course exclude the coupling of 2-D neutrino at string world sheets to 4-D purely right handed neutrinos analogous to the coupling inducing a mixing of sterile neutrino with ordinary neutrinos. Also this could help to achieve the thermal equilibrium with 2-D neutrino species.

Experimental evidence for sterile neutrino?

Many physicists are somewhat disappointed to the results from LHC: the expected discovery of Higgs has been seen as the main achievement of LHC hitherto. Much more was expected. To my opinion there is no reason for disappointment. The exclusion of the standard SUSY at expected energy scale is very far reaching negative result. Also the fact that Higgs mass is too small to be stable without fine tuning is of great theoretical importance. The negative results concerning heavy dark matter candidates are precious guidelines for theoreticians. The non-QCD like behavior in heavy ion collisions and proton-ion collisions is bypassed my mentioning something about AdS/CFT correspondence and non-perturbative QCD effects. I tend to see these effects as direct evidence for M_{89} hadron physics [K29].

In any case, something interesting has emerged quite recently. Resonaances tells that the recent analysis [C3] of X-ray spectrum of galactic clusters claims the presence of monochromatic 3.5 keV photon line. The proposed interpretation is as a decay product of sterile 7 keV neutrino transforming first to a left-handed neutrino and then decaying to photon and neutrino via a loop involving W boson and electron. This is of course only one of the many interpretations. Even the existence of line is highly questionable.

One of the poorly understood aspects of TGD is right-handed neutrino, which is obviously the TGD counterpart of the inert neutrino.

i. The old idea is that covariantly constant right handed neutrino could generate $\mathcal{N} = 2$ super-symmetry in TGD Universe. In fact, all modes of induced spinor field would generate superconformal symmetries but electroweak interactions would break these symmetries for the modes carrying non-vanishing electroweak quantum numbers: they vanish for ν_R. This picture is now well-established at the level of WCW geometry [K80]: super-conformal generators are labelled angular momentum and color representations plus two conformal weights: the conformal weight assignable to the light-like radial coordinate of light-cone boundary and the conformal weight assignable to string coordinate. It seems that these conformal weights are independent. The third integer labelling the states would label genuinely Yangian generators: it would tell the poly-locality of the generator with locus defined by partonic 2-surface: generators acting on single partonic 2-surface, 2 partonic 2-surfaces, ...

ii. It would seem that even the SUSY generated by ν_R must be badly broken unless one is able to invent dramatically different interpretation of SUSY. The scale of SUSY breaking and thus the value of the mass of right-handed neutrino remains open also in TGD. In lack of better one could of course argue that the mass scale must be CP_2 mass scale because right-handed neutrino mixes considerably with the left-handed neutrino (and thus becomes massive) only in this scale. But why this argument does not apply also to left handed neutrino which must also mix with the right-handed one!

iii. One can of course criticize the proposed notion of SUSY: wonder whether fermion + extremely weakly interacting ν_R at same wormhole throat (or interior of 3-surface) can behave as single coherent entity as far spin is considered [K67]?

iv. The condition that the modes of induced spinor field have a well-defined electromagnetic charge eigenvalue [K69] requires that they are localized at 2-D string world sheets or partonic 2-surfaces: without this condition classical W boson fields would mix the em charged and neutral modes with each other. Right-handed neutrino is an exception since it has no electroweak couplings. Unless right-handed neutrino is covariantly constant, the modified gamma matrices can however mix the right-handed neutrino with the left handed one and this can induce transformation to charged mode. This does not happen if each modified gamma matrix can be written as a linear combination of either M^4 or CP_2 gamma matrices and modified Dirac equation is satisfied separately by M^4 and CP_2 parts of the modified Dirac equation.

v. Is the localization of the modes other than covariantly constant neutrino to string world sheets a consequence of dynamics or should one assume this as a separate condition? If one wants similar localization in space-time regions of Euclidian signature - for which CP_2 type vacuum extremal is a good representative - one must assume it as a separate condition. In number theoretic formulation string world sheets/partonic 2-surfaces would be commutative/co-commutative sub-manifolds of space-time surfaces which in turn would be associative or co-associative sub-manifolds of imbedding space possessing (hyper-)octonionic tangent space structure. For this option also right-handed neutrino would be localized to string world sheets. Right-handed neutrino would be covariantly constant only in 2-D sense.
One can consider the possibility that ν_R is de-localized to the entire 4-D space-time sheet. This would certainly modify the interpretation of SUSY since the number of degrees of freedom would be reduced for ν_R.

vi. Non-covariantly constant right-handed neutrinos could mix with left-handed neutrinos but not with charged leptons if the localization to string world sheets is assumed for modes carrying non-vanishing electroweak quantum numbers. This would make possible the decay of right-handed to neutrino plus photon, and one cannot exclude the possibility that ν_R has mass 7 keV.
Could this imply that particles and their spartners differ by this mass only? Could it be possible that practically unbroken SUSY could be there and we would not have observed it? Could one imagine that sfermions have annihilated leaving only states consisting of fundamental fermions? But shouldn't the total rate for the annihilation of photons to hadrons be two times the observed one? This option does not sound plausible.
What if one assumes that given sparticle is charactrized by the same p-adic prime as corresponding particle but is dark in the sense that it corresponds to non-standard value of Planck constant. In this case sfermions would not appear in the same vertex with fermions and one could escape the most obvious contradictions with experimental facts. This leads to the notion of shadron: shadrons would be [K67] obtained by replacing quarks with dark squarks with nearly identical masses. I have asked whether so called X and Y bosons having no natural place in standard model of hadron could be this kind of creatures.

The interpretation of 3.5 keV photons as decay products of right-handed neutrinos is of course totally ad hoc. Another TGD inspired interpretation would be as photons resulting from the decays of excited nuclei to their ground state.

i. Nuclear string model [L2] predicts that nuclei are string like objects formed from nucleons connected by color magnetic flux tubes having quark and antiquark at their ends. These flux tubes are long and define the "magnetic body" of nucleus.

Quark and antiquark have opposite em charges for ordinary nuclei. When they have different charges one obtains exotic state: this predicts entire spectrum of exotic nuclei for which statistic is different from what proton and neutron numbers deduced from em charge and atomic weight would suggest. Exotic nuclei and large values of Planck constant could make also possible cold fusion [K15].

ii. What the mass difference between these states is, is not of course obvious. There is however an experimental finding [C5] (see *Analysis of Gamma Radiation from a Radon Source: Indications of a Solar Influence*) that nuclear decay rates oscillate with a period of year and the rates correlate with the distance from Sun. A possible explanation is that the gamma rays from Sun in few keV range excite the exotic nuclear states with different decay rate so that the average decay rate oscillates [L2]. Note that nuclear excitation energies in keV range would also make possible interaction of nuclei with atoms and molecules.

iii. This allows to consider the possibility that the decays of exotic nuclei in galactic clusters generates 3.5 keV photons. The obvious question is why the spectrum would be concentrated at 3.5 keV in this case (second question is whether the energy is really concentrated at 3.5 keV: a lot of theory is involved with the analysis of the experiments). Do the energies of excited states depend on the color bond only so that they would be essentially same for all nuclei? Or does single excitation dominate in the spectrum? Or is this due to the fact that the thermal radiation leaking from the core of stars excites predominantly single state? Could $E = 3.5$ keV correspond to the maximum intensity for thermal radiation in stellar core? If so, the temperature of the exciting radiation would be about $T \simeq E/3 \simeq 1.2 \times 10^7$ K. This in the temperature around which formation of Helium by nuclear fusion has begun: the temperature at solar core is around 1.57×10^7 K.

10.5 Octo-twistors and twistor space

The basic problem of the twistor approach is that one cannot represent massive momenta in terms of twistors in an elegant manner. One can also consider generalization of the notion of spinor and twistor. I have proposed a possible representation of massive states based on the existence of preferred plane of M^2 in the basic definition of theory allowing to express four-momentum as one of two light-like momenta allowing twistor description. One could however ask whether some more elegant representation of massive M^4 momenta might be possible by generalizing the notion of twistor -perhaps by starting from the number theoretic vision.

The basic idea is obvious: in quantum TGD massive states in M^4 can be regarded as massless states in M^8 and $M^4 \times CP_2$ (recall $M^8 - H$ duality). One can therefore map any massive M^4 momentum to a light-like M^8 momentum and hope that this association could be made in a unique manner. One should assign to a massless 8-momentum an 8-dimensional spinor of fixed chirality. The spinor assigned with the light-like four-momentum is not unique without additional conditions. The existence of covariantly constant right-handed neutrino in CP_2 degrees generating the super-conformal symmetries could allow to eliminate the non-uniqueness. 8-dimensional twistor in M^8 would be a pair of this kind of spinors fixing the momentum of massless particle and the point through which the corresponding light-geodesic goes through: the set of these points forms 8-D light-cone and one can assign to each point a spinor. In $M^4 \times CP_2$ definitions makes also in the case of $M^4 \times CP_2$ and twistor space would also now be a lifting of the space of light-like geodesics.

The possibility to interpret M^8 as hyperoctonionic space suggests also the possibility to define the 8-D counterparts of sigma matrices to hyperoctonions to obtain a representation of sigma matrix algebra which is not a matrix representation. The mapping of gamma matrices to this representation allows to define a notion of hyper-quaternionicity in terms of the modified gamma matrices both in M^8 and H.

The basic challenge is to achieve twistorial description of four-momenta or even $M^4 \times$

CP_2 quantum numbers: this applies both to the momenta of fundamental fermions at the lines of generalized Feynman diagrams and to the massive incoming and outcoming states identified as their composites.

i. A rather attractive way to overcome the problem at the level of fermions propagating along the braid strands at the light-like orbits of partonic 2-surfaces relies on the assumption that generalized Feynman diagrammatics effectively reduces to a form in which all fermions in the propagator lines are massless although they can have non-physical helicity [K44]. One can use ordinary M^4 twistors. This is consistent with the idea that space-time surfaces are quaternionic sub-manifolds of octonionic imbedding space.

ii. Incoming and outgoing states are composites of massless fermions and not massless. They are however massless in 8-D sense. This suggests that they could be described using generalization of twistor formalism from M^4 to M^8 and even betterm to $M^4 \times CP_2$.

In the following two possible twistorializations are considered.

10.5.1 Two manners to twistorialize imbedding space

In the following the generalization of twistor formalism for M^8 or $M^4 \times CP_2$ will be considered in more detail. There are two options to consider.

i. For the first option one assigns to $M^4 \times CP_2$ twistor space as a product of corresponding twistor spaces $T(M_4) = CP_3$ and the flag-manifold $T(CP_2) = SU(3)/U(1) \times U(1)$ parameterizing the choices of quantization axes for $SU(3)$: $T_H = T(M^4) \times T(CP_2)$. Quite remarkably, M^4 and CP_2 are the only 4-D manifolds allowing twistor space with Kähler structure. The twistor space is 12-dimensional. The choice of quantization axis is certainly a physically well-define operation so that $T(CP_2)$ has physical interpretation. If all observable physical states are color singlets situation becomes more complex. If one assumes QCC for color quantum numbers Y and I_3, then also the choice of color quantization axis is fixed at the level of Kähler action from the condition that Y and I_3 have classically their quantal values.

ii. For the second option one generalizes the usual construction for M^8 regarded as tangent space of $M^4 \times CP_2$ (unless one takes $M^8 - H$ duality seriously).

The tangent space option looks like follows.

i. One can map the points of M^8 to octonions. One can consider 2-component spinors with octonionic components and map points of M^8 light-cone to linear combinations of 2×2 Pauli sigma matrices but with octonionic components. By the same arguments as in the deduction of ordinary twistor space one finds that 7-D light-cone boundary is mapped to 7+8 D space since the octonionic 2-spinor/its conjugate can be multiplied/divided by arbitrary octonion without changing the light-like point. By standard argument this space extends to 8+8-D space. The points of M^8 can be identified as 8-D octonionic planes (analogs of complex sphere CP_1 in this space. An attractive identification is as octonionic projective space OP_2. Remarkably, octonions do not allow higher dimensional projective spaces.

ii. If one assumes that the spinors are quaternionic the twistor space should have dimension 7+4+1=12. This dimension is same as for $M^4 \times CP_2$. Does this mean that quaternionicity assumption reduces $T(M^8) = OP_2$ to $T(H) = CP_3 \times SU(3)/U(1) \times U(1)$? Or does it yield 12-D space $G_2/U(1) \times U(1)$, which is also natural since G_2 has 2-D Cartan algebra? Number theoretical compactification would transform $T(M^8) = G_2/U(1) \times U(1)$ to $T(H) = CP_3 \times SU(3)/U(1) \times U(1)$. This would not be surprising since in $M^8 - H$-duality CP_2 parametrizes (hyper)quaternionic planes containing preferred plane M^2.

   Quaternionicity is certainly very natural in TGD framework. Quaternionicity for 8-momenta does not in general imply that they reduce to the observed M^4-momenta unless one identifies M^4 as one particular subspace of M^8. In $M^8 - H$ duality one in principle allows all choices of M^4: it is of course unclear whether this makes any physical difference. Color confinement could be interpreted as a reduction of M^8 momenta to M^4 momenta and would also allow the interpretational problems caused by the fact that CP_2 momenta are not possible.

iii. Since octonions can be regarded as complexified quaternions with non-commuting imaginary unit, one can say that quaternionic spinors in M^8 are "real" and thus analogous to Majorana spinors. Similar interpretation applies at the level of H. Could one can interpret the quaternionicity condition for space-time surfaces and imbedding space spinors as TGD analog of Majorana condition crucial in super string models? This would also be crucial for understanding supersymmetry in TGD sense.

10.5.2 Octotwistorialization of M^8

Consider first the twistorialization in 4-D case. In M^4 one can map light-like momoment to spinors satisfying massless Dirac equation. General point m of M^4 can be mapped to a pair of massless spinors related by incidence relation defining the point m. The essential element of this association is that mass squared can be defined as determinant of the 2×2 matrix resulting in the assignment. Light-likeness is coded to the vanishing of the determinant implying that the spinors defining its rows are linearly independent. The reduction of M^4 inner product to determinant occurs because the 2×2 matrix can be regarded as a matrix representation of complexified quaternion. Massless means that the norm of a complexified quaternion defined as the product of q and its conjugate vanishes. Incidence relation $s_1 = xs_2$ relating point of M^4 and pair of spinors defining the corresponding twistor, can be interpreted in terms of product for complexified quaternions.

The generalization to the 8-D situation is straightforward: replace quaternions with octonions.

i. The transition to M^8 means the replacement of quaternions with octonions. Masslessness corresponds to the vanishing norm for complexified octonion (hyper-octonion).

ii. One should assign to a massless 8-momentum an 8-dimensional spinor identifiable as octonion - or more precisely as hyper-octonion obtained by multiplying the imaginary part of ordinary octonion with commuting imaginary unit j and defining conjugation as a change of sign of j or that of octonionic imaginar units.

iii. This leads to a generalization of the notion of twistor consisting of pair of massless octonion valued spinors (octonions) related by the incidence relation fixing the point of M^8. The incidence relation for Euclidian octonions says $s_1 = xs_2$ and can be interpreted in terms of triality for $SO(8)$ relating conjugate spinor octet to the product of vector octed and spinor octet. For Minkowskian subspace of complexified octonions light-like vectors and s_1 and s_2 can be taken light-like as octonions. Light like x can annihilate s_2.

The possibility to interpret M^8 as hyperoctonionic space suggests also the possibility to define the 8-D counterparts of sigma matrices to hyperoctonions to obtain a representation of sigma matrix algebra which is not a matrix representation. The mapping of gamma matrices to this representation allows to define a notion of hyper-quaternionicity in terms of the modified gamma matrices both in M^8 and H.

10.5.3 Octonionicity, $SO(1,7)$, G_2, and non-associative Malcev group

The symmetries assignable with octonions are rather intricate. First of all, octonions (their hyper-variants defining M^8) have SO(8) (SO(1,7)) as isometries. $G_2 \subset SO(7)$ acts as automorphisms of octonions and $SO(1,7) \rightarrow G_2$ clearly means breaking of Lorentz invariance.

John Baez has described in a lucid manner G_2 geometrically (`http://math.ucr.edu/home/baez/octonions/node14.html`). The basic observation is that that quaternionic sub-space is generated by two linearly independent imaginary units and by their product. By adding a fourth linearly independent imaginary unit, one can generated all octonions. From this and the fact that G_2 represents subgroup of $SO(7)$, one easily deduces that G_2 is 14-dimensional. The Lie algebra of G_2 corresponds to derivations of octonionic algebra as follows infinitesimally from the condition that the image of product is the product of images. The entire algebra $SO(8)$ is direct sum of G_2 and linear transformations generated by right and left multiplication by imaginary octonion: this gives $14 + 14 = 28 = D(SO(8))$. The subgroup $SO(7)$ acting on imaginary octonsions corresponds to the direct sum of derivations and adjoint transformations defined by commutation with imaginary octonions, and has indeed dimension $14 + 7 = 21$.

One can identify also a non-associative group-like structure.

i. In the case of octonionic spinors this group like structure is defined by the analog of phase multiplication of spinor generalizing to a multiplication with octonionic unit expressible as linear combinations of 8 octonionic imaginary units and defining 7-sphere plays appear as analog of automorphisms $o \to uou^{-1} = uou^*$.
   One can associate with these transformations a non-associative Lie group and Lie algebra like structures by defining the commutators just as in the case of matrices that is as $[a, b] = ab - ba$. One 7-D non-associative Lie group like structure with topology of 7-sphere S^7 whereas G_2 is 14-dimensional exceptional Lie group (having S^6 as coset space $S^6 = G_2/SU(3)$). This group like object might be useful in the treatment of octonionic twistors. In the case of quaternions one has genuine group acting as SO(3) rotations.

ii. Octonionic gamma matrices allow to define as their commutators octonionic sigma matrices:
$$\Sigma_{kl} = \frac{i}{2}\left[\gamma_k, \gamma_l\right] \quad . \tag{10.5.1}$$
   This algebra is 14-dimensional thanks to the fact that octonionic gamma matrices are of form $\gamma_0 = \sigma_1 \otimes 1$, $\gamma_i = \sigma_2 \otimes e_i$. Due to the non-associativity of octonions this algebra does not satisfy Jacobi identity - as is easy to verify using Fano triangle - and is therefore not a genuine Lie-algebra. Therefore these sigma matrices do not define a representation of G_2 as I thought first.
   This algebra has decomposition $g = h + t$, $[h, t] \subset t$, $[t, t] \subset h$ characterizing for symmetric spaces. h is the 7-D algebra generated by Σ_{ij} and identical with the non-associative Malcev algebra generated by the commutators of octonionic units. The complement t corresponds to the generators Σ_{0i}. The algebra is clearly an octonionic non-associative analog fo $SO(1, 7)$.

10.5.4 Octonionic spinors in M^8 and real complexified-quaternionic spinors in H?

This above observations about the octonionic sigma matrices raise the problem about the octonionic representation of spinor connection. In $M^8 = M^4 \times E^4$ the spinor connection is trivial but for $M^4 \times CP_2$ not. There are two options.

i. Assume that octonionic spinor structure makes sense for M^8 only and spinor connection is trivial.

ii. An alternative option is to identify M^8 as tangent space of $M^4 \times CP_2$ possessing quaternionic structure defined in terms of octonionic variants of gamma matrices. Should one replace sigma matrices appearing in spinor connection with their octonionic analogs to get a sigma matrix algebra which is pseudo Lie algebra. Or should one map the holonomy algebra of CP_2 spinor connection to a sub-algebra of $G_2 \subset SO(7)$ and define the action of the sigma matrices as ordinary matrix multiplication of octonions rather than octonionic multiplication? This seems to be possible formally.

The replacement of sigma matrices with their octonionic counterparts seems to lead to weird looking results. Octonionic multiplication table implies that the electroweak sigma matrices associated with CP_2 tangent space reduce to M^4 sigma matrices so that the spinor connection is quaternionic. Furthermore, left-handed sigma matrices are mapped to zero so that only the neutral part of spinor connection is non-vanishing. This supports the view that only M^8 gamma matrices make sense and that Dirac equation in M^8 is just free massless Dirac equation leading naturally also to the octonionic twistorialization.
One might think that distinction between different H-chiralities is difficult to make but it turns out that quarks and leptons can be identified as different components of 2-component complexified octonionic spinors.

The natural question is what associativization of octonions gives. This amounts to a condition putting the associator $a(bc)-(ab)c$ to zero. It is enough to consider octonionic imaginary units which are different. By using the decomposition of the octonionic algebra to quaternionic sub-algebra and its complement and general structure of structure constants, one finds that quaternionic sub-algebra remains as such but the products of all imaginary units in the complement with different imaginary units vanish. This means that the complement behaves effectively as 4-D flat space-gamma matrix algebra annihilated by the quaternionic sub-algebra whose imaginary part acts like Lie algebra of $SO(3)$.

10.5.5 What the replacement of $SO(7,1)$ sigma matrices with octonionic sigma matrices could mean?

The basic implication of octonionization is the replacement of $SO(7,1)$ sigma matrices with octonionic sigma matrices. For M^8 this has no consequences since since spinor connection is trivial.
For $M^4\times CP_2$ situation would be different since CP_2 spinor connection would be replaced with its octonionic variant. This has some rather unexpected consequences and suggests that one should not try to octonionize at the level of $M^4 \times CP_2$ but interepret gamma matrices as tensor products of quaternionic gamma matrices, which can be replaced with their matrix representations. There are however some rather intriguing observations which force to keep mind open.

Octonionic representation of 8-D gamma matrices

Consider first the representation of 8-D gamma matrices in terms of tensor products of 7-D gamma matrices and 2-D Pauli sigma matrices.

i. The gamma matrices are given by

$$\gamma^0 = 1 \times \sigma_1 \ , \ \gamma^i = \gamma^i \otimes \sigma_2 \ , \ i = 1,..,7 \ . \tag{10.5.2}$$

7-D gamma matrices in turn can be expressed in terms of 6-D gamma matrices by expressing γ^7 as

$$\gamma^{7)}_{i+1} = \gamma^{6)}_i, i = 1,...,6 \ , \ \gamma^{7)}_1 = \gamma^{6)}_7 = \prod_{i=1}^{6} \gamma^{6)}_i \ . \tag{10.5.3}$$

ii. The octonionic representation is obtained as

$$\gamma_0 = 1 \otimes \sigma_1 \ , \ \gamma_i = e_i \otimes \sigma_2 \ . \tag{10.5.4}$$

where e_i are the octonionic units. $e_i^2 = -1$ guarantees that the M^4 signature of the metric comes out correctly. Note that $\gamma_7 = \prod \gamma_i$ is the counterpart for choosing the preferred octonionic unit and plane M^2.

iii. The octonionic sigma matrices are obtained as commutators of gamma matrices:

$$\Sigma_{0i} = je_i \times \sigma_3 \ , \ \Sigma_{ij} = jf_{ij}{}^{k}e_k \otimes 1 \ . \tag{10.5.5}$$

Here j is commuting imaginary unit. These matrices span G_2 algebra having dimension 14 and rank 2 and having imaginary octonion units and their conjugates as the fundamental representation and its conjugate. The Cartan algebra for the sigma matrices can be chosen to be Σ_{01} and Σ_{23} and belong to a quaternionic sub-algebra.

iv. The lower dimension $D = 14$ of the non-associative version of sigma matrix algebra algebra means that some combinations of sigma matrices vanish. All left or right handed generators of the algebra are mapped to zero: this explains why the dimension is halved from 28 to 14. From the octonionic triangle expressing the multiplication rules for octonion units [A17] one finds $e_4e_5 = e_1$ and $e_6e_7 = -e_1$ and analogous expressions for the cyclic permutations of e_4, e_5, e_6, e_7. From the expression of the left handed sigma matrix $I_L^3 = \sigma_{23} + \sigma^{30}$ representing left handed weak isospin (see the Appendix about the geometry of CP_2 [L1]) one can conclude that this particular sigma matrix and left handed sigma matrices in general are mapped to zero. The quaternionic sub-algebra $SU(2)_L \times SU(2)_R$ is mapped to that for the rotation group $SO(3)$ since in the case of Lorentz group one cannot speak of a decomposition to left and right handed subgroups. The elements of the complement of the quaternionic sub-algebra are expressible in terms of Σ_{ij} in the quaternionic sub-algebra.

Some physical implications of the reduction of $SO(7,1)$ to its octonionic counterpart

The octonization of spinor connection of CP_2 has some weird physical implications forcing to keep mind to the possibility that the octonionic description even at the level of H might have something to do with reality.

i. If $SU(2)_L$ is mapped to zero only the right-handed parts of electro-weak gauge field survive octonionization. The right handed part is neutral containing only photon and Z^0 so that the gauge field becomes Abelian. Z^0 and photon fields become proportional to each other ($Z^0 \rightarrow sin^2(\theta_W)\gamma$) so that classical Z^0 field disappears from the dynamics, and one would obtain just electrodynamics.

ii. The gauge potentials and gauge fields defined by CP_2 spinor connection are mapped to fields in $SO(2) \subset SU(2) \times U(1)$ in quaternionic sub-algebra which in a well-defined sense corresponds to M^4 degrees of freedom and gauge group becomes $SO(2)$ subgroup of rotation group of $E^3 \subset M^4$. This looks like catastrophe. One might say that electroweak interactions are transformed to gravimagnetic interactions.

iii. In very optimistic frame of mind one might ask whether this might be a deeper reason for why electrodynamics is an excellent description of low energy physics and of classical physics. This is consistent with the fact that CP_2 coordinates define 4 field degrees of freedom so that single Abelian gauge field should be enough to describe classical physics. This would remove also the interpretational problems caused by the transitions changing the charge state of fermion induced by the classical W boson fields.

iv. Interestingly, the condition that electromagnetic charge is well-defined quantum number for the modes of the induced spinor field for $X^4 \subset H$ leads to the proposal that the solutions of the modified Dirac equation are localized to string world sheets in Minkowskian regions of space-time surface at least. For CP_2 type vacuum extremals one has massless Dirac and this allows only covariantly constant right-handed neutrino as solution. One has however only a piece of CP_2 (wormhole contact) so that holomorphic solutions annihilated by two complexified gamma matrices are possible in accordance with the conformal symmetries.

Can one assume non-trivial spinor connection in M^8

i. The simplest option encouraged by the requirement of maximal symmetries is that it is absent. Massless 8-momenta would characterize spinor modes in M^8 and this would give physical justification for the octotwistors.

ii. If spinor connection is present at all, it reduces essentially to Kähler connection having different couplings to quarks and leptons identifiable as components of octonionic 2-spinors. It should be $SO(4)$ symmetric and since CP_2 is instant one might argue that now one has also instanton that is self-dual U(1) gauge field in $E^4 \subset M^4 \times E^4$ defining Kähler form. One can loosely say that that one has of constant electric and magnetic fields which are parallel to each other. The rotational symmetry in E^4 would break down to $SO(2)$.

iii. Without spinor connection quarks and leptons are in completely symmetric position at the level of M^8: this is somewhat disturbing. The difference between quarks and leptons in H is made possible by the fact that CP_2 does not allow standard spinor structure. Now this problem is absent. I have also consider the possibility that only leptonic spinor chirality is allowed and quarks result via a kind of anyonization process allowing them to have fractional em charges (see `http://www.tgdtheory.fi/public_html/articles/genesis.pdf`).

iv. If the solutions of the Kähler Dirac equation in Minkowskian regions are localized to two surfaces identifiable as integrable distributions of planes $M^2(x)$ and characterized by a local light-like direction defining the direction of massless momentum, they are holomorphic (in the sense of hyper-complex numbers) such that the second complexified modified gamma matrix annihilates the solution. Same condition makes sense also at the level of M^8 for solutions restricted to string world sheets and the presence or absence of spinor connection does not affect the situation.

   Does this mean that the difference between quarks and leptons becomes visible only at the imbedding space level where ground states of super-conformal representations correspond to to imbedding space spinor harmonics which in CP_2 cm degrees are different for quarks and leptons?

Octo-spinors and their relation to ordinary imbedding space spinors

Octo-spinors are identified as octonion valued 2-spinors with basis

$$\begin{aligned} \Psi_{L,i} &= e_i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ , \\ \Psi_{q,i} &= e_i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ . \end{aligned} \tag{10.5.6}$$

One obtains quark and lepton spinors and conjugation for the spinors transforms quarks to leptons. Note that octospinors can be seen as 2-dimensional spinors with components which have values in the space of complexified octonions.

The leptonic spinor corresponding to real unit and preferred imaginary unit e_1 corresponds naturally to the two spin states of the right handed neutrino. In quark sector this would mean that right handed U quark corresponds to the real unit. The octonions decompose as $1+1+3+\overline{3}$ as representations of $SU(3) \subset G_2$. The concrete representations are given by

$$\begin{array}{ll} \{1 \pm ie_1\} \ , & e_R \text{ and } \nu_R \text{ with spin } 1/2 \ , \\ \{e_2 \pm ie_3\} \ , & e_R \text{ and } \nu_L \text{ with spin } -1/2 \ , \\ \{e_4 \pm ie_5\} & e_L \text{ and } \nu_L \text{ with spin } 1/2 \ , \\ \{e_6 \pm ie_7\} & e_L \text{ and } \nu_L \text{ with spin } 1/2 \ . \end{array} \tag{10.5.7}$$

Instead of spin one could consider helicity. All these spinors are eigenstates of e_1 (and thus of the corresponding sigma matrix) with opposite values for the sign factor $\epsilon = \pm$. The interpretation is in terms of vectorial isospin. States with $\epsilon = 1$ can be interpreted as charged leptons and D type quarks and those with $\epsilon = -1$ as neutrinos and U type quarks. The interpretation would be that the states with vanishing color isospin correspond to right handed fermions and the states with non-vanishing SU(3) isospin (to be not confused with QCD color isospin) and those with non-vanishing SU(3) isospin to left handed fermions.

The importance of this identification is that it allows a unique map of the candidates for the solutions of the octonionic modified Dirac equation to those of ordinary one. There are some delicacies involved due to the possibility to chose the preferred unit e_1 so that the preferred subspace M^2 can corresponds to a sub-manifold $M^2 \subset M^4$.

10.6 About the interpretation of Kähler Dirac equation

The physical interpretation of Kähler Dirac equation is not at all straightforward. The following arguments inspired by effective 2-dimensionality suggest that the modified gamma matrices and corresponding effective metric could allow dual gravitational description of the physics associated with wormhole throats that is holography. This applies in particular to condensed matter physics.

10.6.1 Three Dirac equations

To begin with, Dirac equation appears in three forms in TGD.

i. The Dirac equation in world of classical worlds codes (WCW) for the super Virasoro conditions for the super Kac-Moody and similar representations formed by the states of wormhole contacts forming the counterpart of string like objects (throats correspond to the ends of the string. WCW Dirac operator generalizes the Dirac operator of 8-D imbedding space by bringing in vibrational degrees of freedom. This Dirac equation should give as its solutions zero energy states and corresponding M-matrices generalizing S-matrix and their collection defining the unitary U-matrix whose natural application appears in consciousness theory as a coder of what Penrose calls U-process. The ground states to which super-conformal algebras act correspond to imbedding space spinor modes in accordance with the idea that point like limit gives QFT in imbedding space.
ii. The analog of massless Dirac equation at the level of 8-D imbedding space and satisfied by fermionic ground states of super-conformal representations.
iii. Kähler Dirac equation is satisfied in the interior of space-time. In this equation the gamma matrices are replaced with modified gamma matrices defined by the contractions of canonical momentum currents $T^\alpha_k = \partial L/\partial_\alpha h^k$ with imbedding space gamma matrices Γ_k. This replacement is required by internal consistency and by super-conformal symmetries. The well-definedness of em charge implies that the modes of induced spinor field are localized at 2-D surfaces so that a connection with string theory type approach emerges.

Kähler-Dirac equation defines Dirac equation at space-time level. Consider first K-D equation in the interior of space-time surface.

i. The condition that electromagnetic charge operator defined in terms of em charge expressed in terms of Clifford algebra is well defined for spinor modes (completely analogous to spin defined in terms of sigma matrices) leads to the proposal that induced spinor fields are necessarily localized at 2-dimensional string worlds sheets [K69]. Only the covariantly constant right handed neutrino and its modes assignable to massless extremals (at least) generating super-symmetry (super-conformal symmetries) would form an exception since electroweak couplings would vanish. Note that the modified gamma matrices possess CP_2 and this must vanish in order to have de-localization.
ii. This picture implies stringy realization of super Kac-Moody symmetry elementary particles can be identified as string like objects albeit in different sense than in string models. At light-like 3-surfaces defining the orbits of partonic 2-surfaces spinor fields

carrying electroweak quantum numbers would be located at braid strands as also the notion of finite measurement resolution requires. This picture is also consistent with the puzzling observation that the solutions of the Chern-Simons Dirac equation can be localized on light-like curves inside wormhole throat orbits.

iii. Could Kähler Dirac equation provide a first principle justification for the light-hearted use of effective mass and the analog of Dirac equation in condensed manner physics? This would conform with the holographic philosophy. Partonic 2-surfaces with tangent space data and their light-like orbits would give hologram like representation of physics and the interior of space-time the 4-D representation of physics. Holography would have in the recent situation interpretation also as quantum classical correspondence between representations of physics in terms of quantized spinor fields at the light-like 3-surfaces on one hand and in terms of classical fields on the other hand.

iv. The resulting dispersion relation for the square of the Kähler-Dirac operator assuming that induced like metric, Kähler field, etc. are very slowly varying contains quadratic and linear terms in momentum components plus a term corresponding to magnetic moment coupling. In general massive dispersion relation is obtained as is also clear from the fact that Kähler Dirac gamma matrices are combinations of M^4 and CP_2 gammas so that modified Dirac mixes different M^4 chiralities (basic signal for massivation). If one takes into account the dependence of the induced geometric quantities on space-time point dispersion relations become non-local.

v. Sound as a concept is usually assigned with a rather high level of description. Stringy world sheets could however dramatically raise the status of sound in this respect. The oscillations of string world sheets connecting wormhole throats describe non-local 2-particle interactions. Holography suggests that this interaction just "gravitational" dual for electroweak and color interactions. Could these oscillations inducing the oscillation of the distance between wormhole throats be interpreted at the limit of weak "gravitational" coupling as analogs of sound waves, and could sound velocity correspond to maximal signal velocity assignable to the effective metric?

Various arguments lead to the hypothesis that Kähler-Dirac action contains Chern-Simons-Dirac action localized at partonic orbits as additional term. This term cannot present at the space-like ends of the space-time surfaces. Also Kähler action contains Chern-Simons term and partonic orbits and reduces by field equations to Chern-Simons terms at the space-like ends of space-time surface.

i. The variation of the Kähler-Dirac action gives rise to a boundary term, which is essentially contraction of the normal component of the vector Γ^n defined by Kähler-Dirac gamma matrices. Boundary condition gives $\sqrt{g_4}\Gamma^n\Psi = 0$. Therefore the incoming spinor modes at the boundaries of string world sheets must be massless. A further assumption is that the action of $\sqrt{g_4}\Gamma^n$ equals to that of a massless Dirac operator. By a suitable choice of coordinates this might be achieved. Thus massless Dirac equation in M^4 would emerge for on mass shell states.

ii. At parton orbits of wormhole one can assume that the spinors are generalized eigenstates of C-S-D operator reduces to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator and one would have good hopes that twistor Grassmannian approach works. In TGD based sringy variant of twistor Grassmann approach the integrals over virtual momenta as residue integrals reduce them to 3-D integrals over light-cone subject to momentum conservation constraints at vertices. Virtual fermions are massless but have unphysical polarization. This picture is discussed in detail in [K44].

10.6.2 Does energy metric provide the gravitational dual for condensed matter systems?

The modified gamma matrices define an effective metric via their anti-commutators quadratic in components of energy momentum tensor (canonical momentum densities).

This effective metric vanishes for vacuum extremals. Note that the use of the modified gamma matrices guarantees among other things internal consistency and superconformal symmetries of the theory.

If the above argument is on the right track, this effective metric should have applications in condensed matter theory. The energy metric has a natural interpretation in terms of effective light velocities which depend on direction of propagation. One can diagonalize the energy metric $g_e^{\alpha\beta}$ (contravariant form results from the anti-commutators) and one can denote its eigenvalues by (v_0, v_i) in the case that the signature of the effective metric is $(1, -1, -1, -1)$. The 3-vector v_i/v_0 has interpretation as components of effective light velocity in various directions as becomes clear by thinking the d'Alembert equation for the energy metric. This velocity field could be interpreted as that of hydrodynamic flow. The study of the extremals of Kähler action shows that if this flow is actually Beltrami flow so that the flow parameter associated with the flow lines extends to global coordinate, Kähler action reduces to a 3-D Chern-Simons action and one obtains effective topological QFT. The conserved fermion current $\overline{\Psi}\Gamma_e^\alpha\Psi$ has interpretation as incompressible hydrodynamical flow.

This would give also a nice analogy with AdS/CFT correspondence allowing to describe various kinds of physical systems in terms of higher-dimensional gravitation and black holes are introduced quite routinely to describe condensed matter systems. In TGD framework one would have an analogous situation but with 10-D space-time replaced with the interior of 4-D space-time and the boundary of AdS representing Minkowski space with the light-like 3-surfaces carrying matter. The effective gravitation would correspond to the "energy metric". One can associate with it analogs of curvature tensor, Ricci tensor and Einstein tensor using standard formulas and identify effective energy momentum tensor associated as Einstein tensor with effective Newton's constant appearing as constant of proportionality. Note however that the besides ordinary metric and "energy" metric one would have also the induced classical gauge fields having purely geometric interpretation and action would be Kähler action. This 4-D holography could provide a precise, dramatically simpler, and also a very concrete dual description. This cannot be said about model of graphene based on the introduction of 10-dimensional black holes, branes, and strings chosen in more or less ad hoc manner.

This raises questions. Could this give a general dual gravitational description of dissipative effects in terms of the "energy" metric and induced gauge fields? Does one obtain the analogs of black holes? Do the general theorems of general relativity about the irreversible evolution leading to black holes generalize to describe analogous fate of condensed matter systems caused by dissipation? Can one describe non-equilibrium thermodynamics and self-organization in this manner?

One might argue that the incompressible Beltrami flow defined by the dynamics of the preferred extremals is dissipationless and viscosity must therefore vanish locally. The failure of complete determinism for Kähler action however means generation of entropy since the knowledge about the state decreases gradually. This in turn should have a phenomenological local description in terms of viscosity, which characterizes the transfer of energy to shorter scales and eventually to radiation. The deeper description should be non-local and basically topological and might lead to quantization rules. For instance, one can imagine the quantization of the ratio η/s of the viscosity to entropy density as multiples of a basic unit defined by its lower bound (note that this would be analogous to Quantum Hall effect). For the first M-theory inspired derivation of the lower bound of η/s [D4] . The lower bound for η/s is satisfied in good approximation by what should have been QCD plasma but found to be something different (RHIC and the first evidence for new physics from LHC [K29]).

An encouraring sign comes from the observation that for so called massless extremals representing classically arbitrarily shaped pulses of radiation propagating without dissipation and dispersion along single direction the canonical momentum currents are light-like. The effective contravariant metric vanishes identically so that fermions cannot propate in the interior of massless extremals! This is of course the case also for vacuum extremals. Massless extremals are purely bosonic and represent bosonic radiation. Many-sheeted space-time decomposes into matter containing regions and radiation containing regions. Note that when wormhole contact (particle) is glued to a massless extremal, it is deformed so that CP_2 projection becomes 4-D guaranteeing that the weak form of electric magnetic duality can be satisfied. Therefore massless extremals can be seen as asymptotic regions. Perhaps one could say that dissipation corresponds to a decoherence process creating space-time sheets consisting of matter and radiation. Those containing matter might be even seen as analogs blackholes as far as energy metric is considered.

10.6.3 Preferred extremals as perfect fluids

10.6.4 Preferred extremals as perfect fluids

Almost perfect fluids seems to be abundant in Nature. For instance, QCD plasma was originally thought to behave like gas and therefore have a rather high viscosity to entropy density ratio $x = \eta/s$. Already RHIC found that it however behaves like almost perfect fluid with x near to the minimum predicted by AdS/CFT. The findings from LHC gave additional conform the discovery [C2]. Also Fermi gas is predicted on basis of experimental observations to have at low temperatures a low viscosity roughly 5-6 times the minimal value [D2] . In the following the argument that the preferred extremals of Kähler action are perfect fluids apart from the symmetry breaking to space-time sheets is developed. The argument requires some basic formulas summarized first.

The detailed definition of the viscous part of the stress energy tensor linear in velocity (oddness in velocity relates directly to second law) can be found in [D1] .

i. The symmetric part of the gradient of velocity gives the viscous part of the stress-energy tensor as a tensor linear in velocity. Velocity gradient decomposes to a term traceless tensor term and a term reducing to scalar.

$$\partial_i v_j + \partial_j v_i = \frac{2}{3}\partial_k v^k g_{ij} + (\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v^k g_{ij}) \ . \tag{10.6.1}$$

The viscous contribution to stress tensor is given in terms of this decomposition as

$$\sigma_{visc;ij} = \zeta \partial_k v^k g_{ij} + \eta(\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v^k g_{ij}) \ . \tag{10.6.2}$$

From $dF^i = T^{ij}S_j$ it is clear that bulk viscosity ζ gives to energy momentum tensor a pressure like contribution having interpretation in terms of friction opposing. Shear viscosity η corresponds to the traceless part of the velocity gradient often called just viscosity. This contribution to the stress tensor is non-diagonal and corresponds to momentum transfer in directions not parallel to momentum and makes the flow rotational. This term is essential for the thermal conduction and thermal conductivity vanishes for ideal fluids.

ii. The 3-D total stress tensor can be written as

$$\sigma_{ij} = \rho v_i v_j - p g_{ij} + \sigma_{visc;ij} \ . \tag{10.6.3}$$

The generalization to a 4-D relativistic situation is simple. One just adds terms corresponding to energy density and energy flow to obtain

$$T^{\alpha\beta} = (\rho - p)u^\alpha u^\beta + p g^{\alpha\beta} - \sigma^{\alpha\beta}_{visc} \ . \tag{10.6.4}$$

Here u^α denotes the local four-velocity satisfying $u^\alpha u_\alpha = 1$. The sign factors relate to the concentrations in the definition of Minkowski metric ($(1,-1,-1,-1)$).

iii. If the flow is such that the flow parameters associated with the flow lines integrate to a global flow parameter one can identify new time coordinate t as this flow parameter. This means a transition to a coordinate system in which fluid is at rest everywhere (comoving coordinates in cosmology) so that energy momentum tensor reduces to a diagonal term plus viscous term.

$$T^{\alpha\beta} = (\rho - p)g^{tt}\delta_t^{\alpha}\delta_t^{\beta} + pg^{\alpha\beta} - \sigma_{visc}^{\alpha\beta} \ . \quad (10.6.5)$$

In this case the vanishing of the viscous term means that one has perfect fluid in strong sense.
The existence of a global flow parameter means that one has

$$v_i = \Psi\partial_i\Phi \ . \quad (10.6.6)$$

Ψ and Φ depend on space-time point. The proportionality to a gradient of scalar Φ implies that Φ can be taken as a global time coordinate. If this condition is not satisfied, the perfect fluid property makes sense only locally.

AdS/CFT correspondence allows to deduce a lower limit for the coefficient of shear viscosity as

$$x = \frac{\eta}{s} \geq \frac{\hbar}{4\pi} \ . \quad (10.6.7)$$

This formula holds true in units in which one has $k_B = 1$ so that temperature has unit of energy.

What makes this interesting from TGD view is that in TGD framework perfect fluid property in appropriately generalized sense indeed characterizes locally the preferred extremals of Kähler action defining space-time surface.

i. Kähler action is Maxwell action with U(1) gauge field replaced with the projection of CP_2 Kähler form so that the four CP_2 coordinates become the dynamical variables at QFT limit. This means enormous reduction in the number of degrees of freedom as compared to the ordinary unifications. The field equations for Kähler action define the dynamics of space-time surfaces and this dynamics reduces to conservation laws for the currents assignable to isometries. This means that the system has a hydrodynamic interpretation. This is a considerable difference to ordinary Maxwell equations. Notice however that the "topological" half of Maxwell's equations (Faraday's induction law and the statement that no non-topological magnetic are possible) is satisfied.

ii. Even more, the resulting hydrodynamical system allows an interpretation in terms of a perfect fluid. The general ansatz for the preferred extremals of field equations assumes that various conserved currents are proportional to a vector field characterized by so called Beltrami property. The coefficient of proportionality depends on space-time point and the conserved current in question. Beltrami fields by definition is a vector field such that the time parameters assignable to its flow lines integrate to single global coordinate. This is highly non-trivial and one of the implications is almost topological QFT property due to the fact that Kähler action reduces to a boundary term assignable to wormhole throats which are light-like 3-surfaces at the boundaries of regions of space-time with Euclidian and Minkowskian signatures. The Euclidian regions (or wormhole throats, depends on one's tastes) define what I identify as generalized Feynman diagrams.
Beltrami property means that if the time coordinate for a space-time sheet is chosen to be this global flow parameter, all conserved currents have only time component. In TGD framework energy momentum tensor is replaced with a collection of conserved currents assignable to various isometries and the analog of energy momentum tensor complex constructed in this manner has no counterparts of non-diagonal components. Hence the preferred extremals allow an interpretation in terms of perfect fluid without any viscosity.

This argument justifies the expectation that TGD Universe is characterized by the presence of low-viscosity fluids. Real fluids of course have a non-vanishing albeit small value of x. What causes the failure of the exact perfect fluid property?

i. Many-sheetedness of the space-time is the underlying reason. Space-time surface decomposes into finite-sized space-time sheets containing topologically condensed smaller space-time sheets containing.... Only within given sheet perfect fluid property holds true and fails at wormhole contacts and because the sheet has a finite size. As a consequence, the global flow parameter exists only in given length and time scale. At imbedding space level and in zero energy ontology the phrasing of the same would be in terms of hierarchy of causal diamonds (CDs).

ii. The so called eddy viscosity is caused by eddies (vortices) of the flow. The space-time sheets glued to a larger one are indeed analogous to eddies so that the reduction of viscosity to eddy viscosity could make sense quite generally. Also the phase slippage phenomenon of super-conductivity meaning that the total phase increment of the super-conducting order parameter is reduced by a multiple of 2π in phase slippage so that the average velocity proportional to the increment of the phase along the channel divided by the length of the channel is reduced by a quantized amount.

   The standard arrangement for measuring viscosity involves a lipid layer flowing along plane. The velocity of flow with respect to the surface increases from $v = 0$ at the lower boundary to v_{upper} at the upper boundary of the layer: this situation can be regarded as outcome of the dissipation process and prevails as long as energy is feeded into the system. The reduction of the velocity in direction orthogonal to the layer means that the flow becomes rotational during dissipation leading to this stationary situation.

   This suggests that the elementary building block of dissipation process corresponds to a generation of vortex identifiable as cylindrical space-time sheets parallel to the plane of the flow and orthogonal to the velocity of flow and carrying quantized angular momentum. One expects that vortices have a spectrum labelled by quantum numbers like energy and angular momentum so that dissipation takes in discrete steps by the generation of vortices which transfer the energy and angular momentum to environment and in this manner generate the velocity gradient.

iii. The quantization of the parameter x is suggestive in this framework. If entropy density and viscosity are both proportional to the density n of the eddies, the value of x would equal to the ratio of the quanta of entropy and kinematic viscosity η/n for single eddy if all eddies are identical. The quantum would be $\hbar/4\pi$ in the units used and the suggestive interpretation is in terms of the quantization of angular momentum. One of course expects a spectrum of eddies so that this simple prediction should hold true only at temperatures for which the excitation energies of vortices are above the thermal energy. The increase of the temperature would suggest that gradually more and more vortices come into play and that the ratio increases in a stepwise manner bringing in mind quantum Hall effect. In TGD Universe the value of $\hbar$ can be large in some situations so that the quantal character of dissipation could become visible even macroscopically. Whether this a situation with large $\hbar$ is encountered even in the case of QCD plasma is an interesting question.

The following poor man's argument tries to make the idea about quantization a little bit more concrete.

i. The vortices transfer momentum parallel to the plane from the flow. Therefore they must have momentum parallel to the flow given by the total cm momentum of the vortex. Before continuing some notations are needed. Let the densities of vortices and absorbed vortices be n and n_{abs} respectively. Denote by $v_{\parallel}$ *resp.* $v_{\perp}$ the components of cm momenta parallel to the main flow *resp.* perpendicular to the plane boundary plane. Let m be the mass of the vortex. Denote by S are parallel to the boundary plane.

ii. The flow of momentum component parallel to the main flow due to the absorbed at S is

$$n_{abs} m v_{\parallel} v_{\perp} S \ .$$

This momentum flow must be equal to the viscous force

$$F_{visc} = \eta \frac{v_{\parallel}}{d} \times S \ .$$

From this one obtains

$$\eta = n_{abs} m v_{\perp} d \ .$$

If the entropy density is due to the vortices, it equals apart from possible numerical factors to

$$s = n$$

so that one has

$$\frac{\eta}{s} = m v_{\perp} d \ .$$

This quantity should have lower bound $x = \hbar/4\pi$ and perhaps even quantized in multiples of x, Angular momentum quantization suggests strongly itself as origin of the quantization.

iii. Local momentum conservation requires that the comoving vortices are created in pairs with opposite momenta and thus propagating with opposite velocities $v_{\perp}$. Only one half of vortices is absorbed so that one has $n_{abs} = n/2$. Vortex has quantized angular momentum associated with its internal rotation. Angular momentum is generated to the flow since the vortices flowing downwards are absorbed at the boundary surface.
Suppose that the distance of their center of mass lines parallel to plane is $D = \epsilon d$, ϵ a numerical constant not too far from unity. The vortices of the pair moving in opposite direction have same angular momentum $mv \ \ D/2$ relative to their center of mass line between them. Angular momentum conservation requires that the sum these relative angular momenta cancels the sum of the angular momenta associated with the vortices themselves. Quantization for the total angular momentum for the pair of vortices gives

$$\frac{\eta}{s} = \frac{n\hbar}{\epsilon}$$

Quantization condition would give

$$\epsilon = 4\pi \ .$$

One should understand why $D = 4\pi d$ - four times the circumference for the largest circle contained by the boundary layer- should define the minimal distance between the vortices of the pair. This distance is larger than the distance d for maximally sized vortices of radius $d/2$ just touching. This distance obviously increases as the thickness of the boundary layer increases suggesting that also the radius of the vortices scales like d.

iv. One cannot of course take this detailed model too literally. What is however remarkable that quantization of angular momentum and dissipation mechanism based on vortices identified as space-time sheets indeed could explain why the lower bound for the ratio η/s is so small.

10.6.5 Is the effective metric one- or two-dimensional?

10.6.6 Is the effective metric effectively one- or two-dimensional?

The following argument suggests that the effective metric defined by the anti-commutators of the modified gamma matrices is effectively one- or two-dimensional. Effective one-dimensionality would conform with the observation that the solutions of the modified Dirac equations can be localized to one-dimensional world lines in accordance with the vision that finite measurement resolution implies discretization reducing partonic many-particle states to quantum superpositions of braids. This localization to 1-D curves occurs always at the 3-D orbits of the partonic 2-surfaces.

The argument is based on the following assumptions.

i. The modified gamma matrices for Kähler action are contractions of the canonical momentum densities T^{α}_k with the gamma matrices of H.

ii. The strongest assumption is that the isometry currents

$$J^{A\alpha} = T^{\alpha}_k j^{Ak}$$

for the preferred extremals of Kähler action are of form

$$J^{A\alpha} = \Psi^A (\nabla\Phi)^{\alpha} \quad (10.6.8)$$

with a common function Φ guaranteeing that the flow lines of the currents integrate to coordinate lines of single global coordinate variables (Beltrami property). Index raising is carried out by using the ordinary induced metric.

iii. A weaker assumption is that one has two functions Φ_1 and Φ_2 assignable to the isometry currents of M^4 and CP_2 respectively.:

$$\begin{aligned} J_1^{A\alpha} &= \Psi_1^A (\nabla\Phi_1)^{\alpha} \ , \\ J_2^{A\alpha} &= \Psi_2^A (\nabla\Phi_2)^{\alpha} \ . \end{aligned} \quad (10.6.9)$$

The two functions Φ_1 and Φ_2 could define dual light-like curves spanning string world sheet. In this case one would have effective 2-dimensionality and decomposition to string world sheets [K23] . Isometry invariance does not allow more that two independent scalar functions Φ_i.

Consider now the argument.

i. One can multiply both sides of this equation with j^{Ak} and sum over the index A labeling isometry currents for translations of M^4 and $SU(3)$ currents for CP_2. The tensor quantity $\sum_A j^{Ak} j^{Al}$ is invariant under isometries and must therefore satisfy

$$\sum_A \eta_{AB} j^{Ak} j^{Al} = h^{kl} \ , \quad (10.6.10)$$

where η_{AB} denotes the flat tangent space metric of H. In M^4 degrees of freedom this statement becomes obvious by using linear Minkowski coordinates. In the case of CP_2 one can first consider the simpler case $S^2 = CP_1 = SU(2)/U(1)$. The coset space property implies in standard complex coordinate transforming linearly under $U(1)$ that only the the isometry currents belonging to the complement of $U(1)$ in the sum contribute at the origin and the identity holds true at the origin and by the symmetric space property everywhere. Identity can be verified also directly in standard spherical coordinates. The argument generalizes to the case of $CP_2 = SU(3)/U(2)$ in an obvious manner.

ii. In the most general case one obtains

$$\begin{aligned} T_1^{\alpha k} &= \sum_A \Psi_1^A j^{Ak} \times (\nabla\Phi_1)^{\alpha} \equiv f_1^k (\nabla\Phi_1)^{\alpha} \ , \\ T_2^{\alpha k} &= \sum_A \Psi_1^A j^{Ak} \times (\nabla\Phi_2)^{\alpha} \equiv f_2^k (\nabla\Phi_2)^{\alpha} \ . \end{aligned} \quad (10.6.11)$$

iii. The effective metric given by the anti-commutator of the modified gamma matrices is in turn is given by

$$G^{\alpha\beta} = m_{kl} f_1^k f_1^l (\nabla\Phi_1)^{\alpha} (\nabla\Phi_1)^{\beta} + s_{kl} f_2^k f_2^l (\nabla\Phi_2)^{\alpha} (\nabla\Phi_2)^{\beta} \ . \quad (10.6.12)$$

The covariant form of the effective metric is effectively 1-dimensional for $\Phi_1 = \Phi_2$ in the sense that the only non-vanishing component of the covariant metric $G_{\alpha\beta}$ is diagonal component along the coordinate line defined by $\Phi \equiv \Phi_1 = \Phi_2$. Also the contravariant metric is effectively 1-dimensional since the index raising does not affect the rank of the tensor but depends on the other space-time coordinates. This would correspond to an effective reduction to a dynamics of point-like particles for given selection of braid points. For $\Phi_1 \neq \Phi_2$ the metric is effectively 2-dimensional and would correspond to stringy dynamics.

One can also develop an objection to effective 1- or 2-dimensionality. The proposal for what preferred extremals of Kähler action as deformations of the known extremals of Kähler action could be leads to a beautiful ansatz relying on generalization of conformal invariance and minimal surface equations of string model [K5]. The field equations of TGD reduce to those of classical string model generalized to 4-D context.

If the proposed picture is correct, field equations reduce to purely algebraically conditions stating that the Maxwellian energy momentum tensor for the Kähler action has no common index pairs with the second fundamental form. For the deformations of CP_2 type vacuum extremals T is a complex tensor of type (1,1) and second fundamental form H^k a tensor of type (2,0) and (0,2) so that $Tr(TH^k) =$ is true. This requires that second light-like coordinate of M^4 is constant so that the M^4 projection is 3-dimensional. For Minkowskian signature of the induced metric Hamilton-Jacobi structure replaces conformal structure. Here the dependence of CP_2 coordinates on second light-like coordinate of $M^2(m)$ only plays a fundamental role. Note that now T^{vv} is non-vanishing (and light-like). This picture generalizes to the deformations of cosmic strings and even to the case of vacuum extremals.

There is however an important consistency condition involved. The Maxwell energy momentum tensor for Kähler action must have vanishing covariant divergence. This is satisfied if it is linear combination of Einstein tensor and metric. This gives Einstein's equations with cosmological term in the general case. By the algebraic character of field equations also minimal surface equations are satisfied and Einstein's General Relativity would be exact part of TGD.

In the case of modified Dirac equation the result means that modified gamma matrices are contractions of linear combination of Einstein tensor and metric tensor with the induced gamma matrices so that the TGD counterpart of ordinary Dirac equation would be modified by the addition of a term proportional to Einstein tensor. The condition of effective 1- or 2-dimensionality seems to pose too strong conditions on this combination.

10.7 How to define generalized Feynman diagrams?

S-matrix codes to a high degree the predictions of quantum theories. The longstanding challenge of TGD has been to construct or at least demonstrate the mathematical existence of S-matrix- or actually M-matrix which generalizes this notion in zero energy ontology (ZEO) [K43] . This work has led to the notion of generalized Feynman diagram and the challenge is to give a precise mathematical meaning for this object. The attempt to understand the counterpart of twistors in TGD framework [K59] has inspired several key ideas in this respect but it turned out that twistors themselves need not be absolutely necessary in TGD framework.

i. The notion of generalized Feyman diagram defined by replacing lines of ordinary Feynman diagram with light-like 3-surfaces (elementary particle sized wormhole contacts with throats carrying quantum numbers) and vertices identified as their 2-D ends - I call them partonic 2-surfaces is central. Speaking somewhat loosely, generalized Feynman diagrams (plus background space-time sheets) define the "world of classical worlds" (WCW). These diagrams involve the analogs of stringy diagrams but the interpretation is different: the analogs of stringy loop diagrams have interpretation in terms of particle propagating via two different routes simultaneously (as in the classical double slit experiment) rather than as a decay of particle to two particles. For stringy diagrams the counterparts of vertices are singular as manifolds whereas the entire diagrams are smooth. For generalized Feynman diagrams vertices are smooth but entire diagrams represent singular manifolds just like ordinary Feynman diagrams do. String like objects however emerge in TGD and even ordinary elementary particles are predicted to be magnetic flux tubes of length of order weak gauge boson Compton length with monopoles at their ends as shown in accompanying article. This stringy character should become visible at LHC energies.

ii. Zero energy ontology (ZEO) and causal diamonds (intersections of future and past directed light-cones) define second key ingredient. The crucial observation is that in ZEO it is possible to identify off mass shell particles as pairs of on mass shell fermions at throats of wormhole contact since both positive and negative signs of energy are possible and one obtains also space-like total momenta for wormhole contact behaving as a boson. The localization of fermions to string world sheets and the fact that super-conformal generator G carries fermion number combined with twistorial consideration support the view that the propagators at fermionic lines are of form $(1/G)ip^k\gamma_k(1/G^\dagger + h.c.$ and thus hermitian. In strong models $1/G$ would serve as a propagator and this requires Majorana condition fixing the dimension of the target space to 10 or 11.

iii. A powerful constraint is number theoretic universality requiring the existence of Feynman amplitudes in all number fields when one allows suitable algebraic extensions: roots of unity are certainly required in order to realize p-adic counterparts of plane waves. Also imbedding space, partonic 2-surfaces and WCW must exist in all number fields and their extensions. These constraints are enormously powerful and the attempts to realize this vision have dominated quantum TGD for last two decades.

iv. Representation of 8-D gamma matrices in terms of octonionic units and 2-D sigma matrices is a further important element as far as twistors are considered [K59] . Modified gamma matrices at space-time surfaces are quaternionic/associative and allow a genuine matrix representation. As a matter fact, TGD and WCW could be formulated as study of associative local sub-algebras of the local Clifford algebra of 8-D imbedding space parameterized by quaternionic space-time surfaces.

v. A central conjecture has been that associative (co-associative) 4-surfaces correspond to preferred extremals of Kähler action [K9]. It took long time to realize that in zero energy ontology the notion of preferred extremal might be un-necessary! The reason is that 3-surfaces are now pairs of 3-surfaces at boundaries of causal diamonds and for deterministic dynamics the space-time surface connecting them is expected to be more or less unique. Now the action principle is non-deterministic but the non-determinism would give rise to additional discrete dynamical degrees of freedom naturally assignable to the hierarchy of Planck constants $h_{eff} = n \times h$, n the number of space-time surface with same fixed ends at boundaries of CD and with same values of Kähler action and of conserved quantities. One must be however cautions: this leaves the possibility that there is a gauge symmetry present so that the n sheets correspond to gauge equivalence classes of sheets. Conformal invariance is associated with criticality and is expected to be present also now.

One can of course also ask whether one can assume that the pairs of 3-surfaces at the ends of CD are totally un-correlated. If this assumption is not made then preferred extremal property would make sense also in ZEO and imply additional correlation between the members of these pairs. This kind of correlations would correspond to the Bohr orbit property, which is very attractive space-time correlate for quantum states. This kind of correlates are also expected as space-time counterpart for the correlations between initial and final state in quantum dynamics.

vi. A further conjecture has been that preferred extremals are in some sense critical (second variation of Kähler action could vanish for infinite number of deformations defining a super-conformal algebra). The non-determinism of Kähler action implies this property for $n > 0$ in $h_{eff} = nh$. If the criticality is present, it could correspond to conformal gauge invariance defined by sub-algebras of conformal algebra with conformal weights coming as multiples of n and isomorphic to the conformal algebra itself.

vii. As far as twistors are considered, the first key element is the reduction of the octonionic twistor structure to quaternionic one at space-time surfaces and giving effectively 4-D spinor and twistor structure for quaternionic surfaces.

Quite recently quite a dramatic progress took place in this approach [K18, K59] .

i. The progress was stimulated by the simple observation that on mass shell property puts enormously strong kinematic restrictions on the loop integrations. With mild restrictions on the number of parallel fermion lines appearing in vertices (there can be several since fermionic oscillator operator algebra defining SUSY algebra generates the parton states)- all loops are manifestly finite and if particles has always mass -say small p-adic thermal mass also in case of massless particles and due to IR cutoff due to the presence largest CD- the number of diagrams is finite. Unitarity reduces to Cutkosky rules [B11] automatically satisfied as in the case of ordinary Feynman diagrams.

ii. Ironically, twistors which stimulated all these development do not seem to be absolutely necessary in this approach although they are of course possible. Situation changes if one does not assume small p-adically thermal mass due to the presence of massless particles and one must sum infinite number of diagrams. Here a potential problem is whether the infinite sum respects the algebraic extension in question.

This is about fermionic and momentum space aspects of Feynman diagrams but not yet about the functional (not path-) integral over small deformations of the partonic 2-surfaces. The basic challenges are following.

i. One should perform the functional integral over WCW degrees of freedom for fixed values of on mass shell momenta appearing in the internal lines. After this one must perform integral or summation over loop momenta. Note that the order is important since the space-time surface assigned to the line carries information about the quantum numbers associated with the line by quantum classical correspondence realized in terms of modified Dirac operator.

ii. One must define the functional integral also in the p-adic context. p-Adic Fourier analysis relying on algebraic continuation raises hopes in this respect. p-Adicity suggests strongly that the loop momenta are discretized and ZEO predicts this kind of discretization naturally.

It indeed seems that the functional integrals over WCW could be carried out at general level both in real and p-adic context. This is due to the symmetric space property (maximal number of isometries) of WCW required by the mere mathematical existence of Kähler geometry [K22] in infinite-dimensional context already in the case of much simpler loop spaces [A37] .

i. The p-adic generalization of Fourier analysis allows to algebraize integration- the horrible looking technical challenge of p-adic physics- for symmetric spaces for functions allowing the analog of discrete Fourier decomposition. Symmetric space property is indeed essential also for the existence of Kähler geometry for infinite-D spaces as was learned already from the case of loop spaces. Plane waves and exponential functions expressible as roots of unity and powers of p multiplied by the direct analogs of corresponding exponent functions are the basic building bricks and key functions in harmonic analysis in symmetric spaces. The physically unavoidable finite measurement resolution corresponds to algebraically unavoidable finite algebraic dimension of algebraic extension of p-adics (at least some roots of unity are needed). The cutoff in roots of unity is very reminiscent to that occurring for the representations of quantum groups and is certainly very closely related to these as also to the inclusions of hyper-finite factors of type II¡sub¿1¡/sub¿ defining the finite measurement resolution.

ii. WCW geometrization reduces to that for a single line of the generalized Feynman diagram defining the basic building brick for WCW. Kähler function decomposes to a sum of "kinetic" terms associated with its ends and interaction term associated with the line itself. p-Adicization boils down to the condition that Kähler function, matrix elements of Kähler form, WCW Hamiltonians and their super counterparts, are rational functions of complex WCW coordinates just as they are for those symmetric spaces that I know of. This would allow a continuation to p-adic context.

In the following this vision about generalized Feynman diagrams is discussed in more detail.

10.7.1 Questions

The goal is a proposal for how to perform the integral over WCW for generalized Feynman digrams and the best manner to proceed to to this goal is by making questions.

What does finite measurement resolution mean?

The first question is what finite measurement resolution means.

i. One expects that the algebraic continuation makes sense only for a finite measurement resolution in which case one obtains only finite sums of what one might hope to be algebraic functions. The finiteness of the algebraic extension would be in fact equivalent with the finite measurement resolution.

ii. Finite measurement resolution means a discretization in terms of number theoretic braids. p-Adicization condition suggests that that one must allow only the number theoretic braids. For these the ends of braid at boundary of CD are algebraic points of the imbedding space. This would be true at least in the intersection of real and p-adic worlds.

iii. The question is whether one can localize the points of the braid. The necessity to use momentum eigenstates to achieve quantum classical correspondence in the modified Dirac action [K9] suggests however a de-localization of braid points, that is wave function in space of braid points. In real context one could allow all possible choices for braid points but in p-adic context only algebraic points are possible if one wants to replace integrals with sums. This implies finite measurement resolution analogous to that in lattice. This is also the only possibility in the intersection of real and p-adic worlds.
A non-trivial prediction giving a strong correlation between the geometry of the partonic 2-surface and quantum numbers is that the total number $n_F + n_{\overline{F}}$ of fermions and anti-fermions is bounded above by the number n_{alg} of algebraic points for a given partonic 2-surface: $n_F + n_{\overline{F}} \leq n_{alg}$. Outside the intersection of real and p-adic worlds the problematic aspect of this definition is that small deformations of the partonic 2-surface can radically change the number of algebraic points unless one assumes that the finite measurement resolution means restriction of WCW to a sub-space of algebraic partonic surfaces.

iv. Braids defining propagator lines for fundamental fermions (to be distinguished from observer particles) emerges naturally. Braid strands correspond to the boundaries of string world sheets at which the modes of induced spinor fields are localized from the condition that em charge is well-defined: induced W field and above weak scale also Z^0 field vanish at them.
In order to obtain non-trivial fermion propagator one must add to Kähler-Dirac action Chern-Simons Dirac term located at partonic orbits at which the signature of the induced metric changes. The modes of induced spinor field can be required to be generalized eigenmodes of C-S-D operator with generalized eigenvalue $p^k\gamma_k$ with p^k identified as virtual momentum so that massless Dirac propagator is obtained. p^k is discretized by periodic boundary conditions at opposite boundaries of CD and has IR and UV cutoffs due to the finite size of CD and finite lower limit for the size of sub-CDs.

One has also discretization of the relative position of the second tip of CD at the hyperboloid isometric with mass shell. Only the number of braid points and their momenta would matter, not their positions.
By super-symmetry one must add to Kähler action Chern-Simons term located at partonic orbits and this term must cancel the Chern-Simons term coming from Kähler action by weak form of electric-magnetic duality so that Kähler action reduces to the terms associated with space-like ends of the space-time surface. These terms reduce to Chern-Simons terms if one poses weak form of electric magnetic duality also here. The boundary condition for Kähler-Dirac equations states $\Gamma^n\Psi = 0$ so that incoming fundamental fermions are massless and there is a strong temptation to pose the additional condition $\Gamma^n\Psi = p^k\gamma_k\Psi = 0$
The quantum numbers characterizing positive and negative energy parts of zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse.

How to define integration in WCW degrees of freedom?

The basic question is how to define the integration over WCW degrees of freedom.

i. What comes mind first is Gaussian perturbation theory around the maxima of Kähler function. Gaussian and metric determinants cancel each other and only algebraic expressions remain. Finiteness is not a problem since the Kähler function is non-local functional of 3-surface so that no local interaction vertices are present. One should however assume the vanishing of loops required also by algebraic universality and this assumption look unrealistic when one considers more general functional integrals than that of vacuum functional since free field theory is not in question. The construction of the inverse of the WCW metric defining the propagator is also a very difficult challenge. Duistermaat-Hecke theorem states that something like this known as localization might be possible and one can also argue that something analogous to localization results from a generalization of mean value theorem.

ii. Symmetric space property is more promising since it might reduce the integrations to group theory using the generalization of Fourier analysis for group representations so that there would be no need for perturbation theory in the proposed sense. In finite measurement resolution the symmetric spaces involved would be finite-dimensional. Symmetric space structure of WCW could also allow to define p-adic integration in terms of p-adic Fourier analysis for symmetric spaces. Essentially algebraic continuation of the integration from the real case would be in question with additional constraints coming from the fact that only phase factors corresponding to finite algebraic extensions of rationals are used. Cutoff would emerge automatically from the cutoff for the dimension of the algebraic extension.

How to define generalized Feynman diagrams?

Integration in symmetric spaces could serve as a model at the level of WCW and allow both the understanding of WCW integration and p-adicization as algebraic continuation. In order to get a more realistic view about the problem one must define more precisely what the calculation of the generalized Feynman diagrams means.

i. WCW integration must be carried out separately for all values of the momenta associated with the internal lines. The reason is that the spectrum of eigenvalues λ_i of the modified Dirac operator D depends on the momentum of line and momentum conservation in vertices translates to a correlation of the spectra of D at internal lines.

ii. For tree diagrams algebraic continuation to the p-adic context if the expression involves only the replacement of the generalized eigenvalues of D as functions of momenta with their p-adic counterparts besides vertices. If these functions are algebraically universal and expressible in terms of harmonics of symmetric space , there should be no problems.

iii. If loops are involved, one must integrate/sum over loop momenta. In p-adic context difficulties are encountered if the spectrum of the momenta is continuous. The integration over on mass shell loop momenta is analogous to the integration over sub-CDs, which suggests that internal line corresponds to a $sub-CD$ in which it is at rest. There are excellent reasons to believe that the moduli space for the positions of the upper tip is a discrete subset of hyperboloid of future light-cone. If this is the case, the loop integration indeed reduces to a sum over discrete positions of the tip. p-Adizication would thus give a further good reason why for zero energy ontology.

iv. Propagator is expressible in terms of the inverse of generalized eigenvalue and there is a sum over these for each propagator line. At vertices one has products of WCW harmonics assignable to the incoming lines. The product must have vanishing quantum numbers associated with the phase angle variables of WCW. Non-trivial quantum numbers of the WCW harmonic correspond to WCW quantum numbers assignable to excitations of ordinary elementary particles. WCW harmonics are products of functions depending on the "radial" coordinates and phase factors and the integral over the angles leaves the product of the first ones analogous to Legendre polynomials $P_{l,m}$, These functions are expected to be rational functions or at least algebraic functions involving only square roots.

v. In ordinary QFT incoming and outgoing lines correspond to propagator poles. In the recent case this would mean that incoming stringy lines at the ends of CD correspond to fermions satisfying the stringy mass formula serving as a generalization of masslessness condition.

10.7.2 Generalized Feynman diagrams at fermionic and momentum space level

Negative energy ontology has already led to the idea of interpreting the virtual particles as pairs of positive and negative energy wormhole throats. Hitherto I have taken it as granted that ordinary Feynman diagrammatics generalizes more or less as such. It is however far from clear what really happens in the verties of the generalized Feynman diagrams. The safest approach relies on the requirement that unitarity realized in terms of Cutkosky rules in ordinary Feynman diagrammatics allows a generalization. This requires loop diagrams. In particular, photon-photon scattering can take place only via a fermionic square loop so that it seems that loops must be present at least in the topological sense.

One must be however ready for the possibility that something unexpectedly simple might emerge. For instance, the vision about algebraic physics allows naturally only finite sums for diagrams and does not favor infinite perturbative expansions. Hence the true believer on algebraic physics might dream about finite number of diagrams for a given reaction type. For simplicity generalized Feynman diagrams without the complications brought by the magnetic confinement since by the previous arguments the generalization need not bring in anything essentially new.

The basic idea of duality in early hadronic models was that the lines of the dual diagram representing particles are only re-arranged in the vertices. This however does not allow to get rid of off mass shell momenta. Zero energy ontology encourages to consider a stronger form of this principle in the sense that the virtual momenta of particles could correspond to pairs of on mass shell momenta of particles. If also interacting fermions are pairs of positive and negative energy throats in the interaction region the idea about reducing the construction of Feynman diagrams to some kind of lego rules might work.

Virtual particles as pairs of on mass shell particles in ZEO

The first thing is to try to define more precisely what generalized Feynman diagrams are. The direct generalization of Feynman diagrams implies that both wormhole throats and wormhole contacts join at vertices.

i. A simple intuitive picture about what happens is provided by diagrams obtained by replacing the points of Feynman diagrams (wormhole contacts) with short lines and imagining that the throats correspond to the ends of the line. At vertices where the lines meet the incoming on mass shell quantum numbers would sum up to zero. This approach leads to a straightforward generalization of Feynman diagrams with virtual particles replaced with pairs of on mass shell throat states of type $++$, $--$, and $+-$. Incoming lines correspond to $++$ type lines and outgoing ones to $--$ type lines. The first two line pairs allow only time like net momenta whereas $+-$ line pairs allow also space-like virtual momenta. The sign assigned to a given throat is dictated by the the sign of the on mass shell momentum on the line. The condition that Cutkosky rules generalize as such requires $++$ and $--$ type virtual lines since the cut of the diagram in Cutkosky rules corresponds to on mass shell outgoing or incoming states and must therefore correspond to $++$ or $--$ type lines.

ii. The basic difference as compared to the ordinary Feynman diagrammatics is that loop integrals are integrals over mass shell momenta and that all throats carry on mass shell momenta. In each vertex of the loop mass incoming on mass shell momenta must sum up to on mass shell momentum. These constraints improve the behavior of loop integrals dramatically and give excellent hopes about finiteness. It does not however seem that only a finite number of diagrams contribute to the scattering amplitude besides tree diagrams. The point is that if a the reactions $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$,, where N_i denote particle numbers, are possible in a common kinematical region for N_2-particle states then also the diagrams $N_1 \rightarrow N_2 \rightarrow N_2 \rightarrow N_3$ are possible. The virtual states N_2 include all all states in the intersection of kinematically allow regions for $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$. Hence the dream about finite number possible diagrams is not fulfilled if one allows massless particles. If all particles are massive then the particle number N_2 for given N_1 is limited from above and the dream is realized.

iii. For instance, loops are not possible in the massless case or are highly singular (bringing in mind twistor diagrams) since the conservation laws at vertices imply that the momenta are parallel. In the massive case and allowing mass spectrum the situation is not so simple. As a first example one can consider a loop with three vertices and thus three internal lines. Three on mass shell conditions are present so that the four-momentum can vary in 1-D subspace only. For a loop involving four vertices there are four internal lines and four mass shell conditions so that loop integrals would reduce to discrete sums. Loops involving more than four vertices are expected to be impossible.

iv. The proposed replacement of the elementary fermions with bound states of elementary fermions and monopoles $X_{\pm}$ brings in the analog of stringy diagrammatics. The 2-particle wave functions in the momentum degrees of freedom of fermion and $X_{\pm}$ might allow more flexibility and allow more loops. Note however that there are excellent hopes about the finiteness of the theory also in this case.

Loop integrals are manifestly finite

One can make also more detailed observations about loops.

i. The simplest situation is obtained if only 3-vertices are allowed. In this case conservation of momentum however allows only collinear momenta although the signs of energy need not be the same. Particle creation and annihilation is possible and momentum exchange is possible but is always light-like in the massless case. The scattering matrices of supersymmetric YM theories would suggest something less trivial and this raises the question whether something is missing. Magnetic monopoles are an essential element of also these theories as also massivation and symmetry breaking and this encourages to think that the formation of massive states as fermion $X_\pm$ pairs is needed. Of course, in TGD framework one has also high mass excitations of the massless states making the scattering matrix non-trivial.

ii. In YM theories on mass shell lines would be singular. In TGD framework this is not the case since the propagator is defined as the inverse of the 3-D dimensional reduction of the modified Dirac operator D containing also coupling to four-momentum (this is required by quantum classical correspondence and guarantees stringy propagators),

$$\begin{aligned} D &= i\hat{\Gamma}^\alpha p_\alpha + \hat{\Gamma}^\alpha D_\alpha \ , \\ p_\alpha &= p_k \partial_\alpha h^k \ . \end{aligned} \tag{10.7.1}$$

The propagator does not diverge for on mass shell massless momenta and the propagator lines are well-defined. This is of course of essential importance also in general case. Only for the incoming lines one can consider the possibility that 3-D Dirac operator annihilates the induced spinor fields. All lines correspond to generalized eigenstates of the propagator in the sense that one has $D_3\Psi = \lambda\gamma\Psi$, where γ is modified gamma matrix in the direction of the stringy coordinate emanating from light-like surface and D_3 is the 3-dimensional dimensional reduction of the 4-D modified Dirac operator. The eigenvalue λ is analogous to energy. Note that the eigenvalue spectrum depends on 4-momentum as a parameter.

iii. Massless incoming momenta can decay to massless momenta with both signs of energy. The integration measure $d^2k/2E$ reduces to dx/x where $x \geq 0$ is the scaling factor of massless momentum. Only light-like momentum exchanges are however possible and scattering matrix is essentially trivial. The loop integrals are finite apart from the possible delicacies related to poles since the loop integrands for given massless wormhole contact are proportional to dx/x^3 for large values of x.

iv. Irrespective of whether the particles are massless or not, the divergences are obtained only if one allows too high vertices as self energy loops for which the number of momentum degrees of freedom is $3N-4$ for N-vertex. The construction of SUSY limit of TGD in [K19] led to the conclusion that the parallelly propagating N fermions for given wormhole throat correspond to a product of N fermion propagators with same four-momentum so that for fermions and ordinary bosons one has the standard behavior but for $N > 2$ non-standard so that these excitations are not seen as ordinary particles. Higher vertices are finite only if the total number N_F of fermions propagating in the loop satisfies $N_F > 3N-4$. For instance, a 4-vertex from which $N = 2$ states emanate is finite.

Taking into account magnetic confinement

What has been said above is not quite enough. The weak form of electric-magnetic duality [B2] leads to the picture about elementary particles as pairs of magnetic monopoles inspiring the notions of weak confinement based on magnetic monopole force. Also color confinement would have magnetic counterpart. This means that elementary particles would behave like string like objects in weak boson length scale. Therefore one must also consider the stringy case with wormhole throats replaced with fermion-$X_\pm$ pairs ($X_\pm$ is electromagnetically neutral and $\pm$ refers to the sign of the weak isospin opposite to that of fermion) and their super partners.

i. The simplest assumption in the stringy case is that fermion-$X_\pm$ pairs behave as coherent objects, that is scatter elastically. In more general case only their higher excitations identifiable in terms of stringy degrees of freedom would be created in vertices. The massivation of these states makes possible non-collinear vertices. An open question is how the massivation fermion-$X_\pm$ pairs relates to the existing TGD based description of massivation in terms of Higgs mechanism and modified Dirac operator.

ii. Mass renormalization could come from self energy loops with negative energy lines as also vertex normalization. By very general arguments supersymmetry implies the cancellation of the self energy loops but would allow non-trivial vertex renormalization [K19] .

iii. If only 3-vertices are allowed, the loops containing only positive energy lines are possible if on mass shell fermion-$X_\pm$ pair (or its superpartner) can decay to a pair of positive energy pair particles of same kind. Whether this is possible depends on the masses involved. For ordinary particles these decays are not kinematically possible below intermediate boson mass scale (the decays $F_1 \rightarrow F_2 + \gamma$ are forbidden kinematically or by the absence of flavor changing neutral currents whereas intermediate gauge bosons can decay to on mass shell fermion-anti-fermion pair).

iv. The introduction of IR cutoff for 3-momentum in the rest system associated with the largest CD (causal diamond) looks natural as scale parameter of coupling constant evolution and p-adic length scale hypothesis favors the inverse of the size scale of CD coming in powers of two. This parameter would define the momentum resolution as a discrete parameter of the p-adic coupling constant evolution. This scale does not have any counterpart in standard physics. For electron, d quark, and u quark the proper time distance between the tips of CD corresponds to frequency of 10 Hz, 1280 Hz, and 160 Hz: all these frequencies define fundamental bio-rhythms [K14] .

These considerations have left completely untouched one important aspect of generalized Feynman diagrams: the necessity to perform a functional integral over the deformations of the partonic 2-surfaces at the ends of the lines- that is integration over WCW. Number theoretical universality requires that WCW and these integrals make sense also p-adically and in the following these aspects of generalized Feynman diagrams are discussed.

10.7.3 Harmonic analysis in WCW as a manner to calculate WCW functional integrals

Previous examples suggest that symmetric space property, Kähler and symplectic structure and the use of symplectic coordinates consisting of canonically conjugate pairs of phase angles and corresponding "radial" coordinates are essential for WCW integration and p-adicization. Kähler function, the components of the metric, and therefore also metric determinant and Kähler function depend on the "radial" coordinates only and the possible generalization involves the identification the counterparts of the "radial" coordinates in the case of WCW.

Conditions guaranteeing the reduction to harmonic analysis

The basic idea is that harmonic analysis in symmetric space allows to calculate the functional integral over WCW.

i. Each propagator line corresponds to a symmetric space defined as a coset space G/H of the symplectic group and Kac-Moody group and one might hope that the proposed p-adicization works for it- at least when one considers the hierarchy of measurement resolutions forced by the finiteness of algebraic extensions. This coset space is as a manifold Cartesian product $(G/H) \times (G/H)$ of symmetric spaces G/H associated with ends of the line. Kähler metric contains also an interaction term between the factors of the Cartesian product so that Kähler function can be said to reduce to a sum of "kinetic" terms and interaction term.

ii. Effective 2-dimensionality and ZEO allow to treat the ends of the propagator line independently. This means an enormous simplification. Each line contributes besides propagator a piece to the exponent of Kähler action identifiable as interaction term in action and depending on the propagator momentum. This contribution should be expressible in terms of generalized spherical harmonics. Essentially a sum over the products of pairs of harmonics associated with the ends of the line multiplied by coefficients analogous to $1/(p^2 - m^2)$ in the case of the ordinary propagator would be in question. The optimal situation is that the pairs are harmonics and their conjugates appear so that one has invariance under G analogous to momentum conservation for the lines of ordinary Feynman diagrams.

iii. Momentum conservation correlates the eigenvalue spectra of the modified Dirac operator D at propagator lines [K9] . G-invariance at vertex dictates the vertex as the singlet part of the product of WCW harmonics associated with the vertex and one sums over the harmonics for each internal line. p-Adicization means only the algebraic continuation to real formulas to p-adic context.

iv. The exponent of Kähler function depends on both ends of the line and this means that the geometries at the ends are correlated in the sense that that Kähler form contains interaction terms between the line ends. It is however not quite clear whether it contains separate "kinetic" or self interaction terms assignable to the line ends. For Kähler function the kinetic and interaction terms should have the following general expressions as functions of complex WCW coordinates:

$$\begin{aligned} K_{kin,i} &= \sum_n f_{i,n}(Z_i)\overline{f_{i,n}(Z_i)} + c.c \ , \\ K_{int} &= \sum_n g_{1,n}(Z_1)\overline{g_{2,n}(Z_2)} + c.c \ , i = 1,2 \ . \end{aligned} \tag{10.7.2}$$

Here $K_{kin,i}$ define "kinetic" terms and K_{int} defines interaction term. One would have what might be called holomorphic factorization suggesting a connection with conformal field theories.

Symmetric space property -that is isometry invariance- suggests that one has

$$f_{i,n} = f_{2,n} \equiv f_n \ , \ g_{1,n} = g_{2,n} \equiv g_n \tag{10.7.3}$$

such that the products are invariant under the group H appearing in G/H and therefore have opposite H quantum numbers. The exponent of Kähler function does not factorize although the terms in its Taylor expansion factorize to products whose factors are products of holomorphic and antiholomorphic functions.

v. If one assumes that the exponent of Kähler function reduces to a product of eigenvalues of the modified Dirac operator eigenvalues must have the decomposition

$$\lambda_k = \prod_{i=1,2} exp\left[\sum_n c_{k,n} g_n(Z_i)\overline{g_n(Z_i)} + c.c\right] \times exp\left[\sum_n d_{k,n} g_n(Z_1)\overline{g_n(Z_2)} + c.c\right] \tag{10.7.4}$$

Hence also the eigenvalues coming from the Dirac propagators have also expansion in terms of G/H harmonics so that in principle WCW integration would reduce to Fourier analysis in symmetric space.

Generalization of WCW Hamiltonians

This picture requires a generalization of the view about configuration space Hamiltonians since also the interaction term between the ends of the line is present not taken into account in the previous approach.

i. The proposed representation of WCW Hamiltonians as flux Hamiltonians [K10, K9]

$$\begin{aligned} Q(H_A) &= \int H_A(1+K)Jd^2x \ , \\ J &= \epsilon^{\alpha\beta}J_{\alpha\beta} \ , \ J^{03}\sqrt{g_4} = KJ_{12} \ . \end{aligned} \tag{10.7.5}$$

works for the kinetic terms only since J cannot be the same at the ends of the line. The formula defining K assumes weak form of self-duality (^{03} refers to the coordinates in the complement of X^2 tangent plane in the 4-D tangent plane). K is assumed to be symplectic invariant and constant for given X^2. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K gives the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant. Within experimental uncertainties one has $\alpha_K = g_K^/ 4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

The assumption that Poisson bracket of WCW Hamiltonians reduces to the level of imbedding space - in other words $\{Q(H_A), Q(H_B\} = Q(\{H_A, H_B\})$ - can be justified. One starts from the representation in terms of say flux Hamiltonians $Q(H_A)$ and *defines* $J_{A,B}$ as $J_{A,B} \equiv Q(\{H_A, H_B\})$. One has $\partial H_A/\partial t_B = \{H_B, H_A\}$, where t_B is the parameter associated with the exponentiation of H_B. The inverse J^{AB} of $J_{A,B} = \partial H_B/\partial t_A$ is expressible as $J^{A,B} = \partial t_A/\partial H_B$. From these formulas one can deduce by using chain rule that the bracket $\{Q(H_A), Q(H_B\} = \partial t_C Q(H_A) J^{CD} \partial t_D Q(H_B)$ of flux Hamiltonians equals to the flux Hamiltonian $Q(\{H_A, H_B\})$.

ii. One should be able to assign to WCW Hamiltonians also a part corresponding to the interaction term. The symplectic conjugation associated with the interaction term permutes the WCW coordinates assignable to the ends of the line. One should reduce this apparently non-local symplectic conjugation (if one thinks the ends of line as separate objects) to a non-local symplectic conjugation for $\delta CD \times CP_2$ by identifying the points of lower and upper end of CD related by time reflection and assuming that conjugation corresponds to time reflection. Formally this gives a well defined generalization of the local Poisson brackets between time reflected points at the boundaries of CD. The connection of Hermitian conjugation and time reflection in quantum field theories is is in accordance with this picture.

iii. The only manner to proceed is to assign to the flux Hamiltonian also a part obtained by the replacement of the flux integral over X^2 with an integral over the projection of X^2 to a sphere S^2 assignable to the light-cone boundary or to a geodesic sphere of CP_2, which come as two varieties corresponding to homologically trivial and non-trivial spheres. The projection is defined as by the geodesic line orthogonal to S^2 and going through the point of X^2. The hierarchy of Planck constants assigns to CD a preferred geodesic sphere of CP_2 as well as a unique sphere S^2 as a sphere for which the radial coordinate r_M or the light-cone boundary defined uniquely is constant: this radial coordinate corresponds to spherical coordinate in the rest system defined by the time-like vector connecting the tips of CD. Either spheres or possibly both of them could be relevant.

Recall that also the construction of number theoretic braids and symplectic QFT [K12] led to the proposal that braid diagrams and symplectic triangulations could be defined in terms of projections of braid strands to one of these spheres. One could also consider a weakening for the condition that the points of the number theoretic braid are algebraic by requiring only that the S^2 coordinates of the projection are algebraic and that these coordinates correspond to the discretization of S^2 in terms of the phase angles associated with θ and ϕ.

This gives for the corresponding contribution of the WCW Hamiltonian the expression

$$Q(H_A)_{int} = \int_{S^2_\pm} H_A X \delta^2(s_+, s_-) d^2 s_\pm = \int_{P(X^2_+)\cap P(X^2_-)} \frac{\partial(s^1, s^2)}{\partial(x^1_\pm, x^2_\pm)} d^2 x_\pm \quad (10.7.6)$$

Here the Poisson brackets between ends of the line using the rules involve delta function $\delta^2(s_+, s_-)$ at S^2 and the resulting Hamiltonians can be expressed as a similar integral of $H_{[A,B]}$ over the upper or lower end since the integral is over the intersection of S^2 projections.

The expression must vanish when the induced Kähler form vanishes for either end. This is achieved by identifying the scalar X in the following manner:

$$\begin{aligned} X &= J_+^{kl} J_{kl}^- \ , \\ J_\pm^{kl} &= (1 + K_\pm)\partial_\alpha s^k \partial_\beta s^l J_\pm^{\alpha\beta} \ . \end{aligned} \quad (10.7.7)$$

The tensors are lifts of the induced Kähler form of $X^2_\pm$ to S^2 (not CP_2).

iv. One could of course ask why these Hamiltonians could not contribute also to the kinetic terms and why the brackets with flux Hamiltonians should vanish. This relate to how one *defines* the Kähler form. It was shown above that in case of flux Hamiltonians the definition of Kähler form as brackets gives the basic formula $\{Q(H_A), Q(H_B)\} = Q(\{H_A, H_B\}$ and same should hold true now. In the recent case $J_{A,B}$ would contain an interaction term defined in terms of flux Hamiltonians and the previous argument should go through also now by identifying Hamiltonians as sums of two contributions and by introducing the doubling of the coordinates t_A.
v. The quantization of the modified Dirac operator must be reconsidered. It would seem that one must add to the super-Hamiltonian completely analogous term obtained by replacing $(1+K)J$ with $X\partial(s^1, s^2)/\partial(x^1_\pm, x^2_\pm)$. Besides the anti-commutation relations defining correct anti-commutators to flux Hamiltonians, one should pose anti-commutation relations consistent with the anti-commutation relations of super Hamiltonians. In these anti-commutation relations $(1+K)J\delta^2(x, y)$ would be replaced with $X\delta^2(s^+, s^-)$. This would guarantee that the oscillator operators at the ends of the line are not independent and that the resulting Hamiltonian reduces to integral over either end for $H_{[A,B]}$.
vi. In the case of CP_2 the Hamiltonians generating isometries are rational functions. This should hold true also now so that p-adic variants of Hamiltonians as functions in WCW would make sense. This in turn would imply that the components of the WCW Kähler form are rational functions. Also the exponentiation of Hamiltonians make sense p-adically if one allows the exponents of group parameters to be functions $Exp_p(t)$.

Does the expansion in terms of partial harmonics converge?

The individual terms in the partial wave expansion seem to be finite but it is not at all clear whether the expansion in powers of K actually converges.

i. In the proposed scenario one performs the expansion of the vacuum functional $exp(K)$ in powers of K and therefore in negative powers of α_K. In principle an infinite number of terms can be present. This is analogous to the perturbative expansion based on using magnetic monopoles as basic objects whereas the expansion using the contravariant Kähler metric as a propagator would be in positive powers of α_K and analogous to the expansion in terms of magnetically bound states of wormhole throats with vanishing net value of magnetic charge. At this moment one can only suggest various approaches to how one could understand the situation.
ii. Weak form of self-duality and magnetic confinement could change the situation. Performing the perturbation around magnetic flux tubes together with the assumed slicing of the space-time sheet by stringy world sheets and partonic 2-surfaces could mean that the perturbation corresponds to the action assignable to the electric part of Kähler form proportional to α_K by the weak self-duality. Hence by $K = 4\pi\alpha_K$ relating Kähler electric field to Kähler magnetic field the expansion would come in powers of a term containing sum of terms proportional to α_K^0 and α_K. This would leave to the scattering amplitudes the exponents of Kähler function at the maximum of Kähler function so that the non-analytic dependence on α_K would not disappear.

A further reason to be worried about is that the expansion containing infinite number of terms proportional to α_K^0 could fail to converge.

i. This could be also seen as a reason for why magnetic singlets are unavoidable except perhaps for $\hbar < \hbar_0$. By the holomorphic factorization the powers of the interaction part of Kähler action in powers of $1/\alpha_K$ would naturally correspond to increasing and opposite net values of the quantum numbers assignable to the WCW phase coordinates at the ends of the propagator line. The magnetic bound states could have similar expansion in powers of α_K as pairs of states with arbitrarily high but opposite values of quantum numbers. In the functional integral these quantum numbers would compensate each other. The functional integral would leave only an expansion containing powers of α_K starting from some finite possibly negative (unless one assumes the weak form of self-duality) power. Various gauge coupling strengths are expected to be proportional to α_K and these expansions should reduce to those in powers of α_K.

ii. Since the number of terms in the fermionic propagator expansion is finite, one might hope on basis of super-symmetry that the same is true in the case of the functional integral expansion. By the holomorphic factorization the expansion in powers of K means the appearance of terms with increasingly higher quantum numbers. Quantum number conservation at vertices would leave only a finite number of terms to tree diagrams. In the case of loop diagrams pairs of particles with opposite and arbitrarily high values of quantum numbers could be generated at the vertex and magnetic confinement might be necessary to guarantee the convergence. Also super-symmetry could imply cancellations in loops.

Could one do without flux Hamiltonians?

The fact that the Kähler functions associated with the propagator lines can be regarded as interaction terms inspires the question whether the Kähler function could contain only the interaction terms so that Kähler form and Kähler metric would have components only between the ends of the lines.

i. The basic objection is that flux Hamiltonians too beautiful objects to be left without any role in the theory. One could also argue that the WCW metric would not be positive definite if only the non-diagonal interaction term is present. The simplest example is Hermitian 2×2-matrix with vanishing diagonal for which eigenvalues are real but of opposite sign.

ii. One could of course argue that the expansions of $exp(K)$ and λ_k give in the general powers $(f_n\overline{f_n})^m$ analogous to diverging tadpole diagrams of quantum field theories due to local interaction vertices. These terms do not produce divergences now but the possibility that the exponential series of this kind of terms could diverge cannot be excluded. The absence of the kinetic terms would allow to get rid of these terms and might be argued to be the symmetric space counterpart for the vanishing of loops in WCW integral.

iii. In zero energy ontology this idea does not look completely non-sensical since physical states are pairs of positive and negative energy states. Note also that in quantum theory only creation operators are used to create positive energy states. The manifest non-locality of the interaction terms and absence of the counterparts of kinetic terms would provide a trivial manner to get rid of infinities due to the presence of local interactions. The safest option is however to keep both terms.

Summary

The discussion suggests that one must treat the entire Feynman graph as single geometric object with Kähler geometry in which the symmetric space is defined as product of what could be regarded as analogs of symmetric spaces with interaction terms of the metric coming from the propagator lines. The exponent of Kähler function would be the product of exponents associated with all lines and contributions to lines depend on quantum numbers (momentum and color quantum numbers) propagating in line via the coupling to the modified Dirac operator. The conformal factorization would allow the reduction of integrations to Fourier analysis in symmetric space. What is of decisive importance is that the entire Feynman diagrammatics at WCW level would reduce to the construction of WCW geometry for a single propagator line as a function of quantum numbers propagating on the line.

10.8 Do geometric invariants of preferred extremals define topological invariants of space-time surface and code for quantum physics?

The recent progress in the understanding of preferred extremals [K5] led to a reduction of the field equations to conditions stating for Euclidian signature the existence of Kähler metric. The resulting conditions are a direct generalization of corresponding conditions emerging for the string world sheet and stating that the 2-metric has only non-diagonal components in complex/hypercomplex coordinates. Also energy momentum of Kähler action and has this characteristic (1,1) tensor structure. In Minkowskian signature one obtains the analog of 4-D complex structure combining hyper-complex structure and 2-D complex structure.

The construction lead also to the understanding of how Einstein's equations with cosmological term follow as a consistency condition guaranteeing that the covariant divergence of the Maxwell's energy momentum tensor assignable to Kähler action vanishes. This gives $T = kG + \Lambda g$. By taking trace a further condition follows from the vanishing trace of T:

$$R = \frac{4\Lambda}{k} . \tag{10.8.1}$$

That any preferred extremal should have a constant Ricci scalar proportional to cosmological constant is very strong prediction. Note that the accelerating expansion of the Universe would support positive value of Λ. Note however that both Λ and $k \propto 1/G$ are both parameters characterizing one particular preferred extremal. One could of course argue that the dynamics allowing only constant curvature space-times is too simple. The point is however that particle can topologically condense on several space-time sheets meaning effective superposition of various classical fields defined by induced metric and spinor connection.

The following considerations demonstrate that preferred extremals can be seen as canonical representatives for the constant curvature manifolds playing central role in Thurston's geometrization theorem [A25] known also as hyperbolization theorem implying that geometric invariants of space-time surfaces transform to topological invariants. The generalization of the notion of Ricci flow to Maxwell flow in the space of metrics and further to Kähler flow for preferred extremals in turn gives a rather detailed vision about how preferred extremals organize to one-parameter orbits. It is quite possible that Kähler flow is actually discrete. The natural interpretation is in terms of dissipation and self organization.

Quantum classical correspondence suggests that this line of thought could be continued even further: could the geometric invariants of the preferred extremals could code not only for space-time topology but also for quantum physics? How to calculate the correlation functions and coupling constant evolution has remained a basic unresolved challenge of quantum TGD. Could the correlation functions be reduced to statistical geometric invariants of preferred extemals? The latest (means the end of 2012) and perhaps the most powerful idea hitherto about coupling constant evolution is quantum classical correspondence in statistical sense stating that the statistical properties of a preferred extremal in quantum superposition of them are same as those of the zero energy state in question. This principle would be quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This principle would allow to deduce correlation functions and S-matrix from the statistical properties of single preferred extremal alone using classical intuition. Also coupling constant evolution would be coded by the statistical properties of the representative preferred extremal.

10.8.1 Preferred extremals of Kähler action as manifolds with constant Ricci scalar whose geometric invariants are topological invariants

An old conjecture inspired by the preferred extremal property is that the geometric invariants of space-time surface serve as topological invariants. The reduction of Kähler action to 3-D Chern-Simons terms [K5] gives support for this conjecture as a classical counterpart for the view about TGD as almost topological QFT. The following arguments give a more precise content to this conjecture in terms of existing mathematics.

i. It is not possible to represent the scaling of the induced metric as a deformation of the space-time surface preserving the preferred extremal property since the scale of CP_2 breaks scale invariance. Therefore the curvature scalar cannot be chosen to be equal to one numerically. Therefore also the parameter $R = 4\Lambda/k$ and also Λ and k separately characterize the equivalence class of preferred extremals as is also physically clear.
Also the volume of the space-time sheet closed inside causal diamond CD remains constant along the orbits of the flow and thus characterizes the space-time surface. Λ and even $k \propto 1/G$ can indeed depend on space-time sheet and p-adic length scale hypothesis suggests a discrete spectrum for Λ/k expressible in terms of p-adic length scales: $\Lambda/k \propto 1/L_p^2$ with $p \simeq 2^k$ favored by p-adic length scale hypothesis. During cosmic evolution the p-adic length scale would increase gradually. This would resolve the problem posed by cosmological constant in GRT based theories.

ii. One could also see the preferred extremals as 4-D counterparts of constant curvature 3-manifolds in the topology of 3-manifolds. An interesting possibility raised by the observed negative value of Λ is that most 4-surfaces are constant negative curvature 4-manifolds. By a general theorem coset spaces H^4/Γ, where $H^4 = SO(1,4)/SO(4)$ is hyperboloid of M^5 and Γ a torsion free discrete subgroup of $SO(1,4)$ [A10]. It is not clear to me, whether the constant value of Ricci scalar implies constant sectional curvatures and therefore hyperbolic space property. It could happen that the space of spaces with constant Ricci curvature contain a hyperbolic manifold as an especially symmetric representative. In any case, the geometric invariants of hyperbolic metric are topological invariants.
By Mostow rigidity theorem [A16] finite-volume hyperbolic manifold is unique for $D > 2$ and determined by the fundamental group of the manifold. Since the orbits under the Kähler flow preserve the curvature scalar the manifolds at the orbit must represent different imbeddings of one and hyperbolic 4-manifold. In 2-D case the moduli space for hyperbolic metric for a given genus $g > 0$ is defined by Teichmueller parameters and has dimension $6(g-1)$. Obviously the exceptional character of $D = 2$ case relates to conformal invariance. Note that the moduli space in question plays a key role in p-adic mass calculations [K11].
In the recent case Mostow rigidity theorem could hold true for the Euclidian regions and maybe generalize also to Minkowskian regions. If so then both "topological" and "geometro" in "Topological GeometroDynamics" would be fully justified. The fact that geometric invariants become topological invariants also conforms with "TGD as almost topological QFT" and allows the notion of scale to find its place in topology. Also the dream about exact solvability of the theory would be realized in rather convincing manner.

These conjectures are the main result independent of whether the generalization of the Ricci flow discussed in the sequel exists as a continuous flow or possibly discrete sequence of iterates in the space of preferred extremals of Kähler action. My sincere hope is that the reader could grasp how far reaching these result really are.

10.8.2 Is there a connection between preferred extremals and AdS$_4$/CFT correspondence?

The preferred extremals satisfy Einstein Maxwell equations with a cosmological constant and have negative scalar curvature for negative value of Λ. 4-D space-times with hyperbolic metric provide canonical representation for a large class of four-manifolds and an interesting question is whether these spaces are obtained as preferred extremals and/or vacuum extremals.

4-D hyperbolic space with Minkowski signature is locally isometric with AdS$_4$. This suggests at connection with AdS$_4$/CFT correspondence of M-theory. The boundary of AdS would be now replaced with 3-D light-like orbit of partonic 2-surface at which the signature of the induced metric changes. The metric 2-dimensionality of the light-like surface makes possible generalization of 2-D conformal invariance with the light-like coordinate taking the role of complex coordinate at light-like boundary. AdS could represent a special case of a more general family of space-time surfaces with constant Ricci scalar satistying Einstein-Maxwell equations and generalizing the AdS$_4$/CFT correspondence. There is however a strong objection from cosmology: the accelerated expansion of the Universe requires positive value of Λ and favors De Sitter Space dS_4 instead of AdS_4.

These observations provide motivations for finding whether AdS$_4$ and/or dS_4 allows an imbedding as a vacuum extremal to $M^4\times S^2\subset M^4\times CP_2$, where S^2 is a homologically trivial geodesic sphere of CP_2. It is easy to guess the general form of the imbedding by writing the line elements of, M^4, S^2, and AdS$_4$.

i. The line element of M^4 in spherical Minkowski coordinates (m, r_M, θ, ϕ) reads as

$$ds^2 = dm^2 - dr_M^2 - r_M^2 d\Omega^2 \ . \tag{10.8.2}$$

ii. Also the line element of S^2 is familiar:

$$ds^2 = -R^2(d\Theta^2 + sin^2(\theta)d\Phi^2) \ . \tag{10.8.3}$$

iii. By visiting in Wikipedia one learns that in spherical coordinate the line element of AdS$_4$/dS_4 is given by

$$\begin{aligned} ds^2 &= A(r)dt^2 - \frac{1}{A(r)}dr^2 - r^2 d\Omega^2 \ , \\ A(r) &= 1 + \epsilon y^2 \ , \ y = \frac{r}{r_0} \ , \\ \epsilon &= 1 \ \text{for} \ AdS_4 \ , \ \epsilon = -1 \ \text{for} \ dS_4 \ . \end{aligned} \tag{10.8.4}$$

iv. From these formulas it is easy to see that the ansatz is of the same general form as for the imbedding of Schwartschild-Nordstöm metric:

$$\begin{array}{ll} m = \Lambda t + h(y) \ , & r_M = r \ , \\ \Theta = s(y) \ , & \Phi = \omega(t + f(y)) \ . \end{array} \tag{10.8.5}$$

The non-trivial conditions on the components of the induced metric are given by

$$\begin{aligned} g_{tt} &= \Lambda^2 - x^2 sin^2(\Theta) = A(r) \ , \\ g_{tr} &= \frac{1}{r_0}\left[\Lambda\frac{dh}{dy} - x^2 sin^2(\theta)\frac{df}{dr}\right] = 0 \ , \\ g_{rr} &= \frac{1}{r_0^2}\left[(\frac{dh}{dy})^2 - 1 - x^2 sin^2(\theta)(\frac{df}{dy})^2 - R^2(\frac{d\Theta}{dy})^2\right] = -\frac{1}{A(r)} \ , \\ x &= R\omega \ . \end{aligned} \tag{10.8.6}$$

By some simple algebraic manipulations one can derive expressions for $sin(\Theta)$, df/dr and dh/dr.

i. For $\Theta(r)$ the equation for g_{tt} gives the expression

$$\begin{aligned} sin(\Theta) &= \pm\frac{P^{1/2}}{x} \ , \\ P &= \Lambda^2 - A = \Lambda^2 - 1 - \epsilon y^2 \ . \end{aligned} \qquad (10.8.7)$$

The condition $0 \leq sin^2(\Theta) \leq 1$ gives the conditions

$$\begin{array}{rcll} (\Lambda^2 - x^2 - 1)^{1/2} \leq & y & \leq (\Lambda^2 - 1)^{1/2} & \text{for } \epsilon = 1 \ (AdS_4) \ , \\ (-\Lambda^2 + 1)^{1/2} \leq & y & \leq (x^2 + 1 - \Lambda^2)^{1/2} & \text{for } \epsilon = -1 \ (dS_4) \ . \end{array} \qquad (10.8.8)$$

Only a spherical shell is possible in both cases. The model for the final state of star considered in [K56] predicted similar layer layer like structure and inspired the proposal that stars quite generally have an onion-like structure with radii of various shells characterize by p-adic length scale hypothesis and thus coming in some powers of $\sqrt{2}$. This brings in mind also Titius-Bode law.

ii. From the vanishing of g_{tr} one obtains

$$\frac{dh}{dy} = \frac{P}{\Lambda}\frac{df}{dy} \ . \qquad (10.8.9)$$

iii. The condition for g_{rr} gives

$$(\frac{df}{dy})^2 = \frac{r_0^2}{AP}[A^{-1} - R^2(\frac{d\Theta}{dy})^2] \ . \qquad (10.8.10)$$

Clearly, the right-hand side is positive if $P \geq 0$ holds true and $Rd\Theta/dy$ is small. One can express $d\Theta/dy$ using chain rule as

$$(\frac{d\Theta}{dy})^2 = \frac{x^2y^2}{P(P-x^2)} \ . \qquad (10.8.11)$$

One obtains

$$(\frac{df}{dy})^2 = \Lambda r_0^2\frac{y^2}{AP}\left[\frac{1}{1+y^2} - x^2(\frac{R}{r_0})^2\frac{1}{P(P-x^2)}\right] \ . \qquad (10.8.12)$$

The right hand side of this equation is non-negative for certain range of parameters and variable y. Note that for $r_0 \gg R$ the second term on the right hand side can be neglected. In this case it is easy to integrate $f(y)$.

The conclusion is that both AdS_4 and dS^4 allow a local imbedding as a vacuum extremal. Whether also an imbedding as a non-vacuum preferred extremal to $M^4 \times S^2$, S^2 a homologically non-trivial geodesic sphere is possible, is an interesting question.

10.8.3 Generalizing Ricci flow to Maxwell flow for 4-geometries and Kähler flow for space-time surfaces

The notion of Ricci flow has played a key part in the geometrization of topological invariants of Riemann manifolds. I certainly did not have this in mind when I choose to call my unification attempt "Topological Geometrodynamics" but this title strongly suggests that a suitable generalization of Ricci flow could play a key role in the understanding of also TGD.

Ricci flow and Maxwell flow for 4-geometries

The observation about constancy of 4-D curvature scalar for preferred extremals inspires a generalization of the well-known volume preserving Ricci flow [A22] introduced by Richard Hamilton. Ricci flow is defined in the space of Riemann metrics as

$$\frac{dg_{\alpha\beta}}{dt} = -2R_{\alpha\beta} + 2\frac{R_{avg}}{D}g_{\alpha\beta} \ . \tag{10.8.13}$$

Here R_{avg} denotes the average of the scalar curvature, and D is the dimension of the Riemann manifold. The flow is volume preserving in average sense as one easily checks ($\langle g^{\alpha\beta}dg_{\alpha\beta}/dt\rangle = 0$). The volume preserving property of this flow allows to intuitively understand that the volume of a 3-manifold in the asymptotic metric defined by the Ricci flow is topological invariant. The fixed points of the flow serve as canonical representatives for the topological equivalence classes of 3-manifolds. These 3-manifolds (for instance hyperbolic 3-manifolds with constant sectional curvatures) are highly symmetric. This is easy to understand since the flow is dissipative and destroys all details from the metric.

What happens in the recent case? The first thing to do is to consider what might be called Maxwell flow in the space of all 4-D Riemann manifolds allowing Maxwell field.

i. First of all, the vanishing of the trace of Maxwell's energy momentum tensor codes for the volume preserving character of the flow defined as

$$\frac{dg_{\alpha\beta}}{dt} = T_{\alpha\beta} \ . \tag{10.8.14}$$

Taking covariant divergence on both sides and assuming that d/dt and D_α commute, one obtains that $T^{\alpha\beta}$ is divergenceless.

This is true if one assumes Einstein's equations with cosmological term. This gives

$$\frac{dg_{\alpha\beta}}{dt} = kG_{\alpha\beta} + \Lambda g_{\alpha\beta} = kR_{\alpha\beta} + (-\frac{kR}{2} + \Lambda)g_{\alpha\beta} \ . \tag{10.8.15}$$

The trace of this equation gives that the curvature scalar is constant. Note that the value of the Kähler coupling strength plays a highly non-trivial role in these equations and it is quite possible that solutions exist only for some critical values of α_K. Quantum criticality should fix the allow value triplets (G, Λ, α_K) apart from overall scaling

$$(G, \Lambda, \alpha_K) \rightarrow (xG, \Lambda/x, x\alpha_K \ .$$

Fixing the value of G fixes the values remaining parameters at critical points. The rescaling of the parameter t induces a scaling by x.

ii. By taking trace one obtains the already mentioned condition fixing the curvature to be constant, and one can write

$$\frac{dg_{\alpha\beta}}{dt} = kR_{\alpha\beta} - \Lambda g_{\alpha\beta} \ . \tag{10.8.16}$$

Note that in the recent case $R_{avg} = R$ holds true since curvature scalar is constant. The fixed points of the flow would be Einstein manifolds [A6, A30] satisfying

$$R_{\alpha\beta} = \frac{\Lambda}{k}g_{\alpha\beta} \tag{10.8.17}$$

.

iii. It is by no means obvious that continuous flow is possible. The condition that Einstein-Maxwell equations are satisfied might pick up from a completely general Maxwell flow a discrete subset as solutions of Einstein-Maxwell equations with a cosmological term. If so, one could assign to this subset a sequence of values t_n of the flow parameter t.

iv. I do not know whether 3-dimensionality is somehow absolutely essential for getting the topological classification of closed 3-manifolds using Ricci flow. This ignorance allows me to pose some innocent questions. Could one have a canonical representation of 4-geometries as spaces with constant Ricci scalar? Could one select one particular Einstein space in the class four-metrics and could the ratio Λ/k represent topological invariant if one normalizes metric or curvature scalar suitably. In the 3-dimensional case curvature scalar is normalized to unity. In the recent case this normalization would give $k = 4\Lambda$ in turn giving $R_{\alpha\beta} = g_{\alpha\beta}/4$. Does this mean that there is only single fixed point in local sense, analogous to black hole toward which all geometries are driven by the Maxwell flow? Does this imply that only the 4-volume of the original space would serve as a topological invariant?

Maxwell flow for space-time surfaces

One can consider Maxwell flow for space-time surfaces too. In this case Kähler flow would be the appropriate term and provides families of preferred extremals. Since space-time surfaces inside CD are the basic physical objects are in TGD framework, a possible interpretation of these families would be as flows describing physical dissipation as a four-dimensional phenomenon polishing details from the space-time surface interpreted as an analog of Bohr orbit.

i. The flow is now induced by a vector field $j^k(x,t)$ of the space-time surface having values in the tangent bundle of imbedding space $M^4 \times CP_2$. In the most general case one has Kähler flow without the Einstein equations. This flow would be defined in the space of all space-time surfaces or possibly in the space of all extremals. The flow equations reduce to

$$h_{kl}D_\alpha j^k(x,t)D_\beta h^l = \frac{1}{2}T_{\alpha\beta} . \qquad (10.8.18)$$

The left hand side is the projection of the covariant gradient $D_\alpha j^k(x,t)$ of the flow vector field $j^k(x,t)$ to the tangent space of the space-time surface. D_{alpha} is covariant derivative taking into account that j^k is imbedding space vector field. For a fixed point space-time surface this projection must vanish assuming that this space-time surface reachable. A good guess for the asymptotia is that the divergence of Maxwell energy momentum tensor vanishes and that Einstein's equations with cosmological constant are well-defined.

Asymptotes corresponds to vacuum extremals. In Euclidian regions CP_2 type vacuum extremals and in Minkowskian regions to any space-time surface in any 6-D sub-manifold $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2 having therefore vanishing induced Kähler form. Symplectic transformations of CP_2 combined with diffeomorphisms of M^4 give new Lagrangian manifolds. One would expect that vacuum extremals are approached but never reached at second extreme for the flow.

If one assumes Einstein's equations with a cosmological term, allowed vacuum extremals must be Einstein manifolds. For CP_2 type vacuum extremals this is the case. It is quite possible that these fixed points do not actually exist in Minkowskian sector, and could be replaced with more complex asymptotic behavior such as limit, chaos, or strange attractor.

ii. The flow could be also restricted to the space of preferred extremals. Assuming that Einstein Maxwell equations indeed hold true, the flow equations reduce to

$$h_{kl}D_\alpha j^k(x,t)\partial_\beta h^l = \frac{1}{2}(kR_{\alpha\beta} - \Lambda g_{\alpha\beta}) . \qquad (10.8.19)$$

Preferred extremals would correspond to a fixed sub-manifold of the general flow in the space of all 4-surfaces.

iii. One can also consider a situation in which $j^k(x,t)$ is replaced with $j^k(h,t)$ defining a flow in the entire imbedding space. This assumption is probably too restrictive. In this case the equations reduce to

$$(D_r j_l(x,t) + D_l j_r)\partial_\alpha h^r \partial_\beta h^l = kR_{\alpha\beta} - \Lambda g_{\alpha\beta} . \qquad (10.8.20)$$

Here D_r denotes covariant derivative. Asymptotia is achieved if the tensor $D_k j_l + D_k j_l$ becomes orthogonal to the space-time surface. Note for that Killing vector fields of H the left hand side vanishes identically. Killing vector fields are indeed symmetries of also asymptotic states.

It must be made clear that the existence of a continuous flow in the space of preferred extremals might be too strong a condition. Already the restriction of the general Maxwell flow in the space of metrics to solutions of Einstein-Maxwell equations with cosmological term might lead to discretization, and the assumption about reprentability as 4-surface in $M^4 \times CP_2$ would give a further condition reducing the number of solutions. On the other hand, one might consiser a possibility of a continuous flow in the space of constant Ricci scalar metrics with a fixed 4-volume and having hyperbolic spaces as the most symmetric representative.

Dissipation, self organization, transition to chaos, and coupling constant evolution

A beautiful connection with concepts like dissipation, self-organization, transition to chaos, and coupling constant evolution suggests itself.

i. It is not at all clear whether the vacuum extremal limits of the preferred extremals can correspond to Einstein spaces except in special cases such as CP_2 type vacuum extremals isometric with CP_2. The imbeddability condition however defines a constraint force which might well force asymptotically more complex situations such as limit cycles and strange attractors. In ordinary dissipative dynamics an external energy feed is essential prerequisite for this kind of non-trivial self-organization patterns.
   In the recent case the external energy feed could be replaced by the constraint forces due to the imbeddability condition. It is not too difficult to imagine that the flow (if it exists!) could define something analogous to a transition to chaos taking place in a stepwise manner for critical values of the parameter t. Alternatively, these discrete values could correspond to those values of t for which the preferred extremal property holds true for a general Maxwell flow in the space of 4-metrics. Therefore the preferred extremals of Kähler action could emerge as one-parameter (possibly discrete) families describing dissipation and self-organization at the level of space-time dynamics.

ii. For instance, one can consider the possibility that in some situations Einstein's equations split into two mutually consistent equations of which only the first one is independent

$$\begin{aligned} xJ^{\alpha}_{\ \nu}J^{\nu\beta} &= R^{\alpha\beta} \ , \\ L_K &= xJ^{\alpha}_{\ \nu}J^{\nu\beta} = 4\Lambda \ , \\ x &= \frac{1}{16\pi\alpha_K} \ . \end{aligned} \tag{10.8.21}$$

   Note that the first equation indeed gives the second one by tracing. This happens for CP_2 type vacuum extremals.
   Kähler action density would reduce to cosmological constant which should have a continuous spectrum if this happens always. A more plausible alternative is that this holds true only asymptotically. In this case the flow equation could not lead arbitrary near to vacuum extremal, and one can think of situation in which $L_K = 4\Lambda$ defines an analog of limiting cycle or perhaps even strange attractor. In any case, the assumption would allow to deduce the asymptotic value of the action density which is of utmost importance from calculational point of view: action would be simply $S_K = 4\Lambda V_4$ and one could also say that one has minimal surface with Λ taking the role of string tension.

iii. One of the key ideas of TGD is quantum criticality implying that Kähler coupling strength is analogous to critical temperature. Second key idea is that p-adic coupling constant evolution represents discretized version of continuous coupling constant evolution so that each p-adic prime would correspond a fixed point of ordinary coupling constant evolution in the sense that the 4-volume characterized by the p-adic length scale remains constant. The invariance of the geometric and thus geometric parameters of hyperbolic 4-manifold under the Kähler flow would conform with the interpretation as a flow preserving scale assignable to a given p-adic prime. The continuous evolution in question (if possible at all!) might correspond to a fixed p-adic prime. Also the hierarchy of Planck constants relates to this picture naturally. Planck constant $\hbar_{eff} = n\hbar$ corresponds to a multi-furcation generating n-sheeted structure and certainly affecting the fundamental group.

iv. One can of course question the assumption that a continuous flow exists. The property of being a solution of Einstein-Maxwell equations, imbeddability property, and preferred extremal property might allow allow only discrete sequences of space-time surfaces perhaps interpretable as orbit of an iterated map leading gradually to a fractal limit. This kind of discrete sequence might be also be selected as preferred extremals from the orbit of Maxwell flow without assuming Einstein-Maxwell equations. Perhaps the discrete p-adic coupling constant evolution could be seen in this manner and be regarded as an iteration so that the connection with fractality would become obvious too.

Does a 4-D counterpart of thermodynamics make sense?

The interpretation of the Kähler flow in terms of dissipation, the constancy of R, and almost constancy of L_K suggest an interpretation in terms of 4-D variant of thermodynamics natural in zero energy ontology (ZEO), where physical states are analogs for pairs of initial and final states of quantum event are quantum superpositions of classical time evolutions. Quantum theory becomes a "square root" of thermodynamics so that 4-D analog of thermodynamics might even replace ordinary thermodynamics as a fundamental description. If so this 4-D thermodynamics should be qualitatively consistent with the ordinary 3-D thermodynamics.

i. The first naive guess would be the interpretation of the action density L_K as an analog of energy density $e = E/V_3$ and that of R as the analog to entropy density $s = S/V_3$. The asymptotic states would be analogs of thermodynamical equilibria having constant values of L_K and R.

ii. Apart from an overall sign factor ϵ to be discussed, the analog of the first law $de = Tds - pdV/V$ would be

$$dL_K = kdR + \Lambda\frac{dV_4}{V_4} \quad .$$

One would have the correspondences $S \rightarrow \epsilon RV_4$, $e \rightarrow \epsilon L_K$ and $k \rightarrow T$, $p \rightarrow -\Lambda$. $k \propto 1/G$ indeed appears formally in the role of temperature in Einstein's action defining a formal partition function via its exponent. The analog of second law would state the increase of the magnitude of ϵRV_4 during the Kähler flow.

iii. One must be very careful with the signs and discuss Euclidian and Minkowskian regions separately. Concerning purely thermodynamic aspects at the level of vacuum functional Euclidian regions are those which matter.

   A. For CP_2 type vacuum extremals $L_K \propto E^2 + B^2$, $R = \Lambda/k$, and Λ are positive. In thermodynamical analogy for $\epsilon = 1$ this would mean that pressure is negative.

   B. In Minkowskian regions the value of $R = \Lambda/k$ is negative for $\Lambda < 0$ suggested by the large abundance of 4-manifolds allowing hyperbolic metric and also by cosmological considerations. The asymptotic formula $L_K = 4\Lambda$ considered above suggests that also Kähler action is negative in Minkowskian regions for magnetic flux tubes dominating in TGD inspired cosmology: the reason is that the magnetic contribution to the action density $L_K \propto E^2 - B^2$ dominates.

Consider now in more detail the 4-D thermodynamics interpretation in Euclidian and Minkowskian regions assuming that the the evolution by quantum jumps has Kähler flow as a space-time correlate.

i. In Euclidian regions the choice $\epsilon = 1$ seems to be more reasonable one. In Euclidian regions $-\Lambda$ as the analog of pressure would be negative, and asymptotically (that is for CP_2 type vacuum extremals) its value would be proportional to $\Lambda \propto 1/GR^2$, where R denotes CP_2 radius defined by the length of its geodesic circle.
A possible interpretation for negative pressure is in terms of string tension effectively inducing negative pressure (note that the solutions of the modified Dirac equation indeed assign a string to the wormhole contact). The analog of the second law would require the increase of RV_4 in quantum jumps. The magnitudes of L_K, R, V_4 and Λ would be reduced and approach their asymptotic values. In particular, V_4 would approach asymptotically the volume of CP_2.

ii. In Minkowskian regions Kähler action contributes to the vacuum functional a phase factor analogous to an imaginary exponent of action serving in the role of Morse function so that thermodynamics interpretation can be questioned. Despite this one can check whether thermodynamic interpretation can be considered. The choice $\epsilon = -1$ seems to be the correct choice now. $-\Lambda$ would be analogous to a negative pressure whose gradually decreases. In 3-D thermodynamics it is natural to assign negative pressure to the magnetic flux tube like structures as their effective string tension defined by the density of magnetic energy per unit length. $-R \geq 0$ would entropy and $-L_K \geq 0$ would be the analog of energy density.
$R = \Lambda/k$ and the reduction of Λ during cosmic evolution by quantum jumps suggests that the larger the volume of CD and thus of (at least) Minkowskian space-time sheet the smaller the negative value of Λ.
Assume the recent view about state function reduction explaining how the arrow of geometric time is induced by the quantum jump sequence defining experienced time [K4]. According to this view zero energy states are quantum superpositions over CDs of various size scales but with common tip, which can correspond to either the upper or lower light-like boundary of CD. The sequence of quantum jumps the gradual increase of the average size of CD in the quantum superposition and therefore that of average value of V_4. On the other hand, a gradual decrease of both $-L_K$ and $-R$ looks physically very natural. If Kähler flow describes the effect of dissipation by quantum jumps in ZEO then the space-time surfaces would gradually approach nearly vacuum extremals with constant value of entropy density $-R$ but gradually increasing 4-volume so that the analog of second law stating the increase of $-RV_4$ would hold true.

iii. The interpretation of $-R > 0$ as negentropy density assignable to entanglement is also possible and is consistent with the interpretation in terms of second law. This interpretation would only change the sign factor ϵ in the proposed formula. Otherwise the above arguments would remain as such.

10.8.4 Could correlation functions, S-matrix, and coupling constant evolution be coded the statistical properties of preferred extremals?

Quantum classical correspondence states that all aspects of quantum states should have correlates in the geometry of preferred extremals. In particular, various elementary particle propagators should have a representation as properties of preferred extremals. This would allow to realize the old dream about being able to say something interesting about coupling constant evolution although it is not yet possible to calculate the M-matrices and U-matrix. Hitherto everything that has been said about coupling constant evolution has been rather speculative arguments except for the general vision that it reduces to a discrete evolution defined by p-adic length scales. General first principle definitions are however much more valuable than ad hoc guesses even if the latter give rise to explicit formulas.

In quantum TGD and also at its QFT limit various correlation functions in given quantum state should code for its properties. By quantum classical correspondence these correlation functions should have counterparts in the geometry of preferred extremals.

Even more: these classical counterparts for a given preferred extremal ought to be identical with the quantum correlation functions for the superposition of preferred extremals. This correspondence could be called quantum ergodicity by its analogy with ordinary ergodicity stating that the member of ensemble becomes representative of ensemble.

i. The marvellous implication of quantum ergodicity would be that one could calculate everything solely classically using the classical intuition - the only intuition that we have. Quantum ergodicity would also solve the paradox raised by the quantum classical correspondence for momentum eigenstates. Any preferred extremal in their superposition defining momentum eigenstate should code for the momentum characterizing the superposition itself. This is indeed possible if every extremal in the superposition codes the momentum to the properties of classical correlation functions which are identical for all of them.

ii. The only manner to possibly achieve quantum ergodicity is in terms of the statistical properties of the preferred extremals. It should be possible to generalize the ergodic theorem stating that the properties of statistical ensemble are represented by single space-time evolution in the ensemble of time evolutions. Quantum superposition of classical worlds would effectively reduce to single classical world as far as classical correlation functions are considered. The notion of finite measurement resolution suggests that one must state this more precisely by adding that classical correlation functions are calculated in a given UV and IR resolutions meaning UV cutoff defined by the smallest CD and IR cutoff defined by the largest CD present.

iii. The skeptic inside me immediately argues that TGD Universe is 4-D spin glass so that this quantum ergodic theorem must be broken. In the case of the ordinary spin classes one has not only statistical average for a fixed Hamiltonian but a statistical average over Hamiltonians. There is a probability distribution over the coupling parameters appearing in the Hamiltonian. Maybe the quantum counterpart of this is needed to predict the physically measurable correlation functions.
Could this average be an ordinary classical statistical average over quantum states with different classical correlation functions? This kind of average is indeed taken in density matrix formalism. Or could it be that the square root of thermodynamics defined by ZEO actually gives automatically rise to this average? The eigenvalues of the "hermitian square root " of the density matrix would code for components of the state characterized by different classical correlation functions. One could assign these contributions to different "phases".

iv. Quantum classical correspondence in statistical sense would be very much like holography (now individual classical state represents the entire quantum state). Quantum ergodicity would pose a rather strong constraint on quantum states. This symmetry principle could actually fix the spectrum of zero energy states to a high degree and fix therefore the M-matrices given by the product of hermitian square root of density matrix and unitary S-matrix and unitary U-matrix having M-matrices as its orthonormal rows.

v. In TGD inspired theory of consciousness the counterpart of quantum ergodicity is the postulate that the space-time geometry provides a symbolic representation for the quantum states and also for the contents of consciousness assignable to quantum jumps between quantum states. Quantum ergodicity would realize this strongly self-referential looking condition. The positive and negative energy parts of zero energy state would be analogous to the initial and final states of quantum jump and the classical correlation functions would code for the contents of consciousness like written formulas code for the thoughts of mathematician and provide a sensory feedback.

How classical correlation functions should be defined?

i. General Coordinate Invariance and Lorentz invariance are the basic constraints on the definition. These are achieved for the space-time regions with Minkowskian signature and 4-D M^4 projection if linear Minkowski coordinates are used. This is equivalent with the contraction of the indices of tensor fields with the space-time projections of M^4 Killing vector fields representing translations. Accepting ths generalization, there is no need to restrict oneself to 4-D M^4 projection and one can also consider also Euclidian regions identifiable as lines of generalized Feynman diagrams.

Quantum ergodicity very probably however forces to restrict the consideration to Minkowskian and Euclidian space-time regions and various phases associated with them. Also CP_2 Killing vector fields can be projected to space-time surface and give a representation for classical gluon fields. These in turn can be contracted with M^4 Killing vectors giving rise to gluon fields as analogs of graviton fields but with second polarization index replaced with color index.

ii. The standard definition for the correlation functions associated with classical time evolution is the appropriate starting point. The correlation function $G_{XY}(\tau)$ for two dynamical variables $X(t)$ and $Y(t)$ is defined as the average $G_{XY}(\tau) = \int_T X(t)Y(t+\tau)dt/T$ over an interval of length T, and one can also consider the limit $T \to \infty$. In the recent case one would replace τ with the difference $m_1 - m_2 = m$ of M^4 coordinates of two points at the preferred extremal and integrate over the points of the extremal to get the average. The finite time interval T is replaced with the volume of causal diamond in a given length scale. Zero energy state with given quantum numbers for positive and negative energy parts of the state defines the initial and final states between which the fields appearing in the correlation functions are defined.

iii. What correlation functions should be considered? Certainly one could calculate correlation functions for the induced spinor connection given electro-weak propagators and correlation functions for CP_2 Killing vector fields giving correlation functions for gluon fields using the description in terms of Killing vector fields. If one can uniquely separate from the Fourier transform uniquely a term of form $Z/(p^2 - m^2)$ by its momentum dependence, the coefficient Z can be identified as coupling constant squared for the corresponding gauge potential component and one can in principle deduce coupling constant evolution purely classically. One can imagine of calculating spinorial propagators for string world sheets in the same manner. Note that also the dependence on color quantum numbers would be present so that in principle all that is needed could be calculated for a single preferred extremal without the need to construct QFT limit and to introduce color quantum numbers of fermions as spin like quantum numbers (color quantum numbers corresponds to CP_2 partial wave for the tip of the CD assigned with the particle).

iv. What about Higgs field? TGD in principle allows scalar and pseudo-scalars which could be called Higgs like states. These states are however not necessary for particle massivation although they can represent particle massivation and must do so if one assumes that QFT limit exist. p-Adic thermodynamics however describes particle massivation microscopically.

The problem is that Higgs like field does not seem to have any obvious space-time correlate. The trace of the second fundamental form is the obvious candidate but vanishes for preferred extremals which are both minimal surfaces and solutions of Einstein Maxwell equations with cosmological constant. If the string world sheets at which all spinor components except right handed neutrino are localized for the general solution ansatz of the modified Dirac equation, the corresponding second fundamental form at the level of imbedding space defines a candidate for classical Higgs field. A natural expectation is that string world sheets are minimal surfaces of space-time surface. In general they are however not minimal surfaces of the imbedding space so that one might achieve a microscopic definition of classical Higgs field and its vacuum expectation value as an average of one point correlation function over the string world sheet.

Many detailed speculations about coupling constant evolution to be discussed in the sections below must be taken as innovative guesses doomed to have the eventual fate of guesses. The notion of quantum ergodicity could however be one of the really deep ideas about coupling constant evolution comparable to the notion of p-adic coupling constant evolution. Quantum Ergodicity (briefly QE) would also state something extremely non-trivial also about the construction of correlation functions and S-matrix. Because this principle is so new, the rest of the chapter does not yet contain any applications of QE. This should not lead the reader to under-estimate the potential power of QE.

10.9 Does the exponent of Chern-Simons action reduce to the exponent of the area of minimal surfaces?

As I scanned of hep-th I found an interesting article by Giordano, Peschanski, and Seki [B22] based on AdS/CFT correspondence. What is studied is the high energy behavior of the gluon-gluon and quark-quark scattering amplitudes of $\mathcal{N} = 4$ SUSY.

i. The proposal made earlier by Aldaya and Maldacena [B5] is that gluon-gluon scattering amplitudes are proportional to the imaginary exponent of the area of a minimal surface in AdS_5 whose boundary is identified as *momentum space*. The boundary of the minimal surface would be polygon with light-like edges: this polygon and its dual are familiar from twistor approach.

ii. Giordano, Peschanski, and Seki claim that quark-quark scattering amplitude for heavy quarks corresponds to the exponent of the area for a minimal surface in the *Euclidian* version of AdS_5 which is hyperbolic space (space with a constant negative curvature): it is interpreted as a counterpart of WCW rather than momentum space and amplitudes are obtained by analytic continuation. For instance, a universal Regge behavior is obtained. For general amplitudes the exponent of the area alone is not enough since it does not depend on gluon quantum numbers and vertex operators at the edges of the boundary polygon are needed.

In the following my intention is to consider the formulation of this conjecture in quantum TGD framework. I hasten to inform that I am not a specialist in AdS/CFT and can make only general comments inspired by analogies with TGD.

10.9.1 Why Chern-Simons action should reduce to area for minimal surfaces?

The minimal surface conjectures are highly interesting from TGD point of view. The weak form of electric magnetic duality implies the reduction of Kähler action to 3-D Chern-Simons terms. Effective 2-dimensionality implied by the strong form of General Coordinate Invariance suggests a further reduction of Chern-Simons terms to 2-D terms and the areas of string world sheet and of partonic 2-surface are the only non-topological options that one can imagine. Skeptic could of course argue that the exponent of the minimal surface area results as a characterizer of the quantum state rather than vacuum functional. In the following I defend the minimal interpretation as Chern-Simons terms.

Let us look this conjecture in more detail.

i. In zero energy ontology twistor approach is very natural since all physical states are bound states of massless particles. Also virtual particles are composites of massless states. The possibility to have both signs of energy makes possible space-like momenta for wormhole contacts. Mass shell conditions at internal lines imply extremely strong constraints on the virtual momenta and both UV and IR finiteness are expected to hold true.

ii. The weak form of electric magnetic duality [K18] implies that the exponent of Kähler action reduces to the exponent of Chern-Simons term for 3-D space-like surfaces at the ends of space-time surface inside CD and for light-like 3-surfaces. The coefficient of this term is complex since the contribution of Minkowskian regions of the space-time surface is imaginary ($\sqrt{g_4}$ is imaginary) and that of Euclidian regions (generalized Feynman diagrams) real. The Chern-Simons term from Minkowskian regions is like Morse function and that from Euclidian regions defines Kähler function and stationary phase approximation makes sense. The two contributions are different since the space-like 3-surfaces contributing to Kähler function and Morse function are different.

iii. Electric magnetic duality [K18] leads also to the conclusion that wormhole throats carrying elementary particle quantum numbers are Kähler magnetic monopoles. This forces to identify elementary particles as string like objects with ends having opposite monopole charges. Also more complex configurations are possible.
It is not quite clear what the scale of the stringyness is. The natural first guess inspired by quantum classical correspondence is that it corresponds to the p-adic length scale of the particle characterizing its Compton length. Second possibility is that it corresponds to electroweak scale. For leptons stringyness in Compton length scale might not have any fatal implications since the second end of string contains only neutrinos neutralizing the weak isospin of the state. This kind of monopole pairs could appear even in condensed matter scales: in particular if the proposed hierarchy of Planck constants [K17] is realized.

iv. Strong form of General Coordinate Invariance requires effective 2-dimensionality. In given UV and IR resolutions either partonic 2-surfaces or string world sheets form a finite hierarchy of CDs inside CDs with given CD characterized by a discrete scale coming as an integer multiple of a fundamental scale (essentially CP_2 size). The string world sheets have boundaries consisting of either light-like curves in induced metric at light-like wormhole throats and space-like curves at the ends of CD whose M^4 projections are light-like. These braids intersect partonic 2-surfaces at discrete points carrying fermionic quantum numbers.
This implies a rather concrete analogy with $AdS_5 \times S_5$ duality, which describes gluons as open strings. In zero energy ontology (ZEO) string world sheets are indeed a fundamental notion and the natural conjecture is that these surfaces are minimal surfaces whose area by quantum classical correspondence depends on the quantum numbers of the external particles. String tension in turn should depend on gauge couplings -perhaps only Kähler coupling strength- and geometric parameters like the size scale of CD and the p-adic length scale of the particle.

v. Are the minimal surfaces in question minimal surfaces of the imbedding space $M^4 \times CP_2$ or of the space-time surface X^4? All possible 2-surfaces at the boundary of CD must be allowed so that they cannot correspond to minimal surfaces in $M^4 \times CP_2$ unless one assumes that they emerge in stationary phase approximation only. The boundary conditions at the ends of CD could however be such that *any* partonic 2-surface correspond to a minimal surfaces in X^4. Same applies to string world sheets. One might even hope that these conditions combined with the weak form of electric magnetic duality fixes completely the boundary conditions at wormhole throats and space-like ends of space-time surface.
The trace of the second fundamental form orthogonal to the string world sheet/partonic 2-surface as sub-manifold of space-time surface would vanish: this is nothing but a generalization of the geodesic motion obtained by replacing word line with a 2-D surface. It does not imply the vanishing of the trace of the second fundamental form in $M^4 \times CP_2$ having interpretation as a generalization of particle acceleration [K56]. Effective 2-dimensionality would be realized if Chern-Simons terms reduce to a sum of the areas of these minimal surfaces.

These arguments suggest that scattering amplitudes are proportional to the product of exponents of 2-dimensional actions which can be either imaginary or real. Imaginary exponent would be proportional to the total area of string world sheets and the imaginary unit would come naturally from $\sqrt{g_2}$. Teal exponent proportional to the total area of partonic 2-surfaces. The coefficient of these areas would not in general be same.

The equality of the Minkowskian and Euclidian Chern-Simons terms is suggestive but not necessarily true since there could be also other Chern-Simons contributions than those assignable to wormhole throats and the ends of space-time. The equality would imply that the total area of string world sheets equals to the total area of partonic 2-surfaces suggesting strongly a duality meaning that either Euclidian or Minkowskian regions carry the needed information.

10.9.2 IR cutoff and connection with p-adic physics

In twistor approach the IR cutoff is necessary to get rid of IR divergences. Also in the AdS_5 approach the condition that the minimal surface area is finite requires an IR cutoff. The problem is that there is no natural IR cutoff. In TGD framework zero energy ontology brings in a natural IR cutoff via the finite and quantized size scale of CD guaranteeing that the minimal surfaces involved have a finite area. This implies that also particles usually regarded as massless have a small mass characterized by the size of CD. The size scale of CD would correspond to the scale parameter R assigned with the metric of AdS_5.

i. String tension relates in AdS_5 approach to the gauge coupling g_{YM} and to the number N_c of colors by the formula

$$\lambda = g_{YM}^2 N_c = \frac{R^2}{\alpha'} . \tag{10.9.1}$$

$1/N_c$-expansion is in terms of $1/\sqrt{\lambda}$. The formula has an alternative form as an expression for the string tension

$$\alpha' = \frac{R^2}{\sqrt{g_{YM}^2 N_c}} . \tag{10.9.2}$$

The analog this formula in TGD framework suggests an connection with p-adic length scale hypothesis.

i. As already noticed, the natural counterpart for the scale R could be the discrete value of the size scale of CD. Since the symplectic group assignable to $\delta M^4_\pm \times CP_2$ (or the upper or lower boundary of CD) is the natural generalization of the gauge group, it would seem that $N_c = \infty$ holds true in the absence of cutoff. At the limit $N_c = \infty$ only planar diagrams would contribute to YM scattering amplitudes. Finite measurement resolution must make the effective value of N_c finite so that also λ would be finite. String tension would depend on both the size of CD and the effective number of symplectic colors.

ii. If α' is characterized by the square of the Compton length of the particle, λ would be essentially the square of the ratio of CD size scale given by secondary p-adic lengths and of the primary p-adic length scale associated with the particle: $\lambda = g_{YM}^2\sqrt{p}$, where p is the p-adic prime characterizing the particle. Favored values of the p-adic prime correspond to primes near powers of two. The effective number of symplectic colors would be $N_c = \sqrt{p}/g_{YM}^2$ and the expansion would come in powers of $g_{YM}^2/\sqrt{p}$. For electron one would have $p = M_{127} = 2^{127-1}$ so that the expansion would converge extremely fast. Together with the amazing success of the p-adic mass calculations based on p-adic thermodynamics for the scaling generator L_0 [K31] this suggests a deep connection with p-adic physics and number theoretic universality.

10.9.3 Could Kähler action reduce to Kähler magnetic flux over string world sheets and partonic 2-surfaces?

Can one consider alternative identifications of Kähler action for preferred extremals? The only alternative identification of Kähler function that I can imagine is that Kähler action proportional to the Kähler magnetic flux $\int_{Y^2} J$ or Kähler electric flux $\int_{Y^2} *J$ for string world sheets and possibly also partonic 2-surfaces. These fluxes are dimensionless numbers. If the weak form of electric-magnetic duality holds true also at string world sheets, the two options are equivalent apart from a proportionality constant.

i. For Kähler magnetic flux there would be no explicit dependence on the induced metric. This is in accordance with the almost topological QFT property.

ii. Unless the weak form of electric-magnetic duality holds true, the Kähler electric flux has an explicit dependence on the induced metric but in a scaling invariant manner. The most obvious objection relates to the sign factor of the dual flux which depends on the orientation of the string world sheet and thus changes sign when the orientation of space-time sheet is changed by changing that of the string world sheet. This is in conflict with the independence of Kähler action on orientation. One can however argue that the orientation makes itself actually physically visible via the weak form of electric-magnetic duality and that the change of the orientation as a symmetry is dynamically broken. This breaking would be analogous to parity breaking at the level of imbedding space.

iii. In [K23] it is proposed that braids defined by the boundaries of string world sheets could correspond to Legendrian sub-manifolds, whereas partonic 2-surfaces could the duals of Legendrian manifolds, so that braiding would take place dynamically. The identification of the Kähler action as Kähler magnetic flux associated with string world sheets and possibly also partonic 2-surfaces is consistent with the assumption that the extremal of Kähler action in question. Indeed, the Legendrian property says that the projection of the Kähler gauge potential on braid strand vanishes and this expresses the extremality of the Kähler magnetic flux.

The assumption that Kähler action is proportional to Kähler magnetic flux seems to be consistent with the minimal surface property. The weak form of electric-magnetic duality gives a constraint on the normal derivatives of imbedding space coordinates at the string world sheet and minimal surface property strengthens these constraints. One could perhaps say that space-time surface chooses its shape in such a manner that the string world sheet has a minimal area.

The open questions are following.

i. Does Kähler action for the preferred extremals reduce to the area of the string world sheet or to Kähler flux, or are the representations equivalent so that the induced Kähler form would effectively define area form? If the Kähler form form associated with the induced metric on string world sheet is proportional to the induced Kähler form the Kähler magnetic flux is proportional to the area and Kähler action reduces to genuine area. This condition looks like a natural additional constraint on string world sheets besides minimal surface property.

ii. The proportionality of the induced Kähler form and Kähler form of the induced 2-metric implies as such only the extremal property against the symplectic variations so that one cannot have minimal surface property at imbedding space level. Minimality at space-time level is however possible since space-time surface itself can arrange the situation so that general variations deforming the string world sheet along space-time surface reduce to symplectic variations at the level of the imbedding space.

iii. Does the situation depend on whether the string world sheet is in Minkowskian or Euclidian space-time region? The problem is that in Euclidian regions the value of Kähler action is positive definite and it is not obvious why the Kähler magnetic flux for Euclidian string world sheets should have a fixed sign. Could weak form of electric-magnetic duality fix the sign?

Irrespective whether the Kähler action is proportional to the total area or the Kähler electric flux over string world sheets, the theory would be exactly solvable at string world sheet level (finite measurement resolution).

10.9.4 What is the interpretation of Yangian duality in TGD framework?

Minimal surfaces in both WCW and momentum space are used in the above mentioned two articles [B5, B22]. The possibility of these two descriptions must reflect the Yangian symmetry unifying the conformal symmetries of Minkowski space and momentum space in twistorial approach.

The minimal surfaces in $X^4 \subset M^4 \times CP_2$ are natural in TGD framework. Could also the minimal surfaces in momentum space have some interpretation in TGD framework? Ore more generally, what could be the interpretation of the dual descriptions provided by twistor diagrams with light-like edges and dual twistor diagrams with light-like vertices? One can imagine many interpretations but zero energy ontology suggests an especially attractive and natural interpretation of this duality as the exchange of the roles of wormhole throats carrying always on mass shell massless momenta and wormhole contacts carrying in general off-mass shell momenta and massive momenta in incoming lines.

i. For WCW twistor diagrams vertices correspond to incoming and outgoing light-like momenta. The light-like momenta associated with the wormhole throats of the incoming and outgoing lines of generalized Feynman diagram could correspond to the light-like momenta associated with the vertices of the polygon. The internal lines defined by wormhole contacts carrying virtual off mass shell momenta would naturally correspond to to edges of the twistor diagram.

ii. What about dual twistor diagrams in which light-like momenta correspond to lines? Zero energy ontology implies that virtual wormhole throats carry on mass shell massless momenta whereas incoming wormhole contacts in general carry massive particles: this guarantees the absence of IR divergences. Could one identify the momenta of internal wormhole throats as light-like momenta associated with the lines dual twistor diagrams and the incoming net momenta assignable to wormhole contacts as incoming and outgoing momenta.

Also the transition from Minkowskian to Euclidian signature by Wick rotation could have interpretation in TGD framework. Space-time surfaces decompose into Minkowskian and Euclidian regions. The latter ones represent generalized Feynman diagrams. This suggests a generalization of Wick rotation. The string world sheets in Euclidian regions would define the analogs of the minimal surfaces in Euclidian AdS_5 and the string world sheets in Minkowskian regions the analogs of Minkowskian AdS_5. The magnitudes of the areas would be identical so that they might be seen as analytical continuations of each other in some sense. Note that partonic 2-surfaces would belong to the intersection of Euclidian and Minkowskian space-time regions. This argument tells nothing about possible momentum space analog of $M^4 \times CP_2$.

Chapter 1

Appendix

Originally this appendix was meant to be a purely technical summary of basic facts but in its recent form it tries to briefly summarize those basic visions about TGD which I dare to regarded stabilized. I have added illustrations making it easier to build mental images about what is involved and represented briefly the key arguments. This chapter is hoped to help the reader to get fast grasp about the concepts of TGD.

The basic properties of imbedding space and related spaces are discussed and the relationship of CP_2 to standard model is summarized. The notions of induction of metric and spinor connection, and of spinor structure are discussed. Many-sheeted space-time and related notions such as topological field quantization and the relationship many-sheeted space-time to that of GRT space-time are discussed as well as the recent view about induced spinor fields and the emergence of fermionic strings. Various topics related to p-adic numbers are summarized with a brief definition of p-adic manifold and the idea about generalization of the number concept by gluing real and p-adic number fields to a larger book like structure. Hierarchy of Planck constants can be now understood in terms of the non-determinism of Kähler action and the recent vision about connections to other key ideas is summarized.

A-1 Imbedding space $M^4 \times CP_2$ and related notions

Space-times are regarded as 4-surfaces in $H = M^4 \times CP_2$ the Cartesian product of empty Minkowski space - the space-time of special relativity - and compact 4-D space CP_2 with size scale of order 10^4 Planck lengths. One can say that imbedding space is obtained by replacing each point m of empty Minkowski space with 4-D tiny CP_2. The space-time of general relativity is replaced by a 4-D surface in H which has very complex topology. The notion of many-sheeted space-time gives an idea about what is involved.

Fig. 1. Imbedding space $H = M^4 \times CP_2$ as Cartesian product of Minkowski space M^4 and complex projective space CP_2. `http://www.tgdtheory.fi/appfigures/Hoo.jpg`

Denote by M^4_+ and M^4_- the future and past directed lightcones of M^4. Denote their intersection, which is not unique, by CD. In zero energy ontology (ZEO) causal diamond (CD) is defined as cartesian product $CD \times CP_2$. Often I use CD to refer just to $CD \times CP_2$ since CP_2 factor is relevant from the point of view of ZEO.

Fig. 2. Future and past light-cones M^4_+ and M^4_-. Causal diamonds (CD) are defined as their intersections. `http://www.tgdtheory.fi/appfigures/futurepast.jpg`

Fig. 3. Causal diamond (CD) is highly analogous to Penrose diagram but simpler. `http://www.tgdtheory.fi/appfigures/penrose.jpg`

A rather recent discovery was that CP_2 is the only compact 4-manifold with Euclidian signature of metric allowing twistor space with Kähler structure. M^4 is in turn is the only 4-D space with Minkowskian signature of metric allowing twistor space with Kähler structure so that $H = M^4 \times CP_2$ is twistorially unique.

One can loosely say that quantum states in a given sector of "world of classical worlds" (WCW) are superpositions of space-time surfaces inside CDs and that positive and negative energy parts of zero energy states are localized and past and future boundaries of CDs. CDs form a hierarchy. One can have CDs within CDs and CDs can also overlap. The size of CD is characterized by the proper time distance between its two tips. One can perform both translations and also Lorentz boosts of CD leaving either boundary invariant. Therefore one can assign to CDs a moduli space and speak about wave function in this moduli space.

In number theoretic approach it is natural to restrict the allowed Lorentz boosts to some discrete subgroup of Lorentz group and also the distances between the tips of CDs to multiples of CP_2 radius defined by the length of its geodesic. Therefore the moduli space of CDs discretizes. The quantization of cosmic recession velocities for which there are indications, could relate to this quantization.

A-2 Basic facts about CP_2

CP_2 as a four-manifold is very special. The following arguments demonstrates that it codes for the symmetries of standard models via its isometries and holonomies.

A-2.1 CP_2 as a manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-2.1}$$

Here λ is any non-zero complex number. Note that CP_2 can be also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three coordinate charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1, 2$ the coordinates of Eguchi and Freund [A59] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it \ , \\ \xi^2 &= x + iy \ . \end{aligned} \tag{A-2.2}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi + \Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi - \Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \tag{A-2.3}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connected, with Euler number Euler number 3, Pontryagin number 3 and second $b = 1$.

Fig. 4. CP_2 as manifold. `http://www.tgdtheory.fi/appfigures/cp2.jpg`

A-2.2 Metric and Kähler structure of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \rightarrow exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i\bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5.
The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \tag{A-2.4}$$

where the Hermitian, in fact Kähler metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2\partial_a\partial_{\bar{b}}K \ , \tag{A-2.5}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= log(F) \ , \\ F &= 1+r^2 \ . \end{aligned} \tag{A-2.6}$$

The Kähler function for S^2 has the same form. It gives the S^2 metric $dzd\bar{z}/(1+r^2)^2$ related to its standard form in spherical coordinates by the coordinate transformation $(r,\phi) = (tan(\theta/2),\phi)$.
The representation of the CP_2 metric is deducible from S^5 metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2 + r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2+\sigma_2^2)}{F} \ , \tag{A-2.7}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-2.8}$$

R denotes the radius of the geodesic circle of CP_2. The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2\sum_A e_k^A e_l^A \ , \tag{A-2.9}$$

are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} \ , & e^1 &= \frac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \frac{r\sigma_2}{\sqrt{F}} \ , & e^3 &= \frac{r\sigma_3}{F} \ . \end{aligned} \tag{A-2.10}$$

The explicit representations of vierbein vectors are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} \ , & e^1 &= \frac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\ e^2 &= \frac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} \ , & e^3 &= \frac{r(d\Psi + cos\Theta d\Phi)}{2F} \ . \end{aligned} \tag{A-2.11}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \tag{A-2.12}$$

The vierbein connection satisfying the defining relation

$$de^A = -V^A_B \wedge e^B \ , \tag{A-2.13}$$

is given by

$$\begin{array}{lllllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r - \frac{1}{r})e^3 \ , & V_{12} & = & (2r + \frac{1}{r})e^3 \ . \end{array} \tag{A-2.14}$$

The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{lllllll} R_{01} & = & e^0 \wedge e^1 - e^2 \wedge e^3 \ , & R_{23} & = & e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} & = & e^0 \wedge e^2 - e^3 \wedge e^1 \ , & R_{31} & = & -e^0 \wedge e^2 + e^3 \wedge e^1 \ , \\ R_{03} & = & 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , & R_{12} & = & 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{array} \tag{A-2.15}$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$J = -ig_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \tag{A-2.16}$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$J^k_{\ r}J^{rl} = -s^{kl} \ . \tag{A-2.17}$$

The form J is integer valued and by its covariant constancy satisfies free Maxwell equations. Hence it can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling). Locally one has therefore

$$J = dB \ , \tag{A-2.18}$$

where B is the so called Kähler potential, which is not defined globally since J describes homological magnetic monopole.

It should be noticed that the magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$\begin{array}{lll} B & = & 2re^3 \ , \\ J & = & 2(e^0 \wedge e^3 + e^1 \wedge e^2) = \frac{r}{F^2}dr \wedge (d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F}sin\Theta d\Theta d\Phi \ . \end{array} \tag{A-2.19}$$

The vierbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1,1).

Useful coordinates for CP_2 are the so called canonical coordinates in which Kähler potential and Kähler form have very simple expressions

$$
\begin{aligned}
B &= \sum_{k=1,2} P_k dQ_k \ , \\
J &= \sum_{k=1,2} dP_k \wedge dQ_k \ .
\end{aligned} \tag{A-2.20}
$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$
\begin{aligned}
P_1 &= -\frac{1}{1+r^2} \ , \\
P_2 &= \frac{r^2 cos\Theta}{2(1+r^2)} \ , \\
Q_1 &= \Psi \ , \\
Q_2 &= \Phi \ .
\end{aligned} \tag{A-2.21}
$$

A-2.3 Spinors in CP_2

CP_2 doesn't allow spinor structure in the conventional sense [A54] . However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0,1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallel propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the non-allowed -1- factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential $exp(i2\Phi)$, where Φ is the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

A-2.4 Geodesic sub-manifolds of CP_2

Geodesic sub-manifolds are defined as sub-manifolds having common geodesic lines with the imbedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [A43] a general characterization of the geodesic sub-manifolds for an arbitrary symmetric space G/H is given. Geodesic sub-manifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$[X,[Y,Z]] \in t \ \text{ for } \ X,Y,Z \in t \ . \tag{A-2.22}$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic sub-manifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Phi = 0) \ .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S^2_I. S^2_{II} is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-3 CP_2 geometry and standard model symmetries

A-3.1 Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B15] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e=\pm 1$, CP_2-chirality l,r and M^4-chirality L,R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{A-3.1}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \times \gamma_5$, $1\times\gamma_5$ and $\gamma_5 \times 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e=\pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted.

For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4) = SU(2)_L \times SU(2)_R$.
The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{A-3.2}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.
The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{array}{lllllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r - \frac{1}{r})e^3 \ , & V_{12} & = & (2r + \frac{1}{r})e^3 \ , \end{array} \tag{A-3.3}$$

and

$$B = 2re^3 \ , \tag{A-3.4}$$

respectively. The explicit representation of the vielbein is not needed here.
Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \tag{A-3.5}$$

where one have defined

$$\begin{aligned} I_L^1 &= \frac{(\Sigma_{01} - \Sigma_{23})}{2} \ , \\ I_L^2 &= \frac{(\Sigma_{02} - \Sigma_{13})}{2} \ . \end{aligned} \tag{A-3.6}$$

A_{ch} is clearly left handed so that one can perform the identification

$$W^\pm = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{A-3.7}$$

where $W^\pm$ denotes the charged intermediate vector boson.
Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$\begin{aligned} X &= re^3 \ , \\ Y &= \frac{e^3}{r} \ , \end{aligned} \tag{A-3.8}$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.
To begin with let us define

$$\begin{aligned} \bar{\gamma} &= aX + bY \ , \\ \bar{Z}^0 &= cX + dY \ , \end{aligned} \tag{A-3.9}$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$\begin{aligned} A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\ &+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ . \end{aligned} \tag{A-3.10}$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5\Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \tag{A-3.11}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-3.12}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$\begin{aligned} Q_{em} &= \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} \ , \\ I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ . \end{aligned} \tag{A-3.13}$$

The fields γ and Z^0 are defined via the relations

$$\begin{aligned} \gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\ Z^0 &= 4(a+b)\bar{Z}^0 = 4(X - Y) \ . \end{aligned} \tag{A-3.14}$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \tag{A-3.15}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of Weinberg angle is a dynamical problem. The angle is completely fixed once the YM action is fixed by requiring that action contains no cross term of type γZ^0. Pure symmetry non-broken electro-weak YM action leads to a definite value for the Weinberg angle. One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+1_+ + n_-1_-) \ , \qquad \text{(A-3.16)}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \end{aligned} \qquad \text{(A-3.17)}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \qquad \text{(A-3.18)}$$

Evaluating the expressions above one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \qquad \text{(A-3.19)}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \qquad \text{(A-3.20)}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} \ , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) \ , \end{aligned} \qquad \text{(A-3.21)}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 cross term (this coefficient must vanish) the expression

$$\begin{aligned} X &= -\frac{K}{2g^2} + \frac{fp}{18} \ , \\ K &= Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] \ , \end{aligned} \qquad \text{(A-3.22)}$$

In the general case the value of the coefficient K is given by

$$K = \sum_i \left[-\frac{(18 + 2n_i^2)sin^2\theta_W}{9}\right] \ , \qquad \text{(A-3.23)}$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9\sum_i 1}{(fg^2 + 2\sum_i(18 + n_i^2))} \ . \qquad \text{(A-3.24)}$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9}{(\frac{fg^2}{2} + 28)} \ . \qquad \text{(A-3.25)}$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is quite close to the typical value 9/24 of GUTs [B29] .

A-3.2 Discrete symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:

i. Symmetries must be realized as purely geometric transformations.
ii. Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [B8] .

The action of the reflection P on spinors of is given by

$$\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0 \Psi \ . \tag{A-3.26}$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) \ , \\ \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ . \end{aligned} \tag{A-3.27}$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \Psi^\dagger\gamma^2\gamma^0 \otimes 1 \ . \end{aligned} \tag{A-3.28}$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-4 The relationship of TGD to QFT and string models

TGD could be seen as a generalization of quantum field theory (string models) obtained by replacing pointlike particles (strings) as fundamental objects with 3-surfaces.

Fig. 5. TGD replaces point-like particles with 3-surfaces. `http://www.tgdtheory.fi/appfigures/particletgd.jpg`

The fact that light-like 3-surfaces are effectively metrically 2-dimensional and thus possess generalization of 2-dimensional conformal symmetries with light-like radial coordinate defining the analog of second complex coordinate suggests that this generalization could work and extend the super-conformal symmetries to their 4-D analogs.

The boundary $\delta M^4_+ = S^2 \times R_+$- of 4-D light-cone M^4_+ is also metrically 2-dimensional and allows extended conformal invariance. Also the group of isometries of light-cone boundary and of light-like 3-surfaces is infinite-dimensional since the conformal scalings of S^2 can be compensated by S^2-local scaling of the light-like radial coordinate of R_+. These simple facts mean that 4-dimensional Minkowski space and 4-dimensional space-time surfaces are in completely unique position as far as symmetries are considered.

String like objects obtained as deformations of cosmic strings $X^2 \times Y^2$, where X^2 is minimal surface in M^4 and Y^2 a holomorphic surface of CP_2 are fundamental extremals of Kähler action having string world sheet as M^4 projections. Cosmic strings dominate the primordial cosmology of TGD Universe and inflationary period corresponds to the transition to radiation dominated cosmology for which space-time sheets with 4-D M^4 projection dominate.

Also genuine string like objects emerge from TGD. The conditions that the em charge of modes of induces spinor fields is well-defined requires in the generic case the localization of the modes at 2-D surfaces -string world sheets and possibly also partonic 2-surfaces. This in Minkowskian space-time regions.

Fig. 6. Well-definedness of em charge forces the localization of induced spinor modes to 2-D surfaces in generic situtation in Minkowskian regions of space-time surface. `http://www.tgdtheory.fi/appfigures/fermistring.jpg`

TGD based view about elementary particles has two aspects.

i. The space-time correlates of elementary particles are identified as pairs of wormhole contacts with Euclidian signature of metric and having 4-D CP_2 projection. Their throats behave effectively as Kähler magnetic monopoles so that wormhole throats must be connected by Kähler magnetic flux tubes with monopole flux so that closed flux tubes are obtained.

ii. Fermion number is carried by the modes of the induced spinor field. In Minkowskian space-time regions the modes are localized at string world sheets connecting the wormhole contacts.

Fig. 7. TGD view about elementary particles. a) Particle corresponds 4-D generalization of world line or b) with its light-like 3-D boundary (holography). c) Particle world lines have Euclidian signature of the induced metric. d) They can be identified as wormhole contacts. e) The throats of wormhole contacs carry effective Kähler magnetic charges so that wormhole contacts must appear as pairs in order to obtain closed flux tubes. f) Wormhole contacts are accompnied by fermionic strings connecting the throats at same sheet: the strings do not extend inside the wormhole contacts. `http://www.tgdtheory.fi/appfigures/elparticletgd.jpg`

Particle interactions involve both stringy and QFT aspects.

i. The boundaries of string world sheets correspond to fundamental fermions. This gives rise to massless propagator lines in generalized Feynman diagrammatics. One can speak of "long" string connecting wormhole contacts and having hadronic string as physical counterpart. Long strings should be distinguished from wormhole contacts which due to their super-conformal invariance behave like "short" strings with length scale given by CP_2 size, which is 10^4 times longer than Planck scale characterizing strings in string models.

ii. Wormhole contact defines basic stringy interaction vertex for fermion-fermion scattering. The propagator is essentially the inverse of the superconformal scaling generator L_0. Wormhole contacts containing fermion and antifermion at its opposite throats beheave like virtual bosons so that one has BFF type vertices typically.

iii. In topological sense one has 3-vertices serving as generalizations of 3-vertices of Feynman diagrams. In these vertices 4-D "lines" of generalized Feynman diagrams meet along their 3-D ends. One obtains also the analogs of stringy diagrams but stringy vertices do not have the usual interpretation in terms of particle decays but in terms of propagation of particle along two different routes.

Fig. 8. a) TGD analogs of Feynman and string diagrammatics at the level of space-time topology. b) The 4-D analogs of both string diagrams and QFT diagrams appear but the interpretation of the analogs stringy diagrams is different. `http://www.tgdtheory.fi/appfigures/tgdgraphs.jpg`

A-5 Induction procedure and many-sheeted space-time

Since the classical gauge fields are closely related in TGD framework, it is not possible to have space-time sheets carrying only single kind of gauge field. For instance, em fields are accompanied by Z^0 fields for extremals of Kähler action.

Classical em fields are always accompanied by Z^0 field and some components of color gauge field. For extremals having homologically non-trivial sphere as a CP_2 projection em and Z^0 fields are the only non-vanishing electroweak gauge fields. For homologically trivial sphere only W fields are non-vanishing. Color rotations does not affect the situation.

For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy by 2-dimensionality of the CP_2 projection. Color gauge field has $U(1)$ holonomy for all space-time surfaces and quantum classical correspondence suggest a weak form of color confinement meaning that physical states correspond to color neutral members of color multiplets.

Induction procedure for gauge fields

Induction procedure for gauge potentials and spinor structure is a standard procedure of bundle theory. If one has imbedding of some manifold to the base space of a bundle, the bundle structure can be induced so that it has as base space the imbedded manifold. In the recent case the imbedding of space-time surface to imbedding space defines the induction procedure. The induce gauge potentials and gauge fields are projections of the spinor connection of the imbedding space to the space-time surface. Induction procedure makes sense also for the spinor fields of imbedding space and one obtains geometrization of both electroweak gauge potentials and of spinors.

Fig. 9. Induction of spinor connection and metric as projection to the space-time surface. `http://www.tgdtheory.fi/appfigures/induct.jpg`

Induced gauge fields for space-times for which CP$_2$ projection is a geodesic sphere

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP$_2$ projection, only vacuum extremals and space-time surfaces for which CP$_2$ projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. This can be verified by explicit examples.

$r = \infty$ surface gives rise to a homologically non-trivial geodesic sphere for which e_0 and e_3 vanish imply the vanishing of W field. For space-time sheets for which CP$_2$ projection is $r = \infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \quad .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

$Im(\xi^1) = Im(\xi^2) = 0$ corresponds to homologically trivial geodesic sphere. A more general representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case e^1 and e^3 vanish so that the induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP$_2$ projection color rotations and weak symmetries commute.

A-5.1 Many-sheeted space-time

TGD space-time is many-sheeted: in other words, there are in general several space-sheets which have projection to the same M^4 region. Second manner to say this is that CP_2 coordinates are many-valued functions of M^4 coordinates. The original physical interpretation of many-sheeted space-time time was not correct: it was assumed that single sheet corresponds to GRT space-time and this obviously leads to difficulties since the induced gauge fields are expressible in terms of only four imbedding space coordinates.

Fig. 10. Illustration of many-sheeted space-time of TGD. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg`

Superposition of effects instead of superposition of fields

The first objection against TGD is that superposition is not possible for induced gauge fields and induced metric. The resolution of the problem is that it is effects which need to superpose, not the fields.

Test particle topologically condenses simultaneously to all space-time sheets having a projection to same region of M^4 (that is touches them). The superposition of effects of fields at various space-time sheets replaces the superposition of fields.This is crucial for the understanding also how GRT space-time relates to TGD space-time, which is also in the appendix of this book).

Wormhole contacts

Wormhole contacts are key element of many-sheeted space-time. One does not expect them to be stable unless there is non-trivial Kähler magnetic flux flowing through then so that the throats look like Kähler magnetic monopoles.

Fig. 11. Wormhole contact. `http://www.tgdtheory.fi/appfigures/wormholecontact.jpg`

Since the flow lines of Kähler magnetic field must be closed this requires the presence of another wormhole contact so that one obtains closed monopole flux tube decomposing to two Minkowskian pieces at the two space-time sheets involved and two wormhole contacts with Euclidian signature of the induced metric. These objects are identified as space-time correlates of elementary particles and are clearly analogous to string like objects.

The relationship between the many-sheeted space-time of TGD and of GRT space-time

The space-time of general relativity is single-sheeted and there is no need to regard it as surface in H although the assumption about representability as vacuum extremal gives very powerful constraints in cosmology and astrophysics and might make sense in simple situations.

The space-time of GRT can be regarded as a long length scale approximation obtained by lumping together the sheets of the many-sheeted space-time to a region of M^4 and providing it with an effective metric obtained as sum of M^4 metric and deviations of the induced metrics of various space-time sheets from M^4 metric. Also induced gauge potentials sum up in the similar manner so that also the gauge fields of gauge theories would not be fundamental fields.

Fig. 12. The superposition of fields is replaced with the superposition of their effects in many-sheeted space-time. `http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg`

Space-time surfaces of TGD are considerably simpler objects that the space-times of general relativity and relate to GRT space-time like elementary particles to systems of condensed matter physics. Same can be said about fields since all fields are expressible in terms of imbedding space coordinates and their gradients, and general coordinate invariance means that the number of bosonic field degrees is reduced locally to 4. TGD space-time can be said to be a microscopic description whereas GRT space-time a macroscopic description. In TGD complexity of space-time topology replaces the complexity due to large number of fields in quantum field theory.

Topological field quantization and the notion of magnetic body

Topological field quantization also TGD from Maxwell's theory. TGD predicts topological light rays ("massless extremals (MEs)) as space-time sheets carrying waves or arbitrary shape propagating with maximal signal velocity in single direction only and analogous to laser beams and carrying light-like gauge currents in the generi case. There are also magnetic flux quanta and electric flux quanta. The deformations of cosmic strings with 2-D string orbit as M^4 projection gives rise to magnetic flux tubes carrying monopole flux made possible by CP_2 topology allowing homological Kähler magnetic monopoles.

Fig. 13. Topological quantization for magnetic fields replaces magnetic fields with bundles of them defining flux tubes as topological field quanta. `http://www.tgdtheory.fi/appfigures/field.jpg`

The imbeddability condition for say magnetic field means that the region containing constant magnetic field splits into flux quanta, say tubes and sheets carrying constant magnetic field. Unless one assumes a separate boundary term in Kähler action, boundaries in the usual sense are forbidden except as ends of space-time surfaces at the boundaries of causal diamonds. One obtains typically pairs of sheets glued together along their boundaries giving rise to flux tubes with closed cross section possibly carrying monopole flux.

These kind of flux tubes might make possible magnetic fields in cosmic scales already during primordial period of cosmology since no currents are needed to generate these magnetic fields: cosmic string would be indeed this kind of objects and would dominated during the primordial period. Even superconductors and maybe even ferromagnets could involve this kind of monopole flux tubes.

A-5.2 Imbedding space spinors and induced spinors

One can geometrize also fermionic degrees of freedom by inducing the spinor structure of $M^4 \times CP_2$.

CP_2 does not allow spinor structure in the ordinary sense but one can couple the opposite H-chiralities of H-spinors to an $n = 1$ ($n = 3$) integer multiple of Kähler gauge potential to obtain a respectable modified spinor structure. The em charges of resulting spinors are fractional (integer valued) and the interpretation as quarks (leptons) makes sense since the couplings to the induced spinor connection having interpretation in terms electro-weak gauge potential are identical to those assumed in standard model.

The notion of quark color differs from that of standard model.

i. Spinors do not couple to color gauge potential although the identification of color gauge potential as projection of $SU(3)$ Killing vector fields is possible. This coupling must emerge only at the effective gauge theory limit of TGD.

ii. Spinor harmonics of imbedding space correspond to triality $t = 1$ ($t = 0$) partial waves. The detailed correspondence between color and electroweak quantum numbers is however not correct as such and the interpretation of spinor harmonics of imbedding space is as representations for ground states of super-conformal representations. The wormhole pairs associated with physical quarks and leptons must carry also neutrino pair to neutralize weak quantum numbers above the length scale of flux tube (weak scale or Compton length). The total color quantum numbers or these states must be those of standard model. For instance, the color quantum numbers of fundamental left-hand neutrino and lepton can compensate each other for the physical lepton. For fundamental quark-lepton pair they could sum up to those of physical quark.

The well-definedness of em charge is crucial condition.

i. Although the imbedding space spinor connection carries W gauge potentials one can say that the imbedding space spinor modes have well-defined em charge. One expects that this is true for induced spinor fields inside wormhole contacts with 4-D CP_2 projection and Euclidian signature of the induced metric.

ii. The situation is not the same for the modes of induced spinor fields inside Minkowskian region and one must require that the CP_2 projection of the regions carrying induced spinor field is such that the induced W fields and above weak scale also the induced Z^0 fields vanish in order to avoid large parity breaking effects. This condition forces the CP_2 projection to be 2-dimensional. For a generic Minkowskian space-time region this is achieved only if the spinor modes are localized at 2-D surfaces of space-time surface - string world sheets and possibly also partonic 2-surfaces.

iii. Also the Kähler-Dirac gamma matrices appearing in the modified Dirac equation must vanish in the directions normal to the 2-D surface in order that Kähler-Dirac equation can be satisfied. This does not seem plausible for space-time regions with 4-D CP_2 projection.

iv. One can thus say that strings emerge from TGD in Minkowskian space-time regions. In particular, elementary particles are accompanied by a pair of fermionic strings at the opposite space-time sheets and connecting wormhole contacts. Quite generally, fundamental fermions would propagate at the boundaries of string world sheets as massless particles and wormhole contacts would define the stringy vertices of generalized Feynman diagrams. One obtains geometrized diagrammatics, which brings looks like a combination of stringy and Feynman diagrammatics.

v. This is what happens in the the generic situation. Cosmic strings could serve as examples about surfaces with 2-D CP_2 projection and carrying only em fields and allowing delocalization of spinor modes to the entire space-time surfaces.

A-5.3 Space-time surfaces with vanishing em, Z^0, or Kähler fields

In the following the induced gauge fields are studied for general space-time surface without assuming the extremal property. In fact, extremal property reduces the study to the study of vacuum extremals and surfaces having geodesic sphere as a CP_2 projection and in this sense the following arguments are somewhat obsolete in their generality.

Space-times with vanishing em, Z^0, or Kähler fields

The following considerations apply to a more general situation in which the homologically trivial geodesic sphere and extremal property are not assumed. It must be emphasized that this case is possible in TGD framework only for a vanishing Kähler field.

Using spherical coordinates (r, Θ, Ψ, Φ) for CP_2, the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2}dr \wedge (d\Psi + cos(\Theta)d\Phi) + \frac{r^2}{2F}sin(\Theta)d\Theta \wedge d\Phi \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-5.1}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3+2p)\frac{r}{F^2}dr \wedge (d\Psi + cos(\Theta)d\Phi) + (3+p)\frac{r^2}{2F}sin(\Theta)d\Theta \wedge d\Phi \ , \\ p &= sin^2(\Theta_W) \ , \end{aligned} \tag{A-5.2}$$

where Θ_W denotes Weinberg angle.

i. The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi \ , \\ (3+2p)\frac{1}{r^2F}(d(r^2)/d\Theta)(k + cos(\Theta)) &+ (3+p)sin(\Theta) = 0 \ , \end{aligned} \tag{A-5.3}$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$\begin{aligned}
r &= \sqrt{\frac{X}{1-X}} \ , \\
X &= D\left[|\frac{(k+u}{C}|\right]^{\epsilon} \ , \\
u &\equiv cos(\Theta) \ , \ C = k + cos(\Theta_0) \ , \ D = \frac{r_0^2}{1+r_0^2} \ , \ \epsilon = \frac{3+p}{3+2p} \ ,
\end{aligned} \quad \text{(A-5.4)}$$

where C and D are integration constants. $0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u + k| = [(1 + r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \leq k+1 \ ,$$

where $sign(x)$ denotes the sign of x.
The expressions for Kähler form and Z^0 field are given by

$$\begin{aligned}
J &= -\frac{p}{3+2p} X du \wedge d\Phi \ , \\
Z^0 &= -\frac{6}{p} J \ .
\end{aligned} \quad \text{(A-5.5)}$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

ii. The vanishing of Z^0 fields is achieved by the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically. Also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F} du \wedge d\Phi$ is useful.

iii. The vanishing Kähler field corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$\begin{aligned}
Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u} du \wedge d\Phi = (k+u) du \wedge d\Phi \ , \\
r &= \sqrt{\frac{X}{1-X}} \ , \ X = D|k+u| \ , \\
\gamma &= -\frac{p}{2} Z^0 \ .
\end{aligned} \quad \text{(A-5.6)}$$

For a vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

The effective form of CP_2 metric for surfaces with 2-dimensional CP_2 projection

The effective form of the CP_2 metric for a space-time having vanishing em,Z^0, or Kähler field is of practical value in the case of vacuum extremals and is given by

$$\begin{aligned}
ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta} d\Theta^2 + s^{eff}_{\Phi\Phi} d\Phi^2] \ , \\
s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\
s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ ,
\end{aligned} \quad \text{(A-5.7)}$$

and is useful in the construction of vacuum imbedding of, say Schwartchild metric.

Topological quantum numbers

Space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$\begin{aligned} \Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\ \Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ . \end{aligned} \quad \text{(A-5.8)}$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which space-time surface becomes ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global imbedding for, say a constant magnetic field. Although global imbedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteeing the absence of edges are satisfied.

For instance, the vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \quad \text{(A-5.9)}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

A-6 p-Adic numbers and TGD

A-6.1 p-Adic number fields

p-Adic numbers (p is prime: 2,3,5,...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [A32] . p-Adic numbers are representable as power expansion of the prime number p of form

$$x = \sum_{k\geq k_0} x(k)p^k,\ x(k) = 0,....,p-1 \ . \tag{A-6.1}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \ . \tag{A-6.2}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \ , \tag{A-6.3}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x,y) = |x-y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x,z) \leq max\{d(x,y), d(y,z)\} \ . \tag{A-6.4}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x,y) \leq D \ . \tag{A-6.5}$$

This division of the metric space into classes has following properties:

i. Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.
ii. Distances of points x and y inside single class are smaller than distances between different classes.
iii. Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [B24] . The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

A-6.2 Canonical correspondence between p-adic and real numbers

The basic challenge encountered by p-adic physicist is how to map the predictions of the p-adic physics to real numbers. p-Adic probabilities provide a basic example in this respect. Identification via common rationals and canonical identification and its variants have turned out to play a key role in this respect.

Basic form of canonical identification

There exists a natural continuous map $I : R_p \to R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \to x = \sum_{k<N} y_k p^{-k} \ , \\ y_k &\in \{0,1,..,p-1\} \ . \end{aligned} \tag{A-6.6}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique ($1 = 0.999...$) for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \ , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \ . \end{aligned} \tag{A-6.7}$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^{N} x_k p^k \ , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^k + (x_N - 1)p^N + (p-1)p^{N+1} \sum_{k=0,..} p^k \\ &= y_1 + (x_N - 1)p^N - p^{N+1} \ , \end{aligned} \tag{A-6.8}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

The topology induced by canonical identification

The topology induced by the canonical identification in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see Fig. **??**) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

Fig. 14. The real norm induced by canonical identification from 2-adic norm. `http://www.tgdtheory.fi/appfigures/norm.png`

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common pinary digits. Furthermore, the condition $x +_p y < max\{x, y\}$ holds in general for the p-adic sum of the real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of p. Moreover one has $x \times_p y < x \times y$ in general. The p-Adic negative -1_p associated with p-adic unit 1 is given by $(-1)_p = \sum_k (p-1)p^k$ and defines p-adic negative for each real number x. An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

These results suggest that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |x|_p |y|_R \leq (xy)_R &\leq x_R y_R \ , \end{aligned} \tag{A-6.9}$$

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to the case of $(R_p)^n$ (a linear vector space over the p-adic numbers).

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |\lambda|_p |y|_R \leq (\lambda y)_R &\leq \lambda_R y_R \ , \end{aligned} \tag{A-6.10}$$

where the norm of the vector $x \in T_p^n$ is defined in some manner. The case of Euclidian space suggests the definition

$$(x_R)^2 = (\sum_n x_n^2)_R \ . \tag{A-6.11}$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of p.

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some non-linear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

Modified form of the canonical identification

The original form of the canonical identification is continuous but does not respect symmetries even approximately. This led to a search of variants which would do better in this respect. The modification of the canonical identification applying to rationals only and given by

$$I_Q(q = p^k \times \frac{r}{s}) = p^k \times \frac{I(r)}{I(s)} \tag{A-6.12}$$

is uniquely defined for rationals, maps rationals to rationals, has also a symmetry under exchange of target and domain. This map reduces to a direct identification of rationals for $0 \leq r < p$ and $0 \leq s < p$. It has turned out that it is this map which most naturally appears in the applications. The map is obviously continuous locally since p-adically small modifications of r and s mean small modifications of the real counterparts.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. The predictions for the light elementary particle masses are within extreme accuracy same for I and I_Q but I_Q is theoretically preferred since the real probabilities obtained from p-adic ones by I_Q sum up to one in p-adic thermodynamics.

Generalization of number concept and notion of imbedding space

TGD forces an extension of number concept: roughly a fusion of reals and various p-adic number fields along common rationals is in question. This induces a similar fusion of real and p-adic imbedding spaces. Since finite p-adic numbers correspond always to non-negative reals n-dimensional space R^n must be covered by 2^n copies of the p-adic variant R_p^n of R^n each of which projects to a copy of R_+^n (four quadrants in the case of plane). The common points of p-adic and real imbedding spaces are rational points and most p-adic points are at real infinity.

Real numbers and various algebraic extensions of p-adic number fields are thus glued together along common rationals and also numbers in algebraic extension of rationals whose number belong to the algebraic extension of p-adic numbers. This gives rise to a book like structure with rationals and various algebraic extensions of rationals taking the role of the back of the book. Note that Neper number is exceptional in the sense that it is algebraic number in p-adic number field Q_p satisfying $e^p \ mod \ p = 1$.

Fig. 15. Various number fields combine to form a book like structure. `http://www.tgdtheory.fi/appfigures/book.jpg`

For a given p-adic space-time sheet most points are literally infinite as real points and the projection to the real imbedding space consists of a discrete set of rational points: the interpretation in terms of the unavoidable discreteness of the physical representations of cognition is natural. Purely local p-adic physics implies real p-adic fractality and thus long range correlations for the real space-time surfaces having enough common points with this projection.

p-Adic fractality means that M^4 projections for the rational points of space-time surface X^4 are related by a direct identification whereas CP_2 coordinates of X^4 at these points are related by I, I_Q or some of its variants implying long range correlates for CP_2 coordinates. Since only a discrete set of points are related in this manner, both real and p-adic field equations can be satisfied and there are no problems with symmetries. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs. Also multi-p-fractality is possible.

A-6.3 The notion of p-adic manifold

The notion of p-adic manifold is needed in order to fuse real physics and various p-adic physics to a larger structure which suggests that real and p-adic number fields should be glued together along common rationals bringing in mind adeles. The notion is problematic because p-adic topology is totally disconnected implying that p-adic balls are either disjoint or nested so that ordinary definition of manifold using p-adic chart maps fails. A cure is suggested to be based on chart maps from p-adics to reals rather than to p-adics (see the appendix of the book)

The chart maps are interpreted as cognitive maps, "thought bubbles" with reverse map interpreted as a transformation of intention to action and would be realized in terms of canonical identification or some of its variants.

Fig. 16. The basic idea between p-adic manifold. `http://www.tgdtheory.fi/appfigures/padmanifold.jpg`

There are some problems.

i. Canonical identification does not respect symmetries since it does not commute with second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map arithmetic operations which requires pinary cutoff below which chart map takes rationals to rationals so that commutativity with arithmetics and symmetries is achieved in finite resolution: above the cutoff canonical identification is used

ii. Canonical identification is continuous but does not map smooth p-adic surfaces to smooth real surfaces requiring second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map requiring completion of the image to smooth preferred extremal of Kähler action so that chart map is not unique in accordance with finite measurement resolution

iii. Canonical identification vreaks general coordinate invariance of chart map: (cognition-induced symmetry breaking) minimized if p-adic manifold structure is induced from that for p-adic imbedding space with chart maps to real imbedding space and assuming preferred coordinates made possible by isometries of imbedding space: one however obtains several inequivalent p-adic manifold structures depending on the choice of coordinates: these cognitive representations are not equivalent.

A-7 Hierarchy of Planck constants and dark matter hierarchy

Hierarchy of Planck constants was motivated by the "impossible" quantal effects of ELF em fields on vertebrate cyclotron energies $E = hf = \hbar \times eB/m$ are above thermal energy is possible only if $\hbar$ has value much larger than its standard value. Also Nottale's finding that planetary orbits migh be understood as Bohr orbits for a gigantic gravitational Planck constant.

Hierachy of Planck constant would mean that the values of Planck constant come as integer multiples of ordinary Planck constant: $h_{eff} = n \times h$. The particles at magnetic flux tubes characterized by h_{eff} would correspond to dark matter which would be invisible in the sense that only particle with same value of h_{eff} appear in the same vertex of Feynman diagram.

Hierarchy of Planck constants would be due to the non-determism of the Kähler action predicting huge vacuum degeneracy allowing all space-time surfaces which are sub-manfolds of any $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2. For agiven Y^2 one obtains new manifolds Y^2 by applying symplectic transformations of CP_2.

Non-determinism would mean that the 3-surface at the ends of causal diamond (CD) can be connected by several space-time surfaces carrying same conserved Kähler charges and having same values of Kähler action. Conformal symmetries defined by Kac-Moody algebra associated with the imbedding space isometries could act as gauge transformations and respect the light-likeness property of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian (Minkowskianb space-time region transforms to wormhole contact say). The number of conformal equivalence classes of these surfaces could be finite number n and define discrete physical degree of freedom and one would have $h_{eff} = n \times h$. This degeneracy would mean "second quantization" for the sheets of n-furcation: not only one but several sheets can be realized.

This relates also to quantum criticality postulated to be the basic characteristics of the dynamics of quantum TGD. Quantum criticalities would correspond to an infinite fractal hierarchy of broken conformal symmetries defined by sub-algebras of conformal algebra with conformal weights coming as integer multiples of n. This leads also to connections with quantum criticality and hierarchy of broken conformal symmetries, p-adicity, and negentropic entanglement which by consistency with standard quantum measurement theory would be described in terms of density matrix proportional $n \times n$ identity matrix and being due to unitary entanglement coefficients (typical for quantum computing systems).

Formally the situation could be described by regarding space-time surfaces as surfaces in singular n-fold singular coverings of imbedding space. A stronger assumption would be that they are expressible as as products of n_1 -fold covering of M^4 and n_2-fold covering of CP_2 meaning analogy with multi-sheeted Riemann surfaces and that M^4 coordinates are n_1-valued functions and CP_2 coordinates n_2 -valued functions of space-time coordinates for $n = n_1 \times n_2$. These singular coverings of imbedding space form a book like structure with singularities of the coverings localizable at the boundaries of causal diamonds defining the back of the the book like structure.

Fig. 17. Hierarchy of Planck constants. http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg

A-8 Some notions relevant to TGD inspired consciousness and quantum biology

Below some notions relevant to TGD inspired theory of consciousness and quantum biology.

A-8.1 The notion of magnetic body

Topological field quantization inspires the notion of field body about which magnetic body is especially important example and plays key role in TGD inspired quantum biology and consciousness theory. This is a crucial departure fromt the Maxwellian view. Magnetic body brings in third level to the description of living system as a system interacting strongly with environment. Magnetic body would serve as an intentional agent using biological body as a motor instrument and sensory receptor. EEG would communicated the information from biological body to magnetic body and Libet's findings from time delays of consciousness support this view.
The following pictures illustrate the notion of magnetic body and its dynamics relevant for quantum biology in TGD Universe.

Fig. 18. Magnetic body associated with dipole field. http://www.tgdtheory.fi/appfigures/fluxquant.jpg

Fig. 19. Illustration of the reconnection by magnetic flux loops. http://www.tgdtheory.fi/appfigures/reconnect1.jpg

Fig. 20. Illustration of the reconnection by flux tubes connecting pairs of molecules. http://www.tgdtheory.fi/appfigures/reconect2.jpg

Fig. 21. Flux tube dynamics. a) Reconnection making possible magnetic body to "recognize" the presence of another magnetic body, b) braiding, knotting and linking of flux tubes making possible topological quantum computation, c) contraction of flux tube in phase transition reducing the value of h_{eff} allowing two molecules to find each other in dense molecular soup. http://www.tgdtheory.fi/appfigures/fluxtubedynamics.jpg

A-8.2 Number theoretic entropy and negentropic entanglement

TGD inspired theory of consciousness relies heavily p-Adic norm allows an to define the notion of Shannon entropy for rational probabilities (and even those in algebraic extension of rationals) by replacing the argument of logarithm of probability with its p-adic norm. The resulting entropy can be negative and the interpretation is that number theoretic entanglement entropy defined by this formula for the p-adic prime minimizing its value serves as a measure for conscious information. This negentropy characterizes two-particle system and has nothing to do with the formal negative negentropy assignable to thermodynamic entropy characterizing single particle. Negentropy Maximization Principle (NMP) implies that number theoretic negentropy increases during evolution by quantum jumps. The condition that NMP is consistent with the standard quantum measurement theory requires that negentropic entanglement has a density matrix proportional to unit matrix so that in 2-particle case the entanglement matrix is unitary.

Fig. 22. Schrödinger cat is neither dead or alive. For negentropic entanglement this state would be stable. http://www.tgdtheory.fi/appfigures/cat.jpg

A-8.3 Life as something residing in the intersection of reality and p-adicities

In TGD inspired theory of consciousness p-adic space-time sheets correspond to space-time correlates for thoughts and intentions. The intersections of real and p-adic preferred extremals consist of points whose coordinates are rational or belong to some extension of rational numbers in preferred imbedding space coordinates. They would correspond to the intersection of reality and various p-adicities representing the "mind stuff" of Descartes. There is temptation to assign life to the intersection of realities and p-adicities. The discretization of the chart map assigning to real space-time surface its p-adic counterpart would reflect finite cognitive resolution.

At the level of "world of classical worlds" (WCW) the intersection of reality and various p-adicities would correspond to space-time surfaces (or possibly partonic 2-surfaces) representable in terms of rational functions with polynomial coefficients with are rational or belong to algebraic extension of rationals.

The quantum jump replacing real space-time sheet with p-adic one (vice versa) would correspond to a buildup of cognitive representation (realization of intentional action).

Fig. 23. The quantum jump replacing real space-time surface with corresponding p-adic manifold can be interpreted as formation of though, cognitive representation. Its reversal would correspond to a transformation of intention to action. http://www.tgdtheory.fi/appfigures/padictoreal.jpg

A-8.4 Sharing of mental images

The 3-surfaces serving as correlates for sub-selves can topologically condense to disjoint large space-time sheets representing selves. These 3-surfaces can also have flux tube connections and this makes possible entanglement of sub-selves, which unentangled in the resolution defined by the size of sub-selves. The interpretation for this negentropic entanglement would be in terms of sharing of mental images. This would mean that contents of consciousness are not completely private as assumed in neuroscience.

Fig. 24. Sharing of mental images by entanglement of subselves made possible by flux tube connections between topologically condensed space-time sheets associated with mental images. http://www.tgdtheory.fi/appfigures/sharing.jpg

A-8.5 Time mirror mechanism

Zero energy ontology (ZEO) is crucial part of both TGD and TGD inspired consciousness and leads to the understanding of the relationship between geometric time and experience time and how the arrow of psychological time emerges. One of the basic predictions is the possibiity of negative energy signals propagating backwards in geometric time and having the property that entropy basically associated with subjective time grows in reversed direction of geometric time. Negative energy signals inspire time mirror mechanism (see fig. http://www.tgdtheory.fi/appfigures/timemirror.jpg or fig. 24 in the appendix of this book) providing mechanisms of both memory recall, realization of intentational action initiating action already in geometric past, and remote metabolism. What happens that negative energy signal travels to past and is reflected as positive energy signal and returns to the sender. This process works also in the reverse time direction.

Fig. 25. Zero energy ontology allows time mirror mechanism as a mechanism of memory recall. Essentially "seeing" in time direction is in question. http://www.tgdtheory.fi/appfigures/timemirror.jpg

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Index

TGD: PHYSICS AS INFINITE-DIMENSIONAL GEOMETRY

Topological Geometrodynamics (TGD) is a modification of general relativity inspired by the problems related to the definition of inertial and gravitational energies in general relativity. TGD is also a generalization of super string models. Physical space-times are seen as four-dimensional surfaces in certain 8-dimensional space H. The choice of H is fixed by symmetries of standard model and leads to a geometrization of known classical fields and elementary particle numbers. In fermionic sector strings indeed emerge.

Many-sheeted space-time replaces Einsteinian space-time, which follows as a long length scale approximation in which sheets of the many-sheeted space-time are lumped together. The extension of number concept based on the fusion of real numbers and p-adic number fields implies a further generalisation of the space-time concept allowing to identify space-time correlates of cognition and intentionality.

Zero energy ontology forces an extension of quantum measurement theory to a theory of consciousness and a hierarchy of phases identified as dark matter is predicted with far reaching implications for the understanding of consciousness and living systems. This all implies an elegant theoretical projection of our reality honoring the work by renowned scientists (such as Wheeler, Feynman, Penrose, Einstein, Josephson to name a few) and creating a solid foundation for modeling our Universe in terms of geometry.

Ph.D. Matti Pitkänen • *Hanko, Finland*

Matti Pitkänen started to work with the basic idea of TGD at 1977, published his thesis work about TGD at 1982, and has since then worked to transform the basic vision to a consistent predictive mathematical framework, to solve various interpretational issues, and understand the relationship of TGD with existing theories.

TGD Web Pages: http://www.tgdtheory.com
TGD Diary and Blog: http://matpitka.blogspot.com