TOPOLOGICAL GEOMETRODYNAMICS: PHYSICS AS INFINITE-DIMENSIONAL GEOMETRY

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Preface

This book belongs to a series of online books summarizing the recent state Topological Geometrodynamics (TGD) and its applications. TGD can be regarded as a unified theory of fundamental interactions but is not the kind of unified theory as so called GUTs constructed by graduate students at seventies and eighties using detailed recipes for how to reduce everything to group theory. Nowadays this activity has been completely computerized and it probably takes only a few hours to print out the predictions of this kind of unified theory as an article in the desired format. TGD is something different and I am not ashamed to confess that I have devoted the last 32 years of my life to this enterprise and am still unable to write The Rules.

I got the basic idea of Topological Geometrodynamics (TGD) during autumn 1978, perhaps it was October. What I realized was that the representability of physical space-times as 4-dimensional surfaces of some higher-dimensional space-time obtained by replacing the points of Minkowski space with some very small compact internal space could resolve the conceptual difficulties of general relativity related to the definition of the notion of energy. This belief was too optimistic and only with the advent of what I call zero energy ontology the understanding of the notion of Poincare invariance has become satisfactory.

It soon became clear that the approach leads to a generalization of the notion of space-time with particles being represented by space-time surfaces with finite size so that TGD could be also seen as a generalization of the string model. Much later it became clear that this generalization is consistent with conformal invariance only if space-time is 4-dimensional and the Minkowski space factor of imbedding space is 4-dimensional.

It took some time to discover that also the geometrization of also gauge interactions and elementary particle quantum numbers could be possible in this framework: it took two years to find the unique internal space providing this geometrization involving also the realization that family replication phenomenon for fermions has a natural topological explanation in TGD framework and that the symmetries of the standard model symmetries are much more profound than pragmatic TOE builders have believed them to be. If TGD is correct, main stream particle physics chose the wrong track leading to the recent deep crisis when people decided that quarks and leptons belong to same multiplet of the gauge group implying instability of proton.

There have been also longstanding problems.

- Gravitational energy is well-defined in cosmological models but is not conserved. Hence the conservation of the inertial energy does not seem to be consistent with the Equivalence Principle. Furthermore, the imbeddings of Robertson-Walker cosmologies turned out to be vacuum extremals with respect to the inertial energy. About 25 years was needed to realize that the sign of the inertial energy can be also negative and in cosmological scales the density of inertial energy vanishes: physically acceptable universes are creatable from vacuum. Eventually this led to the notion of zero energy ontology which deviates dramatically from the standard ontology being however consistent with the crossing symmetry of quantum field theories. In this framework the quantum numbers are assigned with zero energy states located at the boundaries of so called causal diamonds defined as intersections of future and past directed light-cones. The notion of energy-momentum becomes length scale dependent since one has a scale hierarchy for causal diamonds. This allows to understand the non-conservation of energy as apparent. Equivalence Principle generalizes and has a formulation in terms of coset representations of Super-Virasoro algebras providing also a justification for p-adic thermodynamics.

- From the beginning it was clear that the theory predicts the presence of long ranged classical electro-weak and color gauge fields and that these fields necessarily accompany classical electromagnetic fields. It took about 26 years to gain the maturity to admit the obvious: these fields are classical correlates for long range color and weak interactions assignable to dark matter. The only possible conclusion is that TGD physics is a fractal consisting of an entire hierarchy of fractal copies of standard model physics. Also the understanding of electro-weak massivation and screening of weak charges has been a long standing problem, and 32 years was needed to discover that what I call weak form of electric-magnetic duality gives a satisfactory solution of the problem and provides also surprisingly powerful insights to the mathematical structure of quantum TGD.

I started the serious attempts to construct quantum TGD after my thesis around 1982. The original optimistic hope was that path integral formalism or canonical quantization might be enough to construct the quantum theory but the first discovery made already during first year of TGD was that these formalisms might be useless due to the extreme non-linearity and enormous vacuum degeneracy of the theory. This turned out to be the case.

- It took some years to discover that the only working approach is based on the generalization of Einstein's program. Quantum physics involves the geometrization of the infinite-dimensional "world of classical worlds" (WCW) identified as 3-dimensional surfaces. Still few years had to pass before I understood that general coordinate invariance leads to a more or less unique solution of the problem and implies that space-time surfaces are analogous to Bohr orbits. Still a coupled of years and I discovered that quantum states of the Universe can be identified as classical spinor fields in WCW. Only quantum jump remains the genuinely quantal aspect of quantum physics.

- During these years TGD led to a rather profound generalization of the space-time concept. Quite general properties of the theory led to the notion of many-sheeted space-time with sheets representing physical subsystems of various sizes. At the beginning of 90s I became dimly aware of the importance of p-adic number fields and soon ended up with the idea that p-adic thermodynamics for a conformally invariant system allows to understand elementary particle massivation with amazingly few input assumptions. The attempts to understand p-adicity from basic principles led gradually to the vision about physics as a generalized number theory as an approach complementary to the physics as an infinite-dimensional spinor geometry of WCW approach. One of its elements was a generalization of the number concept obtained by fusing real numbers and various p-adic numbers along common rationals. The number theoretical trinity involves besides p-adic number fields also quaternions and octonions and the notion of infinite prime.

- TGD inspired theory of consciousness entered the scheme after 1995 as I started to write a book about consciousness. Gradually it became difficult to say where physics ends and consciousness theory begins since consciousness theory could be seen as a generalization of quantum measurement theory by identifying quantum jump as a moment of consciousness and by replacing the observer with the notion of self identified as a system which is conscious as long as it can avoid entanglement with environment. "Everything is conscious and consciousness can be only lost" summarizes the basic philosophy neatly. The idea about p-adic physics as physics of cognition and intentionality emerged also rather naturally and implies perhaps the most dramatic generalization of the space-time concept in which most points of p-adic space-time sheets are infinite in real sense and the projection to the real imbedding space consists of discrete set of points. One of the most fascinating outcomes was the observation that the entropy based on p-adic norm can be negative. This observation led to the vision that life can be regarded as something in the intersection of real and p-adic worlds. Negentropic entanglement has interpretation as a correlate for various positively colored aspects of conscious experience and means also the possibility of strongly correlated states stable under state function reduction and different from the conventional bound states and perhaps playing key role in the energy metabolism of living matter.

- One of the latest threads in the evolution of ideas is only slightly more than six years old. Learning about the paper of Laurent Nottale about the possibility to identify planetary orbits as Bohr orbits with a gigantic value of gravitational Planck constant made once again possible to see the obvious. Dynamical quantized Planck constant is strongly suggested by quantum classical correspondence and the fact that space-time sheets identifiable as quantum coherence regions can have arbitrarily large sizes. During summer 2010 several new insights about the mathematical structure and interpretation of TGD emerged. One of these insights was the realization that the postulated hierarchy of Planck constants might follow from the basic structure of quantum TGD. The point is that due to the extreme non-linearity of the classical action principle the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is one-to-many and the natural description of the situation is in terms of local singular covering spaces of the imbedding space. One could speak about effective value of Planck constant coming as a multiple of its minimal value. The implications of the hierarchy of Planck constants are extremely far reaching so that the significance of the reduction of this hierarchy to the basic mathematical structure distinguishing between TGD and competing theories cannot be under-estimated.

From the point of view of particle physics the ultimate goal is of course a practical construction recipe for the S-matrix of the theory. I have myself regarded this dream as quite too ambitious taking into account how far reaching re-structuring and generalization of the basic mathematical structure of quantum physics is required. It has indeed turned out that the dream about explicit formula is unrealistic before one has understood what happens in quantum jump. Symmetries and general physical principles have turned out to be the proper guide line here. To give some impressions about what is required some highlights are in order.

- With the emergence of zero energy ontology the notion of S-matrix was replaced with M-matrix which can be interpreted as a complex square root of density matrix representable as a diagonal and positive square root of density matrix and unitary S-matrix so that quantum theory in zero energy ontology can be said to define a square root of thermodynamics at least formally.

- A decisive step was the strengthening of the General Coordinate Invariance to the requirement that the formulations of the theory in terms of light-like 3-surfaces identified as 3-surfaces at which the induced metric of space-time surfaces changes its signature and in terms of space-like 3-surfaces are equivalent. This means effective 2-dimensionality in the sense that partonic 2-surfaces defined as intersections of these two kinds of surfaces plus 4-D tangent space data at partonic 2-surfaces code for the physics. Quantum classical correspondence requires the coding of the quantum numbers characterizing quantum states assigned to the partonic 2-surfaces to the geometry of space-time surface. This is achieved by adding to the modified Dirac action a measurement interaction term assigned with light-like 3-surfaces.

- The replacement of strings with light-like 3-surfaces equivalent to space-like 3-surfaces means enormous generalization of the super conformal symmetries of string models. A further generalization of these symmetries to non-local Yangian symmetries generalizing the recently discovered Yangian symmetry of $\mathcal{N} = 4$ supersymmetric Yang-Mills theories is highly suggestive. Here the replacement of point like particles with partonic 2-surfaces means the replacement of conformal symmetry of Minkowski space with infinite-dimensional super-conformal algebras. Yangian symmetry provides also a further refinement to the notion of conserved quantum numbers allowing to define them for bound states using non-local energy conserved currents.

- A further attractive idea is that quantum TGD reduces to almost topological quantum field theory. This is possible if the Kähler action for the preferred extremals defining WCW Kähler function reduces to a 3-D boundary term. This takes place if the conserved currents are so called Beltrami fields with the defining property that the coordinates associated with flow lines extend to single global coordinate variable. This ansatz together with the weak form of electric-magnetic duality reduces the Kähler action to Chern-Simons term with the condition that the 3-surfaces are extremals of Chern-Simons action subject to the constraint force defined by the weak form of electric magnetic duality. It is the latter constraint which prevents the trivialization of the theory to a topological quantum field theory. Also the identification of the Kähler function of WCW as Dirac determinant finds support as well as the description of the scattering amplitudes in terms of braids with interpretation in terms of finite measurement resolution coded to the basic structure of the solutions of field equations.

- In standard QFT Feynman diagrams provide the description of scattering amplitudes. The beauty of Feynman diagrams is that they realize unitarity automatically via the so called Cutkosky rules. In contrast to Feynman's original beliefs, Feynman diagrams and virtual particles are taken only as a convenient mathematical tool in quantum field theories. QFT approach is however plagued by UV and IR divergences and one must keep mind open for the possibility that a genuine progress might mean opening of the black box of the virtual particle.

  In TGD framework this generalization of Feynman diagrams indeed emerges unavoidably. Light-like 3-surfaces replace the lines of Feynman diagrams and vertices are replaced by 2-D partonic

2-surfaces. Zero energy ontology and the interpretation of parton orbits as light-like "wormhole throats" suggests that virtual particle do not differ from on mass shell particles only in that the four- and three- momenta of wormhole throats fail to be parallel. The two throats of the wormhole defining virtual particle would contact carry on mass shell quantum numbers but for virtual particles the four-momenta need not be parallel and can also have opposite signs of energy. Modified Dirac equation suggests a number theoretical quantization of the masses of the virtual particles. The kinematic constraints on the virtual momenta are extremely restrictive and reduce the dimension of the sub-space of virtual momenta and if massless particles are not allowed (IR cutoff provided by zero energy ontology naturally), the number of Feynman diagrams contributing to a particular kind of scattering amplitude is finite and manifestly UV and IR finite and satisfies unitarity constraint in terms of Cutkosky rules. What is remarkable that fermionic propagatos are massless propagators but for on mass shell four-momenta. This gives a connection with the twistor approach and inspires the generalization of the Yangian symmetry to infinite-dimensional super-conformal algebras.

What I have said above is strongly biased view about the recent situation in quantum TGD and I have left all about applications to the introductions of the books whose purpose is to provide a bird's eye of view about TGD as it is now. This vision is single man's view and doomed to contain unrealistic elements as I know from experience. My dream is that young critical readers could take this vision seriously enough to try to demonstrate that some of its basic premises are wrong or to develop an alternative based on these or better premises. I must be however honest and tell that 32 years of TGD is a really vast bundle of thoughts and quite a challenge for anyone who is not able to cheat himself by taking the attitude of a blind believer or a light-hearted debunker trusting on the power of easy rhetoric tricks.

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Neither TGD nor these books would exist without the help and encouragement of many people. The friendship with Heikki and Raija Haila and their family have been kept me in contact with the everyday world and without this friendship I would not have survived through these lonely 32 years most of which I have remained unemployed as a scientific dissident. I am happy that my children have understood my difficult position and like my friends have believed that what I am doing is something valuable although I have not received any official recognition for it.

During last decade Tapio Tammi has helped me quite concretely by providing the necessary computer facilities and being one of the few persons in Finland with whom to discuss about my work. I have had also stimulating discussions with Samuli Penttinen who has also helped to get through the economical situations in which there seemed to be no hope. The continual updating of fifteen online books means quite a heavy bureaucracy at the level of bits and without a systemization one ends up with endless copying and pasting and internal consistency is soon lost. Pekka Rapinoja has offered his help in this respect and I am especially grateful for him for my Python skills. Also Matti Vallinkoski has helped me in computer related problems.

The collaboration with Lian Sidoroff was extremely fruitful and she also helped me to survive economically through the hardest years. The participation to CASYS conferences in Liege has been an important window to the academic world and I am grateful for Daniel Dubois and Peter Marcer for making this participation possible. The discussions and collaboration with Eduardo de Luna and Istvan Dienes stimulated the hope that the communication of new vision might not be a mission impossible after all. Also blog discussions have been very useful. During these years I have received innumerable email contacts from people around the world. In particualr, I am grateful for Mark McWilliams and Ulla Matfolk for providing links to possibly interesting web sites and articles. These contacts have helped me to avoid the depressive feeling of being some kind of Don Quixote of Science and helped me to widen my views: I am grateful for all these people.

In the situation in which the conventional scientific communication channels are strictly closed it is important to have some loop hole through which the information about the work done can at

least in principle leak to the publicity through the iron wall of the academic censorship. Without any exaggeration I can say that without the world wide web I would not have survived as a scientist nor as individual. Homepage and blog are however not enough since only the formally published result is a result in recent day science. Publishing is however impossible without a direct support from power holders- even in archives like arXiv.org.

Situation changed for five years ago as Andrew Adamatsky proposed the writing of a book about TGD when I had already got used to the thought that my work would not be published during my life time. The Prespacetime Journal and two other journals related to quantum biology and consciousness - all of them founded by Huping Hu - have provided this kind of loop holes. In particular, Dainis Zeps, Phil Gibbs, and Arkadiusz Jadczyk deserve my gratitude for their kind help in the preparation of an article series about TGD catalyzing a considerable progress in the understanding of quantum TGD. Also the viXra archive founded by Phil Gibbs and its predecessor Archive Freedom have been of great help: Victor Christianto deserves special thanks for doing the hard work needed to run Archive Freedom. Also the Neuroquantology Journal founded by Sultan Tarlaci deserves a special mention for its publication policy. And last but not least: there are people who experience as a fascinating intellectual challenge to spoil the practical working conditions of a person working with something which might be called unified theory: I am grateful for the people who have helped me to survive through the virus attacks, an activity which has taken roughly one month per year during the last half decade and given a strong hue of grey to my hair.

For a person approaching his sixty year birthday it is somewhat easier to overcome the hard feelings due to the loss of academic human rights than for an inpatient youngster. Unfortunately the economic situation has become increasingly difficult during the twenty years after the economic depression in Finland which in practice meant that Finland ceased to be a constitutional state in the strong sense of the word. It became possible to depose people like me from the society without fear about public reactions and the classification as dropout became a convenient tool of ridicule to circumvent the ethical issues. During last few years when the right wing has held the political power this trend has been steadily strengthening. In this kind of situation the concrete help from individuals has been and will be of utmost importance. Against this background it becomes obvious that this kind of work is not possible without the support from outside and I apologize for not being able to mention all the people who have helped me during these years.

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Chapter 1

Introduction

1.1 Background

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [27]. The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last twenty-three years for the realization of this dream and this has resulted in seven online books about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology.

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

For few yeas ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be.

The fifth thread came with the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and certainly possible in TGD framework. The identification of hierarchy of Planck constants whose values TGD "predicts" in terms of dark matter hierarchy would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The seven online books [1, 2, 5, 6, 3, 4, 7] about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology [8, 9, 10, 11, 12, 15, 13, 14] are warmly recommended to the interested reader.

1.2 Basic Ideas of TGD

The basic physical picture behind TGD was formed as a fusion of two rather disparate approaches: namely TGD is as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model.

1.2.1 TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_+ \times CP_2$, where M^4_+ denotes the interior of the future light cone of the Minkowski space (to be referred as light cone in the sequel) and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [45, 31, 34, 17]. The identification of the space-time as a submanifold [33, 30] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity. The actual choice $H = M^4_+ \times CP_2$ implies the breaking of the Poincare invariance in the cosmological scales but only at the quantum level. It soon however turned out that submanifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

1.2.2 TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

1.2.3 Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there is "vapor phase" that is a "gas" of particle like 3-surfaces (counterpart of the "baby universies" of GRT) and the nonconservation of energy in GRT corresponds to the transfer of energy between the topological condensate and vapor phase.

1.3 The five threads in the development of quantum TGD

The development of TGD has involved four strongly interacting threads: physics as infinite-dimensional geometry; p-adic physics; TGD inspired theory of consciousness and TGD as a generalized number theory. In the following these five threads are briefly described.

1.3.1 Quantum TGD as configuration space spinor geometry

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and are the following ones:

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [35, 32, 29]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \to B + C$. Classically this corresponds to a path of configuration space leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

2. Configuration space is endowed with the metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory.

1.3.2 p-Adic TGD

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzless and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

There were many interpretational and technical questions crying for a definite answer. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *The Physics*? Should one perform p-adicization also at the level of the configuration space of 3-surfaces? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question.

Despite these frustrating uncertainties, the number of the applications of the poorly defined p-adic physics growed steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

1.3.3 TGD as a generalization of physics to a theory consciousness

General coordinate invariance forces the identification of quantum jump as quantum jump between entire deterministic quantum histories rather than time=constant snapshots of single history. The new view about quantum jump forces a generalization of quantum measurement theory such that observer becomes part of the physical system. Thus a general theory of consciousness is unavoidable outcome. This theory is developed in detail in the books [8, 9, 10, 11, 12, 15, 13, 14].

Quantum jump as a moment of consciousness

The identification of quantum jump between deterministic quantum histories (configuration space spinor fields) as a moment of consciousness defines microscopic theory of consciousness. Quantum jump involves the steps

$$\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f \quad ,$$

where U is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution of quantum mechanics. U is however only formally analogous to Schrödinger time evolution of infinite duration although there is *no* real time evolution involved. It is not however clear whether one should regard U-matrix and S-matrix as two different things or not: U-matrix is a completely universal object characterizing the dynamics of evolution by self-organization whereas S-matrix is a highly context dependent concept in wave mechanics and in quantum field theories where it at least formally represents unitary time translation operator at the limit of an infinitely long interaction time. The S-matrix understood in the spirit of superstring models is however something very different and could correspond to U-matrix.

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parameterize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes now a part of quantum theory. Together these requirements imply that the final states of quantum jump correspond to quantum superpositions of space-time surfaces which are macroscopically equivalent. Hence the world of conscious experience looks classical. At least formally quantum jump can be interpreted also as a quantum computation in which matrix U represents unitary quantum computation which is however not identifiable as unitary translation in time direction and cannot be 'engineered'.

The notion of self

The concept of self is absolutely essential for the understanding of the macroscopic and macro-temporal aspects of consciousness. Self corresponds to a subsystem able to remain un-entangled under the sequential informational 'time evolutions' U. Exactly vanishing entanglement is practically impossible in ordinary quantum mechanics and it might be that 'vanishing entanglement' in the condition for self-property should be replaced with 'subcritical entanglement'. On the other hand, if space-time decomposes into p-adic and real regions, and if entanglement between regions representing physics in different number fields vanishes, space-time indeed decomposes into selves in a natural manner.

It is assumed that the experiences of the self after the last 'wake-up' sum up to single average experience. This means that subjective memory is identifiable as conscious, immediate short term memory. Selves form an infinite hierarchy with the entire Universe at the top. Self can be also interpreted as mental images: our mental images are selves having mental images and also we represent mental images of a higher level self. A natural hypothesis is that self S experiences the experiences of its subselves as kind of abstracted experience: the experiences of subselves S_i are not experienced as such but represent kind of averages $\langle S_{ij} \rangle$ of sub-subselves S_{ij}. Entanglement between selves, most naturally realized by the formation of join along boundaries bonds between cognitive or material space-time sheets, provides a possible a mechanism for the fusion of selves to larger selves (for instance, the fusion of the mental images representing separate right and left visual fields to single visual field) and forms wholes from parts at the level of mental images.

Relationship to quantum measurement theory

The third basic element relates TGD inspired theory of consciousness to quantum measurement theory. The assumption that localization occurs in zero modes in each quantum jump implies that the world of conscious experience looks classical. It also implies the state function reduction of the standard quantum measurement theory as the following arguments demonstrate (it took incredibly long time to realize this almost obvious fact!).

1. The standard quantum measurement theory a la von Neumann involves the interaction of brain with the measurement apparatus. If this interaction corresponds to entanglement between microscopic degrees of freedom m with the macroscopic effectively classical degrees of freedom M characterizing the reading of the measurement apparatus coded to brain state, then the reduction of this entanglement in quantum jump reproduces standard quantum measurement theory provide the unitary time evolution operator U acts as flow in zero mode degrees of freedom and correlates completely some orthonormal basis of configuration space spinor fields in non-zero modes with the values of the zero modes. The flow property guarantees that the localization is consistent with unitarity: it also means 1-1 mapping of quantum state basis to classical variables (say, spin direction of the electron to its orbit in the external magnetic field).

2. Since zero modes represent classical information about the geometry of space-time surface (shape, size, classical Kähler field,...), they have interpretation as effectively classical degrees of freedom and are the TGD counterpart of the degrees of freedom M representing the reading of the measurement apparatus. The entanglement between quantum fluctuating non-zero modes and zero modes is the TGD counterpart for the $m-M$ entanglement. Therefore the localization in zero modes is equivalent with a quantum jump leading to a final state where the measurement apparatus gives a definite reading.

This simple prediction is of utmost theoretical importance since the black box of the quantum measurement theory is reduced to a fundamental quantum theory. This reduction is implied by the replacement of the notion of a point like particle with particle as a 3-surface. Also the infinite-dimensionality of the zero mode sector of the configuration space of 3-surfaces is absolutely essential. Therefore the reduction is a triumph for quantum TGD and favors TGD against string models.

Standard quantum measurement theory involves also the notion of state preparation which reduces to the notion of self measurement. Each localization in zero modes is followed by a cascade of self measurements leading to a product state. This process is obviously equivalent with the state preparation process. Self measurement is governed by the so called Negentropy Maximization Principle (NMP) stating that the information content of conscious experience is maximized. In the self measurement the density matrix of some subsystem of a given self localized in zero modes (after ordinary quantum measurement) is measured. The self measurement takes place for that subsystem of self for which the reduction of the entanglement entropy is maximal in the measurement. In p-adic context NMP can be regarded as the variational principle defining the dynamics of cognition. In real context self measurement could be seen as a repair mechanism allowing the system to fight against quantum thermalization by reducing the entanglement for the subsystem for which it is largest (fill the largest hole first in a leaking boat).

Selves self-organize

The fourth basic element is quantum theory of self-organization based on the identification of quantum jump as the basic step of self-organization [25]. Quantum entanglement gives rise to the generation of long range order and the emergence of longer p-adic length scales corresponds to the emergence of larger and larger coherent dynamical units and generation of a slaving hierarchy. Energy (and quantum entanglement) feed implying entropy feed is a necessary prerequisite for quantum self-organization. Zero modes represent fundamental order parameters and localization in zero modes implies that the sequence of quantum jumps can be regarded as hopping in the zero modes so that Haken's classical theory of self organization applies almost as such. Spin glass analogy is a further important element: self-organization of self leads to some characteristic pattern selected by dissipation as some valley of the "energy" landscape.

Dissipation can be regarded as the ultimate Darwinian selector of both memes and genes. The mathematically ugly irreversible dissipative dynamics obtained by adding phenomenological dissipation terms to the reversible fundamental dynamical equations derivable from an action principle can be understood as a phenomenological description replacing in a well defined sense the series of reversible quantum histories with its envelope.

Classical non-determinism of Kähler action

The fifth basic element are the concepts of association sequence and cognitive space-time sheet. The huge vacuum degeneracy of the Kähler action suggests strongly that the absolute minimum space-time is not always unique. For instance, a sequence of bifurcations can occur so that a given space-time branch can be fixed only by selecting a finite number of 3-surfaces with time like(!) separations on the orbit of 3-surface. Quantum classical correspondence suggest an alternative formulation. Space-time surface decomposes into maximal deterministic regions and their temporal sequences have interpretation a space-time correlate for a sequence of quantum states defined by the initial (or final) states of quantum jumps. This is consistent with the fact that the variational principle selects preferred extremals of Kähler action as generalized Bohr orbits.

In the case that non-determinism is located to a finite time interval and is microscopic, this sequence of 3-surfaces has interpretation as a simulation of a classical history, a geometric correlate for contents of consciousness. When non-determinism has long lasting and macroscopic effect one can identify it as volitional non-determinism associated with our choices. Association sequences relate closely with the cognitive space-time sheets defined as space-time sheets having finite time duration and psychological time can be identified as a temporal center of mass coordinate of the cognitive space-time sheet. The gradual drift of the cognitive space-time sheets to the direction of future force by the geometry of the future light cone explains the arrow of psychological time.

p-Adic physics as physics of cognition and intentionality

The sixth basic element adds a physical theory of cognition to this vision. TGD space-time decomposes into regions obeying real and p-adic topologies labelled by primes $p = 2, 3, 5, ...$ p-Adic regions obey the same field equations as the real regions but are characterized by p-adic non-determinism since the functions having vanishing p-adic derivative are pseudo constants which are piecewise constant functions. Pseudo constants depend on a finite number of positive pinary digits of arguments just like numerical predictions of any theory always involve decimal cutoff. This means that p-adic space-time regions are obtained by gluing together regions for which integration constants are genuine constants. The natural interpretation of the p-adic regions is as cognitive representations of real physics. The freedom of imagination is due to the p-adic non-determinism. p-Adic regions perform mimicry and make possible for the Universe to form cognitive representations about itself. p-Adic physics space-time sheets serve also as correlates for intentional action.

A more more precise formulation of this vision requires a generalization of the number concept obtained by fusing reals and p-adic number fields along common rationals (in the case of algebraic extensions among common algebraic numbers). This picture is discussed in [21]. The application this notion at the level of the imbedding space implies that imbedding space has a book like structure with various variants of the imbedding space glued together along common rationals (algebraics). The implication is that genuinely p-adic numbers (non-rationals) are strictly infinite as real numbers so that most points of p-adic space-time sheets are at real infinity, outside the cosmos, and that the projection to the real imbedding space is discrete set of rationals (algebraics). Hence cognition and intentionality are almost completely outside the real cosmos and touch it at a discrete set of points only.

This view implies also that purely local p-adic physics codes for the p-adic fractality characterizing long range real physics and provides an explanation for p-adic length scale hypothesis stating that the primes $p \simeq 2^k$, k integer are especially interesting. It also explains the long range correlations and short term chaos characterizing intentional behavior and explains why the physical realizations of cognition are always discrete (say in the case of numerical computations). Furthermore, a concrete quantum model for how intentions are transformed to actions emerges.

The discrete real projections of p-adic space-time sheets serve also space-time correlate for a logical thought. It is very natural to assign to p-adic pinary digits a p-valued logic but as such this kind of logic does not have any reasonable identification. p-Adic length scale hypothesis suggest that the $p = 2^k - n$ pinary digits represent a Boolean logic B^k with k elementary statements (the points of the k-element set in the set theoretic realization) with n taboos which are constrained to be identically true.

1.3.4 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. For few yeas ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

The role of classical number fields

The vision about the physical role of the classical number fields relies on the notion of number theoretic compactifiction stating that space-time surfaces can be regarded as surfaces of either M^8 or $M^4 \times CP_2$. As surfaces of M^8 identifiable as space of hyper-octonions they are hyper-quaternionic or co-hyper-quaternionic- and thus maximally associative or co-associative. This means that their tangent space is either hyper-quaternionic plane of M^8 or an orthogonal complement of such a plane. These surface can be mapped in natural manner to surfaces in $M^4 \times CP_2$ [20] provided one can assign to each point of tangent space a hyper-complex plane $M^2(x) \subset M^4$. One can also speak about $M^8 - H$ duality.

This vision has very strong predictive power. It predicts that the extremals of Kähler action correspond to either hyper-quaternionic or co-hyper-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane $M^2(x) \subset M^4$. As a consequence, the M^4 projection of space-time surface at each point contains $M^2(x)$ and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets Y^2 and partonic 2-surfaces X^2. The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of $M^2(x)$ is as the space of non-physical polarizations and the plane of local 4-momentum.

One can fairly say, that number theoretical compactification is responsible for most of the understanding of quantum TGD that has emerged during last years. This includes the realization of Equivalence Principle at space-time level, dual formulations of TGD as Minkowskian and Euclidian string model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of configuration space metric and spinor structure in terms of second quantized induced spinor fields with modified Dirac action defined by Kähler action realizing automatically the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type II_1 about which Clifford algebra of configuration space represents an example.

Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory. The work with Riemann hypothesis led to further ideas.

After the realization that infinite primes can be mapped to polynomials representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially satisfying is that p-adic and real regions of the space-time surface could emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably extremely brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

1.3.5 Dynamical quantized Planck constant and dark matter hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale [64] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [28].

Already before learning about Nottale's paper I had proposed the possibility that Planck constant is quantized [23] and the spectrum is given in terms of logarithms of Beraha numbers: the lowest Beraha number B_3 is completely exceptional in that it predicts infinite value of Planck constant. The inverse of the gravitational Planck constant could correspond a gravitational perturbation of this as $1/\hbar_{gr} = v_0/GMm$. The general philosophy would be that when the quantum system would become non-perturbative, a phase transition increasing the value of $\hbar$ occurs to preserve the perturbative character and at the transition $n = 4 \rightarrow 3$ only the small perturbative correction to $1/\hbar(3) = 0$ remains. This would apply to QCD and to atoms with $Z > 137$ as well.

TGD predicts correctly the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n^2-fold: much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. Also a model for the formation of planetary system as a condensation of ordinary matter around quantum coherent dark matter emerges [28].

Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics. An attractive solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

p-Adic and dark matter hierarchies and hierarchy of moments of consciousness

Dark matter hierarchy assigned to a spectrum of Planck constant having arbitrarily large values brings additional elements to the TGD inspired theory of consciousness.

1. Macroscopic quantum coherence can be understood since a particle with a given mass can in principle appear as arbitrarily large scaled up copies (Compton length scales as $\hbar$). The phase transition to this kind of phase implies that space-time sheets of particles overlap and this makes possible macroscopic quantum coherence.

2. The space-time sheets with large Planck constant can be in thermal equilibrium with ordinary ones without the loss of quantum coherence. For instance, the cyclotron energy scale associated with EEG turns out to be above thermal energy at room temperature for the level of dark matter hierarchy corresponding to magnetic flux quanta of the Earth's magnetic field with the size scale of Earth and a successful quantitative model for EEG results [36].

Dark matter hierarchy leads to detailed quantitative view about quantum biology with several testable predictions [36]. The applications to living matter suggests that the basic hierarchy corresponds to a hierarchy of Planck constants coming as $\hbar(k) = \lambda^k(p)\hbar_0$, $\lambda \simeq 2^{11}$ for $p = 2^{127-1}$, $k = 0, 1, 2, ...$ [36]. Also integer valued sub-harmonics and integer valued sub-harmonics of λ might be possible. Each p-adic length scale corresponds to this kind of hierarchy and number theoretical arguments suggest a general formula for the allowed values of Planck constant λ depending logarithmically on p-adic prime [30]. Also the value of $\hbar_0$ has spectrum characterized by Beraha numbers $B_n = 4cos^2(\pi/n)$, $n \geq 3$, varying by a factor in the range $n > 3$ [30]. It must be however emphasized that the relation of this picture to the model of quantized gravitational Planck constant h_{gr} appearing in Nottale's model is not yet completely understood.

The general prediction is that Universe is a kind of inverted Mandelbrot fractal for which each bird's eye of view reveals new structures in long length and time scales representing scaled down copies of standard physics and their dark variants. These structures would correspond to higher levels in self hierarchy. This prediction is consistent with the belief that 75 per cent of matter in the universe is dark.

1. Living matter and dark matter

Living matter as ordinary matter quantum controlled by the dark matter hierarchy has turned out to be a particularly successful idea. The hypothesis has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of EEG [36]. Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [22, 36]. A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [36].

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of $\hbar$ at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

2. Dark matter hierarchy and the notion of self

The vision about dark matter hierarchy leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [18, 36]. The larger the value of Planck constant, the longer the subjectively experienced duration and the average geometric duration $T(k) \propto \lambda^k$ of the quantum jump.

Quantum jumps form also a hierarchy with respect to p-adic and dark hierarchies and the geometric durations of quantum jumps scale like $\hbar$. Dark matter hierarchy suggests also a slight modification of the notion of self. Each self involves a hierarchy of dark matter levels, and one is led to ask whether the highest level in this hierarchy corresponds to single quantum jump rather than a sequence of quantum jumps. The averaging of conscious experience over quantum jumps would occur only for sub-selves at lower levels of dark matter hierarchy and these mental images would be ordered, and single moment of consciousness would be experienced as a history of events. The quantum parallel dissipation at the lower levels would give rise to the experience of flow of time. For instance, hadron as a macro-temporal quantum system in the characteristic time scale of hadron is a dissipating system at quark and gluon level corresponding to shorter p-adic time scales. One can ask whether even entire life cycle could be regarded as a single quantum jump at the highest level so that consciousness would not be completely lost even during deep sleep. This would allow to understand why we seem to know directly that this biological body of mine existed yesterday.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self corresponds at the highest dark matter level to single moment of consciousness. Self would experience the average over the sequence of moments of consciousness associated with each sub-self but there would be no averaging over the separate mental images of this kind, be their parallel or serial. These mental images correspond to sub-selves having shorter wake-up periods than self and would be experienced as being time ordered. Hence the digits in the phone number are experienced as separate mental images and ordered with respect to experienced time.

3. The time span of long term memories as signature for the level of dark matter hierarchy

The simplest dimensional estimate gives for the average increment τ of geometric time in quantum jump $\tau \sim 10^4$ CP_2 times so that $2^{127}-1 \sim 10^{38}$ quantum jumps are experienced during secondary p-adic time scale $T_2(k=127) \simeq 0.1$ seconds which is the duration of physiological moment and predicted to be fundamental time scale of human consciousness [26]. A more refined guess is that $\tau_p = \sqrt{p}\tau$ gives the dependence of the duration of quantum jump on p-adic prime p. By multi-p-fractality predicted by TGD and explaining p-adic length scale hypothesis, one expects that at least $p = 2$-adic level is also always present. For the higher levels of dark matter hierarchy τ_p is scaled up by $\hbar/\hbar_0$. One can understand evolutionary leaps as the emergence of higher levels at the level of individual organism making possible intentionality and memory in the time scale defined τ [22].

Higher levels of dark matter hierarchy provide a neat quantitative view about self hierarchy and its evolution. For instance, EEG time scales corresponds to $k = 4$ level of hierarchy and a time scale of .1 seconds [18, 36], and EEG frequencies correspond at this level dark photon energies above the thermal threshold so that thermal noise is not a problem anymore. Various levels of dark matter hierarchy would naturally correspond to higher levels in the hierarchy of consciousness and the typical duration of life cycle would give an idea about the level in question.

The level would determine also the time span of long term memories as discussed in [36]. $k = 7$ would correspond to a duration of moment of conscious of order human lifetime which suggests that $k = 7$ corresponds to the highest dark matter level relevant to our consciousness whereas higher levels would in general correspond to transpersonal consciousness. $k = 5$ would correspond to time scale of short term memories measured in minutes and $k = 6$ to a time scale of memories measured in days.

The emergence of these levels must have meant evolutionary leap since long term memory is also accompanied by ability to anticipate future in the same time scale. This picture would suggest that the basic difference between us and our cousins is not at the level of genome as it is usually understood but at the level of the hierarchy of magnetic bodies [22, 36]. In fact, higher levels of dark matter hierarchy motivate the introduction of the notions of super-genome and hyper-genome. The genomes of entire organ can join to form super-genome expressing genes coherently. Hyper-genomes would result from the fusion of genomes of different organisms and collective levels of consciousness would express themselves via hyper-genome and make possible social rules and moral.

1.4 Bird's eye of view about the topics of the book

The topics of this book are the purely geometric aspects of the vision about physics as an infinite-dimensional Kähler geometry of the "world of classical worlds", with " classical world" identified either as light-like 3-D surface of the unique Bohr orbit like 4-surface traversing through it. The non-determinism of Kähler action forces to generalize the notion of 3-surface so that unions of space-like surfaces with time like separations must be allowed. Zero energy ontology allows to formulate this picture elegantly in terms of causal diamonds defined as intersections of future and past directed light-cones. Also a a geometric realization of coupling constant evolution and finite measurement resolution emerges.

There are two separate tasks involved.

1. Provide configuration space of 3-surfaces with Kähler geometry which is consistent with 4-dimensional general coordinate invariance so that the metric is Diff^4 degenerate. General coordinate invariance implies that the definition of metric must assign to a given light-like 3-surface X^3 a 4-surface as a kind of Bohr orbit $X^4(X^3)$.

2. Provide the configuration space with a spinor structure. The great idea is to identify configuration space gamma matrices in terms of super algebra generators expressible using second quantized fermionic oscillator operators for induced free spinor fields at the space-time surface assignable to a given 3-surface. The isometry generators and contractions of Killing vectors with gamma matrices would thus form a generalization of Super Kac-Moody algebra.

The condition of mathematical existence poses surprisingly strong conditions on configuration space metric and spinor structure.

1. From the experience with loop spaces one can expect that there is no hope about existence of well-defined Riemann connection unless this space is union of infinite-dimensional symmetric spaces with constant curvature metric and simple considerations requires that vacuum Einstein equations are satisfied by each component in the union. The coordinates labeling these symmetric spaces are zero modes having interpretation as genuinely classical variables which do not quantum fluctuate since they do not contribute to the line element of the configuration space.

2. The construction of the Kähler structure involves also the identification of complex structure. Direct construction of Kähler function as action associated with a preferred Bohr orbit like extremal for some physically motivated action action leads to a unique result. Second approach is group theoretical and is based on a direct guess of isometries of the infinite-dimensional symmetric space formed by 3-surfaces with fixed values of zero modes. The group of isometries is generalization of Kac-Moody group obtained by replacing finite-dimensional Lie group with the group of symplectic transformations of $\delta M^4_+ \times CP_2$, where δM^4_+ is the boundary of 4-dimensional future light-cone. A crucial role is played by the generalized conformal invariance assignable to light-like 3-surfaces and to the boundaries of causal diamond. In particular, a generalization of Equivalence Principle can be formulated in terms of generalized coset construction.

3. Fermionic statistics and quantization of spinor fields can be realized in terms of configuration space spinors structure. Quantum criticality and the idea about space-time surfaces as analogs of Bohr orbits have served as basic guiding lines of Quantum TGD. These notions can be formulated more precisely in terms of the modified Dirac equation for induced spinor fields allowing also realization of super-conformal symmetries and quantum gravitational holography. A rather detailed view about how configuration space Kähler function emerges as Dirac determinant allowing a tentative identification of the preferred extremals of Kähler action as surface for which second variation of Kähler action vanishes for some deformations of the surface. The catastrophe theoretic analog for quantum critical space-time surfaces are the points of space spanned by behavior and control variables at which the determinant defined by the second derivatives of potential function with respect to behavior variables vanishes. Number theoretic

vision leads to rather detailed view about preferred extremals of Kähler action. In particular, preferred extremals should be what I have dubbed as hyper-quaternionic surfaces. It it still an open question whether this characterization is equivalent with quantum criticality or not.

The seven online books about TGD [1, 2, 5, 6, 3, 4, 7] and eight online books about TGD inspired theory of consciousness and quantum biology [8, 9, 10, 11, 12, 15, 13, 14] are warmly recommended for the reader willing to get overall view about what is involved.

1.5 The contents of the book

In the following abstracts of various chapters of the book are given in order to provide overall view.

1.5.1 Identification of the Configuration Space Kähler Function

There are two basic approaches to quantum TGD. The first approach, which is discussed in this chapter, is a generalization of Einstein's geometrization program of physics to an infinite-dimensional context. Second approach is based on the identification of physics as a generalized number theory. The first approach relies on the vision of quantum physics as infinite-dimensional Kähler geometry for the "world of classical worlds" (WCW) identified as the space of 3-surfaces in in certain 8-dimensional space. There are three separate approaches to the challenge of constructing WCW Kähler geometry and spinor structure. The first approach relies on direct guess of Kähler function. Second approach relies on the construction of Kähler form and metric utilizing the huge symmetries of the geometry needed to guarantee the mathematical existence of Riemann connection. The third approach relies on the construction of spinor structure based on the hypothesis that complexified WCW gamma matrices are representable as linear combinations of fermionic oscillator operator for second quantized free spinor fields at space-time surface and on the geometrization of super-conformal symmetries in terms of WCW spinor structure.

In this chapter the proposal for Kähler function based on the requirement of 4-dimensional General Coordinate Invariance implying that its definition must assign to a given 3-surface a unique space-time surface. Quantum classical correspondence requires that this surface is a preferred extremal of some some general coordinate invariant action, and so called Kähler action is a unique candidate in this respect. The preferred extremal has intepretation as an analog of Bohr orbit so that classical physics becomes and exact part of WCW geometry and therefore also quantum physics.

The basic challenge is the explicit identification of WCW Kähler function K. Two assumptions lead to the identification of K as a sum of Chern-Simons type terms associated with the ends of causal diamond and with the light-like wormhole throats at which the signature of the induced metric changes. The first assumption is the weak form of electric magnetic duality. Second assumption is that the Kähler current for preferred extremals satisfies the condition $j_K \wedge dj_K = 0$ implying that the flow parameter of the flow lines of j_K defines a global space-time coordinate. This would mean that the vision about reduction to almost topological QFT would be realized.

Second challenge is the understanding of the space-time correlates of quantum criticality. Electric-magnetic duality helps considerably here. The realization that the hierarchy of Planck constant realized in terms of coverings of the imbedding space follows from basic quantum TGD leads to a further understanding. The extreme non-linearity of canonical momentum densities as functions of time derivatives of the imbedding space coordinates implies that the correspondence between these two variables is not 1-1 so that it is natural to introduce coverings of $CD \times CP_2$. This leads also to a precise geometric characterization of the criticality of the preferred extremals.

1.5.2 Construction of Configuration Space Kähler Geometry from Symmetry Principles

There are three separate approaches to the challenge of constructing WCW Kähler geometry and spinor structure. The first one relies on a direct guess of Kähler function. Second approach relies on the construction of Kähler form and metric utilizing the huge symmetries of the geometry needed to guarantee the mathematical existence of Riemann connection. The third approach relies on the construction of spinor structure assuming that complexified WCW gamma matrices are representable as linear combinations of fermionic oscillator operator for the second quantized free spinor fields at space-time surface and on the geometrization of super-conformal symmetries in terms of spinor structure.

In this chapter the construction of Kähler form and metric based on symmetries is discussed. The basic vision is that WCW can be regarded as the space of generalized Feynman diagrams with lines thickned to light-like 3-surfaces and vertices identified as partonic 2-surfaces. In zero energy ontology the strong form of General Coordinate Invariance (GCI) implies effective 2-dimensionality and the basic objects are pairs partonic 2-surfaces X^2 at opposite light-like boundaries of causal diamonds (CDs).

The hypothesis is that WCW can be regarded as a union of infinite-dimensional symmetric spaces G/H labeled by zero modes having an interpretation as classical, non-quantum fluctuating variables. A crucial role is played by the metric 2-dimensionality of the light-cone boundary δM^4_+ and of light-like 3-surfaces implying a generalization of conformal invariance. The group G acting as isometries of WCW is tentatively identified as the symplectic group of $\delta M^4_+ \times CP_2$ localized with respect to X^2. H is identified as Kac-Moody type group associated with isometries of $H = M^4 \times CP_2$ acting on light-like 3-surfaces and thus on X^2.

An explicit construction for the Hamiltonians of WCW isometry algebra as so called flux Hamiltonians is proposed and also the elements of Kähler form can be constructed in terms of these. Explicit expressions for WCW flux Hamiltonians as functionals of complex coordinates of the Cartesisian product of the infinite-dimensional symmetric spaces having as points the partonic 2-surfaces defining the ends of the the light 3-surface (line of generalized Feynman diagram) are proposed.

1.5.3 Configuration space spinor structure

There are three separate approaches to the challenge of constructing WCW Kähler geometry and spinor structure. The first approach relies on a direct guess of Kähler function. Second approach relies on the construction of Kähler form and metric utilizing the huge symmetries of the geometry needed to guarantee the mathematical existence of Riemann connection. The third approach discussed in this chapter relies on the construction of spinor structure based on the hypothesis that complexified WCW gamma matrices are representable as linear combinations of fermionic oscillator operator for the second quantized free spinor fields at space-time surface and on the geometrization of super-conformal symmetries in terms of spinor structure. This implies a geometrization of fermionic statistics.

The basic philosophy is that at fundamental level the construction of WCW geometry reduces to the second quantization of the induced spinor fields using Dirac action. This assumption is parallel with the bosonic emergence stating that all gauge bosons are pairs of fermion and antifermion at opposite throats of wormhole contact. Vacuum function is identified as Dirac determinant and the conjecture is that it reduces to the exponent of Kähler function. In order to achieve internal consistency induced gamma matrices appearing in Dirac operator must be replaced by the modified gamma matrices defined uniquely by Kähler action and one must also assume that extremals of Kähler action are in question so that the classical space-time dynamics reduces to a consistency condition. This implies also super-symmetries and the fermionic oscillator algebra at partonic 2-surfaces has intepretation as $\mathcal{N} = \infty$ generalization of space-time super-symmetry algebra different however from standard SUSY algebra in that Majorana spinors are not needed. This algebra serves as a building brick of various super-conformal algebras involved.

The requirement that there exist deformations giving rise to conserved Noether charges requires that the preferred extremals are critical in the sense that the second variation of the Kähler action vanishes for these deformations. Thus Bohr orbit property could correspond to criticality or at least involve it.

Quantum classical correspondence demands that quantum numbers are coded to the properties of the preferred extremals given by the Dirac determinant and this requires a linear coupling to the conserved quantum charges in Cartan algebra. Effective 2-dimensionality allows a measurement interaction term only in 3-D Chern-Simons Dirac action assignable to the wormhole throats and the ends of the space-time surfaces at the boundaries of CD. This allows also to have physical propagators reducing to Dirac propagator not possible without the measurement interaction term. An essential point is that the measurement interaction corresponds formally to a gauge transformation for the induced Kähler gauge potential. If one accepts the weak form of electric-magnetic duality Kähler function reduces to a generalized Chern-Simons term and the effect of measurement interaction term to Kähler function reduces effectively to the same gauge transformation.

The basic vision is that WCW gamma matrices are expressible as super-symplectic charges at the boundaries of CD. The basic building brick of WCW is the product of infinite-D symmetric spaces assignable to the ends of the propagator line of the generalized Feynman diagram. WCW Kähler metric has in this case "kinetic" parts associated with the ends and "interaction" part between the ends. General expressions for the super-counterparts of WCW flux Hamiltoniansand for the matrix elements of WCW metric in terms of their anticommutators are proposed on basis of this picture.

1.5.4 Does modified Dirac action define the fundamental action principle?

The construction of the spinor structure for the world of classical worlds (WCW) leads to the vision that second quantized modified Dirac equation codes for the entire quantum TGD. Among other things this would mean that Dirac determinant would define the vacuum functional of the theory having interpretation as the exponent of Kähler function of WCW and Kähler function would reduce to Kähler action for a preferred extremal of Kähler action. In this chapter the recent view about the modified Dirac action are explained in more detail.

1. Identification of the modified Dirac action

The modified Dirac action action involves several terms. The first one is 4-dimensional assignable to Kähler action. Second term is instanton term reducible to an expression restricted to wormhole throats or any light-like 3-surfaces parallel to them in the slicing of space-time surface by light-like 3-surfaces. The third term is assignable to Chern-Simons term and has interpretation as a measurement interaction term linear in Cartan algebra of the isometry group of the imbedding space in order to obtain stringy propagators and also to realize coupling between the quantum numbers associated with super-conformal representations and space-time geometry required by quantum classical correspondence.

This means that 3-D light-like wormhole throats carry induced spinor field which can be regarded as independent degrees of freedom having the spinor fields at partonic 2-surfaces as sources and acting as 3-D sources for the 4-D induced spinor field. The most general measurement interaction would involve the corresponding coupling also for Kähler action but is not physically motivated. There are good arguments in favor of Chern-Simons Dirac action and corresponding measurement interaction.

1. A correlation between 4-D geometry of space-time sheet and quantum numbers is achieved by the identification of exponent of Kähler function as Dirac determinant making possible the entanglement of classical degrees of freedom in the interior of space-time sheet with quantum numbers.

2. Cartan algebra plays a key role not only at quantum level but also at the level of space-time geometry since quantum critical conserved currents vanish for Cartan algebra of isometries and the measurement interaction terms giving rise to conserved currents are possible only for Cartan algebras. Furthermore, modified Dirac equation makes sense only for eigen states of Cartan algebra generators. The hierarchy of Planck constants realized in terms of the book like structure of the generalized imbedding space assigns to each CD (causal diamond) preferred Cartan algebra: in case of Poincare algebra there are two of them corresponding to linear and cylindrical M^4 coordinates.

3. Quantum holography and dimensional reduction hierarchy in which partonic 2-surface defined fermionic sources for 3-D fermionic fields at light-like 3-surfaces Y_l^3 in turn defining fermionic sources for 4-D spinors find an elegant realization. Effective 2-dimensionality is achieved if the replacement of light-like wormhole throat X_l^3 with light-like 3-surface Y_l^3 "parallel" with it in the definition of Dirac determinant corresponds to the $U(1)$ gauge transformation $K \rightarrow K + f + \overline{f}$ for Kähler function of WCW so that WCW Kähler metric is not affected. Here f is holomorphic function of WCW ("world of classical worlds") complex coordinates and arbitrary function of zero mode coordinates.

4. An elegant description of the interaction between super-conformal representations realized at partonic 2-surfaces and dynamics of space-time surfaces is achieved since the values of Cartan

charges are feeded to the 3-D Dirac equation which also receives mass term at the same time. Almost topological QFT at wormhole throats results at the limit when four-momenta vanish: this is in accordance with the original vision about TGD as almost topological QFT.

5. A detailed view about the physical role of quantum criticality results. Quantum criticality fixes the values of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current except in the case of Cartan algebra of isometries. Quantum criticality allows to fix the values of couplings appearing in the measurement interaction by using the condition $K \rightarrow K + f + \overline{f}$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs).

6. The inclusion of imaginary instanton term to the definition of the modified gamma matrices is not consistent with the conjugation of the induced spinor fields. Measurement interaction can be however assigned to both Kähler action and its instanton term. CP breaking, irreversibility and the space-time description of dissipation are closely related and the CP and T oddness of the instanton part of the measurement interaction term could provide first level description for dissipative effects. It must be however emphasized that the mere addition of instanton term to Kähler function could be enough.

7. A radically new view about matter antimatter asymmetry based on zero energy ontology emerges and one could understand the experimental absence of antimatter as being due to the fact antimatter corresponds to negative energy states. The identification of bosons as wormhole contacts is the only possible option in this framework.

8. Almost stringy propagators and a consistency with the identification of wormhole throats as lines of generalized Feynman diagrams is achieved. The notion of bosonic emergence leads to a long sought general master formula for the M-matrix elements. The counterpart for fermionic loop defining bosonic inverse propagator at QFT limit is wormhole contact with fermion and cutoffs in mass squared and hyperbolic angle for loop momenta of fermion and antifermion in the rest system of emitting boson have precise geometric counterpart.

2. Hyper-quaternionicity and quantum criticality

The conjecture that quantum critical space-time surfaces are hyper-quaternionic in the sense that the modified gamma matrices span a quaternionic subspace of complexified octonions at each point of the space-time surface is consistent with what is known about preferred extremals. The condition that both the modified gamma matrices and spinors are quaternionic at each point of the space-time surface leads to a precise ansatz for the general solution of the modified Dirac equation making sense also in the real context. The octonionic version of the modified Dirac equation is very simple since $SO(7,1)$ as vielbein group is replaced with G_2 acting as automorphisms of octonions so that only the neutral Abelian part of the classical electro-weak gauge fields survives the map.

Octonionic gamma matrices provide also a non-associative representation for the 8-D version of Pauli sigma matrices and encourage the identification of 8-D twistors as pairs of octonionic spinors conjectured to be highly relevant also for quantum TGD. Quaternionicity condition implies that octo-twistors reduce to something closely related to ordinary twistors.

3. The exponent of Kähler function as Dirac determinant for the modified Dirac action

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography.

1. The Dirac determinant defined by the product of Dirac determinants associated with the light-like partonic 3-surfaces X^3_l associated with a given space-time sheet X^4 is the simplest candidate for vacuum functional identifiable as the exponent of the Kähler function. Individual Dirac determinant is defined as the product of eigenvalues of the dimensionally reduced modified Dirac operator $D_{K,3}$ and there are good arguments suggesting that the number of eigenvalues is finite. p-Adicization requires that the eigenvalues belong to a given algebraic extension of rationals. This restriction would imply a hierarchy of physics corresponding to different extensions and

could automatically imply the finiteness and algebraic number property of the Dirac determinants if only finite number of eigenvalues would contribute. The regularization would be performed by physics itself if this were the case.

2. It remains to be proven that the product of eigenvalues gives rise to the exponent of Kähler action for the preferred extremal of Kähler action. At this moment the only justification for the conjecture is that this the only thing that one can imagine.

3. A long-standing conjecture has been that the zeros of Riemann Zeta are somehow relevant for quantum TGD. Rieman zeta is however naturally replaced Dirac zeta defined by the eigenvalues of $D_{K,3}$ and closely related to Riemann Zeta since the spectrum consists essentially for the cyclotron energy spectra for localized solutions region of non-vanishing induced Kähler magnetic field and hence is in good approximation integer valued up to some cutoff integer. In zero energy ontology the Dirac zeta function associated with these eigenvalues defines "square root" of thermodynamics assuming that the energy levels of the system in question are expressible as logarithms of the eigenvalues of the modified Dirac operator defining kind of fundamental constants. Critical points correspond to approximate zeros of Dirac zeta and if Kähler function vanishes at criticality as it indeed should, the thermal energies at critical points are in first order approximation proportional to zeros themselves so that a connection between quantum criticality and approximate zeros of Dirac zeta emerges.

4. The discretization induced by the number theoretic braids reduces the world of classical worlds to effectively finite-dimensional space and configuration space Clifford algebra reduces to a finite-dimensional algebra. The interpretation is in terms of finite measurement resolution represented in terms of Jones inclusion $\mathcal{M} \subset \mathcal{N}$ of HFFs with $\mathcal{M}$ taking the role of complex numbers. The finite-D quantum Clifford algebra spanned by fermionic oscillator operators is identified as a representation for the coset space $\mathcal{N}/\mathcal{M}$ describing physical states modulo measurement resolution. In the sectors of generalized imbedding space corresponding to non-standard values of Planck constant quantum version of Clifford algebra is in question.

1.5.5 Miscellaneous topics

This chapter contains topics which do not fit naturally under any umbrella, but which I feel might be of some relevance. Basically TGD inspired comments to the work of the people not terribly relevant to quantum TGD itself are in question. For few years ago Witten's approach to 3-D quantum gravitation raised a considerable interest and this inspired the comparison of this approach with quantum TGD in which light-like 3-surfaces are in a key role. Few years later the entropic gravity of Verlinde stimulated a lot of fuss in blogs and it is interesting to point out how the formal thermodynamical structure (or actually its "square root") emerges in the fundamental formulation of TGD. Lisi's E_8 theory was a further blog favorite and some comments about its failures and possible manners to cure them are discussed. It is also shown ho how E_8 can be seed as being replaced with the Kac-Moody algebra associated standard model symmetry group in TGD framework.

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Chapter 2

Identification of the Configuration Space Kähler Function

2.1 Introduction

The motivation or the construction of configuration space geometry is the postulate that physics reduces to the geometry of classical spinor fields in the the "world of the classical worlds" (WCW) identified as the infinite-dimensional configuration space of 3-surfaces of some subspace of $M^4 \times CP_2$. The first candidates were $M^4_+ \times CP_2$ and $M^4 \times CP_2$, where M^4 and M^4_+ denote Minkowski space and its light cone respectively. The recent identification of WCW is as the the union of sub-WCWs consisting of light-like 3-surface representing generalized Feynman diagrams in $CD \times CP_2$, where CD is intersection of future and past directed light-cones of M^4. The details of this identification will be discussed later.

Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that configuration space possesses Kähler geometry. One of the basic features of the Kähler geometry is that it is solely determined by the so called Kähler function, which defines both the Kähler form J and the components of the Kähler metric g in complex coordinates via the formulas [45]

$$\begin{aligned} J &= i\partial_k\partial_{\bar{l}}K dz^k \wedge d\bar{z}^l \ , \\ ds^2 &= 2\partial_k\partial_{\bar{l}}K dz^k d\bar{z}^l \ . \end{aligned} \tag{2.1.1}$$

Kähler form is covariantly constant two-form and can be regarded as a representation of imaginary unit in the tangent space of the configuration space

$$J_{mr}J^{rn} = -g_m^{\ n} \ . \tag{2.1.2}$$

As a consequence Kähler form defines also symplectic structure in configuration space.

2.1.1 Configuration space Kähler metric from Kähler function

The task of finding Kähler geometry for the configuration space reduces to that of finding the Kähler function. The main constraints on the Kähler function result from the requirement of General Coordinate Invariance (GCI) -or more technically Diff^4 symmetry and Diff degeneracy. GCI requires that the definition of the Kähler function assigns to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$, the generalized Bohr orbit defining the classical physics associated with X^3. The natural guess inspired by quantum classical correspondence is that Kähler function is defined by what might be called Kähler action, which is essentially Maxwell action with Maxwell field expressible in terms of CP_2 coordinates and that the space-time surface corresponds to a preferred extremal of Kähler action.

One can end up with the identification of the preferred extremal via several routes. Kähler action contains Kähler coupling strength as a temperature like parameter and this leads to the idea of quantum criticality fixing this parameter. One could go even even further, and require that space-time surfaces are critical in the sense that there exist an infinite number of vanishing second variations of Kähler action defining conserved Noether charges. The approach based on the modified Dirac action indeed leads naturally to this picture [47]. Kähler coupling strength should be however visible in the solutions of field equations somehow before one can say that these two criticalities have something to do with each other. Since Kähler coupling strength does not appear in field equations it can make its way to field equations only via boundary conditions. This is achieved if one accepts the weak form of self-duality discussed in [22] which roughly states that for the partonic 2-surfaces the induced Kähler electric field is proportional to the Kähler magnetic field strength. The proportionality constant turns out to be essentially the Kähler coupling strength. The simplest hypothesis is that Kähler coupling strength has single universal value for given value of Planck constant and the weak form of self-duality fixes it.

If Kähler action would define a strictly deterministic variational principle, Diff^4 degeneracy and invariance would be achieved by restricting the consideration to 3-surfaces Y^3 at the boundary of M^4_+ and by defining Kähler function for 3-surfaces X^3 at $X^4(Y^3)$ and diffeo-related to Y^3 as $K(X^3) = K(Y^3)$. This reduction might be called quantum gravitational holography. The classical non-determinism of the Kähler action introduces complications which might be overcome in zero energy ontology (ZEO). ZEO and strong from of GCI lead to the effective replacement of X^3 with partonic 2-surfaces at the ends of CD plus the 4-D tangent space distribution associated with them as basic geometric objects so that one can speak about effective 2-dimensionality and strong form of gravitational holography.

2.1.2 Configuration space metric from symmetries

A complementary approach to the problem of constructing configuration space geometry is based on symmetries. The work of Dan Freed [45] has demonstrated that the Kähler geometry of loop spaces is unique from the existence of Riemann connection and fixed completely by the Kac Moody symmetries of the space. In 3-dimensional context one has even better reasons to expect uniqueness. The guess is that configuration space is a union symmetric spaces labeled by zero modes not appearing in the line element as differentials and having interpretations as classical degrees providing a rigorous formulation of quantum measurement theory. The generalized conformal invariance of metrically 2-dimensional light like 3-surfaces acting as causal determinants is the corner stone of the construction. The construction works only for 4-dimensional space-time and imbedding space which is a product of four-dimensional Minkowski space or its future light cone with CP_2.

In this sequel I will first consider the basic properties of the configuration space, propose an identification of the Kähler function and discuss various physical and mathematical motivations behind the proposed definition. The key feature of the Kähler action is the failure of classical determinism in its standard form, and various implications of the failure are discussed.

2.2 Configuration space

The view about configuration space or world of classical worlds (WCW) has developed considerably during the last two decades. Here only the recent view is summarized in order to not load reader with unessential details.

2.2.1 Basic notions

The notions of imbedding space, 3-surface (and 4-surface), and configuration space or "world of classical worlds" (WCW), are central to quantum TGD. The original idea was that 3-surfaces are space-like 3-surfaces of $H = M^4 \times CP_2$ or $H = M^4_+ \times CP_2$, and WCW consists of all possible 3-surfaces in H. The basic idea was that the definition of Kähler metric of WCW assigns to each X^3 a unique space-time surface $X^4(X^3)$ allowing in this manner to realize GCI. During years these notions have however evolved considerably.

The notion of imbedding space

Two generalizations of the notion of imbedding space were forced by number theoretical vision [21, 20, 19].

1. p-Adicization forced to generalize the notion of imbedding space by gluing real and p-adic variants of imbedding space together along rationals and common algebraic numbers. The generalized imbedding space has a book like structure with reals and various p-adic number fields (including their algebraic extensions) representing the pages of the book. As matter fact, this gluing idea generalizes to the level of WCW.

2. With the discovery of zero energy ontology [30, 18] it became clear that the so called causal diamonds (CDs) interpreted as intersections $M^4_+ \cap M^4_-$ of future and past directed light-cones of $M^4 \times CP_2$ define correlates for the quantum states. The position of the "lower" tip of CD characterizes the position of CD in H. If the temporal distance between upper and lower tip of CD is quantized power of 2 multiples of CP_2 length, p-adic length scale hypothesis [29] follows as a consequence. The upper *resp.* lower light-like boundary $\delta M^4_+ \times CP_2$ *resp.* $\delta M^4_- \times CP_2$ of CD can be regarded as the carrier of positive *resp.* negative energy part of the state. All net quantum numbers of states vanish so that everything is creatable from vacuum. Space-time surfaces assignable to zero energy states would would reside inside $CD \times CP_2$s and have their 3-D ends at the light-like boundaries of $CD \times CP_2$. Fractal structure is present in the sense that CDs can contains CDs within CDs, and measurement resolution dictates the length scale below which the sub-CDs are not visible.

3. The realization of the hierarchy of Planck constants [20] led to a further generalization of the notion of imbedding space. Generalized imbedding space is obtained by gluing together Cartesian products of singular coverings and possibly also factor spaces of CD and CP_2 to form a book like structure. There are good physical and mathematical arguments suggesting that only the singular coverings should be allowed [19]. The particles at different pages of this book behave like dark matter relative to each other. This generalization also brings in the geometric correlate for the selection of quantization axes in the sense that the geometry of the sectors of the generalized imbedding space with non-standard value of Planck constant involves symmetry breaking reducing the isometries to Cartan subalgebra. Roughly speaking, each CD and CP_2 is replaced with a union of CDs and CP_2s corresponding to different choices of quantization axes so that no breaking of Poincare and color symmetries occurs at the level of entire WCW.

The notions of 3-surface and space-time surface

The question what one exactly means with 3-surface turned out to be non-trivial and the receont view is an outcome of a long and tedious process involving many hastily done mis-interpretations.

1. The original identification of 3-surfaces was as arbitrary space-like 3-surfaces subject to equivalence implied by GCI. There was a problem related to the realization of GCI since it was not at all obvious why the preferred extremal $X^4(Y^3)$ for Y^3 at $X^4(X^3)$ and Diff^4 related X^3 should satisfy $X^4(Y^3) = X^4(X^3)$.

2. Much later it became clear that light-like 3-surfaces have unique properties for serving as basic dynamical objects, in particular for realizing the GCI in 4-D sense (obviously the identification resolves the above mentioned problem) and understanding the conformal symmetries of the theory. Light-like 3-surfaces can be regarded as orbits of partonic 2-surfaces. Therefore it seems that one must choose between light-like and space-like 3-surfaces or assume generalized GCI requiring that equivalently either space-like 3-surfaces or light-like 3-surfaces at the ends of CDs can be identified as the fundamental geometric objects. General GCI requires that the basic objects correspond to the partonic 2-surfaces identified as intersections of these 3-surfaces plus common 4-D tangent space distribution. At the level of WCW metric this means that the components of the Kähler form and metric can be expressed in terms of data assignable to 2-D partonic surfaces. Since the information about normal space of the 2-surface is needed one has only effective 2-dimensionality. Weak form of self-duality [22] however implies that the normal data (flux Hamiltonians associated with Kähler electric field) reduces to magnetic flux Hamiltonians. This is essential for conformal symmetries and also simplifies the construction enormously.

3. At some stage came the realization that light-like 3-surfaces can have singular topology in the sense that they are analogous to Feynman diagrams. This means that the light-like 3-surfaces

representing lines of Feynman diagram can be glued along their 2-D ends playing the role of vertices to form what I call generalized Feynman diagrams. The ends of lines are located at boundaries of sub-CDs. This brings in also a hierarchy of time scales: the increase of the measurement resolution means introduction of sub-CDs containing sub-Feynman diagrams. As the resolution is improved, new sub-Feynman diagrams emerge so that effective 2-D character holds true in discretized sense and in given resolution scale only.

4. A further but inessential complication relates to the hierarchy of Planck constants forcing to generalize the notion of imbedding space and also to the fact that for non-standard values of Planck constant there is symmetry breaking due to preferred plane M^2 preferred homologically trivial geodesic sphere of CP_2 having interpretation as geometric correlate for the selection of quantization axis. For given sector of CH this means union over choices of this kind.

The basic vision forced by the generalization of GCI has been that space-time surfaces correspond to preferred extremals $X^4(X^3)$ of Kähler action and are thus analogous to Bohr orbits. Kähler function $K(X^3)$ defining the Kähler geometry of the world of classical worlds would correspond to the Kähler action for the preferred extremal. The precise identification of the preferred extremals actually has however remained open.

The study of the modified Dirac equation led to the realization that classical field equations for Kähler action can be seen as consistency conditions for the modified Dirac action and led to the identification of preferred extremals in terms of criticality. This identification which follows naturally also from quantum criticality.

1. The detailed construction of the generalized eigen modes of the dimensional reduction of the modified Dirac operator D_K associated with Kähler action [18] relies on the vision that the generalized eigenvalues of this operator code for information about preferred extremal of Kähler action and that vacuum functional identified as Dirac determinant equals to exponent of Kähler action for a preferred extremal.

2. The next step of progress was the realization that the requirement that the conservation of the Noether currents associated with the modified Dirac equation requires that the second variation of the Kähler action vanishes. In strongest form this condition would be satisfied for all variations and in weak sense only for those defining dynamical symmetries. The interpretation is as a space-time correlate for quantum criticality and the vacuum degeneracy of Kähler action makes the criticality plausible. Weak form of electric-magnetic duality gives a precise formulation for how Kähler coupling strength is visible in the properties of preferred extremals. A generalization of the ideas of the catastrophe theory to infinite-dimensional context results. These conditions make sense also in p-adic context and have a number theoretical universal form.

The notion of number theoretical compactication led to important progress in the understanding of the preferred extremals and the conjectures were consistent with what is known about the known extremals.

1. The conclusion was that one can assign to the 4-D tangent space $T(X^4(X_l^3)) \subset M^8$ a subspace $M^2(x) \subset M^4$ having interpretation as the plane of non-physical polarizations. This in the case that the induced metric has Minkowskian signature. If not, and if co-hyper-quaternionic surface is in question, similar assigned should be possible in normal space. This means a close connection with super string models. Geometrically this would mean that the deformations of 3-surface in the plane of non-physical polarizations would not contribute to the line element of WCW. This is as it must be since complexification does not make sense in M^2 degrees of freedom.

2. In number theoretical framework $M^2(x)$ has interpretation as a preferred hyper-complex sub-space of hyper-octonions defined as 8-D subspace of complexified octonions with the property that the metric defined by the octonionic inner product has signature of M^8. The condition $M^2(x) \subset T(X^4(X_l^3)))$ in principle fixes the tangent space at X_l^3, and one has good hopes that the boundary value problem is well-defined and could fix $X^4(X^3)$ at least partially as a preferred extremal of Kähler action. This picture is rather convincing since the choice $M^2(x) \subset M^4$ plays also other important roles.

3. At the level of H the counterpart for the choice of $M^2(x)$ seems to be following. Suppose that $X^4(X_l^3)$ has Minkowskian signature. One can assign to each point of the M^4 projection $P_{M^4}(X^4(X_l^3))$ a sub-space $M^2(x) \subset M^4$ and its complement $E^2(x)$, and the distributions of these planes are integrable and define what I have called Hamilton-Jacobi coordinates which can be assigned to the known extremals of Kähler with Minkowskian signature. This decomposition allows to slice space-time surfaces by string world sheets and their 2-D partonic duals. Also a slicing to 1-D light-like surfaces and their 3-D light-like duals Y_l^3 parallel to X_l^3 follows under certain conditions on the induced metric of $X^4(X_l^3)$. This decomposition exists for known extremals and has played key role in the recent developments. Physically it means that 4-surface (3-surface) reduces effectively to 3-D (2-D) surface and thus holography at space-time level.

4. The weakest form of number theoretic compactification [20] states that light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^8$, where $X^4(X^3)$ hyper-quaternionic surface in hyper-octonionic M^8 can be mapped to light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^4 \times CP_2$, where $X^4(X^3)$ is now preferred extremum of Kähler action. The natural guess is that $X^4(X^3) \subset M^8$ is a preferred extremal of Kähler action associated with Kähler form of E^4 in the decomposition $M^8 = M^4 \times E^4$, where M^4 corresponds to hyper-quaternions. The conjecture would be that the value of the Kähler action in M^8 is same as in $M^4 \times CP_2$: in fact that 2-surface would have identical induced metric and Kähler form so that this conjecture would follow trivial. $M^8 - H$ duality would in this sense be Kähler isometry.

If one takes $M^- H$ duality seriously, one must conclude that one can choose any partonic 2-surface in the slicing of X^4 as a representative. This means gauge invariance reflect in the definition of Kähler function as $U(1)$ gauge transformation $K \rightarrow K + f + \overline{f}$ having no effect on Kähler metric and Kähler form.

Although the details of this vision might change it can be defended by its ability to fuse together all great visions about quantum TGD. In the sequel the considerations are restricted to 3-surfaces in $M^4_\pm \times CP_2$. The basic outcome is that Kähler metric is expressible using the data at partonic 2-surfaces $X^2 \subset \delta M^4_+ \times CP_2$. The generalization to the actual physical situation requires the replacement of $X^2 \subset \delta M^4_+ \times CP_2$ with unions of partonic 2-surfaces located at light-like boundaries of CDs and sub-CDs.

The notion of configuration space

From the beginning there was a problem related to the precise definition of the configuration space ("world of classical worlds" (WCW)). Should one regard CH as the space of 3-surfaces of $M^4 \times CP_2$ or $M^4_+ \times CP_2$ or perhaps something more delicate.

1. For a long time I believed that the basis question is "M^4_+ or M^4?" and that this question had been settled in favor of M^4_+ by the fact that M^4_+ has interpretation as empty Roberson-Walker cosmology. The huge conformal symmetries assignable to $\delta M^4_+ \times CP_2$ were interpreted as cosmological rather than laboratory symmetries. The work with the conceptual problems related to the notions of energy and time, and with the symmetries of quantum TGD, however led gradually to the realization that there are strong reasons for considering M^4 instead of M^4_+.

2. With the discovery of zero energy ontology it became clear that the so called causal diamonds (CDs) define excellent candidates for the fundamental building blocks of the configuration space or "world of classical worlds" (WCW). The spaces $CD \times CP_2$ regarded as subsets of H defined the sectors of WCW.

3. This framework allows to realize the huge symmetries of $\delta M^4_\pm \times CP_2$ as isometries of WCW. The gigantic symmetries associated with the $\delta M^4_\pm \times CP_2$ are also laboratory symmetries. Poincare invariance fits very elegantly with the two types of super-conformal symmetries of TGD. The first conformal symmetry corresponds to the light-like surfaces $\delta M^4_\pm \times CP_2$ of the imbedding space representing the upper and lower boundaries of CD. Second conformal symmetry corresponds to light-like 3-surface X_l^3, which can be boundaries of X^4 and light-like surfaces separating space-time regions with different signatures of the induced metric. This symmetry is identifiable as the counterpart of the Kac Moody symmetry of string models.

A rather plausible conclusion is that configuration space (WCW) is a union of configuration spaces associated with the spaces $CD \times CP_2$. CDs can contain CDs within CDs so that a fractal like hierarchy having interpretation in terms of measurement resolution results. It must be however emphasized that Kähler function depends on partonic 2-surfaces at both ends of space-time surface so that WCW is topologically Cartesian product of corresponding symmetric spaces. WCW metric must therefore have parts corresponding to the partonic 2-surfaces (free part) and also an interaction term depending on the partonic 2-surface at the opposite ends of the light-like 3-surface. The conclusion is that geometrization reduces to that for single like of generalized Feynman diagram containing partonic 2-surfaces at its ends. Since the complications due to p-adic sectors and hierarchy of Planck constants are not relevant for the basic construction, it reduces to a high degree to a study of a simple special case corresponding to a line of generalized Feynman diagram. One can also deduce the free part of the metric by restricting the consideration to partonic 2-surfaces at single end of generalized Feynman diagram.

A further piece of understanding emerged from the following observations.

1. The induced Kähler form at the partonic 2-surface X^2 - the basic dynamical object if holography is accepted- can be seen as a fundamental symplectic invariant so that the values of $\epsilon^{\alpha\beta}J_{\alpha\beta}$ at X^2 define local symplectic invariants not subject to quantum fluctuations in the sense that they would contribute to the configuration space metric. Hence only induced metric corresponds to quantum fluctuating degrees of freedom at configuration space level and TGD is a genuine theory of gravitation at this level.

2. Configuration space can be divided into slices for which the induced Kähler forms of CP_2 and $\delta M^4_\pm$ at the partonic 2-surfaces X^2 at the light-like boundaries of CDs are fixed. The symplectic group of $\delta M^4_\pm \times CP_2$ parameterizes quantum fluctuating degrees of freedom in given scale (recall the presence of hierarchy of CDs).

3. This leads to the identification of the coset space structure of the sub-configuration space associated with given CD in terms of the generalized coset construction for super-symplectic and super Kac-Moody type algebras (symmetries respecting light-likeness of light-like 3-surfaces). Configuration space in quantum fluctuating degrees of freedom for given values of zero modes can be regarded as being obtained by dividing symplectic group with Kac-Moody group. Equivalently, the local coset space $S^2 \times CP_2$ is in question: this was one of the first ideas about configuration space which I gave up as too naive!

4. Generalized coset construction and coset space structure have very deep physical meaning since they realize Equivalence Principle at quantum level: the identical actions of Super Virasoro generators for super-symplectic and super Kac-Moody algebras implies that inertial and gravitational four-momenta are identical.

2.2.2 Constraints on the configuration space geometry

The constraints on the WCW result both from the infinite dimension of the configuration space and from physically motivated symmetry requirements. There are three basic physical requirements on the configuration space geometry: namely four-dimensional GCI in strong form, Kähler property and the decomposition of configuration space into a union $\cup_i G/H_i$ of symmetric spaces G/H_i, each coset space allowing G-invariant metric such that G is subgroup of some 'universal group' having natural action on 3-surfaces. Together with the infinite dimensionality of the configuration space these requirements pose extremely strong constraints on the configuration space geometry. In the following we shall consider these requirements in more detail.

Diff^4 invariance and Diff^4 degeneracy

Diff^4 plays fundamental role as the gauge group of General Relativity. In string models $Diff^2$ invariance ($Diff^2$ acts on the orbit of the string) plays central role in making possible the elimination of the time like and longitudinal vibrational degrees of freedom of string. Also in the present case the elimination of the tachyons (time like oscillatory modes of 3-surface) is a physical necessity and Diff^4 invariance provides an obvious manner to do the job.

In the standard path l integral formulation the realization of Diff^4 invariance is an easy task at the formal level. The problem is however that path integral over four-surfaces is plagued by divergences and doesn't make sense. In the present case the configuration space consists of 3-surfaces and only $Diff^3$ emerges automatically as the group of re-parameterizations of 3-surface. Obviously one should somehow define the action of Diff^4 in the space of 3-surfaces. Whatever the action of Diff^4 is it must leave the configuration space metric invariant. Furthermore, the elimination of tachyons is expected to be possible only provided the time like deformations of the 3-surface correspond to zero norm vector fields of the configuration space so that 3-surface and its Diff^4 image have zero distance. The conclusion is that configuration space metric should be both Diff^4 invariant and Diff^4 degenerate.

The problem is how to define the action of Diff^4 in $C(H)$. Obviously the only manner to achieve Diff^4 invariance is to require that the very definition of the configuration space metric somehow associates a unique space time surface to a given 3-surface for Diff^4 to act on. The obvious physical interpretation of this space time surface is as "classical space time" so that "Classical Physics" would be contained in configuration space geometry. In fact, this space-time surface is analogous to Bohr orbit so that semiclassical quantization rules become an exact part of the quantum theory. It is this requirement, which has turned out to be decisive concerning the understanding of the WCW geometry.

Decomposition of the configuration space into a union of symmetric spaces G/H

The extremely beautiful theory of finite-dimensional symmetric spaces constructed by Elie Cartan suggests that configuration space should possess decomposition into a union of coset spaces $CH = \cup_i G/H_i$ such that the metric inside each coset space G/H_i is left invariant under the infinite dimensional isometry group G. The metric equivalence of surfaces inside each coset space G/H_i does not mean that 3-surfaces inside G/H_i are physically equivalent. The reason is that the vacuum functional is exponent of Kähler action which is not isometry invariant so that the 3-surfaces, which correspond to maxima of Kähler function for a given orbit, are in a preferred position physically. For instance, one can imagine of calculating functional integral around this maximum perturbatively. Symmetric space property actually allows also much more powerful non-perturbative approach based on harmonic analysis [47]. The sum of over i means actually integration over the zero modes of the metric (zero modes correspond to coordinates not appearing as coordinate differentials in the metric tensor).

The coset space G/H is a symmetric space only under very special Lie-algebraic conditions. Denoting the decomposition of the Lie-algebra g of G to the direct sum of H Lie-algebra h and its complement t by $g = h \oplus t$, one has

$$[h,h] \subset h \quad , \quad [h,t] \subset t \quad , \quad [t,t] \subset h \quad .$$

This decomposition turn out to play crucial role in guaranteing that G indeed acts as isometries and that the metric is Ricci flat.

The four-dimensional $Diff$ invariance indeed suggests to a beautiful solution of the problem of identifying G. The point is that any 3-surface X^3 is $Diff^4$ equivalent to the intersection of $X^4(X^3)$ with the light cone boundary. This in turn implies that 3-surfaces in the space $\delta H = \delta M^4_+ \times CP_2$ should be all what is needed to construct configuration space geometry. The group G can be identified as some subgroup of diffeomorphisms of δH and H_i contains that subgroup of G, which acts as diffeomorphisms of the 3-surface X^3. Since G preserves topology, configuration space must decompose into union $\cup_i G/H_i$, where i labels 3-topologies and various zero modes of the metric. For instance, the elements of the Lie-algebra of G invariant under configuration space complexification correspond to zero modes.

The reduction to the light cone boundary, identifiable as the moment of big bang, looks perhaps odd at first. In fact, it turns out that the classical non-determinism of Kähler action does not allow the complete reduction to the light cone boundary: physically this is a highly desirable implication but means a considerable mathematical challenge.

Kähler property

Kähler property implies that the tangent space of the configuration space allows complexification and that there exists a covariantly constant two-form J_{kl}, which can be regarded as a representation of the imaginary unit in the tangent space of the configuration space:

$$J_k{}^r J_{rl} = -G_{kl} \quad . \tag{2.2.1}$$

There are several physical and mathematical reasons suggesting that configuration space metric should possess Kähler property in some generalized sense.

1. The deepest motivation comes from the need to geometrize hermitian conjugation which is basic mathematical operation of quantum theory.

2. Kähler property turns out to be a necessary prerequisite for defining divergence free configuration space integration. We will leave the demonstration of this fact later although the argument as such is completely general.

3. Kähler property very probably implies an infinite-dimensional isometry group. The study of the loop groups $Map(S^1, G)$ [45] shows that loop group allows only single Kähler metric with well defined Riemann connection and this metric allows local G as its isometries!

   To see this consider the construction of Riemannian connection for $Map(X^3, H)$. The defining formula for the connection is given by the expression

$$\begin{aligned} 2(\nabla_X Y, Z) &= X(Y,Z) + Y(Z,X) - Z(X,Y) \\ &+ ([X,Y],Z) + ([Z,X],Y) - ([Y,Z],X) \end{aligned} \tag{2.2.2}$$

   X, Y, Z are smooth vector fields in $Map(X^3, G)$. This formula defines $\nabla_X Y$ uniquely provided the tangent space of Map is complete with respect to Riemann metric. In the finite-dimensional case completeness means that the inverse of the covariant metric tensor exists so that one can solve the components of connection from the conditions stating the covariant constancy of the metric. In the case of the loop spaces with Kähler metric this is however not the case.

   Now the symmetry comes into the game: if X, Y, Z are left (local gauge) invariant vector fields defined by the Lie-algebra of local G then the first three terms drop away since the scalar products of left invariant vector fields are constants. The expression for the covariant derivative is given by

$$\nabla_X Y = (Ad_X Y - Ad_X^* Y - Ad_Y^* X)/2 \tag{2.2.3}$$

   where Ad_X^* is the adjoint of Ad_X with respect to the metric of the loop space.

   At this point it is important to realize that Freed's argument does not force the isometry group of the configuration space to be $Map(X^3, M^4 \times SU(3))$! Any symmetry group, whose Lie algebra is complete with respect to the configuration space metric (in the sense that any tangent space vector is expressible as superposition of isometry generators modulo a zero norm tangent vector) is an acceptable alternative.

   The Kähler property of the metric is quite essential in one-dimensional case in that it leads to the requirement of left invariance as a mathematical consistency condition and we expect that dimension three makes no exception in this respect. In 3-dimensional case the degeneracy of the metric turns out to be even larger than in 1-dimensional case due to the four-dimensional Diff degeneracy. So we expect that the metric ought to possess some infinite-dimensional isometry group and that the above formula generalizes also to the 3-dimensional case and to the case of local coset space. Note that in M^4 degrees of freedom $Map(X^3, M^4)$ invariance would imply the flatness of the metric in M^4 degrees of freedom.

   The physical implications of the above purely mathematical conjecture should not be underestimated. For example, one natural looking manner to construct physical theory would be based on the idea that configuration space geometry is dynamical and this approach is followed in the attempts to construct string theories [43]. Various physical considerations (in particular the need to obtain oscillator operator algebra) seem to imply that configuration space geometry is necessarily Kähler. The above result however states that configuration space Kähler geometry cannot be dynamical quantity and is dictated solely by the requirement of internal consistency. This result is extremely nice since it has been already found that the definition of the configuration space metric must somehow associate a unique classical space time and "classical physics" to a given 3-surface: uniqueness of the geometry implies the uniqueness of the "classical physics".

4. The choice of the imbedding space becomes highly unique. In fact, the requirement that configuration space is not only symmetric space but also (contact) Kähler manifold inheriting its (degenerate) Kähler structure from the imbedding space suggests that spaces, which are products of four-dimensional Minkowski space with complex projective spaces CP_n, are perhaps the only possible candidates for H. The reason for the unique position of the four-dimensional Minkowski space turns out to be that the boundary of the light cone of D-dimensional Minkowski space is metrically a sphere S^{D-2} despite its topological dimension $D-1$: for $D=4$ one obtains two-sphere allowing Kähler structure and infinite parameter group of conformal symmetries!

5. It seems possible to understand the basic mathematical structures appearing in string model in terms of the Kähler geometry rather nicely.

   (a) The projective representations of the infinite-dimensional isometry group (not necessarily Map!) correspond to the ordinary representations of the corresponding centrally extended group [37]. The representations of Kac Moody group indeed play central role in string models [63, 61] and configuration space approach would explain their occurrence, not as a result of some quantization procedure, but as a consequence of symmetry of the underlying geometric structure.

   (b) The bosonic oscillator operators of string models would correspond to centrally extended Lie-algebra generators of the isometry group acting on spinor fields of the configuration space.

   (c) The "fermionic" fields (Ramond fields, [63, 61]) should correspond to gamma matrices of the configuration space. Fermionic oscillator operators would correspond simply to contractions of isometry generators j_A^k with complexified gamma matrices of configuration space

$$\begin{aligned} \Gamma_A^{\pm} &= j_A^k \Gamma_k^{\pm} \\ \Gamma_k^{\pm} &= (\Gamma^k \pm J^k_{\ l}\Gamma^l)/\sqrt{2} \end{aligned} \tag{2.2.4}$$

   ($J^k_{\ l}$ is the Kähler form of the configuration space) and would create various spin excitations of the configuration space spinor field. $\Gamma_k^{\pm}$ are the complexified gamma matrices, complexification made possible by the Kähler structure of the configuration space.

This suggests that some generalization of the so called Super Kac Moody algebra of string models [63, 61] should be regarded as a spectrum generating algebra for the solutions of field equations in configuration space.

Although the Kähler structure seems to be physically well motivated there is a rather heavy counter argument against the whole idea. Kähler structure necessitates complex structure in the tangent space of the configuration space. In CP_2 degrees of freedom no obvious problems of principle are expected: configuration space should inherit in some sense the complex structure of CP_2.

In Minkowski degrees of freedom the signature of the Minkowski metric seems to pose a serious obstacle for complexification: somehow one should get rid of two degrees of freedom so that only two Euclidian degrees of freedom remain. An analogous difficulty is encountered in quantum field theories: only two of the four possible polarizations of gauge boson correspond to physical degrees of freedom: mathematically the wrong polarizations correspond to zero norm states and transverse states span a complex Hilbert space with Euclidian metric. Also in string model analogous situation occurs: in case of D-dimensional Minkowski space only $D-2$ transversal degrees of freedom are physical. The solution to the problem seems therefore obvious: configuration space metric must be degenerate so that each vibrational mode spans effectively a 2-dimensional Euclidian plane allowing complexification.

We shall find that the definition of Kähler function to be proposed indeed provides a solution to this problem and also to the problems listed before.

1. The definition of the metric doesn't differentiate between 1- and N-particle sectors, avoids spin statistics difficulty and has the physically appealing property that one can associate to each 3-surface a unique classical space time: classical physics is described by the geometry of the configuration space and d the geometry of the configuration space is determined uniquely by the requirement of mathematical consistency.

2. Complexification is possible only provided the dimension of the Minkowski space equals to four and is due to the effective 3-dimensionality of light-cone boundary.

3. It is possible to identify a unique candidate for the necessary infinite-dimensional isometry group G. G is subgroup of the diffeomorphism group of $\delta M^4_+ \times CP_2$. Essential role is played by the fact that the boundary of the four-dimensional light cone, which, despite being topologically 3-dimensional, is metrically two-dimensional Euclidian sphere, and therefore allows infinite-parameter groups of isometries as well as conformal and symplectic symmetries and also Kähler structure unlike the higher-dimensional light cone boundaries. Therefore configuration space metric is Kähler only in the case of four-dimensional Minkowski space and allows symplectic $U(1)$ central extension without conflict with the no-go theorems about higher dimensional central extensions.

   The study of the vacuum degeneracy of Kähler function defined by Kähler action forces to conclude that the isometry group must consist of the symplectic transformations of $\delta H = \delta M^4_+ \times CP_2$. The corresponding Lie algebra can be regarded as a loop algebra associated with the symplectic group of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of light cone boundary. Thus the finite-dimensional group G defining loop group in case of string models extends to an infinite-dimensional group in TGD context. This group has a monstrous size. The radial Virasoro localized with respect to $S^2 \times CP_2$ defines naturally complexification for both G and H. The general form of the Kähler metric deduced on basis of this symmetry has same qualitative properties as that deduced from Kähler function identified as preferred extremal of Kähler action. Also the zero modes, among them isometry invariants, can be identified.

4. The construction of the configuration space spinor structure is based on the identification of the configuration space gamma matrices as linear superpositions of the oscillator operators associated with the second quantized induced spinor fields. The extension of the symplectic invariance to super symplectic invariance fixes the anti-commutation relations of the induced spinor fields, and configuration space gamma matrices correspond directly to the super generators. Physics as number theory vision suggests strongly that configuration space geometry exists for 8-dimensional imbedding space only and that the choice $M^4_+ \times CP_2$ for the imbedding space is the only possible one.

2.3 Identification of the Kähler function

There are three approaches to the construction of the WCW geometry: a direct physics based guess of the Kähler function, a group theoretic approach based on the hypothesis that CH can be regarded as a union of symmetric spaces, and the approach based on the construction of WCW spinor structure first by second quantization of induced spinor fields. Here the first approach is discussed.

2.3.1 Definition of Kähler function

Kähler metric in terms of Kähler function

Quite generally, Kähler function K defines Kähler metric in complex coordinates via the following formula

$$J_{k\bar{l}} = ig_{k\bar{l}} = i\partial_k\partial_{\bar{l}}K \ . \tag{2.3.1}$$

Kähler function is defined only modulo a real part of holomorphic function so that one has the gauge symmetry

$$K \quad \rightarrow \quad K + f + \overline{f} \ . \tag{2.3.2}$$

Let X^3 be a given 3-surface and let X^4 be any four-surface containing X^3 as a sub-manifold: $X^4 \supset X^3$. The 4-surface X^4 possesses in general boundary. If the 3-surface X^3 has nonempty boundary δX^3 then the boundary of X^3 belongs to the boundary of X^4: $\delta X^3 \subset \delta X^4$.

Induced Kähler form and its physical interpretation

Induced Kähler form defines a Maxwell field and it is important to characterize precisely its relationship to the gauge fields as they are defined in gauge theories. Kähler form J is related to the corresponding Maxwell field F via the formula

$$J \quad = \quad xF \ \ , \ \ x = \frac{g_K}{\hbar} \ . \tag{2.3.3}$$

Similar relationship holds true also for the other induced gauge fields. The inverse proportionality of J to $\hbar$ does not matter in the ordinary gauge theory context where one routinely choses units by putting $\hbar = 1$ but becomes very important when one considers a hierachy of Planck constants [20].

Unless one has $J = (g_K/\hbar_0)$, where $\hbar_0$ corresponds to the ordinary value of Planck constant, $\alpha_K = g_K^2/4\pi\hbar$ together the large Planck constant means weaker interactions and convergence of the functional integral defined by the exponent of Kähler function and one can argue that the convergence of the functional integral is what forces the hierarchy of Planck constants. This is in accordance with the vision that Mother Nature likes theoreticians and takes care that the perturbation theory works by making a phase transition increasing the value of the Planck constant in the situation when perturbation theory fails. This leads to a replacement of the M^4 (or more precisely, causal diamond CD) and CP_2 factors of the imbedding space ($CD \times CP_2$) with its $r = \hbar/\hbar_0$-fold singular covering (one can consider also singular factor spaces). If the components of the space-time surfaces at the sheets of the covering are identical, one can interpret r-fold value of Kähler action as a sum of r identical contributions from the sheets of the covering with ordinary value of Planck constant and forget the presence of the covering. Physical states are however different even in the case that one assumes that sheets carry identical quantum states and anyonic phase could correspond to this kind of phase [25].

Kähler action

One can associate to Kähler form Maxwell action and also Chern-Simons anomaly term proportional to $\int_{X^4} J \wedge J$ in well known manner. Chern Simons term is purely topological term and well defined for orientable 4-manifolds, only. Since there is no deep reason for excluding non-orientable space-time surfaces it seems reasonable to drop Chern Simons term from consideration. Therefore Kähler action $S_K(X^4)$ can be defined as

$$S_K(X^4) \quad = \quad k_1 \int_{X^4; X^3 \subset X^4} J \wedge (*J) \ . \tag{2.3.4}$$

The sign of the square root of the metric determinant, appearing implicitly in the formula, is defined in such a manner that the action density is negative for the Euclidian signature of the induced metric and such that for a Minkowskian signature of the induced metric Kähler electric field gives a negative contribution to the action density.

The notational convention

$$k_1 \quad \equiv \quad \frac{1}{16\pi\alpha_K} \ , \tag{2.3.5}$$

where α_K will be referred as Kähler coupling strength will be used in the sequel. If the preferred extremals minimize/maximize [20] the absolute value of the action in each region where action density has a definite sign, the value of α_K can depend on space-time sheet.

Kähler function

One can define the Kähler function in the following manner. Consider first the case $H = M^4_+ \times CP_2$ and neglect for a moment the non-determinism of Kähler action. Let X^3 be a 3-surface at the light-cone boundary $\delta M^4_+ \times CP_2$. Define the value $K(X^3)$ of Kähler function K as the value of the Kähler action for some preferred extremal in the set of four-surfaces containing X^3 as a sub-manifold:

$$K(X^3) \ = \ K(X^4_{pref}) \ , \ X^4_{pref} \subset \{X^4 | X^3 \subset X^4\} \ . \tag{2.3.6}$$

The most plausible identification of preferred extremals is in terms of quantum criticality in the sense that the preferred extremals allow an infinite number of deformations for which the second variation of Kähler action vanishes. Combined with the weak form of electric-magnetic duality forcing appearence of Kähler coupling strength in the boundary conditions at partonic 2-surfaces this condition might be enough to fix preferred extremals completely.

2.3.2 What are the values of the Kähler coupling strength?

Since the vacuum functional of the theory turns out to be essentially the exponent $exp(K)$ of the Kähler function, the dynamics depends on the normalization of the Kähler function. Since the Theory of Everything should be unique it would be highly desirable to find arguments fixing the normalization or equivalently the possible values of the Kähler coupling strength α_K. Also a discrete spectrum of values is acceptable.

The quantization of Kähler form could result in the following manner. It will be found that Abelian extension of the isometry group results by coupling spinors of the configuration space to a multiple of Kähler potential. This means that Kähler potential plays role of gauge connection so that Kähler form must be integer valued by Dirac quantization condition for magnetic charge. So, if Kähler form is co-homologically nontrivial it is quantized.

Unfortunately, the exact definition of renormalization group concept is not at all obvious. There is however a much more general but more or less equivalent manner to formulate the condition fixing the value of α_K. Vacuum functional $exp(K)$ is analogous to the exponent $exp(-H/T)$ appearing in the definition of the partition function of a statistical system and S-matrix elements and other interesting physical quantities are integrals of type $\langle O\rangle = \int exp(K)O\sqrt{G}dV$ and therefore analogous to the thermal averages of various observables. α_K is completely analogous to temperature. The critical points of a statistical system correspond to critical temperatures T_c for which the partition function is nonanalytic function of $T - T_c$ and according RGE hypothesis critical systems correspond to fixed points of renormalization group evolution. Therefore, a mathematically more precise manner to fix the value of α_K is to require that some integrals of type $\langle O\rangle$ (not necessary S-matrix elements) become nonanalytic at $1/\alpha_K - 1/\alpha_K^c$.

This analogy suggests also a physical motivation for the unique value or value spectrum of α_K. Below the critical temperature critical systems suffer something analogous to spontaneous magnetization. At the critical point critical systems are characterized by long range correlations and arbitrarily large volumes of magnetized and non-magnetized phases are present. Spontaneous magnetization might correspond to the generation of Kähler magnetic fields: the most probable 3-surfaces are Kähler magnetized for subcritical values of α_K. At the critical values of α_K the most probable 3-surfaces contain regions dominated by either Kähler electric and or Kähler magnetic fields: by the compactness of CP_2 these regions have in general outer boundaries.

This suggests that 3-space has hierarchical, fractal like structure: 3-surfaces with all sizes (and with outer boundaries) are possible and they have suffered topological condensation on each other. Therefore the critical value of α_K allows the richest possible topological structure for the most probable 3-space. In fact, this hierarchical structure is in accordance with the basic ideas about renormalization group invariance. This hypothesis has highly nontrivial consequences even at the level of ordinary condensed matter physics.

Renormalization group invariance is closely related with criticality. The self duality of the Kähler form and Weyl tensor of CP_2 indeed suggest RG invariance. The point is that in $N = 4$ super-symmetric field theories duality transformation relates the strong coupling limit for ordinary particles with the weak coupling limit for magnetic monopoles and vice versa. If the theory is self-dual these limits must be identical so that action and coupling strength must be RG invariant quantities. This form of self-duality cannot hold true in TGD. The weak form of self-duality discussed in [22] roughly states that for the partonic 2-surface the induce Kähler electric field is proportional to the Kähler magnetic field strength. The proportionality constant is essentially Kähler coupling strength. The simplest hypothesis is that Kähler coupling strength has single universal valiue and the weak form of self-duality fixes it. The proportionality $\alpha_K = g_K^2/4\pi\hbar$ and the proposed quantization of Planck constant requiring a generalization of the imbedding space imply that Kähler coupling strength varies but is constant at a given page of the "Big Book" defined by the generalized imbedding space [20] .

2.3.3 What preferred extremal property means?

The requirement that the 4-surface having given 3-surface as its sub-manifold is absolute minimum of the Kähler action is the most obvious guess for the principle selecting the preferred extremals and has been taken as a working hypothesis for about one and half decades. Quantum criticality of Quantum TGD should have however led to the idea that preferred extremals are critical in the sense that space-time surface allows deformations for which second variation of Kähler action vanishes so that the corresponding Noether currents are conserved.

Further insights emerged through the realization that Noether currents assignable to the modified Dirac equation are conserved only if the first variation of the modified Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X^3_l)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals.

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X^3_l)$ vanishing at the intersections of $X^4(X^3_l)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X^3_l with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of configuration space represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the configuration space metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \to X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X^3_l)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X_l^3 involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

One must be very cautious with what one means with the preferred extremal property and criticality.

1. Does one assign criticality with the partonic 2-surfaces at the ends of CDs? Does one restrict it with the throats for which light-like 3-surface has also degenerate induced 4-metric? Or does one assume stronger form of holography requiring a slicing of space-time surface by partonic 2-surfaces and string world sheets and assign criticality to all partonic 2-surfaces. This kind of slicing is suggested by the study of the extremals [33], required by the number theoretic vision ($M^8 - H$ duality [19]), and also by the purely physical condition that a stringy realization of GCI is possible.

2. What is the exact meaning of the preferred extremal property? The assumption that the variations of Kähler action leaving 3-surfaces at the ends of CDs invariant would not be consistent with the effective 2-dimensionality. The assumption that the critical deformations leave invariant only partonic 2-surfaces would imply genuine 2-dimensionality. Should one assume that critical deformations leave invariant partonic 2-surface and 3-D tangent space in the direction of space-like 3-surface or light-like 3-surface but not both. This would be consistent with effective 3-dimensionality and would explain why Kac-Moody symmetries associated with the light-like 3-surfaces act as gauge symmetries. This is also essential for the realization of Poincare invariance since the quantization of the light-cone proper time distance between CDs implies that infinitesimal Poincare transformations lead out of CD unless compensated by Kac-Moody type transformations acting like gauge transformations. In the similar manner it would explain why symplectic transformations of δCD act like gauge transformations.

3. Could one pose the criticality condition for all partonic 2-surfaces in the slicing or only for the throats of light-like 3-surfaces? This hypothesis looks natural but is not necessary. Light-like throats are very singular objects criticality might apply only to their variations only in the limiting sense and it might be necesary to assume criticality for all partonic 2-surfaces.

2.3.4 Why non-local Kähler function?

Kähler function is nonlocal functional of 3-surface. Non-locality of the Kähler function seems to be at odds with basic assumptions of local quantum field theories. Why this rather radical departure from the basic assumptions of local quantum field theory? The answer is shortly given: configuration space integration appears in the definition of the inner product for WCW spinor fields and this inner product must be free from perturbative divergences. Consider now the argument more closely.

In the case of finite-dimensional symmetric space with Kähler structure the representations of the isometry group necessitate the modification of the integration measure defining the inner product so that the integration measure becomes proportional to the exponent $exp(K)$ of the Kähler function [56]. The generalization to infinite-dimensional case is obvious. Also the requirement of Kac-Moody symmetry leads to the presence of this kind of vacuum functional as will be found later. The exponent is in fact uniquely fixed by finiteness requirement. Configuration space integral is of the following form

$$\int \bar{S}_1 exp(K) S_1 \sqrt{g} dX \quad . \tag{2.3.7}$$

One can develop perturbation theory using local complex coordinates around a given 3-surface in the following manner. The $(1,1)$-part of the second variation of the Kähler function defines the metric and therefore propagagator as contravariant metric and the remaining $(2,0)-$ and $(0,2)$-parts of the second variation are treated perturbatively. The most natural choice for the 3-surface are obviously the 3-surfaces, which correspond to extrema of the Kähler function.

When perturbation theory is developed around the 3-surface one obtains two ill-defined determinants.

1. The Gaussian determinant coming from the exponent, which is just the inverse square root for the matrix defined by the metric defining $(1,1)$-part of the second variation of the Kähler function in local coordinates.

2. The metric determinant. The matrix representing covariant metric is however same as the matrix appearing in Gaussian determinant by the defining property of the Kähler metric: in local complex coordinates the matrix defined by second derivatives is of type $(1,1)$. Therefore these two ill defined determinants (recall the presence of Diff degeneracy) cancel each other exactly for a unique choice of the vacuum functional!

Of course, the cancellation of the determinants is not enough. For an arbitrary local action one encounters the standard perturbative divergences. Since most local actions (Chern-Simons term is perhaps an exception [49]) for induced geometric quantities are extremely nonlinear there is no hope of obtaining a finite theory. For nonlocal action the situation is however completely different. There are no local interaction vertices and therefore no products of delta functions in perturbation theory.

A further nice feature of the perturbation theory is that the propagator for small deformations is nothing but the contravariant metric. Also the various vertices of the theory are closely related to the metric of the configuration space since they are determined by the Kähler function so that perturbation theory would have a beautiful geometric interpretation. Furthermore, since four-dimensional Diff degeneracy implies that the propagator doesn't couple to un-physical modes.

It should be noticed that divergence cancellation arguments do not necessarily exclude Chern Simons term from vacuum functional defined as imaginary exponent of $exp(ik_2 \int_{X^4} J \wedge J)$. The term is not well defined for non-orientable space-time surfaces and one must assume that k_2 vanishes for these surfaces. The presence of this term might provide first principle explanation for CP breaking. If k_2 is integer multiple of $1/(8\pi)$ Chern Simons term gives trivial contribution for closed space-time surfaces since instanton number is in question. By adding a suitable boundary term of form $exp(ik_3 \int_{\delta X^3} J \wedge A)$ it is possible to guarantee that the exponent is integer valued for 4-surfaces with boundary, too.

There are two arguments suggesting that local Chern Simons term would not introduce divergences. First, 3-dimensional Chern Simons term for ordinary Abelian gauge field is known to define a divergence free field theory [49]. The term doesn't depend at all on the induced metric and therefore contains no dimensional parameters (CP_2 radius) and its expansion in terms of CP_2 coordinate variables is of the form allowed by renormalizable field theory in the sense that only quartic terms appear. This is seen by noticing that there always exist symplectic coordinates, where the expression of the Kähler potential is of the form

$$A = \sum_k P_k dQ^k \ . \tag{2.3.8}$$

The expression for Chern-Simons term in these coordinates is given by

$$k_2 \int_{X^3} \sum_{k,l} P_l dP_k \wedge dQ^k \wedge dQ^l \ , \tag{2.3.9}$$

and clearly quartic CP_2 coordinates. A further nice property of the Chern Simons term is that this term is invariant under symplectic transformations of CP_2, which are realized as $U(1)$ gauge transformation for the Kähler potential.

2.4 Some properties of Kähler action

In this section some properties of Kähler action and Kähler function are discussed in light of experienced gained during about 15 years after the introduction of the notion.

2.4.1 Vacuum degeneracy and some of its implications

The vacuum degeneracy is perhaps the most characteristic feature of the Kähler action. Although it is not associated with the preferred extremals of Kähler action, there are good reasons to expect that it has deep consequences concerning the structure of the theory.

Vacuum degeneracy of the Kähler action

The basic reason for choosing Kähler action is its enormous vacuum degeneracy, which makes long range interactions possible (the well known problem of the membrane theories is the absence of massless particles [46]). The Kähler form of CP_2 defines symplectic structure and any 4-surface for which CP_2 projection is so called Lagrangian manifold (at most two dimensional manifold with vanishing induced Kähler form), is vacuum extremal due to the vanishing of the induced Kähler form. More explicitly, in the local coordinates, where the vector potential A associated with the Kähler form reads as $A = \sum_k P_k dQ^k$. Lagrangian manifolds are expressible locally in the following form

$$P_k = \partial_k f(Q^i) \ . \tag{2.4.1}$$

where the function f is arbitrary. Notice that for the general YM action surfaces with one-dimensional CP_2 projection are vacuum extremals but for Kähler action one obtains additional degeneracy.

There is also a second kind of vacuum degeneracy, which is relevant to the elementary particle physics. The so called CP_2 type vacuum extremals are warped imbeddings X^4 of CP_2 to H such that Minkowski coordinates are functions of a single CP_2 coordinate, and the one-dimensional projection of X^4 is random light like curve. These extremals have a non-vanishing action but vanishing Poincare charges. Their small deformations are identified as space-time counterparts of fermions and their super partners. Wormhole throats identified as pieces of these extremals are identified as bosons and their super partners.

The conditions stating light likeness are equivalent with the Virasoro conditions of string models and this actually led to the eventualo realization that conformal invariance is a basic symmetry of TGD and that WCW can be regarded as a union of symmetric spaces with isometry groups having identification as symplectic and Kac-Moody type groups assignable to the partonic 2-surfaces.

Approximate symplectic invariance

Vacuum extremals have diffeomorphisms of M^4_+ and M^4_+ local symplectic transformations as symmetries. For non-vacuum extremals these symmetries leave induced Kähler form invariant and only induced metric breaks these symmetries. Symplectic transformations of CP_2 act on the Maxwell field defined by the induced Kähler form in the same manner as ordinary $U(1)$ gauge symmetries. They are however not gauge symmetries since gauge invariance is still present. In fact, the construction of the configuration space geometry relies on the assumption that symplectic transformations of $\delta M^4_+ \times CP_2$ which infinitesimally correspond to combinations of M^4_+ local CP_2 symplectic and CP_2-local M^4_+ symplectic transformations act as isometries of the configuration space. In zero energy ontology these transformations act simultanoeusly on all partonic 2-surfaces characterizing the space-time sheet representing a generalized Feynman diagram inside CD.

The fact that CP_2 symplectic transformations do not act as genuine gauge transformations means that $U(1)$ gauge invariance is effectively broken. This has non-trivial implications. The field equations allow purely geometric vacuum 4-currents not possible in Maxwell's electrodynamics [33]. For the known extremals (massless extremals) they are light-like and a possible interpretation is in terms of Bose-Einstein condensates of collinear massless bosons.

Spin glass degeneracy

Vacuum degeneracy means that all surfaces belonging to $M^4_+ \times Y^2$, Y^2 any Lagrangian sub-manifold of CP_2 are vacua irrespective of the topology and that symplectic transformations of CP_2 generate new surfaces Y^2. If preferred extremals are obtained as small deformations of vacuum extremals (for which the criticality is maximal), one expects therefore enormous ground state degeneracy, which could be seen as 4-dimensional counterpart of the spin glass degeneracy. This degeneracy corresponds to the hypothesis that configuration space is a union of symmetric spaces labeled by zero modes which do not appear at the line-element of the configuration space metric.

Zero modes define what might be called the counterpart of spin glass energy landscape and the maxima Kähler function as a function of zero modes define a discrete set which might be called reduced configuration space. Spin glass degeneracy turns out to be crucial element for understanding how macro-temporal quantum coherence emerges in TGD framework. One of the basic ideas about p-adicization is that the maxima of Kähler function define the TGD counterpart of spin glass energy landscape [21, 23]. The hierarchy of discretizations of the symmetric spaces corresponding to a hierarchy of measurement resolutions [47] could allow an identification in terms of a hierarchy spin glass energy landscapes so that the algebraic points of the WCW would correspond to the maxima of Kähler function. The hierarchical structure would be due to the failure of strict non-determinism of Kähler action allowing in zero energy ontology to add endlessly details to the space-time sheets representing zero energy states in shorter scale.

Generalized quantum gravitational holography

The original naive belief was that the construction of the configuration space geometry reduces to $\delta H = \delta M^4_+ \times CP_2$. An analogous idea in string model context became later known as quantum gravitational holography. The basic implication of the vacuum degeneracy is classical non-determinism, which is expected to reflect itself as the properties of the Kähler function and configuration space geometry. Obviously classical non-determinism challenges the notion of quantum gravitational holography.

The hope was that a generalization of the notion of 3-surface is enough to get rid of the degeneracy and save quantum gravitational holography in its simplest form. This would mean that one just replaces space-like 3-surfaces with "association sequences" consisting of sequences of space-like 3-surfaces with time like separations as causal determinants. This would mean that the absolute minima of Kähler function would become degenerate: same space-like 3-surface at δH would correspond to several association sequences with the same value of Kähler function.

The life turned out to be more complex than this. CP_2 type extremals have Euclidian signature of the induced metric and therefore CP_2 type extremals glued to space-time sheet with Minkowskian signature of the induced metric are surrounded by light like surfaces X^3_l, which might be called elementary particle horizons. The non-determinism of the CP_2 type extremals suggests strongly that also elementary particle horizons behave non-deterministically and must be regarded as causal determinants having time like projection in M^4_+. Pieces of CP_2 type extremals are good candidates for the wormhole contacts connecting a space-time sheet to a larger space-time sheet and are also surrounded by an elementary particle horizons and non-determinism is also now present. That this non-determinism would allow the proposed simple description seems highly implausible.

Zero energy ontology realized in terms of a hierarchy of CDs seems to provide the most plausible treatment of the non-determinism and has indeed led to a breakthrough in the construction and understanding of quantum TGD. At the level of generalized Feynman diagrams sub-CDs containing zero energy states represent a hierarchy of radiative corrections so that the classical determinism is direct correlate for the quantum non-determinism. Determinism makes sense only when one has specified the length scale of measurement resolution. One can always add a CD containing a vacuum extremal to get a new zero energy state and a preferred extremal containing more details.

Classical non-determinism saves the notion of time

Although classical non-determinism represents a formidable mathematical challenge it is a must for several reasons. Quantum classical correspondence, which has become a basic guide line in the development of TGD, states that all quantum phenomena have classical space-time correlates. This is not new as far as properties of quantum states are considered. What is new that also quantum jumps and quantum jump sequences which define conscious existence in TGD Universe, should have classical space-time correlates: somewhat like written language is correlate for the contents of consciousness of the writer. Classical non-determinism indeed makes this possible. Classical non-determinism makes also possible the realization of statistical ensembles as ensembles formed by strictly deterministic pieces of the space-time sheet so that even thermodynamics has space-time representations. Space-time surface can thus be seen as symbolic representations for the quantum existence.

In canonically quantized general relativity the loss of time is fundamental problem. If quantum gravitational holography would work in the most strict sense, time would be lost also in TGD since all relevant information about quantum states would be determined by the moment of big bang. More precisely, geometro-temporal localization for the contents of conscious experience would not be possible. Classical non-determinism together with quantum-classical correspondence however suggests that it is possible to have quantum jumps in which non-determinism is concentrated in space-time region so that also conscious experience contains information about this region only.

2.4.2 Four-dimensional General Coordinate Invariance

The proposed definition of the Kähler function is consistent with GCI and implies also 4-dimensional Diff degeneracy of the Kähler metric. Zero energy ontology inspires strengthening of the GCI in the sense that space-like 3-surfaces at the boundaries of CD are physically equivalent with the light-like 3-surfaces connecting the ends. This imples that basic geometric objects are partonic 2-surfaces at the boundaries of CDs identified as the intersections of these two kinds of surfaces. Besides this the distribution of 4-D tangent planes at partonic 2-surfaces would code for physics so that one would have only effective 2-dimensionality. The failure of the non-determinism of Kähler action in the standard sense of the word affects the situation also and one must allow a fractal hierarchy of CDs inside CDs having interpretation in terms of radiative corrections.

Resolution of tachyon difficulty and absence of Diff anomalies

In TGD as in string models the tachyon difficulty is potentially present: unless the time like vibrational excitations possess zero norm they contribute tachyonic term to the mass squared operator of Super Kac Moody algebra. This difficulty is familiar already from string models [63, 61].

The degeneracy of the metric with respect to the time like vibrational excitations guarantees that time like excitations do not contribute to the mass squared operator so that mass spectrum is tachyon free. It also implies the decoupling of the tachyons from physical states: the propagator of the theory corresponds essentially to the inverse of the Kähler metric and therefore decouples from time like vibrational excitations. The experience with string model suggests that if metric is degenerate with respect to diffeomorphisms of $X^4(X^3)$ there are indeed good hopes that time like excitations possess vanishing norm with respect to configuration space metric.

The four-dimensional Diff invariance of the Kähler function implies that Diff invariance is guaranteed in the strong sense since the scalar product of two Diff vector fields given by the matrix associated with $(1,1)$ part of the second variation of the Kähler action vanishes identically. This property gives hopes of obtaining theory, which is free from Diff anomalies: in fact loop space metric is not Diff degenerate and this might be the underlying reason to the problems encountered in string models [63, 61].

Complexification of the configuration space

Strong form of GCI plays a fundamental role in the complexification of the configuration space. GCI in strong form reduces the basic building brick of WCW to the pairs of partonic 2-surfaces and their 4-D tangent space data associated with ends of light-like 3-surface at light-like boundaries of CD. At boths end the imbedding space is effectively reduces to $\delta M^4_+ \times CP_2$ (forgetting the complications due to non-determinism of Kähler action). Light cone boundary in turn is metrically 2-dimensional Euclidian sphere allowing infinite-dimensional group of conformal symmetries and Kähler structure. Therefore one can say that in certain sense configuration space metric inherits the Kähler structure of $S^2 \times CP_2$. This mechanism works in case of four-dimensional Minkowski space only: higher-dimensional spheres do not possess even Kähler structure. In fact, it turns out that the quantum fluctuating degrees of freedom can be regarded in well-defined sense as a local variant of $S^2 \times CP_2$ and thus as an infinite-dimensional analog of symmetric space as the considerations of [22] demonstrate.

The details of the complexification were understood only after the construction of configuration space geometry and spinor structure in terms of second quantized induced spinor fields [18]. This also allows to make detailed statements about complexification [22].

Contravariant metric and Diff^4 degeneracy

Diff degeneracy implies that the definition of the contravariant metric, which corresponds to the propagator associated to small deformations of minimizing surface is not quite straightforward. We believe that this problem is only technical. Certainly this problem is not new, being encountered in both GRT and gauge theories [45, 44]. In TGD a solution of the problem is provided by the existence of infinite-dimensional isometry group. If the generators of this group form a complete set in the sense that any vector of the tangent space is expressible as as sum of these generators plus some zero norm vector fields then one can restrict the consideration to this subspace and in this subspace the matrix $g(X, Y)$ defined by the components of the metric tensor indeed indeed possesses well defined inverse

$g^{-1}(X,Y)$. This procedure is analogous to gauge fixing conditions in gauge theories and coordinate fixing conditions in General Relativity.

It has turned that the representability of WCW as a union of symmetric spaces makes possible an approach to WCW integration based on harmonic analysis replacing the perturbative approach based on perturbative functional integral. This approach allows also a p-adic variant and leads an effective discretization in terms of discrete variants of WCW for which the points of symmetric space consist of algebraic points. There is an infinite number of these discretizations [21] and the interpretation is in terms of finite measurement resolution. This gives a connection with the p-adicization program, infinite primes, inclusions of hyper-finite factors as representation of the finite measurement resolution, and the hierarchy of Planck constants [19] so that various approaches to quantum TGD converge nicely.

General Coordinate Invariance and WCW spinor fields

GCI applies also at the level of quantum states. WCW spinor fields are Diff^4 invariant. This in fact fixes not only classical but also quantum dynamics completely. The point is that the values of the configuration space spinor fields must be essentially same for all Diff^4 related 3-surfaces at the orbit X^4 associated with a given 3-surface. This would mean that the time development of Diff^4 invariant configuration spinor field is completely determined by its initial value at the moment of the big bang!

This is of course a naive over statement. The non-determinism of Kähler action and zero energy ontology force to take the causal diamond (CD) defined by the intersection of future and past directed light-cones as the basic structural unit of configuration space, and there is fractal hierarchy of CDs within CDs so that the above statement makes sense only for giving CD in measurement resolution neglecting the presence of smaller CDs. Strong form of GCI also implies factorization of WCW spinor fields into a sum of products associated with various partonic 2-surfaces. In particular, one obtains time-like entanglement between positive and negative energy parts of zero energy states and entanglement coefficients define what can be identified as M-matrix expressible as a "complex square root" of density matrix and reducing to a product of positive definite diagonal square root of density matrix and unitary S-matrix. The collection of orthonormal M-matrices in turn define unitary U-matrix between zero energy states. M-matrix is the basic object measured in particle physics laboratory.

2.4.3 Configuration space geometry, generalized catastrophe theory, and phase transitions

The definition of configuration space geometry has nice catastrophe theoretic interpretation. To understand the connection consider first the definition of the ordinary catastrophe theory [36].

1. In catastrophe theory one considers extrema of the potential function depending on dynamical variables x as function of external parameters c. The basic space decomposes locally into cartesian product $E = C \times X$ of control variables c, appearing as parameters in potential function $V(c,x)$ and of state variables x appearing as dynamical variables. Equilibrium states of the system correspond to the extrema of the potential $V(x,c)$ with respect to the variables x and in the absence of symmetries they form a sub-manifold of M with dimension equal to that of the parameter space C. In some regions of C there are several extrema of potential function and the extremum value of x as a function of c is multi-valued. These regions of $C \times X$ are referred to as catastrophes. The simplest example is cusp catastrophe (see Fig. 2.4.3) with two control parameters and one state variable.

2. In catastrophe regions the actual equilibrium state must be selected by some additional physical requirement. If system obeys flow dynamics defined by first order differential equations the catastrophic jumps take place along the folds of the cusp catastrophe (delay rule). On the other hand, the Maxwell rule obeyed by thermodynamic phase transitions states that the equilibrium state corresponds to the absolute minimum of the potential function and the state of system changes in discontinuous manner along the Maxwell line in the middle between the folds of the cusp (see Fig. 2.4.3).

3. As far as discontinuous behavior is considered fold catastrophe is the basic catastrophe: all catastrophes contain folds as there 'satellites' and one aim of the catastrophe theory is to derive

all possible manners for the stable organization of folds into higher catastrophes. The fundamental result of the catastrophe theory is that for dimensions d of C smaller than 5 there are only 7 basic catastrophes and polynomial potential functions provide a canonical representation for the catastrophes: fold catastrophe corresponds to third order polynomial (in fold the two real roots become a pair of complex conjugate roots), cusp to fourth order polynomial, etc.

Consider now the TGD counterpart of this. TGD allows allows two kinds of catastrophe theories.

1. The first one is related to Kähler action as a local functional of 4-surface. The nature of this catastrophe theory depends on what one means with the preferred extremals.

2. Second catastrophe theory corresponds to Kähler function a non-local functional of 3-surface. The maxima of the vacuum functional defined as the exponent of Kähler function define what might called effective space-times, and discontinuous jumps changing the values of the parameters characterizing the maxima are possible.

Consider first the option based on Kähler action.

1. Potential function corresponds to Kähler action restricted to the solutions of Euler Lagrange equations. Catastrophe surface corresponds to the four-surfaces found by extremizing Kähler action with respect to to the variables of X (time derivatives of coordinates of C specifying X^3 in H_a) keeping the variables of C specifying 3-surface X^3 fixed. Preferred extremal property is analogous to the Bohr quantization since canonical momenta cannot be chosen freely as in the ordinary initial value problems of the classical physics. Preferred extremals are by definition at criticality. Behavior variables correspond to the deformations of the 4-surface keeping partonic 2-surfaces and 3-D tangent space data fixed and preserving extremal property. Control variables would correspond to these data.

2. At criticality the rank of the infinite-dimensional matrix defined by the second functional derivatives of the Kähler action is reduced. Catastrophes form a hierarchy characterized by the reduction of the rank of this matrix and Thom's catastrophe theory generalizes to infinite-dimensional context. Criticality in this sense would be one aspect of quantum criticality having also other aspects. No discrete jumps would occur and system would only move along the critical surface becoming more or less critical.

3. There can exist however several critical extremals assignable to a given partonic 2-surface but have nothing to do with the catastrophes as defined in Thom's approach. In presence of degeneracy one should be able to choose one of the critical extremals or replace this kind of regions of WCW by their multiple coverings so that single partonic 2-surface is replaced with its multiple copy. The degeneracy of the preferred extremals could be actually a deeper reason for the hierarchy of Planck constants involving in its most plausible version n-fold singular coverings of CD and CP_2. This interpretation is very satisfactory since the generalization of the imbedding space and hierarchy of Planck constants would follow naturally from quantum criticality rather than as separate hypothesis.

4. The existence of the catastrophes is implied by the vacuum degeneracy of the Kähler action. For example, for pieces of Minkowski space in $M^4_+ \times CP_2$ the second variation of the Kähler action vanishes identically and only the fourth variation is non-vanishing: these 4-surfaces are analogous to the tip of the cusp catastrophe. There are also space-time surfaces for which the second variation is non-vanishing but degenerate and a hierarchy of subsets in the space of extremal 4-surfaces with decreasing degeneracy of the second variation defines the boundaries of the projection of the catastrophe surface to the space of 3-surfaces. The space-times for which second variation is degenerate contain as subset the critical and initial value sensitive absolute minimum space-times.

Consider next the catastrophe theory defined by Kähler function.

1. In this case the most obvious identification for the behavior variables would be in terms of the space of all 3-surfaces in $CD \times CP_2$ - and if one believes in holography and zero energy ontology - the 2-surfaces assignable the boundaries of causal diamonds (CDs).

2. The natural control variables are zero modes whereas behavior variables would correspond to quantum fluctuating degrees of freedom contributing to the configuration space metric. The induced Kähler form at partonic 2-surface would define infinitude of purely classical control variables. There is also a correlation between zero modes identified as degrees of freedom assignable to the interior of 3-surface and quantum fluctuating degrees of freedom assigned to the partonic 2-surfaces. This is nothing but holography and effective 2-dimensionality justifying the basic assumption of quantum measurement theory about the correspondence between classical and quantum variables. The absence of several maxima implies also the presence of saddle surfaces at which the rank of the matrix defined by the second derivatives is reduced. This could lead to a non-positive definite metric. It seems that it is possible to have maxima of Kähler function without losing positive definiteness of the metric since metric is defined as (1,1)-type derivatives with respect to complex coordinates. In case of CP_2 however Kähler function has single degenerate maximum corresponding to the homologically trivial geodesic sphere at $r = \infty$. It might happen that also in the case of infinite-D symmetric space finite maxima are impossible.

3. The criticality of Kähler function would be analogous to thermodynamical criticality and to the criticality in the sense of catastrophe theory. In this case Maxwell's rule is possible and even plausible since quantum jump replaces the dynamics defined by a continuous flow.

Cusp catastrophe provides a simple concretization of the situation for the criticality of Kähler action (as distinguished from that for Kähler function).

1. The set M of the critical 4-surfaces corresponds to the V-shaped boundary of the 2-D cusp catastrophe in 3-D space to plane. In general case it forms codimension one set in configuration space. In TGD Universe physical system would reside at this line or its generalization to higher dimensional catastrophes. For the criticality associated with Kähler action the transitions would be smooth transitions between different criticalities characterized by the rank defined above: in the case of cusp from the tip of cusp to the vertex of cusp or vice versa. Evolution could mean a gradual increase of criticality in this sense. If preferred extremals are not unique, cusp catastrophe does not provide any analogy. The strong form of criticality would mean that the system would be always "at the tip of cusp" in metaphoric sense. Vacuum extremals are maximally critical in trivial sense, and the deformations of vacuum extremals could define the hierarchy of criticalities.

2. For the criticality of Kähler action Maxwell's rule stating that discontinuous jumps occur along the middle line of the cusp is in conflict with catastrophe theory predicting that jumps occurs along at criticality. For the criticality of Kähler function -if allowed at all by symmetric space property- Maxwell's rule can hold true but cannot be regarded as a fundamental law. It is of course known that phase transitions can occur in different manners (super heating and super cooling).

Figure 2.1: Cusp catastrophe

2.5 Weak form electric-magnetic duality and its implications

The notion of electric-magnetic duality [51] was proposed first by Olive and Montonen and is central in $\mathcal{N}=4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [22]. What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2,-1,-1)$ and could be proportional to color hyper charge.

3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects in electro-weak scale: this should become manifest at LHC energies.

4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found. The solution ansatz applies also to the extremals of Chern-Simons action and and to the conserved currents associated with the modified Dirac equation defined as contractions of the modified gamma matrices between the solutions of the modified Dirac equation. The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

6. The general solution ansatz works for induced Kähler Dirac equation and Chern-Simons Dirac equation and reduces them to ordinary differential equations along flow lines. The induced spinor fields are simply constant along flow lines of indued spinor field for Dirac equation in suitable gauge. Also the generalized eigen modes of the modified Chern-Simons Dirac operator can be deduced explicitly if the throats and the ends of space-time surface at the boundaries of CD are extremals of Chern-Simons action. Chern-Simons Dirac equation reduces to ordinary differential equations along flow lines and one can deduce the general form of the spectrum and the explicit representation of the Dirac determinant in terms of geometric quantities characterizing the 3-surface (eigenvalues are inversely proportional to the lengths of strands of the flow lines in the effective metric defined by the modified gamma matrices).

2.5.1 Could a weak form of electric-magnetic duality hold true?

Holography means that the initial data at the partonic 2-surfaces should fix the configuration space metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} \ . \tag{2.5.1}$$

A more general form of this duality is suggested by the considerations of [34] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [56] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} \ . \tag{2.5.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the configuration space metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1+K)J \ , \tag{2.5.3}$$

where J can denotes the Kähler magnetic flux, makes it possible to have a non-trivial configuration space metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [39] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \tag{2.5.4}$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \quad (2.5.5)$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \quad (2.5.6)$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi} \times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \quad (2.5.7)$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the modified Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [25] supports this interpretation.

3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

The weak form of electric-magnetic duality has surprisingly strong implications for basic view about quantum TGD as following considerations show.

2.5.2 Magnetic confinement, the short range of weak forces, and color confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L \overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L \nu_R$. $\nu_L \overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hyper-charge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four Gaussian Mersennes corresponding to $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [65].

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [23]. The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_{\pm}$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats

are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

2. The addition of the particles $X^{\pm}$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

3. How should one describe the bound state formed by the fermion and $X^{\pm}$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [38]. If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [31].

Should $J + J_1$ appear in Kähler action?

The presence of the S^2 Kähler form J_1 in the weak form of electric-magnetic duality was originally suggested by an erratic argument about the reduction to almost topological QFT to be described in the next subsection. In any case this argument raises the question whether one could replace J with $J + J_1$ in the Kähler action. This would not affect the basic non-vacuum extremals but would modify the vacuum degeneracy of the Kähler action. Canonically imbedded M^4 would become a monopole configuration with an infinite magnetic energy and Kähler action due to the monopole singularity at the line connecting tips of the CD. Action and energy can be made small by drilling a small hole around origin. This is however not consistent with the weak form of electro-weak duality. Amusingly, the modified Dirac equation reduces to ordinary massless Dirac equation in M^4.

This extremal can be transformed to a vacuum extremal by assuming that the solution is also a CP_2 magnetic monopole with opposite contribution to the magnetic charge so that $J + J_1 = 0$ holds true. This is achieved if one can regard space-time surface as a map $M^4 \to CP_2$ reducing to a map $(\Theta, \Phi) = (\theta, \pm\phi)$ with the sign chosen by properly projecting the homologically non-trivial $r_M = constant$ spheres of CD to the homologically non-trivial geodesic sphere of CP_2. Symplectic transformations of $S^2 \times CP_2$ produce new vacuum extremals of this kind. Using Darboux coordinates in which one has $J = \sum_{k=1,2} P_k dQ^k$ and assuming that (P_1, Q_1) corresponds to the CP_2 image of S^2, one can take Q_2 to be arbitrary function of P^2, which in turn is an arbitrary function of M^4 coordinates to obtain even more general vacuum extremals with 3-D CP_2 projection. Therefore the spectrum of vacuum extremals, which is very relevant for the TGD based description of gravitation in long length scales because it allows to satisfy Einstein's equations as an additional condition, looks much richer than for the original option, and it is natural to ask whether this option might make sense.

An objection is that J_1 is a radial monopole field and this breaks Lorentz invariance to $SO(3)$. Lorentz invariance is broken to $SO(3)$ for a given CD also by the presence of the preferred time direction defined by the time-like line connecting the tips of the CD becoming carrying the monopole charge but is compensated since Lorentz boosts of CDs are possible. Could one consider similar compensation also now? Certainly the extremely small breaking of Lorentz invariance and the vanishing of the monopole charge for the vacuum extremals is all that is needed at the space-time level. No new gauge fields would be introduced since only the Kähler field part of photon and Z^0 boson would receive an additional contribution.

The ultimate fate of the modification depends on whether it is consistent with the general relativistic description of gravitation. Since a breaking of spherical symmetry is involved, it is not at all clear whether one can find vacuum extremals which represent small deformations of the Reissner-Nordström metric and Robertson-Walker metric. The argument below shows that this option does not allow the imbedding of small deformations of physically plausible space-time metrics as vacuum extremals.

The basic vacuum extremal whose deformations should give vacuum extremals allowing interpretation as solutions of Einstein's equations is given by a map $M^4 \rightarrow CP_2$ projecting the r_M constant spheres S^2 of M^2 to the homologically non-trivial geodesic sphere of CP_2. The winding number of this map is -1 in order to achieve vanishing of the induced Kähler form $J + J_1$. For instance, the following two canonical forms of the map are possible

$$\begin{aligned} (\Theta, \Psi) &= (\theta_M, -\phi_M) \ , \\ (\Theta, \Psi) &= (\pi - \theta_M, \phi_M) \ . \end{aligned} \tag{2.5.8}$$

Here (Θ, Ψ) refers to the geodesic sphere of CP_2 and (θ_M, ϕ_M) to the sphere of M^4.
The resulting space-time surface is not flat and Einstein tensor is non-vanishing. More complex metrics can be constructed from this metric by a deformation making the CP_2 projection 3-dimensional.

Using the expression of the CP_2 line element in Eguchi-Hanson coordinates [41]

$$\frac{ds^2}{R^2} = \frac{dr^2}{F^2} + \frac{r^2}{F}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + fracr^2 4F sin^2\Theta d\Phi^2) \tag{2.5.9}$$

and s the relationship $r = tan(\Theta)$, one obtains following expression for the CP_2 metric

$$\frac{ds^2}{R^2} = d\theta_M^2 + sin^2(\theta_M)\left[(d\phi_M + cos(\theta)d\Phi)^2 + \frac{1}{4}(d\theta^2 + sin^2(\theta)d\Phi^2\right] \ . \tag{2.5.10}$$

The resulting metric is obtained from the metric of S^2 by replacing $d\phi^2$ which 3-D line element. The factor $sin^2(\theta_M)$ implies that the induced metric becomes singular at North and South poles of S^2. In particular, the gravitational potential is proportional to $sin^2(\theta_M)$ so that gravitational force in the radial direction vanishes at equators. It is very difficult to imagine any manner to produce a small deformation of Reissner-Nordstrm metric or Robertson-Walker metric. Hence it seems that the vacuum extremals produce by $J + J_1$ option are not physical.

2.5.3 Could Quantum TGD reduce to almost topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the modified Dirac action action. I gave up this proposal but the following argument shows that

Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^\alpha A_\alpha$ plus and integral of the boundary term $J^{n\beta}A_\beta\sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta}A_\beta\sqrt{g_4}$ over the ends of the 3-surface.

2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K\epsilon^{n\beta\gamma\delta}J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K\epsilon^{0\beta\gamma\delta}J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $\hbar_0 \rightarrow r\hbar_0$ would effectively describe this. Boundary conditions would however give $1/r$ factor so that $\hbar$ would disappear from the Kähler function! The original attempt to realize quantum TGD as an almost topological QFT was in terms of Chern-Simons action but was given up. It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [33]) so that the Coulombic term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

2. The original naive conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha(J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma}J_{\beta\gamma})\sqrt{g_4}d^3x \ . \tag{2.5.11}$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.

3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta}K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^\alpha \partial_\alpha \phi = -j^\alpha A_\alpha \ . \tag{2.5.12}$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j_K^\alpha$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j_\beta^K \partial_\gamma j_\delta^K \quad = \quad 0 \ . \tag{2.5.13}$$

j_K is a four-dimensional counterpart of Beltrami field [55] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [33]. The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^\alpha \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j_K^\alpha \phi$ and $j_I^\alpha \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha \phi) \quad = \quad 0 \ . \tag{2.5.14}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_\phi^e = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the imons type term is trivial if the total weighted Kähler magnetic flux $Q_\phi^m = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler potential couples to the modified

Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function. Kähler function is identified as a Dirac determinant for Chern-Simons Dirac action and the spectrum of this operator should not be invariant under these gauge transformations if this picture is correct. This is is achieved if the gauge transformation is carried only in the Dirac action corresponding to the Chern-Simons term: this assumption is motivated by the breaking of time reversal invariance induced by quantum measurements. The modification of Kähler action can be guessed to correspond just to the Chern-Simons contribution from the instanton term.

7. A reasonable looking guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q_ϕ^m and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

2.5.4 A general solution ansatz based on almost topological QFT property

The basic vision behind the ansatz is the reduction of quantum TGD to almost topological field theory. This requires that the flow parameters associated with the flow lines of isometry currents and Kähler current extend to global coordinates. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when weak electro-weak duality is applied as boundary conditions. The strongest form of the hydrodynamical interpretation requires that all conserved currents are parallel to Kähler current. In the more general case one would have several hydrodynamic flows. Also the braidings (several of them for the most general ansatz) assigned with the light-like 3-surfaces are naturally defined by the flow lines of conserved currents. The independent behavior of particles at different flow lines can be seen as a realization of the complete integrability of the theory. In free quantum field theories on mass shell Fourier components are in a similar role but the geometric interpretation in terms of flow is of course lacking. This picture should generalize also to the solution of the modified Dirac equation.

Basic field equations

Consider first the equations at general level.

1. The breaking of the Poincare symmetry due to the presence of monopole field occurs and leads to the isometry group $T\times SO(3)\times SU(3)$ corresponding to time translations, rotations, and color group. The Cartan algebra is four-dimensional and field equations reduce to the conservation laws of energy E, angular momentum J, color isospin I_3, and color hypercharge Y.

2. Quite generally, one can write the field equations as conservation laws for I, J, I_3, and Y.

$$D_\alpha\left[D_\beta(J^{\alpha\beta}H_A) - j_K^\alpha H^A + T^{\alpha\beta} j_A^l h_{kl}\partial_\beta h^l\right] \quad = \quad 0 \ . \qquad (2.5.15)$$

The first term gives a contraction of the symmetric Ricci tensor with antisymmetric Kähler form and vanishes so that one has

$$D_\alpha \left[j_K^\alpha H^A - T^{\alpha\beta} j_A^k h_{kl} \partial_\beta h^l \right] = 0 \ . \tag{2.5.16}$$

For energy one has $H_A = 1$ and energy current associated with the flow lines is proportional to the Kähler current. Its divergence vanishes identically.

3. One can express the divergence of the term involving energy momentum tensor as as sum of terms involving $j_K^\alpha J_{\alpha\beta}$ and contraction of second fundamental form with energy momentum tensor so that one obtains

$$j_K^\alpha D_\alpha H^A = j_K^\alpha J_\alpha{}^\beta j_\beta^A + T^{\alpha\beta} H_{\alpha\beta}^k j_k^A \ . \tag{2.5.17}$$

Hydrodynamical solution ansatz

The characteristic feature of the solution ansatz would be the reduction of the dynamics to hydrodynamics analogous to that for a continuous distribution of particles iniatially at the end of X^3 of the light-like 3-surface moving along flow lines defined by currents j_A satisfying the integrability condition $j_A \wedge dj_A = 0$. Field theory would reduce effectively to particle mechanics along flow lines with conserved charges defined by various isometry currents. The strongest condition is that all isometry currents j_A and also Kähler current j_K are proportional to the same current j. The more general option corresponds to multi-hydrodynamics.

1. Solution ansatz

Conserved currents are analogous to hydrodynamical currents in the sense that the flow parameter along flow lines extends to a global space-time coordinate. The conserved current is proportional to the gradient $\nabla\Phi$ of the coordinate varying along the flow lines: $J = \Psi\nabla\Phi$ and by a proper choice of Ψ one can allow to have conservation. The initial values of Ψ and Φ can be selected freely along the flow lines beginning from either the end of the space-time surface or from wormhole throats.

If one requires hydrodynamics also for Chern-Simons action (effective 2-dimensionality is required for preferred extremals), the initial values of scalar functions can be chosen freely only at the partonic 2-surfaces. The freedom to chose the intial values of the charges conserved along flow lines at the partonic 2-surfaces means the existence of an infinite number of conserved charges so that the theory would be integrable and even in two different coordinate directions. The basic difference as compared to ordinary conservation laws is that the conserved currents are parallel and their flow parameter extends to a global coordinate.

1. The most general assumption is that the conserved isometry currents

$$J_A^\alpha = j_K^\alpha H^A - T^{\alpha\beta} j_A^k h_{kl} \partial_\beta h^l \tag{2.5.18}$$

and Kähler current as well as instanto current are integrable in the sense that $J_A \wedge J_A = 0$ and $j_K \wedge j_K = 0$ hold true. One could imagine the possibility that the currents are not parallel. If instanton current and Kähler current are proportional to each other, Coulomb interaction term in the Kähler action vanishes and almost topological QFT property is achieved.

2. The integrability condition $dJ_A \wedge J_A = 0$ is satisfied if one one has

$$J_A = \Psi_A d\Phi_A \ . \tag{2.5.19}$$

The ansatz allows a gauge transformation induced by a symplectic transformation of S^2.Φ_A is same for Kähler current and instanton current.

3. The conservation of J_A gives

$$d*(\Psi_A d\Phi_A) = 0 \ . \tag{2.5.20}$$

This would mean separate hydrodynamics for each of the currents involved. In principle there is not need to assume any further conditions and one can imagine infinite basis of scalar function pairs (Ψ_A, Φ_A) since criticality implies infinite number deformations implying conserved Noether currents.

4. The conservation condition reduces to d'Alembert equation in the induced metric if one assumes that $\nabla\Psi_A$ is orthogonal with every $d\Phi_A$.

$$d*d\Phi_A = 0 \ , \ d\Psi_A\cdot d\Phi_A = 0 \ . \tag{2.5.21}$$

Taking $x = \Phi_A$ as a coordinate the orthogonality condition states $g^{xj}\partial_j\Psi_A = 0$ and in the general case one cannot solve the condition by simply assuming that Ψ_A depends on the coordinates transversal to Φ_A only. These conditions bring in mind $p\cdot p = 0$ and $p\cdot e$ condition for massless modes of Maxwell field having fixed momentum and polarization. $d\Phi_A$ would correspond to p and $d\Psi_A$ to polarization. The condition that each isometry current corresponds its own pair (Ψ_A, Φ_A) would mean that each isometry current corresponds to independent light-like momentum and polarization. Ordinary free quantum field theory would support this view whereas hydrodynamics and QFT limit of TGD would support single flow.

These are the most general hydrodynamical conditions that one can assume. One can consider also more restricted scenarios.

1. The strongest ansatz is inspired by the hydrodynamical picture in which all conserved isometry charges flow along same flow lines so that one would have

$$J_A = \Psi_A d\Phi \ . \tag{2.5.22}$$

In this case same Φ would satisfy simultaneously the d'Alembert type equations.

$$d*d\Phi = 0 \ , \ d\Psi_A\cdot d\Phi = 0. \tag{2.5.23}$$

This would mean that the massless modes associated with isometry currents move in parallel manner but can have different polarizations. The spinor modes associated with light-light like 3-surfaces carry parallel four-momenta, which suggest that this option is correct. This allows a very general family of solutions and one can have a complete 3-dimensional basis of functions Ψ_A with gradient orthogonal to $d\Phi$.

2. Isometry invariance under $T\times SO(3)\times SU(3)$ allows to consider the possibility that one has

$$J_A = k_A\Psi_A d\Phi_{G(A)} \ , \ d*(d\Phi_G(A)) = 0 \ , \ d\Psi_A\cdot d\Phi_G(A)) = 0 \ . \tag{2.5.24}$$

where $G(A)$ is T for energy current, $SO(3)$ for angular momentum currents and $SU(3)$ for color currents. Energy would thus flow along its own flux lines, angular momentum along its own flow lines, and color quantum numbers along their own flow lines. For instance, color currents would differ from each other only by a numerical constant. The replacement of Ψ_A with $\Psi_{G(A)}$ would be too strong a condition since Killing vector fields are not related by a constant factor.

To sum up, the most general option is that each conserved current J_A defines its own integrable flow lines defined by the scalar function pair (Ψ_A, Φ_A). A complete basis of scalar functions satisfying the d'Alembert type equation guaranteing current conservation could be imagined with restrictions coming from the effective 2-dimensionality reducing the scalar function basis effectively to the partonic 2-surface. The diametrically opposite option corresponds to the basis obtained by assuming that only single Φ is involved. The ansatz does not distinguish between J and $J+J_1$ options.

The proposed solution ansatz can be compared to the earlier ansatz [34] stating that Kähler current is topologized in the sense that for $D(CP_2)=3$ it is proportional to the identically conserved instanton current (so that 4-D Lorentz force vanishes) and vanishes for $D(CP_2)=4$ (Maxwell phase). This hypothesis requires that instanton current is Beltrami field for $D(CP_2)=3$. In the recent case the assumption that also instanton current satisfies the Beltrami hypothesis in strong sense (single function Φ) generalizes the topologization hypothesis for $D(CP_2)=3$ and guarantees that Coulomb term in Kähler action vanishes identically. A weaker form is obtained by replacing Kähler potential by its gauge transform in which case one also obtains a boundary term. As a matter fact, the topologization hypothesis applies to isometry currents also for $D(CP_2)=4$ although instanton current is not conserved anymore. One can consider variants of instanton current since both (A_1, J_1) and (A, J) are available.

Can one require the extremal property in the case of Chern-Simons action?

Effective 2-dimensionality is achieved if the ends and wormhole throats are extremals of Chern-Simons action. The strongest condition would be that space-time surfaces allow orthogonal slicings by 3-surfaces which are extremals of Chern-Simons action.

Also in this case one can require that the flow parameter associated with the flow lines of the isometry currents extends to a global coordinate. Kähler magnetic field $B=*J$ defines a conserved current so that all conserved currents would flow along the field lines of B and one would have 3-D Beltrami flow. Note that in magnetohydrodynamics the standard assumption is that currents flow along the field lines of the magnetic field.

For wormhole throats light-likeness causes some complications since the induced metric is degenerate and the contravariant metric must be restricted to the complement of the light-like direction. This means that d'Alembert equation reduces to 2-dimensional Laplace equation. For space-like 3-surfaces one obtains the counterpart of Laplace equation with partonic 2-surfaces serving as sources. The interpretation in terms of analogs of Coulomb potentials created by 2-D charge distributions would be natural.

If $J+J1$ appears in Kähler action the extremals need not have 2-dimensional CP_2 projection as they must have for J option, and one can hope of obtaining large enough solution family consistent with effective 2-dimensionality. The field equations can be reduced to conservation conditions for the isometry currents for $SO(3)\times SU(3)$ along flow lines.

2.5.5 Holomorphic factorization of Kähler function

One can guess the general form of the core part of the Kähler function as function of complex coordinates assignable to the partonic surfaces at positive and negative energy ends of CD. It its convenient to restrict the consideration to the simplest possible non-trivial case which is represented by single propagator line connecting the ends of CD.

1. The propagator line corresponds to a symmetric space defined as a coset space G/H of the symplectic group and Kac-Moody group. This coset space is as a manifold Cartesian product $(G/H)\times(G/H)$ of symmetric spaces G/H associated with ends of the line. Kähler metric contains also an interaction term between the factors of the Cartesian product so that Kähler function can be said to reduce to a sum of "kinetic" terms and interaction term.

2. The exponent of Kähler function depends on both ends of the line and this means that the geometries at the ends are correlated in the sense that that Kähler form contains interaction terms between the line ends. It is however not quite clear whether it contains separate "kinetic" or self interaction terms assignable to the line ends. For Kähler function the kinetic and interaction terms should have the following general expressions as functions of complex WCW coordinates:

$$\begin{aligned} K_{kin,i} &= \sum_n f_{i,n}(Z_i)\overline{f_{i,n}(Z_i)} + c.c \ , \\ K_{int} &= \sum_n g_{1,n}(Z_1)\overline{g_{2,n}(Z_2)} + c.c \ , i=1,2 \ . \end{aligned} \tag{2.5.25}$$

Here $K_{kin,i}$ define "kinetic" terms and K_{int} defines interaction term. One would have what might be called holomorphic factorization suggesting a connection with conformal field theories. K_{kin} would correspond to the Chern-Simons term assignable to the ends of the line and K_{int} to the Chern-Simons terms assignable to the wormhole throats.

2.5.6 Could the dynamics of Kähler action predict the hierarchy of Planck constants?

The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. The formula for the Planck constant involves heuristic guess work and physical plausibility arguments. There are good arguments in favor of the hypothesis that only coverings are possible. Only a finite number of pages of the Big Book correspond to a given value of Planck constant, biological evolution corresponds to a gradual dispersion to the pages of the Big Book with larger Planck constant, and a connection with the hierarchy of infinite primes and p-adicization program based on the mathematical realization of finite measurement resolution emerges.

One can however ask whether this hierarchy could emerge directly from the basic quantum TGD rather than as a separate hypothesis. The following arguments suggest that this might be possible. One finds also a precise geometric interpretation of preferred extremal property interpreted as criticality in zero energy ontology.

1-1 correspondence between canonical momentum densities and time derivatives fails for Kähler action

The basic motivation for the geometrization program was the observation that canonical quantization for TGD fails. To see what is involved let us try to perform a canonical quantization in zero energy ontology at the 3-D surfaces located at the light-like boundaries of $CD \times CP_2$.

1. In canonical quantization canonical momentum densities $\pi^0_k \equiv \pi_k = \partial L_K/\partial(\partial_0 h^k)$, where $\partial_0 h^k$ denotes the time derivative of imbedding space coordinate, are the physically natural quantities in terms of which to fix the initial values: once their value distribution is fixed also conserved charges are fixed. Also the weak form of electric-magnetic duality given by $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and a mild generalization of this condition to be discussed below can be interpreted as a manner to fix the values of conserved gauge charges (not Noether charges) to their quantized values since Kähler magnetic flux equals to the integer giving the homology class of the (wormhole) throat. This condition alone need not characterize criticality, which requires an infinite number of deformations of X^4 for which the second variation of the Kähler action vanishes and implies infinite number conserved charges. This in fact gives hopes of replacing π_k with these conserved Noether charges.

2. Canonical quantization requires that $\partial_0 h^k$ in the energy is expressed in terms of π_k. The equation defining π_k in terms of $\partial_0 h^k$ is however highly non-linear although algebraic. By taking squares the equations reduces to equations for rational functions of $\partial_0 h^k$. $\partial_0 h^k$ appears in contravariant and covariant metric at most quadratically and in the induced Kähler electric field linearly and by multplying the equations by $det(g_4)^3$ one can transform the equations to a polynomial form so that in principle $\partial_0 h^k$ can obtained as a solution of polynomial equations.

3. One can always eliminate one half of the coordinates by choosing 4 imbedding space coordinates as the coordinates of the spacetime surface so that the initial value conditions reduce to those for the canonical momentum densities associated with the remaining four coordinates. For instance, for space-time surfaces representable as map $M^4 \rightarrow CP_2$ M^4 coordinates are natural and the time derivatives $\partial_0 s^k$ of CP_2 coordinates are multivalued. One would obtain four polynomial equations with $\partial_0 s^k$ as unknowns. In regions where CP_2 projection is 4-dimensional -in particular for the deformations of CP_2 vacuum extremals the natural coordinates are CP_2 coordinates and one can regard $\partial_0 m^k$ as unknows. For the deformations of cosmic strings, which are of form $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, one can use coordinates of $M^2 \times S^2$, where S^2 is geodesic sphere as natural coordinates and regard as unknowns E^2 coordinates and remaining CP_2 coordinates.

4. One can imagine solving one of the four polynomials equations for time derivaties in terms of other obtaining N roots. Then one would substitute these roots to the remaining 3 conditions to obtain algebraic equations from which one solves then second variable. Obviously situation is very complex without additional symmetries. The criticality of the preferred extremals might however give additional conditions allowing simplifications. The reasons for giving up the canonical quantization program was following. For the vacuum extremals of Kähler action π_k are however identically vanishing and this means that there is an infinite number of value distributions for $\partial_0 h^k$. For small deformations of vacuum extremals one might however hope a finite number of solutions to the conditions and thus finite number of space-time surfaces carrying same conserved charges.

If one assumes that physics is characterized by the values of the conserved charges one must treat the the many-valuedness of $\partial_0 h^k$. The most obvious guess is that one should replace the space of space-like 4-surfaces corresponding to different roots $\partial_0 h^k = F^k(\pi_l)$ with four-surfaces in the covering space of $CD \times CP_2$ corresponding to different branches of the many-valued function $\partial_0 h^k = F(\pi_l)$ co-inciding at the ends of CD.

Do the coverings forces by the many-valuedness of $\partial_0 h^k$ correspond to the coverings associated with the hierarchy of Planck constants?

The obvious question is whether this covering space actually corresponds to the covering spaces associated with the hierarchy of Planck constants. This would conform with quantum classical correspondence. The hierarchy of Planck constants and hierarchy of covering spaces was introduced to cure the failure of the perturbation theory at quantum level. At classical level the multivaluedness of $\partial_0 h^k$ means a failure of perturbative canonical quantization and forces the introduction of the covering spaces. The interpretation would be that when the density of matter becomes critical the space-time surface splits to several branches so that the density at each branches is sub-critical. It is of course not at all obvious whether the proposed structure of the Big Book is really consistent with this hypothesis and one also consider modifications of this structure if necessary. The manner to proceed is by making questions.

1. The proposed picture would give only single integer characterizing the covering. Two integers assignable to CD and CP_2 degrees of freedom are however needed. How these two coverings could emerge?

   (a) One should fix also the values of $\pi_k^n = \partial L_K/\partial h_n^k$, where n refers to space-like normal coordinate at the wormhole throats. If one requires that charges do not flow between regions with different signatures of the metric the natural condition is $\pi_k^n = 0$ and allows also multi-valued solution. Since wormhole throats carry magnetic charge and since weak form of electric-magnetic duality is assumed, one can assume that CP_2 projection is four-dimensional so that one can use CP_2 coordinates and regard $\partial_0 m^k$ as un-knows. The basic idea about topological condensation in turn suggests that M^4 projection can be assumed to be 4-D inside space-like 3-surfaces so that here $\partial_0 s^k$ are the unknowns. At partonic 2-surfaces one would have conditions for both π_k^0 and π_k^n. One might hope that the numbers of solutions are finite for preferred extremals because of their symmetries and given by n_a for $\partial_0 m^k$ and by n_b for $\partial_0 s^k$. The optimistic guess is that n_a and n_b corresponds to the numbers of sheets for singular coverings of CD and CP_2. The covering could be visualized as replacement of space-time surfaces with space-time surfaces which have $n_a n_b$ branches. n_b branches would degenerate to single branch at the ends of diagrams of the generaled Feynman graph and n_a branches would degenerate to single one at wormhole throats.

(b) This picture is not quite correct yet. The fixing of π_k^0 and π_k^n should relate closely to the effective 2-dimensionality as an additional condition perhaps crucial for criticality. One could argue that both π_k^0 and π_k^n must be fixed at X^3 and X_l^3 in order to effectively bring in dynamics in two directions so that X^3 could be interpreted as a an orbit of partonic 2-surface in space-like direction and X_l^3 as its orbit in light-like direction. The additional conditions could be seen as gauge conditions made possible by symplectic and Kac-Moody type conformal symmetries. The conditions for π_0^k would give n_b branches in CP_2 degrees of freedom and the conditions for π_k^n would split each of these branches to n_a branches.

(c) The existence of these two kinds of conserved charges (possibly vanishing for π_k^n) could relate also very closely to the slicing of the space-time sheets by string world sheets and partonic 2-surfaces.

2. Should one then treat these branches as separate space-time surfaces or as a single space-time surface? The treatment as a single surface seems to be the correct thing to do. Classically the conserved changes would be $n_a n_b$ times larger than for single branch. Kähler action need not (but could!) be same for different branches but the total action is $n_a n_b$ times the average action and this effectively corresponds to the replacement of the $\hbar_0/g_K^2$ factor of the action with $\hbar/g_K^2$, $r \equiv \hbar/\hbar_0 = n_a n_b$. Since the conserved quantum charges are proportional to $\hbar$ one could argue that $r = n_a n_b$ tells only that the charge conserved charge is $n_a n_b$ times larger than without multi-valuedness. $\hbar$ would be only effectively $n_a n_b$ fold. This is of course poor man's argument but might catch something essential about the situation.

3. How could one interpret the condition $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and its generalization to be discussed below in this framework? The first observation is that the total Kähler electric charge is by $\alpha_K \propto 1/(n_a n_b)$ same always. The interpretation would be in terms of charge fractionization meaning that each branch would carry Kähler electric charge $Q_K = n g_K/n_a n_b$. I have indeed suggested explanation of charge fractionization and quantum Hall effect based on this picture.

4. The vision about the hierarchy of Planck constants involves also assumptions about imbedding space metric. The assumption that the M^4 covariant metric is proportional to $\hbar^2$ follows from the physical idea about $\hbar$ scaling of quantum lengths as what Compton length is. One can always introduce scaled M^4 coordinates bringing M^4 metric into the standard form by scaling up the M^4 size of CD. It is not clear whether the scaling up of CD size follows automatically from the proposed scenario. The basic question is why the M^4 size scale of the critical extremals must scale like $n_a n_b$? This should somehow relate to the weak self-duality conditions implying that Kähler field at each branch is reduced by a factor $1/r$ at each branch. Field equations should posses a dynamical symmetry involving the scaling of CD by integer k and $J^{0\beta}\sqrt{g_4}$ and $J^{n\beta}\sqrt{g_4}$ by $1/k$. The scaling of CD should be due to the scaling up of the M^4 time interval during which the branched light-like 3-surface returns back to a non-branched one.

5. The proposed view about hierarchy of Planck constants is that the singular coverings reduce to single-sheeted coverings at $M^2 \subset M^4$ for CD and to $S^2 \subset CP_2$ for CP_2. Here S^2 is any homologically trivial geodesic sphere of CP_2 and has vanishing Kähler form. Weak self-duality condition is indeed consistent with any value of $\hbar$ and impies that the vacuum property for the partonic 2-surface implies vacuum property for the entire space-time sheet as holography indeed requires. This condition however generalizes. In weak self-duality conditions the value of $\hbar$ is free for any 2-D Lagrangian sub-manifold of CP_2.

   The branching along M^2 would mean that the branches of preferred extremals always collapse to single branch when their M^4 projection belongs to M^2. Magnetically charged light-light-like throats cannot have M^4 projection in M^2 so that self-duality conditions for different values of $\hbar$ do not lead to inconsistencies. For spacelike 3-surfaces at the boundaries of CD the condition would mean that the M^4 projection becomes light-like geodesic. Straight cosmic strings would have M^2 as M^4 projection. Also CP_2 type vacuum extremals for which the random light-like projection in M^4 belongs to M^2 would represent this of situation. One can ask whether the degeneration of branches actually takes place along any string like object $X^2 \times Y^2$, where X^2 defines a minimal surface in M^4. For these the weak self-duality condition would imply $\hbar = \infty$ at the ends of the string. It is very plausible that string like objects feed their magnetic fluxes to larger space-times sheets through wormhole contacts so that these conditions are not encountered.

Connection with the criticality of preferred extremals

Also a connection with quantum criticality and the criticality of the preferred extremals suggests itself. Criticality for the preferred extremals must be a property of space-like 3-surfaces and light-like 3-surfaces with degenerate 4-metric and the degeneration of the $n_a n_b$ branches of the space-time surface at the its ends and at wormhole throats is exactly what happens at criticality. For instance, in catastrophe theory roots of the polynomial equation giving extrema of a potential as function of control parameters co-incide at criticality. If this picture is correct the hierarchy of Planck constants would be an outcome of criticality and of preferred extremal property and preferred extremals would be just those multi-branched space-time surfaces for which branches co-incide at the the boundaries of $CD \times CP_2$ and at the throats.

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Chapter 3

Construction of Configuration Space Kähler Geometry from Symmetry Principles

3.1 Introduction

The most general expectation is that configuration space can be regarded as a union of coset spaces which are infinite-dimensional symmetric spaces with Kähler structure: $C(H) = \cup_i G/H(i)$. Index i labels 3-topology and zero modes. The group G, which can depend on 3-surface, can be identified as a subgroup of diffeomorphisms of $\delta M^4_+ \times CP_2$ and H must contain as its subgroup a group, whose action reduces to $Diff(X^3)$ so that these transformations leave 3-surface invariant.

The task is to identify plausible candidate for G and H and to show that the tangent space of the configuration space allows Kähler structure, in other words that the Lie-algebras of G and $H(i)$ allow complexification. One must also identify the zero modes and construct integration measure for the functional integral in these degrees of freedom. Besides this one must deduce information about the explicit form of configuration space metric from symmetry considerations combined with the hypothesis that Kähler function is Kähler action for a preferred extremal of Kähler action. One must of course understand what "preferred" means.

3.1.1 General Coordinate Invariance and generalized quantum gravitational holography

The basic motivation for the construction of configuration space geometry is the vision that physics reduces to the geometry of classical spinor fields in the infinite-dimensional configuration space of 3-surfaces of $M^4_+ \times CP_2$ or of $M^4 \times CP_2$. Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that configuration space possesses Kähler geometry. Kähler geometry is coded into Kähler function.

The original belief was that the four-dimensional general coordinate invariance of Kähler function reduces the construction of the geometry to that for the boundary of configuration space consisting of 3-surfaces on $\delta M^4_+ \times CP_2$, the moment of big bang. The proposal was that Kähler function $K(Y^3)$ could be defined as a preferred extremal of so called Kähler action for the unique space-time surface $X^4(Y^3)$ going through given 3-surface Y^3 at $\delta M^4_+ \times CP_2$. For Diff^4 transforms of Y^3 at $X^4(Y^3)$ Kähler function would have the same value so that Diff^4 invariance and degeneracy would be the outcome. The proposal was that the preferred extremals are absolute minima of Kähler action.

This picture turned out to be too simple.

1. I have already described the recent view about light-like 3-surfaces as generalized Feynman diagrams and space-time surfaces as preferred extremals of Kähler action and will not repeat what has been said.
2. It has also become obvious that the gigantic symmetries associated with $\delta M^4_\pm \times CP_2 \subset CD \times CP_2$ manifest themselves as the properties of propagators and vertices. Cosmological considerations,

Poincare invariance, and the new view about energy favor the decomposition of the configuration space to a union of configuration spaces assignable to causal diamonds CDs defined as intersections of future and past directed light-cones. The minimum assumption is that CDs label the sectors of CH: the nice feature of this option is that the considerations of this chapter restricted to $\delta M^4_+ \times CP_2$ generalize almost trivially. This option is beautiful because the center of mass degrees of freedom associated with the different sectors of CH would correspond to M^4 itself and its Cartesian powers.

The definition of the Kähler function requires that the many-to-one correspondence $X^3 \rightarrow X^4(X^3)$ must be replaced by a bijective correspondence in the sense that X^3_l as light-like 3-surface is unique among all its Diff^4 translates. This also allows physically preferred "gauge fixing" allowing to get rid of the mathematical complications due to Diff4 degeneracy. The internal geometry of the space-time sheet must define the preferred 3-surface X^3_l.

The realization of this vision means a considerable mathematical challenge. The effective metric 2-dimensionality of 3-dimensional light-like surfaces X^3_l of M^4 implies generalized conformal and symplectic symmetries allowing to generalize quantum gravitational holography from light like boundary so that the complexities due to the non-determinism can be taken into account properly.

3.1.2 Light like 3-D causal determinants and effective 2-dimensionality

The light like 3-surfaces X^3_l of space-time surface appear as 3-D causal determinants. Basic examples are boundaries and elementary particle horizons at which Minkowskian signature of the induced metric transforms to Euclidian one. This brings in a second conformal symmetry related to the metric 2-dimensionality of the 3-D light-like 3-surface. This symmetry is identifiable as TGD counterpart of the Kac Moody symmetry of string models. The challenge is to understand the relationship of this symmetry to configuration space geometry and the interaction between the two conformal symmetries.

1. Field-particle duality is realized. Light-like 3-surfaces X^3_l -generalized Feynman diagrams - correspond to the particle aspect of field-particle duality whereas the physics in the interior of space-time surface $X^4(X^3_l)$ would correspond to the field aspect. Generalized Feynman diagrams in 4-D sense could be identified as regions of space-time surface having Euclidian signature.

2. One could also say that light-like 3-surfaces X^3_l and the space-like 3-surfaces X^3 in the intersections of $X^4(X^3_l) \cap CD \times CP_2$ where the causal diamond CD is defined as the intersections of future and past directed light-cones provide dual descriptions.

3. Generalized coset construction implies that the differences of super-symplectic and Super Kac-Moody type Super Virasoro generators annihilated physical states. This implies Equivalence Principle. This construction in turn led to the realization that configuration space for fixed values of zero modes - in particular the values of the induced Kähler form of $\delta M^4_\pm \times CP_2$ - allows identification as a coset space obtained by dividing the symplectic group of $\delta M^4_\pm \times CP_2$ with Kac-Moody group, whose generators vanish at $X^2 = X^3_l \times \delta M^4_\pm \times CP_2$. One can say that quantum fluctuating degrees of freedom in a very concrete sense correspond to the local variant of $S^2 \times CP_2$.

The analog of conformal invariance in the light-like direction of X^3_l and in the light-like radial direction of $\delta M^4_\pm$ implies that the data at either X^3 or X^3_l should be enough to determine configuration space geometry. This implies that the relevant data is contained to their intersection X^2 at least for finite regions of X^3. This is the case if the deformations of X^3_l not affecting X^2 and preserving light likeness corresponding to zero modes or gauge degrees of freedom and induce deformations of X^3 also acting as zero modes. The outcome is effective 2-dimensionality. One must be however cautious in order to not make over-statements. The reduction to 2-D theory in global sense would trivialize the theory and the reduction to 2-D theory must takes places for finite region of X^3 only so one has in well defined sense three-dimensionality in discrete sense. A more precise formulation of this vision is in terms of hierarchy of CDs containing CDs containing.... The introduction of sub-CD:s brings in improved measurement resolution and means also that effective 2-dimensionality is realized in the scale of sub-CD only.

One cannot over-emphasize the importance of the effective 2-dimensionality. It indeed simplifies dramatically the earlier formulas for configuration space metric involving 3-dimensional integrals over

$X^3 \subset M^4_+ \times CP_2$ reducing now to 2-dimensional integrals. Note that X^3 is determined by preferred extremal property of $X^4(X^3_l)$ once X^3_l is fixed and one can hope that this mapping is one-to-one.

3.1.3 Magic properties of light cone boundary and isometries of configuration space

The special conformal, metric and symplectic properties of the light cone of four-dimensional Minkowski space: δM^4_+, the boundary of four-dimensional light cone is metrically 2-dimensional(!) sphere allowing infinite-dimensional group of conformal transformations and isometries(!) as well as Kähler structure. Kähler structure is not unique: possible Kähler structures of light cone boundary are paramet3rized by Lobatchevski space $SO(3,1)/SO(3)$. The requirement that the isotropy group $SO(3)$ of S^2 corresponds to the isotropy group of the unique classical 3-momentum assigned to $X^4(Y^3)$ defined as a preferred extremum of Kähler action, fixes the choice of the complex structure uniquely. Therefore group theoretical approach and the approach based on Kähler action complement each other.

1. The allowance of an infinite-dimensional group of isometries isomorphic to the group of conformal transformations of 2-sphere is completely unique feature of the 4-dimensional light cone boundary. Even more, in case of $\delta M^4_+ \times CP_2$ the isometry group of δM^4_+ becomes localized with respect to CP_2! Furthermore, the Kähler structure of δM^4_+ defines also symplectic structure.

   Hence any function of $\delta M^4_+ \times CP_2$ would serve as a Hamiltonian transformation acting in both CP_2 and δM^4_+ degrees of freedom. These transformations obviously differ from ordinary local gauge transformations. This group leaves the symplectic form of $\delta M^4_+ \times CP_2$, defined as the sum of light cone and CP_2 symplectic forms, invariant. The group of symplectic transformations of $\delta M^4_+ \times CP_2$ is a good candidate for the isometry group of the configuration space.

2. The approximate symplectic invariance of Kähler action is broken only by gravitational effects and is exact for vacuum extremals. If Kähler function were exactly invariant under the symplectic transformations of CP_2, CP_2 symplectic transformations wiykd correspond to zero modes having zero norm in the Kähler metric of configuration space. This does not make sense since symplectic transformations of $\delta M^4 \times CP_2$ actually parameterize the quantum fluctuation degrees of freedom.

3. The groups G and H, and thus configuration space itself, should inherit the complex structure of the light cone boundary. The diffeomorphims of M^4 act as dynamical symmetries of vacuum extremals. The radial Virasoro localized with respect to $S^2 \times CP_2$ could in turn act in zero modes perhaps inducing conformal transformations: note that these transformations lead out from the symmetric space associated with given values of zero modes.

3.1.4 Symplectic transformations of $\delta M^4_+ \times CP_2$ as isometries of configuration space

The symplectic transformations of $\delta M^4_+ \times CP_2$ are excellent candidates for inducing symplectic transformations of the configuration space acting as isometries. There are however deep differences with respect to the Kac Moody algebras.

1. The conformal algebra of the configuration space is gigantic when compared with the Virasoro + Kac Moody algebras of string models as is clear from the fact that the Lie-algebra generator of a symplectic transformation of $\delta M^4_+ \times CP_2$ corresponding to a Hamiltonian which is product of functions defined in δM^4_+ and CP_2 is sum of generator of δM^4_+-local symplectic transformation of CP_2 and CP_2-local symplectic transformations of δM^4_+. This means also that the notion of local gauge transformation generalizes.

2. The physical interpretation is also quite different: the relevant quantum numbers label the unitary representations of Lorentz group and color group, and the four-momentum labeling the states of Kac Moody representations is not present. Physical states carrying no energy and momentum at quantum level are predicted. The appearance of a new kind of angular momentum not assignable to elementary particles might shed some light to the longstanding problem of baryonic spin (quarks are not responsible for the entire spin of proton). The possibility of a new kind of color might have implications even in macroscopic length scales.

3. The central extension induced from the natural central extension associated with $\delta M^4_+ \times CP_2$ Poisson brackets is anti-symmetric with respect to the generators of the symplectic algebra rather than symmetric as in the case of Kac Moody algebras associated with loop spaces. At first this seems to mean a dramatic difference. For instance, in the case of CP_2 symplectic transformations localized with respect to δM^4_+ the central extension would vanish for Cartan algebra, which means a profound physical difference. For $\delta M^4_+ \times CP_2$ symplectic algebra a generalization of the Kac Moody type structure however emerges naturally.

   The point is that δM^4_+-local CP_2 symplectic transformations are accompanied by CP_2 local δM^4_+ symplectic transformations. Therefore the Poisson bracket of two δM^4_+ local CP_2 Hamiltonians involves a term analogous to a central extension term symmetric with respect to CP_2 Hamiltonians, and resulting from the δM^4_+ bracket of functions multiplying the Hamiltonians. This additional term could give the entire bracket of the configuration space Hamiltonians at the maximum of the Kähler function where one expects that CP_2 Hamiltonians vanish and have a form essentially identical with Kac Moody central extension because it is indeed symmetric with respect to indices of the symplectic group.

3.1.5 Does the symmetric space property reduce to coset construction for Super Virasoro algebras?

The idea about symmetric space is extremely beautiful but it took a long time and several false alarms before the time was ripe for identifying the precise form of the Cartan decomposition $g = t + h$ satisfying the defining conditions

$$g = t + h \ , \quad [t,t] \subset h \ , \quad [h,t] \subset t \ . \tag{3.1.1}$$

The ultimate solution of the puzzle turned out to be amazingly simple and came only after quantum TGD was understood well enough.

Configuration space geometry allows two super-conformal symmetries. The first one corresponds to super-symplectic transformations acting at the level of imbedding space. The second one corresponds to super Kac-Moody symmetry acting as deformations of light-like 3-surfaces respecting their light-likeness. Super Kac-Moody algebra can be regarded as sub-algebra of super-symplectic algebra, and quantum states correspond to the coset representations for these two algebras so that the differences of the corresponding super-Virasoro generators annihilate physical states. This obviously generalizes Goddard-Olive-Kent construction [48]. The physical interpretation is in terms of Equivalence Principle. After having realized this it took still some time to realize that this coset representation and therefore also Equivalence Principle also corresponds to the coset structure of the configuration space!

The conclusion would be that t corresponds to super-symplectic algebra made also local with respect to X^3 and h corresponds to super Kac-Moody algebra. The experience with finite-dimensional coset spaces would suggest that super Kac-Moody generators interpreted in terms of h leave the points of configuration space analogous to the origin of say CP_2 invariant and in fact vanish at this point. Therefore super Kac-Moody generators should vanish for those 3-surfaces X^3_l which correspond to the origin of coset space. The maxima of Kähler function could correspond to this kind of points and could play also an essential role in the integration over configuration space by generalizing the Gaussian integration of free quantum field theories.

3.1.6 What effective 2-dimensionality and holography really mean?

Concerning the interpretation of Kac-Moody algebra there are some poorly understood points, which directly relate to what one means with holography.

1. The strongest view about effective 2-dimensionality (holography) is that for preferred extremals the partonic 2-surfaces X^2 at the ends of CD act as causal determinants fixing X^3_l in the resolution defined by CD. A weaker view about holography is that light-like 3-surfaces with fixed ends give rise to same configuration space metric and the deformations of these surfaces by Kac-Moody algebra correspond to zero modes just like the interior degrees of freedom for space-like 3-surface do. Which of these options is the correct one? The same question can be posed in the case of space-like 3-surfaces.

2. The non-trivial action of Kac-Moody algebra in the interior of X_l^3 together with effective 2-dimensionality and holography would encourage the interpretation of Kac-Moody symmetries acting trivially at X^2 as gauge symmetries. Light-like 3-surfaces having fixed partonic 2-surfaces at their ends would be equivalent physically and effective 2-dimensionality and holography would be realized modulo gauge transformations.

3. There are also Kac-Moody generators which do not vanish at the ends of the X_l^3, and these would act as physical symmetries and their action would reduce at X^2 to symplectic action. This Kac-Moody algebra should appear in p-adic mass calculations. This seems to be in conflict with the idea that coset construction corresponds to coset space construction. Perhaps strict correspondence is too naive an assumption. Why couldn't one use the larger Kac-Moody algebra in coset construction and smaller Kac-Moody algebra in coset space construction?

4. Gauge symmetry property means that the Kähler metric of the configuration space is same for all gauge equivalent choices of X_l^3 and Kac-Moody deformations correspond to zero modes. Kähler function could differ by a real part of a holomorphic function of configuration space coordinates representing now Kac-Moody transforms of X_l^3. If Dirac determinant gives the exponent of Kähler function, the eigenvalues of the modified Dirac action can differ only by scalings with are products of holomorphic function of configuration space coordinates and its conjugates labeling different Kac-Moody transforms of X_l^3. This condition makes sense if one restricts the consideration to the finite number of eigenvalues λ_k assigned to D_K. The introduction of instanton term transforming the eigenvalues to $\lambda_k + \sqrt{n}$ would not allow his scaling.

   Either one must assume more general spectrum of form $\lambda_k + \sqrt{n}x_k$ with λ_k and x_k scaling in identical manner or that $n = 0$ modes are enough to define Kähler function. The latter option might be correct since the preferred extremal realizes effective 2-dimensionality at space-time level and conformal excitations break it so that they should not contribute to Kähler function. Also number theoretic universality favors this option. One cannot however exclude the first option. It must be admitted that the situation is not completely understood.

3.1.7 About the relationship between super-symplectic and super Kac-Moody algebras

The relationship between Kac-Moody and symplectic algebras is now relatively well understood but the physical interpretation of Kac-Moody algebra deserves attention. There are two Kac-Moody algebras: the smaller one leaves partonic 2-surfaces invariant and second one affects also them. Both of them are in dual relation to the symplectic algebra and these relations correspond to coset space construction and coset construction.

TGD inspired quantum measurement theory suggests that the super-symplectic algebra and smaller Kac-Moody algebra correspond to each other like classical and quantal degrees of freedom. Hence smaller Kac-Moody algebra would act in the zero modes of the configuration space metric. In the proposed construction this indeed is the case for Kac Moody algebra elements leaving partonic 2-surface invariant and appearing in the *coset space construction* but not for those Kac-Moody algebra elements affecting partonic 2-surface and allowing interpretation as sub-algebra of symplectic algebra and appearing in *coset construction*. This interpretation conforms also with the fact that Kac-Moody algebra generates massive excitations in p-adic thermodynamics.

In TGD inspired quantum measurement theory zero modes correspond to classical non-quantum fluctuating dynamical variables in 1-1 correspondence with quantum fluctuating degrees of freedom like the positions of the pointer of the measurement apparatus with the directions of spin of electron. Hence Kac-Moody algebra would define configuration space coordinates in terms of the map induced by correlation between classical and quantal degrees of freedom induced by entanglement. The choice of gauge selecting one particular light-like 3-surface X_l^3 could have thus interpretation as a map mapping quantum degrees of freedom to classical ones. This choice of gauge could be achieved by the addition of phase factor depending on quantum numbers assigned with the braid strands so that stationary phase approximation would select the preferred 3-surface with fluctuations around them allowed.

The dual relation between super symplectic algebra and bigger Kac-Moody algebra is realized in terms of coset construction. The idea inspired by Olive-Goddard-Kent coset construction is that the generators of Super Virasoro algebra corresponds to the differences of those associated with Super Kac-Moody and super-symplectic algebras. The justification comes from the miraculous geometry of the light cone boundary implying that Super Kac-Moody conformal symmetries of X^2 can be compensated by super-symplectic local radial scalings so that the differences of corresponding Super Virasoro generators annihilate physical states. If the central extension parameters are same, the resulting central extension is trivial. What is done is to construct first a state with a non-positive conformal weight using super-symplectic generators, and then to apply Super-Kac Moody generators to compensate this conformal weight to get a state with vanishing conformal weight. Mass squared would however correspond to either Super-Kac Moody or super-symplectic mass. The identity of these masses gives rise to Equivalence Principle as a one manifestation of the coset representation.

3.1.8 Attempts to identify configuration space Hamiltonians

I have made several attempts to identify configuration space Hamiltonians. The first two candidates referred to as magnetic and electric Hamiltonians, emerged in a relatively early stage. The third candidate identifies Hamiltonians as Noether charges and is motivated by the QFT analogy. Magnetic option is the simplest one and the only one consistent with the interpretation of Kac-Moody symmetries leaving the ends of X_l^3 invariant.

Magnetic Hamiltonians

Assuming that the elements of the radial Virasoro algebra of δM_+^4 have zero norm, one ends up with an explicit identification of the symplectic structures of the configuration space. There is almost unique identification for the symplectic structure. Configuration space counterparts of $\delta M^4 \times CP_2$ Hamiltonians are defined by the generalized signed and and unsigned Kähler magnetic fluxes

$$\begin{aligned}
Q_m(H_A, X^2) &= Z \int_{X^2} H_A J \sqrt{g_2} d^2x \ , \\
Q_m^+(H_A, r_M) &= Z \int_{X^2} H_A |J| \sqrt{g_2} d^2x \ , \\
J &\equiv \epsilon^{\alpha\beta} J_{\alpha\beta} \ .
\end{aligned}$$

H_A is CP_2 Hamiltonian multiplied by a function of coordinates of light cone boundary belonging to a unitary representation of the Lorentz group. Z is a conformal factor depending on symplectic invariants. The symplectic structure is induced by the symplectic structure of CP_2.

The most general flux is superposition of signed and unsigned fluxes Q_m and Q_m^+.

$$Q_m^{\alpha,\beta}(H_A, X^2) = \alpha Q_m(H_A, X^2) + \beta Q_m^+(H_A, X^2) \ .$$

Thus it seems that symmetry arguments fix the form of the configuration space metric apart from the presence of a conformal factor Z multiplying the magnetic flux and the degeneracy related to the signed and unsigned fluxes.

Holography requires that the relevant data about configuration space geometry is contained by 2-D surfaces X^2 at the intersections of light-like 3-surfaces $\delta M_\pm^4 + \times CP_2$ defining the boundaries of causal diamonds. In this case the entire Hamiltonian could be defined as the sum of magnetic fluxes over surfaces $X_i^2 \subset X^3$.

The key feature of these Hamiltonians is that they depend on X^2 only. This conforms with the interpretation of Kac-Moody transformations leaving X^2 invariant as gauge symmetries deforming light-like 3-surfaces and leaving configuration space metric as such. By the identify $g_{k\bar{l}} = iJ_{k\bar{l}}$ the half brackets $j^{Ak} J_{k\bar{l}} j^{B\bar{l}} = \partial_k H_A J^{k\bar{l}} \partial_{\bar{l}} H^B$ would define the matrix elements of both Kähler metric and Kähler form: this means a tight constraint if Kähler action defines the metric and magnetic Hamiltonians are the correct choice.

Electric Hamiltonians and electric-magnetic duality

Preferred extremal property allows to consider the possibility that one can identify configuration space Hamiltonians as classical charges $Q_e(H_A)$ associated with the Hamiltonians of the symplectic transformations of the light cone boundary, that is as variational derivatives of the Kähler action with respect to the infinitesimal deformations induced by $\delta M_+^4 \times CP_2$ Hamiltonians.

Alternatively, one might simply replace Kähler magnetic field J with Kähler electric field defined by space-time dual $*J$ in the formulas of previous section. These Hamiltonians are analogous to Kähler electric charge and the hypothesis motivated by the experience with the instantons of the Euclidian Yang Mills theories and 'Yin-Yang' principle, as well as by the duality of CP_2 geometry, is that for the preferred extremals of the Kähler action these Hamiltonians are affinely related:

$$Q_e(H_A) = Z\left[Q_m(H_A) + q_e(H_A)\right] \quad .$$

Here Z and q_e are constants depending on symplectic invariants only. Thus the equivalence of the two approaches to the construction of configuration space geometry boils down to the hypothesis of a physically well motivated electric-magnetic duality.

The crucial technical idea is to regard configuration space metric as a quadratic form in the entire Lie-algebra of the isometry group G such that the matrix elements of the metric vanish in the sub-algebra H of G acting as $Diff^3(X^3)$. The Lie-algebra of G with degenerate metric in the sense that H vector fields possess zero norm, can be regarded as a tangent space basis for the configuration space at point X^3 at which H acts as an isotropy group: at other points of the configuration space H is different. For given values of zero modes the maximum of Kähler function is the best candidate for X^3. This picture applies also in symplectic degrees of freedom.

There are objections against electric representation.

1. Without additional assumptions the Hamiltonians obtained by replacing induced Kähler form with its dual brings in the dependence on the induced metric of space-time surface at X^2 so that configuration space Hamiltonians do not transform nicely under symplectic transformations. Only if the contravariant Kähler electric field defines a symplectic invariant - maybe the preferred extremal property could guarantee this- electric representation of the Hamiltonians looks attractive. Electric-magnetic duality would follow trivially if the self duality of the induced Kähler form of CP_2 is preserved in the induction procedure at X^2.

2. Kac-Moody transformations vanishing at X^2 are not expected to leave the Hamiltonians invariant since they affect the induced metric. This is however highly desirable if effective 2-dimensionality holds true as gauge invariance.

3.1.9 For the reader

Few words about the representation of ideas are in order. For a long time the books about TGD served as kind of lab note books - a bottom-up representation providing kind of a ladder making clear the evolution of ideas. This led gradually to a rather chaotic situation in which it was difficult for me to control the internal consistency and for the possible reader to distinguish between the big ideas and ad hoc guesses, most of them related to the detailed realization of big visions. Therefore I decided to clean up a lot of the ad hoc stuff. I have also changed the representation so that it is more top-down and tries to achieve over-all views.

There are several visions about what TGD is and I have worked hardly to achieve a fusion of this visions. Hence simple linear representation in which reader climbs to a tree of wisdom is impossible. I must summarize overall view from the beginning and refer to the results deduced in chapters towards the end of the book and also to ideas discussed in other books. For instance, the construction of configuration space spinor structure discussed in the last chapter [18] provides the understanding necessary to make the construction of configuration space geometry more detailed. Also number theoretical vision discussed in another book [6] is necessary. Somehow it seems that a graphic representation emphasizing visually the big picture should be needed to make the representation more comprehensible.

3.2 How to generalize the construction of configuration space geometry to take into account the classical non-determinism?

If the imbedding space were $H_+ = M^4_+ \times CP_2$ and if Kähler action were deterministic, the construction of configuration space geometry reduces to $\delta M^4_+ \times CP_2$. Thus in this limit quantum holography principle [59, 55] would be satisfied also in TGD framework and actually reduce to the general coordinate invariance. The classical non-determinism of Kähler action however means that this construction is not quite enough and the challenge is to generalize the construction.

3.2.1 Quantum holography in the sense of quantum gravity theories

In string theory context quantum holography is more or less synonymous with Maldacena conjecture [59] which (very roughly) states that string theory in Anti-de-Sitter space AdS is equivalent with a conformal field theory at the boundary of AdS. In purely quantum gravitational context [55], quantum holography principle states that quantum gravitational interactions at high energy limit in AdS can be described using a topological field theory reducing to a conformal (and non-gravitational) field theory defined at the *time like* boundary of the AdS. Thus the time like boundary plays the role of a dynamical hologram containing all information about correlation functions of $d+1$ dimensional theory. This reduction also conforms with the fact that black hole entropy is proportional to the horizon area rather than the volume inside horizon.

Holography principle reduces to general coordinate invariance in TGD. If the action principle assigning space-time surface to a given 3-surface X^3 at light cone boundary were completely deterministic, four-dimensional general coordinate invariance would reduce the construction of the configuration geometry for the space of 3-surfaces in $M^4_+ \times CP_2$ to the construction of the geometry at the boundary of the configuration space consisting of 3-surfaces in $\delta M^4_+ \times CP_2$ (moment of big bang). Also the quantum theory would reduce to the boundary of the future light cone.

The classical non-determinism of Kähler action however implies that quantum holography in this strong form fails. This is very desirable from the point of view of both physics and consciousness theory. Classical determinism would also mean that time would be lost in TGD as it is lost in GRT. Classical non-determinism is also absolutely essential for quantum consciousness and makes possible conscious experiences with contents localized into finite time interval despite the fact that quantum jumps occur between configuration space spinor fields defining what I have used to call quantum histories. Classical non-determinism makes it also possible to generalize quantum-classical correspondence in the sense that classical non-determinism at the space-time level provides correlate for quantum non-determinism. The failure of classical determinism is a difficult challenge for the construction of the configuration space geometry. One might however hope that the notion of quantum holography generalizes.

3.2.2 How the classical determinism fails in TGD?

One might hope that determinism in a generalized sense might be achieved by generalizing the notion of 3-surface by allowing unions of space-like 3-surfaces with time like separations with very strong but not complete correlations between the space-like 3-surfaces. In this case the non-determinism would mean that the 3-surfaces Y^3 at light cone boundary correspond to at most enumerable number of preferred extremals $X^4(Y^3)$ of Kähler action so that one would get finite or at most enumerably infinite number of replicas of a given configuration space region and the construction would still reduce to the light cone boundary.

1. This is probably quite too simplistic view. Any 4-surface which has CP_2 projection which belongs to so called Lagrange manifold of CP_2 having by definition vanishing induced Kähler form is vacuum extremal. Thus there is an infinite variety of 6-dimensional sub-manifolds of H for which all extremals of Kähler action are vacua.

2. CP_2 type vacuum extremals are different since they possess non-vanishing Kähler form and Kähler action. They are identifiable as classical counterparts of elementary particles have M^4_+ projection which is a random light like curve (this in fact gives rise to conformal invariance identifiable as counterpart of quaternion conformal invariance). Thus there are good reasons to suspect that classical non-determinism might destroy the dream about complete reduction to the light cone boundary.

3. The wormhole contacts connecting different space-time sheets together can be seen as pieces of CP_2 type extremals and one expects that the non-determinism is still there and that the metrically 2-dimensional elementary particle horizons (light like 3-surfaces of H surrounding wormhole contacts and having time-like M^4_+ projection) might be a crucial element in the understanding of quantum TGD. The non-determinism of CP_2 type extremals is absolutely crucial

for the ordinary elementary particle physics. It seems that the conformal symmetries responsible for the ordinary elementary particle quantum numbers acting in these degrees of freedom do not contribute to the configuration space metric line element.

4. The possibility of space-time sheets with a negative time orientation with ensuing negative sign of classical energy is a further blow against δM^4_+ reductionism. Space-time sheets can be created as pairs of positive and negative energy space-time sheet from vacuum and this forces to modify radically the ontology of physics. Crossing symmetry allows to interpret particle reactions as a creation of zero energy states from vacuum, and the identification of the gravitational energy as the difference between positive and negative energies of matter supports the view that the net inertial (conserved Poincare-) energy of the universe vanishes both in quantal and classical sense. This option resolves unpleasant questions about net conserved quantum numbers of Universe, and provides an elegant interpretation of the vacuum extremals as correlates for systems with vanishing Poincare energy. This option is the only possible alternative from the point of view of TGD inspired cosmology where Robertson-Walker metrics are vacuum extremals with respect to inertial energy. In particular, super-symplectic invariance transforms to a fundamental symmetry of elementary particle physics besides the conformal symmetry associated with 3-D light like causal determinants which means a dramatic departure from string models unless it is somehow equivalent with the super-symplectic symmetry.

The treatment of the non-determinism in a framework in which the prediction of time evolution is seen as initial value problem, seems to be difficult. Also the notion of configuration space becomes a messy concept. Zero energy ontology changes the situation completely. Light-like 3-surfaces become representations of generalized Feynman diagrams and brings in the notion of finite time resolution. One obtains adirect connection with the concepts of quantum field theory with path integral with cutoff replaced with a sum over various preferred extremals with cutoff in time resolution.

3.2.3 The notions of imbedding space, 3-surface, and configuration space

The notions of imbedding space, 3-surface (and 4-surface), and configuration space (world of classical worlds (WCW)) are central to quantum TGD. The original idea was that 3-surfaces are space-like 3-surfaces of $H = M^4 \times CP_2$ or $H = M^4_+ \times CP_2$, and WCW consists of all possible 3-surfaces in H. The basic idea was that the definition of Kähler metric of WCW assigns to each X^3 a unique space-time surface $X^4(X^3)$ allowing in this manner to realize general coordinate invariance. During years these notions have however evolved considerably. Therefore it seems better to begin directly from the recent picture.

The notion of imbedding space

Two generalizations of the notion of imbedding space were forced by number theoretical vision [21, 20, 19].

1. p-Adicization forced to generalize the notion of imbedding space by gluing real and p-adic variants of imbedding space together along rationals and common algebraic numbers. The generalized imbedding space has a book like structure with reals and various p-adic number fields (including their algebraic extensions) representing the pages of the book.

2. With the discovery of zero energy ontology [30, 18] it became clear that the so called causal diamonds (CDs) interpreted as intersections $M^4_+ \cap M^4_-$ of future and past directed light-cones of $M^4 \times CP_2$ define correlates for the quantum states. The position of the "lower" tip of CD characterizes the position of CD in H. If the temporal distance between upper and lower tip of CD is quantized power of 2 multiples of CP_2 length, p-adic length scale hypothesis [25] follows as a consequence. The upper *resp.* lower light-like boundary $\delta M^4_+ \times CP_2$ *resp.* $\delta M^4_- \times CP_2$ of CD can be regarded as the carrier of positive *resp.* negative energy part of the state. All net quantum numbers of states vanish so that everything is creatable from vacuum. Space-time surfaces assignable to zero energy states would would reside inside $CD \times CP_2$s and have their 3-D ends at the light-like boundaries of $CD \times CP_2$. Fractal structure is present in the sense that CDs can contains CDs within CDs, and measurement resolution dictates the length scale below which the sub-CDs are not visible.

3. The realization of the hierarchy of Planck constants [20] led to a further generalization of the notion of imbedding space. Generalized imbedding space is obtained by gluing together Cartesian products of singular coverings and factor spaces of CD and CP_2 to form a book like structure. The particles at different pages of this book behave like dark matter relative to each other. This generalization also brings in the geometric correlate for the selection of quantization axes in the sense that the geometry of the sectors of the generalized imbedding space with non-standard value of Planck constant involves symmetry breaking reducing the isometries to Cartan subalgebra. Roughly speaking, each CD and CP_2 is replaced with a union of CDs and CP_2s corresponding to different choices of quantization axes so that no breaking of Poincare and color symmetries occurs at the level of entire WCW.

4. The construction of quantum theory at partonic level brings in very important delicacies related to the Kähler gauge potential of CP_2. Kähler gauge potential must have what one might call pure gauge parts in M^4 in order that the theory does not reduce to mere topological quantum field theory. Hence the strict Cartesian product structure $M^4 \times CP_2$ breaks down in a delicate manner. These additional gauge components -present also in CP_2- play key role in the model of anyons, charge fractionization, and quantum Hall effect [25].

The notions of 3-surface and space-time surface

The question what one exactly means with 3-surface turned out to be non-trivial.

1. The original identification of 3-surfaces was as arbitrary space-like 3-surfaces subject to Equivalence implied by General Coordinate Invariance. There was a problem related to the realization of General Coordinate Invariance since it was not at all obvious why the preferred extremal $X^4(Y^3)$ for Y^3 at $X^4(X^3)$ and Diff^4 related X^3 should satisfy $X^4(Y^3) = X^4(X^3)$.

2. Much later it became clear that light-like 3-surfaces have unique properties for serving as basic dynamical objects, in particular for realizing the General Coordinate Invariance in 4-D sense (obviously the identification resolves the above mentioned problem) and understanding the conformal symmetries of the theory. On basis of these symmetries light-like 3-surfaces can be regarded as orbits of partonic 2-surfaces so that the theory is locally 2-dimensional. It is however important to emphasize that this indeed holds true only locally. At the level of WCW metric this means that the components of the Kähler form and metric can be expressed in terms of data assignable to 2-D partonic surfaces. It is however essential that information about normal space of the 2-surface is needed.

3. At some stage came the realization that light-like 3-surfaces can have singular topology in the sense that they are analogous to Feynman diagrams. This means that the light-like 3-surfaces representing lines of Feynman diagram can be glued along their 2-D ends playing the role of vertices to form what I call generalized Feynman diagrams. The ends of lines are located at boundaries of sub-CDs. This brings in also a hierarchy of time scales: the increase of the measurement resolution means introduction of sub-CDs containing sub-Feynman diagrams. As the resolution is improved, new sub-Feynman diagrams emerge so that effective 2-D character holds true in discretized sense and in given resolution scale only.

4. A further complication relates to the hierarchy of Planck constants forcing to generalize the notion of imbedding space and also to the fact that for non-standard values of Planck constant there is symmetry breaking due to preferred plane M^2 preferred homologically trivial geodesic sphere of CP_2 having interpretation as geometric correlate for the selection of quantization axis. For given sector of CH this means union over choices of this kind.

The basic vision forced by the generalization of General Coordinate Invariance has been that space-time surfaces correspond to preferred extremals $X^4(X^3)$ of Kähler action and are thus analogous to Bohr orbits. Kähler function $K(X^3)$ defining the Kähler geometry of the world of classical worlds would correspond to the Kähler action for the preferred extremal. The precise identification of the preferred extremals actually has however remained open.

The obvious but rather ad hoc guess motivated by physical intuition was that preferred extremals correspond to the absolute minima of Kähler action for space-time surfaces containing X^3. This choice has some nice implications. For instance, one can develop an argument for the existence of an infinite number of conserved charges. If X^3 is light-like surface- either light-like boundary of X^4 or light-like 3-surface assignable to a wormhole throat at which the induced metric of X^4 changes its signature- this identification circumvents the obvious objections. This option however failed to have a direct analog in the p-adic sectors of the world of classical worlds (WCW). The reason is that minimization does not make sense for the p-adic valued counterpart of Kähler action since it is not even well-defined although the field equations make sense p-adically. Therefore, if absolute minimization makes sense it must have expression as purely algebraic conditions.

Much later number theoretical compactication led to important progress in the understanding of the preferred extremals and the conjectures were consistent with what is known about the known extremals.

1. The conclusion was that one can assign to the 4-D tangent space $T(X^4(X^3_l)) \subset M^8$ a subspace $M^2(x) \subset M^4$ having interpretation as the plane of non-physical polarizations. This in the case that the induced metric has Minkowskian signature. If not, and if co-hyper-quaternionic surface is in question, similar assigned should be possible in normal space. This means a close connection with super string models. Geometrically this would mean that the deformations of 3-surface in the plane of non-physical polarizations would not contribute to the line element of WCW. This is as it must be since complexification does not make sense in M^2 degrees of freedom.

2. In number theoretical framework $M^2(x)$ has interpretation as a preferred hyper-complex subspace of hyper-octonions defined as 8-D subspace of complexified octonions with the property that the metric defined by the octonionic inner product has signature of M^8. The condition $M^2(x) \subset T(X^4(X^3_l)))$ in principle fixes the tangent space at X^3_l, and one has good hopes that the boundary value problem is well-defined and could fix $X^4(X^3)$ at least partially as a preferred extremal of Kähler action. This picture is rather convincing since the choice $M^2(x) \subset M^4$ plays also other important roles.

3. At the level of H the counterpart for the choice of $M^2(x)$ seems to be following. Suppose that $X^4(X^3_l)$ has Minkowskian signature. One can assign to each point of the M^4 projection $P_{M^4}(X^4(X^3_l))$ a sub-space $M^2(x) \subset M^4$ and its complement $E^2(x)$, and the distributions of these planes are integrable and define what I have called Hamilton-Jacobi coordinates which can be assigned to the known extremals of Kähler with Minkowskian signature. This decomposition allows to slice space-time surfaces by string world sheets and their 2-D partonic duals. Also a slicing to 1-D light-like surfaces and their 3-D light-like duals Y^3_l parallel to X^3_l follows under certain conditions on the induced metric of $X^4(X^3_l)$. This decomposition exists for known extremals and has played key role in the recent developments. Physically it means that 4-surface (3-surface) reduces effectively to 3-D (2-D) surface and thus holography at space-time level.

4. The weakest form of number theoretic compactification [20] states that light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^8$, where $X^4(X^3)$ hyper-quaternionic surface in hyper-octonionic M^8 can be mapped to light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^4 \times CP_2$, where $X^4(X^3)$ is now preferred extremum of Kähler action. The natural guess is that $X^4(X^3) \subset M^8$ is a preferred extremal of Kähler action associated with Kähler form of E^4 in the decomposition $M^8 = M^4 \times E^4$, where M^4 corresponds to hyper-quaternions. The conjecture would be that the value of the Kähler action in M^8 is same as in $M^4 \times CP_2$: in fact that 2-surface would have identical induced metric and Kähler form so that this conjecture would follow trivial. $M^8 - H$ duality would in this sense be Kähler isometry.

The study of the modified Dirac equation meant further steps of progress and lead to a rather detailed view about what preferred extremals are.

1. The detailed construction of the generalized eigen modes of the modified Dirac operator D_K associated with Kähler action [18] relies on the vision that the generalized eigenvalues of this operator code for information about preferred extremal of Kähler action. The view about TGD as almost topological QFT is realized if the eigenmodes correspond to the solutions of D_K, which are effectively 3-dimensional. Otherwise almost topological QFT property would require Chern-Simons action alone and this choice is definitely un-physical. The first guess was that the

eigenmodes are restricted to X^3_l and therefore analogous to spinorial shock waves. As I realized that number theoretical compactification requires the slicing of $X^4(X^3_l)$ by light-like 3-surfaces Y^3_l parallel to X^3_l, it became clear that super-conformal gauge invariance with respect to the coordinate labeling the slices is a more natural manner to realized effective 3-dimensionality and guarantees that Y^3_l is gauge equivalent with X^3_l (General Coordinate Invariance).

2. The eigen modes of the modified Dirac operator D_K have the defining property that they are localized in regions of X^3_l, where the induced Kähler gauge field is non-vanishing. This guarantees that the number of generalized eigen modes is finite so that Dirac determinant is also finite and algebraic number if eigenvalues are algebraic numbers, and therefore makes sense also in p-adic context although Kähler action itself does not make sense p-adically.

3. The construction of the configuration space geometry in terms of modified Dirac action strengthens also the boundary conditions to the condition that there exists space-time coordinates in which the induced CP_2 Kähler form and induced metric satisfy the conditions $J_{ni} = 0$, $g_{ni} = 0$ hold at X^3_l. One could say that at X^3_l situation is static both metrically and for the Maxwell field defined by the induced Kähler form.

4. The final step in the rapid evolution of ideas that too place during three months - at least I hope so since I do not want to continue this updating endlessly - was the realization that the introduction of imaginary CP breaking instanton part to the Kähler action is possible and also necessary if one wants a stringy variant of Feynman rules. Imaginary part does not contribute to the configuration space metric. This enriches the spectrum of the modified Dirac operator with an infinite number of conformal excitations breaking the effective 2-dimensionality of 3-surfaces and exact holography. Conformal excitations make possible stringy Feynman diagrammatics [16]. A breaking of effective 3-dimensionality of space-time surface comes through the non-determinism of Kähler action which indeed is the mechanism breaking the effective 2-dimensionality. Dirac determinant can be defined in terms of zeta function regularization using Riemann Zeta. Finite measurement resolution realized in terms of braids defined on basis of purely physical criteria however forces a cutoff in conformal weight and finiteness so that number theoretical universality is not lost.

5. This picture relying crucially on the the slicing of $X^4(X^3)$ did not yet fix the definition of preferred extremals analytically at the level of field equations. The next step of progress was the realization that the requirement that the conservation of the Noether currents associated with the modified Dirac equation requires that the second variation of the Kähler action vanishes. In strongest form this condition would be satisfied for all variations and in weak sense only for those defining dynamical symmetries. The interpretation is as space-time correlate for quantum criticality and the vacuum degeneracy of Kähler action makes the criticality plausible. A generalization of the ideas of the catastrophe theory to infinite-dimensional context results [34]. These conditions make sense also in p-adic context and have a number theoretical universal form.

Although the details of this vision might change it can be defended by its ability to fuse together all great visions about quantum TGD. In the sequel the considerations are restricted to 3-surfaces in $M^4_+ \times CP_2$. The basic outcome is that Kähler metric is expressible using the data at partonic 2-surfaces $X^2 \subset \delta M^4_+ \times CP_2$. The generalization to the actual physical situation requires the replacement of $X^2 \subset \delta M^4_\pm \times CP_2$ with unions of partonic 2-surfaces located at light-like boundaries of CDs and sub-CDs.

The notion of configuration space

From the beginning there was a problem related to the precise definition of the configuration space ("world of classical worlds" (WCW)). Should one regard CH as the space of 3-surfaces of $M^4 \times CP_2$ or $M^4_+ \times CP_2$ or perhaps something more delicate.

1. For a long time I believed that the question "M^4_+ or M^4?" had been settled in favor of M^4_+ by the fact that M^4_+ has interpretation as empty Roberson-Walker cosmology. The huge conformal symmetries assignable to $\delta M^4_+ \times CP_2$ were interpreted as cosmological rather than laboratory symmetries. The work with the conceptual problems related to the notions of energy and time,

and with the symmetries of quantum TGD, however led gradually to the realization that there are strong reasons for considering M^4 instead of M^4_+.

2. With the discovery of zero energy ontology it became clear that the so called causal diamonds (CDs) define excellent candidates for the fundamental building blocks of the configuration space or "world of classical worlds" (WCW). The spaces $CD \times CP_2$ regarded as subsets of H defined the sectors of WCW.

3. This framework allows to realize the huge symmetries of $\delta M^4_\pm \times CP_2$ as isometries of WCW. The gigantic symmetries associated with the $\delta M^4_\pm \times CP_2$ are also laboratory symmetries. Poincare invariance fits very elegantly with the two types of super-conformal symmetries of TGD. The first conformal symmetry corresponds to the light-like surfaces $\delta M^4_\pm \times CP_2$ of the imbedding space representing the upper and lower boundaries of CD. Second conformal symmetry corresponds to light-like 3-surface X^3_l, which can be boundaries of X^4 and light-like surfaces separating space-time regions with different signatures of the induced metric. This symmetry is identifiable as the counterpart of the Kac Moody symmetry of string models.

A rather plausible conclusion is that configuration space (WCW) is a union of configuration spaces associated with the spaces $CD \times CP_2$. CDs can contain CDs within CDs so that a fractal like hierarchy having interpretation in terms of measurement resolution results. Since the complications due to p-adic sectors and hierarchy of Planck constants are not relevant for the basic construction, it reduces to a high degree to a study of a simple special case $\delta M^4_+ \times CP_2$.

A further piece of understanding emerged from the following observations.

1. The induced Kähler form at the partonic 2-surface X^2 - the basic dynamical object if holography is accepted- can be seen as a fundamental symplectic invariant so that the values of $\epsilon^{\alpha\beta}J_{\alpha\beta}$ at X^2 define local symplectic invariants not subject to quantum fluctuations in the sense that they would contribute to the configuration space metric. Hence only induced metric corresponds to quantum fluctuating degrees of freedom at configuration space level and TGD is a genuine theory of gravitation at this level.

2. Configuration space can be divided into slices for which the induced Kähler forms of CP_2 and $\delta M^4_\pm$ at the partonic 2-surfaces X^2 at the light-like boundaries of CDs are fixed. The symplectic group of $\delta M^4_\pm \times CP_2$ parameterizes quantum fluctuating degrees of freedom in given scale (recall the presence of hierarchy of CDs).

3. This leads to the identification of the coset space structure of the sub-configuration space associated with given CD in terms of the generalized coset construction for super-symplectic and super Kac-Moody type algebras (symmetries respecting light-likeness of light-like 3-surfaces). Configuration space in quantum fluctuating degrees of freedom for given values of zero modes can be regarded as being obtained by dividing symplectic group with Kac-Moody group. Equivalently, the local coset space $S^2 \times CP_2$ is in question: this was one of the first ideas about configuration space which I gave up as too naive!

4. Generalized coset construction and coset space structure have very deep physical meaning since they realize Equivalence Principle at quantum level: the identical actions of Super Virasoro generators for super-symplectic and super Kac-Moody algebras implies that inertial and gravitational four-momenta are identical.

3.2.4 The treatment of non-determinism of Kähler action in zero energy ontology

The non-determinism of Kähler action means that the reduction of the construction of the configuration space geometry to the light cone boundary fails. Besides degeneracy of the preferred extrema of Kähler action, the non-determinism should manifest itself as a presence of causal determinants also other than light cone boundary.

One can imagine two kinds of causal determinants.

1. Elementary particle horizons and light-like boundaries $X_l^3 \subset X^4$ of 4-surfaces representing wormhole throats act as causal determinants for the space-time dynamics defined by Kähler action. The boundary values of this dynamics have been already considered.

2. At imbedding space level causal determinants correspond to light like CD forming a fractal hierarchy of CDs within CDs. These causal determinants determine the dynamics of zero energy states having interpretation as pairs of initial and final states in standard quantum theory.

The manner to treat the classical non-determinism would be roughly following.

1. The replacement of space-like 3-surface X^3 with X_l^3 transforms initial value problem for X^3 to a boundary value problem for X_l^3. In principle one can also use the surfaces $X^3 \subset \delta CD \times CP_2$ if X_l^3 fixes $X^4(X_l^3)$ and thus X^3 uniquely. For years an important question was whether both X^3 and X_l^3 contribute separately to the configuration space geometry or whether they provide descriptions, which are in some sense dual. This lead to the notion of 7-3 duality and I even considered the possibility that $\delta M^4_+ \times CP_2$ could be replaced with a more general surface $X_l^3 \times CP_2$ allowing also generalized symplectic and conformal symmetries. 7-3 duality is not a good term since the actual duality actually relates descriptions based on space-like 3-surfaces X^3 and light-like 3-surfaces X_l^3. Hence it seems that the proper place for 7-3 duality is in paper basked.

2. Only Super-Kac-Moody type conformal algebra makes sense in the interior of X_l^3. In the 2-D intersections of X_l^3 with the boundary of causal diamond (CD) defined as intersection of future and past directed light-cones super-symplectic algebra makes sense. This implies effective two-dimensionality which is broken by the non-determinism represented using the hierarchy of CDs meaning that the data from these 2-D surfaces and their normal spaces at boundaries of CDs in various scales determine the configuration space metric.

3. An important question has been whether Kac-Moody and super-symplectic algebras provide descriptions which are dual in some sense. At the level of Super-Virasoro algebras duality seems to be satisfied in the sense of generalized coset construction meaning that the differences of Super Virasoro generators of super-symplectic and super Kac-Moody algebras annihilate physical states. Among other things this means that four-momenta assignable to the two Super Virasoro representations are identical. T he interpretation is in terms of a generalization of Equivalence Principle [18, 30]. This gives also a justification for p-adic thermodynamics applying only to Super Kac-Moody algebra.

4. Light-like 3-surfaces can be regarded also as generalized Feynman diagrams. The finite length resolution mean means also a cutoff in the number of generalized Feynman diagrams and this number remains always finite if the light-like 3-surfaces identifiable as maxima of Kähler function correspond to the diagrams. The finiteness of this number is also essential for number theoretic universality since it guarantees that the elements of M-matrix are algebraic numbers if momenta and other quantum numbers have this property. The introduction of new sub-CDs means also introduction of zero energy states in corresponding time scale.

5. The notion of finite measurement resolution expressed in terms of hierarchy of CDs within CDs is important for the treatment of classical non-determinism. In a given resolution the non-determinism of Kähler action remains invisible below the time scale assigned to the smallest CDs. One could also say that complete non-determinism characterized in terms path integral with cutoff is replaced in TGD framework with the partial failure of classical non-determinism leading to generalized Feynman diagrams. This gives rise to to discrete coupling constant evolution and avoids the mathematical ill-definedness and infinities plaguing path integral formalism since the functional integral over 3-surfaces is well defined.

6. Dirac determinant defining vacuum functional is assumed to correspond to exponent of Kähler action for its preferred extremal. Dirac determinant is defined as a product of finite number of eigenvalues of the transverse part $D_K(X^2)$ of the modified Dirac operator D_K assumed to have decomposition $D_K = D_K(X^2) + D_K(Y^2)$ reflecting the dual slicings of X^4 to string world sheets Y^2 and partonic 2-surfaces X^2. The existence of the slicing is supported by the properties of known extremals of Kähler action and strongly suggested by number theoretical

compactification, and it implies among other things dimensional reduction to Minkowskian string model like theory and its Euclidian equivalent allowing to understand how Equivalence Principle is realized at space-time level. Finite number for the eigenvalues raises even hope that in a given resolution the functional integral reduces to Gaussian integral over a finite-dimensional space of logarithms of eigenvalues.

7. One can ask why Kähler action and playing with all these delicacies related to the failure of complete determinism. After all, one could formally replace Kähler action with 4-volume as in brane models. Space-time surfaces would be minimal surfaces and Dirac operator would be standard Dirac operator for the induced metric. Dirac determinant would however become infinite since the modes would not be anymore analogs of cyclotron states necessarily localized to a finite region of X_l^3. Recall that for Kähler action X_l^3 indeed decomposes into patches inside with induced Kähler form is non-vanishing and Dirac determinant defining the exponent of Kähler function is well-defined and finite without any regularization procedure. Hence Kähler action is completely unique.

3.2.5 Category theory and configuration space geometry

Due the effects caused by the classical non-determinism even classical TGD universes are very far from simple Cartesian clockworks, and the understanding of the general structure of the configuration space is a formidable challenge. Category theory is a branch of mathematics which is basically a theory about universal aspects of mathematical structures. Thus category theoretical thinking might help in disentangling the complexities of the configuration space geometry and the basic ideas of category theory are discussed in this spirit and as an innocent layman. It indeed turns out that the approach makes highly non-trivial predictions.

In zero energy ontology the effects of non-determinism are taken into account in terms of causal diamonds forming a hierarchical fractal structure. One must allow also the unions of CDs, CDs within CDs, and probably also overlapping of CDs, and there are good reasons to expert that CDs and corresponding algebraic structures could define categories. If one does not allow overlapping CDs then set theoretic inclusion map defines a natural arrow. If one allows both unions and intersections then CDs would form a structure analogous to the set of open sets used in set theoretic topology. One could indeed see CDs (or rather their Cartesian products with CP_2) as analogs of open sets in Minkowskian signature.

So called ribbon categories seem to be tailor made for the formulation of quantum TGD and allow to build bridge to topological and conformal field theories. This discussion based on standard ontology. In [17] rather detailed category theoretical constructions are discussed. Important role is played by the notion of operad [46, 53]: this structure can be assigned with both generalized Feynman diagrams and with the hierarchy of symplectic fusion algebras realizing symplectic analogs of the fusion rules of conformal field theories.

3.3 Identification of the symmetries and coset space structure of the configuration space

In this section the identification of the isometry group of the configuration space will be discussed at general level.

3.3.1 Reduction to the light cone boundary

The reduction to the light cone boundary would occur exactly if Kähler action were strictly deterministic. This is not the case but it is possible to generalize the construction at light cone boundary to the general case if causal diamonds define the basic structural units of the configuration space.

Old argument

The identification of the configuration space follows as a consequence of 4-dimensional Diff invariance. The right question to ask is the following one. How could one coordinatize the physical(!) vibrational degrees of freedom for 3-surfaces in Diff^4 invariant manner: coordinates should have same values for all Diff^4 related 3-surfaces belonging to the orbit of X^3? The answer is following:

1. Fix some 3-surface (call it Y^3) on the orbit of X^3 in Diff^4 invariant manner.
2. Use as configuration space coordinates of X^3 and all its diffeomorphs the coordinates parameterizing small deformations of Y^3. This kind of replacement is physically acceptable since metrically the configuration space is equivalent with $Map/Diff^4$.
3. Require that the fixing procedure is Lorentz invariant, where Lorentz transformations in question leave light M^4_+ invariant and thus act as isometries.

The simplest choice of Y^3 is the intersection of the orbit of 3-surface (X^4) with the set $\delta M^4_+ \times CP_2$, where δM^4_+ denotes the boundary of the light cone (moment of big bang):

$$Y^3 \quad = \quad X^4 \cap \delta M^4_+ \times CP_2 \tag{3.3.1}$$

Lorentz invariance allows also the choice $X \times CP_2$, where X corresponds to the hyperboloid $a = \sqrt{(m^0)^2 - r_M^2} = constant$ but only the proposed choice ($a = 0$) leads to a natural complexification in M^4 degrees of freedom. This choice is also cosmologically very natural and completely analogous to the quantum gravitational holography of string theories.

Configuration space has a fiber space structure. Base space consists of 3-surfaces $Y^3 \subset \delta M^4_+ \times CP_2$ and fiber consists of 3-surfaces on the orbit of Y^3 , which are Diff^4 equivalent with Y^3. The distance between the surfaces in the fiber is vanishing in configuration space metric. An elegant manner to avoid difficulties caused by Diff^4 degeneracy in configuration space integration is to *define* integration measure as integral over the reduced configuration space consisting of 3-surfaces Y^3 at the light cone boundary.

Situation is however quite not so simple. The vacuum degeneracy of Kähler action suggests strongly classical non-determinism so that there are several, possibly, infinite number of preferred extremals $X^4(Y^3)$ associated with given Y^3 on light cone boundary. This implies additional degeneracy.

One might hope that the reduced configuration space could be replaced by its covering space so that given Y^3 corresponds to several points of the covering space and configuration space has many-sheeted structure. Obviously the copies of Y^3 have identical geometric properties. Configuration space integral would decompose into a sum of integrals over different sheets of the reduced configuration space. Note that configuration space spinor fields are in general different on different sheets of the reduced configuration space.

Even this is probably not enough: it is quite possible that all light like surfaces of M^4 possessing Hamilton Jacobi structure (and thus interpretable as light fronts) are involved with the construction of the configuration space geometry. Because of their metric two-dimensionality the proposed construction should generalize. This would mean that configuration space geometry has also local laboratory scale aspects and that the general ideas might allow testing.

New version of the argument

This is was the argument for two decades ago. A more elegant formulation would in terms of light-like 3-surfaces connecting the boundaries of causal diamond taken as basic geometric objects and identified as generalized Feynman diagrams so that they are singular as manifolds at the vertices.

If both formulations are required to be correct, the only conclusion is that effective 2-dimensionality must hold true in the scale of given CD. In other words, the intersection $X^2 = X^3_l \cap X^3$ at the boundary of CD is effectively the basic dynamical unit. The failure of strict non-determinism however forces to introduce entire hierarchy of CDs responsible also for coupling constant evolution defined in terms of the measurement resolution identified as the size of the smallest CD present.

3.3.2 Configuration space as a union of symmetric spaces

In finite-dimensional context globally symmetric spaces are of form G/H and connection and curvature are independent of the metric, provided it is left invariant under G. The hope is that same holds true in infinite-dimensional context. The most one can hope of obtaining is the decomposition $C(H) =$

$\cup_i G/H_i$ over orbits of G. One could allow also symmetry breaking in the sense that G and H depend on the orbit: $C(H) = \cup_i G_i/H_i$ but it seems that G can be chosen to be same for all orbits. What is essential is that these groups are infinite-dimensional. The basic properties of the coset space decomposition give very strong constraints on the group H, which certainly contains the subgroup of G, whose action reduces to diffeomorphisms of X^3.

Consequences of the decomposition

If the decomposition to a union of coset spaces indeed occurs, the consequences for the calculability of the theory are enormous since it suffices to find metric and curvature tensor for single representative 3-surface on a given orbit (contravariant form of metric gives propagator in perturbative calculation of matrix elements as functional integrals over the configuration space). The representative surface can be chosen to correspond to the maximum of Kähler function on a given orbit and one obtains perturbation theory around this maximum (Kähler function is not isometry invariant).

The task is to identify the infinite-dimensional groups G and H and to understand the zero mode structure of the configuration space. Almost twenty (seven according to long held belief!) years after the discovery of the candidate for the Kähler function defining the metric, it became finally clear that these identifications follow quite nicely from $Diff^4$ invariance and $Diff^4$ degeneracy as well as special properties of the Kähler action.

The guess (not the first one!) would be following. G corresponds to the symplectic transformations of $\delta M^4_\pm \times CP_2$ leaving the induced Kähler form invariant. If G acts as isometries the values of Kähler form at partonic 2-surfaces (remember effective 2-dimensionality) are zero modes and configuration space allows slicing to symplectic orbits of the partonic 2-surface with fixed induced Kähler form. Quantum fluctuating degrees of freedom would correspond to symplectic group and to the fluctuations of the induced metric. The group H dividing G would in turn correspond to the Kac-Moody symmetries respecting light-likeness of X^3_l and acting in X^3_l but trivially at the partonic 2-surface X^2. This coset structure was originally discovered via coset construction for super Virasoro algebras of super-symplectic and super Kac-Moody algebras and realizes Equivalence Principle at quantum level.

Configuration space isometries as a subgroup of $Diff(\delta M^4_+ \times CP_2)$

The reduction to light cone boundary leads to the identification of the isometry group as some subgroup of for the group G for the diffeomorphisms of $\delta M^4_+ \times CP_2$. These diffeomorphisms indeed act in a natural manner in δCH, the the space of 3-surfaces in $\delta M^4_+ \times CP_2$. Configuration space is expected to decompose to a union of the coset spaces G/H_i, where H_i corresponds to some subgroup of G containing the transformations of G acting as diffeomorphisms for given X^3. Geometrically the vector fields acting as diffeomorphisms of X^3 are tangential to the 3-surface. H_i could depend on the topology of X^3 and since G does not change the topology of 3-surface each 3-topology defines separate orbit of G. Therefore, the union involves sum over all topologies of X^3 plus possibly other 'zero modes'. Different topologies are naturally glued together since singular 3-surfaces intermediate between two 3-topologies correspond to points common to the two sectors with different topologies.

3.3.3 Isometries of configuration space geometry as symplectic transformations of $\delta M^4_+ \times CP_2$

During last decade I have considered several candidates for the group G of isometries of the configuration space as the sub-algebra of the subalgebra of $Diff(\delta M^4_+ \times CP_2)$. To begin with let us write the general decomposition of $diff(\delta M^4_+ \times CP_2)$:

$$diff(\delta M^4_+ \times CP_2) = S(CP_2) \times diff(\delta M^4_+) \oplus S(\delta M^4_+) \times diff(CP_2) \ . \tag{3.3.2}$$

Here $S(X)$ denotes the scalar function basis of space X. This Lie-algebra is the direct sum of light cone diffeomorphisms made local with respect to CP_2 and CP_2 diffeomorphisms made local with respect to light cone boundary.

The idea that entire diffeomorphism group would act as isometries looks unrealistic since the theory should be more or less equivalent with topological field theory in this case. Consider now the various candidates for G.

1. The fact that symplectic transformations of CP_2 and M^4_+ diffeomorphisms are dynamical symmetries of the vacuum extremals suggests the possibility that the diffeomorphisms of the light cone boundary and symplectic transformations of CP_2 could leave Kähler function invariant and thus correspond to zero modes. The symplectic transformations of CP_2 localized with respect to light cone boundary acting as symplectic transformations of CP_2 have interpretation as local color transformations and are a good candidate for the isometries. The fact that local color transformations are not even approximate symmetries of Kähler action is not a problem: if they were exact symmetries, Kähler function would be invariant and zero modes would be in question.

2. CP_2 local conformal transformations of the light cone boundary act as isometries of δM^4_+. Besides this there is a huge group of the symplectic symmetries of $\delta M^4_+ \times CP_2$ if light cone boundary is provided with the symplectic structure. Both groups must be considered as candidates for groups of isometries. $\delta M^4_+ \times CP_2$ option exploits fully the special properties of $\delta M^4_+ \times CP_2$, and one can develop simple argument demonstrating that $\delta M^4_+ \times CP_2$ symplectic invariance is the correct option. Also the construction of configuration space gamma matrices as super-symplectic charges supports $\delta M^4_+ \times CP_2$ option.

This picture remained same for a long time. The discovery that Kac-Moody algebra consisting of X^2 local symmetries generated by Hamiltonians of isometry sub-algebra of symplectic algebra forced to challenge this picture and ask whether also X^2-local transformations of symplectic group could be involved.

1. The basic condition is that the X^2 local transformation acts leaves induced Kähler form invariant apart from diffeomorphism. Denote the infinitesimal generator of X^2 local symplecto morphism by $\Phi_A(x)j^{Ak}$, where A labels Hamiltonians in the sum and by j^α the generator of X^2 diffeomorphism.

2. The invariance of $J = \epsilon^{\alpha\beta}J_{\alpha\beta}\sqrt{g_2}$ modulo diffeomorphism under the infinitesimal symplectic transformation gives

$$\{H^A, \Phi_A\} \equiv \partial_\alpha H^A \epsilon^{\alpha\beta}\partial_\beta \Phi_A = \partial_\alpha J j^\alpha \quad . \tag{3.3.3}$$

3. Note that here the Poisson bracket is not defined by $J^\alpha\beta$ but $\epsilon^{\alpha\beta}$ defined by the induced metric. Left hand side reflects the failure of symplectomorphism property due to the dependence of $\Phi_A(x)$ on X^2 coordinate which and comes from the gradients of $\delta M^4 \times CP_2$ coordinates in the expression of the induced Kähler form. Right hand side corresponds to the action of infinitesimal diffeomorphism.

4. Let us assume that one can restrict the consideration to single Hamiltonian so that the transformation is generated by $\Phi(x)H_A$ and that to each $\Phi(x)$ there corresponds a diffeomorphism of X^2, which is a symplectic transformation of X^2 with respect to symplectic form $\epsilon^{\alpha\beta}$ and generated by Hamiltonian $\Psi(x)$. This transforms the invariance condition to

$$\{H^A, \Phi\} \equiv \partial_\alpha H^A \epsilon^{\alpha\beta}\partial_\beta \Phi = \partial_\alpha J \epsilon^{\alpha\beta}\partial_\beta \Psi_A = \{J, \Psi_A\} \quad . \tag{3.3.4}$$

This condition can be solved identically by assuming that Φ_A and Ψ are proportional to arbitrary smooth function of J:

$$\Phi = f(J) \quad , \quad \Psi_A = -f(J)H_A \quad . \tag{3.3.5}$$

Therefore the X^2 local symplectomorphisms of H reduce to symplectic transformations of X^2 with Hamiltonians depending on single coordinate J of X^2. The analogy with conformal invariance for which transformations depend on single coordinate z is obvious. As far as the anti-commutation relations for induced spinor fields are considered this means that $J = consant$ curves behave as points points. For extrema of J appearing as candidates for points of number theoretic braids $J = constant$ curves reduce to points.

5. From the structure of the conditions it is easy to see that the transformations generate a Lie-algebra. For the transformations $\Phi_A^1 H^A$ $\Phi_A^2 H^A$ the commutator is

$$\Phi_A^{[1,2]} = f_A^{\ BC}\Phi_B\Phi_C \ , \tag{3.3.6}$$

where $f_A^{\ BC}$ are the structure constants for the symplectic algebra of $\delta M_\pm^4 \times CP_2$. From this form it is easy to check that Jacobi identifies are satisfied. The commutator has same form as the commutator of gauge algebra generators. BRST gauge symmetry is perhaps the nearest analog of this symmetry. In the case of isometries these transforms realized local color gauge symmetry in TGD sense.

6. If space-time surface allows a slicing to light-like 3-surfaces Y_l^3 parallel to X_l^3, these conditions make sense also for the partonic 2-surfaces defined by the intersections of Y_l^3 with $\delta M_\pm^4 \times CP_2$ and "parallel" to X^2. The local symplectic transformations also generalize to their local variants in X_l^3. Light-likeness of X_l^3 means effective metric 2-dimensionality so that 2-D Kähler metric and symplectic form as well as the invariant $J = \epsilon^{\alpha\beta}J_{\alpha\beta}$ exist. A straightforward calculation shows that the the notion of local symplectic transformation makes sense also now and formulas are exactly the same as above.

3.3.4 Identification of Kac-Moody symmetries

The Kac-Moody algebra of symmetries acting as symmetries respecting the light-likeness of 3-surfaces plays a crucial role in the identification of quantum fluctuating configuration space degrees of freedom contributing to the metric.

Identification of Kac-Moody algebra

The generators of bosonic super Kac-Moody algebra leave the light-likeness condition $\sqrt{g_3} = 0$ invariant. This gives the condition

$$\delta g_{\alpha\beta} Cof(g^{\alpha\beta}) = 0 \ , \tag{3.3.7}$$

Here Cof refers to matrix cofactor of $g_{\alpha\beta}$ and summation over indices is understood. The conditions can be satisfied if the symmetries act as combinations of infinitesimal diffeomorphisms $x^\mu \rightarrow x^\mu + \xi^\mu$ of X^3 and of infinitesimal conformal symmetries of the induced metric

$$\delta g_{\alpha\beta} = \lambda(x)g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha\xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \tag{3.3.8}$$

Ansatz as an X^3-local conformal transformation of imbedding space

Write δh^k as a super-position of X^3-local infinitesimal diffeomorphisms of the imbedding space generated by vector fields $J^A = j^{A,k}\partial_k$:

$$\delta h^k = c_A(x)j^{A,k} \ . \tag{3.3.9}$$

This gives

$$\begin{aligned} c_A(x) \quad & \left[D_k j_l^A + D_l j_k^A\right]\partial_\alpha h^k \partial_\beta h^l + 2\partial_\alpha c_A h_{kl} j^{A,k}\partial_\beta h^l \\ = \quad & \lambda(x)g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha\xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \end{aligned} \tag{3.3.10}$$

If an X^3-local variant of a conformal transformation of the imbedding space is in question, the first term is proportional to the metric since one has

$$D_k j_l^A + D_l j_k^A = 2h_{kl} \ . \tag{3.3.11}$$

The transformations in question includes conformal transformations of $H_\pm$ and isometries of the imbedding space H.

The contribution of the second term must correspond to an infinitesimal diffeomorphism of X^3 reducible to infinitesimal conformal transformation ψ^μ:

$$2\partial_\alpha c_A h_{kl} j^{A,k} \partial_\beta h^l = \xi^\mu \partial_\mu g_{\alpha\beta} + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta \xi^\mu \ . \tag{3.3.12}$$

A rough analysis of the conditions

One could consider a strategy of fixing c_A and solving solving ξ^μ from the differential equations. In order to simplify the situation one could assume that $g_{ir} = g_{rr} = 0$. The possibility to cast the metric in this form is plausible since generic 3-manifold allows coordinates in which the metric is diagonal.

1. The equation for g_{rr} gives

$$\partial_r c_A h_{kl} j^{Ak} \partial_r h^k = 0 \ . \tag{3.3.13}$$

   The radial derivative of the transformation is orthogonal to X^3. No condition on ξ^α results. If c_A has common multiplicative dependence on $c_A = f(r) d_A$ by a one obtains

$$d_A h_{kl} j^{Ak} \partial_r h^k = 0 \ . \tag{3.3.14}$$

   so that J^A is orthogonal to the light-like tangent vector $\partial_r h^k$ X^3 which is the counterpart for the condition that Kac-Moody algebra acts in the transversal degrees of freedom only. The condition also states that the components g_{ri} is not changed in the infinitesimal transformation.

   It is possible to choose $f(r)$ freely so that one can perform the choice $f(r) = r^n$ and the notion of radial conformal weight makes sense. The dependence of c_A on transversal coordinates is constrained by the transversality condition only. In particular, a common scale factor having free dependence on the transversal coordinates is possible meaning that X^3- local conformal transformations of H are in question.

2. The equation for g_{ri} gives

$$\partial_r \xi^i = \partial_r c_A h_{kl} j^{Ak} h^{ij} \partial_j h^k \ . \tag{3.3.15}$$

   The equation states that g_{ri} are not affected by the symmetry. The radial dependence of ξ^i is fixed by this differential equation. No condition on ξ^r results. These conditions imply that the local gauge transformations are dynamical with the light-like radial coordinate r playing the role of the time variable. One should be able to fix the transformation more or less arbitrarily at the partonic 2-surface X^2.

3. The three independent equations for g_{ij} give

$$\xi^\alpha \partial_\alpha g_{ij} + g_{kj}\partial_i \xi^k + g_{ki}\partial_j \xi^k = \partial_i c_A h_{kl} j^{Ak} \partial_j h^l \ . \tag{3.3.16}$$

   These are 3 differential equations for 3 functions ξ^α on 2 independent variables x^i with r appearing as a parameter. Note however that the derivatives of ξ^r do not appear in the equation.

At least formally equations are not over-determined so that solutions should exist for arbitrary choices of c_A as functions of X^3 coordinates satisfying the orthogonality conditions. If this is the case, the Kac-Moody algebra can be regarded as a local algebra in X^3 subject to the orthogonality constraint.

This algebra contains as a subalgebra the analog of Kac-Moody algebra for which all c_A except the one associated with time translation and fixed by the orthogonality condition depends on the radial coordinate r only. The larger algebra decomposes into a direct sum of representations of this algebra.

Commutators of infinitesimal symmetries

The commutators of infinitesimal symmetries need not be what one might expect since the vector fields ξ^μ are functionals c_A and of the induced metric and also c_A depends on induced metric via the orthogonality condition. What this means that $j^{A,k}$ in principle acts also to ϕ_B in the commutator $[c_A J^A, c_B J^B]$.

$$\left[c_A J^A, c_B J^B\right] \quad = \quad c_A c_B J^{[A,B]} + J^A \circ c_B J^B - J^B \circ c_A J^A \quad , \tag{3.3.17}$$

where $\circ$ is a short hand notation for the change of c_B induced by the effect of the conformal transformation J^A on the induced metric.

Luckily, the conditions in the case $g_{rr} = g_{ir} = 0$ state that the components g_{rr} and g_{ir} of the induced metric are unchanged in the transformation so that the condition for c_A resulting from g_{rr} component of the metric is not affected. Also the conditions coming from $g_{ir} = 0$ remain unchanged. Therefore the commutation relations of local algebra apart from constraint from transversality result.

The commutator algebra of infinitesimal symmetries should also close in some sense. The orthogonality to the light-like tangent vector creates here a problem since the commutator does not obviously satisfy this condition automatically. The problem can be solved by following the recipes of non-covariant quantization of string model.

1. Make a choice of gauge by choosing time translation P^0 in a preferred M^4 coordinate frame to be the preferred generator $J^{A_0} \equiv P^0$, whose coefficient $\Phi_{A_0} \equiv \Psi(P^0)$ is solved from the orthogonality condition. This assumption is analogous with the assumption that time coordinate is non-dynamical in the quantization of strings. The natural basis for the algebra is obtained by allowing only a single generator J^A besides P^0 and putting $d_A = 1$.

2. This prescription must be consistent with the well-defined radial conformal weight for the $J^A \neq P^0$ in the sense that the proportionality of d_A to r^n for $J^A \neq P^0$ must be consistent with commutators. SU(3) part of the algebra is of course not a problem. From the Lorentz vector property of P^k it is clear that the commutators resulting in a repeated commutation have well-defined radial conformal weights only if one restricts $SO(3,1)$ to $SO(3)$ commuting with P^0. Also D could be allowed without losing well-defined radial conformal weights but the argument below excludes it. This picture conforms with the earlier identification of the Kac-Moody algebra.

   Conformal algebra contains besides Poincare algebra and the dilation $D = m^k\partial_{m^k}$ the mutually commuting generators $K^k = (m^r m_r \partial_{m^k} - 2m^k m^l \partial_{m^l})/2$. The commutators involving added generators are

   $$\begin{array}{ll} \left[D, K^k\right] = -K^k \ , & \left[D, P^k\right] = P^k \ , \\ \left[K^k, K^l\right] = 0 \ , & \left[K^k, P^l\right] = m^{kl} D - M^{kl} \ . \end{array} \tag{3.3.18}$$

   From the last commutation relation it is clear that the inclusion of K^k would mean loss of well-defined radial conformal weights.

3. The coefficient dm^0/dr of $\Psi(P^0)$ in the equation

   $$\Psi(P^0)\frac{dm^0}{dr} = -J^{Ak} h_{kl} \partial_r h^l$$

is always non-vanishing due to the light-likeness of r. Since P^0 commutes with generators of $SO(3)$ (but not with D so that it is excluded!), one can *define* the commutator of two generators as a commutator of the remaining part and identify $\Psi(P^0)$ from the condition above.

4. Of course, also the more general transformations act as Kac-Moody type symmetries but the interpretation would be that the sub-algebra plays the same role as $SO(3)$ in the case of Lorentz group: that is gives rise to generalized spin degrees of freedom whereas the entire algebra divided by this sub-algebra would define the coset space playing the role of orbital degrees of freedom. In fact, also the Kac-Moody type symmetries for which c_A depends on the transversal coordinates of X^3 would correspond to orbital degrees of freedom. The presence of these orbital degrees of freedom arranging super Kac-Moody representations into infinite multiplets labeled by function basis for X^2 means that the number of degrees of freedom is much larger than in string models.

5. It is possible to replace the preferred time coordinate m^0 with a preferred light-like coordinate. There are good reasons to believe that orbifold singularity for phases of matter involving non-standard value of Planck constant corresponds to a preferred light-ray going through the tip of $\delta M^4_\pm$. Thus it would be natural to assume that the preferred M^4 coordinate varies along this light ray or its dual. The Kac-Moody group $SO(3)\times E^3$ respecting the radial conformal weights would reduce to $SO(2)\times E^2$ as in string models. E^2 would act in tangent plane of $S^2_\pm$ along this ray defining also $SO(2)$ rotation axis.

3.3.5 Coset space structure for a symmetric space

The key ingredient in the theory of symmetric spaces is that the Lie-algebra of G has the following decomposition

$$
\begin{array}{l}
g = h + t \ , \\
{[h,h]} \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ .
\end{array}
$$

In present case this has highly nontrivial consequences. The commutator of *any* two infinitesimal generators generating nontrivial deformation of 3-surface belongs to h and thus vanishing norm in the configuration space metric at the point which is left invariant by H. In fact, this same condition follows from Ricci flatness requirement and guarantees also that G acts as isometries of the configuration space. This generalization is supported by the properties of the unitary representations of Lorentz group at the light cone boundary and by number theoretical considerations.

The algebras suggesting themselves as candidates are symplectic algebra of $\delta M^\pm \times CP_2$ and Kac-Moody algebra mapping light-like 3-surfaces to light-like 3-surfaces to be discussed in the next section.

The identification of the precise form of the coset space structure is however somewhat delicate.

1. The essential point is that both symplectic and Kac-Moody algebras allow representation in terms of X^3_l-local Hamiltonians. The general expression for the Hamilton of Kac-Moody algebra is

$$
H = \sum \Phi_A(x) H^A \ . \tag{3.3.19}
$$

Here H^A are Hamiltonians of $SO(3)\times SU(3)$ acting in $\delta X^3_l \times CP_2$. For symplectic algebra any Hamiltonian is allowed. If x corresponds to any point of X^3_l, one must assume a slicing of the causal diamond CD by translates of $\delta M^4_\pm$.

2. For symplectic generators the dependence of form on r^Δ on light-like coordinate of $\delta X^3_l \times CP_2$ is allowed. Δ is complex parameter whose modulus squared is interpreted as conformal weight. Δ is identified as analogous quantum number labeling the modes of induced spinor field.

3. One can wonder whether the choices of the $r_M = constant$ sphere S^2 is the only choice. The Hamiltonin-Jacobi coordinate for $X^4_{X^3_l}$ suggest an alternative choice as E^2 in the decomposition of $M^4 = M^2(x)\times E^2(x)$ required by number theoretical compactification and present for known extremals of Kähler action with Minkowskian signature of induced metric. In this case $SO(3)$ would be replaced with $SO(2)$. It however seems that the radial light-like coordinate u of $X^4(X^3_l)$ would remain the same since any other curve along light-like boundary would be space-like.

4. The vector fields for representing Kac-Moody algebra must vanish at the partonic 2-surface $X^2 \subset \delta M^4_\pm \times CP_2$. The corresponding vector field must vanish at each point of X^2:

$$j^k \quad = \quad \sum \Phi_A(x) J^{kl} H^A_l = 0 \ . \tag{3.3.20}$$

   This means that the vector field corresponds to $SO(2)\times U(2)$ defining the isotropy group of the point of $S^2 \times CP_2$.

   This expression could be deduced from the idea that the surfaces X^2 are analogous to origin of CP_2 at which $U(2)$ vector fields vanish. Configuration space at X^2 could be also regarded as the analog of the origin of local $S^2\times CP_2$. This interpretation is in accordance with the original idea which however was given up in the lack of proper realization. The same picture can be deduced from braiding in which case the Kac-Moody algebra corresponds to local $SO(2)\times U(2)$ for each point of the braid at X^2. The condition that Kac-Moody generators with positive conformal weight annihilate physical states could be interpreted by stating effective 2-dimensionality in the sense that the deformations of X^3_l preserving its light-likeness do not affect the physics. Note however that Kac-Moody type Virasoro generators do not annihilate physical states.

5. Kac-Moody algebra generator must leave induced Kähler form invariant at X^2. This is of course trivial since the action leaves each point invariant. The conditions of Cartan decomposition are satisfied. The commutators of the Kac-Moody vector fields with symplectic generators are non-vanishing since the action of symplectic generator on Kac-Moody generator restricted to X^2 gives a non-vanishing result belonging to the symplectic algebra. Also the commutators of Kac-Moody generators are Kac-Moody generators.

3.4 Complexification

A necessary prerequisite for the Kähler geometry is the complexification of the tangent space in vibrational degrees of freedom. What this means in recent context is non-trivial.

3.4.1 Why complexification is needed?

The Minkowskian signature of M^4 metric seems however to represent an insurmountable obstacle for the complexification of M^4 type vibrational degrees of freedom. On the other hand, complexification seems to have deep roots in the actual physical reality.

1. In the perturbative quantization of gauge fields one associates to each gauge field excitation polarization vector e and massless four-momentum vector p ($p^2 = 0$, $p\cdot e = 0$). These vectors define the decomposition of the tangent space of M^4: $M^4 = M^2 \times E^2$, where M^2 type polarizations correspond to zero norm states and E^2 type polarizations correspond to physical states with non-vanishing norm. Same type of decomposition occurs also in the linearized theory of gravitation. The crucial feature is that E^2 allows complexification! The general conclusion is that the modes of massless, linear, boson fields define always complexification of M^4 (or its tangent space) by effectively reducing it to E^2. Also in string models similar situation is encountered. For a string in D-dimensional space only D-2 transversal Euclidian degrees of freedom are physical.

2. Since symplectically extended isometry generators are expected to create physical states in TGD approach same kind of physical complexification should take place for them, too: this indeed takes place in string models in critical dimension. Somehow one should be able to associate polarization vector and massless four momentum vector to the deformations of a given 3-surface so that these vectors define the decomposition $M^4 = M^2 \times E^2$ for each mode. Configuration space metric should be degenerate: the norm of M^2 deformations should vanish as opposed to the norm of E^2 deformations.

Consider now the implications of this requirement.

1. In order to associate four-momentum and polarization (or at least the decomposition $M^4 = M^2 \times E^2$) to the deformations of the 3-surface one should have field equations, which determine the time development of the 3-surface uniquely. Furthermore, the time development for small deformations should be such that it makes sense to associate four momentum and polarization or at least the decomposition $M^4 = M^2 \times E^2$ to the deformations in suitable basis.

   The solution to this problem is afforded by the proposed definition of the Kähler function. The definition of the Kähler function indeed associates to a given 3-surface a unique four-surface as the preferred extremal of the Kähler action. Therefore one can associate a unique time development to the deformations of the surface X^3 and if TGD describes the observed world this time development should describe the evolution of photon, gluon, graviton, etc. states and so we can hope that tangent space complexification could be defined.

2. We have found that M^2 part of the deformation should have zero norm. In particular, the time like vibrational modes have zero norm in configuration space metric. This is true if Kähler function is not only $Diff^3$ invariant but also Diff^4 invariant in the sense that Kähler function has same value for all 3-surfaces belonging to the orbit of X^3 and related to X^3 by diffeomorphism of X^4. This is indeed the case.

3. Even this is not enough. One expects the presence of massive modes having also longitudinal polarization and for these states the number of physical vibrational degrees of freedom is 3 so that complexification seems to be impossible by odd dimension.

The reduction to the light cone boundary implied by $Diff^4$ invariance makes possible to identify the complexification. Crucial role is played by the special properties of the boundary of 4-dimensional light cone, which is metrically two-sphere and thus allows generalized complex and Kähler structure.

3.4.2 The metric, conformal and symplectic structures of the light cone boundary

The special metric properties of the light cone boundary play a crucial role in the complexification. The point is that the boundary of the light cone has degenerate metric: although light cone boundary is topologically 3-dimensional it is metrically 2-dimensional: effectively sphere. In standard spherical Minkowski coordinates light cone boundary is defined by the equation $r_M = m^0$ and induced metric reads

$$ds^2 = -r_M^2 d\Omega^2 = -r_M^2 dz d\bar{z}/(1+z\bar{z})^2 \ , \tag{3.4.1}$$

and has Euclidian signature. Since S^2 allows complexification and thus also Kähler structure (and as a by-product also symplectic structure) there are good hopes of obtaining just the required type of complexification in non-degenerate M^4 degrees of freedom: configuration space would effectively inherit its Kähler structure from $S^2 \times CP_2$.

By its effective two-dimensionality the boundary of the four-dimensional light cone has infinite-dimensional group of (local) conformal transformations. Using complex coordinate z for S^2 the general local conformal transformation reads

$$\begin{aligned} r &\rightarrow f(r_M, z, \bar{z}) \ , \\ z &\rightarrow g(z) \ , \end{aligned} \tag{3.4.2}$$

where f is an arbitrary real function and g is an arbitrary analytic function with a finite number of poles. The infinitesimal generators of this group span an algebra, call it C, analogous to Virasoro algebra. This algebra is semidirect sum of two algebras L and R given by

$$\begin{aligned} C &= L \oplus R \ , \\ [L, R] &\subset R \ , \end{aligned} \tag{3.4.3}$$

where L denotes standard Virasoro algebra of the two- sphere generated by the generators

$$L_n = z^{n+1}d/dz \quad (3.4.4)$$

and R denotes the algebra generated by the vector fields

$$R_n = f_n(z, \bar{z}, r_M)\partial_{r_M} \ , \quad (3.4.5)$$

where $f(z, \bar{z}, r_M)$ forms complete real scalar function basis for light cone boundary. The vector fields of R have the special property that they have vanishing norm in M^4 metric.

This modification of conformal group implies that the Virasoro generator L_0 becomes $L_0 = zd/dz - r_M d/dr_M$ so that the scaling momentum becomes the difference $n - m$ or S^2 and radial scaling momenta. One could achieve conformal invariance by requiring that S^2 and radial scaling quantum numbers compensate each other.

Of crucial importance is that light cone boundary allows infinite dimensional group of isometries! An arbitrary conformal transformation $z \to f(z)$ induces to the metric a conformal factor given by $|df/dz|^2$. The compensating radial scaling $r_M \to r_M/|df/dz|$ compensates this factor so that the line element remains invariant.

The Kähler structure of light cone boundary defines automatically symplectic structure. The symplectic form is degenerate and just the area form of S^2 given by

$$J = r_M^2 sin(\theta) d\theta \wedge d\phi,$$

in standard spherical coordinates, there is infinite-dimensional group of symplectic transformations leaving the symplectic form of the light cone boundary (that is S^2) invariant. These transformations are local with respect to the radial coordinate r_M. The symplectic and Kähler structures of light cone boundary are not unique: different structures are labeled by the coset space $SO(3,1)/SO(3)$. One can however associate with a given 3-surface Y^3 a unique structure by requiring that the the corresponding subgroup $SO(3)$ of Lorentz group acts as the isotropy group of the conserved classical four-momentum assigned to Y^3 by the preferred extremal property.

In case of $\delta M^4_+ \times CP_2$ both the conformal transformations, isometries and symplectic transformations of the light cone boundary can be made local also with respect to CP_2. The idea that the infinite-dimensional algebra of symplectic transformations of $\delta M^4_+ \times CP_2$ act as isometries of the configuration space and that radial vector fields having zero norm in the metric of light cone boundary possess zero norm also in configuration space metric, looks extremely attractive.

In the case of $\delta M^4_+ \times CP_2$ one could combine the symplectic and Kähler structures of δM^4_+ and CP_2 to single symplectic/Kähler structure. The symplectic transformations leaving this symplectic structure invariant would be generated by the function algebra of $\delta M^4_+ \times CP_2$ such that a arbitrary function serves as a Hamiltonian of a symplectic transformation. This group serves as a candidate for the isometry group of the configuration space. An alternative identification for the isometry algebra is as symplectic symmetries of CP_2 localized with respect to the light cone boundary. Hamiltonians would be also now elements of the function algebra of $\delta M^4_+ \times CP_2$ but their Poisson brackets would be defined using only CP_2 symplectic form.

The problem is to decide which option is correct. There is a simple argument fixing the latter option. The symplecticly imbedded CP_2 would be left invariant under δM^4_+ local symplectic transformations of CP_2. This seems strange. Under symplectic algebra of $\delta M^4_+ \times CP_2$ also symplecticly imbedded CP_2 is deformed and this sounds more realistic. The isometry algebra is therefore assumed to be the group $can(\delta M^4_+ \times CP_2)$ generated by the scalar function basis $S(\delta M^4_+ \times CP_2) = S(\delta M^4_+) \times S(CP_2)$ of the light cone boundary using the Poisson brackets to be discussed in more detail later.

There are some no-go theorems associated with higher-dimensional Abelian extensions [51], and although the contexts are quite different, it is interesting to consider the recent situation in light of these theorems.

1. Conformal invariance is an essentially 2-dimensional notion. Light cone boundary is however metrically and conformally 2-sphere, and therefore the conformal algebra is effectively that associated with the 2-sphere. In the same manner, the quaternion conformal algebra associated

with the metrically 2-dimensional elementary particle horizons surrounding wormhole contacts allows the usual Kac Moody algebra and actually also contributes to the configuration space metric.

2. In dimensions $D > 2$ Abelian extensions of the gauge algebra are extensions by an infinite-dimensional Abelian group rather than central extensions by the group $U(1)$. This result has an analog at the level of configuration space geometry. The extension associated with the symplectic algebra of CP_2 localized with respect to the light cone boundary is analogous a symplectic extension defined by Poisson bracket $\{p,q\} = 1$. The central extension is the function space associated with δM^4_+ and indeed infinite-dimensional if only only CP_2 symplectic structure induces the Poisson bracket but one-dimensional if $\delta M^4_+ \times CP_2$ Poisson bracket induces the extension. In the latter case the symmetries fix the metric completely at the point corresponding to the origin of symmetric space (presumably the maximum of Kähler function for given values of zero modes).

3. $D > 2$ extensions possess no unitary faithful representations (satisfying certain well motivated physical constraints) [51]. It might be that the degeneracy of the configuration space metric is the analog for the loss of faithful representations.

3.4.3 Complexification and the special properties of the light cone boundary

In case of Kähler metric G and H Lie-algebras must allow complexification so that the isometries can act as holomorphic transformations. Since G and H can be regarded as subalgebras of the vector fields of $\delta M^4_+ \times CP_2$, they inherit in a natural manner the complex structure of the light cone boundary.

There are two candidates for the configuration space complexification. The simplest, and also the correct, alternative is that complexification is induced by natural complexification of vector field basis on $\delta M^4_+ \times CP_2$. In CP_2 degrees of freedom there is natural complexification

$$\xi \rightarrow \bar{\xi} \ .$$

In δM^4_+ degrees of freedom this could involve the transformation

$$z \rightarrow \bar{z}$$

and certainly involves complex conjugation for complex scalar function basis in the radial direction:

$$f(r_M) \rightarrow \overline{f(r_M)} \ ,$$

which turns out to play same role as the function basis of circle in the Kähler geometry of loop groups [45].

The requirement that the functions are eigen functions of radial scalings favors functions $(r_M/r_0)^k$, where k is in general a complex number. The function can be expressed as a product of real power of r_M and logarithmic plane wave. It turns out that the radial complexification alternative is the correct manner to obtain Kähler structure. The reason is that symplectic transformations leave the value of r_M invariant. Radial Virasoro invariance plays crucial role in making the complexification possible.

One could consider also a second alternative assumed in the earlier formulation of the configuration space geometry. The close analogy with string models and conformal field theories suggests that for Virasoro generators the complexification must reduce to the hermitian conjugation of the conformal field theories: $L_n \rightarrow L_{-n} = L_n^\dagger$. Clearly this complexification is induced from the transformation $z \rightarrow \frac{1}{z}$ and differs from the complexification induced by complex conjugation $z \rightarrow \bar{z}$. The basis would be polynomial in z and $\bar{z}$. Since radial algebra could be also seen as Virasoro algebra localized with respect to $S^2 \times CP_2$ one could consider the possibility that also in radial direction the inversion $r_M \rightarrow \frac{1}{r_M}$ is involved.

The essential prerequisite for the Kähler structure is that both G and H allow same complexification so that the isometries in question can be regarded as holomorphic transformations. In finite-dimensional case this essentially what is needed since metric can be constructed by parallel translation along the orbit of G from H-invariant Kähler metric at a representative point. The requirement of H-invariance forces the radial complexification based on complex powers r_M^k: radial complexification works since symplectic transformations leave r_M invariant.

Some comments on the properties of the proposed complexification are in order.

1. The proposed complexification, which is analogous to the choice of gauge in gauge theories is not Lorentz invariant unless one can fix the coordinates of the light cone boundary apart from $SO(3)$ rotation not affecting the value of the radial coordinate r_M (if the imaginary part of k in r_M^k is always non-vanishing). This is possible as will be explained later.

2. It turns out that the function basis of light-cone boundary multiplying CP_2 Hamiltonians corresponds to unitary representations of the Lorentz group at light cone boundary so that the Lorentz invariance is rather manifest.

3. There is a nice connection with the proposed physical interpretation of the complexification. At the moment of the big bang all particles move with the velocity of light and therefore behave as massless particles. To a given point of the light cone boundary one can associate a unique direction of massless four-momentum by semiclassical considerations: at the point $m^k = (m^0, m^i)$ momentum is proportional to the vector $(m^0, -m^i)$. Since the particles are massless only two polarization vectors are possible and these correspond to the tangent vectors to the sphere $m^0 = r_M$. Of course, one must always fix polarizations at some point of tangent space but since massless polarization vectors are not physical this doesn't imply difficulties: different choices correspond to different gauges.

4. Complexification in the proposed manner is not possible except in the case of four-dimensional Minkowski space. Non-zero norm deformations correspond to vector fields of the light cone boundary acting on the sphere S^{D-2} and the decomposition to $(1,0)$ and $(0,1)$ parts is possible only when the sphere in question is two-dimensional since other spheres do allow neither complexification nor Kähler structure.

3.4.4 How to fix the complex and symplectic structures in a Lorentz invariant manner?

One can assign to light-cone boundary a symplectic structure since it reduces effectively to S^2. The possible symplectic structures of δM^4_+ are parameterized by the coset space $SO(3,1)/SO(3))$, where H is the isotropy group $SO(3)$ of a time like vector. Complexification also fixes the choice of the spherical coordinates apart from rotations around the quantization axis of angular momentum.

The selection of some preferred symplectic structure in an ad hoc manner breaks manifest Lorentz invariance but is possible if physical theory remains Lorentz invariant. The more natural possibility is that 3-surface Y^3 itself fixes in some natural manner the choice of the symplectic structure so that there is unique subgroup $SO(3)$ of $SO(3,1)$ associated with Y^3. If configuration space Kähler function corresponds to a preferred extremal of Kähler action, this is indeed the case. One can associate unique conserved four-momentum $P^k(Y^3)$ to the preferred extremal $X^4(Y^3)$ of the Kähler action and the requirement that the rotation group $SO(3)$ leaving the symplectic structure invariant leaves also $P^k(Y^3)$ invariant, fixes the symplectic structure associated with Y^3 uniquely.

Therefore configuration space decomposes into a union of symplectic spaces labeled by $SO(3,1)/SO(3)$ isomorphic to $a = constant$ hyperboloid of light cone. The direction of the classical angular momentum vector $w^k = \epsilon^{klmn} P_l J_{mn}$ determined by the classical angular momentum tensor of associated with Y^3 fixes one coordinate axis and one can require that $SO(2)$ subgroup of $SO(3)$ acting as rotation around this coordinate axis acts as phase transformation of the complex coordinate z of S^2. Other rotations act as nonlinear holomorphic transformations respecting the complex structure.

Clearly, the coordinates are uniquely fixed modulo $SO(2)$ rotation acting as phase multiplication in this case. If $P^k(Y^3)$ is light like, one can only require that the rotation group $SO(2)$ serving as the isotropy group of 3-momentum belongs to the group $SO(3)$ characterizing the symplectic structure and it seems that symplectic structure cannot be uniquely fixed without additional constraints in this case. Probably this has no practical consequences since the 3-surfaces considered have actually infinite size and 4-momentum is most probably time like for them. Note however that the direction of 3-momentum defines unique axis such that $SO(2)$ rotations around this axis are represented as phase multiplication.

Similar almost unique frame exists also in CP_2 degrees of freedom and corresponds to the complex coordinates transforming linearly under $U(2)$ acting as isotropy group of the Lie-algebra element defined by classical color charges Q_a of Y^3. One can fix unique Cartan subgroup of $U(2)$ by noticing that $SU(3)$ allows completely symmetric structure constants d_{abc} such that $R_a = d_a^{\ bc} Q_b Q_c$ defines

Lie-algebra element commuting with Q_a. This means that R_a and Q_a span in generic case $U(1)\times U(1)$ Cartan subalgebra and there are unique complex coordinates for which this subgroup acts as phase multiplications. The space of nonequivalent frames is isomorphic with $CP(2)$ so that one can say that cm degrees of freedom correspond to Cartesian product of $SO(3,1)/SO(3)$ hyperboloid and CP_2 whereas coordinate choices correspond to the Cartesian product of $SO(3,1)/SO(2)$ and $SU(3)/U(1)\times U(1)$.

Symplectic transformations leave the value of δM^4_+ radial coordinate r_M invariant and this implies large number of additional zero modes characterizing the size and shape of the 3-surface. Besides this Kähler magnetic fluxes through the $r_M = constant$ sections of X^3 as a function of r_M provide additional invariants, which are functions rather than numbers. The Fourier components for the magnetic fluxes provide infinite number of symplectic invariants. The presence of these zero modes imply that 3-surfaces behave much like classical objects in the sense that neither their shape nor form nor classical Kähler magnetic fields, are subject to Gaussian fluctuations. Of course, quantum superpositions of 3-surfaces with different values of these invariants are possible.

There are reasons to expect that at least certain infinitesimal symplectic transformations correspond to zero modes of the Kähler metric (symplectic transformations act as dynamical symmetries of the vacuum extremals of the Kähler action). If this is indeed the case, one can ask whether it is possible to identify an integration measure for them.

If one can associate symplectic structure with zero modes, the symplectic structure defines integration measure in a standard manner (for 2n-dimensional symplectic manifold the integration measure is just the n-fold wedge power $J\wedge J...\wedge J$ of the symplectic form J). Unfortunately, in infinite-dimensional context this is not enough since divergence free functional integral analogous to a Gaussian integral is needed and it seems that it is not possible to integrate in zero modes and that this relates in a deep manner to state function reduction. If all symplectic transformations of $\delta M^4_+ \times CP_2$ are represented as symplectic transformations of the configuration space, then the existence of symplectic structure decomposing into Kähler (and symplectic) structure in complexified degrees of freedom and symplectic (but not Kähler) structure in zero modes, is an automatic consequence.

3.4.5 The general structure of the isometry algebra

There are three options for the isometry algebra of configuration space

1. Isometry algebra as the algebra of CP_2 symplectic transformations leaving invariant the symplectic form of CP_2 localized with respect to δM^4_+.

2. Certainly the configuration space metric in δM^4_+ must be non-trivial and actually given by the magnetic flux Hamiltonians defining symplectic invariants. Furthermore, the super-symplectic generators constructed from quarks automatically give as anti-commutators this part of the configuration space metric. One could interpret these symplectic invariants as configuration space Hamiltonians for δM^4_+ symplectic transformations obtained when CP_2 Hamiltonian is constant.

3. Isometry algebra consists of $\delta M^4_+ \times CP_2$ symplectic transformations. In this case a local color transformation involves necessarily a local S^2 transformation. Unfortunately, it is difficult to decide at this stage which of these options is correct.

The eigen states of the rotation generator and Lorentz boost in the same direction defining a unitary representation of the Lorentz group at light cone boundary define the most natural function basis for the light cone boundary. The elements of this bases have also well defined scaling quantum numbers and define also a unitary representation of the conformal algebra. The product of the basic functions is very simple in this basis since various quantum numbers are additive.

Spherical harmonics of S^2 provide an alternative function basis for the light cone boundary:

$$H^m_{jk} \equiv Y_{jm}(\theta,\phi) r_M^k \quad . \tag{3.4.6}$$

One can criticize this basis for not having nice properties under Lorentz group.

The product of basis functions is given by Glebch-Gordan coefficients for symmetrized tensor product of two representation of the rotation group. Poisson bracket in turn reduces to the Glebch-Gordans of anti-symmetrized tensor product. The quantum numbers m and k are additive. The basis is eigen-function basis for the imaginary part of the Virasoro generator L_0 generating rotations around quantization axis of angular momentum. In fact, only the imaginary part of the Virasoro generator $L_0 = zd/dz = \rho\partial_\rho - \frac{2}{2}\partial_\phi$ has global single valued Hamiltonian, whereas the corresponding representation for the transformation induced by the real part of L_0, with a compensating radial scaling added, cannot be realized as a global symplectic transformation.

The Poisson bracket of two functions $H^m_{j_1k_1}$ and $H^m_{j_2k_2}$ can be calculated and is of the general form

$$\{H^{m_1}_{j_1k}, H^{m_2}_{j_2k_2}\} \equiv C(j_1m_1j_2m_2|j, m_1+m_2)_A H^{m_1+m_2}_{j,k_1+k_2} \quad . \tag{3.4.7}$$

The coefficients are Glebch-Gordan coefficients for the anti-symmetrized tensor product for the representations of the rotation group.

The isometries of the light cone boundary correspond to conformal transformations accompanied by a local radial scaling compensating the conformal factor coming from the conformal transformations having parametric dependence of radial variable and CP_2 coordinates. It seems however that isometries cannot in general be realized as symplectic transformations. The first difficulty is that symplectic transformations cannot affect the value of the radial coordinate. For rotation algebra the representation as symplectic transformations is however possible.

In CP_2 degrees of freedom scalar function basis having definite color transformation properties is desirable. Scalar function basis can be obtained as the algebra generated by the Hamiltonians of color transformations by multiplication. The elements of basis can be typically expressed as monomials of color Hamiltonians H^A_c

$$H^A_D = \sum_{\{B_j\}} C^A_{DB_1B_2....B_N} \prod_{B_i} H^{B_i}_c \quad , \tag{3.4.8}$$

where summation over all index combinations $\{B_i\}$ is understood. The coefficients $C^A_{DB_1B_2....B_N}$ are Glebch-Gordan coefficients for completely symmetric N:th power $8 \otimes 8... \otimes 8$ of octet representations. The representation is not unique since $\sum_A H^A_c H^A_c = 1$ holds true. One can however find for each representation D some minimum value of N.

The product of Hamiltonians $H^{D_1}_A$ and $H^B_{D_2}$ can be decomposed by Glebch-Gordan coefficients of the symmetrized representation $(D_1 \otimes D_2)_S$ as

$$H^A_{D_1} H^B_{D_2} = C^{ABD}_{D_1D_2DC}(S) H^C_D \quad , \tag{3.4.9}$$

where $'S'$ indicates that the symmetrized representation is in question. In the similar manner one can decompose the Poisson bracket of two Hamiltonians

$$\{H^A_{D_1}, H^B_{D_2}\} = C^{ABD}_{D_1D_2DC}(A) H^C_D \quad . \tag{3.4.10}$$

Here $'A'$ indicates that Glebch-Gordan coefficients for the anti-symmetrized tensor product of the representations D_1 and D_2 are in question.

One can express the infinitesimal generators of CP_2 symplectic transformations in terms of the color isometry generators J^B_c using the expansion of the Hamiltonian in terms of the monomials of color Hamiltonians:

$$\begin{aligned} j^A_{DN} &= F^A_{DB} J^B_c \quad , \\ F^A_{DB} &= N \sum_{\{B_j\}} C^A_{DB_1B_2...B_{N-1}B} \prod_j H^{B_j}_c \quad , \end{aligned} \tag{3.4.11}$$

where summation over all possible $\{B_j\}$:s appears. Therefore, the interpretation as a color group localized with respect to CP_2 coordinates is valid in the same sense as the interpretation of space-time diffeomorphism group as local Poincare group. Thus one can say that TGD color is localized with respect to the entire $\delta M^4_+ \times CP_2$.

A convenient basis for the Hamiltonians of $\delta M^4_+ \times CP_2$ is given by the functions

$$H^{mA}_{jkD} = H^m_{jk} H^A_D \ .$$

The symplectic transformation generated by H^{mA}_{jkD} acts both in M^4 and CP_2 degrees of freedom and the corresponding vector field is given by

$$J^r = H^A_D J^{rl}(\delta M^4_+)\partial_l H^m_{jk} + H^m_{jk} J^{rl}(CP_2)\partial_l H^A_D \ . \qquad (3.4.12)$$

The general form for their Poisson bracket is:

$$\begin{aligned} &\{H^{m_1A_1}_{j_1k_1D_1}, H^{m_2A_2}_{j_2k_2D_2}\} = H^{A_1}_{D_1}H^{A_2}_{D_2}\{H^{m_1}_{j_1k_1}, H^{m_2}_{j_2k_2}\} + H^{m_1}_{j_1k_1}H^{m_2}_{j_2k_2}\{H^{A_1}_{D_1}, H^{A_2}_{D_2}\} \\ &= \left[C^{A_1A_2A}_{D_1D_2D}(S)C(j_1m_1j_2m_2|jm)_A + C^{A_1A_2A}_{D_1D_2D}(A)C(j_1m_1j_2m_2|jm)_S\right] H^{mA}_{j,k_1+k_2,D} \ . \end{aligned} \qquad (3.4.13)$$

What is essential that radial 'momenta' and angular momentum are additive in δM^4_+ degrees of freedom and color quantum numbers are additive in CP_2 degrees of freedom.

3.4.6 Representation of Lorentz group and conformal symmetries at light cone boundary

A guess deserving testing is that the representations of the Lorentz group at light cone boundary might provide natural building blocks for the construction of the configuration space Hamiltonians. In the following the explicit representation of the Lorentz algebra at light cone boundary is deduced, and a function basis giving rise to the representations of Lorentz group and having very simple properties under modified Poisson bracket of δM^4_+ is constructed.

Explicit representation of Lorentz algebra

It is useful to write the explicit expressions of Lorentz generators using complex coordinates for S^2. The expression for the $SU(2)$ generators of the Lorentz group are

$$\begin{aligned} J_x &= (z^2-1)d/dz + c.c. = L_1 - L_{-1} + c.c. \ , \\ J_y &= (iz^2+1)d/dz + c.c. = iL_1 + iL_{-1} + c.c. \ , \\ J_z &= iz\frac{d}{dz} + c.c. = iL_z + c.c. \ . \end{aligned} \qquad (3.4.14)$$

The expressions for the generators of Lorentz boosts can be derived easily. The boost in m^3 direction corresponds to an infinitesimal transformation

$$\begin{aligned} \delta m^3 &= -\varepsilon r_M \ , \\ \delta r_M &= -\varepsilon m^3 = -\varepsilon\sqrt{r_M^2 - (m^1)^2 - (m^2)^2} \ . \end{aligned} \qquad (3.4.15)$$

The relationship between complex coordinates of S^2 and M^4 coordinates m^k is given by stereographic projection

$$\begin{aligned} z &= \frac{(m^1+im^2)}{(r_M-\sqrt{r_M^2-(m^1)^2-(m^2)^2})} \\ &= \frac{sin(\theta)(cos\phi+isin\phi)}{(1-cos\theta)} \ , \\ cot(\theta/2) &= \rho=\sqrt{z\bar{z}} \ , \\ tan(\phi) &= \frac{m^2}{m^1} \ . \end{aligned} \tag{3.4.16}$$

This implies that the change in z coordinate doesn't depend at all on r_M and is of the following form

$$\delta z = -\frac{\varepsilon}{2}(1+\frac{z(z+\bar{z})}{2})(1+z\bar{z}) \ . \tag{3.4.17}$$

The infinitesimal generator for the boosts in z-direction is therefore of the following form

$$L_z = [\frac{2z\bar{z}}{(1+z\bar{z})}-1]r_M\frac{\partial}{\partial_{r_M}}-iJ_z \ . \tag{3.4.18}$$

Generators of L_x and L_y are most conveniently obtained as commutators of $[L_z,J_y]$ and $[L_z,J_x]$. For L_y one obtains the following expression:

$$L_y = 2\frac{(z\bar{z}(z+\bar{z})+i(z-\bar{z}))}{(1+z\bar{z})^2}r_M\frac{\partial}{\partial_{r_M}}-iJ_y \ , \tag{3.4.19}$$

For L_x one obtains analogous expressions. All Lorentz boosts are of the form $L_i=-iJ_i+local\ radial\ scaling$ and of zeroth degree in radial variable so that their action on the general generator $X^{klm}\propto z^k\bar{z}^l r_M^m$ doesn't change the value of the label m being a mere local scaling transformation in radial direction. If radial scalings correspond to zero norm isometries this representation is metrically equivalent with the representations of Lorentz boosts as Möbius transformations.

Representations of the Lorentz group reduced with respect to $SO(3)$

The ordinary harmonics of S^2 define in a natural manner infinite series of representation functions transformed to each other in Lorentz transformations. The inner product defined by the integration measure $r_M^2 d\Omega dr_M/r_M$ remains invariant under Lorentz boosts since the scaling of r_M induced by the Lorentz boost compensates for the conformal scaling of $d\Omega$ induced by a Lorentz transformation represented as a Möbius transformation. Thus unitary representations of Lorentz group are in question.

The unitary main series representations of the Lorentz group are characterized by half-integer m and imaginary number $k_2=i\rho$, where ρ is any real number [52]. A natural guess is that $m=0$ holds true for all representations realizable at the light cone boundary and that radial waves are of form r_M^k, $k=k_1+ik_2=-1+i\rho$ and thus eigen states of the radial scaling so that the action of Lorentz boosts is simple in the angular momentum basis. The inner product in radial degrees of freedom reduces to that for ordinary plane waves when $log(r_M)$ is taken as a new integration variable. The complexification is well-defined for non-vanishing values of ρ.

It is also possible to have non-unitary representations of the Lorentz group and the realization of the symmetric space structure suggests that one must have $k=k_1+ik_2$, k_1 half-integer. For these representations unitarity fails because the inner product in the radial degrees of freedom is non-unitary. A possible physical interpretation consistent with the general ideas about conformal invariance is that the representations $k=-1+i\rho$ correspond to the unitary ground state representations and $k=-1+n/2+i\rho$, $n=\pm1,\pm2,...$, to non-unitary representations. The general view about conformal invariance suggests that physical states constructed as tensor products satisfy the condition $\sum_i n_i=0$ completely analogous to Virasoro conditions.

Representations of the Lorentz group with $E^2 \times SO(2)$ as isotropy group

One can construct representations of Lorentz group and conformal symmetries at the light cone boundary. Since $SL(2,C)$ is the group generated by the generators L_0 and $L_\pm$ of the conformal algebra, it is clear that infinite-dimensional representations of Lorentz group can be also regarded as representations of the conformal algebra. One can require that the basis corresponds to eigen functions of the rotation generator J_z and corresponding boost generator L_z. For functions which do not depend on r_M these generators are completely analogous to the generators L_0 generating scalings and iL_0 generating rotations. Also the generator of radial scalings appears in the formulas and one must consider the possibility that it corresponds to the generator L_0.

In order to construct scalar function eigen basis of L_z and J_z, one can start from the expressions

$$\begin{aligned} L_3 &\equiv i(L_z + L_{\bar{z}}) = 2i[\frac{2z\bar{z}}{(1+z\bar{z})} - 1]r_M\frac{\partial}{\partial_{r_M}} + i\rho\partial_\rho \ , \\ J_3 &\equiv iL_z - iL_{\bar{z}} = i\partial_\phi \ . \end{aligned} \tag{3.4.20}$$

If the eigen functions do not depend on r_M, one obtains the usual basis z^n of Virasoro algebra, which however is not normalizable basis. The eigenfunctions of the generators L_3, J_3 and $L_0 = ir_M d/dr_M$ satisfying

$$\begin{aligned} J_3 f_{m,n,k} &= m f_{m,n,k} \ , \\ L_3 f_{m,n,k} &= n f_{m,n,k} \ , \\ L_0 f_{m,n,k} &= k f_{m,n,k} \ . \end{aligned} \tag{3.4.21}$$

are given by

$$f_{m,n,k} = e^{im\phi}\frac{\rho^{n-k}}{(1+\rho^2)^k} \times (\frac{r_M}{r_0})^k \ . \tag{3.4.22}$$

$n = n_1 + in_2$ and $k = k_1 + ik_2$ are in general complex numbers. The condition

$$n_1 - k_1 \geq 0$$

is required by regularity at the origin of S^2 The requirement that the integral over S^2 defining norm exists (the expression for the differential solid angle is $d\Omega = \frac{\rho}{(1+\rho^2)^2}d\rho d\phi$) implies

$$n_1 < 3k_1 + 2 \ .$$

From the relationship $(cos(\theta), sin(\theta)) = (\rho^2-1)/(\rho^2+1), 2\rho/(\rho^2+1))$ one can conclude that for $n_2 = k_2 = 0$ the representation functions are proportional to f $sin(\theta)^{n-k}(cos(\theta)-1)^{n-k}$. Therefore they have in their decomposition to spherical harmonics only spherical harmonics with angular momentum $l < 2(n-k)$. This suggests that the condition

$$|m| \leq 2(n-k) \tag{3.4.23}$$

is satisfied quite generally.

The emergence of the three quantum numbers (m,n,k) can be understood. Light cone boundary can be regarded as a coset space $SO(3,1)/E^2 \times SO(2)$, where $E^2 \times SO(2)$ is the group leaving the light like vector defined by a particular point of the light cone invariant. The natural choice of the Cartan group is therefore $E^2 \times SO(2)$. The three quantum numbers (m,n,k) have interpretation as quantum numbers associated with this Cartan algebra.

The representations of the Lorentz group are characterized by one half-integer valued and one complex parameter. Thus k_2 and n_2, which are Lorentz invariants, might not be independent parameters, and the simplest option is $k_2 = n_2$.

The nice feature of the function basis is that various quantum numbers are additive under multiplication:

$$f(m_a, n_a, k_a) \times f(m_b, n_b, k_b) = f(m_a + m_b, n_a + n_b, k_a + k_b) \ .$$

These properties allow to cast the Poisson brackets of the symplectic algebra of the configuration space into an elegant form.

The Poisson brackets for the δM^4_+ Hamiltonians defined by f_{mnk} can be written using the expression $J^{\rho\phi} = (1+\rho^2)/\rho$ as

$$\begin{aligned} \{f_{m_a,n_a,k_a}, f_{m_b,n_b,k_b}\} &= i\,[(n_a - k_a)m_b - (n_b - k_b)m_a] \times f_{m_a+m_b,n_a+n_b-2,k_a+k_b} \\ &+ 2i\,[(2-k_a)m_b - (2-k_b)m_a] \times f_{m_a+m_b,n_a+n_b-1,k_a+k_b-1} \ . \end{aligned} \tag{3.4.24}$$

Can one find unitary light-like representations of Lorentz group?

It is interesting to compare the representations in question to the unitary representations of Lorentz group discussed in [52].

1. The unitary representations discussed in [52] are characterized by are constructed by deducing the explicit representations for matrix elements of the rotation generators J_x, J_y, J_z and boost generators L_x, L_y, L_z by decomposing the representation into series of representations of $SU(2)$ defining the isotropy subgroup of a time like momentum. Therefore the states are labeled by eigenvalues of J_z. In the recent case the isotropy group is $E^2 \times SO(2)$ leaving light like point invariant. States are therefore labeled by three different quantum numbers.

2. The representations of [52] are realized the space of complex valued functions of complex coordinates ξ and $\overline{\xi}$ labeling points of complex plane. These functions have complex degrees $n_+ = m/2 - 1 + l_1$ with respect to ξ and $n_- = -m/2 - 1 + l_1$ with respect to $\overline{\xi}$. l_0 is complex number in the general case but for unitary representations of main series it is given by $l_1 = i\rho$ and for the representations of supplementary series l_1 is real and satisfies $0 < |l_1| < 1$. The main series representation is derived from a representation space consisting of homogenous functions of variables z^0, z^1 of degree n_+ and of $\overline{z}^0$ and $\overline{z}^1$ of degrees $n_\pm$. One can separate express these functions as product of $(z^1)^{n^+}$ $(\overline{z}^1)^{n_-}$ and a polynomial of $\xi = z^1/z^2$ and $\overline{\xi}$ with degrees n_+ and n_-. Unitarity reduces to the requirement that the integration measure of complex plane is invariant under the Lorentz transformations acting as Moebius transformations of the complex plane. Unitarity implies $l_1 = -1 + i\rho$.

3. For the representations at δM^4_+ formal unitarity reduces to the requirement that the integration measure of $r_M^2 d\Omega dr_M/r_M$ of δM^4_+ remains invariant under Lorentz transformations. The action of Lorentz transformation on the complex coordinates of S^2 induces a conformal scaling which can be compensated by an S^2 local radial scaling. At least formally the function space of δM^4_+ thus defines a unitary representation. For the function basis f_{mnk} $k = -1+i\rho$ defines a candidate for a unitary representation since the logarithmic waves in the radial coordinate are completely analogous to plane waves for $k_1 = -1$. This condition would be completely analogous to the vanishing of conformal weight for the physical states of super conformal representations. The problem is that for $k_1 = -1$ guaranteing square integrability in S^2 implies $-2 < n_1 < -2$ so that unitarity is possible only for the function basis consisting of spherical harmonics.

   There is no deep reason against non-unitary representations and symmetric space structure indeed requires that k_1 is half-integer valued. First of all, configuration space spinor fields are analogous to ordinary spinor fields in M^4, which also define non-unitary representations of Lorentz group. Secondly, if 3-surfaces at the light cone boundary are finite-sized, the integrals defined by f_{mnk} over 3-surfaces Y^3 are always well-defined. Thirdly, the continuous spectrum of k_2 could be transformed to a discrete spectrum when k_1 becomes half-integer valued.

Hermitian form for light cone Hamiltonians involves also the integration over S^2 degrees of freedom and the non-unitarity of the inner product reflects itself as non-orthogonality of the the eigen function basis. Introducing the variable $u = \rho^2 + 1$ as a new integration variable, one can express the inner product in the form

$$
\begin{aligned}
\langle m_a, n_a, k_a | m_b, n_b, k_b \rangle &= \pi\delta(k_{2a} - k_{2b}) \times \delta_{m_1,m_2} \times I \ , \\
I &= \int_1^{\infty} f(u) du \ , \\
f(u) &= \frac{(u-1)^{\frac{(N-K)+i\Delta}{2}}}{u^{K+2}} \ .
\end{aligned} \tag{3.4.25}
$$

The integrand has cut from $u = 1$ to infinity along real axis. The first thing to observe is that for $N = K$ the exponent of the integral reduces to very simple form and integral exists only for $K = k_{1a} + k_{1b} > -1$. For $k_{1i} = -1/2$ the integral diverges.

The discontinuity of the integrand due to the cut at the real axis is proportional to the integrand and given by

$$
\begin{aligned}
f(u) - f(e^{i2\pi}u) &= \left[1 - e^{-\pi\Delta}\right] f(u) \ , \\
\Delta &= n_{1a} - k_{1a} - n_{1b} + k_{1b} \ .
\end{aligned} \tag{3.4.26}
$$

This means that one can transform the integral to an integral around the cut. This integral can in turn completed to an integral over closed loop by adding the circle at infinity to the integration path. The integrand has $K+1$-fold pole at $u = 0$.

Under these conditions one obtains

$$
\begin{aligned}
I &= \frac{2\pi i}{1 - e^{-\pi\Delta}} \times R \times (R-1) .... \times (R-K-1) \times (-1)^{\frac{N-K}{2} - K - 1} \ , \\
R &\equiv \frac{N-K}{2} + i\Delta \ .
\end{aligned} \tag{3.4.27}
$$

This expression is non-vanishing for $\Delta \neq 0$. Thus it is not possible to satisfy orthogonality conditions without the un-physical $n = k, k_1 = 1/2$ constraint. The result is finite for $K > -1$ so that $k_1 > -1/2$ must be satisfied and if one allows only half-integers in the spectrum, one must have $k_1 \geq 0$, which is very natural if real conformal weights which are half integers are allowed.

3.4.7 How the complex eigenvalues of the radial scaling operator relate to conformal weights?

Complexified Hamiltonians can be chosen to be eigenmodes of the radial scaling operator $r_M d/dr_M$, and the first guess was that the correct interpretation is as conformal weights. The problem is however that the eigenvalues are complex. Second problem is that general arguments are not enough to fix the spectrum of eigenvalues. There should be a direct connection to the dynamics defined by Kähler action with instanton term included and the modified Dirac action defined by it.

The construction of configuration space spinor structure in terms of second quantized induced spinor fields [18] leads to the conclusion that the modes of induced spinor fields are labeled by generalized eigenvalues λ such that $|\lambda|^2$ has interpretation as a conformal weight and λ itself is analogous to Higgs expectation value. Coset construction requires that super-symplectic and super Kac-Moody conformal weights $|\lambda|^2$ are same. This is achieved if the Hamiltonians are generalized eigen modes of $D = \gamma^x d/dx$, $x = log(r/r_0)$, satisfying $DH = \lambda\gamma^x H$ and thus of form $exp(\lambda x) = (r/r_0)^\lambda$ with the same spectrum of complex eigenvalues λ as associated with the modified Dirac operator. That $log(r/r_0)$ naturally corresponds to the coordinate u assignable to the generalized eigen modes of modified Dirac operator supports this interpretation.

If the Kähler action and modified Dirac action involve also the CP breaking instanton term, the eigenvalues λ are complex and complexity relates directly also to the breaking of time reversal invariance.

3.5 Magnetic and electric representations of the configuration space Hamiltonians

Symmetry considerations lead to the hypothesis that configuration space Hamiltonians are apart from a factor depending on symplectic invariants equal to magnetic flux Hamiltonians. On the other hand, the hypothesis that Kähler function corresponds to a preferred extremal of Kähler action leads to the hypothesis that configuration space Hamiltonians corresponds to classical charges associated with the Hamiltonians of the light cone boundary. These charges are very much like electric charges. The requirement that two approaches are equivalent leads to the hypothesis that magnetic and electric Hamiltonians are identical apart from a factor depending on isometry invariants. At the level of CP_2 corresponding duality corresponds to the self-duality of Kähler form stating that the magnetic and electric parts of Kähler form are identical.

3.5.1 Radial symplectic invariants

All $\delta M^4_+ \times CP_2$ symplectic transformations leave invariant the value of the radial coordinate r_M. Therefore the radial coordinate r_M of X^3 regarded as a function of $S^2 \times CP_2$ coordinates serves as height function. The number, type, ordering and values for the extrema for this height function in the interior and boundary components are isometry invariants. These invariants characterize not only the topology but also the size and shape of the 3-surface. The result implies that configuration space metric indeed differentiates between 3-surfaces with the size of Planck length and with the size of galaxy. The characterization of these invariants reduces to Morse theory. The extrema correspond to topology changes for the two-dimensional (one-dimensional) $r_M = constant$ section of 3-surface (boundary of 3-surface). The height functions of sphere and torus serve as a good illustrations of the situation. A good example about non-topological extrema is provided by a sphere with two horns.

There are additional symplectic invariants. The 'magnetic fluxes' associated with the δM^4_+ symplectic form

$$J_{S^2} = r_M^2 sin(\theta) d\theta \wedge d\phi$$

over any $X^2 \subset X^3$ are symplectic invariants. In particular, the integrals over $r_M = constant$ sections (assuming them to be 2-dimensional) are symplectic invariants. They give simply the solid angle $\Omega(r_M)$ spanned by $r_M = constant$ section and thus $r_M^2 \Omega(r_M)$ characterizes transversal geometric size of the 3-surface. A convenient manner to discretize these invariants is to consider the Fourier components of these invariants in radial logarithmic plane wave basis discussed earlier:

$$\Omega(k) = \int_{r_{min}}^{r_{max}} (r_M/r_{max})^k \Omega(r_M) \frac{dr_M}{r_M} \ , \ k = k_1 + ik_2 \ , \ perk_1 \geq 0 \ . \tag{3.5.1}$$

One must take into account that for each section in which the topology of $r_M = constant$ section remains constant one must associate invariants with separate components of the two-dimensional section. For a given value of r_M, r_M constant section contains several components (to visualize the situation consider torus as an example).

Also the quantities

$$\Omega^+(X^2) = \int_{X^2} |J| \equiv \int |\epsilon^{\alpha\beta} J_{\alpha\beta}| \sqrt{g_2} d^2x$$

are symplectic invariants and provide additional geometric information about 3-surface. These fluxes are non-vanishing also for closed surfaces and give information about the geometry of the boundary components of 3-surface (signed fluxes vanish for boundary components unless they enclose the dip of the light cone).

Since zero norm generators remain invariant under complexification, their contribution to the Kähler metric vanishes. It is not at all obvious whether the configuration space integration measure in these degrees of freedom exists at all. A localization in zero modes occurring in each quantum jump seems a more plausible and under suitable additional assumption it would have interpretation as a state function reduction. In string model similar situation is encountered; besides the functional integral determined by string action, one has integral over the moduli space.

If the effective 2-dimensionality implied by the strong form of general coordinate invariance discussed in the introduction is accepted, there is no need to integrate over the variable r_M and just the fluxes over the 2-surfaces X^2_i identified as intersections of light like 3-D causal determinants with X^3 contain the data relevant for the construction of the configuration space geometry. Also the symplectic invariants associated with these surfaces are enough.

3.5.2 Kähler magnetic invariants

The Kähler magnetic fluxes defined both the normal component of the Kähler magnetic field and by its absolute value

$$\begin{aligned} Q_m(X^2) &= \int_{X^2} J_{CP_2} = J_{\alpha\beta}\epsilon^{\alpha\beta}\sqrt{g_2}d^2x \ , \\ Q^+_m(X^2) &= \int_{X^2} |J_{CP_2}| \equiv \int_{X^2} |J_{\alpha\beta}\epsilon^{\alpha\beta}|\sqrt{g_2}d^2x \ , \end{aligned} \tag{3.5.2}$$

over suitably defined 2-surfaces are invariants under both Lorentz isometries and the symplectic transformations of CP_2 and can be calculated once X^3 is given.

For a closed surface $Q_m(X^2)$ vanishes unless the homology equivalence class of the surface is nontrivial in CP_2 degrees of freedom. In this case the flux is quantized. $Q^+_M(X^2)$ is non-vanishing for closed surfaces, too. Signed magnetic fluxes over non-closed surfaces depend on the boundary of X^2 only:

$$\begin{array}{l} \int_{X^2} J = \int_{\delta X^2} A \ . \\ J = dA \ . \end{array}$$

Un-signed fluxes can be written as sum of similar contributions over the boundaries of regions of X^2 in which the sign of J remains fixed.

$$\begin{aligned} Q_m(X^2) &= \int_{X^2} J_{CP_2} = J_{\alpha\beta}\epsilon^{\alpha\beta}\sqrt{g_2}d^2x \ , \\ Q^+_m(X^2) &= \int_{X^2} |J_{CP_2}| \equiv \int_{X^2} |J_{\alpha\beta}\epsilon^{\alpha\beta}|\sqrt{g_2}d^2x \ , \end{aligned} \tag{3.5.3}$$

There are also symplectic invariants, which are Lorentz covariants and defined as

$$\begin{aligned} Q_m(K,X^2) &= \int_{X^2} f_K J_{CP_2} \ , \\ Q^+_m(K,X^2) &= \int_{X^2} f_K |J_{CP_2}| \ , \\ f_{K\equiv(s,n,k)} &= e^{is\phi}\times\frac{\rho^{n-k}}{(1+\rho^2)^k}\times(\frac{r_M}{r_0})^k \end{aligned} \tag{3.5.4}$$

These symplectic invariants transform like an infinite-dimensional unitary representation of Lorentz group.

There must exist some minimal number of symplectically non-equivalent 2-surfaces of X^3, and the magnetic fluxes over the representatives these surfaces give thus good candidates for zero modes.

1. If effective 2-dimensionality is accepted, the surfaces X^2_i defined by the intersections of light like 3-D causal determinants X^3_l and X^3 provide a natural identification for these 2-surfaces.

2. Without effective 2-dimensionality the situation is more complex. Since symplectic transformations leave r_M invariant, a natural set of 2-surfaces X^2 appearing in the definition of fluxes are separate pieces for $r_M = constant$ sections of 3-surface. For a generic 3-surface, these surfaces are 2-dimensional and there is continuum of them so that discrete Fourier transforms of these invariants are needed. One must however notice that $r_M = constant$ surfaces could be be 3-dimensional in which case the notion of flux is not well-defined.

3.5.3 Isometry invariants and spin glass analogy

The presence of isometry invariants implies coset space decomposition $\cup_i G/H$. This means that quantum states are characterized, not only by the vacuum functional, which is just the exponential $exp(K)$ of Kähler function (Gaussian in lowest approximation) but also by a wave function in vacuum modes. Therefore the functional integral over the configuration space decomposes into an integral over zero modes for approximately Gaussian functionals determined by $exp(K)$. The weights for the various vacuum mode contributions are given by the probability density associated with the zero modes. The integration over the zero modes is a highly problematic notion and it could be eliminated if a localization in the zero modes occurs in quantum jumps. The localization would correspond to a state function reduction and zero modes would be effectively classical variables correlated in one-one manner with the quantum numbers associated with the quantum fluctuating degrees of freedom.

For a given orbit K depends on zero modes and thus one has mathematical similarity with spin glass phase for which one has probability distribution for Hamiltonians appearing in the partition function $exp(-H/T)$. In fact, since TGD Universe is also critical, exact similarity requires that also the temperature is critical for various contributions to the average partition function of spin glass phase. The characterization of isometry invariants and zero modes of the Kähler metric provides a precise characterization for how TGD Universe is quantum analog of spin glass.

The spin glass analogy has been the basic starting point in the construction of p-adic field theory limit of TGD. The ultra-metric topology for the free energy minima of spin glass phase motivates the hypothesis that effective quantum average space-time possesses ultra-metric topology. This approach leads to excellent predictions for elementary particle masses and predicts even new branches of physics [31, 26]. As a matter fact, an entire fractal hierarchy of copies of standard physics is predicted.

3.5.4 Magnetic flux representation of the symplectic algebra

Accepting the strong form of general coordinate invariance implying effective two-dimensionality WCW Hamiltonians correspond to the fluxes associated with various 2-surfaces X^2_i defined by the intersections of light-like light-like 3-surfaces $X^3_{l,i}$ with X^3 at the boundaries of CD considered. Bearing in mind that zero energy ontology is the correct approach, one can restrict the consideration on fluxes at $\delta M^4_+ \times CP_2$ One must also remember that if the proposed symmetries hold true, it is in principle choose any partonic 2-surface in the conjectured slicing of the Minkowskian space-time sheet to partonic 2-surfaces parametrized by the points of stringy world sheets.

Generalized magnetic fluxes

Isometry invariants are just special case of the fluxes defining natural coordinate variables for the configuration space. Symplectic transformations of CP_2 act as $U(1)$ gauge transformations on the Kähler potential of CP_2 (similar conclusion holds at the level of $\delta M^4_+ \times CP_2$).

One can generalize these transformations to local symplectic transformations by allowing the Hamiltonians to be products of the CP_2 Hamiltonians with the real and imaginary parts of the functions $f_{m,n,k}$ (see Eq. 3.12.22) defining the Lorentz covariant function basis H_A, $A \equiv (a, m, n, k)$ at the light cone boundary: $H_A = H_a \times f(m, n, k)$, where a labels the Hamiltonians of CP_2.

One can associate to any Hamiltonian H^A of this kind both signed and unsigned magnetic flux via the following formulas:

$$\begin{aligned} Q_m(H_A|X^2) &= \int_{X^2} H_A J \ , \\ Q_m^+(H_A|X^2) &= \int_{X^2} H_A|J| \ . \end{aligned} \tag{3.5.5}$$

Here X^2 corresponds to any surface X^2_i resulting as intersection of X^3 with $X^3_{l,i}$. Both signed and unsigned magnetic fluxes and their superpositions

$$Q_m^{\alpha,\beta}(H_A|X^2) = \alpha Q_m(H_A|X^2) + \beta Q_m^+(H_A|X^2) \ , \ A \equiv (a, s, n, k) \tag{3.5.6}$$

provide representations of Hamiltonians. Note that symplectic invariants $Q_m^{\alpha,\beta}$ correspond to $H^A = 1$ and $H^A = f_{s,n,k}$. $H^A = 1$ can be regarded as a natural central term for the Poisson bracket algebra. Therefore, the isometry invariance of Kähler magnetic and electric gauge fluxes follows as a natural consequence.

The obvious question concerns about the correct values of the parameters α and β. One possibility is that the flux is an unsigned flux so that one has $\alpha = 0$. This option is favored by the construction of the configuration space spinor structure involving the construction of the fermionic super charges anti-commuting to configuration space Hamiltonians: super charges contain the square root of flux, which must be therefore unsigned. Second possibility is that magnetic fluxes are signed fluxes so that β vanishes.

One can define also the electric counterparts of the flux Hamiltonians by replacing J in the defining formulas with its dual $*J$

$$*J_{\alpha\beta} = \epsilon_{\alpha\beta}^{\ \ \gamma\delta} J_{\gamma\delta} .$$

For $H_A = 1$ these fluxes reduce to ordinary Kähler electric fluxes. These fluxes are however not symplectic covariants since the definition of the dual involves the induced metric, which is not symplectic invariant. The electric gauge fluxes for Hamiltonians in various representations of the color group ought to be important in the description of hadrons, not only as string like objects, but quite generally. These degrees of freedom would be identifiable as non-perturbative degrees of freedom involving genuinely classical Kähler field whereas quarks and gluons would correspond to the perturbative degrees of freedom, that is the interactions between CP_2 type extremals.

Poisson brackets

From the symplectic invariance of the radial component of Kähler magnetic field it follows that the Lie-derivative of the flux $Q_m^{\alpha,\beta}(H_A)$ with respect to the vector field $X(H_B)$ is given by

$$X(H_B) \cdot Q_m^{\alpha,\beta}(H_A) = Q_m^{\alpha,\beta}(\{H_B, H_A\}) \ . \tag{3.5.7}$$

The transformation properties of $Q_m^{\alpha,\beta}(H_A)$ are very nice if the basis for H_B transforms according to appropriate irreducible representation of color group and rotation group. This in turn implies that the fluxes $Q_m^{\alpha,\beta}(H_A)$ as functionals of 3-surface on given orbit provide a representation for the Hamiltonian as a functional of 3-surface. For a given surface X^3, the Poisson bracket for the two fluxes $Q_m^{\alpha,\beta}(H_A)$ and $Q_m^{\alpha,\beta}(H_B)$ can be defined as

$$\begin{aligned} \{Q_m^{\alpha,\beta}(H_A), Q_m^{\alpha,\beta}(H_B)\} &\equiv X(H_B) \cdot Q_m^{\alpha,\beta}(H_A) \\ &= Q_m^{\alpha,\beta}(\{H_A, H_B\}) = Q_m^{\alpha,\beta}(\{H_A, H_B\}) \ . \end{aligned} \tag{3.5.8}$$

The study of configuration space gamma matrices identifiable as symplectic super charges demonstrates that the supercharges associated with the radial deformations vanish identically so that radial deformations correspond to zero norm degrees of freedom as one might indeed expect on physical grounds. The reason is that super generators involve the invariants $j^{ak}\gamma_k$ which vanish by $\gamma_{r_M} = 0$.

The natural central extension associated with the symplectic group of CP_2 ($\{p,q\} = 1!$) induces a central extension of this algebra. The central extension term resulting from $\{H_A, H_B\}$ when CP_2 Hamiltonians have $\{p,q\} = 1$ equals to the symplectic invariant $Q_m^{\alpha,\beta}(f(m_a + m_b, n_a + n_b, k_a + k_b))$ on the right hand side. This extension is however anti-symmetric in symplectic degrees of freedom rather than in loop space degrees of freedom and therefore does not lead to the standard Kac Moody type algebra.

Quite generally, the Virasoro and Kac Moody algebras of string models are replaced in TGD context by much larger symmetry algebras. Kac Moody algebra corresponds to the the deformations of light-like 3-surfaces respecting their light-likeness and leaving partonic 2-surfaces at δCD intact and are highly relevant to the elementary particle physics. This algebra allows a representation in terms of X_l^3 local Hamiltonians generating isometries of $\delta M^4_\pm \times CP_2$. Hamiltonian representation is essential for super-symmetrization since fermionic super charges anti-commute to Hamiltonians rather than vector fields: this is one of the deep differences between TGD and string models. Kac-Moody algebra does not contribute to configuration space metric since by definition the generators vanish at partonic 2-surfaces. This is essential for the coset space property.

A completely new algebra is the CP_2 symplectic algebra localized with respect to the light cone boundary and relevant to the configuration space geometry. This extends to $S^2 \times CP_2$ -or rather $\delta M^4_\pm \times CP_2$ symplectic algebra and this gives the strongest predictions concerning configuration space metric. The local radial Virasoro localized with respect to $S^2 \times CP_2$ acts in zero modes and has automatically vanishing norm with respect to configuration space metric defined by super charges.

3.5.5 Symplectic transformations of $\delta M^4_\pm \times CP_2$ as isometries and electric-magnetic duality

According to the construction of Kähler metric, symplectic transformations of $\delta M^4_\pm \times CP_2$ act as isometries whereas radial Virasoro algebra localized with respect to CP_2 has zero norm in the configuration space metric.

Hamiltonians can be organized into light like unitary representations of $so(3,1) \times su(3)$ and the symmetry condition $Zg(X,Y) = 0$ requires that the component of the metric is $so(3,1) \times su(3)$ invariant and this condition is satisfied if the component of metric between two different representations D_1 and D_2 of $so(3,1) \times su(3)$ is proportional to Glebch-Gordan coefficient $C_{D_1 D_2, D_S}$ between $D_1 \otimes D_2$ and singlet representation D_S. In particular, metric has components only between states having identical $so(3,1) \times su(3)$ quantum numbers.

Magnetic representation of configuration space Hamiltonians means the action of the symplectic transformations of the light cone boundary as configuration space isometries is an intrinsic property of the light cone boundary. If electric-magnetic duality holds true, the preferred extremal property only determines the conformal factor of the metric depending on zero modes. This is precisely as it should be if the group theoretical construction works. Hence it should be possible by a direct calculation check whether the metric defined by the magnetic flux Hamiltonians as half Poisson brackets in complex coordinates is invariant under isometries. Symplectic invariance of the metric means that matrix elements of the metric are left translates of the metric along geodesic lines starting from the origin of coordinates, which now naturally corresponds to the preferred extremal of Kähler action. Since metric derives from symplectic form this means that the matrix elements of symplectic form given by Poisson brackets of Hamiltonians must be left translates of their values at origin along geodesic line. The matrix elements in question are given by flux Hamiltonians and since symplectic transforms of flux Hamiltonian is flux Hamiltonian for the symplectic transform of Hamiltonian, it seems that the conditions are satisfied.

3.6 General expressions for the symplectic and Kähler forms

One can derive general expressions for symplectic and Kähler forms as well as Kähler metric of the configuration space. The fact that these expressions involve only first variation of the Kähler action implies huge simplification of the basic formulas. Duality hypothesis leads to further simplifications of the formulas.

3.6.1 Closedness requirement

The fluxes of Kähler magnetic and electric fields for the Hamiltonians of $\delta M^4_+ \times CP_2$ suggest a general representation for the components of the symplectic form of the configuration space. The basic requirement is that Kähler form satisfies the defining condition

$$X \cdot J(Y,Z) + J([X,Y],Z) + J(X,[Y,Z]) = 0 \ , \tag{3.6.1}$$

where X, Y, Z are now vector fields associated with Hamiltonian functions defining configuration space coordinates.

3.6.2 Matrix elements of the symplectic form as Poisson brackets

Quite generally, the matrix element of $J(X(H_A), X(H_B))$ between vector fields $X(H_A))$ and $X(H_B))$ defined by the Hamiltonians H_A and H_B of $\delta M^4_+ \times CP_2$ isometries is expressible as Poisson bracket

$$J^{AB} = J(X(H_A), X(H_B)) = \{H_A, H_B\} \ . \tag{3.6.2}$$

J^{AB} denotes contravariant components of the symplectic form in coordinates given by a subset of Hamiltonians. The magnetic flux Hamiltonians $Q_m^{\alpha,\beta}(H_{A,k})$ of Eq. 4.6.1 provide an explicit representation for the Hamiltonians at the level of configuration space so that the components of the symplectic form of the configuration space are expressible as classical charges for the Poisson brackets of the Hamiltonians of the light cone boundary:

$$J(X(H_A), X(H_B)) = Q_m^{\alpha,\beta}(\{H_A, H_B\}) \ . \tag{3.6.3}$$

Recall that the superscript α, β refers the coefficients of J and $|J|$ in the superposition of these Kähler magnetic fluxes. Note that $Q_m^{\alpha,\beta}$ contains unspecified conformal factor depending on symplectic invariants characterizing Y^3 and is unspecified superposition of signed and unsigned magnetic fluxes.

This representation does not carry information about the tangent space of space-time surface at the partonic 2-surface, which motivates the proposal that also electric fluxes are present and proportional to magnetic fluxes with a factor K, which is symplectic invariant so that commutators of flux Hamiltonians come out correctly. This would give

$$Q_m^{\alpha,\beta}(H_A)_{em} = Q_e^{\alpha,\beta}(H_A) + Q_m^{\alpha,\beta}(H_A) = (1+K)Q_m^{\alpha,\beta}(H_A) \ . \tag{3.6.4}$$

Since Kähler form relates to the standard field tensor by a factor $e/\hbar$, flux Hamiltonians are dimensionless so that commutators do not involve $\hbar$. The commutators would come as

$$Q_{em}^{\alpha,\beta}(\{H_A, H_B\}) \rightarrow (1+K)Q_m^{\alpha,\beta}(\{H_A, H_B\}) \ . \tag{3.6.5}$$

The factor $1 + K$ plays the same role as Planck constant in the commutators.

WCW Hamiltonians vanish for the extrema of the Kähler function as variational derivatives of the Kähler action. Hence Hamiltonians are good candidates for the coordinates appearing as coordinates in the perturbative functional integral around extrema (with maxima giving dominating contribution). It is clear that configuration space coordinates around a given extremum include only those Hamiltonians, which vanish at extremum (that is those Hamiltonians which span the tangent space of G/H) In Darboux coordinates the Poisson brackets reduce to the symplectic form

$$\begin{aligned} \{P^I, Q^J\} &= J^{IJ} = J_I \delta^{I,J} \ . \\ J_I &= 1 \ . \end{aligned} \tag{3.6.6}$$

It is not clear whether Darboux coordinates with $J_I = 1$ are possible in the recent case: probably the unit matrix on right hand side of the defining equation is replaced with a diagonal matrix depending on symplectic invariants so that one has $J_I \neq 1$. The integration measure is given by the symplectic volume element given by the determinant of the matrix defined by the Poisson brackets of the Hamiltonians appearing as coordinates. The value of the symplectic volume element is given by the matrix formed by the Poisson brackets of the Hamiltonians and reduces to the product

$$Vol = \prod_I J_I$$

in generalized Darboux coordinates.

Kähler potential (that is gauge potential associated with Kähler form) can be written in Darboux coordinates as

$$A = \sum_I J_I P_I dQ^I \ . \tag{3.6.7}$$

3.6.3 General expressions for Kähler form, Kähler metric and Kähler function

The expressions of Kähler form and Kähler metric in complex coordinates can obtained by transforming the contravariant form of the symplectic form from symplectic coordinates provided by Hamiltonians to complex coordinates:

$$J^{Z^i\bar{Z}^j} = iG^{Z^i\bar{Z}^j} \quad = \quad \partial_{H^A}Z^i\partial_{H^B}\bar{Z}^j J^{AB} \quad , \tag{3.6.8}$$

where J^{AB} is given by the classical Kahler charge for the light cone Hamiltonian $\{H^A, H^B\}$. Complex coordinates correspond to linear coordinates of the complexified Lie-algebra providing exponentiation of the isometry algebra via exponential mapping. What one must know is the precise relationship between allowed complex coordinates and Hamiltonian coordinates: this relationship is in principle calculable. In Darboux coordinates the expressions become even simpler:

$$J^{Z^i\bar{Z}^j} = iG^{Z^i\bar{Z}^j} \quad = \quad \sum_I J(I)(\partial_{P^i}Z^i\partial_{Q^I}\bar{Z}^j - \partial_{Q^I}Z^i\partial_{P^I}\bar{Z}^j) \quad . \tag{3.6.9}$$

Kähler function can be formally integrated from the relationship

$$\begin{aligned} A_{Z^i} &= i\partial_{Z^i}K \quad , \\ A_{\bar{Z}^i} &= -i\partial_{Z^i}K \quad . \end{aligned} \tag{3.6.10}$$

holding true in complex coordinates. Kähler function is obtained formally as integral

$$K \quad = \quad \int_0^Z (A_{Z^i}dZ^i - A_{\bar{Z}^i}d\bar{Z}^i) \quad . \tag{3.6.11}$$

3.6.4 $Diff(X^3)$ invariance and degeneracy and conformal invariances of the symplectic form

$J(X(H_A), X(H_B))$ defines symplectic form for the coset space G/H only if it is $Diff(X^3)$ degenerate. This means that the symplectic form $J(X(H_A), X(H_B))$ vanishes whenever Hamiltonian H_A or H_B is such that it generates diffeomorphism of the 3-surface X^3. If effective 2-dimensionality holds true, $J(X(H_A), X(H_B))$ vanishes if H_A or H_B generates two-dimensional diffeomorphism $d(H_A)$ at the surface X_i^2.

One can always write

$$J(X(H_A), X(H_B)) = X(H_A)Q(H_B|X_i^2) \quad .$$

If H_A generates diffeomorphism, the action of $X(H_A)$ reduces to the action of the vector field X_A of some X_i^2-diffeomorphism. Since $Q(H_B|r_M)$ is manifestly invariant under the diffemorphisms of X^2, the result is vanishing:

$$X_A Q(H_B|X_i^2) = 0 \quad ,$$

so that $Diff^2$ invariance is achieved.

The radial diffeomorphisms possibly generated by the radial Virasoro algebra do not produce trouble. The change of the flux integrand X under the infinitesimal transformation $r_M \rightarrow r_M + \epsilon r_M^n$ is given by $r_M^n dX/dr_M$. Replacing r_M with $r_M^{-n+1}/(-n+1)$ as variable, the integrand reduces to a total divergence dX/du the integral of which vanishes over the closed 2-surface X_i^2. Hence radial Virasoro generators having zero norm annihilate all matrix elements of the symplectic form. The induced metric of X_i^2 induces a unique conformal structure and since the conformal transformations of X_i^2 can be interpreted as a mere coordinate changes, they leave the flux integrals invariant.

3.6.5 Complexification and explicit form of the metric and Kähler form

The identification of the Kähler form and Kähler metric in symplectic degrees of freedom follows trivially from the identification of the symplectic form and definition of complexification. The requirement that Hamiltonians are eigen states of angular momentum (and possibly Lorentz boost generator), isospin and hypercharge implies physically natural complexification. In order to fix the complexification completely one must introduce some convention fixing which states correspond to 'positive' frequencies and which to 'negative frequencies' and which to zero frequencies that is to decompose the generators of the symplectic algebra to three sets Can_+, Can_- and Can_0. One must distinguish between Can_0 and zero modes, which are not considered here at all. For instance, CP_2 Hamiltonians correspond to zero modes.

The natural complexification relies on the imaginary part of the radial conformal weight whereas the real part defines the $g = t + h$ decomposition naturally. The wave vector associated with the radial logarithmic plane wave corresponds to the angular momentum quantum number associated with a wave in S^1 in the case of Kac Moody algebra. One can imagine three options.

1. It is quite possible that the spectrum of k_2 does not contain $k_2 = 0$ at all so that the sector Can_0 could be empty. This complexification is physically very natural since it is manifestly invariant under $SU(3)$ and $SO(3)$ defining the preferred spherical coordinates. The choice of $SO(3)$ is unique if the classical four-momentum associated with the 3-surface is time like so that there are no problems with Lorentz invariance.

2. If $k_2 = 0$ is possible one could have

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k=k1+ik_2}, k_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = 0\} \ . \end{aligned} \tag{3.6.12}$$

3. If it is possible to $n_2 \neq 0$ for $k_2 = 0$, one could define the decomposition as

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k}, k_2 > 0 \ \text{ or } \ k_2 = 0, n_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0 \ \text{ or} k_2 = 0, n_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = n_2 = 0\} \ . \end{aligned} \tag{3.6.13}$$

   In this case the complexification is unique and Lorentz invariance guaranteed if one can fix the $SO(2)$ subgroup uniquely. The quantization axis of angular momentum could be chosen to be the direction of the classical angular momentum associated with the 3-surface in its rest system.

The only thing needed to get Kähler form and Kähler metric is to write the half Poisson bracket defined by Eq. 3.14.15

$$\begin{aligned} J_f(X(H_A), X(H_B)) &= 2Im\,(iQ_f(\{H_A, H_B\}_{-+})) \ , \\ G_f(X(H_A), X(H_B)) &= 2Re\,(iQ_f(\{H_A, H_B\}_{-+})) \ . \end{aligned} \tag{3.6.14}$$

Symplectic form, and thus also Kähler form and Kähler metric, could contain a conformal factor depending on the isometry invariants characterizing the size and shape of the 3-surface. At this stage one cannot say much about the functional form of this factor.

3.6.6 Comparison of CP_2 Kähler geometry with configuration space geometry

The explicit discussion of the role of $g = t+h$ decomposition of the tangent space of the configuration space provides deep insights to the metric of the symmetric space. There are indeed many questions to be answered. To what point of configuration space (that is 3-surface) the proposed $g = t + h$ decomposition corresponds to? Can one derive the components of the metric and Kähler form from the Poisson brackets of complexified Hamiltonians? Can one characterize the point in question in terms of the properties of configuration space Hamiltonians? Does the central extension of the configuration space reduce to the symplectic central extension of the symplectic algebra or can one consider also other options?

Cartan decomposition for CP_2

A good manner to gain understanding is to consider the CP_2 metric and Kähler form at the origin of complex coordinates for which the sub-algebra $h = u(2)$ defines the Cartan decomposition.

1. $g = t + h$ decomposition depends on the point of the symmetric space in general. In case of CP_2 $u(2)$ sub-algebra transforms as $g \circ u(2) \circ g^{-1}$ when the point s is replaced by gsg^{-1}. This is expected to hold true also in case of configuration space (unless it is flat) so that the task is to identify the point of the configuration space at which the proposed decomposition holds true.
2. The Killing vector fields of h sub-algebra vanish at the origin of CP_2 in complex coordinates. The corresponding Hamiltonians need not vanish but their Poisson brackets must vanish. It is possible to add suitable constants to the Hamiltonians in order to guarantee that they vanish at origin.
3. It is convenient to introduce complex coordinates and decompose isometry generators to holomorphic components $J^a_+ = j^{ak}\partial_k$ and $j^a_- = j^{a\bar{k}}\partial_{\bar{k}}$. One can introduce what might be called half Poisson bracket and half inner product defined as

$$\begin{aligned}
\{H^a, H^b\}_{-+} &\equiv \partial_{\bar{k}} H^a J^{\bar{k}l}\partial_l H^b \\
&= j^{ak} J_{k\bar{l}} j^{b\bar{l}} = -i(j^a_+, j^b_-) \ .
\end{aligned} \tag{3.6.15}$$

One can express Poisson bracket of Hamiltonians and the inner product of the corresponding Killing vector fields in terms of real and imaginary parts of the half Poisson bracket:

$$\begin{aligned}
\{H^a, H^b\} &= 2Im\left(i\{H^a, H^b\}_{-+}\right) \ , \\
(j^a, j^b) &= 2Re\left(i(j^a_+, j^b_-)\right) = 2Re\left(i\{H^a, H^b\}_{-+}\right) \ .
\end{aligned} \tag{3.6.16}$$

What this means that Hamiltonians and their half brackets code all information about metric and Kähler form. Obviously this is of utmost importance in the case of the configuration space metric whose symplectic structure and central extension are derived from those of CP_2.

Consider now the properties of the metric and Kähler form at the origin.

1. The relations satisfied by the half Poisson brackets can be written symbolically as

$$\begin{aligned}
&\{h, h\}_{-+} = 0 \ , \\
&Re\left(i\{h, t\}_{-+}\right) = 0 \ , \quad Im\left(i\{h, t\}_{-+}\right) = 0 \ , \\
&Re\left(i\{t, t\}_{-+}\right) \neq 0 \ , \quad Im\left(i\{t, t\}_{-+}\right) \neq 0 \ .
\end{aligned} \tag{3.6.17}$$

2. The first two conditions state that h vector fields have vanishing inner products at the origin. The first condition states also that the Hamiltonians for the commutator algebra $[h,h] = SU(2)$ vanish at origin whereas the Hamiltonian for $U(1)$ algebra corresponding to the color hyper charge need not vanish although it can be made vanishing. The third condition implies that the Hamiltonians of t vanish at origin.

3. The last two conditions state that the Kähler metric and form are non-vanishing between the elements of t. Since the Poisson brackets of t Hamiltonians are Hamiltonians of h, the only possibility is that $\{t,t\}$ Poisson brackets reduce to a non-vanishing $U(1)$ Hamiltonian at the origin or that the bracket at the origin is due to the symplectic central extension. The requirement that all Hamiltonians vanish at origin is very attractive aesthetically and forces to interpret $\{t,t\}$ brackets at origin as being due to a symplectic central extension. For instance, for S^2 the requirement that Hamiltonians vanish at origin would mean the replacement of the Hamiltonian $H = cos(\theta)$ representing a rotation around z-axis with $H_3 = cos(\theta) - 1$ so that the Poisson bracket of the generators H_1 and H_2 can be interpreted as a central extension term.

4. The conditions for the Hamiltonians of $u(2)$ sub-algebra state that their variations with respect to g vanish at origin. Thus $u(2)$ Hamiltonians have extremum value at origin.

5. Also the Kähler function of CP_2 has extremum at the origin. This suggests that in the case of the configuration space the counterpart of the origin corresponds to the maximum of the Kähler function.

Cartan algebra decomposition at the level of configuration space

The discussion of the properties of CP_2 Kähler metric at origin provides valuable guide lines in an attempt to understand what happens at the level of the configuration space. The use of the half bracket for the configuration space Hamiltonians in turn allows to calculate the matrix elements of the configuration space metric and Kähler form explicitly in terms of the magnetic or electric flux Hamiltonians.

The earlier construction was rather tricky and formula-rich and not very convincing physically. Cartan decomposition had to be assigned with something and in lack of anything better it was assigned with Super Virasoro algebra, which indeed allows this kind of decompositions but without any strong physical justification. The realization that super-symplectic and super Kac-Moody symmetries define coset construction at the level of basic quantum TGD, and that this construction provides a realization of Equivalence Principle at microscopic level, forced eventually the realization that also the coset space decomposition of configuration space realizes Equivalence Principle geometrically.

It must be however emphasized that holography implying effective 2-dimensionality of 3-surfaces in some length scale resolution is absolutely essential for this construction since it allows to effectively reduce Kac-Moody generators associated with X^3_l to $X^2 = X^3_l \cap \delta M^4_\pm \times CP_2$. In the similar manner super-symplectic generators can be dimensionally reduced to X^2. Number theoretical compactification forces the dimensional reduction and the known extremals are consistent with it [33]. The construction of configuration space spinor structure and metric in terms of the second quantized spinor fields [18] relies to this picture as also the recent view about M-matrix [16].

In this framework the coset space decomposition becomes trivial.

1. The algebra g is labeled by color quantum numbers of CP_2 Hamiltonians and by the label (m,n,k) labeling the function basis of the light cone boundary. Also a localization with respect to X^2 is needed. This is a new element as compared to the original view.

2. Super Kac-Moody algebra is labeled by color octet Hamiltonians and function basis of X^2. Since Lie-algebra action does not lead out of irreps, this means that Cartan algebra decomposition is satisfied.

3.6.7 Comparison with loop groups

It is useful to compare the recent approach to the geometrization of the loop groups consisting of maps from circle to Lie group G [45], which served as the inspirer of the configuration space geometry approach but later turned out to not apply as such in TGD framework.

In the case of loop groups the tangent space T corresponds to the local Lie-algebra $T(k, A) = exp(ik\phi)T_A$, where T_A generates the finite-dimensional Lie-algebra g and ϕ denotes the angle variable of circle; k is integer. The complexification of the tangent space corresponds to the decomposition

$$T = \{X(k > 0, A)\} \oplus \{X(k < 0, A)\} \oplus \{X(k = 0, A)\} = T_+ \oplus T_- \oplus T_0$$

of the tangent space. Metric corresponds to the central extension of the loop algebra to Kac Moody algebra and the Kähler form is given by

$$J(X(k_1 < 0, A), X(k_2 > 0, B)) = k_2\delta(k_1 + k_2)\delta(A, B) \ .$$

In present case the finite dimensional Lie algebra g is replaced with the Lie-algebra of the symplectic transformations of $\delta M^4_+ \times CP_2$ centrally extended using symplectic extension. The scalar function basis on circle is replaced with the function basis on an interval of length Δr_M with periodic boundary conditions; effectively one has circle also now.

The basic difference is that one can consider two kinds of central extensions now.

1. Central extension is most naturally induced by the natural central extension ($\{p, q\} = 1$) defined by Poisson bracket. This extension is anti-symmetric with respect to the generators of the symplectic group: in the case of the Kac Moody central extension it is symmetric with respect to the group G. The symplectic transformations of CP_2 might correspond to non-zero modes also because they are not exact symmetries of Kähler action. The situation is however rather delicate since $k = 0$ light cone harmonic has a diverging norm due to the radial integration unless one poses both lower and upper radial cutoffs although the matrix elements would be still well defined for typical 3-surfaces. For Kac Moody group $U(1)$ transformations correspond to the zero modes. Light cone function algebra can be regarded as a local $U(1)$ algebra defining central extension in the case that only CP_2 symplectic transformations local with respect to δM^4_+ act as isometries: for Kac Moody algebra the central extension corresponds to an ordinary $U(1)$ algebra. In the case that entire light cone symplectic algebra defines the isometries the central extension reduces to a $U(1)$ central extension.

3.6.8 Symmetric space property implies Ricci flatness and isometric action of symplectic transformations

The basic structure of symmetric spaces is summarized by the following structural equations

$$\begin{aligned} &g = h + t \ , \\ &[h, h] \subset h \ , \quad [h, t] \subset t \ , \quad [t, t] \subset h \ . \end{aligned} \tag{3.6.18}$$

In present case the equations imply that all commutators of the Lie-algebra generators of $Can(\neq 0)$ having non-vanishing integer valued radial quantum number n_2, possess zero norm. This condition is extremely strong and guarantees isometric action of $Can(\delta M^4_+ \times CP_2)$ as well as Ricci flatness of the configuration space metric.

The requirement $[t, t] \subset h$ and $[h, t] \subset t$ are satisfied if the generators of the isometry algebra possess generalized parity P such that the generators in t have parity $P = -1$ and the generators belonging to h have parity $P = +1$. Conformal weight n must somehow define this parity. The first possibility to come into mind is that odd values of n correspond to $P = -1$ and even values to $P = 1$. Since n is additive in commutation, this would automatically imply $h \oplus t$ decomposition with the required properties. This assumption looks however somewhat artificial. TGD however forces a generalization of Super Algebras and N-S and Ramond type algebras can be combined to a larger algebra containing also Virasoro and Kac Moody generators labeled by half-odd integers. This suggests strongly that isometry generators are labeled by half integer conformal weight and that half-odd integer conformal weight corresponds to parity $P = -1$ whereas integer conformal weight corresponds to parity $P = 1$. Coset space would structure would state conformal invariance of the theory since super-symplectic generators with integer weight would correspond to zero modes.

Quite generally, the requirement that the metric is invariant under the flow generated by vector field X leads together with the covariant constancy of the metric to the Killing conditions

$$X \cdot g(Y,Z) = 0 = g([X,Y],Z) + g(Y,[X,Z]) \ . \tag{3.6.19}$$

If the commutators of the complexified generators in $Can(\neq 0)$ have zero norm then the two terms on the right hand side of Eq. (3.14.19) vanish separately. This is true if the conditions

$$Q_m^{\alpha,\beta}(\{H^A,\{H^B,H^C\}\}) = 0 \ , \tag{3.6.20}$$

are satisfied for all triplets of Hamiltonians in $Can_{\neq 0}$. These conditions follow automatically from the $[t,t] \subset h$ property and guarantee also Ricci flatness as will be found later.

It must be emphasized that for Kähler metric defined by purely magnetic fluxes, one cannot pose the conditions of Eq. (3.14.20) as consistency conditions on the initial values of the time derivatives of imbedding space coordinates whereas in general case this is possible. If the consistency conditions are satisfied for a single surface on the orbit of symplectic group then they are satisfied on the entire orbit. Clearly, isometry and Ricci flatness requirements and the requirement of time reversal invariance might well force Kähler electric alternative.

3.6.9 How to find Kähler function?

If one has found the expansion of configuration space Kähler form in terms of electric fluxes one can solve also the Kähler function from the defining partial differential equations $J_{k\bar{l}} = \partial k \partial_{\bar{l}} K$. The solution is not unique since the equation allows the symmetry

$$K \rightarrow K + f(z^k) + \overline{f(z^k)} \ ,$$

where f is arbitrary holomorphic function of z^k. This non-uniqueness is probably eliminated by the requirement that Kähler function vanishes for vacuum extremals. This in turn makes in principle possible to find the maxima of Kähler function and to perform functional integration perturbatively around them.

Electric-magnetic duality implies that, apart from conformal factor depending on isometry invariants, one can solve Kähler metric without any knowledge on the initial values of the time derivatives of the imbedding space coordinates. Apart from conformal factor the resulting geometry is purely intrinsic to δCH. The role of Kähler action is only to to define $Diff^4$ invariance and give the rule how the metric is translated to metric on arbitrary point of CH. The degeneracy of the preferred extrema also implies that configuration space has multi-sheeted structure analogous to that encountered in case of Riemann surfaces.

As shown in [34], very general assumptions inspired by the light-likeness of Kähler current for the known extremals combined with electric-magnetic duality imply the reduction of Kähler action for the preferred extremals to Chern-Simons terms at the ends of CD and at wormhole throats plus boundary term depending on induced metric so that one has almost topological QFT. The latter is due to the possibility to choose the gauge for Kähler potential to code information about conserved quantum numbers to the Kähler function and is the counterpart for the measurement interaction term in Dirac action. This term should correspond to a real part of a holomorphic function so that it does not contribute to the Kähler metric.

Also a promising concrete construction recipe for Kähler function is in terms of the modified Dirac operator [18]. The recipe is described briefly in the introduction. If the conjecture that Dirac determinant coincides with the exponent of Kähler action for a preferred extremal is correct, the value of the Kähler coupling strength follows as a prediction of the theory. From the construction it is clear that Dirac determinant involves only a finite number of eigenvalues of the modified Dirac operator and can thus be an algebraic or even rational number if eigenvalues have this property. The most satisfactory property of the construction is that one can use the intuition from the behavior of 2-D magnetic systems.

3.7 Ricci flatness and divergence cancelation

Divergence cancelation in configuration space integration requires Ricci flatness and in this section the arguments in favor of Ricci flatness are discussed in detail.

3.7.1 Inner product from divergence cancelation

Forgetting the delicacies related to the non-determinism of the Kähler action, the inner product is given by integrating the usual Fock space inner product defined at each point of the configuration space over the reduced configuration space containing only the 3-surfaces Y^3 belonging to $\delta H = \delta M^4_+ \times CP_2$ ('lightcone boundary') using the exponent $exp(K)$ as a weight factor:

$$\begin{aligned} \langle \Psi_1 | \Psi_2 \rangle &= \int \overline{\Psi}_1(Y^3)\Psi_2(Y^3)exp(K)\sqrt{G}dY^3 \ , \\ \overline{\Psi_1}(Y^3)\Psi_2(Y^3) &\equiv \langle \Psi_1(Y^3)|\Psi_2(Y^3)\rangle_{Fock} \ . \end{aligned} \tag{3.7.1}$$

The degeneracy for the preferred extremals of Kähler action implies additional summation over the degenerate extremals associated with Y^3. The restriction of the integration on light cone boundary is Diff^4 invariant procedure and resolves in elegant manner the problems related to the integration over Diff^4 degrees of freedom. A variant of the inner product is obtained dropping the bosonic vacuum functional $exp(K)$ from the definition of the inner product and by assuming that it is included into the spinor fields themselves. Probably it is just a matter of taste how the necessary bosonic vacuum functional is included into the inner product: what is essential that the vacuum functional $exp(K)$ is somehow present in the inner product.

The unitarity of the inner product follows from the unitary of the Fock space inner product and from the unitarity of the standard L^2 inner product defined by configuration space integration in the set of the L^2 integrable scalar functions. It could well occur that $Diff^4$ invariance implies the reduction of the configuration space integration to $C(\delta H)$.

Consider next the bosonic integration in more detail. The exponent of the Kähler function appears in the inner product also in the context of the finite dimensional group representations. For the representations of the noncompact groups (say $SL(2,R)$) in coset spaces (now $SL(2,R)/U(1)$ endowed with Kähler metric) the exponent of Kähler function is necessary in order to get square integrable representations [56]. The scalar product for two complex valued representation functions is defined as

$$(f,g) = \int \overline{f}gexp(nK)\sqrt{g}dV \ . \tag{3.7.2}$$

By unitarity, the exponent is an integer multiple of the Kähler function. In the present case only the possibility $n=1$ is realized if one requires a complete cancelation of the determinants. In finite dimensional case this corresponds to the restriction to single unitary representation of the group in question.

The sign of the action appearing in the exponent is of decisive importance in order to make theory stable. The point is that the theory must be well defined at the limit of infinitely large system. Minimization of action is expected to imply that the action of infinitely large system is bound from above: the generation of electric Kähler fields gives negative contributions to the action. This implies that at the limit of the infinite system the average action per volume is non-positive. For systems having negative average density of action vacuum functional $exp(K)$ vanishes so that only configurations with vanishing average action per volume have significant probability. On the other hand, the choice $exp(-K)$ would make theory unstable: probability amplitude would be infinite for all configurations having negative average action per volume. In the fourth part of the book it will be shown that the requirement that average Kähler action per volume cancels has important cosmological consequences.

Consider now the divergence cancelation in the bosonic integration. One can develop the Kähler function as a Taylor series around maximum of Kähler function and use the contravariant Kähler metric as a propagator. Gaussian and metric determinants cancel each other for a unique vacuum functional. Ricci flatness guarantees that metric determinant is constant in complex coordinates so that one avoids divergences coming from it. The non-locality of the Kähler function as a functional of the 3-surface serves as an additional regulating mechanism: if $K(X^3)$ were a local functional of X^3 one would encounter divergences in the perturbative expansion.

The requirement that quantum jump corresponds to a quantum measurement in the sense of quantum field theories implies that quantum jump involves localization in zero modes. Localization in the zero modes implies automatically p-adic evolution since the decomposition of the configuration space into sectors D_P labeled by the infinite primes P is determined by the corresponding decomposition in zero modes. Localization in zero modes would suggest that the calculation of the physical predictions does not involve integration over zero modes: this would dramatically simplify the calculational apparatus of the theory. Probably this simplification occurs at the level of practical calculations if U-matrix separates into a product of matrices associated with zero modes and fiber degrees of freedom.

One must also calculate the predictions for the ratios of the rates of quantum transitions to different values of zero modes and here one cannot actually avoid integrals over zero modes. To achieve this one is forced to define the transition probabilities for quantum jumps involving a localization in zero modes as

$$P(x,\alpha\rightarrow y,\beta)=\sum_{r,s}|S(r,\alpha\rightarrow s,\beta)|^2|\Psi_r(x)|^2|\Psi_s(y)|^2 \ ,$$

where x and y correspond to the zero mode coordinates and r and s label a complete state functional basis in zero modes and $S(r,m\rightarrow s,n)$ involves integration over zero modes. In fact, only in this manner the notion of the localization in the zero modes makes mathematically sense at the level of S-matrix. In this case also unitarity conditions are well-defined. In zero modes state function basis can be freely constructed so that divergence difficulties could be avoided. An open question is whether this construction is indeed possible.

Some comments about the actual evaluation of the bosonic functional integral are in order.

1. Since configuration space metric is degenerate and the bosonic propagator is essentially the contravariant metric, bosonic integration is expected to reduce to an integration over the zero modes. For instance, isometry invariants are variables of this kind. These modes are analogous to the parameters describing the conformal equivalence class of the orbit of the string in string models.

2. α_K is a natural small expansion parameter in configuration space integration. It should be noticed that α_K, when defined by the criticality condition, could also depend on the coordinates parameterizing the zero modes.

3. Semiclassical approximation, which means the expansion of the functional integral as a sum over the extrema of the Kähler function, is a natural approach to the calculation of the bosonic integral. Symmetric space property suggests that for the given values of the zero modes there is only single extremum and corresponds to the maximum of the Kähler function. There are theorems (Duistermaat-Hecke theorem) stating that semiclassical approximation is exact for certain systems (for example for integrable systems [48]). Symmetric space property suggests that Kähler function might possess the properties guaranteing the exactness of the semiclassical approximation. This would mean that the calculation of the integral $\int exp(K)\sqrt{G}dY^3$ and even more complex integrals involving configuration space spinor fields would be completely analogous to a Gaussian integration of free quantum field theory. This kind of reduction actually occurs in string models and is consistent with the criticality of the Kähler coupling constant suggesting that all loop integrals contributing to the renormalization of the Kähler action should vanish. Also the condition that configuration space integrals are continuable to p-adic number fields requires this kind of reduction.

3.7.2 Why Ricci flatness

It has been already found that the requirement of divergence cancelation poses extremely strong constraints on the metric of the configuration space. The results obtained hitherto are the following.

1. If the vacuum functional is the exponent of Kähler function one gets rid of the divergences resulting from the Gaussian determinants and metric determinants: determinants cancel each other.

2. The non-locality of the Kähler action gives good hopes of obtaining divergence free perturbation theory.

The following arguments show that Ricci flatness of the metric is a highly desirable property.

1. Dirac operator should be a well defined operator. In particular its square should be well defined. The problem is that the square of Dirac operator contains curvature scalar, which need not be finite since it is obtained via two infinite-dimensional trace operations from the curvature tensor. In case of loop spaces [45] the Kähler property implies that even Ricci tensor is only conditionally convergent. In fact, loop spaces with Kähler metric are Einstein spaces (Ricci tensor is proportional to metric) and Ricci scalar is infinite.

   In 3-dimensional case situation is even worse since the trace operation involves 3 summation indices instead of one! The conclusion is that Ricci tensor had better to vanish in vibrational degrees of freedom.

2. For Ricci flat metric the determinant of the metric is constant in geodesic complex coordinates as is seen from the expression for Ricci tensor [45]

$$R_{k\bar{l}} = \partial_k \partial_{\bar{l}} ln(det(g)) \tag{3.7.3}$$

   in Kähler metric. This obviously simplifies considerably functional integration over the configuration space: one obtains just the standard perturbative field theory in the sense that metric determinant gives no contributions to the functional integration.

3. The constancy of the metric determinant results not only in calculational simplifications: it also eliminates divergences. This is seen by expanding the determinant as a functional Taylor series with respect to the coordinates of the configuration space. In local complex coordinates the first term in the expansion of the metric determinant is determined by Ricci tensor

$$\delta\sqrt{g} \propto R_{k\bar{l}} z^k \bar{z}^l \quad . \tag{3.7.4}$$

   In configuration space integration using standard rules of Gaussian integration this term gives a contribution proportional to the contraction of the propagator with Ricci tensor. But since the propagator is just the contravariant metric one obtains Ricci scalar as result. So, in order to avoid divergences, Ricci scalar must be finite: this is certainly guaranteed if Ricci tensor vanishes.

4. The following group theoretic argument suggests that Ricci tensor either vanishes or is divergent. The holonomy group of the configuration space is a subgroup of $U(n = \infty)$ ($D = 2n$ is the dimension of the Kähler manifold) by Kähler property and Ricci flatness is guaranteed if the $U(1)$ factor is absent from the holonomy group. In fact Ricci tensor is proportional to the trace of the $U(1)$ generator and since this generator corresponds to an infinite dimensional unit matrix the trace diverges: therefore given element of the Ricci tensor is either infinite or vanishes. Therefore the vanishing of the Ricci tensor seems to be a mathematical necessity. This naive argument doesn't hold true in the case of loop spaces, for which Kähler metric with finite non-vanishing Ricci tensor exists [45]. Note however that also in this case the sum defining Ricci tensor is only conditionally convergent.

There are indeed good hopes that Ricci tensor vanishes. By the previous argument the vanishing of the Ricci tensor is equivalent with the absence of divergences in configuration space integration. That divergences are absent is suggested by the non-locality of the Kähler function as a functional of 3-surface: the divergences of local field theories result from the locality of interaction vertices. Ricci flatness in vibrational degrees of freedom is not only necessary mathematically. It is also appealing physically: one can regard Ricci flat configuration space as a vacuum solution of Einstein's equations $G^{\alpha\beta} = 0$.

3.7.3 Ricci flatness and Hyper Kähler property

Ricci flatness property is guaranteed if configuration space geometry is Hyper Kähler [49, 50] (there exists 3 covariantly constant antisymmetric tensor fields, which can be regarded as representations of quaternionic imaginary units). Hyper Kähler property guarantees Ricci flatness because the contractions of the curvature tensor appearing in the components of the Ricci tensor transform to traces over Lie algebra generators, which are $SU(n)$ generators instead of $U(n)$ generators so that the traces vanish. In the case of the loop spaces left invariance implies that Ricci tensor in the vibrational degrees is a multiple of the metric tensor so that Ricci scalar has an infinite value. This is basically due to the fact that Kac-Moody algebra has $U(1)$ central extension.

Consider now the arguments in favor of Ricci flatness of the configuration space.

1. The symplectic algebra of δM^4_+ takes effectively the role of the $U(1)$ extension of the loop algebra. More concretely, the $SO(2)$ group of the rotation group $SO(3)$ takes the role of $U(1)$ algebra. Since volume preserving transformations are in question, the traces of the symplectic generators vanish identically and in finite-dimensional this should be enough for Ricci flatness even if Hyper Kähler property is not achieved.

2. The comparison with CP_2 allows to link Ricci flatness with conformal invariance. The elements of the Ricci tensor are expressible in terms of traces of the generators of the holonomy group $U(2)$ at the origin of CP_2, and since $U(1)$ generator is non-vanishing at origin, the Ricci tensor is non-vanishing. In recent case the origin of CP_2 is replaced with the maximum of Kähler function and holonomy group corresponds to super-symplectic generators labelled by integer valued real parts k_1 of the conformal weights $k = k_1 + i\rho$. If generators with $k_1 = n$ vanish at the maximum of the Kähler function, the curvature scalar should vanish at the maximum and by the symmetric space property everywhere. These conditions correspond to Virasoro conditions in super string models.

   A possible source of difficulties are the generators having $k_1 = 0$ and resulting as commutators of generators with opposite real parts of the conformal weights. It might be possible to assume that only the conformal weights $k = k_1 + i\rho$, $k_1 = 0, 1, ...$ are possible since it is the imaginary part of the conformal weight which defines the complexification in the recent case. This would mean that the commutators involve only positive values of k_1.

3. In the infinite-dimensional case the Ricci tensor involves also terms which are non-vanishing even when the holonomy algebra does not contain $U(1)$ factor. It will be found that symmetric space property guarantees Ricci flatness even in this case and the reason is essentially the vanishing of the generators having $k_1 = n$ at the maximum of Kähler function.

There are also arguments in favor of the Hyper Kähler property.

1. The dimensions of the imbedding space and space-time are 8 and 4 respectively so that the dimension of configuration space in vibrational modes is indeed multiple of four as required by Hyper Kähler property. Hyper Kähler property requires a quaternionic structure in the tangent space of the configuration space. Since any direction on the sphere S^2 defined by the linear combinations of quaternionic imaginary units with unit norm defines a particular complexification physically, Hyper Kähler property means the possibility to perform complexification in S^2-fold manners.

2. S^2-fold degeneracy is indeed associated with the definition of the complex structure of the configuration space. First of all, the direction of the quantization axis for the spherical harmonics or for the eigen states of Lorentz Cartan algebra at δM^4_+ can be chosen in S^2-fold manners. Quaternion conformal invariance means Hyper Kähler property almost by definition and the S^2-fold degeneracy for the complexification is obvious in this case.

If these naive arguments survive a more critical inspection, the conclusion would be that the effective 2-dimensionality of light like 3-surfaces implying generalized conformal and symplectic symmetries would also imply Hyper Kähler property of the configuration space and make the theory well-defined mathematically. This obviously fixes the dimension of space-time surfaces as well as the dimension of Minkowski space factor of the imbedding space.

In the sequel we shall show that Ricci flatness is guaranteed provided that the holonomy group of the configuration space is isomorphic to some subgroup of $SU(n=\infty)$ instead of $U(n=\infty)$ (n is the complex dimension of the configuration space) implied by the Kähler property of the metric. We also derive an expression for the Ricci tensor in terms of the structure constants of the isometry algebra and configuration space metric. The expression for the Ricci tensor is formally identical with that obtained by Freed for loop spaces: the only difference is that the structure constants of the finite-dimensional group are replaced with the group $Can(\delta H)$. Also the arguments in favor of Hyper Kähler property are discussed in more detail.

3.7.4 The conditions guaranteing Ricci flatness

In the case of Kähler geometry Ricci flatness condition can be characterized purely Lie-algebraically: the holonomy group of the Riemann connection, which in general is subgroup of $U(n)$ for Kähler manifold of complex dimension n, must be subgroup of $SU(n)$ so that the Lie-algebra of this group consists of traceless matrices. This condition is easy to derive using complex coordinates. Ricci tensor is given by the following expression in complex vielbein basis

$$R^{A\bar{B}} = R^{A\bar{C}B}_{\quad\ \bar{C}} \ , \tag{3.7.5}$$

where the latter summation is only over the antiholomorphic indices $\bar{C}$. Using the cyclic identities

$$\sum_{cycl\ \bar{C}B\bar{D}} R^{A\bar{C}B\bar{D}} = 0 \ , \tag{3.7.6}$$

the expression for Ricci tensor reduces to the form

$$R^{A\bar{B}} = R^{A\bar{B}C}_{\quad\ C} \ , \tag{3.7.7}$$

where the summation is only over the holomorphic indices C. This expression can be regarded as a trace of the curvature tensor in the holonomy algebra of the Riemann connection. The trace is taken over holomorphic indices only: the traces over holomorphic and anti-holomorphic indices cancel each other by the antisymmetry of the curvature tensor. For Kähler manifold holonomy algebra is subalgebra of $U(n)$, when the complex dimension of manifold is n and Ricci tensor vanishes if and only if the holonomy Lie-algebra consists of traceless matrices, or equivalently: holonomy group is subgroup of $SU(n)$. This condition is expected to generalize also to the infinite-dimensional case.

We shall now show that if configuration space metric is Kähler and possesses infinite-dimensional isometry algebra with the property that its generators form a complete basis for the tangent space (every tangent vector is expressible as a superposition of the isometry generators plus zero norm vector) it is possible to derive a representation for the Ricci tensor in terms of the structure constants of the isometry algebra and of the components of the metric and its inverse in the basis formed by the isometry generators and that Ricci tensor vanishes identically for the proposed complexification of the configuration space provided the generators $\{H_{A,m\neq 0}, H_{B,n\neq 0}\}$ correspond to zero norm vector fields of configuration space.

The general definition of the curvature tensor as an operator acting on vector fields reads

$$R(X,Y)Z = [\nabla_X, \nabla_Y]Z - \nabla_{[X,Y]}Z \ . \tag{3.7.8}$$

If the vector fields considered are isometry generators the covariant derivative operator is given by the expression

$$\begin{aligned} \nabla_X Y &= (Ad_X Y - Ad^*_X Y - Ad^*_Y X)/2 \ , \\ (Ad^*_X Y, Z) &= (Y, Ad_X Z) \ , \end{aligned} \tag{3.7.9}$$

where $Ad_X Y = [X,Y]$ and Ad^*_X denotes the adjoint of Ad_X with respect to configuration space metric.

In the sequel we shall assume that the vector fields in question belong to the basis formed by the isometry generators. The matrix representation of Ad_X in terms of the structure constants $C_{X,Y:Z}$ of the isometry algebra is given by the expression

$$\begin{aligned} Ad^m_{Xn} &= C_{X,Y:Z}\hat{Y}_n Z^m \ , \\ [X,Y] &= C_{X,Y:Z}Z \ , \\ \hat{Y} &= g^{-1}(Y,V)V \ , \end{aligned} \tag{3.7.10}$$

where the summation takes place over the repeated indices and $\hat{Y}$ denotes the dual vector field of Y with respect to the configuration space metric. From its definition one obtains for Ad^*_X the matrix representation

$$\begin{aligned} Ad^{*m}_{Xn} &= C_{X,Y:Z}\hat{Y}^m Z_n \ , \\ Ad^*_X Y &= C_{X,U:V}g(Y,U)g^{-1}(V,W)W = g(Y,U)g^{-1}([X,U],W)W \ , \end{aligned} \tag{3.7.11}$$

where the summation takes place over the repeated indices.

Using the representations of ∇_X in terms of Ad_X and its adjoint and the representations of Ad_X and Ad^*_X in terms of the structure constants and some obvious identities (such as $C_{[X,Y],Z:V} = C_{X,Y:U}C_{U,Z:V}$) one can by a straightforward but tedious calculation derive a more detailed expression for the curvature tensor and Ricci tensor. Straightforward calculation of the Ricci tensor has however turned to be very tedious even in the case of the diagonal metric and in the following we shall use a more convenient representation [45] of the curvature tensor applying in case of the Kähler geometry.

The expression of the curvature tensor is given in terms of the so called Toeplitz operators T_X defined as linear operators in the "positive energy part" G_+ of the isometry algebra spanned by the $(1,0)$ parts of the isometry generators. In present case the positive and negative energy parts and cm part of the algebra can be defined just as in the case of loop spaces:

$$\begin{aligned} G_+ &= \{H^{Ak}|k>0\} \ , \\ G_- &= \{H^{Ak}|k<0\} \ , \\ G_0 &= \{H^{Ak}|k=0\} \ . \end{aligned} \tag{3.7.12}$$

Here H^{Ak} denote the Hamiltonians generating the symplectic transformations of δH. The positive energy generators with non-vanishing norm have positive radial scaling dimension: $k \geq 0$, which corresponds to the imaginary part of the scaling momentum $K = k_1 + i\rho$ associated with the factors $(r_M/r_0)^K$. A priori the spectrum of ρ is continuous but it is quite possible that the spectrum of ρ is discrete and $\rho = 0$ does not appear at all in the spectrum in the sense that the flux Hamiltonians associated with $\rho = 0$ elements vanish for the maximum of Kähler function which can be taken to be the point where the calculations are done.

T_X differs from Ad_X in that the negative energy part of $Ad_X Y = [X,Y]$ is dropped away:

$$\begin{aligned} T_X : G_+ &\rightarrow G_+ \ , \\ Y &\rightarrow [X,Y]_+ \ . \end{aligned} \tag{3.7.13}$$

Here " + " denotes the projection to "positive energy" part of the algebra. Using Toeplitz operators one can associate to various isometry generators linear operators $\Phi(X_0)$, $\Phi(X_-)$ and $\Phi(X_+)$ acting on G_+:

$$\begin{aligned} \Phi(X_0) &= T_{X_0} \ , \ X_0 \varepsilon G_0 \ , \\ \Phi(X_-) &= T_{X_-} \ , \ X_- \varepsilon G_- \ , \\ \Phi(X_+) &= -T^*_{X_-} \ , \ X_+ \varepsilon G_+ \ . \end{aligned} \tag{3.7.14}$$

Here "*" denotes hermitian conjugate in the diagonalized metric: the explicit representation $\Phi(X_+)$ is given by the expression [45]

$$\begin{aligned}\Phi(X_+) &= D^{-1}T_{X_-}D \ , \\ DX_+ &= d(X)X_- \ , \\ d(X) &= g(X_-,X_+) \ . \end{aligned} \tag{3.7.15}$$

Here $d(X)$ is just the diagonal element of metric assumed to be diagonal in the basis used. denotes the conformal factor associated with the metric.

The representations for the action of ,$\Phi(X_0)$, $\Phi(X_-)$ and $\Phi(X_+)$ in terms of metric and structure constants of the isometry algebra are in the case of the diagonal metric given by the expressions

$$\begin{aligned}\Phi(X_0)Y_+ &= C_{X_0,Y_+:U_+}U_+ \ , \\ \Phi(X_-)Y_+ &= C_{X_-,Y_+:U_+}U_+ \ , \\ \Phi(X_+)Y_+ &= \frac{d(Y)}{d(U)}C_{X_-,Y_-:U_-}U_+ \ . \end{aligned} \tag{3.7.16}$$

The expression for the action of the curvature tensor in positive energy part G_+ of the isometry algebra in terms of the these operators is given as [45]:

$$R(X,Y)Z_+ = \{[\Phi(X),\Phi(Y)] - \Phi([X,Y])\}Z_+ \ . \tag{3.7.17}$$

The calculation of the Ricci tensor is based on the observation that for Kähler manifolds Ricci tensor is a tensor of type $(1,1)$, and therefore it is possible to calculate Ricci tensor as the trace of the curvature tensor with respect to indices associated with G_+.

$$Ricci(X_+,Y_-) = (\hat{Z}_+, R(X_+,Y_-)Z_+) \equiv Trace(R(X_+,Y_-)) \ , \tag{3.7.18}$$

where the summation over Z_+ generators is performed.

Using the explicit representations of the operators Φ one obtains the following explicit expression for the Ricci tensor

$$\begin{aligned}Ricci(X_+,Y_-) &= Trace\{[D^{-1}T_{X_+}D,T_{Y_-}] - T_{[X_+,Y_-]_{|G_0+G_-}} \\ &- D^{-1}T_{[X_+,Y_-]_{|G_+}}D\} \ . \end{aligned} \tag{3.7.19}$$

This expression is identical to that encountered in case of loop spaces and the following arguments are repetition of those applying in the case of loop spaces.

The second term in the Ricci tensor is the only term present in the finite-dimensional case. This term vanishes if the Lie-algebra in question consists of traceless matrices. Since symplectic transformations are volume-preserving the traces of Lie-algebra generators vanish so that this term is absent. The last term gives a non-vanishing contribution to the trace for the same reason.

The first term is quadratic in structure constants and does not vanish in case of loop spaces. It can be written explicitly using the explicit representations of the various operators appearing in the formula:

$$\begin{aligned}Trace\{[D^{-1}T_{X_-}D,T_{Y_-}]\} &= \sum_{Z_+,U_+}[C_{X_-,U_-:Z_-}C_{Y_-,Z_+:U_+}\frac{d(U)}{d(Z)} \\ &- C_{X_-,Z_-:U_-}C_{Y_-,U_+:Z_+}\frac{d(Z)}{d(U)}] \ . \end{aligned} \tag{3.7.20}$$

Each term is antisymmetric under the exchange of U and Z and one might fail to conclude that the sum vanishes identically. This is not the case. By the diagonality of the metric with respect to radial quantum number, one has $m(X_-) = m(Y_-)$ for the non-vanishing elements of the Ricci tensor. Furthermore, one has $m(U) = m(Z) - m(Y)$, which eliminates summation over $m(U)$ in the first term and summation over $m(Z)$ in the second term. Note however, that summation over other labels related to symplectic algebra are present.

By performing the change $U \to Z$ in the second term one can combine the sums together and as a result one has finite sum

$$\begin{aligned} \sum_{0<m(Z)<m(X)} [C_{X_-,U_-:Z_-} C_{Y_-,Z_+:U_+} \frac{d(U)}{d(Z)} &= C \sum_{0<m(Z)<m(X)} \frac{m(X)}{m(Z)-m(X)} \ , \\ C &= \sum_{Z,U} C_{X,U:Z} C_{Y,Z:U} \frac{d_0(U)}{d_0(Z)} \ . \end{aligned} \tag{3.7.21}$$

Here the dependence of $d(X) = |m(X)|d_0(X)$ on $m(X)$ is factored out; $d_0(X)$ does not depend on k_X. The dependence on $m(X)$ in the resulting expression factorizes out, and one obtains just the purely group theoretic term C, which should vanish for the space to be Ricci flat.

The sum is quadratic in structure constants and can be visualized as a loop sum. It is instructive to write the sum in terms of the metric in the symplectic degrees of freedom to see the geometry behind the Ricci flatness:

$$C = \sum_{Z,U} g([Y,Z],U) g^{-1}([X,U],Z) \ . \tag{3.7.22}$$

Each term of this sum involves a commutator of two generators with a non-vanishing norm. Since tangent space complexification is inherited from the local coset space, the non-vanishing commutators in complexified basis are always between generators in $Can_{\neq 0}$; that is they do not not belong to rigid $su(2) \times su(3)$.

The condition guaranteing Ricci flatness at the maximum of Kähler function and thus everywhere is simple. All elements of type $[X_{\neq 0}, Y_{\neq 0}]$ vanish or have vanishing norm. In case of CP_2 Kähler geometry this would correspond to the vanishing of the $U(2)$ generators at the origin of CP_2 (note that the holonomy group is $U(2)$ in case of CP_2). At least formally stronger condition is that the algebra generated by elements of this type, the commutator algebra associated with $Can_{\neq 0}$, consist of elements of zero norm. Already the (possibly) weaker condition implies that adjoint map $Ad_{X \neq 0}$ and its hermitian adjoint $Ad^*_{X_{\neq 0}}$ create zero norm states. Since isometry conditions involve also adjoint action the condition also implies that $Can_{\neq 0}$ acts as isometries. More concrete form for the condition is that all flux factors involving double Poisson bracket and three generators in $Can_{\neq 0}$ vanish:

$$Q_e(\{H_A, \{H_B, H_C\}\}) = 0 \ , \ \text{for} \ H_A, H_B, H_C \ \text{in} \ Can_{\neq 0} \ . \tag{3.7.23}$$

The vanishing of fluxes involving two Poisson brackets and three Hamiltonians guarantees isometry invariance and Ricci flatness and, as found in [22], is implied by the $[t,t] \subset h$ property of the Lie-algebra of coset space G/H having symmetric space structure.

The conclusion is that the mere existence of the proposed isometry group (guaranteed by the symmetric space property) implies the vanishing of the Ricci tensor and vacuum Einstein equations. The existence of the infinite parameter isometry group in turn follows basically from the condition guaranteing the existence of the Riemann connection. Therefore vacuum Einstein equations seem to arise, not only as a consequence of a physically motivated variational principle but as a mathematical consistency condition in infinite dimensional Kähler geometry. The flux representation seems to provide elegant manner to formulate and solve these conditions and isometry invariance implies Ricci flatness.

3.7.5 Is configuration space metric Hyper Kähler?

The requirement that configuration space integral integration is divergence free implies that configuration space metric is Ricci flat. The so called Hyper-Kähler metrics [50, 49, 60] are particularly nice representatives of Ricci flat metrics. In the following the basic properties of Hyper-Kähler metrics are briefly described and the problem whether Hyper Kähler property could realized in case of $M^4_+ \times CP_2$ is considered.

Hyper-Kähler property

Hyper-Kähler metric is a generalization of the Kähler metric. For Kähler metric metric tensor and Kähler form correspond to the complex numbers 1 and i and therefore define complex structure in the tangent space of the manifold. For Hyper Kähler metric tangent space allows three closed Kähler forms I, J, K, which with respect to the multiplication obey the algebra of quaternionic imaginary units and have square equal to - 1, which corresponds to the metric of Hyper Kähler space.

$$I^2 \;=\; J^2 = K^2 = -1 \;\; IJ = -JI = K, \text{ etc. } . \tag{3.7.24}$$

To define Kähler structure one must choose one of the Kähler forms or any linear combination of I, J and K with unit norm. The group $SO(3)$ rotates different Kähler structures to each other playing thus the role of quaternion automorphisms. This group acts also as coordinate transformations in Hyper Kähler manifold but in general fails to act as isometries.

If K is chosen to define complex structure then K is tensor of type $(1,1)$ in complex coordinates, I and J being tensors of type $(2,0)+(0,2)$. The forms $I+iJ$ and $I-iJ$ are holomorphic and anti-holomorphic forms of type $(2,0)$ and $(0,2)$ respectively and defined standard step operators I_+ and I_- of $SU(2)$ algebra. The holonomy group of Hyper-Kähler metric is always $Sp(k)$, $k \leq dimM/4$, the group of $k \times k$ unitary matrices with quaternionic entries. This group is indeed subgroup of SU(2k), so that its generators are traceless and Hyper Kähler metric is therefore Ricci flat.

Hyper Kähler metrics have been encountered in the context of 3-dimensional super symmetric sigma models: a necessary prerequisite for obtaining $N = 4$ super-symmetric sigma model is that target space allows Hyper Kähler metric [58, 60]. In particular, it has been found that Hyper Kähler property is decisive for the divergence cancelation.

Hyper-Kähler metrics arise also in monopole and instanton physics [50]. The moduli spaces for monopoles have Hyper Kähler property. This suggests that Hyper Kähler property is characteristic for the configuration (or moduli) spaces of 4-dimensional Yang Mills types systems. Since YM action appears in the definition of configuration space metric there are hopes that also in present case the metric possesses Hyper-Kähler property.

CP_2 allows what might be called almost Hyper-Kähler structure known as quaternionion structure. This means that the Weil tensor of CP_2 consists of three components in one-one correspondence with components of iso-spin and only one of them- the one corresponding to Kähler form- is covariantly constant. The physical interpretation is in terms of electroweak symmetry breaking selecting one isospin direction as a favored direction.

Does the 'almost' Hyper-Kähler structure of CP_2 lift to a genuine Hyper-Kähler structure in configuration space?

The Hyper-Kähler property of configuration space metric does not seem to be in conflict with the general structure of TGD.

1. In string models the dimension of the "space-time" is two and Weyl invariance and complex structures play a decisive role in the theory. In present case the dimension of the space-time is four and one therefore might hope that quaternions play a similar role. Indeed, Weyl invariance implies YM action in dimension 4 and as already mentioned moduli spaces of instantons and monopoles enjoy the Hyper Kähler property.

2. Also the dimension of the imbedding space is important. The dimension of Hyper Kähler manifold must be multiple of 4. The dimension of configuration space is indeed infinite multiple of 8: each vibrational mode giving one "8".

3. The complexification of the configuration space in symplectic degrees of freedom is inherited from $S^2 \times CP_2$ and CP_2 Kähler form defines the symplectic form of configuration space. The point is that CP_2 Weyl tensor has 3 covariantly constant components, having as their square

metric apart from sign. One of them is Kähler form, which is closed whereas the other two are non-closed forms and therefore fail to define Kähler structure. The group $SU(2)$ of electro-weak isospin rotations rotate these forms to each other. It would not be too suprising if one could identify the configuration space counterparts of these forms as representations of quaternionic units at the level of configuration space. The failure of the Hyper Kähler property at the level of CP_2 geometry is due to the electro-weak symmetry breaking and physical intuition (in particular, p-adic mass calculations [4]) suggests that electro-weak symmetry might not be broken at the level of configuration space geometry).

A possible topological obstruction for the Hyper Kähler property is related to the cohomology of the configuration space: the three Kähler forms must be co-homologically trivial as is clear from the following argument. If any of 3 quaternionic 2-form is cohomologically nontrivial then by $SO(3)$ symmetry rotating Kähler forms to each other all must be co-homologically nontrivial. On the other hand, electro-weak isospin rotation leads to a linear combination of 3 Kähler forms and the flux associated with this form is in general not integer valued. The point is however that Kähler form forms only the $(1,1)$ part of the symplectic form and must be co-homologically trivial whereas the zero mode part is same for all complexifications and can be co-homologically nontrivial. The co-homological non-triviality of the zero mode part of the symplectic form is indeed a nice feature since it fixes the normalization of the Kähler function apart from a multiplicative integer. On the other hand the hypothesis that Kähler coupling strength is analogous to critical temperature provides a dynamical (and perhaps equivalent) manner to fix the normalization of the Kähler function.

Since the properties of the configuration space metric are inherited from $M^4_+ \times CP_2$ then also the Hyper Kähler property should be understandable in terms of the imbedding space geometry. In particular, the complex structure in CP_2 vibrational degrees of freedom is inherited from CP_2. Hyper Kähler property implies the existence of a continuum (sphere S^2) of complex structures: any linear superposition of 3 independent Kähler forms defines a respectable complex structure. Therefore also CP_2 should have this continuum of complex structures and this is certainly not the case.

Indeed, if we had instead of CP_2 Hyper Kähler manifold with 3 covariantly constant 2-forms then it would be easy to understand the Hyper Kähler structure of configuration space. Given the Kähler structure of the configuration space would be obtained by replacing induced Kähler electric and magnetic fields in the definition of flux factors $Q(H_{A,m})$ with the appropriate component of the induced Weyl tensor. CP_2 indeed manages to be very nearly Hyper Kähler manifold!

How CP_2 fails to be Hyper Kähler manifold can be seen in the following manner. The Weyl tensor of CP_2 allows three independent components, which are self dual as 2-forms and rotated to each other by vielbein rotations.

$$\begin{aligned} W_{03} &= W_{12} \equiv 2I_3 = 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ W_{01} &= W_{23} \equiv I_1 = -e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ W_{02} &= W_{31} \equiv I_2 = -e^0 \wedge e^2 - e^3 \wedge e^1 \ . \end{aligned} \tag{3.7.25}$$

The component I_3 is just the Kähler form of CP_2. Remaining components are covariantly constant only with respect to spinor connection and not closed forms so that they cannot be interpreted as Maxwell fields. Their squares equal however apart from sign with the metric of CP_2, when appropriate normalization factor is used. If these forms were covariantly constant Kähler action defined by any linear superposition of these forms would indeed define Kähler structure in configuration space and the group $SO(3)$ would rotate these forms to each other. The projections of the components of the Weyl tensor on 3-surface define 3 vector fields as their duals and only one of these vector fields (Kähler magnetic field) is divergenceless. One might regard these 3 vector fields as counter parts of quaternion units associated with the broken Hyper Kähler structure, that is quaternion structure. The interpretation in terms of electro-weak symmetry breaking is obvious.

One cannot exclude the possibility that the symplectic invariance of the induced Kähler electric field implies that the electric parts of the other two components of induced Weyl tensor are symplectic invariants. This is the minimum requirement. What is however obvious is that the magnetic parts cannot be closed forms for arbitrary 3-surfaces at light cone boundary. One counter example is enough and CP_2 type extremals seem to provide this counter example: the components of the induced Weyl tensor are just the same as they are for CP_2 and clearly not symplecticly invariant.

Thus it seems that configuration space could allow Hyper Kähler structure broken by electro-weak interactions but it cannot be inherited from CP_2. An open question is whether it allows genuine quaternionic structure. Good prospects for obtaining quaternionic structure are provided by the quaternionic counterpart QP_2 of CP_2, which is 8-dimensional and has coset space structure $QP_2 = Sp(3)/Sp(2) \times Sp(1)$. This choice does not seem to be consistent with the symmetries of the standard model. Note however that the over all symmetry group is obtained by replacing complex numbers with quaternions on the matrix representation of the standard model group.

Could different complexifications for M^4_+ and light like surfaces induce Hyper Kähler structure for configuration space?

Quaternionic structure means also the existence of a family of complex structures parameterized by a sphere S^2. The complex structure of the configuration space is inherited from the complex structure of some light like surface.

In the case of the light cone boundary δM^4_+ the complex structure corresponds to the choice of quantization axis of angular momentum for the sphere $r_M = constant$ so that the coordinates orthogonal to the quantization axis define a complex coordinate: the sphere S^2 parameterizes these choices. Thus there is a temptation to identify the choice of quantization axis with a particular imaginary unit and Hyper Kähler structure would directly relate to the properties rotation group. This would bring an additional item to the list of miraculous properties of light like surfaces of 4-dimensional space-times.

This might relate to the fact that configuration space geometry is not determined by the symplectic algebra of CP_2 localized with respect to the light cone boundary as one might first expect but consists of $M^4_+ \times CP_2$ Hamiltonians so that infinitesimal symplectic transformation of CP_2 involves always also M^4_+-symplectic transformation. M^4_+ Hamiltonians are defined by a function basis generated as products of the Hamiltonians H_3 and $H_1 \pm iH_2$ generating rotations with respect to three orthogonal axes, and two of these Hamiltonians are complexified.

Also the light like 3-surfaces X^3_l associated with quaternion conformal invariance are determined by some 2-surface X^2 and the choice of complex coordinates and if X^2 is sphere the choices are labelled by S^2. In this case, the presence of quaternion conformal structure would be almost obvious since it is possible to choose some complex coordinate in several manners and the choices are labelled by S^2. The choice of the complex coordinate in turn fixes 2-surface X^2 as a surface for which the remaining coordinates are constant. X^2 need not however be located at the elementary particle horizon unless one poses additional constraint. One might hope that different choices of X^2 resulting in this manner correspond to all possible different selections of the complex structure and that this choice could fix uniquely the conformal equivalence class of X^2 appearing as argument in elementary particle vacuum functionals. If X^2 has a more complex topology the identification is not so clear but since conformal algebra SL(2,C) containing algebra of rotation group is involved, one might argue that the choice of quantization axis also now involves S^2 degeneracy. If these arguments are correct one could conclude that Hyper Kähler structure is implicitly involved and guarantees Ricci flatness of the configuration space metric.

3.8 Consistency conditions on metric

In this section various consistency conditions on the configuration space metric are discussed. In particular, it will be found that the conditions guaranteing the existence of Riemann connection in the set of all(!) vector fields (including zero norm vector fields) gives very strong constraints on the general form of the metric and that these constraints are indeed satisfied for the proposed metric.

3.8.1 Consistency conditions on Riemann connection

To study the consequences of the consistency conditions, it is most convenient to consider matrix elements of the metric in the basis formed by the isometry generators themselves. The consistency conditions state the covariant constancy of the metric tensor

$$\nabla_Z g(X,Y) = g(\nabla_Z X, Y) + g(X, \nabla_Z Y) = Z \cdot g(X,Y) \ . \tag{3.8.1}$$

$Z \cdot g(X,Y)$ vanishes, when Z generates isometries so that conditions state the covariant constancy of the matrix elements in this case. It must be emphasized that the ill defined-ness of the inner products of form $g(\nabla_Z X, Y)$ is just the reason for requiring infinite-dimensional isometry group. The point is that $\nabla_Z X$ need not to belong to the Hilbert space spanned by the tangent vector fields since the terms of type $Zg(X,Y)$ do not necessarily exist mathematically [45]. The elegant solution to the problem is that all tangent space vector fields act as isometries so that these quantities vanish identically.

The conditions of Eq. (3.16.1) can be written explicitly by using the general expression for the covariant derivative

$$\begin{aligned} g(\nabla_Z X, Y) &= [Zg(X,Y) + Xg(Z,Y) - Yg(Z,X) \\ &+ g(Ad_Z X - Ad^*_Z X - Ad^*_X Z, Y)]/2 \ . \end{aligned} \tag{3.8.2}$$

What happens is that the terms depending on the derivatives of the matrix elements (terms of type $Zg(X,Y)$) cancel each other (these terms vanish for the metric invariant under isometries), and one obtains the following consistency conditions

$$g(Ad_Z X - Ad^*_Z X - Ad^*_X Z, Y) + g(X, Ad_Z Y - Ad^*_Z Y - Ad^*_Y Z) = 0 \ . \tag{3.8.3}$$

Using the explicit representations of $Ad_Z X$ and $Ad *_Z X$ in terms of structure constants

$$\begin{aligned} Ad_Z X &= [Z,X] = C_{Z,X:U} U \ . \\ Ad^*_Z X &= C_{Z,U:V} g(X,V) g^{-1}(U,W) W = g(X,[Z,U]) g^{-1}(U,W) W \ . \end{aligned} \tag{3.8.4}$$

where the summation over repeated "indices" is performed, one finds that consistency conditions are identically satisfied provided the generators X and Y have a non-vanishing norm. The reason is that the contributions coming from $\nabla_Z X$ and $\nabla_Z Y$ cancel each other.

When one of the generators, say X, appearing in the inner product has a vanishing norm so that one has $g(X,Y) = 0$, for any generator Y, situation changes! The contribution of $\nabla_Z Y$ term to the consistency conditions drops away and using Eqs. (3.16.3) and (3.16.4) one obtains the following consistency conditions

$$C_{Z,X:U} g(U,Y) + C_{X,Y:U} g(U,Z) = -X \cdot g(Z,Y) \ . \tag{3.8.5}$$

Note that summation over U is carried out. If X is isometry generator (this need not be the case always) the condition reduces to a simpler form:

$$C_{X,Z:U} g(U,Y) + C_{X,Y:U} g(Z,U) = g([X,Z].Y) + g(Z,[X,Y]) = 0 \ . \tag{3.8.6}$$

These conditions have nice geometric interpretation. If the matrix elements are regarded as ordinary Hilbert space products between the isometry generators the conditions state that the metric defining the inner product behaves as a scalar in the general case.

3.8.2 Consistency conditions for the radial Virasoro algebra

The action of the radial Virasoro in nontrivial manner in the zero modes. Therefore isometry interpretation is excluded and consistency conditions do not make sense in this case. One can however consider the possibility that metric is invariant or suffers only an overall scaling under the action of the radial scaling generated by $L_0 = r_M d/dr_M$. Since the radial integration measure is scaling invariant and only powers of r_M/r_0 appear in Hamiltonians, the effect of the scaling $r_M \to \lambda r_M$ on the matrix elements of the metric is a scaling by $\lambda^{k_a+\bar{k}_b}$). One can interpret this by saying that scaling changes the values of zero modes and hence leads outside the symmetric space in question.

Invariance of reduced matrix element obtained by dividing away the powers of the scaling factor is achieved if the metric contains the conformal factor

$$S = \frac{1}{\Delta u} f(\frac{r_i}{r_j}) \ , \tag{3.8.7}$$

where r_i are the extrema of r_M interpreted as height function of X^3 and f is a priori arbitrary positive definite function. Since the presence of f presumably gives rise to renormalization corrections depending on the size and shape of 3-surface by scaling the propagator defined by the contravariant metric, the dependence on the ratios r_i/r_j should be slow, logarithmic dependence. Also the dependence on the Fourier components of the solid angles $\Omega(r_M)$ associated with the $r_M = constant$ sections is possible.

3.8.3 Explicit conditions for the isometry invariance

The identification of the Lie-algebra of isometry generators has been proposed but cannot provide any proof for the existence of the infinite parameter symmetry group at this stage. What one can do at this stage is to formulate explicitly the conditions guaranteing isometry invariance of the metric and try to see whether there are any hopes that these conditions are satisfied. It has been already found that the expression of the metric reduces for light cone alternative to the sum of two boundary terms coming from infinite future and from the boundary of the light cone. If the contribution from infinitely distant future vanishes, as one might expect, then only the contribution from the boundary of the light cone remains.

A tedious but straightforward evaluation of the second variation (see Appendix of the book) for Kähler action implies the following form for the second variation of the Kähler action

$$\delta^2 S = \int_{a=0}^{a=\infty} I^{n\beta}_{kl} \delta h^k D_\beta \delta h^l \ , \tag{3.8.8}$$

where the tensor $I^{\alpha\beta}_{kl}$ is defined as partial derivatives of the Kähler Lagrangian with respect to the derivatives $\partial_\alpha h^k$

$$I^{\alpha\beta}_{kl} = \partial_{\partial_\alpha h^k} \partial_{\partial_\beta h^l} L_M \ . \tag{3.8.9}$$

If the upper limit $a = \sqrt{(m^0)^2 - r_M^2} = \infty$ in the substitution vanishes then one can calculate second variation and therefore metric from the knowledge of the time derivatives $\partial_n h^k$ and $\partial_n \delta h^k$ on the boundary of the light cone only.

Kähler metric can be identified as the $(1,1)$ part of the second variation. This means that one can express the deformation as an element of the isometry algebra plus a arbitrary deformation in radial direction of the light cone boundary interpretable as conformal transformation of the light cone boundary. Radial contributions to the second variation are dropped (by definition of Kähler metric) and what remains is essentially a deformation in S^2 degrees of freedom.

The left invariance of the metric under the deformations of the isometry algebra implies an infinite number of conditions of the form

$$J^C g(J^A, J^B) = 0 \ , \tag{3.8.10}$$

where J^A, J^B and J^C denote the generators of the isometry group. These conditions ought to fix completely the time derivatives of the coordinates h^k for each 3-surface at light cone boundary and therefore in principle the whole minimizing four-surface provided the initial value problem associated with the Kähler action possesses a unique solution. What is nice that the requirement of isometry invariance in principle would provides solution to the problem of finding preferred extremals of the Kähler action.

These conditions, when written explicitly give infinite number of conditions for the time derivative of the generator J^C (we assume for a moment that C is held fixed and let A and B run) at the boundary of the light cone. Time derivatives are in principle determined also by the requirement that deformed surface corresponds to an absolute minimum of the Kähler action. The basis of δH scalar functions respecting color and rotational symmetries is the most promising one.

3.8.4 Direct consistency checks

If duality holds true, the most general form of the configuration space metric is defined by the fluxes $Q_m^{\alpha,\beta}$, where α and β are the coefficients of signed and unsigned magnetic fluxes. Present is also a conformal factor depending on those zero modes, which do not appear in the symplectic form and which characterize the size and shape of the 3-surface. $[t,t] \subset h$ property implying Ricci flatness and isometry property of symplectic transformations, requires the vanishing of the fluxes $Q_m^{\alpha,\beta}(\{H_{A,m\neq 0},\{H_{B,n\neq 0},H_{C,p\neq 0}\}\})$ associated with double commutators and poses strong consistency conditions on the metric. If n labelling symplectic generators has half integer values then the conditions simply state conformal invariance: generators labelled by integers have vanishing norm whereas half-odd integers correspond to non-vanishing norm. Isometry invariance gives additional conditions on fluxes $Q_m^{\alpha,\beta}$. Lorentz invariance strengthens these conditions further. It could be that these conditions fix the initial values of the imbedding space coordinates completely.

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3.9 Introduction

The most general expectation is that configuration space can be regarded as a union of coset spaces which are infinite-dimensional symmetric spaces with Kähler structure: $C(H) = \cup_i G/H(i)$. Index i labels 3-topology and zero modes. The group G, which can depend on 3-surface, can be identified as a subgroup of diffeomorphisms of $\delta M^4_+ \times CP_2$ and H must contain as its subgroup a group, whose action reduces to $Diff(X^3)$ so that these transformations leave 3-surface invariant.

The task is to identify plausible candidate for G and H and to show that the tangent space of the configuration space allows Kähler structure, in other words that the Lie-algebras of G and $H(i)$ allow complexification. One must also identify the zero modes and construct integration measure for the functional integral in these degrees of freedom. Besides this one must deduce information about the explicit form of configuration space metric from symmetry considerations combined with the hypothesis that Kähler function is Kähler action for a preferred extremal of Kähler action. One must of course understand what "preferred" means.

3.9.1 General Coordinate Invariance and generalized quantum gravitational holography

The basic motivation for the construction of configuration space geometry is the vision that physics reduces to the geometry of classical spinor fields in the infinite-dimensional configuration space of 3-surfaces of $M^4_+ \times CP_2$ or of $M^4 \times CP_2$. Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that configuration space possesses Kähler geometry. Kähler geometry is coded into Kähler function.

The original belief was that the four-dimensional general coordinate invariance of Kähler function reduces the construction of the geometry to that for the boundary of configuration space consisting of 3-surfaces on $\delta M^4_+ \times CP_2$, the moment of big bang. The proposal was that Kähler function $K(Y^3)$ could be defined as a preferred extremal of so called Kähler action for the unique space-time surface $X^4(Y^3)$ going through given 3-surface Y^3 at $\delta M^4_+ \times CP_2$. For Diff^4 transforms of Y^3 at $X^4(Y^3)$ Kähler function would have the same value so that Diff^4 invariance and degeneracy would be the outcome. The proposal was that the preferred extremals are absolute minima of Kähler action.

This picture turned out to be too simple.

1. I have already described the recent view about light-like 3-surfaces as generalized Feynman diagrams and space-time surfaces as preferred extremals of Kähler action and will not repeat what has been said.

2. It has also become obvious that the gigantic symmetries associated with $\delta M^4_\pm \times CP_2 \subset CD \times CP_2$ manifest themselves as the properties of propagators and vertices. Cosmological considerations, Poincare invariance, and the new view about energy favor the decomposition of the configuration space to a union of configuration spaces assignable to causal diamonds CDs defined as intersections of future and past directed light-cones. The minimum assumption is that CDs label the sectors of CH: the nice feature of this option is that the considerations of this chapter restricted to $\delta M^4_+ \times CP_2$ generalize almost trivially. This option is beautiful because the center of mass degrees of freedom associated with the different sectors of CH would correspond to M^4 itself and its Cartesian powers.

The definition of the Kähler function requires that the many-to-one correspondence $X^3 \rightarrow X^4(X^3)$ must be replaced by a bijective correspondence in the sense that X^3_l as light-like 3-surface is unique among all its Diff^4 translates. This also allows physically preferred "gauge fixing" allowing to get rid of the mathematical complications due to Diff4 degeneracy. The internal geometry of the space-time sheet must define the preferred 3-surface X^3_l.

The realization of this vision means a considerable mathematical challenge. The effective metric 2-dimensionality of 3-dimensional light-like surfaces X^3_l of M^4 implies generalized conformal and symplectic symmetries allowing to generalize quantum gravitational holography from light like boundary so that the complexities due to the non-determinism can be taken into account properly.

3.9.2 Light like 3-D causal determinants and effective 2-dimensionality

The light like 3-surfaces X^3_l of space-time surface appear as 3-D causal determinants. Basic examples are boundaries and elementary particle horizons at which Minkowskian signature of the induced metric transforms to Euclidian one. This brings in a second conformal symmetry related to the metric 2-dimensionality of the 3-D light-like 3-surface. This symmetry is identifiable as TGD counterpart of the Kac Moody symmetry of string models. The challenge is to understand the relationship of this symmetry to configuration space geometry and the interaction between the two conformal symmetries.

1. Field-particle duality is realized. Light-like 3-surfaces X^3_l -generalized Feynman diagrams - correspond to the particle aspect of field-particle duality whereas the physics in the interior of space-time surface $X^4(X^3_l)$ would correspond to the field aspect. Generalized Feynman diagrams in 4-D sense could be identified as regions of space-time surface having Euclidian signature.

2. One could also say that light-like 3-surfaces X^3_l and the space-like 3-surfaces X^3 in the intersections of $X^4(X^3_l) \cap CD \times CP_2$ where the causal diamond CD is defined as the intersections of future and past directed light-cones provide dual descriptions.

3. Generalized coset construction implies that the differences of super-symplectic and Super Kac-Moody type Super Virasoro generators annihilated physical states. This implies Equivalence Principle. This construction in turn led to the realization that configuration space for fixed values of zero modes - in particular the values of the induced Kähler form of $\delta M^4_\pm \times CP_2$ - allows identification as a coset space obtained by dividing the symplectic group of $\delta M^4_\pm \times CP_2$ with Kac-Moody group, whose generators vanish at $X^2 = X^3_l \times \delta M^4_\pm \times CP_2$. One can say that quantum fluctuating degrees of freedom in a very concrete sense correspond to the local variant of $S^2 \times CP_2$.

The analog of conformal invariance in the light-like direction of X_l^3 and in the light-like radial direction of $\delta M_\pm^4$ implies that the data at either X^3 or X_l^3 should be enough to determine configuration space geometry. This implies that the relevant data is contained to their intersection X^2 at least for finite regions of X^3. This is the case if the deformations of X_l^3 not affecting X^2 and preserving light likeness corresponding to zero modes or gauge degrees of freedom and induce deformations of X^3 also acting as zero modes. The outcome is effective 2-dimensionality. One must be however cautious in order to not make over-statements. The reduction to 2-D theory in global sense would trivialize the theory and the reduction to 2-D theory must takes places for finite region of X^3 only so one has in well defined sense three-dimensionality in discrete sense. A more precise formulation of this vision is in terms of hierarchy of CDs containing CDs containing.... The introduction of sub-CD:s brings in improved measurement resolution and means also that effective 2-dimensionality is realized in the scale of sub-CD only.

One cannot over-emphasize the importance of the effective 2-dimensionality. It indeed simplifies dramatically the earlier formulas for configuration space metric involving 3-dimensional integrals over $X^3 \subset M_+^4 \times CP_2$ reducing now to 2-dimensional integrals. Note that X^3 is determined by preferred extremal property of $X^4(X_l^3)$ once X_l^3 is fixed and one can hope that this mapping is one-to-one.

3.9.3 Magic properties of light cone boundary and isometries of configuration space

The special conformal, metric and symplectic properties of the light cone of four-dimensional Minkowski space: δM_+^4, the boundary of four-dimensional light cone is metrically 2-dimensional(!) sphere allowing infinite-dimensional group of conformal transformations and isometries(!) as well as Kähler structure. Kähler structure is not unique: possible Kähler structures of light cone boundary are paramet3rized by Lobatchevski space $SO(3,1)/SO(3)$. The requirement that the isotropy group $SO(3)$ of S^2 corresponds to the isotropy group of the unique classical 3-momentum assigned to $X^4(Y^3)$ defined as a preferred extremum of Kähler action, fixes the choice of the complex structure uniquely. Therefore group theoretical approach and the approach based on Kähler action complement each other.

1. The allowance of an infinite-dimensional group of isometries isomorphic to the group of conformal transformations of 2-sphere is completely unique feature of the 4-dimensional light cone boundary. Even more, in case of $\delta M_+^4 \times CP_2$ the isometry group of δM_+^4 becomes localized with respect to CP_2! Furthermore, the Kähler structure of δM_+^4 defines also symplectic structure.

   Hence any function of $\delta M_+^4 \times CP_2$ would serve as a Hamiltonian transformation acting in both CP_2 and δM_+^4 degrees of freedom. These transformations obviously differ from ordinary local gauge transformations. This group leaves the symplectic form of $\delta M_+^4 \times CP_2$, defined as the sum of light cone and CP_2 symplectic forms, invariant. The group of symplectic transformations of $\delta M_+^4 \times CP_2$ is a good candidate for the isometry group of the configuration space.

2. The approximate symplectic invariance of Kähler action is broken only by gravitational effects and is exact for vacuum extremals. If Kähler function were exactly invariant under the symplectic transformations of CP_2, CP_2 symplectic transformations wiykd correspond to zero modes having zero norm in the Kähler metric of configuration space. This does not make sense since symplectic transformations of $\delta M^4 \times CP_2$ actually parameterize the quantum fluctuation degrees of freedom.

3. The groups G and H, and thus configuration space itself, should inherit the complex structure of the light cone boundary. The diffeomorphims of M^4 act as dynamical symmetries of vacuum extremals. The radial Virasoro localized with respect to $S^2 \times CP_2$ could in turn act in zero modes perhaps inducing conformal transformations: note that these transformations lead out from the symmetric space associated with given values of zero modes.

3.9.4 Symplectic transformations of $\delta M_+^4 \times CP_2$ as isometries of configuration space

The symplectic transformations of $\delta M_+^4 \times CP_2$ are excellent candidates for inducing symplectic transformations of the configuration space acting as isometries. There are however deep differences with respect to the Kac Moody algebras.

1. The conformal algebra of the configuration space is gigantic when compared with the Virasoro + Kac Moody algebras of string models as is clear from the fact that the Lie-algebra generator of a symplectic transformation of $\delta M^4_+ \times CP_2$ corresponding to a Hamiltonian which is product of functions defined in δM^4_+ and CP_2 is sum of generator of δM^4_+-local symplectic transformation of CP_2 and CP_2-local symplectic transformations of δM^4_+. This means also that the notion of local gauge transformation generalizes.

2. The physical interpretation is also quite different: the relevant quantum numbers label the unitary representations of Lorentz group and color group, and the four-momentum labeling the states of Kac Moody representations is not present. Physical states carrying no energy and momentum at quantum level are predicted. The appearance of a new kind of angular momentum not assignable to elementary particles might shed some light to the longstanding problem of baryonic spin (quarks are not responsible for the entire spin of proton). The possibility of a new kind of color might have implications even in macroscopic length scales.

3. The central extension induced from the natural central extension associated with $\delta M^4_+ \times CP_2$ Poisson brackets is anti-symmetric with respect to the generators of the symplectic algebra rather than symmetric as in the case of Kac Moody algebras associated with loop spaces. At first this seems to mean a dramatic difference. For instance, in the case of CP_2 symplectic transformations localized with respect to δM^4_+ the central extension would vanish for Cartan algebra, which means a profound physical difference. For $\delta M^4_+ \times CP_2$ symplectic algebra a generalization of the Kac Moody type structure however emerges naturally.

   The point is that δM^4_+-local CP_2 symplectic transformations are accompanied by CP_2 local δM^4_+ symplectic transformations. Therefore the Poisson bracket of two δM^4_+ local CP_2 Hamiltonians involves a term analogous to a central extension term symmetric with respect to CP_2 Hamiltonians, and resulting from the δM^4_+ bracket of functions multiplying the Hamiltonians. This additional term could give the entire bracket of the configuration space Hamiltonians at the maximum of the Kähler function where one expects that CP_2 Hamiltonians vanish and have a form essentially identical with Kac Moody central extension because it is indeed symmetric with respect to indices of the symplectic group.

3.9.5 Does the symmetric space property reduce to coset construction for Super Virasoro algebras?

The idea about symmetric space is extremely beautiful but it took a long time and several false alarms before the time was ripe for identifying the precise form of the Cartan decomposition $g = t+h$ satisfying the defining conditions

$$g = t + h \ , \quad [t,t] \subset h \ , \quad [h,t] \subset t \ . \tag{3.9.1}$$

The ultimate solution of the puzzle turned out to be amazingly simple and came only after quantum TGD was understood well enough.

Configuration space geometry allows two super-conformal symmetries. The first one corresponds to super-symplectic transformations acting at the level of imbedding space. The second one corresponds to super Kac-Moody symmetry acting as deformations of light-like 3-surfaces respecting their light-likeness. Super Kac-Moody algebra can be regarded as sub-algebra of super-symplectic algebra, and quantum states correspond to the coset representations for these two algebras so that the differences of the corresponding super-Virasoro generators annihilate physical states. This obviously generalizes Goddard-Olive-Kent construction [48]. The physical interpretation is in terms of Equivalence Principle. After having realized this it took still some time to realize that this coset representation and therefore also Equivalence Principle also corresponds to the coset structure of the configuration space!

The conclusion would be that t corresponds to super-symplectic algebra made also local with respect to X^3 and h corresponds to super Kac-Moody algebra. The experience with finite-dimensional coset spaces would suggest that super Kac-Moody generators interpreted in terms of h leave the points of configuration space analogous to the origin of say CP_2 invariant and in fact vanish at this point. Therefore super Kac-Moody generators should vanish for those 3-surfaces X^3_l which correspond to the origin of coset space. The maxima of Kähler function could correspond to this kind of points and could play also an essential role in the integration over configuration space by generalizing the Gaussian integration of free quantum field theories.

3.9.6 What effective 2-dimensionality and holography really mean?

Concerning the interpretation of Kac-Moody algebra there are some poorly understood points, which directly relate to what one means with holography.

1. The strongest view about effective 2-dimensionality (holography) is that for preferred extremals the partonic 2-surfaces X^2 at the ends of CD act as causal determinants fixing X_l^3 in the resolution defined by CD. A weaker view about holography is that light-like 3-surfaces with fixed ends give rise to same configuration space metric and the deformations of these surfaces by Kac-Moody algebra correspond to zero modes just like the interior degrees of freedom for space-like 3-surface do. Which of these options is the correct one? The same question can be posed in the case of space-like 3-surfaces.

2. The non-trivial action of Kac-Moody algebra in the interior of X_l^3 together with effective 2-dimensionality and holography would encourage the interpretation of Kac-Moody symmetries acting trivially at X^2 as gauge symmetries. Light-like 3-surfaces having fixed partonic 2-surfaces at their ends would be equivalent physically and effective 2-dimensionality and holography would be realized modulo gauge transformations.

3. There are also Kac-Moody generators which do not vanish at the ends of the X_l^3, and these would act as physical symmetries and their action would reduce at X^2 to symplectic action. This Kac-Moody algebra should appear in p-adic mass calculations. This seems to be in conflict with the idea that coset construction corresponds to coset space construction. Perhaps strict correspondence is too naive an assumption. Why couldn't one use the larger Kac-Moody algebra in coset construction and smaller Kac-Moody algebra in coset space construction?

4. Gauge symmetry property means that the Kähler metric of the configuration space is same for all gauge equivalent choices of X_l^3 and Kac-Moody deformations correspond to zero modes. Kähler function could differ by a real part of a holomorphic function of configuration space coordinates representing now Kac-Moody transforms of X_l^3. If Dirac determinant gives the exponent of Kähler function, the eigenvalues of the modified Dirac action can differ only by scalings with are products of holomorphic function of configuration space coordinates and its conjugates labeling different Kac-Moody transforms of X_l^3. This condition makes sense if one restricts the consideration to the finite number of eigenvalues λ_k assigned to D_K. The introduction of instanton term transforming the eigenvalues to $\lambda_k + \sqrt{n}$ would not allow his scaling.

   Either one must assume more general spectrum of form $\lambda_k + \sqrt{n}x_k$ with λ_k and x_k scaling in identical manner or that $n = 0$ modes are enough to define Kähler function. The latter option might be correct since the preferred extremal realizes effective 2-dimensionality at space-time level and conformal excitations break it so that they should not contribute to Kähler function. Also number theoretic universality favors this option. One cannot however exclude the first option. It must be admitted that the situation is not completely understood.

3.9.7 About the relationship between super-symplectic and super Kac-Moody algebras

The relationship between Kac-Moody and symplectic algebras is now relatively well understood but the physical interpretation of Kac-Moody algebra deserves attention. There are two Kac-Moody algebras: the smaller one leaves partonic 2-surfaces invariant and second one affects also them. Both of them are in dual relation to the symplectic algebra and these relations correspond to coset space construction and coset construction.

TGD inspired quantum measurement theory suggests that the super-symplectic algebra and smaller Kac-Moody algebra correspond to each other like classical and quantal degrees of freedom. Hence smaller Kac-Moody algebra would act in the zero modes of the configuration space metric. In the proposed construction this indeed is the case for Kac Moody algebra elements leaving partonic 2-surface invariant and appearing in the *coset space construction* but not for those Kac-Moody algebra elements affecting partonic 2-surface and allowing interpretation as sub-algebra of symplectic algebra and appearing in *coset construction*. This interpretation conforms also with the fact that Kac-Moody algebra generates massive excitations in p-adic thermodynamics.

In TGD inspired quantum measurement theory zero modes correspond to classical non-quantum fluctuating dynamical variables in 1-1 correspondence with quantum fluctuating degrees of freedom like the positions of the pointer of the measurement apparatus with the directions of spin of electron. Hence Kac-Moody algebra would define configuration space coordinates in terms of the map induced by correlation between classical and quantal degrees of freedom induced by entanglement. The choice of gauge selecting one particular light-like 3-surface X_l^3 could have thus interpretation as a map mapping quantum degrees of freedom to classical ones. This choice of gauge could be achieved by the addition of phase factor depending on quantum numbers assigned with the braid strands so that stationary phase approximation would select the preferred 3-surface with fluctuations around them allowed.

The dual relation between super symplectic algebra and bigger Kac-Moody algebra is realized in terms of coset construction. The idea inspired by Olive-Goddard-Kent coset construction is that the generators of Super Virasoro algebra corresponds to the differences of those associated with Super Kac-Moody and super-symplectic algebras. The justification comes from the miraculous geometry of the light cone boundary implying that Super Kac-Moody conformal symmetries of X^2 can be compensated by super-symplectic local radial scalings so that the differences of corresponding Super Virasoro generators annihilate physical states. If the central extension parameters are same, the resulting central extension is trivial. What is done is to construct first a state with a non-positive conformal weight using super-symplectic generators, and then to apply Super-Kac Moody generators to compensate this conformal weight to get a state with vanishing conformal weight. Mass squared would however correspond to either Super-Kac Moody or super-symplectic mass. The identity of these masses gives rise to Equivalence Principle as a one manifestation of the coset representation.

3.9.8 Attempts to identify configuration space Hamiltonians

I have made several attempts to identify configuration space Hamiltonians. The first two candidates referred to as magnetic and electric Hamiltonians, emerged in a relatively early stage. The third candidate identifies Hamiltonians as Noether charges and is motivated by the QFT analogy. Magnetic option is the simplest one and the only one consistent with the interpretation of Kac-Moody symmetries leaving the ends of X_l^3 invariant.

Magnetic Hamiltonians

Assuming that the elements of the radial Virasoro algebra of δM_+^4 have zero norm, one ends up with an explicit identification of the symplectic structures of the configuration space. There is almost unique identification for the symplectic structure. Configuration space counterparts of $\delta M^4 \times CP_2$ Hamiltonians are defined by the generalized signed and and unsigned Kähler magnetic fluxes

$$\begin{aligned} Q_m(H_A, X^2) &= Z \int_{X^2} H_A J \sqrt{g_2} d^2x \ , \\ Q_m^+(H_A, r_M) &= Z \int_{X^2} H_A |J| \sqrt{g_2} d^2x \ , \\ J &\equiv \epsilon^{\alpha\beta} J_{\alpha\beta} \ . \end{aligned}$$

H_A is CP_2 Hamiltonian multiplied by a function of coordinates of light cone boundary belonging to a unitary representation of the Lorentz group. Z is a conformal factor depending on symplectic invariants. The symplectic structure is induced by the symplectic structure of CP_2.

The most general flux is superposition of signed and unsigned fluxes Q_m and Q_m^+.

$$Q_m^{\alpha,\beta}(H_A, X^2) = \alpha Q_m(H_A, X^2) + \beta Q_m^+(H_A, X^2) \ .$$

Thus it seems that symmetry arguments fix the form of the configuration space metric apart from the presence of a conformal factor Z multiplying the magnetic flux and the degeneracy related to the signed and unsigned fluxes.

Holography requires that the relevant data about configuration space geometry is contained by 2-D surfaces X^2 at the intersections of light-like 3-surfaces $\delta M_\pm^4 + \times CP_2$ defining the boundaries of causal diamonds. In this case the entire Hamiltonian could be defined as the sum of magnetic fluxes over surfaces $X_i^2 \subset X^3$.

The key feature of these Hamiltonians is that they depend on X^2 only. This conforms with the interpretation of Kac-Moody transformations leaving X^2 invariant as gauge symmetries deforming light-like 3-surfaces and leaving configuration space metric as such. By the identify $g_{k\bar{l}} = iJ_{k\bar{l}}$ the half brackets $j^{Ak}J_{k\bar{l}}j^{B\bar{l}} = \partial_k H_A J^{k\bar{l}}\partial_{\bar{l}} H^B$ would define the matrix elements of both Kähler metric and Kähler form: this means a tight constraint if Kähler action defines the metric and magnetic Hamiltonians are the correct choice.

Electric Hamiltonians and electric-magnetic duality

Preferred extremal property allows to consider the possibility that one can identify configuration space Hamiltonians as classical charges $Q_e(H_A)$ associated with the Hamiltonians of the symplectic transformations of the light cone boundary, that is as variational derivatives of the Kähler action with respect to the infinitesimal deformations induced by $\delta M^4_+ \times CP_2$ Hamiltonians.

Alternatively, one might simply replace Kähler magnetic field J with Kähler electric field defined by space-time dual $*J$ in the formulas of previous section. These Hamiltonians are analogous to Kähler electric charge and the hypothesis motivated by the experience with the instantons of the Euclidian Yang Mills theories and 'Yin-Yang' principle, as well as by the duality of CP_2 geometry, is that for the preferred extremals of the Kähler action these Hamiltonians are affinely related:

$$Q_e(H_A) = Z\left[Q_m(H_A) + q_e(H_A)\right] \quad .$$

Here Z and q_e are constants depending on symplectic invariants only. Thus the equivalence of the two approaches to the construction of configuration space geometry boils down to the hypothesis of a physically well motivated electric-magnetic duality.

The crucial technical idea is to regard configuration space metric as a quadratic form in the entire Lie-algebra of the isometry group G such that the matrix elements of the metric vanish in the sub-algebra H of G acting as $Diff^3(X^3)$. The Lie-algebra of G with degenerate metric in the sense that H vector fields possess zero norm, can be regarded as a tangent space basis for the configuration space at point X^3 at which H acts as an isotropy group: at other points of the configuration space H is different. For given values of zero modes the maximum of Kähler function is the best candidate for X^3. This picture applies also in symplectic degrees of freedom.

There are objections against electric representation.

1. Without additional assumptions the Hamiltonians obtained by replacing induced Kähler form with its dual brings in the dependence on the induced metric of space-time surface at X^2 so that configuration space Hamiltonians do not transform nicely under symplectic transformations. Only if the contravariant Kähler electric field defines a symplectic invariant - maybe the preferred extremal property could guarantee this- electric representation of the Hamiltonians looks attractive. Electric-magnetic duality would follow trivially if the self duality of the induced Kähler form of CP_2 is preserved in the induction procedure at X^2.

2. Kac-Moody transformations vanishing at X^2 are not expected to leave the Hamiltonians invariant since they affect the induced metric. This is however highly desirable if effective 2-dimensionality holds true as gauge invariance.

3.9.9 For the reader

Few words about the representation of ideas are in order. For a long time the books about TGD served as kind of lab note books - a bottom-up representation providing kind of a ladder making clear the evolution of ideas. This led gradually to a rather chaotic situation in which it was difficult for me to control the internal consistency and for the possible reader to distinguish between the big ideas and ad hoc guesses, most of them related to the detailed realization of big visions. Therefore I decided to clean up a lot of the ad hoc stuff. I have also changed the representation so that it is more top-down and tries to achieve over-all views.

There are several visions about what TGD is and I have worked hardly to achieve a fusion of this visions. Hence simple linear representation in which reader climbs to a tree of wisdom is impossible.

I must summarize overall view from the beginning and refer to the results deduced in chapters towards the end of the book and also to ideas discussed in other books. For instance, the construction of configuration space spinor structure discussed in the last chapter [18] provides the understanding necessary to make the construction of configuration space geometry more detailed. Also number theoretical vision discussed in another book [6] is necessary. Somehow it seems that a graphic representation emphasizing visually the big picture should be needed to make the representation more comprehensible.

3.10 How to generalize the construction of configuration space geometry to take into account the classical non-determinism?

If the imbedding space were $H_+ = M^4_+ \times CP_2$ and if Kähler action were deterministic, the construction of configuration space geometry reduces to $\delta M^4_+ \times CP_2$. Thus in this limit quantum holography principle [59, 55] would be satisfied also in TGD framework and actually reduce to the general coordinate invariance. The classical non-determinism of Kähler action however means that this construction is not quite enough and the challenge is to generalize the construction.

3.10.1 Quantum holography in the sense of quantum gravity theories

In string theory context quantum holography is more or less synonymous with Maldacena conjecture [59] which (very roughly) states that string theory in Anti-de-Sitter space AdS is equivalent with a conformal field theory at the boundary of AdS. In purely quantum gravitational context [55], quantum holography principle states that quantum gravitational interactions at high energy limit in AdS can be described using a topological field theory reducing to a conformal (and non-gravitational) field theory defined at the *time like* boundary of the AdS. Thus the time like boundary plays the role of a dynamical hologram containing all information about correlation functions of $d+1$ dimensional theory. This reduction also conforms with the fact that black hole entropy is proportional to the horizon area rather than the volume inside horizon.

Holography principle reduces to general coordinate invariance in TGD. If the action principle assigning space-time surface to a given 3-surface X^3 at light cone boundary were completely deterministic, four-dimensional general coordinate invariance would reduce the construction of the configuration geometry for the space of 3-surfaces in $M^4_+ \times CP_2$ to the construction of the geometry at the boundary of the configuration space consisting of 3-surfaces in $\delta M^4_+ \times CP_2$ (moment of big bang). Also the quantum theory would reduce to the boundary of the future light cone.

The classical non-determinism of Kähler action however implies that quantum holography in this strong form fails. This is very desirable from the point of view of both physics and consciousness theory. Classical determinism would also mean that time would be lost in TGD as it is lost in GRT. Classical non-determinism is also absolutely essential for quantum consciousness and makes possible conscious experiences with contents localized into finite time interval despite the fact that quantum jumps occur between configuration space spinor fields defining what I have used to call quantum histories. Classical non-determinism makes it also possible to generalize quantum-classical correspondence in the sense that classical non-determinism at the space-time level provides correlate for quantum non-determinism. The failure of classical determinism is a difficult challenge for the construction of the configuration space geometry. One might however hope that the notion of quantum holography generalizes.

3.10.2 How the classical determinism fails in TGD?

One might hope that determinism in a generalized sense might be achieved by generalizing the notion of 3-surface by allowing unions of space-like 3-surfaces with time like separations with very strong but not complete correlations between the space-like 3-surfaces. In this case the non-determinism would mean that the 3-surfaces Y^3 at light cone boundary correspond to at most enumerable number of preferred extremals $X^4(Y^3)$ of Kähler action so that one would get finite or at most enumerably infinite number of replicas of a given configuration space region and the construction would still reduce to the light cone boundary.

1. This is probably quite too simplistic view. Any 4-surface which has CP_2 projection which belongs to so called Lagrange manifold of CP_2 having by definition vanishing induced Kähler

form is vacuum extremal. Thus there is an infinite variety of 6-dimensional sub-manifolds of H for which all extremals of Kähler action are vacua.

2. CP_2 type vacuum extremals are different since they possess non-vanishing Kähler form and Kähler action. They are identifiable as classical counterparts of elementary particles have M^4_+ projection which is a random light like curve (this in fact gives rise to conformal invariance identifiable as counterpart of quaternion conformal invariance). Thus there are good reasons to suspect that classical non-determinism might destroy the dream about complete reduction to the light cone boundary.

3. The wormhole contacts connecting different space-time sheets together can be seen as pieces of CP_2 type extremals and one expects that the non-determinism is still there and that the metrically 2-dimensional elementary particle horizons (light like 3-surfaces of H surrounding wormhole contacts and having time-like M^4_+ projection) might be a crucial element in the understanding of quantum TGD. The non-determinism of CP_2 type extremals is absolutely crucial for the ordinary elementary particle physics. It seems that the conformal symmetries responsible for the ordinary elementary particle quantum numbers acting in these degrees of freedom do not contribute to the configuration space metric line element.

4. The possibility of space-time sheets with a negative time orientation with ensuing negative sign of classical energy is a further blow against δM^4_+ reductionism. Space-time sheets can be created as pairs of positive and negative energy space-time sheet from vacuum and this forces to modify radically the ontology of physics. Crossing symmetry allows to interpret particle reactions as a creation of zero energy states from vacuum, and the identification of the gravitational energy as the difference between positive and negative energies of matter supports the view that the net inertial (conserved Poincare-) energy of the universe vanishes both in quantal and classical sense. This option resolves unpleasant questions about net conserved quantum numbers of Universe, and provides an elegant interpretation of the vacuum extremals as correlates for systems with vanishing Poincare energy. This option is the only possible alternative from the point of view of TGD inspired cosmology where Robertson-Walker metrics are vacuum extremals with respect to inertial energy. In particular, super-symplectic invariance transforms to a fundamental symmetry of elementary particle physics besides the conformal symmetry associated with 3-D light like causal determinants which means a dramatic departure from string models unless it is somehow equivalent with the super-symplectic symmetry.

The treatment of the non-determinism in a framework in which the prediction of time evolution is seen as initial value problem, seems to be difficult. Also the notion of configuration space becomes a messy concept. Zero energy ontology changes the situation completely. Light-like 3-surfaces become representations of generalized Feynman diagrams and brings in the notion of finite time resolution. One obtains adirect connection with the concepts of quantum field theory with path integral with cutoff replaced with a sum over various preferred extremals with cutoff in time resolution.

3.10.3 The notions of imbedding space, 3-surface, and configuration space

The notions of imbedding space, 3-surface (and 4-surface), and configuration space (world of classical worlds (WCW)) are central to quantum TGD. The original idea was that 3-surfaces are space-like 3-surfaces of $H = M^4 \times CP_2$ or $H = M^4_+ \times CP_2$, and WCW consists of all possible 3-surfaces in H. The basic idea was that the definition of Kähler metric of WCW assigns to each X^3 a unique space-time surface $X^4(X^3)$ allowing in this manner to realize general coordinate invariance. During years these notions have however evolved considerably. Therefore it seems better to begin directly from the recent picture.

The notion of imbedding space

Two generalizations of the notion of imbedding space were forced by number theoretical vision [21, 20, 19].

1. p-Adicization forced to generalize the notion of imbedding space by gluing real and p-adic variants of imbedding space together along rationals and common algebraic numbers. The

generalized imbedding space has a book like structure with reals and various p-adic number fields (including their algebraic extensions) representing the pages of the book.

2. With the discovery of zero energy ontology [30, 18] it became clear that the so called causal diamonds (CDs) interpreted as intersections $M^4_+ \cap M^4_-$ of future and past directed light-cones of $M^4 \times CP_2$ define correlates for the quantum states. The position of the "lower" tip of CD characterizes the position of CD in H. If the temporal distance between upper and lower tip of CD is quantized power of 2 multiples of CP_2 length, p-adic length scale hypothesis [25] follows as a consequence. The upper *resp.* lower light-like boundary $\delta M^4_+ \times CP_2$ *resp.* $\delta M^4_- \times CP_2$ of CD can be regarded as the carrier of positive *resp.* negative energy part of the state. All net quantum numbers of states vanish so that everything is creatable from vacuum. Space-time surfaces assignable to zero energy states would would reside inside $CD \times CP_2$s and have their 3-D ends at the light-like boundaries of $CD \times CP_2$. Fractal structure is present in the sense that CDs can contains CDs within CDs, and measurement resolution dictates the length scale below which the sub-CDs are not visible.

3. The realization of the hierarchy of Planck constants [20] led to a further generalization of the notion of imbedding space. Generalized imbedding space is obtained by gluing together Cartesian products of singular coverings and factor spaces of CD and CP_2 to form a book like structure. The particles at different pages of this book behave like dark matter relative to each other. This generalization also brings in the geometric correlate for the selection of quantization axes in the sense that the geometry of the sectors of the generalized imbedding space with non-standard value of Planck constant involves symmetry breaking reducing the isometries to Cartan subalgebra. Roughly speaking, each CD and CP_2 is replaced with a union of CDs and CP_2s corresponding to different choices of quantization axes so that no breaking of Poincare and color symmetries occurs at the level of entire WCW.

4. The construction of quantum theory at partonic level brings in very important delicacies related to the Kähler gauge potential of CP_2. Kähler gauge potential must have what one might call pure gauge parts in M^4 in order that the theory does not reduce to mere topological quantum field theory. Hence the strict Cartesian product structure $M^4 \times CP_2$ breaks down in a delicate manner. These additional gauge components -present also in CP_2- play key role in the model of anyons, charge fractionization, and quantum Hall effect [25].

The notions of 3-surface and space-time surface

The question what one exactly means with 3-surface turned out to be non-trivial.

1. The original identification of 3-surfaces was as arbitrary space-like 3-surfaces subject to Equivalence implied by General Coordinate Invariance. There was a problem related to the realization of General Coordinate Invariance since it was not at all obvious why the preferred extremal $X^4(Y^3)$ for Y^3 at $X^4(X^3)$ and Diff^4 related X^3 should satisfy $X^4(Y^3) = X^4(X^3)$.

2. Much later it became clear that light-like 3-surfaces have unique properties for serving as basic dynamical objects, in particular for realizing the General Coordinate Invariance in 4-D sense (obviously the identification resolves the above mentioned problem) and understanding the conformal symmetries of the theory. On basis of these symmetries light-like 3-surfaces can be regarded as orbits of partonic 2-surfaces so that the theory is locally 2-dimensional. It is however important to emphasize that this indeed holds true only locally. At the level of WCW metric this means that the components of the Kähler form and metric can be expressed in terms of data assignable to 2-D partonic surfaces. It is however essential that information about normal space of the 2-surface is needed.

3. At some stage came the realization that light-like 3-surfaces can have singular topology in the sense that they are analogous to Feynman diagrams. This means that the light-like 3-surfaces representing lines of Feynman diagram can be glued along their 2-D ends playing the role of vertices to form what I call generalized Feynman diagrams. The ends of lines are located at boundaries of sub-CDs. This brings in also a hierarchy of time scales: the increase of the measurement resolution means introduction of sub-CDs containing sub-Feynman diagrams. As

the resolution is improved, new sub-Feynman diagrams emerge so that effective 2-D character holds true in discretized sense and in given resolution scale only.

4. A further complication relates to the hierarchy of Planck constants forcing to generalize the notion of imbedding space and also to the fact that for non-standard values of Planck constant there is symmetry breaking due to preferred plane M^2 preferred homologically trivial geodesic sphere of CP_2 having interpretation as geometric correlate for the selection of quantization axis. For given sector of CH this means union over choices of this kind.

The basic vision forced by the generalization of General Coordinate Invariance has been that space-time surfaces correspond to preferred extremals $X^4(X^3)$ of Kähler action and are thus analogous to Bohr orbits. Kähler function $K(X^3)$ defining the Kähler geometry of the world of classical worlds would correspond to the Kähler action for the preferred extremal. The precise identification of the preferred extremals actually has however remained open.

The obvious but rather ad hoc guess motivated by physical intuition was that preferred extremals correspond to the absolute minima of Kähler action for space-time surfaces containing X^3. This choice has some nice implications. For instance, one can develop an argument for the existence of an infinite number of conserved charges. If X^3 is light-like surface- either light-like boundary of X^4 or light-like 3-surface assignable to a wormhole throat at which the induced metric of X^4 changes its signature- this identification circumvents the obvious objections. This option however failed to have a direct analog in the p-adic sectors of the world of classical worlds (WCW). The reason is that minimization does not make sense for the p-adic valued counterpart of Kähler action since it is not even well-defined although the field equations make sense p-adically. Therefore, if absolute minimization makes sense it must have expression as purely algebraic conditions.

Much later number theoretical compactication led to important progress in the understanding of the preferred extremals and the conjectures were consistent with what is known about the known extremals.

1. The conclusion was that one can assign to the 4-D tangent space $T(X^4(X^3_l)) \subset M^8$ a subspace $M^2(x) \subset M^4$ having interpretation as the plane of non-physical polarizations. This in the case that the induced metric has Minkowskian signature. If not, and if co-hyper-quaternionic surface is in question, similar assigned should be possible in normal space. This means a close connection with super string models. Geometrically this would mean that the deformations of 3-surface in the plane of non-physical polarizations would not contribute to the line element of WCW. This is as it must be since complexification does not make sense in M^2 degrees of freedom.

2. In number theoretical framework $M^2(x)$ has interpretation as a preferred hyper-complex sub-space of hyper-octonions defined as 8-D subspace of complexified octonions with the property that the metric defined by the octonionic inner product has signature of M^8. The condition $M^2(x) \subset T(X^4(X^3_l)))$ in principle fixes the tangent space at X^3_l, and one has good hopes that the boundary value problem is well-defined and could fix $X^4(X^3)$ at least partially as a preferred extremal of Kähler action. This picture is rather convincing since the choice $M^2(x) \subset M^4$ plays also other important roles.

3. At the level of H the counterpart for the choice of $M^2(x)$ seems to be following. Suppose that $X^4(X^3_l)$ has Minkowskian signature. One can assign to each point of the M^4 projection $P_{M^4}(X^4(X^3_l))$ a sub-space $M^2(x) \subset M^4$ and its complement $E^2(x)$, and the distributions of these planes are integrable and define what I have called Hamilton-Jacobi coordinates which can be assigned to the known extremals of Kähler with Minkowskian signature. This decomposition allows to slice space-time surfaces by string world sheets and their 2-D partonic duals. Also a slicing to 1-D light-like surfaces and their 3-D light-like duals Y^3_l parallel to X^3_l follows under certain conditions on the induced metric of $X^4(X^3_l)$. This decomposition exists for known extremals and has played key role in the recent developments. Physically it means that 4-surface (3-surface) reduces effectively to 3-D (2-D) surface and thus holography at space-time level.

4. The weakest form of number theoretic compactification [20] states that light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^8$, where $X^4(X^3)$ hyper-quaternionic surface in hyper-octonionic M^8 can be mapped to light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^4 \times CP_2$, where $X^4(X^3)$ is now preferred

extremum of Kähler action. The natural guess is that $X^4(X^3) \subset M^8$ is a preferred extremal of Kähler action associated with Kähler form of E^4 in the decomposition $M^8 = M^4 \times E^4$, where M^4 corresponds to hyper-quaternions. The conjecture would be that the value of the Kähler action in M^8 is same as in $M^4 \times CP_2$: in fact that 2-surface would have identical induced metric and Kähler form so that this conjecture would follow trivial. $M^8 - H$ duality would in this sense be Kähler isometry.

The study of the modified Dirac equation meant further steps of progress and lead to a rather detailed view about what preferred extremals are.

1. The detailed construction of the generalized eigen modes of the modified Dirac operator D_K associated with Kähler action [18] relies on the vision that the generalized eigenvalues of this operator code for information about preferred extremal of Kähler action. The view about TGD as almost topological QFT is realized if the eigenmodes correspond to the solutions of D_K, which are effectively 3-dimensional. Otherwise almost topological QFT property would require Chern-Simons action alone and this choice is definitely un-physical. The first guess was that the eigenmodes are restricted to X^3_l and therefore analogous to spinorial shock waves. As I realized that number theoretical compactification requires the slicing of $X^4(X^3_l)$ by light-like 3-surfaces Y^3_l parallel to X^3_l, it became clear that super-conformal gauge invariance with respect to the coordinate labeling the slices is a more natural manner to realized effective 3-dimensionality and guarantees that Y^3_l is gauge equivalent with X^3_l (General Coordinate Invariance).

2. The eigen modes of the modified Dirac operator D_K have the defining property that they are localized in regions of X^3_l, where the induced Kähler gauge field is non-vanishing. This guarantees that the number of generalized eigen modes is finite so that Dirac determinant is also finite and algebraic number if eigenvalues are algebraic numbers, and therefore makes sense also in p-adic context although Kähler action itself does not make sense p-adically.

3. The construction of the configuration space geometry in terms of modified Dirac action strengthens also the boundary conditions to the condition that there exists space-time coordinates in which the induced CP_2 Kähler form and induced metric satisfy the conditions $J_{ni} = 0$, $g_{ni} = 0$ hold at X^3_l. One could say that at X^3_l situation is static both metrically and for the Maxwell field defined by the induced Kähler form.

4. The final step in the rapid evolution of ideas that too place during three months - at least I hope so since I do not want to continue this updating endlessly - was the realization that the introduction of imaginary CP breaking instanton part to the Kähler action is possible and also necessary if one wants a stringy variant of Feynman rules. Imaginary part does not contribute to the configuration space metric. This enriches the spectrum of the modified Dirac operator with an infinite number of conformal excitations breaking the effective 2-dimensionality of 3-surfaces and exact holography. Conformal excitations make possible stringy Feynman diagrammatics [16]. A breaking of effective 3-dimensionality of space-time surface comes through the non-determinism of Kähler action which indeed is the mechanism breaking the effective 2-dimensionality. Dirac determinant can be defined in terms of zeta function regularization using Riemann Zeta. Finite measurement resolution realized in terms of braids defined on basis of purely physical criteria however forces a cutoff in conformal weight and finiteness so that number theoretical universality is not lost.

5. This picture relying crucially on the the slicing of $X^4(X^3)$ did not yet fix the definition of preferred extremals analytically at the level of field equations. The next step of progress was the realization that the requirement that the conservation of the Noether currents associated with the modified Dirac equation requires that the second variation of the Kähler action vanishes. In strongest form this condition would be satisfied for all variations and in weak sense only for those defining dynamical symmetries. The interpretation is as space-time correlate for quantum criticality and the vacuum degeneracy of Kähler action makes the criticality plausible. A generalization of the ideas of the catastrophe theory to infinite-dimensional context results [34]. These conditions make sense also in p-adic context and have a number theoretical universal form.

Although the details of this vision might change it can be defended by its ability to fuse together all great visions about quantum TGD. In the sequel the considerations are restricted to 3-surfaces in $M^4_+ \times CP_2$. The basic outcome is that Kähler metric is expressible using the data at partonic 2-surfaces $X^2 \subset \delta M^4_+ \times CP_2$. The generalization to the actual physical situation requires the replacement of $X^2 \subset \delta M^4_\pm \times CP_2$ with unions of partonic 2-surfaces located at light-like boundaries of CDs and sub-CDs.

The notion of configuration space

From the beginning there was a problem related to the precise definition of the configuration space ("world of classical worlds" (WCW)). Should one regard CH as the space of 3-surfaces of $M^4 \times CP_2$ or $M^4_+ \times CP_2$ or perhaps something more delicate.

1. For a long time I believed that the question "M^4_+ or M^4?" had been settled in favor of M^4_+ by the fact that M^4_+ has interpretation as empty Roberson-Walker cosmology. The huge conformal symmetries assignable to $\delta M^4_+ \times CP_2$ were interpreted as cosmological rather than laboratory symmetries. The work with the conceptual problems related to the notions of energy and time, and with the symmetries of quantum TGD, however led gradually to the realization that there are strong reasons for considering M^4 instead of M^4_+.

2. With the discovery of zero energy ontology it became clear that the so called causal diamonds (CDs) define excellent candidates for the fundamental building blocks of the configuration space or "world of classical worlds" (WCW). The spaces $CD \times CP_2$ regarded as subsets of H defined the sectors of WCW.

3. This framework allows to realize the huge symmetries of $\delta M^4_\pm \times CP_2$ as isometries of WCW. The gigantic symmetries associated with the $\delta M^4_\pm \times CP_2$ are also laboratory symmetries. Poincare invariance fits very elegantly with the two types of super-conformal symmetries of TGD. The first conformal symmetry corresponds to the light-like surfaces $\delta M^4_\pm \times CP_2$ of the imbedding space representing the upper and lower boundaries of CD. Second conformal symmetry corresponds to light-like 3-surface X^3_l, which can be boundaries of X^4 and light-like surfaces separating space-time regions with different signatures of the induced metric. This symmetry is identifiable as the counterpart of the Kac Moody symmetry of string models.

A rather plausible conclusion is that configuration space (WCW) is a union of configuration spaces associated with the spaces $CD \times CP_2$. CDs can contain CDs within CDs so that a fractal like hierarchy having interpretation in terms of measurement resolution results. Since the complications due to p-adic sectors and hierarchy of Planck constants are not relevant for the basic construction, it reduces to a high degree to a study of a simple special case $\delta M^4_+ \times CP_2$.

A further piece of understanding emerged from the following observations.

1. The induced Kähler form at the partonic 2-surface X^2 - the basic dynamical object if holography is accepted- can be seen as a fundamental symplectic invariant so that the values of $\epsilon^{\alpha\beta} J_{\alpha\beta}$ at X^2 define local symplectic invariants not subject to quantum fluctuations in the sense that they would contribute to the configuration space metric. Hence only induced metric corresponds to quantum fluctuating degrees of freedom at configuration space level and TGD is a genuine theory of gravitation at this level.

2. Configuration space can be divided into slices for which the induced Kähler forms of CP_2 and $\delta M^4_\pm$ at the partonic 2-surfaces X^2 at the light-like boundaries of CDs are fixed. The symplectic group of $\delta M^4_\pm \times CP_2$ parameterizes quantum fluctuating degrees of freedom in given scale (recall the presence of hierarchy of CDs).

3. This leads to the identification of the coset space structure of the sub-configuration space associated with given CD in terms of the generalized coset construction for super-symplectic and super Kac-Moody type algebras (symmetries respecting light-likeness of light-like 3-surfaces). Configuration space in quantum fluctuating degrees of freedom for given values of zero modes can be regarded as being obtained by dividing symplectic group with Kac-Moody group. Equivalently, the local coset space $S^2 \times CP_2$ is in question: this was one of the first ideas about configuration space which I gave up as too naive!

4. Generalized coset construction and coset space structure have very deep physical meaning since they realize Equivalence Principle at quantum level: the identical actions of Super Virasoro generators for super-symplectic and super Kac-Moody algebras implies that inertial and gravitational four-momenta are identical.

3.10.4 The treatment of non-determinism of Kähler action in zero energy ontology

The non-determinism of Kähler action means that the reduction of the construction of the configuration space geometry to the light cone boundary fails. Besides degeneracy of the preferred extrema of Kähler action, the non-determinism should manifest itself as a presence of causal determinants also other than light cone boundary.

One can imagine two kinds of causal determinants.

1. Elementary particle horizons and light-like boundaries $X^3_l \subset X^4$ of 4-surfaces representing wormhole throats act as causal determinants for the space-time dynamics defined by Kähler action. The boundary values of this dynamics have been already considered.

2. At imbedding space level causal determinants correspond to light like CD forming a fractal hierarchy of CDs within CDs. These causal determinants determine the dynamics of zero energy states having interpretation as pairs of initial and final states in standard quantum theory.

The manner to treat the classical non-determinism would be roughly following.

1. The replacement of space-like 3-surface X^3 with X^3_l transforms initial value problem for X^3 to a boundary value problem for X^3_l. In principle one can also use the surfaces $X^3 \subset \delta CD \times CP_2$ if X^3_l fixes $X^4(X^3_l)$ and thus X^3 uniquely. For years an important question was whether both X^3 and X^3_l contribute separately to the configuration space geometry or whether they provide descriptions, which are in some sense dual. This lead to the notion of 7-3 duality and I even considered the possibility that $\delta M^4_+ \times CP_2$ could be replaced with a more general surface $X^3_l \times CP_2$ allowing also generalized symplectic and conformal symmetries. 7-3 duality is not a good term since the actual duality actually relates descriptions based on space-like 3-surfaces X^3 and light-like 3-surfaces X^3_l. Hence it seems that the proper place for 7-3 duality is in paper basked.

2. Only Super-Kac-Moody type conformal algebra makes sense in the interior of X^3_l. In the 2-D intersections of X^3_l with the boundary of causal diamond (CD) defined as intersection of future and past directed light-cones super-symplectic algebra makes sense. This implies effective two-dimensionality which is broken by the non-determinism represented using the hierarchy of CDs meaning that the data from these 2-D surfaces and their normal spaces at boundaries of CDs in various scales determine the configuration space metric.

3. An important question has been whether Kac-Moody and super-symplectic algebras provide descriptions which are dual in some sense. At the level of Super-Virasoro algebras duality seems to be satisfied in the sense of generalized coset construction meaning that the differences of Super Virasoro generators of super-symplectic and super Kac-Moody algebras annihilate physical states. Among other things this means that four-momenta assignable to the two Super Virasoro representations are identical. T he interpretation is in terms of a generalization of Equivalence Principle [18, 30]. This gives also a justification for p-adic thermodynamics applying only to Super Kac-Moody algebra.

4. Light-like 3-surfaces can be regarded also as generalized Feynman diagrams. The finite length resolution mean means also a cutoff in the number of generalized Feynman diagrams and this number remains always finite if the light-like 3-surfaces identifiable as maxima of Kähler function correspond to the diagrams. The finiteness of this number is also essential for number theoretic universality since it guarantees that the elements of M-matrix are algebraic numbers if momenta and other quantum numbers have this property. The introduction of new sub-CDs means also introduction of zero energy states in corresponding time scale.

5. The notion of finite measurement resolution expressed in terms of hierarchy of CDs within CDs is important for the treatment of classical non-determinism. In a given resolution the non-determinism of Kähler action remains invisible below the time scale assigned to the smallest CDs. One could also say that complete non-determinism characterized in terms path integral with cutoff is replaced in TGD framework with the partial failure of classical non-determinism leading to generalized Feynman diagrams. This gives rise to to discrete coupling constant evolution and avoids the mathematical ill-definedness and infinities plaguing path integral formalism since the functional integral over 3-surfaces is well defined.

6. Dirac determinant defining vacuum functional is assumed to correspond to exponent of Kähler action for its preferred extremal. Dirac determinant is defined as a product of finite number of eigenvalues of the transverse part $D_K(X^2)$ of the modified Dirac operator D_K assumed to have decomposition $D_K = D_K(X^2) + D_K(Y^2)$ reflecting the dual slicings of X^4 to string world sheets Y^2 and partonic 2-surfaces X^2. The existence of the slicing is supported by the properties of known extremals of Kähler action and strongly suggested by number theoretical compactification, and it implies among other things dimensional reduction to Minkowskian string model like theory and its Euclidian equivalent allowing to understand how Equivalence Principle is realized at space-time level. Finite number for the eigenvalues raises even hope that in a given resolution the functional integral reduces to Gaussian integral over a finite-dimensional space of logarithms of eigenvalues.

7. One can ask why Kähler action and playing with all these delicacies related to the failure of complete determinism. After all, one could formally replace Kähler action with 4-volume as in brane models. Space-time surfaces would be minimal surfaces and Dirac operator would be standard Dirac operator for the induced metric. Dirac determinant would however become infinite since the modes would not be anymore analogs of cyclotron states necessarily localized to a finite region of X^3_l. Recall that for Kähler action X^3_l indeed decomposes into patches inside with induced Kähler form is non-vanishing and Dirac determinant defining the exponent of Kähler function is well-defined and finite without any regularization procedure. Hence Kähler action is completely unique.

3.10.5 Category theory and configuration space geometry

Due the effects caused by the classical non-determinism even classical TGD universes are very far from simple Cartesian clockworks, and the understanding of the general structure of the configuration space is a formidable challenge. Category theory is a branch of mathematics which is basically a theory about universal aspects of mathematical structures. Thus category theoretical thinking might help in disentangling the complexities of the configuration space geometry and the basic ideas of category theory are discussed in this spirit and as an innocent layman. It indeed turns out that the approach makes highly non-trivial predictions.

In zero energy ontology the effects of non-determinism are taken into account in terms of causal diamonds forming a hierarchical fractal structure. One must allow also the unions of CDs, CDs within CDs, and probably also overlapping of CDs, and there are good reasons to expert that CDs and corresponding algebraic structures could define categories. If one does not allow overlapping CDs then set theoretic inclusion map defines a natural arrow. If one allows both unions and intersections then CDs would form a structure analogous to the set of open sets used in set theoretic topology. One could indeed see CDs (or rather their Cartesian products with CP_2) as analogs of open sets in Minkowskian signature.

So called ribbon categories seem to be tailor made for the formulation of quantum TGD and allow to build bridge to topological and conformal field theories. This discussion based on standard ontology. In [17] rather detailed category theoretical constructions are discussed. Important role is played by the notion of operad [46, 53]: this structure can be assigned with both generalized Feynman diagrams and with the hierarchy of symplectic fusion algebras realizing symplectic analogs of the fusion rules of conformal field theories.

3.11 Identification of the symmetries and coset space structure of the configuration space

In this section the identification of the isometry group of the configuration space will be discussed at general level.

3.11.1 Reduction to the light cone boundary

The reduction to the light cone boundary would occur exactly if Kähler action were strictly deterministic. This is not the case but it is possible to generalize the construction at light cone boundary to the general case if causal diamonds define the basic structural units of the configuration space.

Old argument

The identification of the configuration space follows as a consequence of 4-dimensional Diff invariance. The right question to ask is the following one. How could one coordinatize the physical(!) vibrational degrees of freedom for 3-surfaces in Diff^4 invariant manner: coordinates should have same values for all Diff^4 related 3-surfaces belonging to the orbit of X^3? The answer is following:

1. Fix some 3-surface (call it Y^3) on the orbit of X^3 in Diff^4 invariant manner.
2. Use as configuration space coordinates of X^3 and all its diffeomorphs the coordinates parameterizing small deformations of Y^3. This kind of replacement is physically acceptable since metrically the configuration space is equivalent with $Map/Diff^4$.
3. Require that the fixing procedure is Lorentz invariant, where Lorentz transformations in question leave light M^4_+ invariant and thus act as isometries.

The simplest choice of Y^3 is the intersection of the orbit of 3-surface (X^4) with the set $\delta M^4_+ \times CP_2$, where δM^4_+ denotes the boundary of the light cone (moment of big bang):

$$Y^3 = X^4 \cap \delta M^4_+ \times CP_2 \tag{3.11.1}$$

Lorentz invariance allows also the choice $X \times CP_2$, where X corresponds to the hyperboloid $a = \sqrt{(m^0)^2 - r_M^2} = constant$ but only the proposed choice ($a = 0$) leads to a natural complexification in M^4 degrees of freedom. This choice is also cosmologically very natural and completely analogous to the quantum gravitational holography of string theories.

Configuration space has a fiber space structure. Base space consists of 3-surfaces $Y^3 \subset \delta M^4_+ \times CP_2$ and fiber consists of 3-surfaces on the orbit of Y^3 , which are Diff^4 equivalent with Y^3. The distance between the surfaces in the fiber is vanishing in configuration space metric. An elegant manner to avoid difficulties caused by Diff^4 degeneracy in configuration space integration is to *define* integration measure as integral over the reduced configuration space consisting of 3-surfaces Y^3 at the light cone boundary.

Situation is however quite not so simple. The vacuum degeneracy of Kähler action suggests strongly classical non-determinism so that there are several, possibly, infinite number of preferred extremals $X^4(Y^3)$ associated with given Y^3 on light cone boundary. This implies additional degeneracy.

One might hope that the reduced configuration space could be replaced by its covering space so that given Y^3 corresponds to several points of the covering space and configuration space has many-sheeted structure. Obviously the copies of Y^3 have identical geometric properties. Configuration space integral would decompose into a sum of integrals over different sheets of the reduced configuration space. Note that configuration space spinor fields are in general different on different sheets of the reduced configuration space.

Even this is probably not enough: it is quite possible that all light like surfaces of M^4 possessing Hamilton Jacobi structure (and thus interpretable as light fronts) are involved with the construction of the configuration space geometry. Because of their metric two-dimensionality the proposed construction should generalize. This would mean that configuration space geometry has also local laboratory scale aspects and that the general ideas might allow testing.

New version of the argument

This is was the argument for two decades ago. A more elegant formulation would in terms of light-like 3-surfaces connecting the boundaries of causal diamond taken as basic geometric objects and identified as generalized Feynman diagrams so that they are singular as manifolds at the vertices.

If both formulations are required to be correct, the only conclusion is that effective 2-dimensionality must hold true in the scale of given CD. In other words, the intersection $X^2 = X^3_l \cap X^3$ at the boundary of CD is effectively the basic dynamical unit. The failure of strict non-determinism however forces to introduce entire hierarchy of CDs responsible also for coupling constant evolution defined in terms of the measurement resolution identified as the size of the smallest CD present.

3.11.2 Configuration space as a union of symmetric spaces

In finite-dimensional context globally symmetric spaces are of form G/H and connection and curvature are independent of the metric, provided it is left invariant under G. The hope is that same holds true in infinite-dimensional context. The most one can hope of obtaining is the decomposition $C(H) = \cup_i G/H_i$ over orbits of G. One could allow also symmetry breaking in the sense that G and H depend on the orbit: $C(H) = \cup_i G_i/H_i$ but it seems that G can be chosen to be same for all orbits. What is essential is that these groups are infinite-dimensional. The basic properties of the coset space decomposition give very strong constraints on the group H, which certainly contains the subgroup of G, whose action reduces to diffeomorphisms of X^3.

Consequences of the decomposition

If the decomposition to a union of coset spaces indeed occurs, the consequences for the calculability of the theory are enormous since it suffices to find metric and curvature tensor for single representative 3-surface on a given orbit (contravariant form of metric gives propagator in perturbative calculation of matrix elements as functional integrals over the configuration space). The representative surface can be chosen to correspond to the maximum of Kähler function on a given orbit and one obtains perturbation theory around this maximum (Kähler function is not isometry invariant).

The task is to identify the infinite-dimensional groups G and H and to understand the zero mode structure of the configuration space. Almost twenty (seven according to long held belief!) years after the discovery of the candidate for the Kähler function defining the metric, it became finally clear that these identifications follow quite nicely from $Diff^4$ invariance and $Diff^4$ degeneracy as well as special properties of the Kähler action.

The guess (not the first one!) would be following. G corresponds to the symplectic transformations of $\delta M^4_\pm \times CP_2$ leaving the induced Kähler form invariant. If G acts as isometries the values of Kähler form at partonic 2-surfaces (remember effective 2-dimensionality) are zero modes and configuration space allows slicing to symplectic orbits of the partonic 2-surface with fixed induced Kähler form. Quantum fluctuating degrees of freedom would correspond to symplectic group and to the fluctuations of the induced metric. The group H dividing G would in turn correspond to the Kac-Moody symmetries respecting light-likeness of X^3_l and acting in X^3_l but trivially at the partonic 2-surface X^2. This coset structure was originally discovered via coset construction for super Virasoro algebras of super-symplectic and super Kac-Moody algebras and realizes Equivalence Principle at quantum level.

Configuration space isometries as a subgroup of $Diff(\delta M^4_+ \times CP_2)$

The reduction to light cone boundary leads to the identification of the isometry group as some subgroup of for the group G for the diffeomorphisms of $\delta M^4_+ \times CP_2$. These diffeomorphisms indeed act in a natural manner in δCH, the the space of 3-surfaces in $\delta M^4_+ \times CP_2$. Configuration space is expected to decompose to a union of the coset spaces G/H_i, where H_i corresponds to some subgroup of G containing the transformations of G acting as diffeomorphisms for given X^3. Geometrically the vector fields acting as diffeomorphisms of X^3 are tangential to the 3-surface. H_i could depend on the topology of X^3 and since G does not change the topology of 3-surface each 3-topology defines separate orbit of G. Therefore, the union involves sum over all topologies of X^3 plus possibly other 'zero modes'. Different topologies are naturally glued together since singular 3-surfaces intermediate between two 3-topologies correspond to points common to the two sectors with different topologies.

3.11.3 Isometries of configuration space geometry as symplectic transformations of $\delta M^4_+ \times CP_2$

During last decade I have considered several candidates for the group G of isometries of the configuration space as the sub-algebra of the subalgebra of $Diff(\delta M^4_+ \times CP_2)$. To begin with let us write the general decomposition of $diff(\delta M^4_+ \times CP_2)$:

$$diff(\delta M^4_+ \times CP_2) \quad = \quad S(CP_2) \times diff(\delta M^4_+) \oplus S(\delta M^4_+) \times diff(CP_2) \ . \tag{3.11.2}$$

Here $S(X)$ denotes the scalar function basis of space X. This Lie-algebra is the direct sum of light cone diffeomorphisms made local with respect to CP_2 and CP_2 diffeomorphisms made local with respect to light cone boundary.

The idea that entire diffeomorphism group would act as isometries looks unrealistic since the theory should be more or less equivalent with topological field theory in this case. Consider now the various candidates for G.

1. The fact that symplectic transformations of CP_2 and M^4_+ diffeomorphisms are dynamical symmetries of the vacuum extremals suggests the possibility that the diffeomorphisms of the light cone boundary and symplectic transformations of CP_2 could leave Kähler function invariant and thus correspond to zero modes. The symplectic transformations of CP_2 localized with respect to light cone boundary acting as symplectic transformations of CP_2 have interpretation as local color transformations and are a good candidate for the isometries. The fact that local color transformations are not even approximate symmetries of Kähler action is not a problem: if they were exact symmetries, Kähler function would be invariant and zero modes would be in question.

2. CP_2 local conformal transformations of the light cone boundary act as isometries of δM^4_+. Besides this there is a huge group of the symplectic symmetries of $\delta M^4_+ \times CP_2$ if light cone boundary is provided with the symplectic structure. Both groups must be considered as candidates for groups of isometries. $\delta M^4_+ \times CP_2$ option exploits fully the special properties of $\delta M^4_+ \times CP_2$, and one can develop simple argument demonstrating that $\delta M^4_+ \times CP_2$ symplectic invariance is the correct option. Also the construction of configuration space gamma matrices as super-symplectic charges supports $\delta M^4_+ \times CP_2$ option.

This picture remained same for a long time. The discovery that Kac-Moody algebra consisting of X^2 local symmetries generated by Hamiltonians of isometry sub-algebra of symplectic algebra forced to challenge this picture and ask whether also X^2-local transformations of symplectic group could be involved.

1. The basic condition is that the X^2 local transformation acts leaves induced Kähler form invariant apart from diffeomorphism. Denote the infinitesimal generator of X^2 local symplecto morphism by $\Phi_A(x)j^{Ak}$, where A labels Hamiltonians in the sum and by j^α the generator of X^2 diffeomorphism.

2. The invariance of $J = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ modulo diffeomorphism under the infinitesimal symplectic transformation gives

$$\{H^A, \Phi_A\} \quad \equiv \quad \partial_\alpha H^A \epsilon^{\alpha\beta}\partial_\beta \Phi_A = \partial_\alpha J j^\alpha \ . \tag{3.11.3}$$

3. Note that here the Poisson bracket is not defined by $J^\alpha\beta$ but $\epsilon^{\alpha\beta}$ defined by the induced metric. Left hand side reflects the failure of symplectomorphism property due to the dependence of $\Phi_A(x)$ on X^2 coordinate which and comes from the gradients of $\delta M^4 \times CP_2$ coordinates in the expression of the induced Kähler form. Right hand side corresponds to the action of infinitesimal diffeomorphism.

4. Let us assume that one can restrict the consideration to single Hamiltonian so that the transformation is generated by $\Phi(x)H_A$ and that to each $\Phi(x)$ there corresponds a diffeomorphism of X^2, which is a symplectic transformation of X^2 with respect to symplectic form $\epsilon^{\alpha\beta}$ and generated by Hamiltonian $\Psi(x)$. This transforms the invariance condition to

$$\{H^A,\Phi\} \equiv \partial_\alpha H^A\epsilon^{\alpha\beta}\partial_\beta\Phi = \partial_\alpha J\epsilon^{\alpha\beta}\partial_\beta\Psi_A = \{J,\Psi_A\} \ . \tag{3.11.4}$$

   This condition can be solved identically by assuming that Φ_A and Ψ are proportional to arbitrary smooth function of J:

$$\Phi = f(J) \ , \quad \Psi_A = -f(J)H_A \ . \tag{3.11.5}$$

   Therefore the X^2 local symplectomorphisms of H reduce to symplectic transformations of X^2 with Hamiltonians depending on single coordinate J of X^2. The analogy with conformal invariance for which transformations depend on single coordinate z is obvious. As far as the anti-commutation relations for induced spinor fields are considered this means that $J = consant$ curves behave as points points. For extrema of J appearing as candidates for points of number theoretic braids $J = constant$ curves reduce to points.

5. From the structure of the conditions it is easy to see that the transformations generate a Lie-algebra. For the transformations $\Phi^1_AH^A$ $\Phi^2_AH^A$ the commutator is

$$\Phi_A^{[1,2]} = f_A^{\ BC}\Phi_B\Phi_C \ , \tag{3.11.6}$$

   where $f_A^{\ BC}$ are the structure constants for the symplectic algebra of $\delta M^4_\pm \times CP_2$. From this form it is easy to check that Jacobi identifies are satisfied. The commutator has same form as the commutator of gauge algebra generators. BRST gauge symmetry is perhaps the nearest analog of this symmetry. In the case of isometries these transforms realized local color gauge symmetry in TGD sense.

6. If space-time surface allows a slicing to light-like 3-surfaces Y^3_l parallel to X^3_l, these conditions make sense also for the partonic 2-surfaces defined by the intersections of Y^3_l with $\delta M^4_\pm \times CP_2$ and "parallel" to X^2. The local symplectic transformations also generalize to their local variants in X^3_l. Light-likeness of X^3_l means effective metric 2-dimensionality so that 2-D Kähler metric and symplectic form as well as the invariant $J = \epsilon^{\alpha\beta}J_{\alpha\beta}$ exist. A straightforward calculation shows that the the notion of local symplectic transformation makes sense also now and formulas are exactly the same as above.

3.11.4 Identification of Kac-Moody symmetries

The Kac-Moody algebra of symmetries acting as symmetries respecting the light-likeness of 3-surfaces plays a crucial role in the identification of quantum fluctuating configuration space degrees of freedom contributing to the metric.

Identification of Kac-Moody algebra

The generators of bosonic super Kac-Moody algebra leave the light-likeness condition $\sqrt{g_3} = 0$ invariant. This gives the condition

$$\delta g_{\alpha\beta}Cof(g^{\alpha\beta}) = 0 \ , \tag{3.11.7}$$

Here Cof refers to matrix cofactor of $g_{\alpha\beta}$ and summation over indices is understood. The conditions can be satisfied if the symmetries act as combinations of infinitesimal diffeomorphisms $x^\mu \rightarrow x^\mu + \xi^\mu$ of X^3 and of infinitesimal conformal symmetries of the induced metric

$$\delta g_{\alpha\beta} = \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \tag{3.11.8}$$

Ansatz as an X^3-local conformal transformation of imbedding space

Write δh^k as a super-position of X^3-local infinitesimal diffeomorphisms of the imbedding space generated by vector fields $J^A = j^{A,k}\partial_k$:

$$\delta h^k = c_A(x) j^{A,k} \ . \tag{3.11.9}$$

This gives

$$\begin{aligned} c_A(x) & \left[D_k j^A_l + D_l j^A_k\right] \partial_\alpha h^k \partial_\beta h^l + 2\partial_\alpha c_A h_{kl} j^{A,k} \partial_\beta h^l \\ = & \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha\xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \end{aligned} \tag{3.11.10}$$

If an X^3-local variant of a conformal transformation of the imbedding space is in question, the first term is proportional to the metric since one has

$$D_k j^A_l + D_l j^A_k = 2h_{kl} \ . \tag{3.11.11}$$

The transformations in question includes conformal transformations of $H_\pm$ and isometries of the imbedding space H.

The contribution of the second term must correspond to an infinitesimal diffeomorphism of X^3 reducible to infinitesimal conformal transformation ψ^μ:

$$2\partial_\alpha c_A h_{kl} j^{A,k}\partial_\beta h^l = \xi^\mu \partial_\mu g_{\alpha\beta} + g_{\mu\beta}\partial_\alpha\xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \tag{3.11.12}$$

A rough analysis of the conditions

One could consider a strategy of fixing c_A and solving solving ξ^μ from the differential equations. In order to simplify the situation one could assume that $g_{ir} = g_{rr} = 0$. The possibility to cast the metric in this form is plausible since generic 3-manifold allows coordinates in which the metric is diagonal.

1. The equation for g_{rr} gives

$$\partial_r c_A h_{kl} j^{Ak}\partial_r h^k = 0 \ . \tag{3.11.13}$$

The radial derivative of the transformation is orthogonal to X^3. No condition on ξ^α results. If c_A has common multiplicative dependence on $c_A = f(r)d_A$ by a one obtains

$$d_A h_{kl} j^{Ak}\partial_r h^k = 0 \ . \tag{3.11.14}$$

so that J^A is orthogonal to the light-like tangent vector $\partial_r h^k$ X^3 which is the counterpart for the condition that Kac-Moody algebra acts in the transversal degrees of freedom only. The condition also states that the components g_{ri} is not changed in the infinitesimal transformation.

It is possible to choose $f(r)$ freely so that one can perform the choice $f(r) = r^n$ and the notion of radial conformal weight makes sense. The dependence of c_A on transversal coordinates is constrained by the transversality condition only. In particular, a common scale factor having free dependence on the transversal coordinates is possible meaning that X^3- local conformal transformations of H are in question.

2. The equation for g_{ri} gives

$$\partial_r \xi^i = \partial_r c_A h_{kl} j^{Ak} h^{ij} \partial_j h^k \quad . \tag{3.11.15}$$

The equation states that g_{ri} are not affected by the symmetry. The radial dependence of ξ^i is fixed by this differential equation. No condition on ξ^r results. These conditions imply that the local gauge transformations are dynamical with the light-like radial coordinate r playing the role of the time variable. One should be able to fix the transformation more or less arbitrarily at the partonic 2-surface X^2.

3. The three independent equations for g_{ij} give

$$\xi^\alpha \partial_\alpha g_{ij} + g_{kj} \partial_i \xi^k + g_{ki} \partial_j \xi^k = \partial_i c_A h_{kl} j^{Ak} \partial_j h^l \quad . \tag{3.11.16}$$

These are 3 differential equations for 3 functions ξ^α on 2 independent variables x^i with r appearing as a parameter. Note however that the derivatives of ξ^r do not appear in the equation. At least formally equations are not over-determined so that solutions should exist for arbitrary choices of c_A as functions of X^3 coordinates satisfying the orthogonality conditions. If this is the case, the Kac-Moody algebra can be regarded as a local algebra in X^3 subject to the orthogonality constraint.

This algebra contains as a subalgebra the analog of Kac-Moody algebra for which all c_A except the one associated with time translation and fixed by the orthogonality condition depends on the radial coordinate r only. The larger algebra decomposes into a direct sum of representations of this algebra.

Commutators of infinitesimal symmetries

The commutators of infinitesimal symmetries need not be what one might expect since the vector fields ξ^μ are functionals c_A and of the induced metric and also c_A depends on induced metric via the orthogonality condition. What this means that $j^{A,k}$ in principle acts also to ϕ_B in the commutator $[c_A J^A, c_B J^B]$.

$$\left[c_A J^A, c_B J^B\right] = c_A c_B J^{[A,B]} + J^A \circ c_B J^B - J^B \circ c_A J^A \quad , \tag{3.11.17}$$

where $\circ$ is a short hand notation for the change of c_B induced by the effect of the conformal transformation J^A on the induced metric.

Luckily, the conditions in the case $g_{rr} = g_{ir} = 0$ state that the components g_{rr} and g_{ir} of the induced metric are unchanged in the transformation so that the condition for c_A resulting from g_{rr} component of the metric is not affected. Also the conditions coming from $g_{ir} = 0$ remain unchanged. Therefore the commutation relations of local algebra apart from constraint from transversality result.

The commutator algebra of infinitesimal symmetries should also close in some sense. The orthogonality to the light-like tangent vector creates here a problem since the commutator does not obviously satisfy this condition automatically. The problem can be solved by following the recipes of non-covariant quantization of string model.

1. Make a choice of gauge by choosing time translation P^0 in a preferred M^4 coordinate frame to be the preferred generator $J^{A_0} \equiv P^0$, whose coefficient $\Phi_{A_0} \equiv \Psi(P^0)$ is solved from the orthogonality condition. This assumption is analogous with the assumption that time coordinate is non-dynamical in the quantization of strings. The natural basis for the algebra is obtained by allowing only a single generator J^A besides P^0 and putting $d_A = 1$.

2. This prescription must be consistent with the well-defined radial conformal weight for the $J^A \neq P^0$ in the sense that the proportionality of d_A to r^n for $J^A \neq P^0$ must be consistent with commutators. SU(3) part of the algebra is of course not a problem. From the Lorentz vector property of P^k it is clear that the commutators resulting in a repeated commutation have well-defined radial conformal weights only if one restricts $SO(3,1)$ to $SO(3)$ commuting with P^0. Also D could be allowed without losing well-defined radial conformal weights but the argument below excludes it. This picture conforms with the earlier identification of the Kac-Moody algebra.

   Conformal algebra contains besides Poincare algebra and the dilation $D = m^k\partial_{m^k}$ the mutually commuting generators $K^k = (m^r m_r \partial_{m^k} - 2m^k m^l \partial_{m^l})/2$. The commutators involving added generators are

$$\begin{array}{ll} \left[D, K^k\right] = -K^k \ , & \left[D, P^k\right] = P^k \ , \\ \left[K^k, K^l\right] = 0 \ , & \left[K^k, P^l\right] = m^{kl} D - M^{kl} \ . \end{array} \qquad (3.11.18)$$

   From the last commutation relation it is clear that the inclusion of K^k would mean loss of well-defined radial conformal weights.

3. The coefficient dm^0/dr of $\Psi(P^0)$ in the equation

$$\Psi(P^0)\frac{dm^0}{dr} = -J^{Ak}h_{kl}\partial_r h^l$$

   is always non-vanishing due to the light-likeness of r. Since P^0 commutes with generators of $SO(3)$ (but not with D so that it is excluded!), one can *define* the commutator of two generators as a commutator of the remaining part and identify $\Psi(P^0)$ from the condition above.

4. Of course, also the more general transformations act as Kac-Moody type symmetries but the interpretation would be that the sub-algebra plays the same role as $SO(3)$ in the case of Lorentz group: that is gives rise to generalized spin degrees of freedom whereas the entire algebra divided by this sub-algebra would define the coset space playing the role of orbital degrees of freedom. In fact, also the Kac-Moody type symmetries for which c_A depends on the transversal coordinates of X^3 would correspond to orbital degrees of freedom. The presence of these orbital degrees of freedom arranging super Kac-Moody representations into infinite multiplets labeled by function basis for X^2 means that the number of degrees of freedom is much larger than in string models.

5. It is possible to replace the preferred time coordinate m^0 with a preferred light-like coordinate. There are good reasons to believe that orbifold singularity for phases of matter involving non-standard value of Planck constant corresponds to a preferred light-ray going through the tip of $\delta M^4_\pm$. Thus it would be natural to assume that the preferred M^4 coordinate varies along this light ray or its dual. The Kac-Moody group $SO(3)\times E^3$ respecting the radial conformal weights would reduce to $SO(2)\times E^2$ as in string models. E^2 would act in tangent plane of $S^2_\pm$ along this ray defining also $SO(2)$ rotation axis.

3.11.5 Coset space structure for a symmetric space

The key ingredient in the theory of symmetric spaces is that the Lie-algebra of G has the following decomposition

$$\begin{array}{l} g = h + t \ , \\ [h,h] \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ . \end{array}$$

In present case this has highly nontrivial consequences. The commutator of *any* two infinitesimal generators generating nontrivial deformation of 3-surface belongs to h and thus vanishing norm in the configuration space metric at the point which is left invariant by H. In fact, this same condition follows from Ricci flatness requirement and guarantees also that G acts as isometries of the configuration space. This generalization is supported by the properties of the unitary representations of Lorentz group at the light cone boundary and by number theoretical considerations.

The algebras suggesting themselves as candidates are symplectic algebra of $\delta M^{\pm} \times CP_2$ and Kac-Moody algebra mapping light-like 3-surfaces to light-like 3-surfaces to be discussed in the next section.

The identification of the precise form of the coset space structure is however somewhat delicate.

1. The essential point is that both symplectic and Kac-Moody algebras allow representation in terms of X^3_l-local Hamiltonians. The general expression for the Hamilton of Kac-Moody algebra is

$$H = \sum \Phi_A(x) H^A \quad . \tag{3.11.19}$$

   Here H^A are Hamiltonians of $SO(3) \times SU(3)$ acting in $\delta X^3_l \times CP_2$. For symplectic algebra any Hamiltonian is allowed. If x corresponds to any point of X^3_l, one must assume a slicing of the causal diamond CD by translates of $\delta M^4_{\pm}$.

2. For symplectic generators the dependence of form on r^{Δ} on light-like coordinate of $\delta X^3_l \times CP_2$ is allowed. Δ is complex parameter whose modulus squared is interpreted as conformal weight. Δ is identified as analogous quantum number labeling the modes of induced spinor field.

3. One can wonder whether the choices of the $r_M = constant$ sphere S^2 is the only choice. The Hamiltonin-Jacobi coordinate for $X^4_{X^3_l}$ suggest an alternative choice as E^2 in the decomposition of $M^4 = M^2(x) \times E^2(x)$ required by number theoretical compactification and present for known extremals of Kähler action with Minkowskian signature of induced metric. In this case $SO(3)$ would be replaced with $SO(2)$. It however seems that the radial light-like coordinate u of $X^4(X^3_l)$ would remain the same since any other curve along light-like boundary would be space-like.

4. The vector fields for representing Kac-Moody algebra must vanish at the partonic 2-surface $X^2 \subset \delta M^4_{\pm} \times CP_2$. The corresponding vector field must vanish at each point of X^2:

$$j^k = \sum \Phi_A(x) J^{kl} H^A_l = 0 \quad . \tag{3.11.20}$$

   This means that the vector field corresponds to $SO(2) \times U(2)$ defining the isotropy group of the point of $S^2 \times CP_2$.

   This expression could be deduced from the idea that the surfaces X^2 are analogous to origin of CP_2 at which $U(2)$ vector fields vanish. Configuration space at X^2 could be also regarded as the analog of the origin of local $S^2 \times CP_2$. This interpretation is in accordance with the original idea which however was given up in the lack of proper realization. The same picture can be deduced from braiding in which case the Kac-Moody algebra corresponds to local $SO(2) \times U(2)$ for each point of the braid at X^2. The condition that Kac-Moody generators with positive conformal weight annihilate physical states could be interpreted by stating effective 2-dimensionality in the sense that the deformations of X^3_l preserving its light-likeness do not affect the physics. Note however that Kac-Moody type Virasoro generators do not annihilate physical states.

5. Kac-Moody algebra generator must leave induced Kähler form invariant at X^2. This is of course trivial since the action leaves each point invariant. The conditions of Cartan decomposition are satisfied. The commutators of the Kac-Moody vector fields with symplectic generators are non-vanishing since the action of symplectic generator on Kac-Moody generator restricted to X^2 gives a non-vanishing result belonging to the symplectic algebra. Also the commutators of Kac-Moody generators are Kac-Moody generators.

3.12 Complexification

A necessary prerequisite for the Kähler geometry is the complexification of the tangent space in vibrational degrees of freedom. What this means in recent context is non-trivial.

3.12.1 Why complexification is needed?

The Minkowskian signature of M^4 metric seems however to represent an insurmountable obstacle for the complexification of M^4 type vibrational degrees of freedom. On the other hand, complexification seems to have deep roots in the actual physical reality.

1. In the perturbative quantization of gauge fields one associates to each gauge field excitation polarization vector e and massless four-momentum vector p ($p^2 = 0$, $p \cdot e = 0$). These vectors define the decomposition of the tangent space of M^4: $M^4 = M^2 \times E^2$, where M^2 type polarizations correspond to zero norm states and E^2 type polarizations correspond to physical states with non-vanishing norm. Same type of decomposition occurs also in the linearized theory of gravitation. The crucial feature is that E^2 allows complexification! The general conclusion is that the modes of massless, linear, boson fields define always complexification of M^4 (or its tangent space) by effectively reducing it to E^2. Also in string models similar situation is encountered. For a string in D-dimensional space only D-2 transversal Euclidian degrees of freedom are physical.

2. Since symplectically extended isometry generators are expected to create physical states in TGD approach same kind of physical complexification should take place for them, too: this indeed takes place in string models in critical dimension. Somehow one should be able to associate polarization vector and massless four momentum vector to the deformations of a given 3-surface so that these vectors define the decomposition $M^4 = M^2 \times E^2$ for each mode. Configuration space metric should be degenerate: the norm of M^2 deformations should vanish as opposed to the norm of E^2 deformations.

Consider now the implications of this requirement.

1. In order to associate four-momentum and polarization (or at least the decomposition $M^4 = M^2 \times E^2$) to the deformations of the 3-surface one should have field equations, which determine the time development of the 3-surface uniquely. Furthermore, the time development for small deformations should be such that it makes sense to associate four momentum and polarization or at least the decomposition $M^4 = M^2 \times E^2$ to the deformations in suitable basis.

   The solution to this problem is afforded by the proposed definition of the Kähler function. The definition of the Kähler function indeed associates to a given 3-surface a unique four-surface as the preferred extremal of the Kähler action. Therefore one can associate a unique time development to the deformations of the surface X^3 and if TGD describes the observed world this time development should describe the evolution of photon, gluon, graviton, etc. states and so we can hope that tangent space complexification could be defined.

2. We have found that M^2 part of the deformation should have zero norm. In particular, the time like vibrational modes have zero norm in configuration space metric. This is true if Kähler function is not only $Diff^3$ invariant but also Diff^4 invariant in the sense that Kähler function has same value for all 3-surfaces belonging to the orbit of X^3 and related to X^3 by diffeomorphism of X^4. This is indeed the case.

3. Even this is not enough. One expects the presence of massive modes having also longitudinal polarization and for these states the number of physical vibrational degrees of freedom is 3 so that complexification seems to be impossible by odd dimension.

The reduction to the light cone boundary implied by $Diff^4$ invariance makes possible to identify the complexification. Crucial role is played by the special properties of the boundary of 4-dimensional light cone, which is metrically two-sphere and thus allows generalized complex and Kähler structure.

3.12.2 The metric, conformal and symplectic structures of the light cone boundary

The special metric properties of the light cone boundary play a crucial role in the complexification. The point is that the boundary of the light cone has degenerate metric: although light cone boundary is topologically 3-dimensional it is metrically 2-dimensional: effectively sphere. In standard spherical

Minkowski coordinates light cone boundary is defined by the equation $r_M = m^0$ and induced metric reads

$$ds^2 = -r_M^2 d\Omega^2 = -r_M^2 dz d\bar{z}/(1+z\bar{z})^2 \ , \tag{3.12.1}$$

and has Euclidian signature. Since S^2 allows complexification and thus also Kähler structure (and as a by-product also symplectic structure) there are good hopes of obtaining just the required type of complexification in non-degenerate M^4 degrees of freedom: configuration space would effectively inherit its Kähler structure from $S^2 \times CP_2$.

By its effective two-dimensionality the boundary of the four-dimensional light cone has infinite-dimensional group of (local) conformal transformations. Using complex coordinate z for S^2 the general local conformal transformation reads

$$\begin{aligned} r &\rightarrow f(r_M, z, \bar{z}) \ , \\ z &\rightarrow g(z) \ , \end{aligned} \tag{3.12.2}$$

where f is an arbitrary real function and g is an arbitrary analytic function with a finite number of poles. The infinitesimal generators of this group span an algebra, call it C, analogous to Virasoro algebra. This algebra is semidirect sum of two algebras L and R given by

$$\begin{aligned} C &= L \oplus R \ , \\ [L, R] &\subset R \ , \end{aligned} \tag{3.12.3}$$

where L denotes standard Virasoro algebra of the two- sphere generated by the generators

$$L_n = z^{n+1} d/dz \tag{3.12.4}$$

and R denotes the algebra generated by the vector fields

$$R_n = f_n(z, \bar{z}, r_M)\partial_{r_M} \ , \tag{3.12.5}$$

where $f(z, \bar{z}, r_M)$ forms complete real scalar function basis for light cone boundary. The vector fields of R have the special property that they have vanishing norm in M^4 metric.

This modification of conformal group implies that the Virasoro generator L_0 becomes $L_0 = zd/dz - r_M d/dr_M$ so that the scaling momentum becomes the difference $n - m$ or S^2 and radial scaling momenta. One could achieve conformal invariance by requiring that S^2 and radial scaling quantum numbers compensate each other.

Of crucial importance is that light cone boundary allows infinite dimensional group of isometries! An arbitrary conformal transformation $z \rightarrow f(z)$ induces to the metric a conformal factor given by $|df/dz|^2$. The compensating radial scaling $r_M \rightarrow r_M/|df/dz|$ compensates this factor so that the line element remains invariant.

The Kähler structure of light cone boundary defines automatically symplectic structure. The symplectic form is degenerate and just the area form of S^2 given by

$$J = r_M^2 sin(\theta) d\theta \wedge d\phi,$$

in standard spherical coordinates, there is infinite-dimensional group of symplectic transformations leaving the symplectic form of the light cone boundary (that is S^2) invariant. These transformations are local with respect to the radial coordinate r_M. The symplectic and Kähler structures of light cone boundary are not unique: different structures are labeled by the coset space $SO(3,1)/SO(3)$. One can however associate with a given 3-surface Y^3 a unique structure by requiring that the the corresponding subgroup $SO(3)$ of Lorentz group acts as the isotropy group of the conserved classical four-momentum assigned to Y^3 by the preferred extremal property.

In case of $\delta M^4_+ \times CP_2$ both the conformal transformations, isometries and symplectic transformations of the light cone boundary can be made local also with respect to CP_2. The idea that the infinite-dimensional algebra of symplectic transformations of $\delta M^4_+ \times CP_2$ act as isometries of the configuration space and that radial vector fields having zero norm in the metric of light cone boundary possess zero norm also in configuration space metric, looks extremely attractive.

In the case of $\delta M^4_+ \times CP_2$ one could combine the symplectic and Kähler structures of δM^4_+ and CP_2 to single symplectic/Kähler structure. The symplectic transformations leaving this symplectic structure invariant would be generated by the function algebra of $\delta M^4_+ \times CP_2$ such that a arbitrary function serves as a Hamiltonian of a symplectic transformation. This group serves as a candidate for the isometry group of the configuration space. An alternative identification for the isometry algebra is as symplectic symmetries of CP_2 localized with respect to the light cone boundary. Hamiltonians would be also now elements of the function algebra of $\delta M^4_+ \times CP_2$ but their Poisson brackets would be defined using only CP_2 symplectic form.

The problem is to decide which option is correct. There is a simple argument fixing the latter option. The symplecticly imbedded CP_2 would be left invariant under δM^4_+ local symplectic transformations of CP_2. This seems strange. Under symplectic algebra of $\delta M^4_+ \times CP_2$ also symplecticly imbedded CP_2 is deformed and this sounds more realistic. The isometry algebra is therefore assumed to be the group $can(\delta M^4_+ \times CP_2)$ generated by the scalar function basis $S(\delta M^4_+ \times CP_2) = S(\delta M^4_+) \times S(CP_2)$ of the light cone boundary using the Poisson brackets to be discussed in more detail later.

There are some no-go theorems associated with higher-dimensional Abelian extensions [51], and although the contexts are quite different, it is interesting to consider the recent situation in light of these theorems.

1. Conformal invariance is an essentially 2-dimensional notion. Light cone boundary is however metrically and conformally 2-sphere, and therefore the conformal algebra is effectively that associated with the 2-sphere. In the same manner, the quaternion conformal algebra associated with the metrically 2-dimensional elementary particle horizons surrounding wormhole contacts allows the usual Kac Moody algebra and actually also contributes to the configuration space metric.

2. In dimensions $D > 2$ Abelian extensions of the gauge algebra are extensions by an infinite-dimensional Abelian group rather than central extensions by the group $U(1)$. This result has an analog at the level of configuration space geometry. The extension associated with the symplectic algebra of CP_2 localized with respect to the light cone boundary is analogous a symplectic extension defined by Poisson bracket $\{p, q\} = 1$. The central extension is the function space associated with δM^4_+ and indeed infinite-dimensional if only only CP_2 symplectic structure induces the Poisson bracket but one-dimensional if $\delta M^4_+ \times CP_2$ Poisson bracket induces the extension. In the latter case the symmetries fix the metric completely at the point corresponding to the origin of symmetric space (presumably the maximum of Kähler function for given values of zero modes).

3. $D > 2$ extensions possess no unitary faithful representations (satisfying certain well motivated physical constraints) [51]. It might be that the degeneracy of the configuration space metric is the analog for the loss of faithful representations.

3.12.3 Complexification and the special properties of the light cone boundary

In case of Kähler metric G and H Lie-algebras must allow complexification so that the isometries can act as holomorphic transformations. Since G and H can be regarded as subalgebras of the vector fields of $\delta M^4_+ \times CP_2$, they inherit in a natural manner the complex structure of the light cone boundary.

There are two candidates for the configuration space complexification. The simplest, and also the correct, alternative is that complexification is induced by natural complexification of vector field basis on $\delta M^4_+ \times CP_2$. In CP_2 degrees of freedom there is natural complexification

$$\xi \to \bar{\xi} \ .$$

In δM^4_+ degrees of freedom this could involve the transformation

$$z \to \bar{z}$$

and certainly involves complex conjugation for complex scalar function basis in the radial direction:

$$f(r_M) \to \overline{f(r_M)} \ ,$$

which turns out to play same role as the function basis of circle in the Kähler geometry of loop groups [45].

The requirement that the functions are eigen functions of radial scalings favors functions $(r_M/r_0)^k$, where k is in general a complex number. The function can be expressed as a product of real power of r_M and logarithmic plane wave. It turns out that the radial complexification alternative is the correct manner to obtain Kähler structure. The reason is that symplectic transformations leave the value of r_M invariant. Radial Virasoro invariance plays crucial role in making the complexification possible.

One could consider also a second alternative assumed in the earlier formulation of the configuration space geometry. The close analogy with string models and conformal field theories suggests that for Virasoro generators the complexification must reduce to the hermitian conjugation of the conformal field theories: $L_n \to L_{-n} = L_n^\dagger$. Clearly this complexification is induced from the transformation $z \to \frac{1}{z}$ and differs from the complexification induced by complex conjugation $z \to \bar{z}$. The basis would be polynomial in z and $\bar{z}$. Since radial algebra could be also seen as Virasoro algebra localized with respect to $S^2 \times CP_2$ one could consider the possibility that also in radial direction the inversion $r_M \to \frac{1}{r_M}$ is involved.

The essential prerequisite for the Kähler structure is that both G and H allow same complexification so that the isometries in question can be regarded as holomorphic transformations. In finite-dimensional case this essentially what is needed since metric can be constructed by parallel translation along the orbit of G from H-invariant Kähler metric at a representative point. The requirement of H-invariance forces the radial complexification based on complex powers r_M^k: radial complexification works since symplectic transformations leave r_M invariant.

Some comments on the properties of the proposed complexification are in order.

1. The proposed complexification, which is analogous to the choice of gauge in gauge theories is not Lorentz invariant unless one can fix the coordinates of the light cone boundary apart from $SO(3)$ rotation not affecting the value of the radial coordinate r_M (if the imaginary part of k in r_M^k is always non-vanishing). This is possible as will be explained later.

2. It turns out that the function basis of light-cone boundary multiplying CP_2 Hamiltonians corresponds to unitary representations of the Lorentz group at light cone boundary so that the Lorentz invariance is rather manifest.

3. There is a nice connection with the proposed physical interpretation of the complexification. At the moment of the big bang all particles move with the velocity of light and therefore behave as massless particles. To a given point of the light cone boundary one can associate a unique direction of massless four-momentum by semiclassical considerations: at the point $m^k = (m^0, m^i)$ momentum is proportional to the vector $(m^0, -m^i)$. Since the particles are massless only two polarization vectors are possible and these correspond to the tangent vectors to the sphere $m^0 = r_M$. Of course, one must always fix polarizations at some point of tangent space but since massless polarization vectors are not physical this doesn't imply difficulties: different choices correspond to different gauges.

4. Complexification in the proposed manner is not possible except in the case of four-dimensional Minkowski space. Non-zero norm deformations correspond to vector fields of the light cone boundary acting on the sphere S^{D-2} and the decomposition to $(1,0)$ and $(0,1)$ parts is possible only when the sphere in question is two-dimensional since other spheres do allow neither complexification nor Kähler structure.

3.12.4 How to fix the complex and symplectic structures in a Lorentz invariant manner?

One can assign to light-cone boundary a symplectic structure since it reduces effectively to S^2. The possible symplectic structures of δM_+^4 are parameterized by the coset space $SO(3,1)/SO(3))$, where H is the isotropy group $SO(3)$ of a time like vector. Complexification also fixes the choice of the spherical coordinates apart from rotations around the quantization axis of angular momentum.

The selection of some preferred symplectic structure in an ad hoc manner breaks manifest Lorentz invariance but is possible if physical theory remains Lorentz invariant. The more natural possibility is that 3-surface Y^3 itself fixes in some natural manner the choice of the symplectic structure so that there is unique subgroup $SO(3)$ of $SO(3,1)$ associated with Y^3. If configuration space Kähler function corresponds to a preferred extremal of Kähler action, this is indeed the case. One can associate unique conserved four-momentum $P^k(Y^3)$ to the preferred extremal $X^4(Y^3)$ of the Kähler action and the requirement that the rotation group $SO(3)$ leaving the symplectic structure invariant leaves also $P^k(Y^3)$ invariant, fixes the symplectic structure associated with Y^3 uniquely.

Therefore configuration space decomposes into a union of symplectic spaces labeled by $SO(3,1)/SO(3)$ isomorphic to $a = constant$ hyperboloid of light cone. The direction of the classical angular momentum vector $w^k = \epsilon^{klmn} P_l J_{mn}$ determined by the classical angular momentum tensor of associated with Y^3 fixes one coordinate axis and one can require that $SO(2)$ subgroup of $SO(3)$ acting as rotation around this coordinate axis acts as phase transformation of the complex coordinate z of S^2. Other rotations act as nonlinear holomorphic transformations respecting the complex structure.

Clearly, the coordinates are uniquely fixed modulo $SO(2)$ rotation acting as phase multiplication in this case. If $P^k(Y^3)$ is light like, one can only require that the rotation group $SO(2)$ serving as the isotropy group of 3-momentum belongs to the group $SO(3)$ characterizing the symplectic structure and it seems that symplectic structure cannot be uniquely fixed without additional constraints in this case. Probably this has no practical consequences since the 3-surfaces considered have actually infinite size and 4-momentum is most probably time like for them. Note however that the direction of 3-momentum defines unique axis such that $SO(2)$ rotations around this axis are represented as phase multiplication.

Similar almost unique frame exists also in CP_2 degrees of freedom and corresponds to the complex coordinates transforming linearly under $U(2)$ acting as isotropy group of the Lie-algebra element defined by classical color charges Q_a of Y^3. One can fix unique Cartan subgroup of $U(2)$ by noticing that $SU(3)$ allows completely symmetric structure constants d_{abc} such that $R_a = d_a^{\ bc} Q_b Q_c$ defines Lie-algebra element commuting with Q_a. This means that R_a and Q_a span in generic case $U(1)\times U(1)$ Cartan subalgebra and there are unique complex coordinates for which this subgroup acts as phase multiplications. The space of nonequivalent frames is isomorphic with $CP(2)$ so that one can say that cm degrees of freedom correspond to Cartesian product of $SO(3,1)/SO(3)$ hyperboloid and CP_2 whereas coordinate choices correspond to the Cartesian product of $SO(3,1)/SO(2)$ and $SU(3)/U(1)\times U(1)$.

Symplectic transformations leave the value of δM^4_+ radial coordinate r_M invariant and this implies large number of additional zero modes characterizing the size and shape of the 3-surface. Besides this Kähler magnetic fluxes through the $r_M = constant$ sections of X^3 as a function of r_M provide additional invariants, which are functions rather than numbers. The Fourier components for the magnetic fluxes provide infinite number of symplectic invariants. The presence of these zero modes imply that 3-surfaces behave much like classical objects in the sense that neither their shape nor form nor classical Kähler magnetic fields, are subject to Gaussian fluctuations. Of course, quantum superpositions of 3-surfaces with different values of these invariants are possible.

There are reasons to expect that at least certain infinitesimal symplectic transformations correspond to zero modes of the Kähler metric (symplectic transformations act as dynamical symmetries of the vacuum extremals of the Kähler action). If this is indeed the case, one can ask whether it is possible to identify an integration measure for them.

If one can associate symplectic structure with zero modes, the symplectic structure defines integration measure in a standard manner (for 2n-dimensional symplectic manifold the integration measure is just the n-fold wedge power $J\wedge J...\wedge J$ of the symplectic form J). Unfortunately, in infinite-dimensional context this is not enough since divergence free functional integral analogous to a Gaussian integral is needed and it seems that it is not possible to integrate in zero modes and that this relates in a deep manner to state function reduction. If all symplectic transformations of $\delta M^4_+ \times CP_2$ are represented as symplectic transformations of the configuration space, then the existence of symplectic structure decomposing into Kähler (and symplectic) structure in complexified degrees of freedom and symplectic (but not Kähler) structure in zero modes, is an automatic consequence.

3.12.5 The general structure of the isometry algebra

There are three options for the isometry algebra of configuration space

1. Isometry algebra as the algebra of CP_2 symplectic transformations leaving invariant the symplectic form of CP_2 localized with respect to δM^4_+.

2. Certainly the configuration space metric in δM^4_+ must be non-trivial and actually given by the magnetic flux Hamiltonians defining symplectic invariants. Furthermore, the super-symplectic generators constructed from quarks automatically give as anti-commutators this part of the configuration space metric. One could interpret these symplectic invariants as configuration space Hamiltonians for δM^4_+ symplectic transformations obtained when CP_2 Hamiltonian is constant.

3. Isometry algebra consists of $\delta M^4_+ \times CP_2$ symplectic transformations. In this case a local color transformation involves necessarily a local S^2 transformation. Unfortunately, it is difficult to decide at this stage which of these options is correct.

The eigen states of the rotation generator and Lorentz boost in the same direction defining a unitary representation of the Lorentz group at light cone boundary define the most natural function basis for the light cone boundary. The elements of this bases have also well defined scaling quantum numbers and define also a unitary representation of the conformal algebra. The product of the basic functions is very simple in this basis since various quantum numbers are additive.

Spherical harmonics of S^2 provide an alternative function basis for the light cone boundary:

$$H^m_{jk} \equiv Y_{jm}(\theta,\phi) r^k_M \ . \tag{3.12.6}$$

One can criticize this basis for not having nice properties under Lorentz group.

The product of basis functions is given by Glebch-Gordan coefficients for symmetrized tensor product of two representation of the rotation group. Poisson bracket in turn reduces to the Glebch-Gordans of anti-symmetrized tensor product. The quantum numbers m and k are additive. The basis is eigen-function basis for the imaginary part of the Virasoro generator L_0 generating rotations around quantization axis of angular momentum. In fact, only the imaginary part of the Virasoro generator $L_0 = zd/dz = \rho\partial_\rho - \frac{2}{2}\partial_\phi$ has global single valued Hamiltonian, whereas the corresponding representation for the transformation induced by the real part of L_0, with a compensating radial scaling added, cannot be realized as a global symplectic transformation.

The Poisson bracket of two functions $H^m_{j_1k_1}$ and $H^m_{j_2k_2}$ can be calculated and is of the general form

$$\{H^{m_1}_{j_1k}, H^{m_2}_{j_2k_2}\} \equiv C(j_1m_1j_2m_2|j, m_1+m_2)_A H^{m_1+m_2}_{j,k_1+k_2} \ . \tag{3.12.7}$$

The coefficients are Glebch-Gordan coefficients for the anti-symmetrized tensor product for the representations of the rotation group.

The isometries of the light cone boundary correspond to conformal transformations accompanied by a local radial scaling compensating the conformal factor coming from the conformal transformations having parametric dependence of radial variable and CP_2 coordinates. It seems however that isometries cannot in general be realized as symplectic transformations. The first difficulty is that symplectic transformations cannot affect the value of the radial coordinate. For rotation algebra the representation as symplectic transformations is however possible.

In CP_2 degrees of freedom scalar function basis having definite color transformation properties is desirable. Scalar function basis can be obtained as the algebra generated by the Hamiltonians of color transformations by multiplication. The elements of basis can be typically expressed as monomials of color Hamiltonians H^A_c

$$H^A_D = \sum_{\{B_j\}} C^A_{DB_1B_2....B_N} \prod_{B_i} H^{B_i}_c \ , \tag{3.12.8}$$

where summation over all index combinations $\{B_i\}$ is understood. The coefficients $C^A_{DB_1B_2....B_N}$ are Glebch-Gordan coefficients for completely symmetric N:th power $8 \otimes 8 ... \otimes 8$ of octet representations.

The representation is not unique since $\sum_A H_c^A H_c^A = 1$ holds true. One can however find for each representation D some minimum value of N.

The product of Hamiltonians $H_A^{D_1}$ and $H_{D_2}^B$ can be decomposed by Glebch-Gordan coefficients of the symmetrized representation $(D_1 \otimes D_2)_S$ as

$$H_{D_1}^A H_{D_2}^B = C_{D_1 D_2 DC}^{ABD}(S) H_D^C \ , \tag{3.12.9}$$

where $'S'$ indicates that the symmetrized representation is in question. In the similar manner one can decompose the Poisson bracket of two Hamiltonians

$$\{H_{D_1}^A, H_{D_2}^B\} = C_{D_1 D_2 DC}^{ABD}(A) H_D^C \ . \tag{3.12.10}$$

Here $'A'$ indicates that Glebch-Gordan coefficients for the anti-symmetrized tensor product of the representations D_1 and D_2 are in question.

One can express the infinitesimal generators of CP_2 symplectic transformations in terms of the color isometry generators J_c^B using the expansion of the Hamiltonian in terms of the monomials of color Hamiltonians:

$$\begin{aligned} j_{DN}^A &= F_{DB}^A J_c^B \ , \\ F_{DB}^A &= N \sum_{\{B_j\}} C_{DB_1 B_2 \ldots B_{N-1} B}^A \prod_j H_c^{B_j} \ , \end{aligned} \tag{3.12.11}$$

where summation over all possible $\{B_j\}$:s appears. Therefore, the interpretation as a color group localized with respect to CP_2 coordinates is valid in the same sense as the interpretation of space-time diffeomorphism group as local Poincare group. Thus one can say that TGD color is localized with respect to the entire $\delta M_+^4 \times CP_2$.

A convenient basis for the Hamiltonians of $\delta M_+^4 \times CP_2$ is given by the functions

$$H_{jkD}^{mA} = H_{jk}^m H_D^A \ .$$

The symplectic transformation generated by H_{jkD}^{mA} acts both in M^4 and CP_2 degrees of freedom and the corresponding vector field is given by

$$J^r = H_D^A J^{rl}(\delta M_+^4) \partial_l H_{jk}^m + H_{jk}^m J^{rl}(CP_2) \partial_l H_D^A \ . \tag{3.12.12}$$

The general form for their Poisson bracket is:

$$\begin{aligned} &\{H_{j_1 k_1 D_1}^{m_1 A_1}, H_{j_2 k_2 D_2}^{m_2 A_2}\} = H_{D_1}^{A_1} H_{D_2}^{A_2} \{H_{j_1 k_1}^{m_1}, H_{j_2 k_2}^{m_2}\} + H_{j_1 k_1}^{m_1} H_{j_2 k_2}^{m_2} \{H_{D_1}^{A_1}, H_{D_2}^{A_2}\} \\ &= \left[C_{D_1 D_2 D}^{A_1 A_2 A}(S) C(j_1 m_1 j_2 m_2 | jm)_A + C_{D_1 D_2 D}^{A_1 A_2 A}(A) C(j_1 m_1 j_2 m_2 | jm)_S \right] H_{j,k_1+k_2,D}^{mA} \ . \end{aligned} \tag{3.12.13}$$

What is essential that radial 'momenta' and angular momentum are additive in δM_+^4 degrees of freedom and color quantum numbers are additive in CP_2 degrees of freedom.

3.12.6 Representation of Lorentz group and conformal symmetries at light cone boundary

A guess deserving testing is that the representations of the Lorentz group at light cone boundary might provide natural building blocks for the construction of the configuration space Hamiltonians. In the following the explicit representation of the Lorentz algebra at light cone boundary is deduced, and a function basis giving rise to the representations of Lorentz group and having very simple properties under modified Poisson bracket of δM_+^4 is constructed.

Explicit representation of Lorentz algebra

It is useful to write the explicit expressions of Lorentz generators using complex coordinates for S^2. The expression for the $SU(2)$ generators of the Lorentz group are

$$\begin{aligned}
J_x &= (z^2-1)d/dz + c.c. = L_1 - L_{-1} + c.c. \ , \\
J_y &= (iz^2+1)d/dz + c.c. = iL_1 + iL_{-1} + c.c. \ , \\
J_z &= iz\frac{d}{dz} + c.c. = iL_z + c.c. \ .
\end{aligned} \tag{3.12.14}$$

The expressions for the generators of Lorentz boosts can be derived easily. The boost in m^3 direction corresponds to an infinitesimal transformation

$$\begin{aligned}
\delta m^3 &= -\varepsilon r_M \ , \\
\delta r_M &= -\varepsilon m^3 = -\varepsilon\sqrt{r_M^2-(m^1)^2-(m^2)^2} \ .
\end{aligned} \tag{3.12.15}$$

The relationship between complex coordinates of S^2 and M^4 coordinates m^k is given by stereographic projection

$$\begin{aligned}
z &= \frac{(m^1+im^2)}{(r_M-\sqrt{r_M^2-(m^1)^2-(m^2)^2})} \\
&= \frac{sin(\theta)(cos\phi+isin\phi)}{(1-cos\theta)} \ , \\
cot(\theta/2) &= \rho = \sqrt{z\bar{z}} \ , \\
tan(\phi) &= \frac{m^2}{m^1} \ .
\end{aligned} \tag{3.12.16}$$

This implies that the change in z coordinate doesn't depend at all on r_M and is of the following form

$$\delta z = -\frac{\varepsilon}{2}(1+\frac{z(z+\bar{z})}{2})(1+z\bar{z}) \ . \tag{3.12.17}$$

The infinitesimal generator for the boosts in z-direction is therefore of the following form

$$L_z = [\frac{2z\bar{z}}{(1+z\bar{z})}-1]r_M\frac{\partial}{\partial_{r_M}} - iJ_z \ . \tag{3.12.18}$$

Generators of L_x and L_y are most conveniently obtained as commutators of $[L_z, J_y]$ and $[L_z, J_x]$. For L_y one obtains the following expression:

$$L_y = 2\frac{(z\bar{z}(z+\bar{z})+i(z-\bar{z}))}{(1+z\bar{z})^2}r_M\frac{\partial}{\partial_{r_M}} - iJ_y \ , \tag{3.12.19}$$

For L_x one obtains analogous expressions. All Lorentz boosts are of the form $L_i = -iJ_i + local\ radial\ scaling$ and of zeroth degree in radial variable so that their action on the general generator $X^{klm} \propto z^k\bar{z}^l r_M^m$ doesn't change the value of the label m being a mere local scaling transformation in radial direction. If radial scalings correspond to zero norm isometries this representation is metrically equivalent with the representations of Lorentz boosts as Möbius transformations.

Representations of the Lorentz group reduced with respect to $SO(3)$

The ordinary harmonics of S^2 define in a natural manner infinite series of representation functions transformed to each other in Lorentz transformations. The inner product defined by the integration measure $r_M^2 d\Omega dr_M/r_M$ remains invariant under Lorentz boosts since the scaling of r_M induced by the Lorentz boost compensates for the conformal scaling of $d\Omega$ induced by a Lorentz transformation represented as a Möbius transformation. Thus unitary representations of Lorentz group are in question.

The unitary main series representations of the Lorentz group are characterized by half-integer m and imaginary number $k_2 = i\rho$, where ρ is any real number [52]. A natural guess is that $m = 0$ holds true for all representations realizable at the light cone boundary and that radial waves are of form r_M^k, $k = k_1 + ik_2 = -1 + i\rho$ and thus eigen states of the radial scaling so that the action of Lorentz boosts is simple in the angular momentum basis. The inner product in radial degrees of freedom reduces to that for ordinary plane waves when $log(r_M)$ is taken as a new integration variable. The complexification is well-defined for non-vanishing values of ρ.

It is also possible to have non-unitary representations of the Lorentz group and the realization of the symmetric space structure suggests that one must have $k = k_1 + ik_2$, k_1 half-integer. For these representations unitarity fails because the inner product in the radial degrees of freedom is non-unitary. A possible physical interpretation consistent with the general ideas about conformal invariance is that the representations $k = -1 + i\rho$ correspond to the unitary ground state representations and $k = -1 + n/2 + i\rho$, $n = \pm 1, \pm 2, ...$, to non-unitary representations. The general view about conformal invariance suggests that physical states constructed as tensor products satisfy the condition $\sum_i n_i = 0$ completely analogous to Virasoro conditions.

Representations of the Lorentz group with $E^2 \times SO(2)$ as isotropy group

One can construct representations of Lorentz group and conformal symmetries at the light cone boundary. Since $SL(2,C)$ is the group generated by the generators L_0 and $L_\pm$ of the conformal algebra, it is clear that infinite-dimensional representations of Lorentz group can be also regarded as representations of the conformal algebra. One can require that the basis corresponds to eigen functions of the rotation generator J_z and corresponding boost generator L_z. For functions which do not depend on r_M these generators are completely analogous to the generators L_0 generating scalings and iL_0 generating rotations. Also the generator of radial scalings appears in the formulas and one must consider the possibility that it corresponds to the generator L_0.

In order to construct scalar function eigen basis of L_z and J_z, one can start from the expressions

$$\begin{aligned} L_3 &\equiv i(L_z + L_{\bar{z}}) = 2i[\frac{2z\bar{z}}{(1+z\bar{z})} - 1]r_M\frac{\partial}{\partial_{r_M}} + i\rho\partial_\rho \ , \\ J_3 &\equiv iL_z - iL_{\bar{z}} = i\partial_\phi \ . \end{aligned} \tag{3.12.20}$$

If the eigen functions do not depend on r_M, one obtains the usual basis z^n of Virasoro algebra, which however is not normalizable basis. The eigenfunctions of the generators L_3, J_3 and $L_0 = ir_M d/dr_M$ satisfying

$$\begin{aligned} J_3 f_{m,n,k} &= m f_{m,n,k} \ , \\ L_3 f_{m,n,k} &= n f_{m,n,k} \ , \\ L_0 f_{m,n,k} &= k f_{m,n,k} \ . \end{aligned} \tag{3.12.21}$$

are given by

$$f_{m,n,k} = e^{im\phi}\frac{\rho^{n-k}}{(1+\rho^2)^k} \times (\frac{r_M}{r_0})^k \ . \tag{3.12.22}$$

$n = n_1 + in_2$ and $k = k_1 + ik_2$ are in general complex numbers. The condition

$$n_1 - k_1 \geq 0$$

is required by regularity at the origin of S^2 The requirement that the integral over S^2 defining norm exists (the expression for the differential solid angle is $d\Omega = \frac{\rho}{(1+\rho^2)^2} d\rho d\phi$) implies

$$n_1 < 3k_1 + 2 \ .$$

From the relationship $(cos(\theta), sin(\theta)) = (\rho^2-1)/(\rho^2+1), 2\rho/(\rho^2+1))$ one can conclude that for $n_2 = k_2 = 0$ the representation functions are proportional to f $sin(\theta)^{n-k}(cos(\theta)-1)^{n-k}$. Therefore they have in their decomposition to spherical harmonics only spherical harmonics with angular momentum $l < 2(n-k)$. This suggests that the condition

$$|m| \leq 2(n-k) \tag{3.12.23}$$

is satisfied quite generally.

The emergence of the three quantum numbers (m, n, k) can be understood. Light cone boundary can be regarded as a coset space $SO(3,1)/E^2 \times SO(2)$, where $E^2 \times SO(2)$ is the group leaving the light like vector defined by a particular point of the light cone invariant. The natural choice of the Cartan group is therefore $E^2 \times SO(2)$. The three quantum numbers (m, n, k) have interpretation as quantum numbers associated with this Cartan algebra.

The representations of the Lorentz group are characterized by one half-integer valued and one complex parameter. Thus k_2 and n_2, which are Lorentz invariants, might not be independent parameters, and the simplest option is $k_2 = n_2$.

The nice feature of the function basis is that various quantum numbers are additive under multiplication:

$$f(m_a, n_a, k_a) \times f(m_b, n_b, k_b) = f(m_a + m_b, n_a + n_b, k_a + k_b) \ .$$

These properties allow to cast the Poisson brackets of the symplectic algebra of the configuration space into an elegant form.

The Poisson brackets for the δM^4_+ Hamiltonians defined by f_{mnk} can be written using the expression $J^{\rho\phi} = (1+\rho^2)/\rho$ as

$$\begin{aligned} \{f_{m_a,n_a,k_a}, f_{m_b,n_b,k_b}\} &= i\left[(n_a-k_a)m_b - (n_b-k_b)m_a\right] \times f_{m_a+m_b,n_a+n_b-2,k_a+k_b} \\ &+ 2i\left[(2-k_a)m_b - (2-k_b)m_a\right] \times f_{m_a+m_b,n_a+n_b-1,k_a+k_b-1} \ . \end{aligned} \tag{3.12.24}$$

Can one find unitary light-like representations of Lorentz group?

It is interesting to compare the representations in question to the unitary representations of Lorentz group discussed in [52].

1. The unitary representations discussed in [52] are characterized by are constructed by deducing the explicit representations for matrix elements of the rotation generators J_x, J_y, J_z and boost generators L_x, L_y, L_z by decomposing the representation into series of representations of $SU(2)$ defining the isotropy subgroup of a time like momentum. Therefore the states are labeled by eigenvalues of J_z. In the recent case the isotropy group is $E^2 \times SO(2)$ leaving light like point invariant. States are therefore labeled by three different quantum numbers.

2. The representations of [52] are realized the space of complex valued functions of complex coordinates ξ and $\overline{\xi}$ labeling points of complex plane. These functions have complex degrees $n_+ = m/2 - 1 + l_1$ with respect to ξ and $n_- = -m/2 - 1 + l_1$ with respect to $\overline{\xi}$. l_0 is complex number in the general case but for unitary representations of main series it is given by $l_1 = i\rho$ and for the representations of supplementary series l_1 is real and satisfies $0 < |l_1| < 1$. The main series representation is derived from a representation space consisting of homogenous functions of variables z^0, z^1 of degree n_+ and of $\overline{z}^0$ and $\overline{z}^1$ of degrees $n_\pm$. One can separate express these functions as product of $(z^1)^{n^+}$ $(\overline{z}^1)^{n_-}$ and a polynomial of $\xi = z^1/z^2$ and $\overline{\xi}$ with degrees n_+ and n_-. Unitarity reduces to the requirement that the integration measure of complex plane is invariant under the Lorentz transformations acting as Moebius transformations of the complex plane. Unitarity implies $l_1 = -1 + i\rho$.

3. For the representations at δM^4_+ formal unitarity reduces to the requirement that the integration measure of $r_M^2 d\Omega dr_M/r_M$ of δM^4_+ remains invariant under Lorentz transformations. The action of Lorentz transformation on the complex coordinates of S^2 induces a conformal scaling which can be compensated by an S^2 local radial scaling. At least formally the function space of δM^4_+ thus defines a unitary representation. For the function basis f_{mnk} $k=-1+i\rho$ defines a candidate for a unitary representation since the logarithmic waves in the radial coordinate are completely analogous to plane waves for $k_1=-1$. This condition would be completely analogous to the vanishing of conformal weight for the physical states of super conformal representations. The problem is that for $k_1=-1$ guaranteing square integrability in S^2 implies $-2<n_1<-2$ so that unitarity is possible only for the function basis consisting of spherical harmonics.

   There is no deep reason against non-unitary representations and symmetric space structure indeed requires that k_1 is half-integer valued. First of all, configuration space spinor fields are analogous to ordinary spinor fields in M^4, which also define non-unitary representations of Lorentz group. Secondly, if 3-surfaces at the light cone boundary are finite-sized, the integrals defined by f_{mnk} over 3-surfaces Y^3 are always well-defined. Thirdly, the continuous spectrum of k_2 could be transformed to a discrete spectrum when k_1 becomes half-integer valued.

Hermitian form for light cone Hamiltonians involves also the integration over S^2 degrees of freedom and the non-unitarity of the inner product reflects itself as non-orthogonality of the the eigen function basis. Introducing the variable $u=\rho^2+1$ as a new integration variable, one can express the inner product in the form

$$\begin{aligned}
\langle m_a,n_a,k_a|m_b,n_b,k_b\rangle &= \pi\delta(k_{2a}-k_{2b})\times\delta_{m_1,m_2}\times I \ ,\\
I &= \int_1^\infty f(u)du \ ,\\
f(u) &= \frac{(u-1)^{\frac{(N-K)+i\Delta}{2}}}{u^{K+2}} \ .
\end{aligned} \tag{3.12.25}$$

The integrand has cut from $u=1$ to infinity along real axis. The first thing to observe is that for $N=K$ the exponent of the integral reduces to very simple form and integral exists only for $K=k_{1a}+k_{1b}>-1$. For $k_{1i}=-1/2$ the integral diverges.

The discontinuity of the integrand due to the cut at the real axis is proportional to the integrand and given by

$$\begin{aligned}
f(u)-f(e^{i2\pi}u) &= \left[1-e^{-\pi\Delta}\right]f(u) \ ,\\
\Delta &= n_{1a}-k_{1a}-n_{1b}+k_{1b} \ .
\end{aligned} \tag{3.12.26}$$

This means that one can transform the integral to an integral around the cut. This integral can in turn completed to an integral over closed loop by adding the circle at infinity to the integration path. The integrand has $K+1$-fold pole at $u=0$.

Under these conditions one obtains

$$\begin{aligned}
I &= \frac{2\pi i}{1-e^{-\pi\Delta}}\times R\times(R-1)....\times(R-K-1)\times(-1)^{\frac{N-K}{2}-K-1} \ ,\\
R &\equiv \frac{N-K}{2}+i\Delta \ .
\end{aligned} \tag{3.12.27}$$

This expression is non-vanishing for $\Delta\neq 0$. Thus it is not possible to satisfy orthogonality conditions without the un-physical $n=k, k_1=1/2$ constraint. The result is finite for $K>-1$ so that $k_1>-1/2$ must be satisfied and if one allows only half-integers in the spectrum, one must have $k_1\geq 0$, which is very natural if real conformal weights which are half integers are allowed.

3.12.7 How the complex eigenvalues of the radial scaling operator relate to conformal weights?

Complexified Hamiltonians can be chosen to be eigenmodes of the radial scaling operator $r_M d/dr_M$, and the first guess was that the correct interpretation is as conformal weights. The problem is however that the eigenvalues are complex. Second problem is that general arguments are not enough to fix the spectrum of eigenvalues. There should be a direct connection to the dynamics defined by Kähler action with instanton term included and the modified Dirac action defined by it.

The construction of configuration space spinor structure in terms of second quantized induced spinor fields [18] leads to the conclusion that the modes of induced spinor fields are labeled by generalized eigenvalues λ such that $|\lambda|^2$ has interpretation as a conformal weight and λ itself is analogous to Higgs expectation value. Coset construction requires that super-symplectic and super Kac-Moody conformal weights $|\lambda|^2$ are same. This is achieved if the Hamiltonians are generalized eigen modes of $D = \gamma^x d/dx$, $x = log(r/r_0)$, satisfying $DH = \lambda\gamma^x H$ and thus of form $exp(\lambda x) = (r/r_0)^\lambda$ with the same spectrum of complex eigenvalues λ as associated with the modified Dirac operator. That $log(r/r_0)$ naturally corresponds to the coordinate u assignable to the generalized eigen modes of modified Dirac operator supports this interpretation.

If the Kähler action and modified Dirac action involve also the CP breaking instanton term, the eigenvalues λ are complex and complexity relates directly also to the breaking of time reversal invariance.

3.13 Magnetic and electric representations of the configuration space Hamiltonians

Symmetry considerations lead to the hypothesis that configuration space Hamiltonians are apart from a factor depending on symplectic invariants equal to magnetic flux Hamiltonians. On the other hand, the hypothesis that Kähler function corresponds to a preferred extremal of Kähler action leads to the hypothesis that configuration space Hamiltonians corresponds to classical charges associated with the Hamiltonians of the light cone boundary. These charges are very much like electric charges. The requirement that two approaches are equivalent leads to the hypothesis that magnetic and electric Hamiltonians are identical apart from a factor depending on isometry invariants. At the level of CP_2 corresponding duality corresponds to the self-duality of Kähler form stating that the magnetic and electric parts of Kähler form are identical.

3.13.1 Radial symplectic invariants

All $\delta M^4_+ \times CP_2$ symplectic transformations leave invariant the value of the radial coordinate r_M. Therefore the radial coordinate r_M of X^3 regarded as a function of $S^2 \times CP_2$ coordinates serves as height function. The number, type, ordering and values for the extrema for this height function in the interior and boundary components are isometry invariants. These invariants characterize not only the topology but also the size and shape of the 3-surface. The result implies that configuration space metric indeed differentiates between 3-surfaces with the size of Planck length and with the size of galaxy. The characterization of these invariants reduces to Morse theory. The extrema correspond to topology changes for the two-dimensional (one-dimensional) $r_M = constant$ section of 3-surface (boundary of 3-surface). The height functions of sphere and torus serve as a good illustrations of the situation. A good example about non-topological extrema is provided by a sphere with two horns.

There are additional symplectic invariants. The 'magnetic fluxes' associated with the δM^4_+ symplectic form

$$J_{S^2} = r_M^2 sin(\theta) d\theta \wedge d\phi$$

over any $X^2 \subset X^3$ are symplectic invariants. In particular, the integrals over $r_M = constant$ sections (assuming them to be 2-dimensional) are symplectic invariants. They give simply the solid angle $\Omega(r_M)$ spanned by $r_M = constant$ section and thus $r_M^2\Omega(r_M)$ characterizes transversal geometric size of the 3-surface. A convenient manner to discretize these invariants is to consider the Fourier components of these invariants in radial logarithmic plane wave basis discussed earlier:

$$\Omega(k) = \int_{r_{min}}^{r_{max}} (r_M/r_{max})^k \Omega(r_M) \frac{dr_M}{r_M} \quad , \quad k = k_1 + ik_2 \ , \ perk_1 \geq 0 \ . \tag{3.13.1}$$

One must take into account that for each section in which the topology of $r_M = constant$ section remains constant one must associate invariants with separate components of the two-dimensional section. For a given value of r_M, r_M constant section contains several components (to visualize the situation consider torus as an example).

Also the quantities

$$\Omega^+(X^2) = \int_{X^2} |J| \equiv \int |\epsilon^{\alpha\beta} J_{\alpha\beta}| \sqrt{g_2} d^2x$$

are symplectic invariants and provide additional geometric information about 3-surface. These fluxes are non-vanishing also for closed surfaces and give information about the geometry of the boundary components of 3-surface (signed fluxes vanish for boundary components unless they enclose the dip of the light cone).

Since zero norm generators remain invariant under complexification, their contribution to the Kähler metric vanishes. It is not at all obvious whether the configuration space integration measure in these degrees of freedom exists at all. A localization in zero modes occurring in each quantum jump seems a more plausible and under suitable additional assumption it would have interpretation as a state function reduction. In string model similar situation is encountered; besides the functional integral determined by string action, one has integral over the moduli space.

If the effective 2-dimensionality implied by the strong form of general coordinate invariance discussed in the introduction is accepted, there is no need to integrate over the variable r_M and just the fluxes over the 2-surfaces X^2_i identified as intersections of light like 3-D causal determinants with X^3 contain the data relevant for the construction of the configuration space geometry. Also the symplectic invariants associated with these surfaces are enough.

3.13.2 Kähler magnetic invariants

The Kähler magnetic fluxes defined both the normal component of the Kähler magnetic field and by its absolute value

$$\begin{aligned} Q_m(X^2) &= \int_{X^2} J_{CP_2} = J_{\alpha\beta}\epsilon^{\alpha\beta}\sqrt{g_2}d^2x \ , \\ Q_m^+(X^2) &= \int_{X^2} |J_{CP_2}| \equiv \int_{X^2} |J_{\alpha\beta}\epsilon^{\alpha\beta}|\sqrt{g_2}d^2x \ , \end{aligned} \tag{3.13.2}$$

over suitably defined 2-surfaces are invariants under both Lorentz isometries and the symplectic transformations of CP_2 and can be calculated once X^3 is given.

For a closed surface $Q_m(X^2)$ vanishes unless the homology equivalence class of the surface is nontrivial in CP_2 degrees of freedom. In this case the flux is quantized. $Q_M^+(X^2)$ is non-vanishing for closed surfaces, too. Signed magnetic fluxes over non-closed surfaces depend on the boundary of X^2 only:

$$\begin{aligned} &\textstyle\int_{X^2} J = \int_{\delta X^2} A \ . \\ &J = dA \ . \end{aligned}$$

Un-signed fluxes can be written as sum of similar contributions over the boundaries of regions of X^2 in which the sign of J remains fixed.

$$\begin{aligned} Q_m(X^2) &= \int_{X^2} J_{CP_2} = J_{\alpha\beta}\epsilon^{\alpha\beta}\sqrt{g_2}d^2x \ , \\ Q_m^+(X^2) &= \int_{X^2} |J_{CP_2}| \equiv \int_{X^2} |J_{\alpha\beta}\epsilon^{\alpha\beta}|\sqrt{g_2}d^2x \ , \end{aligned} \tag{3.13.3}$$

There are also symplectic invariants, which are Lorentz covariants and defined as

$$\begin{aligned} Q_m(K,X^2) &= \int_{X^2} f_K J_{CP_2} \ , \\ Q_m^+(K,X^2) &= \int_{X^2} f_K |J_{CP_2}| \ , \\ f_{K\equiv(s,n,k)} &= e^{is\phi}\times\frac{\rho^{n-k}}{(1+\rho^2)^k}\times(\frac{r_M}{r_0})^k \end{aligned} \tag{3.13.4}$$

These symplectic invariants transform like an infinite-dimensional unitary representation of Lorentz group.

There must exist some minimal number of symplectically non-equivalent 2-surfaces of X^3, and the magnetic fluxes over the representatives these surfaces give thus good candidates for zero modes.

1. If effective 2-dimensionality is accepted, the surfaces X_i^2 defined by the intersections of light like 3-D causal determinants X_l^3 and X^3 provide a natural identification for these 2-surfaces.

2. Without effective 2-dimensionality the situation is more complex. Since symplectic transformations leave r_M invariant, a natural set of 2-surfaces X^2 appearing in the definition of fluxes are separate pieces for $r_M = constant$ sections of 3-surface. For a generic 3-surface, these surfaces are 2-dimensional and there is continuum of them so that discrete Fourier transforms of these invariants are needed. One must however notice that $r_M = constant$ surfaces could be be 3-dimensional in which case the notion of flux is not well-defined.

3.13.3 Isometry invariants and spin glass analogy

The presence of isometry invariants implies coset space decomposition $\cup_i G/H$. This means that quantum states are characterized, not only by the vacuum functional, which is just the exponential $exp(K)$ of Kähler function (Gaussian in lowest approximation) but also by a wave function in vacuum modes. Therefore the functional integral over the configuration space decomposes into an integral over zero modes for approximately Gaussian functionals determined by $exp(K)$. The weights for the various vacuum mode contributions are given by the probability density associated with the zero modes. The integration over the zero modes is a highly problematic notion and it could be eliminated if a localization in the zero modes occurs in quantum jumps. The localization would correspond to a state function reduction and zero modes would be effectively classical variables correlated in one-one manner with the quantum numbers associated with the quantum fluctuating degrees of freedom.

For a given orbit K depends on zero modes and thus one has mathematical similarity with spin glass phase for which one has probability distribution for Hamiltonians appearing in the partition function $exp(-H/T)$. In fact, since TGD Universe is also critical, exact similarity requires that also the temperature is critical for various contributions to the average partition function of spin glass phase. The characterization of isometry invariants and zero modes of the Kähler metric provides a precise characterization for how TGD Universe is quantum analog of spin glass.

The spin glass analogy has been the basic starting point in the construction of p-adic field theory limit of TGD. The ultra-metric topology for the free energy minima of spin glass phase motivates the hypothesis that effective quantum average space-time possesses ultra-metric topology. This approach leads to excellent predictions for elementary particle masses and predicts even new branches of physics [31, 26]. As a matter fact, an entire fractal hierarchy of copies of standard physics is predicted.

3.13.4 Magnetic flux representation of the symplectic algebra

Accepting the strong form of general coordinate invariance implying effective two-dimensionality WCW Hamiltonians correspond to the fluxes associated with various 2-surfaces X_i^2 defined by the intersections of light-like light-like 3-surfaces $X_{l,i}^3$ with X^3 at the boundaries of CD considered. Bearing in mind that zero energy ontology is the correct approach, one can restrict the consideration on fluxes at $\delta M_+^4 \times CP_2$ One must also remember that if the proposed symmetries hold true, it is in principle choose any partonic 2-surface in the conjectured slicing of the Minkowskian space-time sheet to partonic 2-surfaces parametrized by the points of stringy world sheets.

Generalized magnetic fluxes

Isometry invariants are just special case of the fluxes defining natural coordinate variables for the configuration space. Symplectic transformations of CP_2 act as $U(1)$ gauge transformations on the Kähler potential of CP_2 (similar conclusion holds at the level of $\delta M^4_+ \times CP_2$).

One can generalize these transformations to local symplectic transformations by allowing the Hamiltonians to be products of the CP_2 Hamiltonians with the real and imaginary parts of the functions $f_{m,n,k}$ (see Eq. 3.12.22) defining the Lorentz covariant function basis H_A, $A \equiv (a,m,n,k)$ at the light cone boundary: $H_A = H_a \times f(m,n,k)$, where a labels the Hamiltonians of CP_2.

One can associate to any Hamiltonian H^A of this kind both signed and unsigned magnetic flux via the following formulas:

$$\begin{aligned} Q_m(H_A|X^2) &= \int_{X^2} H_A J \ , \\ Q_m^+(H_A|X^2) &= \int_{X^2} H_A|J| \ . \end{aligned} \tag{3.13.5}$$

Here X^2 corresponds to any surface X^2_i resulting as intersection of X^3 with $X^3_{l,i}$. Both signed and unsigned magnetic fluxes and their superpositions

$$Q_m^{\alpha,\beta}(H_A|X^2) = \alpha Q_m(H_A|X^2) + \beta Q_m^+(H_A|X^2) \ , \ \ A \equiv (a,s,n,k) \tag{3.13.6}$$

provide representations of Hamiltonians. Note that symplectic invariants $Q_m^{\alpha,\beta}$ correspond to $H^A = 1$ and $H^A = f_{s,n,k}$. $H^A = 1$ can be regarded as a natural central term for the Poisson bracket algebra. Therefore, the isometry invariance of Kähler magnetic and electric gauge fluxes follows as a natural consequence.

The obvious question concerns about the correct values of the parameters α and β. One possibility is that the flux is an unsigned flux so that one has $\alpha = 0$. This option is favored by the construction of the configuration space spinor structure involving the construction of the fermionic super charges anti-commuting to configuration space Hamiltonians: super charges contain the square root of flux, which must be therefore unsigned. Second possibility is that magnetic fluxes are signed fluxes so that β vanishes.

One can define also the electric counterparts of the flux Hamiltonians by replacing J in the defining formulas with its dual $*J$

$$*J_{\alpha\beta} = \epsilon_{\alpha\beta}^{\ \ \gamma\delta} J_{\gamma\delta}.$$

For $H_A = 1$ these fluxes reduce to ordinary Kähler electric fluxes. These fluxes are however not symplectic covariants since the definition of the dual involves the induced metric, which is not symplectic invariant. The electric gauge fluxes for Hamiltonians in various representations of the color group ought to be important in the description of hadrons, not only as string like objects, but quite generally. These degrees of freedom would be identifiable as non-perturbative degrees of freedom involving genuinely classical Kähler field whereas quarks and gluons would correspond to the perturbative degrees of freedom, that is the interactions between CP_2 type extremals.

Poisson brackets

From the symplectic invariance of the radial component of Kähler magnetic field it follows that the Lie-derivative of the flux $Q_m^{\alpha,\beta}(H_A)$ with respect to the vector field $X(H_B)$ is given by

$$X(H_B) \cdot Q_m^{\alpha,\beta}(H_A) = Q_m^{\alpha,\beta}(\{H_B,H_A\}) \ . \tag{3.13.7}$$

The transformation properties of $Q_m^{\alpha,\beta}(H_A)$ are very nice if the basis for H_B transforms according to appropriate irreducible representation of color group and rotation group. This in turn implies that the fluxes $Q_m^{\alpha,\beta}(H_A)$ as functionals of 3-surface on given orbit provide a representation for the Hamiltonian as a functional of 3-surface. For a given surface X^3, the Poisson bracket for the two fluxes $Q_m^{\alpha,\beta}(H_A)$ and $Q_m^{\alpha,\beta}(H_B)$ can be defined as

$$\begin{aligned} \{Q_m^{\alpha,\beta}(H_A), Q_m^{\alpha,\beta}(H_B)\} &\equiv X(H_B)\cdot Q_m^{\alpha,\beta}(H_A) \\ &= Q_m^{\alpha,\beta}(\{H_A,H_B\}) = Q_m^{\alpha,\beta}(\{H_A,H_B\}) \ . \end{aligned} \tag{3.13.8}$$

The study of configuration space gamma matrices identifiable as symplectic super charges demonstrates that the supercharges associated with the radial deformations vanish identically so that radial deformations correspond to zero norm degrees of freedom as one might indeed expect on physical grounds. The reason is that super generators involve the invariants $j^{ak}\gamma_k$ which vanish by $\gamma_{r_M}=0$.

The natural central extension associated with the symplectic group of CP_2 ($\{p,q\}=1!$) induces a central extension of this algebra. The central extension term resulting from $\{H_A,H_B\}$ when CP_2 Hamiltonians have $\{p,q\}=1$ equals to the symplectic invariant $Q_m^{\alpha,\beta}(f(m_a+m_b,n_a+n_b,k_a+k_b))$ on the right hand side. This extension is however anti-symmetric in symplectic degrees of freedom rather than in loop space degrees of freedom and therefore does not lead to the standard Kac Moody type algebra.

Quite generally, the Virasoro and Kac Moody algebras of string models are replaced in TGD context by much larger symmetry algebras. Kac Moody algebra corresponds to the the deformations of light-like 3-surfaces respecting their light-likeness and leaving partonic 2-surfaces at δCD intact and are highly relevant to the elementary particle physics. This algebra allows a representation in terms of X_l^3 local Hamiltonians generating isometries of $\delta M_\pm^4\times CP_2$. Hamiltonian representation is essential for super-symmetrization since fermionic super charges anti-commute to Hamiltonians rather than vector fields: this is one of the deep differences between TGD and string models. Kac-Moody algebra does not contribute to configuration space metric since by definition the generators vanish at partonic 2-surfaces. This is essential for the coset space property.

A completely new algebra is the CP_2 symplectic algebra localized with respect to the light cone boundary and relevant to the configuration space geometry. This extends to $S^2\times CP_2$ -or rather $\delta M_\pm^4\times CP_2$ symplectic algebra and this gives the strongest predictions concerning configuration space metric. The local radial Virasoro localized with respect to $S^2\times CP_2$ acts in zero modes and has automatically vanishing norm with respect to configuration space metric defined by super charges.

3.13.5 Symplectic transformations of $\delta M_\pm^4\times CP_2$ as isometries and electric-magnetic duality

According to the construction of Kähler metric, symplectic transformations of $\delta M_\pm^4\times CP_2$ act as isometries whereas radial Virasoro algebra localized with respect to CP_2 has zero norm in the configuration space metric.

Hamiltonians can be organized into light like unitary representations of $so(3,1)\times su(3)$ and the symmetry condition $Zg(X,Y)=0$ requires that the component of the metric is $so(3,1)\times su(3)$ invariant and this condition is satisfied if the component of metric between two different representations D_1 and D_2 of $so(3,1)\times su(3)$ is proportional to Glebch-Gordan coefficient $C_{D_1D_2,D_S}$ between $D_1\otimes D_2$ and singlet representation D_S. In particular, metric has components only between states having identical $so(3,1)\times su(3)$ quantum numbers.

Magnetic representation of configuration space Hamiltonians means the action of the symplectic transformations of the light cone boundary as configuration space isometries is an intrinsic property of the light cone boundary. If electric-magnetic duality holds true, the preferred extremal property only determines the conformal factor of the metric depending on zero modes. This is precisely as it should be if the group theoretical construction works. Hence it should be possible by a direct calculation check whether the metric defined by the magnetic flux Hamiltonians as half Poisson brackets in complex coordinates is invariant under isometries. Symplectic invariance of the metric means that matrix elements of the metric are left translates of the metric along geodesic lines starting from the origin of coordinates, which now naturally corresponds to the preferred extremal of Kähler action. Since metric derives from symplectic form this means that the matrix elements of symplectic form given by Poisson brackets of Hamiltonians must be left translates of their values at origin along geodesic line. The matrix elements in question are given by flux Hamiltonians and since symplectic transforms of flux Hamiltonian is flux Hamiltonian for the symplectic transform of Hamiltonian, it seems that the conditions are satisfied.

3.14 General expressions for the symplectic and Kähler forms

One can derive general expressions for symplectic and Kähler forms as well as Kähler metric of the configuration space. The fact that these expressions involve only first variation of the Kähler action implies huge simplification of the basic formulas. Duality hypothesis leads to further simplifications of the formulas.

3.14.1 Closedness requirement

The fluxes of Kähler magnetic and electric fields for the Hamiltonians of $\delta M^4_+ \times CP_2$ suggest a general representation for the components of the symplectic form of the configuration space. The basic requirement is that Kähler form satisfies the defining condition

$$X \cdot J(Y,Z) + J([X,Y],Z) + J(X,[Y,Z]) = 0 \ , \tag{3.14.1}$$

where X, Y, Z are now vector fields associated with Hamiltonian functions defining configuration space coordinates.

3.14.2 Matrix elements of the symplectic form as Poisson brackets

Quite generally, the matrix element of $J(X(H_A), X(H_B))$ between vector fields $X(H_A))$ and $X(H_B))$ defined by the Hamiltonians H_A and H_B of $\delta M^4_+ \times CP_2$ isometries is expressible as Poisson bracket

$$J^{AB} = J(X(H_A), X(H_B)) = \{H_A, H_B\} \ . \tag{3.14.2}$$

J^{AB} denotes contravariant components of the symplectic form in coordinates given by a subset of Hamiltonians. The magnetic flux Hamiltonians $Q^{\alpha,\beta}_m(H_{A,k})$ of Eq. 4.6.1 provide an explicit representation for the Hamiltonians at the level of configuration space so that the components of the symplectic form of the configuration space are expressible as classical charges for the Poisson brackets of the Hamiltonians of the light cone boundary:

$$J(X(H_A), X(H_B)) = Q^{\alpha,\beta}_m(\{H_A, H_B\}) \ . \tag{3.14.3}$$

Recall that the superscript α, β refers the coefficients of J and $|J|$ in the superposition of these Kähler magnetic fluxes. Note that $Q^{\alpha,\beta}_m$ contains unspecified conformal factor depending on symplectic invariants characterizing Y^3 and is unspecified superposition of signed and unsigned magnetic fluxes.

This representation does not carry information about the tangent space of space-time surface at the partonic 2-surface, which motivates the proposal that also electric fluxes are present and proportional to magnetic fluxes with a factor K, which is symplectic invariant so that commutators of flux Hamiltonians come out correctly. This would give

$$Q^{\alpha,\beta}_m(H_A)_{em} = Q^{\alpha,\beta}_e(H_A) + Q^{\alpha,\beta}_m(H_A) = (1+K)Q^{\alpha,\beta}_m(H_A) \ . \tag{3.14.4}$$

Since Kähler form relates to the standard field tensor by a factor $e/\hbar$, flux Hamiltonians are dimensionless so that commutators do not involve $\hbar$. The commutators would come as

$$Q^{\alpha,\beta}_{em}(\{H_A, H_B\}) \rightarrow (1+K)Q^{\alpha,\beta}_m(\{H_A, H_B\}) \ . \tag{3.14.5}$$

The factor $1+K$ plays the same role as Planck constant in the commutators.

WCW Hamiltonians vanish for the extrema of the Kähler function as variational derivatives of the Kähler action. Hence Hamiltonians are good candidates for the coordinates appearing as coordinates in the perturbative functional integral around extrema (with maxima giving dominating contribution). It is clear that configuration space coordinates around a given extremum include only those Hamiltonians, which vanish at extremum (that is those Hamiltonians which span the tangent space of G/H) In Darboux coordinates the Poisson brackets reduce to the symplectic form

$$\begin{aligned} \{P^I,Q^J\} &= J^{IJ} = J_I\delta^{I,J} \ . \\ J_I &= 1 \ . \end{aligned} \tag{3.14.6}$$

It is not clear whether Darboux coordinates with $J_I = 1$ are possible in the recent case: probably the unit matrix on right hand side of the defining equation is replaced with a diagonal matrix depending on symplectic invariants so that one has $J_I \neq 1$. The integration measure is given by the symplectic volume element given by the determinant of the matrix defined by the Poisson brackets of the Hamiltonians appearing as coordinates. The value of the symplectic volume element is given by the matrix formed by the Poisson brackets of the Hamiltonians and reduces to the product

$$Vol = \prod_I J_I$$

in generalized Darboux coordinates.

Kähler potential (that is gauge potential associated with Kähler form) can be written in Darboux coordinates as

$$A = \sum_I J_I P_I dQ^I \ . \tag{3.14.7}$$

3.14.3 General expressions for Kähler form, Kähler metric and Kähler function

The expressions of Kähler form and Kähler metric in complex coordinates can obtained by transforming the contravariant form of the symplectic form from symplectic coordinates provided by Hamiltonians to complex coordinates:

$$J^{Z^i\bar{Z}^j} = iG^{Z^i\bar{Z}^j} = \partial_{H^A}Z^i\partial_{H^B}\bar{Z}^jJ^{AB} \ , \tag{3.14.8}$$

where J^{AB} is given by the classical Kahler charge for the light cone Hamiltonian $\{H^A,H^B\}$. Complex coordinates correspond to linear coordinates of the complexified Lie-algebra providing exponentiation of the isometry algebra via exponential mapping. What one must know is the precise relationship between allowed complex coordinates and Hamiltonian coordinates: this relationship is in principle calculable. In Darboux coordinates the expressions become even simpler:

$$J^{Z^i\bar{Z}^j} = iG^{Z^i\bar{Z}^j} = \sum_I J(I)(\partial_{P^i}Z^i\partial_{Q^I}\bar{Z}^j - \partial_{Q^I}Z^i\partial_{P^I}\bar{Z}^j) \ . \tag{3.14.9}$$

Kähler function can be formally integrated from the relationship

$$\begin{aligned} A_{Z^i} &= i\partial_{Z^i}K \ , \\ A_{\bar{Z}^i} &= -i\partial_{Z^i}K \ . \end{aligned} \tag{3.14.10}$$

holding true in complex coordinates. Kähler function is obtained formally as integral

$$K = \int_0^Z (A_{Z^i}dZ^i - A_{\bar{Z}^i}d\bar{Z}^i) \ . \tag{3.14.11}$$

3.14.4 $Diff(X^3)$ invariance and degeneracy and conformal invariances of the symplectic form

$J(X(H_A), X(H_B))$ defines symplectic form for the coset space G/H only if it is $Diff(X^3)$ degenerate. This means that the symplectic form $J(X(H_A), X(H_B))$ vanishes whenever Hamiltonian H_A or H_B is such that it generates diffeomorphism of the 3-surface X^3. If effective 2-dimensionality holds true, $J(X(H_A), X(H_B))$ vanishes if H_A or H_B generates two-dimensional diffeomorphism $d(H_A)$ at the surface X_i^2.

One can always write

$$J(X(H_A), X(H_B)) = X(H_A)Q(H_B|X_i^2) \ .$$

If H_A generates diffeomorphism, the action of $X(H_A)$ reduces to the action of the vector field X_A of some X_i^2-diffeomorphism. Since $Q(H_B|r_M)$ is manifestly invariant under the diffemorphisms of X^2, the result is vanishing:

$$X_A Q(H_B|X_i^2) = 0 \ ,$$

so that $Diff^2$ invariance is achieved.

The radial diffeomorphisms possibly generated by the radial Virasoro algebra do not produce trouble. The change of the flux integrand X under the infinitesimal transformation $r_M \rightarrow r_M + \epsilon r_M^n$ is given by $r_M^n dX/dr_M$. Replacing r_M with $r_M^{-n+1}/(-n+1)$ as variable, the integrand reduces to a total divergence dX/du the integral of which vanishes over the closed 2-surface X_i^2. Hence radial Virasoro generators having zero norm annihilate all matrix elements of the symplectic form. The induced metric of X_i^2 induces a unique conformal structure and since the conformal transformations of X_i^2 can be interpreted as a mere coordinate changes, they leave the flux integrals invariant.

3.14.5 Complexification and explicit form of the metric and Kähler form

The identification of the Kähler form and Kähler metric in symplectic degrees of freedom follows trivially from the identification of the symplectic form and definition of complexification. The requirement that Hamiltonians are eigen states of angular momentum (and possibly Lorentz boost generator), isospin and hypercharge implies physically natural complexification. In order to fix the complexification completely one must introduce some convention fixing which states correspond to 'positive' frequencies and which to 'negative frequencies' and which to zero frequencies that is to decompose the generators of the symplectic algebra to three sets Can_+, Can_- and Can_0. One must distinguish between Can_0 and zero modes, which are not considered here at all. For instance, CP_2 Hamiltonians correspond to zero modes.

The natural complexification relies on the imaginary part of the radial conformal weight whereas the real part defines the $g = t + h$ decomposition naturally. The wave vector associated with the radial logarithmic plane wave corresponds to the angular momentum quantum number associated with a wave in S^1 in the case of Kac Moody algebra. One can imagine three options.

1. It is quite possible that the spectrum of k_2 does not contain $k_2 = 0$ at all so that the sector Can_0 could be empty. This complexification is physically very natural since it is manifestly invariant under $SU(3)$ and $SO(3)$ defining the preferred spherical coordinates. The choice of $SO(3)$ is unique if the classical four-momentum associated with the 3-surface is time like so that there are no problems with Lorentz invariance.

2. If $k_2 = 0$ is possible one could have

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k=k1+ik_2}, k_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = 0\} \ . \end{aligned} \tag{3.14.12}$$

3. If it is possible to $n_2 \neq 0$ for $k_2 = 0$, one could define the decomposition as

$$
\begin{aligned}
Can_+ &= \{H^a_{m,n,k}, k_2 > 0 \text{ or } k_2 = 0, n_2 > 0\} \ , \\
Can_- &= \{H^a_{m,n,k}, k_2 < 0 \text{ or} k_2 = 0, n_2 < 0\} \ , \\
Can_0 &= \{H^a_{m,n,k}, k_2 = n_2 = 0\} \ .
\end{aligned} \tag{3.14.13}
$$

In this case the complexification is unique and Lorentz invariance guaranteed if one can fix the $SO(2)$ subgroup uniquely. The quantization axis of angular momentum could be chosen to be the direction of the classical angular momentum associated with the 3-surface in its rest system.

The only thing needed to get Kähler form and Kähler metric is to write the half Poisson bracket defined by Eq. 3.14.15

$$
\begin{aligned}
J_f(X(H_A), X(H_B)) &= 2Im\,(iQ_f(\{H_A, H_B\}_{-+})) \ , \\
G_f(X(H_A), X(H_B)) &= 2Re\,(iQ_f(\{H_A, H_B\}_{-+})) \ .
\end{aligned} \tag{3.14.14}
$$

Symplectic form, and thus also Kähler form and Kähler metric, could contain a conformal factor depending on the isometry invariants characterizing the size and shape of the 3-surface. At this stage one cannot say much about the functional form of this factor.

3.14.6 Comparison of CP_2 Kähler geometry with configuration space geometry

The explicit discussion of the role of $g = t + h$ decomposition of the tangent space of the configuration space provides deep insights to the metric of the symmetric space. There are indeed many questions to be answered. To what point of configuration space (that is 3-surface) the proposed $g = t + h$ decomposition corresponds to? Can one derive the components of the metric and Kähler form from the Poisson brackets of complexified Hamiltonians? Can one characterize the point in question in terms of the properties of configuration space Hamiltonians? Does the central extension of the configuration space reduce to the symplectic central extension of the symplectic algebra or can one consider also other options?

Cartan decomposition for CP_2

A good manner to gain understanding is to consider the CP_2 metric and Kähler form at the origin of complex coordinates for which the sub-algebra $h = u(2)$ defines the Cartan decomposition.

1. $g = t + h$ decomposition depends on the point of the symmetric space in general. In case of CP_2 $u(2)$ sub-algebra transforms as $g \circ u(2) \circ g^{-1}$ when the point s is replaced by gsg^{-1}. This is expected to hold true also in case of configuration space (unless it is flat) so that the task is to identify the point of the configuration space at which the proposed decomposition holds true.

2. The Killing vector fields of h sub-algebra vanish at the origin of CP_2 in complex coordinates. The corresponding Hamiltonians need not vanish but their Poisson brackets must vanish. It is possible to add suitable constants to the Hamiltonians in order to guarantee that they vanish at origin.

3. It is convenient to introduce complex coordinates and decompose isometry generators to holomorphic components $J^a_+ = j^{ak}\partial_k$ and $j^a_- = j^{a\bar{k}}\partial_{\bar{k}}$. One can introduce what might be called half Poisson bracket and half inner product defined as

$$
\begin{aligned}
\{H^a, H^b\}_{-+} &\equiv \partial_{\bar{k}} H^a J^{\bar{k}l} \partial_l H^b \\
&= j^{ak} J_{k\bar{l}} j^{b\bar{l}} = -i(j^a_+, j^b_-) \ .
\end{aligned} \tag{3.14.15}
$$

One can express Poisson bracket of Hamiltonians and the inner product of the corresponding Killing vector fields in terms of real and imaginary parts of the half Poisson bracket:

$$\begin{aligned} \{H^a, H^b\} &= 2Im\left(i\{H^a, H^b\}_{-+}\right) \ , \\ (j^a, j^b) &= 2Re\left(i(j^a_+, j^b_-)\right) = 2Re\left(i\{H^a, H^b\}_{-+}\right) \ . \end{aligned} \tag{3.14.16}$$

What this means that Hamiltonians and their half brackets code all information about metric and Kähler form. Obviously this is of utmost importance in the case of the configuration space metric whose symplectic structure and central extension are derived from those of CP_2.

Consider now the properties of the metric and Kähler form at the origin.

1. The relations satisfied by the half Poisson brackets can be written symbolically as

$$\begin{aligned} &\{h, h\}_{-+} = 0 \ , \\ &Re\left(i\{h, t\}_{-+}\right) = 0 \ , \quad Im\left(i\{h, t\}_{-+}\right) = 0 \ , \\ &Re\left(i\{t, t\}_{-+}\right) \neq 0 \ , \quad Im\left(i\{t, t\}_{-+}\right) \neq 0 \ . \end{aligned} \tag{3.14.17}$$

2. The first two conditions state that h vector fields have vanishing inner products at the origin. The first condition states also that the Hamiltonians for the commutator algebra $[h, h] = SU(2)$ vanish at origin whereas the Hamiltonian for $U(1)$ algebra corresponding to the color hyper charge need not vanish although it can be made vanishing. The third condition implies that the Hamiltonians of t vanish at origin.

3. The last two conditions state that the Kähler metric and form are non-vanishing between the elements of t. Since the Poisson brackets of t Hamiltonians are Hamiltonians of h, the only possibility is that $\{t, t\}$ Poisson brackets reduce to a non-vanishing $U(1)$ Hamiltonian at the origin or that the bracket at the origin is due to the symplectic central extension. The requirement that all Hamiltonians vanish at origin is very attractive aesthetically and forces to interpret $\{t, t\}$ brackets at origin as being due to a symplectic central extension. For instance, for S^2 the requirement that Hamiltonians vanish at origin would mean the replacement of the Hamiltonian $H = cos(\theta)$ representing a rotation around z-axis with $H_3 = cos(\theta) - 1$ so that the Poisson bracket of the generators H_1 and H_2 can be interpreted as a central extension term.

4. The conditions for the Hamiltonians of $u(2)$ sub-algebra state that their variations with respect to g vanish at origin. Thus $u(2)$ Hamiltonians have extremum value at origin.

5. Also the Kähler function of CP_2 has extremum at the origin. This suggests that in the case of the configuration space the counterpart of the origin corresponds to the maximum of the Kähler function.

Cartan algebra decomposition at the level of configuration space

The discussion of the properties of CP_2 Kähler metric at origin provides valuable guide lines in an attempt to understand what happens at the level of the configuration space. The use of the half bracket for the configuration space Hamiltonians in turn allows to calculate the matrix elements of the configuration space metric and Kähler form explicitly in terms of the magnetic or electric flux Hamiltonians.

The earlier construction was rather tricky and formula-rich and not very convincing physically. Cartan decomposition had to be assigned with something and in lack of anything better it was assigned with Super Virasoro algebra, which indeed allows this kind of decompositions but without any strong physical justification. The realization that super-symplectic and super Kac-Moody symmetries define coset construction at the level of basic quantum TGD, and that this construction provides a realization of Equivalence Principle at microscopic level, forced eventually the realization that also the coset space decomposition of configuration space realizes Equivalence Principle geometrically.

It must be however emphasized that holography implying effective 2-dimensionality of 3-surfaces in some length scale resolution is absolutely essential for this construction since it allows to effectively reduce Kac-Moody generators associated with X^3_l to $X^2 = X^3_l \cap \delta M^4_\pm \times CP_2$. In the similar manner super-symplectic generators can be dimensionally reduced to X^2. Number theoretical compactification forces the dimensional reduction and the known extremals are consistent with it [33]. The construction of configuration space spinor structure and metric in terms of the second quantized spinor fields [18] relies to this picture as also the recent view about M-matrix [16].

In this framework the coset space decomposition becomes trivial.

1. The algebra g is labeled by color quantum numbers of CP_2 Hamiltonians and by the label (m, n, k) labeling the function basis of the light cone boundary. Also a localization with respect to X^2 is needed. This is a new element as compared to the original view.

2. Super Kac-Moody algebra is labeled by color octet Hamiltonians and function basis of X^2. Since Lie-algebra action does not lead out of irreps, this means that Cartan algebra decomposition is satisfied.

3.14.7 Comparison with loop groups

It is useful to compare the recent approach to the geometrization of the loop groups consisting of maps from circle to Lie group G [45], which served as the inspirer of the configuration space geometry approach but later turned out to not apply as such in TGD framework.

In the case of loop groups the tangent space T corresponds to the local Lie-algebra $T(k, A) = exp(ik\phi)T_A$, where T_A generates the finite-dimensional Lie-algebra g and ϕ denotes the angle variable of circle; k is integer. The complexification of the tangent space corresponds to the decomposition

$$T = \{X(k > 0, A)\} \oplus \{X(k < 0, A)\} \oplus \{X(k = 0, A)\} = T_+ \oplus T_- \oplus T_0$$

of the tangent space. Metric corresponds to the central extension of the loop algebra to Kac Moody algebra and the Kähler form is given by

$$J(X(k_1 < 0, A), X(k_2 > 0, B)) = k_2\delta(k_1 + k_2)\delta(A, B) \ .$$

In present case the finite dimensional Lie algebra g is replaced with the Lie-algebra of the symplectic transformations of $\delta M^4_+ \times CP_2$ centrally extended using symplectic extension. The scalar function basis on circle is replaced with the function basis on an interval of length Δr_M with periodic boundary conditions; effectively one has circle also now.

The basic difference is that one can consider two kinds of central extensions now.

1. Central extension is most naturally induced by the natural central extension ($\{p, q\} = 1$) defined by Poisson bracket. This extension is anti-symmetric with respect to the generators of the symplectic group: in the case of the Kac Moody central extension it is symmetric with respect to the group G. The symplectic transformations of CP_2 might correspond to non-zero modes also because they are not exact symmetries of Kähler action. The situation is however rather delicate since $k = 0$ light cone harmonic has a diverging norm due to the radial integration unless one poses both lower and upper radial cutoffs although the matrix elements would be still well defined for typical 3-surfaces. For Kac Moody group $U(1)$ transformations correspond to the zero modes. Light cone function algebra can be regarded as a local $U(1)$ algebra defining central extension in the case that only CP_2 symplectic transformations local with respect to δM^4_+ act as isometries: for Kac Moody algebra the central extension corresponds to an ordinary $U(1)$ algebra. In the case that entire light cone symplectic algebra defines the isometries the central extension reduces to a $U(1)$ central extension.

3.14.8 Symmetric space property implies Ricci flatness and isometric action of symplectic transformations

The basic structure of symmetric spaces is summarized by the following structural equations

$$g = h + t \ ,$$
$$[h,h] \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ . \tag{3.14.18}$$

In present case the equations imply that all commutators of the Lie-algebra generators of $Can(\neq 0)$ having non-vanishing integer valued radial quantum number n_2, possess zero norm. This condition is extremely strong and guarantees isometric action of $Can(\delta M^4_+ \times CP_2)$ as well as Ricci flatness of the configuration space metric.

The requirement $[t,t] \subset h$ and $[h,t] \subset t$ are satisfied if the generators of the isometry algebra possess generalized parity P such that the generators in t have parity $P = -1$ and the generators belonging to h have parity $P = +1$. Conformal weight n must somehow define this parity. The first possibility to come into mind is that odd values of n correspond to $P = -1$ and even values to $P = 1$. Since n is additive in commutation, this would automatically imply $h \oplus t$ decomposition with the required properties. This assumption looks however somewhat artificial. TGD however forces a generalization of Super Algebras and N-S and Ramond type algebras can be combined to a larger algebra containing also Virasoro and Kac Moody generators labeled by half-odd integers. This suggests strongly that isometry generators are labeled by half integer conformal weight and that half-odd integer conformal weight corresponds to parity $P = -1$ whereas integer conformal weight corresponds to parity $P = 1$. Coset space would structure would state conformal invariance of the theory since super-symplectic generators with integer weight would correspond to zero modes.

Quite generally, the requirement that the metric is invariant under the flow generated by vector field X leads together with the covariant constancy of the metric to the Killing conditions

$$X \cdot g(Y,Z) \ = \ 0 = g([X,Y],Z) + g(Y,[X,Z]) \ . \tag{3.14.19}$$

If the commutators of the complexified generators in $Can(\neq 0)$ have zero norm then the two terms on the right hand side of Eq. (3.14.19) vanish separately. This is true if the conditions

$$Q_m^{\alpha,\beta}(\{H^A,\{H^B,H^C\}\}) \ = \ 0 \ , \tag{3.14.20}$$

are satisfied for all triplets of Hamiltonians in $Can_{\neq 0}$. These conditions follow automatically from the $[t,t] \subset h$ property and guarantee also Ricci flatness as will be found later.

It must be emphasized that for Kähler metric defined by purely magnetic fluxes, one cannot pose the conditions of Eq. (3.14.20) as consistency conditions on the initial values of the time derivatives of imbedding space coordinates whereas in general case this is possible. If the consistency conditions are satisfied for a single surface on the orbit of symplectic group then they are satisfied on the entire orbit. Clearly, isometry and Ricci flatness requirements and the requirement of time reversal invariance might well force Kähler electric alternative.

3.14.9 How to find Kähler function?

If one has found the expansion of configuration space Kähler form in terms of electric fluxes one can solve also the Kähler function from the defining partial differential equations $J_{k\bar{l}} = \partial k \partial_{\bar{l}} K$. The solution is not unique since the equation allows the symmetry

$$K \to K + f(z^k) + \overline{f(z^k)} \ ,$$

where f is arbitrary holomorphic function of z^k. This non-uniqueness is probably eliminated by the requirement that Kähler function vanishes for vacuum extremals. This in turn makes in principle possible to find the maxima of Kähler function and to perform functional integration perturbatively around them.

Electric-magnetic duality implies that, apart from conformal factor depending on isometry invariants, one can solve Kähler metric without any knowledge on the initial values of the time derivatives of the imbedding space coordinates. Apart from conformal factor the resulting geometry is purely intrinsic to δCH. The role of Kähler action is only to to define $Diff^4$ invariance and give the rule how the metric is translated to metric on arbitrary point of CH. The degeneracy of the preferred extrema also implies that configuration space has multi-sheeted structure analogous to that encountered in case of Riemann surfaces.

As shown in [34], very general assumptions inspired by the light-likeness of Kähler current for the known extremals combined with electric-magnetic duality imply the reduction of Kähler action for the preferred extremals to Chern-Simons terms at the ends of CD and at wormhole throats plus boundary term depending on induced metric so that one has almost topological QFT. The latter is due to the possibility to choose the gauge for Kähler potential to code information about conserved quantum numbers to the Kähler function and is the counterpart for the measurement interaction term in Dirac action. This term should correspond to a real part of a holomorphic function so that it does not contribute to the Kähler metric.

Also a promising concrete construction recipe for Kähler function is in terms of the modified Dirac operator [18]. The recipe is described briefly in the introduction. If the conjecture that Dirac determinant coincides with the exponent of Kähler action for a preferred extremal is correct, the value of the Kähler coupling strength follows as a prediction of the theory. From the construction it is clear that Dirac determinant involves only a finite number of eigenvalues of the modified Dirac operator and can thus be an algebraic or even rational number if eigenvalues have this property. The most satisfactory property of the construction is that one can use the intuition from the behavior of 2-D magnetic systems.

3.15 Ricci flatness and divergence cancelation

Divergence cancelation in configuration space integration requires Ricci flatness and in this section the arguments in favor of Ricci flatness are discussed in detail.

3.15.1 Inner product from divergence cancelation

Forgetting the delicacies related to the non-determinism of the Kähler action, the inner product is given by integrating the usual Fock space inner product defined at each point of the configuration space over the reduced configuration space containing only the 3-surfaces Y^3 belonging to $\delta H = \delta M^4_+ \times CP_2$ ('lightcone boundary') using the exponent $exp(K)$ as a weight factor:

$$\begin{aligned} \langle \Psi_1 | \Psi_2 \rangle &= \int \overline{\Psi}_1(Y^3)\Psi_2(Y^3) exp(K)\sqrt{G} dY^3 \ , \\ \overline{\Psi_1}(Y^3)\Psi_2(Y^3) &\equiv \langle \Psi_1(Y^3)|\Psi_2(Y^3)\rangle_{Fock} \ . \end{aligned} \tag{3.15.1}$$

The degeneracy for the preferred extremals of Kähler action implies additional summation over the degenerate extremals associated with Y^3. The restriction of the integration on light cone boundary is Diff^4 invariant procedure and resolves in elegant manner the problems related to the integration over Diff^4 degrees of freedom. A variant of the inner product is obtained dropping the bosonic vacuum functional $exp(K)$ from the definition of the inner product and by assuming that it is included into the spinor fields themselves. Probably it is just a matter of taste how the necessary bosonic vacuum functional is included into the inner product: what is essential that the vacuum functional $exp(K)$ is somehow present in the inner product.

The unitarity of the inner product follows from the unitary of the Fock space inner product and from the unitarity of the standard L^2 inner product defined by configuration space integration in the set of the L^2 integrable scalar functions. It could well occur that $Diff^4$ invariance implies the reduction of the configuration space integration to $C(\delta H)$.

Consider next the bosonic integration in more detail. The exponent of the Kähler function appears in the inner product also in the context of the finite dimensional group representations. For the representations of the noncompact groups (say $SL(2,R)$) in coset spaces (now $SL(2,R)/U(1)$ endowed with Kähler metric) the exponent of Kähler function is necessary in order to get square integrable representations [56]. The scalar product for two complex valued representation functions is defined as

$$(f,g) = \int \overline{f} g exp(nK)\sqrt{g} dV \ . \tag{3.15.2}$$

By unitarity, the exponent is an integer multiple of the Kähler function. In the present case only the possibility $n = 1$ is realized if one requires a complete cancelation of the determinants. In finite dimensional case this corresponds to the restriction to single unitary representation of the group in question.

The sign of the action appearing in the exponent is of decisive importance in order to make theory stable. The point is that the theory must be well defined at the limit of infinitely large system. Minimization of action is expected to imply that the action of infinitely large system is bound from above: the generation of electric Kähler fields gives negative contributions to the action. This implies that at the limit of the infinite system the average action per volume is non-positive. For systems having negative average density of action vacuum functional $exp(K)$ vanishes so that only configurations with vanishing average action per volume have significant probability. On the other hand, the choice $exp(-K)$ would make theory unstable: probability amplitude would be infinite for all configurations having negative average action per volume. In the fourth part of the book it will be shown that the requirement that average Kähler action per volume cancels has important cosmological consequences.

Consider now the divergence cancelation in the bosonic integration. One can develop the Kähler function as a Taylor series around maximum of Kähler function and use the contravariant Kähler metric as a propagator. Gaussian and metric determinants cancel each other for a unique vacuum functional. Ricci flatness guarantees that metric determinant is constant in complex coordinates so that one avoids divergences coming from it. The non-locality of the Kähler function as a functional of the 3-surface serves as an additional regulating mechanism: if $K(X^3)$ were a local functional of X^3 one would encounter divergences in the perturbative expansion.

The requirement that quantum jump corresponds to a quantum measurement in the sense of quantum field theories implies that quantum jump involves localization in zero modes. Localization in the zero modes implies automatically p-adic evolution since the decomposition of the configuration space into sectors D_P labeled by the infinite primes P is determined by the corresponding decomposition in zero modes. Localization in zero modes would suggest that the calculation of the physical predictions does not involve integration over zero modes: this would dramatically simplify the calculational apparatus of the theory. Probably this simplification occurs at the level of practical calculations if U-matrix separates into a product of matrices associated with zero modes and fiber degrees of freedom.

One must also calculate the predictions for the ratios of the rates of quantum transitions to different values of zero modes and here one cannot actually avoid integrals over zero modes. To achieve this one is forced to define the transition probabilities for quantum jumps involving a localization in zero modes as

$$P(x,\alpha \to y,\beta) = \sum_{r,s} |S(r,\alpha \to s,\beta)|^2 |\Psi_r(x)|^2 |\Psi_s(y)|^2 \ ,$$

where x and y correspond to the zero mode coordinates and r and s label a complete state functional basis in zero modes and $S(r,m \to s,n)$ involves integration over zero modes. In fact, only in this manner the notion of the localization in the zero modes makes mathematically sense at the level of S-matrix. In this case also unitarity conditions are well-defined. In zero modes state function basis can be freely constructed so that divergence difficulties could be avoided. An open question is whether this construction is indeed possible.

Some comments about the actual evaluation of the bosonic functional integral are in order.

1. Since configuration space metric is degenerate and the bosonic propagator is essentially the contravariant metric, bosonic integration is expected to reduce to an integration over the zero modes. For instance, isometry invariants are variables of this kind. These modes are analogous to the parameters describing the conformal equivalence class of the orbit of the string in string models.

2. α_K is a natural small expansion parameter in configuration space integration. It should be noticed that α_K, when defined by the criticality condition, could also depend on the coordinates parameterizing the zero modes.

3. Semiclassical approximation, which means the expansion of the functional integral as a sum over the extrema of the Kähler function, is a natural approach to the calculation of the bosonic

integral. Symmetric space property suggests that for the given values of the zero modes there is only single extremum and corresponds to the maximum of the Kähler function. There are theorems (Duistermaat-Hecke theorem) stating that semiclassical approximation is exact for certain systems (for example for integrable systems [48]). Symmetric space property suggests that Kähler function might possess the properties guaranteing the exactness of the semiclassical approximation. This would mean that the calculation of the integral $\int exp(K)\sqrt{G}dY^3$ and even more complex integrals involving configuration space spinor fields would be completely analogous to a Gaussian integration of free quantum field theory. This kind of reduction actually occurs in string models and is consistent with the criticality of the Kähler coupling constant suggesting that all loop integrals contributing to the renormalization of the Kähler action should vanish. Also the condition that configuration space integrals are continuable to p-adic number fields requires this kind of reduction.

3.15.2 Why Ricci flatness

It has been already found that the requirement of divergence cancelation poses extremely strong constraints on the metric of the configuration space. The results obtained hitherto are the following.

1. If the vacuum functional is the exponent of Kähler function one gets rid of the divergences resulting from the Gaussian determinants and metric determinants: determinants cancel each other.

2. The non-locality of the Kähler action gives good hopes of obtaining divergence free perturbation theory.

The following arguments show that Ricci flatness of the metric is a highly desirable property.

1. Dirac operator should be a well defined operator. In particular its square should be well defined. The problem is that the square of Dirac operator contains curvature scalar, which need not be finite since it is obtained via two infinite-dimensional trace operations from the curvature tensor. In case of loop spaces [45] the Kähler property implies that even Ricci tensor is only conditionally convergent. In fact, loop spaces with Kähler metric are Einstein spaces (Ricci tensor is proportional to metric) and Ricci scalar is infinite.

   In 3-dimensional case situation is even worse since the trace operation involves 3 summation indices instead of one! The conclusion is that Ricci tensor had better to vanish in vibrational degrees of freedom.

2. For Ricci flat metric the determinant of the metric is constant in geodesic complex coordinates as is seen from the expression for Ricci tensor [45]

$$R_{k\bar{l}} \;=\; \partial_k\partial_{\bar{l}}ln(det(g)) \tag{3.15.3}$$

   in Kähler metric. This obviously simplifies considerably functional integration over the configuration space: one obtains just the standard perturbative field theory in the sense that metric determinant gives no contributions to the functional integration.

3. The constancy of the metric determinant results not only in calculational simplifications: it also eliminates divergences. This is seen by expanding the determinant as a functional Taylor series with respect to the coordinates of the configuration space. In local complex coordinates the first term in the expansion of the metric determinant is determined by Ricci tensor

$$\delta\sqrt{g} \propto R_{k\bar{l}}z^k\bar{z}^l \quad . \tag{3.15.4}$$

   In configuration space integration using standard rules of Gaussian integration this term gives a contribution proportional to the contraction of the propagator with Ricci tensor. But since the propagator is just the contravariant metric one obtains Ricci scalar as result. So, in order to avoid divergences, Ricci scalar must be finite: this is certainly guaranteed if Ricci tensor vanishes.

4. The following group theoretic argument suggests that Ricci tensor either vanishes or is divergent. The holonomy group of the configuration space is a subgroup of $U(n=\infty)$ ($D=2n$ is the dimension of the Kähler manifold) by Kähler property and Ricci flatness is guaranteed if the $U(1)$ factor is absent from the holonomy group. In fact Ricci tensor is proportional to the trace of the $U(1)$ generator and since this generator corresponds to an infinite dimensional unit matrix the trace diverges: therefore given element of the Ricci tensor is either infinite or vanishes. Therefore the vanishing of the Ricci tensor seems to be a mathematical necessity. This naive argument doesn't hold true in the case of loop spaces, for which Kähler metric with finite non-vanishing Ricci tensor exists [45]. Note however that also in this case the sum defining Ricci tensor is only conditionally convergent.

There are indeed good hopes that Ricci tensor vanishes. By the previous argument the vanishing of the Ricci tensor is equivalent with the absence of divergences in configuration space integration. That divergences are absent is suggested by the non-locality of the Kähler function as a functional of 3-surface: the divergences of local field theories result from the locality of interaction vertices. Ricci flatness in vibrational degrees of freedom is not only necessary mathematically. It is also appealing physically: one can regard Ricci flat configuration space as a vacuum solution of Einstein's equations $G^{\alpha\beta}=0$.

3.15.3 Ricci flatness and Hyper Kähler property

Ricci flatness property is guaranteed if configuration space geometry is Hyper Kähler [49, 50] (there exists 3 covariantly constant antisymmetric tensor fields, which can be regarded as representations of quaternionic imaginary units). Hyper Kähler property guarantees Ricci flatness because the contractions of the curvature tensor appearing in the components of the Ricci tensor transform to traces over Lie algebra generators, which are $SU(n)$ generators instead of $U(n)$ generators so that the traces vanish. In the case of the loop spaces left invariance implies that Ricci tensor in the vibrational degrees is a multiple of the metric tensor so that Ricci scalar has an infinite value. This is basically due to the fact that Kac-Moody algebra has $U(1)$ central extension.

Consider now the arguments in favor of Ricci flatness of the configuration space.

1. The symplectic algebra of δM^4_+ takes effectively the role of the $U(1)$ extension of the loop algebra. More concretely, the $SO(2)$ group of the rotation group $SO(3)$ takes the role of $U(1)$ algebra. Since volume preserving transformations are in question, the traces of the symplectic generators vanish identically and in finite-dimensional this should be enough for Ricci flatness even if Hyper Kähler property is not achieved.

2. The comparison with CP_2 allows to link Ricci flatness with conformal invariance. The elements of the Ricci tensor are expressible in terms of traces of the generators of the holonomy group $U(2)$ at the origin of CP_2, and since $U(1)$ generator is non-vanishing at origin, the Ricci tensor is non-vanishing. In recent case the origin of CP_2 is replaced with the maximum of Kähler function and holonomy group corresponds to super-symplectic generators labelled by integer valued real parts k_1 of the conformal weights $k=k_1+i\rho$. If generators with $k_1=n$ vanish at the maximum of the Kähler function, the curvature scalar should vanish at the maximum and by the symmetric space property everywhere. These conditions correspond to Virasoro conditions in super string models.

   A possible source of difficulties are the generators having $k_1=0$ and resulting as commutators of generators with opposite real parts of the conformal weights. It might be possible to assume that only the conformal weights $k=k_1+i\rho$, $k_1=0,1,...$ are possible since it is the imaginary part of the conformal weight which defines the complexification in the recent case. This would mean that the commutators involve only positive values of k_1.

3. In the infinite-dimensional case the Ricci tensor involves also terms which are non-vanishing even when the holonomy algebra does not contain $U(1)$ factor. It will be found that symmetric space

property guarantees Ricci flatness even in this case and the reason is essentially the vanishing of the generators having $k_1 = n$ at the maximum of Kähler function.

There are also arguments in favor of the Hyper Kähler property.

1. The dimensions of the imbedding space and space-time are 8 and 4 respectively so that the dimension of configuration space in vibrational modes is indeed multiple of four as required by Hyper Kähler property. Hyper Kähler property requires a quaternionic structure in the tangent space of the configuration space. Since any direction on the sphere S^2 defined by the linear combinations of quaternionic imaginary units with unit norm defines a particular complexification physically, Hyper Kähler property means the possibility to perform complexification in S^2-fold manners.

2. S^2-fold degeneracy is indeed associated with the definition of the complex structure of the configuration space. First of all, the direction of the quantization axis for the spherical harmonics or for the eigen states of Lorentz Cartan algebra at δM^4_+ can be chosen in S^2-fold manners. Quaternion conformal invariance means Hyper Kähler property almost by definition and the S^2-fold degeneracy for the complexification is obvious in this case.

If these naive arguments survive a more critical inspection, the conclusion would be that the effective 2-dimensionality of light like 3-surfaces implying generalized conformal and symplectic symmetries would also imply Hyper Kähler property of the configuration space and make the theory well-defined mathematically. This obviously fixes the dimension of space-time surfaces as well as the dimension of Minkowski space factor of the imbedding space.

In the sequel we shall show that Ricci flatness is guaranteed provided that the holonomy group of the configuration space is isomorphic to some subgroup of $SU(n = \infty)$ instead of $U(n = \infty)$ (n is the complex dimension of the configuration space) implied by the Kähler property of the metric. We also derive an expression for the Ricci tensor in terms of the structure constants of the isometry algebra and configuration space metric. The expression for the Ricci tensor is formally identical with that obtained by Freed for loop spaces: the only difference is that the structure constants of the finite-dimensional group are replaced with the group $Can(\delta H)$. Also the arguments in favor of Hyper Kähler property are discussed in more detail.

3.15.4 The conditions guaranteing Ricci flatness

In the case of Kähler geometry Ricci flatness condition can be characterized purely Lie-algebraically: the holonomy group of the Riemann connection, which in general is subgroup of $U(n)$ for Kähler manifold of complex dimension n, must be subgroup of $SU(n)$ so that the Lie-algebra of this group consists of traceless matrices. This condition is easy to derive using complex coordinates. Ricci tensor is given by the following expression in complex vielbein basis

$$R^{A\bar{B}} = R^{A\bar{C}B}{}_{\bar{C}} \ , \tag{3.15.5}$$

where the latter summation is only over the antiholomorphic indices $\bar{C}$. Using the cyclic identities

$$\sum_{cycl\ \bar{C}B\bar{D}} R^{A\bar{C}B\bar{D}} = 0 \ , \tag{3.15.6}$$

the expression for Ricci tensor reduces to the form

$$R^{A\bar{B}} = R^{A\bar{B}C}{}_{C} \ , \tag{3.15.7}$$

where the summation is only over the holomorphic indices C. This expression can be regarded as a trace of the curvature tensor in the holonomy algebra of the Riemann connection. The trace is taken over holomorphic indices only: the traces over holomorphic and anti-holomorphic indices cancel each other by the antisymmetry of the curvature tensor. For Kähler manifold holonomy algebra is subalgebra of $U(n)$, when the complex dimension of manifold is n and Ricci tensor vanishes if and only if the holonomy Lie-algebra consists of traceless matrices, or equivalently: holonomy group is subgroup of $SU(n)$. This condition is expected to generalize also to the infinite-dimensional case.

We shall now show that if configuration space metric is Kähler and possesses infinite-dimensional isometry algebra with the property that its generators form a complete basis for the tangent space (every tangent vector is expressible as a superposition of the isometry generators plus zero norm vector) it is possible to derive a representation for the Ricci tensor in terms of the structure constants of the isometry algebra and of the components of the metric and its inverse in the basis formed by the isometry generators and that Ricci tensor vanishes identically for the proposed complexification of the configuration space provided the generators $\{H_{A,m\neq 0}, H_{B,n\neq 0}\}$ correspond to zero norm vector fields of configuration space.

The general definition of the curvature tensor as an operator acting on vector fields reads

$$R(X,Y)Z = [\nabla_X,\nabla_Y]Z - \nabla_{[X,Y]}Z \ . \tag{3.15.8}$$

If the vector fields considered are isometry generators the covariant derivative operator is given by the expression

$$\begin{aligned} \nabla_X Y &= (Ad_X Y - Ad^*_X Y - Ad^*_Y X)/2 \ , \\ (Ad^*_X Y, Z) &= (Y, Ad_X Z) \ , \end{aligned} \tag{3.15.9}$$

where $Ad_X Y = [X,Y]$ and Ad^*_X denotes the adjoint of Ad_X with respect to configuration space metric.

In the sequel we shall assume that the vector fields in question belong to the basis formed by the isometry generators. The matrix representation of Ad_X in terms of the structure constants $C_{X,Y:Z}$ of the isometry algebra is given by the expression

$$\begin{aligned} Ad^m_{Xn} &= C_{X,Y:Z}\hat{Y}_n Z^m \ , \\ [X,Y] &= C_{X,Y:Z} Z \ , \\ \hat{Y} &= g^{-1}(Y,V)V \ , \end{aligned} \tag{3.15.10}$$

where the summation takes place over the repeated indices and $\hat{Y}$ denotes the dual vector field of Y with respect to the configuration space metric. From its definition one obtains for Ad^*_X the matrix representation

$$\begin{aligned} Ad^{*m}_{Xn} &= C_{X,Y:Z}\hat{Y}^m Z_n \ , \\ Ad^*_X Y &= C_{X,U:V}\, g(Y,U) g^{-1}(V,W) W = g(Y,U)g^{-1}([X,U],W)W \ , \end{aligned} \tag{3.15.11}$$

where the summation takes place over the repeated indices.

Using the representations of ∇_X in terms of Ad_X and its adjoint and the representations of Ad_X and Ad^*_X in terms of the structure constants and some obvious identities (such as $C_{[X,Y],Z:V} = C_{X,Y:U}C_{U,Z:V}$) one can by a straightforward but tedious calculation derive a more detailed expression for the curvature tensor and Ricci tensor. Straightforward calculation of the Ricci tensor has however turned to be very tedious even in the case of the diagonal metric and in the following we shall use a more convenient representation [45] of the curvature tensor applying in case of the Kähler geometry.

The expression of the curvature tensor is given in terms of the so called Toeplitz operators T_X defined as linear operators in the "positive energy part" G_+ of the isometry algebra spanned by the $(1,0)$ parts of the isometry generators. In present case the positive and negative energy parts and cm part of the algebra can be defined just as in the case of loop spaces:

$$\begin{aligned} G_+ &= \{H^{Ak} | k > 0\} \ , \\ G_- &= \{H^{Ak} | k < 0\} \ , \\ G_0 &= \{H^{Ak} | k = 0\} \ . \end{aligned} \tag{3.15.12}$$

Here H^{Ak} denote the Hamiltonians generating the symplectic transformations of δH. The positive energy generators with non-vanishing norm have positive radial scaling dimension: $k \geq 0$, which corresponds to the imaginary part of the scaling momentum $K = k_1 + i\rho$ associated with the factors $(r_M/r_0)^K$. A priori the spectrum of ρ is continuous but it is quite possible that the spectrum of ρ is discrete and $\rho = 0$ does not appear at all in the spectrum in the sense that the flux Hamiltonians associated with $\rho = 0$ elements vanish for the maximum of Kähler function which can be taken to be the point where the calculations are done.

T_X differs from Ad_X in that the negative energy part of $Ad_X Y = [X, Y]$ is dropped away:

$$\begin{aligned} T_X : G_+ &\rightarrow G_+ \ , \\ Y &\rightarrow [X, Y]_+ \ . \end{aligned} \tag{3.15.13}$$

Here " + " denotes the projection to "positive energy" part of the algebra. Using Toeplitz operators one can associate to various isometry generators linear operators $\Phi(X_0)$, $\Phi(X_-)$ and $\Phi(X_+)$ acting on G_+:

$$\begin{aligned} \Phi(X_0) &= T_{X_0} \ , \ X_0 \varepsilon G_0 \ , \\ \Phi(X_-) &= T_{X_-} \ , \ X_- \varepsilon G_- \ , \\ \Phi(X_+) &= -T^*_{X_-} \ , \ X_+ \varepsilon G_+ \ . \end{aligned} \tag{3.15.14}$$

Here "*" denotes hermitian conjugate in the diagonalized metric: the explicit representation $\Phi(X_+)$ is given by the expression [45]

$$\begin{aligned} \Phi(X_+) &= D^{-1} T_{X_-} D \ , \\ D X_+ &= d(X) X_- \ , \\ d(X) &= g(X_-, X_+) \ . \end{aligned} \tag{3.15.15}$$

Here $d(X)$ is just the diagonal element of metric assumed to be diagonal in the basis used. denotes the conformal factor associated with the metric.

The representations for the action of ,$\Phi(X_0)$, $\Phi(X_-)$ and $\Phi(X_+)$ in terms of metric and structure constants of the isometry algebra are in the case of the diagonal metric given by the expressions

$$\begin{aligned} \Phi(X_0) Y_+ &= C_{X_0, Y_+ : U_+} U_+ \ , \\ \Phi(X_-) Y_+ &= C_{X_-, Y_+ : U_+} U_+ \ , \\ \Phi(X_+) Y_+ &= \frac{d(Y)}{d(U)} C_{X_-, Y_- : U_-} U_+ \ . \end{aligned} \tag{3.15.16}$$

The expression for the action of the curvature tensor in positive energy part G_+ of the isometry algebra in terms of the these operators is given as [45]:

$$R(X, Y) Z_+ = \{[\Phi(X), \Phi(Y)] - \Phi([X, Y])\} Z_+ \ . \tag{3.15.17}$$

The calculation of the Ricci tensor is based on the observation that for Kähler manifolds Ricci tensor is a tensor of type $(1, 1)$, and therefore it is possible to calculate Ricci tensor as the trace of the curvature tensor with respect to indices associated with G_+.

$$Ricci(X_+, Y_-) = (\hat{Z}_+, R(X_+, Y_-) Z_+) \equiv Trace(R(X_+, Y_-)) \ , \tag{3.15.18}$$

where the summation over Z_+ generators is performed.

Using the explicit representations of the operators Φ one obtains the following explicit expression for the Ricci tensor

$$
\begin{aligned}
Ricci(X_+,Y_-) &= Trace\{[D^{-1}T_{X_+}D,T_{Y_-}] - T_{[X_+,Y_-]_{|G_0+G_-}} \\
&- D^{-1}T_{[X_+,Y_-]_{|G_+}}D\} \ . \qquad (3.15.19)
\end{aligned}
$$

This expression is identical to that encountered in case of loop spaces and the following arguments are repetition of those applying in the case of loop spaces.

The second term in the Ricci tensor is the only term present in the finite-dimensional case. This term vanishes if the Lie-algebra in question consists of traceless matrices. Since symplectic transformations are volume-preserving the traces of Lie-algebra generators vanish so that this term is absent. The last term gives a non-vanishing contribution to the trace for the same reason.

The first term is quadratic in structure constants and does not vanish in case of loop spaces. It can be written explicitly using the explicit representations of the various operators appearing in the formula:

$$
\begin{aligned}
Trace\{[D^{-1}T_{X_-}D,T_{Y_-}]\} &= \sum_{Z_+,U_+}[C_{X_-,U_-:Z_-}C_{Y_-,Z_+:U_+}\frac{d(U)}{d(Z)} \\
&- C_{X_-,Z_-:U_-}C_{Y_-,U_+:Z_+}\frac{d(Z)}{d(U)}] \ . \qquad (3.15.20)
\end{aligned}
$$

Each term is antisymmetric under the exchange of U and Z and one might fail to conclude that the sum vanishes identically. This is not the case. By the diagonality of the metric with respect to radial quantum number, one has $m(X_-) = m(Y_-)$ for the non-vanishing elements of the Ricci tensor. Furthermore, one has $m(U) = m(Z) - m(Y)$, which eliminates summation over $m(U)$ in the first term and summation over $m(Z)$ in the second term. Note however, that summation over other labels related to symplectic algebra are present.

By performing the change $U \to Z$ in the second term one can combine the sums together and as a result one has finite sum

$$
\begin{aligned}
\sum_{0<m(Z)<m(X)}[C_{X_-,U_-:Z_-}C_{Y_-,Z_+:U_+}\frac{d(U)}{d(Z)} &= C\sum_{0<m(Z)<m(X)}\frac{m(X)}{m(Z)-m(X)} \ , \\
C &= \sum_{Z,U}C_{X,U:Z}C_{Y,Z:U}\frac{d_0(U)}{d_0(Z)} \ . \qquad (3.15.21)
\end{aligned}
$$

Here the dependence of $d(X) = |m(X)|d_0(X)$ on $m(X)$ is factored out; $d_0(X)$ does not depend on k_X. The dependence on $m(X)$ in the resulting expression factorizes out, and one obtains just the purely group theoretic term C, which should vanish for the space to be Ricci flat.

The sum is quadratic in structure constants and can be visualized as a loop sum. It is instructive to write the sum in terms of the metric in the symplectic degrees of freedom to see the geometry behind the Ricci flatness:

$$
C = \sum_{Z,U}g([Y,Z],U)g^{-1}([X,U],Z) \ . \qquad (3.15.22)
$$

Each term of this sum involves a commutator of two generators with a non-vanishing norm. Since tangent space complexification is inherited from the local coset space, the non-vanishing commutators in complexified basis are always between generators in $Can_{\neq 0}$; that is they do not not belong to rigid $su(2)\times su(3)$.

The condition guaranteing Ricci flatness at the maximum of Kähler function and thus everywhere is simple. All elements of type $[X_{\neq 0},Y_{\neq 0}]$ vanish or have vanishing norm. In case of CP_2 Kähler geometry this would correspond to the vanishing of the $U(2)$ generators at the origin of CP_2 (note that the holonomy group is $U(2)$ in case of CP_2). At least formally stronger condition is that the algebra generated by elements of this type, the commutator algebra associated with $Can_{\neq 0}$, consist of elements of zero norm. Already the (possibly) weaker condition implies that adjoint map $Ad_{X\neq 0}$ and its hermitian adjoint $Ad^*_{X_{\neq 0}}$ create zero norm states. Since isometry conditions involve also adjoint action the condition also implies that $Can_{\neq 0}$ acts as isometries. More concrete form for the condition is that all flux factors involving double Poisson bracket and three generators in $Can_{\neq 0}$ vanish:

$$Q_e(\{H_A,\{H_B,H_C\}\}) \;=\; 0 \;,\; \text{for } H_A,H_B,H_C \text{ in } Can_{\neq 0} \;. \tag{3.15.23}$$

The vanishing of fluxes involving two Poisson brackets and three Hamiltonians guarantees isometry invariance and Ricci flatness and, as found in [22], is implied by the $[t,t] \subset h$ property of the Lie-algebra of coset space G/H having symmetric space structure.

The conclusion is that the mere existence of the proposed isometry group (guaranteed by the symmetric space property) implies the vanishing of the Ricci tensor and vacuum Einstein equations. The existence of the infinite parameter isometry group in turn follows basically from the condition guaranteing the existence of the Riemann connection. Therefore vacuum Einstein equations seem to arise, not only as a consequence of a physically motivated variational principle but as a mathematical consistency condition in infinite dimensional Kähler geometry. The flux representation seems to provide elegant manner to formulate and solve these conditions and isometry invariance implies Ricci flatness.

3.15.5 Is configuration space metric Hyper Kähler?

The requirement that configuration space integral integration is divergence free implies that configuration space metric is Ricci flat. The so called Hyper-Kähler metrics [50, 49, 60] are particularly nice representatives of Ricci flat metrics. In the following the basic properties of Hyper-Kähler metrics are briefly described and the problem whether Hyper Kähler property could realized in case of $M^4_+ \times CP_2$ is considered.

Hyper-Kähler property

Hyper-Kähler metric is a generalization of the Kähler metric. For Kähler metric metric tensor and Kähler form correspond to the complex numbers 1 and i and therefore define complex structure in the tangent space of the manifold. For Hyper Kähler metric tangent space allows three closed Kähler forms I,J,K, which with respect to the multiplication obey the algebra of quaternionic imaginary units and have square equal to - 1, which corresponds to the metric of Hyper Kähler space.

$$I^2 \;=\; J^2 = K^2 = -1 \;\; IJ = -JI = K, \text{ etc. } . \tag{3.15.24}$$

To define Kähler structure one must choose one of the Kähler forms or any linear combination of I,J and K with unit norm. The group $SO(3)$ rotates different Kähler structures to each other playing thus the role of quaternion automorphisms. This group acts also as coordinate transformations in Hyper Kähler manifold but in general fails to act as isometries.

If K is chosen to define complex structure then K is tensor of type $(1,1)$ in complex coordinates, I and J being tensors of type $(2,0)+(0,2)$. The forms $I+iJ$ and $I-iJ$ are holomorphic and anti-holomorphic forms of type $(2,0)$ and $(0,2)$ respectively and defined standard step operators I_+ and I_- of $SU(2)$ algebra. The holonomy group of Hyper-Kähler metric is always $Sp(k)$, $k \leq dimM/4$, the group of $k\times k$ unitary matrices with quaternionic entries. This group is indeed subgroup of SU(2k), so that its generators are traceless and Hyper Kähler metric is therefore Ricci flat.

Hyper Kähler metrics have been encountered in the context of 3-dimensional super symmetric sigma models: a necessary prerequisite for obtaining $N=4$ super-symmetric sigma model is that target space allows Hyper Kähler metric [58, 60]. In particular, it has been found that Hyper Kähler property is decisive for the divergence cancelation.

Hyper-Kähler metrics arise also in monopole and instanton physics [50]. The moduli spaces for monopoles have Hyper Kähler property. This suggests that Hyper Kähler property is characteristic for the configuration (or moduli) spaces of 4-dimensional Yang Mills types systems. Since YM action appears in the definition of configuration space metric there are hopes that also in present case the metric possesses Hyper-Kähler property.

CP_2 allows what might be called almost Hyper-Kähler structure known as quaternionion structure. This means that the Weil tensor of CP_2 consists of three components in one-one correspondence with components of iso-spin and only one of them- the one corresponding to Kähler form- is covariantly constant. The physical interpretation is in terms of electroweak symmetry breaking selecting one isospin direction as a favored direction.

Does the 'almost' Hyper-Kähler structure of CP_2 lift to a genuine Hyper-Kähler structure in configuration space?

The Hyper-Kähler property of configuration space metric does not seem to be in conflict with the general structure of TGD.

1. In string models the dimension of the "space-time" is two and Weyl invariance and complex structures play a decisive role in the theory. In present case the dimension of the space-time is four and one therefore might hope that quaternions play a similar role. Indeed, Weyl invariance implies YM action in dimension 4 and as already mentioned moduli spaces of instantons and monopoles enjoy the Hyper Kähler property.

2. Also the dimension of the imbedding space is important. The dimension of Hyper Kähler manifold must be multiple of 4. The dimension of configuration space is indeed infinite multiple of 8: each vibrational mode giving one "8".

3. The complexification of the configuration space in symplectic degrees of freedom is inherited from $S^2 \times CP_2$ and CP_2 Kähler form defines the symplectic form of configuration space. The point is that CP_2 Weyl tensor has 3 covariantly constant components, having as their square metric apart from sign. One of them is Kähler form, which is closed whereas the other two are non-closed forms and therefore fail to define Kähler structure. The group $SU(2)$ of electro-weak isospin rotations rotate these forms to each other. It would not be too suprising if one could identify the configuration space counterparts of these forms as representations of quaternionic units at the level of configuration space. The failure of the Hyper Kähler property at the level of CP_2 geometry is due to the electro-weak symmetry breaking and physical intuition (in particular, p-adic mass calculations [4]) suggests that electro-weak symmetry might not be broken at the level of configuration space geometry).

A possible topological obstruction for the Hyper Kähler property is related to the cohomology of the configuration space: the three Kähler forms must be co-homologically trivial as is clear from the following argument. If any of 3 quaternionic 2-form is cohomologically nontrivial then by $SO(3)$ symmetry rotating Kähler forms to each other all must be co-homologically nontrivial. On the other hand, electro-weak isospin rotation leads to a linear combination of 3 Kähler forms and the flux associated with this form is in general not integer valued. The point is however that Kähler form forms only the $(1,1)$ part of the symplectic form and must be co-homologically trivial whereas the zero mode part is same for all complexifications and can be co-homologically nontrivial. The co-homological non-triviality of the zero mode part of the symplectic form is indeed a nice feature since it fixes the normalization of the Kähler function apart from a multiplicative integer. On the other hand the hypothesis that Kähler coupling strength is analogous to critical temperature provides a dynamical (and perhaps equivalent) manner to fix the normalization of the Kähler function.

Since the properties of the configuration space metric are inherited from $M^4_+ \times CP_2$ then also the Hyper Kähler property should be understandable in terms of the imbedding space geometry. In particular, the complex structure in CP_2 vibrational degrees of freedom is inherited from CP_2. Hyper Kähler property implies the existence of a continuum (sphere S^2) of complex structures: any linear superposition of 3 independent Kähler forms defines a respectable complex structure. Therefore also CP_2 should have this continuum of complex structures and this is certainly not the case.

Indeed, if we had instead of CP_2 Hyper Kähler manifold with 3 covariantly constant 2-forms then it would be easy to understand the Hyper Kähler structure of configuration space. Given the Kähler structure of the configuration space would be obtained by replacing induced Kähler electric and magnetic fields in the definition of flux factors $Q(H_{A,m})$ with the appropriate component of the induced Weyl tensor. CP_2 indeed manages to be very nearly Hyper Kähler manifold!

How CP_2 fails to be Hyper Kähler manifold can be seen in the following manner. The Weyl tensor of CP_2 allows three independent components, which are self dual as 2-forms and rotated to each other by vielbein rotations.

$$\begin{aligned} W_{03} &= W_{12} \equiv 2I_3 = 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ W_{01} &= W_{23} \equiv I_1 = -e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ W_{02} &= W_{31} \equiv I_2 = -e^0 \wedge e^2 - e^3 \wedge e^1 \ . \end{aligned} \tag{3.15.25}$$

The component I_3 is just the Kähler form of CP_2. Remaining components are covariantly constant only with respect to spinor connection and not closed forms so that they cannot be interpreted as Maxwell fields. Their squares equal however apart from sign with the metric of CP_2, when appropriate normalization factor is used. If these forms were covariantly constant Kähler action defined by any linear superposition of these forms would indeed define Kähler structure in configuration space and the group $SO(3)$ would rotate these forms to each other. The projections of the components of the Weyl tensor on 3-surface define 3 vector fields as their duals and only one of these vector fields (Kähler magnetic field) is divergenceless. One might regard these 3 vector fields as counter parts of quaternion units associated with the broken Hyper Kähler structure, that is quaternion structure. The interpretation in terms of electro-weak symmetry breaking is obvious.

One cannot exclude the possibility that the symplectic invariance of the induced Kähler electric field implies that the electric parts of the other two components of induced Weyl tensor are symplectic invariants. This is the minimum requirement. What is however obvious is that the magnetic parts cannot be closed forms for arbitrary 3-surfaces at light cone boundary. One counter example is enough and CP_2 type extremals seem to provide this counter example: the components of the induced Weyl tensor are just the same as they are for CP_2 and clearly not symplecticly invariant.

Thus it seems that configuration space could allow Hyper Kähler structure broken by electro-weak interactions but it cannot be inherited from CP_2. An open question is whether it allows genuine quaternionic structure. Good prospects for obtaining quaternionic structure are provided by the quaternionic counterpart QP_2 of CP_2, which is 8-dimensional and has coset space structure $QP_2 = Sp(3)/Sp(2) \times Sp(1)$. This choice does not seem to be consistent with the symmetries of the standard model. Note however that the over all symmetry group is obtained by replacing complex numbers with quaternions on the matrix representation of the standard model group.

Could different complexifications for M^4_+ and light like surfaces induce Hyper Kähler structure for configuration space?

Quaternionic structure means also the existence of a family of complex structures parameterized by a sphere S^2. The complex structure of the configuration space is inherited from the complex structure of some light like surface.

In the case of the light cone boundary δM^4_+ the complex structure corresponds to the choice of quantization axis of angular momentum for the sphere $r_M = constant$ so that the coordinates orthogonal to the quantization axis define a complex coordinate: the sphere S^2 parameterizes these choices. Thus there is a temptation to identify the choice of quantization axis with a particular imaginary unit and Hyper Kähler structure would directly relate to the properties rotation group. This would bring an additional item to the list of miraculous properties of light like surfaces of 4-dimensional space-times.

This might relate to the fact that configuration space geometry is not determined by the symplectic algebra of CP_2 localized with respect to the light cone boundary as one might first expect but consists of $M^4_+ \times CP_2$ Hamiltonians so that infinitesimal symplectic transformation of CP_2 involves always also M^4_+-symplectic transformation. M^4_+ Hamiltonians are defined by a function basis generated as products of the Hamiltonians H_3 and $H_1 \pm iH_2$ generating rotations with respect to three orthogonal axes, and two of these Hamiltonians are complexified.

Also the light like 3-surfaces X^3_l associated with quaternion conformal invariance are determined by some 2-surface X^2 and the choice of complex coordinates and if X^2 is sphere the choices are labelled by S^2. In this case, the presence of quaternion conformal structure would be almost obvious since it is possible to choose some complex coordinate in several manners and the choices are labelled by S^2. The choice of the complex coordinate in turn fixes 2-surface X^2 as a surface for which the remaining coordinates are constant. X^2 need not however be located at the elementary particle horizon unless one poses additional constraint. One might hope that different choices of X^2 resulting in this manner correspond to all possible different selections of the complex structure and that this choice could fix uniquely the conformal equivalence class of X^2 appearing as argument in elementary particle vacuum functionals. If X^2 has a more complex topology the identification is not so clear but since conformal algebra SL(2,C) containing algebra of rotation group is involved, one might argue that the choice of quantization axis also now involves S^2 degeneracy. If these arguments are correct one could conclude that Hyper Kähler structure is implicitly involved and guarantees Ricci flatness of the configuration space metric.

3.16 Consistency conditions on metric

In this section various consistency conditions on the configuration space metric are discussed. In particular, it will be found that the conditions guaranteing the existence of Riemann connection in the set of all(!) vector fields (including zero norm vector fields) gives very strong constraints on the general form of the metric and that these constraints are indeed satisfied for the proposed metric.

3.16.1 Consistency conditions on Riemann connection

To study the consequences of the consistency conditions, it is most convenient to consider matrix elements of the metric in the basis formed by the isometry generators themselves. The consistency conditions state the covariant constancy of the metric tensor

$$\nabla_Z g(X,Y) = g(\nabla_Z X,Y) + g(X,\nabla_Z Y) = Z\cdot g(X,Y) \ . \tag{3.16.1}$$

$Z\cdot g(X,Y)$ vanishes, when Z generates isometries so that conditions state the covariant constancy of the matrix elements in this case. It must be emphasized that the ill defined-ness of the inner products of form $g(\nabla_Z X,Y)$ is just the reason for requiring infinite-dimensional isometry group. The point is that $\nabla_Z X$ need not to belong to the Hilbert space spanned by the tangent vector fields since the terms of type $Zg(X,Y)$ do not necessarily exist mathematically [45]. The elegant solution to the problem is that all tangent space vector fields act as isometries so that these quantities vanish identically.

The conditions of Eq. (3.16.1) can be written explicitly by using the general expression for the covariant derivative

$$\begin{aligned} g(\nabla_Z X,Y) &= [Zg(X,Y)+Xg(Z,Y)-Yg(Z,X) \\ &+ g(Ad_Z X - Ad^*_Z X - Ad^*_X Z,Y)]/2 \ . \end{aligned} \tag{3.16.2}$$

What happens is that the terms depending on the derivatives of the matrix elements (terms of type $Zg(X,Y)$) cancel each other (these terms vanish for the metric invariant under isometries), and one obtains the following consistency conditions

$$g(Ad_Z X - Ad^*_Z X - Ad^*_X Z,Y) + g(X, Ad_Z Y - Ad^*_Z Y - Ad^*_Y Z) = 0 \ . \tag{3.16.3}$$

Using the explicit representations of $Ad_Z X$ and $Ad*_Z\, X$ in terms of structure constants

$$\begin{aligned} Ad_Z X &= [Z,X] = C_{Z,X:U}U \ . \\ Ad^*_Z X &= C_{Z,U:V}\, g(X,V) g^{-1}(U,W)W = g(X,[Z,U])g^{-1}(U,W)W \ . \end{aligned} \tag{3.16.4}$$

where the summation over repeated "indices" is performed, one finds that consistency conditions are identically satisfied provided the generators X and Y have a non-vanishing norm. The reason is that the contributions coming from $\nabla_Z X$ and $\nabla_Z Y$ cancel each other.

When one of the generators, say X, appearing in the inner product has a vanishing norm so that one has $g(X,Y)=0$, for any generator Y, situation changes! The contribution of $\nabla_Z Y$ term to the consistency conditions drops away and using Eqs. (3.16.3) and (3.16.4) one obtains the following consistency conditions

$$C_{Z,X:U}g(U,Y) + C_{X,Y:U}g(U,Z) = -X \cdot g(Z,Y) \ . \tag{3.16.5}$$

Note that summation over U is carried out. If X is isometry generator (this need not be the case always) the condition reduces to a simpler form:

$$C_{X,Z:U}g(U,Y) + C_{X,Y:U}g(Z,U) = g([X,Z].Y) + g(Z,[X,Y]) = 0 \ . \tag{3.16.6}$$

These conditions have nice geometric interpretation. If the matrix elements are regarded as ordinary Hilbert space products between the isometry generators the conditions state that the metric defining the inner product behaves as a scalar in the general case.

3.16.2 Consistency conditions for the radial Virasoro algebra

The action of the radial Virasoro in nontrivial manner in the zero modes. Therefore isometry interpretation is excluded and consistency conditions do not make sense in this case. One can however consider the possibility that metric is invariant or suffers only an overall scaling under the action of the radial scaling generated by $L_0 = r_M d/dr_M$. Since the radial integration measure is scaling invariant and only powers of r_M/r_0 appear in Hamiltonians, the effect of the scaling $r_M \rightarrow \lambda r_M$ on the matrix elements of the metric is a scaling by $\lambda^{k_a+\bar{k}_b}$). One can interpret this by saying that scaling changes the values of zero modes and hence leads outside the symmetric space in question.

Invariance of reduced matrix element obtained by dividing away the powers of the scaling factor is achieved if the metric contains the conformal factor

$$S = \frac{1}{\Delta u} f(\frac{r_i}{r_j}) \ , \tag{3.16.7}$$

where r_i are the extrema of r_M interpreted as height function of X^3 and f is a priori arbitrary positive definite function. Since the presence of f presumably gives rise to renormalization corrections depending on the size and shape of 3-surface by scaling the propagator defined by the contravariant metric, the dependence on the ratios r_i/r_j should be slow, logarithmic dependence. Also the dependence on the Fourier components of the solid angles $\Omega(r_M)$ associated with the $r_M = constant$ sections is possible.

3.16.3 Explicit conditions for the isometry invariance

The identification of the Lie-algebra of isometry generators has been proposed but cannot provide any proof for the existence of the infinite parameter symmetry group at this stage. What one can do at this stage is to formulate explicitly the conditions guaranteing isometry invariance of the metric and try to see whether there are any hopes that these conditions are satisfied. It has been already found that the expression of the metric reduces for light cone alternative to the sum of two boundary terms coming from infinite future and from the boundary of the light cone. If the contribution from infinitely distant future vanishes, as one might expect, then only the contribution from the boundary of the light cone remains.

A tedious but straightforward evaluation of the second variation (see Appendix of the book) for Kähler action implies the following form for the second variation of the Kähler action

$$\delta^2 S = \int_{a=0}^{a=\infty} I^{n\beta}_{kl} \delta h^k D_\beta \delta h^l \ , \tag{3.16.8}$$

where the tensor $I^{\alpha\beta}_{kl}$ is defined as partial derivatives of the Kähler Lagrangian with respect to the derivatives $\partial_\alpha h^k$

$$I_{kl}^{\alpha\beta} = \partial_{\partial_\alpha h^k}\partial_{\partial_\beta h^l} L_M \ . \tag{3.16.9}$$

If the upper limit $a = \sqrt{(m^0)^2 - r_M^2} = \infty$ in the substitution vanishes then one can calculate second variation and therefore metric from the knowledge of the time derivatives $\partial_n h^k$ and $\partial_n \delta h^k$ on the boundary of the light cone only.

Kähler metric can be identified as the $(1,1)$ part of the second variation. This means that one can express the deformation as an element of the isometry algebra plus a arbitrary deformation in radial direction of the light cone boundary interpretable as conformal transformation of the light cone boundary. Radial contributions to the second variation are dropped (by definition of Kähler metric) and what remains is essentially a deformation in S^2 degrees of freedom.

The left invariance of the metric under the deformations of the isometry algebra implies an infinite number of conditions of the form

$$J^C g(J^A, J^B) = 0 \ , \tag{3.16.10}$$

where J^A, J^B and J^C denote the generators of the isometry group. These conditions ought to fix completely the time derivatives of the coordinates h^k for each 3-surface at light cone boundary and therefore in principle the whole minimizing four-surface provided the initial value problem associated with the Kähler action possesses a unique solution. What is nice that the requirement of isometry invariance in principle would provides solution to the problem of finding preferred extremals of the Kähler action.

These conditions, when written explicitly give infinite number of conditions for the time derivative of the generator J^C (we assume for a moment that C is held fixed and let A and B run) at the boundary of the light cone. Time derivatives are in principle determined also by the requirement that deformed surface corresponds to an absolute minimum of the Kähler action. The basis of δH scalar functions respecting color and rotational symmetries is the most promising one.

3.16.4 Direct consistency checks

If duality holds true, the most general form of the configuration space metric is defined by the fluxes $Q_m^{\alpha,\beta}$, where α and β are the coefficients of signed and unsigned magnetic fluxes. Present is also a conformal factor depending on those zero modes, which do not appear in the symplectic form and which characterize the size and shape of the 3-surface. $[t,t] \subset h$ property implying Ricci flatness and isometry property of symplectic transformations, requires the vanishing of the fluxes $Q_m^{\alpha,\beta}(\{H_{A,m\neq 0}, \{H_{B,n\neq 0}, H_{C,p\neq 0}\}\})$ associated with double commutators and poses strong consistency conditions on the metric. If n labelling symplectic generators has half integer values then the conditions simply state conformal invariance: generators labelled by integers have vanishing norm whereas half-odd integers correspond to non-vanishing norm. Isometry invariance gives additional conditions on fluxes $Q_m^{\alpha,\beta}$. Lorentz invariance strengthens these conditions further. It could be that these conditions fix the initial values of the imbedding space coordinates completely.

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Books about TGD

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Chapter 4

Configuration Space Spinor Structure

4.1 Introduction

Quantum TGD should be reducible to the classical spinor geometry of the configuration space. In particular, physical states should correspond to the modes of the configuration space spinor fields. The immediate consequence is that configuration space spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the configuration space spinor structure there are some important clues.

4.1.1 Geometrization of fermionic statistics in terms of configuration space spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the configuration space spinor structure in the sense that the anti-commutation relations for configuration space gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the configuration space spinor structure. [63] has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the configuration space and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their naturally derives from the anti-commutativity of the fermionic oscillator operators.

   As a consequence, configuration space spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of configuration space spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the 'orbital' degrees of freedom of the ordinary spinor field.

2. The classical theory for the bosonic fields is an essential part of the configuration space geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the configuration space spinor structure somehow. The properties of the associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. Contrary to the long held belief it seems that covariantly constant right handed neutrino does not generate . The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the configuration space geometry. This is indeed true if the complexified configuration space gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and its boundaries. There is actually no deep reason forbidding the gamma matrices of the configuration space to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with

$$\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB} \ ,$$

where J_{AB} denotes the matrix elements of the Kähler form of the configuration space. The presence of the Hermitian conjugation is necessary because configuration space gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the configuration space Dirac operator comes out correctly.

5. TGD as a generalized number theory vision leads to the understanding of how the second quantization of the induced spinor fields should be carried out and space-time conformal symmetries allow to explicitly solve the Dirac equation associated with the modified Dirac action in the interior and at the 3-D light like causal determinants. An essentially new element is the notion of number theoretic braid forced by the fact that the modified Dirac operator allows only finite number of generalized eigen modes so that the number of fermionic oscillator operators is finite. As a consequence, anticommutation relations can be satisfied only for a finite set of points defined by the number theoretic braid, which is uniquely identifiable. The interpretation is in terms of finite measurement resolution. The finite Clifford algebra spanned by the fermionic oscillator operators is interpreted as the factor space $\mathcal{M}/\mathcal{N}$ of infinite hyper-finite factors of type II_1 defined by configuration space Clifford algebra $\mathcal{N}$ and included Clifford algebra $\mathcal{M} \subset \mathcal{N}$ interpreted as the characterizer of the finite measurement resolution. Note that the finite number of eigenvalues guarantees that Dirac determinant identified as the exponent of Kähler function is finite. Finite number of eigenvalues is also essential for number theoretic universality.

4.1.2 Modified Dirac equation for induced classical spinor fields

The earlier approach to the definition of the configuration space spinor structure relied on the second quantized ordinary massless Dirac action for the induced spinors. This action had some anomalous looking features. The first anomaly was the appearance of the effective tachyonic mass term proportional to the trace of the second fundamental form vanishing only for minimal surfaces. The breaking of $N = 2$ super symmetry generated by right-handed neutrinos for other than minimal surfaces was the second anomalous feature. It became also clear that the divergences of the fermionic isometry currents can have a non-vanishing c-number anomaly unless one varies Dirac action also with respect to the configuration space coordinates. This anomaly obviously might destroy the definition of the configuration space spinor structure.

The vision about quantum TGD as a generalized number theory [21, 20, 19] comes in rescue here. One of its outcomes was the realization that, in order to achieve exact super-symmetry, one must modify Dirac action so that its variation with respect to the imbedding space coordinates gives the field equations derivable from the action principle in question. By taking the modified Dirac action as the fundamental action, one can identify vacuum functional as the Dirac determinant. If this determinant equals to exponent of Kähler action for the preferred extremal containing partonic 3-surfaces, one can predict even the value of the Kähler coupling constant.

Chern-Simons - or Kähler Dirac action?

Two alternative choices represented themselves as candidates for the modified Dirac action: either the 3-D Chern-Simons Dirac action or 4-D Kähler action. Eventually came the realization that the addition of a measurement interaction term to either Chern-Simons action or Kähler action is needed to resolve a bundle of conceptual problems. It took still some time to conclude that Kähler action with instanton term is the correct choice since the measurement interaction term assigned to Chern-Simons-Dirac action creates more problems than it solves.

1. *Basic implications*

1. A correlation between 4-D geometry of space-time sheet and quantum numbers is achieved by the identification of exponent of Kähler function as Dirac determinant making possible the entanglement of classical degrees of freedom in the interior of space-time sheet with quantum numbers.

2. Cartan algebra plays a key role not only at quantum level but also at the level of space-time geometry since quantum critical conserved currents vanish for Cartan algebra of isometries and the measurement interaction terms giving rise to conserved currents are possible only for Cartan algebras. Furthermore, modified Dirac equation makes sense only for eigen states of Cartan algebra generators. The hierarchy of Planck constants realized in terms of the book like structure of the generalized imbedding space assigns to each CD (causal diamond) preferred Cartan algebra: in case of Poincare algebra there are two of them corresponding to linear and cylindrical M^4 coordinates.

3. Quantum holography and dimensional reduction hierarchy in which partonic 2-surface defined fermionic sources for 3-D fermionic fields at light-like 3-surfaces Y_l^3 in turn defining fermionic sources for 4-D spinors find an elegant realization. Effective 2-dimensionality is achieved if the replacement of light-like wormhole throat X_l^3 with light-like 3-surface Y_l^3 "parallel" with it in the definition of Dirac determinant corresponds to the $U(1)$ gauge transformation $K \rightarrow K + f + \overline{f}$ for Kähler function of WCW so that WCW Kähler metric is not affected. Here f is holomorphic function of WCW ("world of classical worlds") complex coordinates and arbitrary function of zero mode coordinates.

4. An elegant description of the interaction between super-conformal representations realized at partonic 2-surfaces and dynamics of space-time surfaces is achieved since the values of Cartan charges are feeded to the 3-D Dirac equation which also receives mass term at the same time. Almost topological QFT at wormhole throats results at the limit when four-momenta vanish: this is in accordance with the original vision about TGD as almost topological QFT.

5. A detailed view about the physical role of quantum criticality results. Quantum criticality fixes the values of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current except in the case of Cartan algebra of isometries. Quantum criticality allows to fix the values of couplings appearing in the measurement interaction by using the condition $K \rightarrow K + f + \overline{f}$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs).

6. The inclusion of imaginary instanton term to the definition of the modified gamma matrices is not consistent with the conjugation of the induced spinor fields. Measurement interaction can be however assigned to both Kähler action and its instanton term. CP breaking, irreversibility and the space-time description of dissipation are closely related and the CP and T oddness of the instanton part of the measurement interaction term could provide first level description for dissipative effects. It must be however emphasized that the mere addition of instanton term to Kähler function could be enough.

7. A radically new view about matter antimatter asymmetry based on zero energy ontology emerges and one could understand the experimental absence of antimatter as being due to the fact antimatter corresponds to negative energy states. The identification of bosons as wormhole contacts is the only possible option in this framework.

8. Almost stringy propagators and a consistency with the identification of wormhole throats as lines of generalized Feynman diagrams is achieved. The notion of bosonic emergence leads to a long sought general master formula for the M-matrix elements. The counterpart for fermionic loop defining bosonic inverse propagator at QFT limit is wormhole contact with fermion and cutoffs in mass squared and hyperbolic angle for loop momenta of fermion and antifermion in the rest system of emitting boson have precise geometric counterpart.

2. Hyper-quaternionicity and quantum criticality

The conjecture that quantum critical space-time surfaces are hyper-quaternionic in the sense that the modified gamma matrices span a quaternionic subspace of complexified octonions at each point of the space-time surface is consistent with what is known about preferred extremals. The condition that both the modified gamma matrices and spinors are quaternionic at each point of the space-time surface leads to a precise ansatz for the general solution of the modified Dirac equation making sense also in the real context. The octonionic version of the modified Dirac equation is very simple since $SO(7,1)$ as vielbein group is replaced with G_2 acting as automorphisms of octonions so that only the neutral Abelian part of the classical electro-weak gauge fields survives the map.

Octonionic gamma matrices provide also a non-associative representation for the 8-D version of Pauli sigma matrices and encourage the identification of 8-D twistors as pairs of octonionic spinors conjectured to be highly relevant also for quantum TGD. Quaternionicity condition implies that octo-twistors reduce to something closely related to ordinary twistors.

Super-conformal symmetries of modified Dirac action

The modified Dirac equation allows large number of super-conformal gauge symmetries as zero modes of $D_K(Y_l^3)$ and are interpreted as generators of exact $N=4$ super-conformal gauge symmetries in both quark and lepton sectors. These super-symmetries correspond to pure super gauge transformations and state the the effective 3-dimensionality of space-time dynamics.

Super-symplectic and super Kac-Moody transformations respecting the light-likeness of light-like 3-surfaces define dynamical super conformal symmetries with covariantly constant right handed neutrino spinor serving as the generator of super symmetries. These are crucial for p-adic thermodynamics. No spartners of ordinary particles are predictedin particular $N=2$ space-time super-symmetry is generated by the righthanded neutrino is absent contrary to the earlier beliefs. There is no need to emphasize the experimental implications of this finding.

An essential difference with respect to the standard super-conformal symmetries is that Majorana condition is not satisfied and the usual super-space formalism does not apply. The notion of super-space is un-necessary since fermionic super-generators do not anticommute to vector fields of symmetries but to their Hamiltonians.

Identification of configuration space gamma matrices

Configuration space gamma matrices identified as super generators of super-symplectic or super Kac-Moody algebras (depending on CH coordinates used) are expressible in terms of the oscillator operators associated with the eigen modes of the modified Dirac operator. Super-symplectic and super Kac-Moody charges are expressible as integrals over 2-dimensional partonic surfaces X^2 and interior degrees of freedom of X^4 can be regarded as zero modes representing classical variables in one-one correspondence with quantal degrees of freedom at X_l^3 as indeed required by quantum measurement theory. The resulting situation is highly reminiscent of WZW model and the results imply that at technical level the methods of 2-D conformal field theories should allow to construct quantum TGD.

4.1.3 The exponent of Kähler function as Dirac determinant for the modified Dirac action?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography.

1. The Dirac determinant defined by the product of Dirac determinants associated with the light-like partonic 3-surfaces X_l^3 associated with a given space-time sheet X^4 is the simplest candidate for vacuum functional identifiable as the exponent of the Kähler function. One can of course worry about the finiteness of the Dirac determinant. p-Adicization requires that the eigenvalues belong to a given algebraic extension of rationals. This restriction would imply a hierarchy of physics corresponding to different extensions and could automatically imply the finiteness and algebraic number property of the Dirac determinants if only finite number of eigenvalues would contribute. The regularization would be performed by physics itself if this were the case.

2. The basic problem has been how to feed in the information about the preferred extremal of Kähler action to the eigenvalue spectrum of the Dirac operator in question. The identification of the preferred extremal associated with X_l^3 became possible via the boundary conditions at X_l^3 dictated by number theoretical compactification, which also predicted the dual slicings of the M^4 projection of space-time surface by string world sheets and partonic 2-surfaces. The basic observation is that the Dirac equation associated with the 4-D Dirac operator D_K associated with by Kähler action can be seen as a conservation law for a super current. The slicing of $X^4(X_l^3)$ by the parallel light-like 3-surfaces Y_l^3 allows solutions for which the super current flows along Y_l^3 and has no component in normal direction. The zero modes of D_K reducing to effectively 3-D solutions of D_K at each Y_l^3 give a family of holographic copies of X_l^3. The effective 3-dimensionality is due to the super-conformal gauge invariance in the direction of light-like coordinate u labeling the 3-surfaces Y_l^3.

3. The spectrum of eigenvalues corresponds to the "energy" spectrum of D_K and the product of the eigenvalues defines the Dirac determinant in standard manner. If the eigenmodes are restricted to those localized to regions of strong induced electro-weak magnetic field, the number of eigen modes is finite and therefore also Dirac determinant is finite.

4. The requirement that the Noether currents associated with Dirac Kähler action are conserved is that preferred extremals of Kähler action correspond to extremals for which the second variation of Kähler action vanishes at least for the deformations associated with the conserved currents. Obviously this is nothing but the formulation of quantum criticality at space-time level!

5. The physical analog is energy spectrum for Dirac operator in external magnetic field. The effective metric appearing in the modified Dirac operator corresponds to $\hat{g}^{\alpha\beta} = \partial L_K/\partial h^k_\alpha \partial L_K/\partial h^l_\beta h_{kl}$, and vanishes at the boundaries of regions carrying non-vanishing Kähler magnetic field. Hence the modes must be localized to regions $X^3_{l,i}$ containing a non-vanishing Kähler magnetic field. Cyclotron states in constant magnetic field serve as a good analog for the situation and only a finite number of cyclotron states are possible since for higher cyclotron states the wave function -essentially harmonic oscillator wave function- would concentrate outside $X^3_{l,i}$.

6. A more precise argument goes as follows. Assume that it is induced Kähler magnetic field B_K that matters. The vanishing of the effective contravariant metric near the boundary of $X^3_{l,i}$ corresponds to an infinite effective mass for massive particle in constant magnetic field so that the counterpart for the cyclotron frequency scale eB/m reduces to zero. The radius of the cyclotron orbit is proportional to $1/\sqrt{eB}$ and approaches to infinity. Hence the required localization is not possible only for cyclotron states for which the cyclotron radius is below that the transversal size scale of $X^3_{l,i}$.

7. It remains to be proven that the product of eigenvalues gives rise to the exponent of Kähler action for the preferred extremal of Kähler action. At this moment the only justification for the conjecture is that this the only thing that one can imagine.

4.1.4 Super-conformal symmetries

The almost topological QFT property of partonic formulation based on modified Dirac Kähler action allows a rich structure of $N = 4$ super-conformal symmetries. In particular, the generalized Kac-Moody symmetries leave corresponding X^3-local isometries respecting the light-likeness condition. A rather detailed view about various aspects of super-conformal symmetries emerge leading to identification of fermionic anti-commutation relations and explicit expressions for configuration space gamma matrices and Kähler metric. This picture is consistent with the conditions posed by p-adic mass calculations.

The relationship between super-symplectic (SC) and Super Kac-Moody (SKM) symmetries has been one of the central themes in the development of TGD. The progress in the understanding of the number theoretical aspects of TGD gives good hopes of lifting $SKMV$ (V denotes Virasoro) to a subalgebra of SCV so that coset construction works meaning that the differences of SCV and $SKMV$ generators annihilate physical states. This condition has interpretation in terms of Equivalence Principle with coset Super Virasoro conditions defining a generalization of Einstein's equations in TGD framework. Also p-adic thermodynamics finds a justification since the expectation values of SKM conformal weights can be non-vanishing in physical states.

Number theoretical considerations play a key role and lead to the picture in which effective discretization occurs so that partonic two-surface is effectively replaced by a discrete set of algebraic points belonging to the intersection of the real partonic 2-surface and its p-adic counterpart obeying the same algebraic equations. This implies effective discretization of super-conformal field theory giving N-point functions defining vertices via discrete versions of stringy formulas.

Before continuing I must represent apologies for the reader. This chapter is just now under updating due to the dramatic simplifications related to identification of the eigenvalue spectrum of the modified Dirac operator and the definition of the Dirac determinant. The new vision is briefly discussed but a lot of mammoth bones remains to be eliminated.

4.2 Configuration space spinor structure: general definition

The basic problem in constructing configuration space spinor structure is clearly the construction of the explicit representation for the gamma matrices of the configuration space. One should be able to identify the space, where these gamma matrices act as well as the counterparts of the "free" gamma matrices, in terms of which the gamma matrices would be representable using generalized vielbein coefficients.

4.2.1 Defining relations for gamma matrices

The ordinary definition of the gamma matrix algebra is in terms of the anti-commutators

$$\{\gamma_A, \gamma_B\} = 2g_{AB} \quad .$$

This definition served implicitly also as a basic definition of the gamma matrix algebra in TGD context until the difficulties related to the understanding of the configuration space d'Alembertian defined in terms of the square of the Dirac operator forced to reconsider the definition. If configuration space allows Kähler structure, the most general definition allows to replace the metric any covariantly constant Hermitian form. In particular, g_{AB} can be replaced with

$$\{\Gamma_A^\dagger, \Gamma_B\} = iJ_{AB} \quad , \tag{4.2.1}$$

where J_{AB} denotes the matrix element of the Kähler form of the configuration space. The reason is that gamma matrices carry fermion number and are non-hermitian in all coordinate systems. This definition is numerically equivalent with the standard one in the complex coordinates but in arbitrary coordinates situation is different since in general coordinates iJ_{kl} is a nontrivial positive square root of g_{kl}. The realization of this delicacy is necessary in order to understand how the square of the configuration space Dirac operator comes out correctly. Obviously, what one must do is the equivalent of replacing $D^2 = (\Gamma^k D_k)^2$ with $D\hat{D}$ with $\hat{D}$ defined as

$$\hat{D} = iJ^{kl}\Gamma_l^\dagger D_k \quad .$$

4.2.2 General vielbein representations

There are two ideas, which make the solution of the problem obvious.

1. Since the classical time development in bosonic degrees of freedom (induced gauge fields) is coded into the geometry of the configuration space it seems natural to expect that same applies in the case of the spinor structure. The time development of the induced spinor fields dictated by TGD counterpart of the massless Dirac action should be coded into the definition of the configuration space spinor structure. This leads to the challenge of defining what classical spinor field means.

2. Since classical scalar field in the configuration space corresponds to second quantized boson fields of the imbedding space same correspondence should apply in the case of the fermions, too. The spinor fields of configuration space should correspond to second quantized fermion field of the imbedding space and the space of the configuration space spinors should be more or less identical with the Fock space of the second quantized fermion field of imbedding space or $X^4(X^3)$. Since classical spinor fields at space-time surface are obtained by restricting the spinor structure to the space-time surface, one might consider the possibility that life is really simple: the second quantized spinor field corresponds to the free spinor field of the imbedding space satisfying the counterpart of the massless Dirac equation and more or less standard anti-commutation relations. Unfortunately life is not *so* simple as the construction of configuration space spinor structure demonstrates: second quantization must be performed for induced spinor fields.

It is relatively simple to fill in the details once these basic ideas are accepted.

1. The only natural candidate for the second quantized spinor field is just the on X^4. Since this field is free field, one can indeed perform second quantization and construct fermionic oscillator operator algebra with unique anti-commutation relations. The space of the configuration space spinors can be identified as the associated with these oscillator operators. This space depends on 3-surface and strictly speaking one should speak of the Fock bundle having configuration space as its base space.

2. The gamma matrices of the configuration space (or rather fermionic Kac Moody generators) are representable as super positions of the fermionic oscillator algebra generators:

$$\begin{aligned} \Gamma_A^+ &= E_A^n a_n^\dagger \\ \Gamma_A^- &= \bar{E}_A^n a_n \\ iJ_{A\bar{B}} &= \sum_n E_A^n \bar{E}_B^n \end{aligned} \tag{4.2.2}$$

   where E_A^n are the vielbein coefficients. Induced spinor fields can possess zero modes and there is no oscillator operators associated with these modes. Since oscillator operators are spin 1/2 objects, configuration space gamma matrices are analogous to spin 3/2 spinor fields (in a very general sense). Therefore the generalized vielbein and configuration space metric is analogous to the pair of spin 3/2 and spin 2 fields encountered in super gravitation! Notice that the contractions $j^{Ak}\Gamma_k$ of the complexified gamma matrices with the isometry generators are genuine spin 1/2 objects labeled by the quantum numbers labeling isometry generators. In particular, in CP_2 degrees of freedom these fermions are color octets.

3. A further great idea inspired by the symplectic and Kähler structures of the configuration space is that configuration gamma matrices are actually generators of super-symplectic symmetries. This simplifies enormously the construction allows to deduce explicit formulas for the gamma matrices.

4.2.3 Inner product for configuration space spinor fields

The conjugation operation for configuration space spinors corresponds to the standard *ket* $\rightarrow$ *bra* operation for the states of the Fock space:

$$\begin{aligned}\Psi &\leftrightarrow |\Psi\rangle \\ \bar{\Psi} &\leftrightarrow \langle\Psi|\end{aligned} \tag{4.2.3}$$

The inner product for configuration space spinors at a given point of the configuration space is just the standard Fock space inner product, which is unitary.

$$\bar{\Psi}_1(X^3)\Psi_2(X^3) = \langle\Psi_1|\Psi_2\rangle_{|X^3} \tag{4.2.4}$$

Configuration space inner product for two configuration space spinor fields is obtained as the integral of the Fock space inner product over the whole configuration space using the vacuum functional $exp(K)$ as a weight factor

$$\langle\Psi_1|\Psi_2\rangle = \int \langle\Psi_1|\Psi_2\rangle_{|X^3} exp(K)\sqrt{G}dX^3 \tag{4.2.5}$$

This inner product is obviously unitary. A modified form of the inner product is obtained by including the factor $exp(K/2)$ in the definition of the spinor field. In fact, the construction of the central extension for the isometry algebra leads automatically to the appearance of this factor in vacuum spinor field.

The inner product differs from the standard inner product for, say, Minkowski space spinors in that integration is over the entire configuration space rather than over a time= constant slice of the configuration space. Also the presence of the vacuum functional makes it different from the finite dimensional inner product. These are not un-physical features. The point is that (apart from classical non-determinism forcing to generalized the concept of 3-surface) $Diff^4$ invariance dictates the behavior of the configuration space spinor field completely: it is determined form its values at the moment of the big bang. Therefore there is no need to postulate any Dirac equation to determine the behavior and therefore no need to use the inner product derived from dynamics.

4.2.4 Holonomy group of the vielbein connection

Generalized vielbein allows huge gauge symmetry. An important constraint on physical observables is that they do not depend at all on the gauge chosen to represent the gamma matrices. This is indeed achieved using vielbein connection, which is now quadratic in fermionic oscillator operators. The holonomy group of the vielbein connection is the configuration space counterpart of the electro-weak gauge group and its algebra is expected to have same general structure as the algebra of the configuration space isometries. In particular, the generators of this algebra should be labeled by conformal weights like the elements of Kac Moody algebras. In present case however conformal weights are complex as the construction of the configuration space geometry demonstrates.

4.2.5 Realization of configuration space gamma matrices in terms of super symmetry generators

In string models super symmetry generators behave effectively as gamma matrices and it is very tempting to assume that configuration space gamma matrices can be regarded as generators of the symplectic algebra extended to super-symplectic Kac Moody type algebra. The experience with string models suggests also that radial Virasoro algebra extends to Super Virasoro algebra. There are good reasons to expect that configuration space Dirac operator and its square give automatically a realization of this algebra. It this is indeed the case, then configuration space spinor structure as well as Dirac equation reduces to mere group theory.

One can actually guess the general form of the super-symplectic algebra. The form is a direct generalization of the ordinary super Kac Moody algebra. The complexified super generators S_A are identifiable as configuration space gamma matrices:

$$\Gamma_A = S_A \ . \tag{4.2.6}$$

The anti-commutators $\{\Gamma_A^\dagger, \Gamma_B\}_+ = i2J_{A,B}$ define a Hermitian matrix, which is proportional to the Kähler form of the configuration space rather than metric as usually. Only in complex coordinates the anti-commutators equal to the metric numerically. This is, apart from the multiplicative constant n, is expressible as the Poisson bracket of the configuration space Hamiltonians H_A and H_B. Therefore one should be able to identify super generators $S_A(r_M)$ for each values of r_M as the counterparts of fluxes. The anti-commutators between the super generators S_A and their Hermitian conjugates should read as

$$\{S_A, S_B^\dagger\}_+ \quad = \quad iQ_m(H_{[A,B]}) \ . \tag{4.2.7}$$

and should be induced directly from the anti-commutation relations of free second quantized spinor fields of the imbedding space restricted to the light cone boundary.

The commutation relations between s and super generators follow solely from the transformation properties of the super generators under symplectic transformations, which are same as for the Hamiltonians themselves

$$\{H_{Am}, S_{Bn}\}_- \quad = \quad S_{[Am,Bn]} \ , \tag{4.2.8}$$

and are of the same form as in the case of Super-Kac-Moody algebra.

The task is to derive an explicit representation for the super generators S_A in both cases. For obvious reason the spinor fields restricted to the 3-surfaces on the light cone boundary $\delta M^4_+ \times CP_2$ can be used. Leptonic/quark like oscillator operators are used to construct Ramond/NS type algebra.

What is then the strategy that one should follow?

1. Configuration space Hamiltonians correspond to either magnetic or electric flux Hamiltonians and the conjecture is that these representations are equivalent. It turns out that this electric-magnetic duality generalizes to the level of super charges. It also turns out that quark representation is the only possible option whereas leptonic super charges super-symmetrize the ordinary function algebra of the light cone boundary.

2. The simplest option would be that second quantized imbedding space spinors could be used in the definition of super charges. This turns out to not work and one must second quantize the induced spinor fields.

3. The task is to identify a super-symmetric variational principle for the induced spinors: ordinary Dirac action does not work. It turns out that in the most plausible scenario the modified Dirac action varied with respect to *both* imbedding space coordinates and spinor fields is the fundamental action principle. The c-number parts of the conserved symplectic charges associated with this action give rise to bosonic conserved charges defining configuration space Hamiltonians. The second quantization of the spinor fields reduces to the requirement that super charges and Hamiltonians generate super-symplectic algebra determining the anti-commutation relations for the induced spinor fields.

4.2.6 Central extension as symplectic extension at configuration space level

The earlier attempts to understand the emergence of central extension of super-symplectic algebra were based on the notion of symplectic extension. This general view is not given up although it seems that this abstract approach is not very practical. Symplectic extension emerged originally in the attempts to construct formal expression for the configuration space Dirac equation. The rather obvious idea was that the Dirac equation reduces to super Virasoro conditions with Super Virasoro generators involving the Dirac operator of the imbedding space. The basic difficulty was the necessity to assign to the gamma matrices of the imbedding space fermion number. In the recent formulation the Dirac operator of H does not appear in in the Super Virasoro conditions so that this problem disappears.

The proposal that Super Virasoro conditions should replaced with conditions stating that the commutator of super-symplectic and super Kac-Moody algebras annihilates physical states, looks rather feasible. One could call these conditions as configuration space Dirac equation but at this moment I feel that this would be just play with words and mask the group theoretical content of these conditions. In any case, the formulas for the symplectic extension and action of isometry generators on configuration space spinor deserve to be summarized.

Symplectic extension

The Abelian extension of the super-symplectic algebra is obtained by an extremely simple trick. Replace the ordinary derivatives appearing in the definition of, say spinorial isometry generator, by the covariant derivatives defined by a coupling to a multiple of the Kähler potential.

$$\begin{aligned} j^{Ak}\partial_k &\rightarrow j^{Ak}D_k \ , \\ D_k &= \partial_k + ikA_k/2 \ . \end{aligned} \tag{4.2.9}$$

where A_k denotes Kähler potential. The reality of the parameter k is dictated by the Hermiticity requirement and also by the requirement that Abelian extension reduces to the standard form in Cartan algebra. k is expected to be integer also by the requirement that covariant derivative corresponds to connection (quantization of magnetic charge).

The commutation relations for the centrally extended generators J^A read:

$$[J^A, J^B] = J^{[A,B]} + ikj^{Ak}J_{kl}j^{Bl} \equiv J^{[A,B]} + ikJ_{AB} \ . \tag{4.2.10}$$

Since Kähler form defines symplectic structure in configuration space one can express Abelian extension term as a Poisson bracket of two Hamiltonians

$$J_{AB} \equiv j^{Ak}J_{kl}j^{Bl} = \{H^A, H^B\} \ . \tag{4.2.11}$$

Notice that Poisson bracket is well defined also when Kähler form is degenerate.

The extension indeed has acceptable properties:

1. Jacobi-identities reduce to the form

$$\sum_{cyclic} H^{[A,[B,C]]} = 0 \ , \tag{4.2.12}$$

and therefore to the Jacobi identities of the original Lie- algebra in Hamiltonian representation.

2. In the Cartan algebra Abelian extension reduces to a constant term since the Poisson bracket for two commuting generators must be a multiple of a unit matrix. This feature is clearly crucial for the non-triviality of the Abelian extension and is encountered already at the level of ordinary (q,p) Poisson algebra: although the differential operators ∂_p and ∂_q commute the Poisson bracket of the corresponding Hamiltonians p and q is nontrivial: $\{p,q\} = 1$. Therefore the extension term commutes with the generators of the Cartan subalgebra. Extension is also local $U(1)$ extension since Poisson algebra differs from the Lie-algebra of the vector fields in that it contains constant Hamiltonian ("1" in the commutator), which commutes with all other Hamiltonians and corresponds to a vanishing vector field.

3. For the generators not belonging to Cartan sub-algebra of CH isometries Abelian extension term is not annihilated by the generators of the original algebra and in this respect the extension differs from the standard central extension for the loop algebras. It must be however emphasized that for the super-symplectic algebra generators correspond to products of δM^4_+ and CP_2 Hamiltonians and this means that generators of say δM^4_+-local $SU(3)$ Cartan algebra are non-commuting and the commutator is completely analogous to central extension term since it is symmetric with respect to $SU(3)$ generators.

4. The proposed method yields a trivial extension in the case of $Diff^4$. The reason is the (four-dimensional!) Diff degeneracy of the Kähler form. Abelian extension term is given by the contraction of the $Diff^4$ generators with the Kähler potential

$$j^{Ak}J_{kl}j^{Bl} = 0 \ , \tag{4.2.13}$$

   which vanishes identically by the Diff degeneracy of the Kähler form. Therefore neither 3- or 4-dimensional Diff invariance is not expected to cause any difficulties. Recall that 4-dimensional Diff degeneracy is what is needed to eliminate time like vibrational excitations from the spectrum of the theory. By the way, the fact that the loop space metric is not Diff degenerate makes understandable the emergence of Diff anomalies in string models [63, 61].

5. The extension is trivial also for the other zero norm generators of the tangent space algebra, in particular for the $k_2 = Im(k) = 0$ symplectic generators possible present so that these generators indeed act as genuine $U(1)$ transformations.

6. Concerning the solution of configuration space Dirac equation the maximum of Kähler function is expected to be special, much like origin of Minkowski space and symmetric space property suggests that the construction of solutions reduces to this point. At this point the generators and Hamiltonians of the algebra h in the defining Cartan decomposition $g = h+t$ should vanish. h corresponds to integer values of $k_1 = Re(k)$ for Cartan algebra of super-symplectic algebra and integer valued conformal weights n for Super Kac-Moody algebra. The algebra reduces at the maximum to an exceptionally simple form since only central extension contributes to the metric and Kähler form. In the ideal case the elements of the metric and Kähler form could be even diagonal. The degeneracy of the metric might of course pose additional complications.

Super symplectic action on configuration space spinors

The generators of symplectic transformations are obtained in the spinor representation of the isometry group of the configuration space by the following formal construction. Take isometry generator in the spinor representation and add to the covariant derivative D_k defined by vielbein connection the coupling to the multiple of the Kähler potential: $D_k \rightarrow D_k + ikAk/2$.

$$\begin{aligned} J^A &= j^{Ak}D_k + D_l j_k \Sigma^{kl}/2 \ , \\ &\rightarrow \ \hat{J}^A = j^{Ak}(D_k + ikA_k/2) + D_l j_k^A \Sigma^{kl}/2 \ , \end{aligned} \tag{4.2.14}$$

This induces the required central term to the commutation relations. Introduce complex coordinates and define bosonic creation and annihilation operators as $(1,0)$ and $(0,1)$ parts of the modified isometry generators

$$\begin{aligned} B_A^\dagger &= J_+^A = j^{Ak}(D_k + ... \ , \\ B_A &= J_-^A = j^{A\bar{k}}(D_{\bar{k}} + ... \ . \end{aligned} \tag{4.2.15}$$

where "k" refers now to complex coordinates and "$\bar{k}$" to their conjugates.

Fermionic generators are obtained as the contractions of the complexified gamma matrices with the isometry generators

$$\begin{aligned} \Gamma_A^\dagger &= j^{Ak}\Gamma_k \ , \\ \Gamma_A &= j^{A\bar{k}}\Gamma_{\bar{k}} \ . \end{aligned} \tag{4.2.16}$$

Notice that the bosonic Cartan algebra generators obey standard oscillator algebra commutation relations and annihilate fermionic Cartan algebra generators. Hermiticity condition holds in the sense that creation type generators are hermitian conjugates of the annihilation operator type generators. There are two kinds of representations depending on whether one uses leptonic or quark like oscillator operators to construct the gammas. These will be assumed to correspond to Ramond and NS type generators with the radial plane waves being labeled by integer and half odd integer indices respectively.

The non-vanishing commutators between the Cartan algebra bosonic generators are given by the matrix elements of the Kähler form in the basis of formed by the isometry generators

$$[B_A^\dagger, B_B] = J(j^{A\dagger}, j^B) \equiv J_{\bar{A}B} \ . \tag{4.2.17}$$

and are isometry invariant quantities. The commutators between local $SU(3)$ generators not belonging to Cartan algebra are just those of the local gauge algebra with Abelian extension term added.

The anti-commutators between the fermionic generators are given by the elements of the metric (as opposed to Kähler form in the case of bosonic generators) in the basis formed by the isometry generators

$$\{\Gamma_A\dagger, \Gamma_B\} = 2g(j^{A\dagger}, j^B) \equiv 2g_{\bar{A}B} \ . \tag{4.2.18}$$

and are invariant under isometries. Numerically the commutators and anti-commutators differ only the presence of the imaginary unit and the scale factor R relating the metric and Kähler form to each other (the factor R is same for CP_2 metric and Kähler form).

The commutators between bosonic and fermionic generators are given by

$$[B_A, \Gamma_B] = \Gamma_{[A,B]} \ . \tag{4.2.19}$$

The presence of vielbein and rotation terms in the representation of the isometry generators is essential for obtaining these nice commutations relations. The commutators vanish identically for Cartan algebra generators. From the commutation relations it is clear that Super Kac Moody algebra structure is directly related to the Kähler structure of the configuration space: the anti-commutator of fermionic generators is proportional to the metric and the commutator of the bosonic generators is proportional to the Kähler form. It is this algebra, which should generate the solutions of the field equations of the theory.

The vielbein and rotational parts of the bosonic isometry generators are quadratic in the fermionic oscillator operators and this suggests the interpretation as the fermionic contribution to the isometry currents. This means that the action of the bosonic generators is essentially non-perturbative since it creates fermion antifermion pairs besides exciting bosonic degrees of freedom.

4.2.7 Configuration space Clifford algebra as a hyper-finite factor of type II_1

The naive expectation is that the trace of the unit matrix associated with the Clifford algebra spanned by configuration space sigma matrices is infinite and thus defines an excellent candidate for a source of divergences in perturbation theory. This potential source of infinities remained un-noticed until it became clear that there is a connection with von Neumann algebras [52]. In fact, for a separable Hilbert space defines a standard representation for so called [54]. This guarantees that the trace of the unit matrix equals to unity and there is no danger about divergences.

Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [54].

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless.

von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac based on the notion of delta function, plus the emergence of Feynman graphs, the possibility to formulate the notion of delta function rigorously in terms of distributions, and the emergence of path integral approach meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [51, 43] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [55, 46] relate closely to type II_1 factors. In topological quantum computation [60] based on braid groups [49] modular S-matrices they play an especially important role.

Clifford algebra of configuration space as von Neumann algebra

The Clifford algebra of the configuration space provides a school example of a hyper-finite factor of type II_1, which means that fermionic sector does not produce divergence problems. Super-symmetry means that also "orbital" degrees of freedom corresponding to the deformations of 3-surface define similar factor. The general theory of hyper-finite factors of type II_1 is very rich and leads to rather detailed understanding of the general structure of S-matrix in TGD framework. For instance, there is a unitary evolution operator intrinsic to the von Neumann algebra defining in a natural manner single particle time evolution. Also a connection with 3-dimensional topological quantum field theories and knot theory, conformal field theories, braid groups, quantum groups, and quantum counterparts of quaternionic and octonionic division algebras emerges naturally. These aspects are discussed in detail in [30].

4.3 Hierarchy of Planck constants and the generalization of the notion of imbedding space

In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is summarized. The question is whether it might be possible in some sense to replace

H or its Cartesian factors by their necessarily singular multiple coverings and factor spaces. One can consider two options: either M^4 or the causal diamond CD. The latter one is the more plausible option from the point of view of WCW geometry.

4.3.1 The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The starting point was the proposal of Nottale [64] that the orbits of inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [28, 16] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.

2. Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [24]. TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or over-critical mass density and the "pressure" associated with these cosmologies is negative.

3. The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the imbedding space obtained by gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.

4. Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X^3_l leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [38].

5. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase. If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [64] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr}G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

6. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and aminoacids, and to

the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [44, 38].

4.3.2 The most general option for the generalized imbedding space

Simple physical arguments pose constraints on the choice of the most general form of the imbedding space.

1. The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is un-acceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the singular coverings and factor spaces of CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.

    (a) The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the modified gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.

    (b) The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

3. For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD} \hat{\times} G_a) \times (\hat{CP}_2 \hat{\times} G_b)$, $(\hat{CD} \hat{\times} G_a) \times \hat{CP}_2/G_b$, $\hat{CD}/G_a \times (\hat{CP}_2 \hat{\times} G_b)$, and $\hat{CD}/G_a \times \hat{CP}_2/G_b$.

4. The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tedrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tedrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

4.3.3 About the phase transitions changing Planck constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunneling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

4.3.4 How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

1. One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

2. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.

3. The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to $n_a n_b$ sheets and multiplies YM action action by $n_a n_b$ which is equivalent with the $\hbar = n_a n_b \hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_a n_b \hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.

   This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	$n_a n_b$	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_a n_b}$

4.3.5 Preferred values of Planck constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} seem to be especially favored as values of n_a in living matter [36].

4.3.6 How Planck constants are visible in Kähler action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anticommutation relations of various superconformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck constants. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

4.3.7 Could the dynamics of Kähler action predict the hierarchy of Planck constants?

The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. The formula for the Planck constant involves heuristic guess work and physical plausibility arguments. There are good arguments in favor of the hypothesis that only coverings are possible. Only a finite number of pages of the Big Book correspond to a given value of Planck constant, biological evolution corresponds to a gradual dispersion to the pages of the Big Book with larger Planck constant, and a connection with the hierarchy of infinite primes and p-adicization program based on the mathematical realization of finite measurement resolution emerges.

One can however ask whether this hierarchy could emerge directly from the basic quantum TGD rather than as a separate hypothesis. The following arguments suggest that this might be possible. One finds also a precise geometric interpretation of preferred extremal property interpreted as criticality in zero energy ontology.

1-1 correspondence between canonical momentum densities and time derivatives fails for Kähler action

The basic motivation for the geometrization program was the observation that canonical quantization for TGD fails. To see what is involved let us try to perform a canonical quantization in zero energy ontology at the 3-D surfaces located at the light-like boundaries of $CD \times CP_2$.

1. In canonical quantization canonical momentum densities $\pi^0_k \equiv \pi_k = \partial L_K/\partial(\partial_0 h^k)$, where $\partial_0 h^k$ denotes the time derivative of imbedding space coordinate, are the physically natural quantities in terms of which to fix the initial values: once their value distribution is fixed also conserved charges are fixed. Also the weak form of electric-magnetic duality given by $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and a mild generalization of this condition to be discussed below can be interpreted as a manner to fix the values of conserved gauge charges (not Noether charges) to their quantized values

since Kähler magnetic flux equals to the integer giving the homology class of the (wormhole) throat. This condition alone need not characterize criticality, which requires an infinite number of deformations of X^4 for which the second variation of the Kähler action vanishes and implies infinite number conserved charges. This in fact gives hopes of replacing π_k with these conserved Noether charges.

2. Canonical quantization requires that $\partial_0 h^k$ in the energy is expressed in terms of π_k. The equation defining π_k in terms of $\partial_0 h^k$ is however highly non-linear although algebraic. By taking squares the equations reduces to equations for rational functions of $\partial_0 h^k$. $\partial_0 h^k$ appears in contravariant and covariant metric at most quadratically and in the induced Kähler electric field linearly and by multplying the equations by $det(g_4)^3$ one can transform the equations to a polynomial form so that in principle $\partial_0 h^k$ can obtained as a solution of polynomial equations.

3. One can always eliminate one half of the coordinates by choosing 4 imbedding space coordinates as the coordinates of the spacetime surface so that the initial value conditions reduce to those for the canonical momentum densities associated with the remaining four coordinates. For instance, for space-time surfaces representable as map $M^4 \to CP_2$ M^4 coordinates are natural and the time derivatives $\partial_0 s^k$ of CP_2 coordinates are multivalued. One would obtain four polynomial equations with $\partial_0 s^k$ as unknowns. In regions where CP_2 projection is 4-dimensional -in particular for the deformations of CP_2 vacuum extremals the natural coordinates are CP_2 coordinates and one can regard $\partial_0 m^k$ as unknows. For the deformations of cosmic strings, which are of form $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, one can use coordinates of $M^2 \times S^2$, where S^2 is geodesic sphere as natural coordinates and regard as unknowns E^2 coordinates and remaining CP_2 coordinates.

4. One can imagine solving one of the four polynomials equations for time derivaties in terms of other obtaining N roots. Then one would substitute these roots to the remaining 3 conditions to obtain algebraic equations from which one solves then second variable. Obviously situation is very complex without additional symmetries. The criticality of the preferred extremals might however give additional conditions allowing simplifications. The reasons for giving up the canonical quantization program was following. For the vacuum extremals of Kähler action π_k are however identically vanishing and this means that there is an infinite number of value distributions for $\partial_0 h^k$. For small deformations of vacuum extremals one might however hope a finite number of solutions to the conditions and thus finite number of space-time surfaces carrying same conserved charges.

If one assumes that physics is characterized by the values of the conserved charges one must treat the the many-valuedness of $\partial_0 h^k$. The most obvious guess is that one should replace the space of space-like 4-surfaces corresponding to different roots $\partial_0 h^k = F^k(\pi_l)$ with four-surfaces in the covering space of $CD \times CP_2$ corresponding to different branches of the many-valued function $\partial_0 h^k = F(\pi_l)$ co-inciding at the ends of CD.

Do the coverings forces by the many-valuedness of $\partial_0 h^k$ correspond to the coverings associated with the hierarchy of Planck constants?

The obvious question is whether this covering space actually corresponds to the covering spaces associated with the hierarchy of Planck constants. This would conform with quantum classical correspondence. The hierarchy of Planck constants and hierarchy of covering spaces was introduced to cure the failure of the perturbation theory at quantum level. At classical level the multivaluedness of $\partial_0 h^k$ means a failure of perturbative canonical quantization and forces the introduction of the covering spaces. The interpretation would be that when the density of matter becomes critical the space-time surface splits to several branches so that the density at each branches is sub-critical. It is of course not at all obvious whether the proposed structure of the Big Book is really consistent with this hypothesis and one also consider modifications of this structure if necessary. The manner to proceed is by making questions.

1. The proposed picture would give only single integer characterizing the covering. Two integers assignable to CD and CP_2 degrees of freedom are however needed. How these two coverings could emerge?

(a) One should fix also the values of $\pi_k^n = \partial L_K/\partial h_n^k$, where n refers to space-like normal coordinate at the wormhole throats. If one requires that charges do not flow between regions with different signatures of the metric the natural condition is $\pi_k^n = 0$ and allows also multi-valued solution. Since wormhole throats carry magnetic charge and since weak form of electric-magnetic duality is assumed, one can assume that CP_2 projection is four-dimensional so that one can use CP_2 coordinates and regard $\partial_0 m^k$ as un-knows. The basic idea about topological condensation in turn suggests that M^4 projection can be assumed to be 4-D inside space-like 3-surfaces so that here $\partial_0 s^k$ are the unknowns. At partonic 2-surfaces one would have conditions for both π_k^0 and π_k^n. One might hope that the numbers of solutions are finite for preferred extremals because of their symmetries and given by n_a for $\partial_0 m^k$ and by n_b for $\partial_0 s^k$. The optimistic guess is that n_a and n_b corresponds to the numbers of sheets for singular coverings of CD and CP_2. The covering could be visualized as replacement of space-time surfaces with space-time surfaces which have $n_a n_b$ branches. n_b branches would degenerate to single branch at the ends of diagrams of the generaled Feynman graph and n_a branches would degenerate to single one at wormhole throats.

(b) This picture is not quite correct yet. The fixing of π_k^0 and π_k^n should relate closely to the effective 2-dimensionality as an additional condition perhaps crucial for criticality. One could argue that both π_k^0 and π_k^n must be fixed at X^3 and X_l^3 in order to effectively bring in dynamics in two directions so that X^3 could be interpreted as a an orbit of partonic 2-surface in space-like direction and X_l^3 as its orbit in light-like direction. The additional conditions could be seen as gauge conditions made possible by symplectic and Kac-Moody type conformal symmetries. The conditions for π_0^k would give n_b branches in CP_2 degrees of freedom and the conditions for π_k^n would split each of these branches to n_a branches.

(c) The existence of these two kinds of conserved charges (possibly vanishing for π_k^n) could relate also very closely to the slicing of the space-time sheets by string world sheets and partonic 2-surfaces.

2. Should one then treat these branches as separate space-time surfaces or as a single space-time surface? The treatment as a single surface seems to be the correct thing to do. Classically the conserved changes would be $n_a n_b$ times larger than for single branch. Kähler action need not (but could!) be same for different branches but the total action is $n_a n_b$ times the average action and this effectively corresponds to the replacement of the $\hbar_0/g_K^2$ factor of the action with $\hbar/g_K^2$, $r \equiv \hbar/\hbar_0 = n_a n_b$. Since the conserved quantum charges are proportional to $\hbar$ one could argue that $r = n_a n_b$ tells only that the charge conserved charge is $n_a n_b$ times larger than without multi-valuedness. $\hbar$ would be only effectively $n_a n_b$ fold. This is of course poor man's argument but might catch something essential about the situation.

3. How could one interpret the condition $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and its generalization to be discussed below in this framework? The first observation is that the total Kähler electric charge is by $\alpha_K \propto 1/(n_a n_b)$ same always. The interpretation would be in terms of charge fractionization meaning that each branch would carry Kähler electric charge $Q_K = n g_K/n_a n_b$. I have indeed suggested explanation of charge fractionization and quantum Hall effect based on this picture.

4. The vision about the hierarchy of Planck constants involves also assumptions about imbedding space metric. The assumption that the M^4 covariant metric is proportional to $\hbar^2$ follows from the physical idea about $\hbar$ scaling of quantum lengths as what Compton length is. One can always introduce scaled M^4 coordinates bringing M^4 metric into the standard form by scaling up the M^4 size of CD. It is not clear whether the scaling up of CD size follows automatically from the proposed scenario. The basic question is why the M^4 size scale of the critical extremals must scale like $n_a n_b$? This should somehow relate to the weak self-duality conditions implying that Kähler field at each branch is reduced by a factor $1/r$ at each branch. Field equations should posses a dynamical symmetry involving the scaling of CD by integer k and $J^{0\beta}\sqrt{g_4}$ and $J^{n\beta}\sqrt{g_4}$ by $1/k$. The scaling of CD should be due to the scaling up of the M^4 time interval during which the branched light-like 3-surface returns back to a non-branched one.

5. The proposed view about hierarchy of Planck constants is that the singular coverings reduce to single-sheeted coverings at $M^2 \subset M^4$ for CD and to $S^2 \subset CP_2$ for CP_2. Here S^2 is any

homologically trivial geodesic sphere of CP_2 and has vanishing Kähler form. Weak self-duality condition is indeed consistent with any value of $\hbar$ and impies that the vacuum property for the partonic 2-surface implies vacuum property for the entire space-time sheet as holography indeed requires. This condition however generalizes. In weak self-duality conditions the value of $\hbar$ is free for any 2-D Lagrangian sub-manifold of CP_2.

The branching along M^2 would mean that the branches of preferred extremals always collapse to single branch when their M^4 projection belongs to M^2. Magnetically charged light-light-like throats cannot have M^4 projection in M^2 so that self-duality conditions for different values of $\hbar$ do not lead to inconsistencies. For spacelike 3-surfaces at the boundaries of CD the condition would mean that the M^4 projection becomes light-like geodesic. Straight cosmic strings would have M^2 as M^4 projection. Also CP_2 type vacuum extremals for which the random light-like projection in M^4 belongs to M^2 would represent this of situation. One can ask whether the degeneration of branches actually takes place along any string like object $X^2 \times Y^2$, where X^2 defines a minimal surface in M^4. For these the weak self-duality condition would imply $\hbar = \infty$ at the ends of the string. It is very plausible that string like objects feed their magnetic fluxes to larger space-times sheets through wormhole contacts so that these conditions are not encountered.

Connection with the criticality of preferred extremals

Also a connection with quantum criticality and the criticality of the preferred extremals suggests itself. Criticality for the preferred extremals must be a property of space-like 3-surfaces and light-like 3-surfaces with degenerate 4-metric and the degeneration of the $n_a n_b$ branches of the space-time surface at the its ends and at wormhole throats is exactly what happens at criticality. For instance, in catastrophe theory roots of the polynomial equation giving extrema of a potential as function of control parameters co-incide at criticality. If this picture is correct the hierarchy of Planck constants would be an outcome of criticality and of preferred extremal property and preferred extremals would be just those multi-branched space-time surfaces for which branches co-incide at the the boundaries of $CD \times CP_2$ and at the throats.

4.4 Number theoretic compactification and $M^8 - H$ duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to number theory. In strong form $M^8 - H$ duality boils down to the assumption that space-time surfaces can be regarded either as surfaces of H or as surfaces of M^8 composed of hyper-quaternionic and co-hyper-quaternionic regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric.

4.4.1 Basic idea behind $M^8 - M^4 \times CP_2$ duality

The hopes of giving $M^4 \times CP_2$ hyper-octonionic structure are meager. This circumstance forces to ask whether four-surfaces $X^4 \subset M^8$ could under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly so that the spontaneous compactification of super string models would correspond in TGD to two different manners to interpret the space-time surface. The following arguments suggest that this is indeed the case.

The hard mathematical fact behind number theoretical compactification is that the quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space) are parameterized by CP_2 just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of CP_2, as the automorphism sub-group of octonions, and as color group.

1. The space of complex structures of the octonion space is parameterized by S^6. The subgroup $SU(3)$ of the full automorphism group G_2 respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it e_1. Hyper-quaternions can be identified as $U(2)$ Lie-algebra but it is obvious that hyper-octonions do not allow an identification as $SU(3)$ Lie algebra. Rather, octonions decompose as $1 \oplus 1 \oplus 3 \oplus \overline{3}$ to the irreducible representations of $SU(3)$.

2. Geometrically the choice of a preferred complex (quaternionic) structure means fixing of complex (quaternionic) sub-space of octonions. The fixing of a hyper-quaternionic structure of hyper-octonionic M^8 means a selection of a fixed hyper-quaternionic sub-space $M^4 \subset M^8$ implying the decomposition $M^8 = M^4 \times E^4$. If M^8 is identified as the tangent space of $H = M^4 \times CP_2$, this decomposition results naturally. It is also possible to select a fixed hyper-complex structure, which means a further decomposition $M^4 = M^2 \times E^2$.

3. The basic result behind number theoretic compactification and $M^8 - H$ duality is that hyper-quaternionic sub-spaces $M^4 \subset M^8$ containing a fixed hyper-complex sub-space $M^2 \subset M^4$ or its light-like line $M_\pm$ are parameterized by CP_2. The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of e_1) are labeled by $U(2) \subset SU(3)$. The choice of e_2 and e_3 amounts to fixing $e_2 \pm \sqrt{-1}e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of e_1 and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having e_2 and e_3 components. Hence all possible completions of $1, e_1$ by adding e_2, e_3 doublet are labeled by $SU(3)/U(2) = CP_2$.

4. Space-time surface $X^4 \subset M^8$ is by the standard definition hyper-quaternionic if the tangent spaces of X^4 are hyper-quaternionic planes. Co-hyper-quaternionicity means the same for normal spaces. The presence of fixed hyper-complex structure means at space-time level that the tangent space of X^4 contains fixed M^2 at each point. Under this assumption one can map the points $(m, e) \in M^8$ to points $(m, s) \in H$ by assigning to the point (m, e) of X^4 the point (m, s), where $s \in CP_2$ characterize $T(X^4)$ as hyper-quaternionic plane. This definition is not the only one and even the appropriate one in TGD context the replacement of the tangent plane with the 4-D plane spanned by modified gamma matrices defined by Kähler action is a more natural choice. This plane is not parallel to tangent plane in general. In the sequel $T(X^4)$ denotes the preferred 4-plane which co-incides with tangent plane of X^4 only if the action defining modified gamma matrices is 4-volume.

5. The choice of M^2 can be made also local in the sense that one has $T(X^4) \supset M^2(x) \subset M^4 \subset H$. It turns out that strong form of number theoretic compactification requires this kind of generalization. In this case one must be able to fix the convention how the point of CP_2 is assigned to a hyper-quaternionic plane so that it applies to all possible choices of $M^2 \subset M^4$. Since $SO(3)$ hyper-quaternionic rotation relates the hyper-quaternionic planes to each other, the natural assumption is hyper-quaternionic planes related by $SO(3)$ rotation correspond to the same point of CP_2. Under this assumption it is possible to map hyper-quaternionic surfaces of M^8 for which $M^2 \subset M^4$ depends on point of X^4 to H.

4.4.2 Hyper-octonionic Pauli "matrices" and modified definition of hyper-quaternionicity

Hyper-octonionic Pauli matrices suggest an interesting possibility to define precisely what hyper-quaternionicity means at space-time level (for background see [28]).

1. According to the standard definition space-time surface X^4 is hyper-quaternionic if the tangent space at each point of X^4 in $X^4 \subset M^8$ picture is hyper-quaternionic. What raises worries is that this definition involves in no manner the action principle so that it is far from obvious that this identification is consistent with the vacuum degeneracy of Kähler action. It also unclear how one should formulate hyper-quaternionicity condition in $X^4 \subset M^4 \times CP_2$ picture.

2. The idea is to map the modified gamma matrices $\Gamma^\alpha = \frac{\partial L_K}{\partial h^k_\alpha}\Gamma^k$, $\Gamma_k = e^A_k \gamma_A$, to hyper-octonionic Pauli matrices σ^α by replacing γ_A with hyper-octonion unit. Hyper-quaternionicity would state that the hyper-octonionic Pauli matrices σ^α obtained in this manner span complexified quaternion sub-algebra at each point of space-time. These conditions would provide a number theoretic manner to select preferred extremals of Kähler action. Remarkably, this definition applies both in case of M^8 and $M^4 \times CP_2$.

3. Modified Pauli matrices span the tangent space of X^4 if the action is four-volume because one has $\frac{\partial L_K}{\partial h^k_\alpha} = \sqrt{g}g^{\alpha\beta}\partial h^l_\beta h_{kl}$. Modified gamma matrices reduce to ordinary induced gamma matrices in this case: 4-volume indeed defines a super-conformally symmetric action for ordinary gamma

matrices since the mass term of the Dirac action given by the trace of the second fundamental form vanishes for minimal surfaces.

4. For Kähler action the hyper-quaternionic sub-space does not coincide with the tangent space since $\frac{\partial L_K}{\partial h^k_\alpha}$ contains besides the gravitational contribution coming from the induced metric also the "Maxwell contribution" from the induced Kähler form not parallel to space-time surface. Modified gamma matrices are required by super conformal symmetry for the extremals of Kähler action and they also guarantee that vacuum extremals defined by surfaces in $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2, are trivially hyper-quaternionic surfaces. The modified definition of hyper-quaternionicity does not affect in any manner $M^8 \leftrightarrow M^4 \times CP_2$ duality allowing purely number theoretic interpretation of standard model symmetries.

A side comment not strictly related to hyper-quaternionicity is in order. The anticommutators of the modified gamma matrices define an effective Riemann metric and one can assign to it the counterparts of Riemann connection, curvature tensor, geodesic line, volume, etc... One would have two different metrics associated with the space-time surface. Only if the action defining space-time surface is identified as the volume in the ordinary metric, these metrics are equivalent. The index raising for the effective metric could be defined also by the induced metric and it is not clear whether one can define Riemann connection also in this case. Could this effective metric have concrete physical significance and play a deeper role in quantum TGD? For instance, AdS-CFT duality leads to ask whether interactions be coded in terms of the gravitation associated with the effective metric.

4.4.3 Minimal form of $M^8 - H$ duality

The basic problem in the construction of quantum TGD has been the identification of the preferred extremals of Kähler action playing a key role in the definition of the theory. The most elegant manner to do this is by fixing the 4-D tangent space $T(X^4(X^3_l))$ of $X^4(X^3_l)$ at each point of X^3_l so that the boundary value problem is well defined. What I called number theoretical compactification allows to achieve just this although I did not fully realize this in the original vision. The minimal picture is following.

1. The basic observations are following. Let M^8 be endowed with hyper-octonionic structure. For hyper-quaternionic space-time surfaces in M^8 tangent spaces are by definition hyper-quaternionic. If they contain a preferred plane $M^2 \subset M^4 \subset M^8$ in their tangent space, they can be mapped to 4-surfaces in $M^4 \times CP_2$. The reason is that the hyper-quaternionic planes containing preferred the hyper-complex plane M^2 of $M_\pm \subset M^2$ are parameterized by points of CP_2. The map is simply $(m, e) \rightarrow (m, s(m, e))$, where m is point of M^4, e is point of E^4, and $s(m, 2)$ is point of CP_2 representing the hyperquaternionic plane. The inverse map assigns to each point (m, s) in $M^4 \times CP_2$ point m of M^4, undetermined point e of E^4 and 4-D plane. The requirement that the distribution of planes containing the preferred M^2 or $M_\pm$ corresponds to a distribution of planes for 4-D surface is expected to fix the points e. The physical interpretation of M^2 is in terms of plane of non-physical polarizations so that gauge conditions have purely number theoretical interpretation.

2. In principle, the condition that $T(X^4)$ contains M^2 can be replaced with a weaker condition that either of the two light-like vectors of M^2 is contained in it since already this condition assigns to $T(X^4)$ M^2 and the map $H \rightarrow M^8$ becomes possible. Only this weaker form applies in the case of massless extremals [33] as will be found.

3. The original idea was that hyper-quaternionic 4-surfaces in M^8 containing $M^2 \subset M^4$ in their tangent space could correspond to preferred extremals of Kähler action. This condition does not seem to be consistent with what is known about the extremals of Kähler action. The weaker form of the hypothesis is that hyper-quaternionicity holds only for 4-D tangent spaces of $X^3_l \subset H = M^4 \times CP_2$ identified as wormhole throats or boundary components lifted to 3-surfaces in 8-D tangent space M^8 of H. The minimal hypothesis would be that only $T(X^4(X^3_l))$ at X^3_l is associative that is hyper-quaternionic for fixed M^2. $X^3_l \subset M^8$ and $T(X^4(X^3_l))$ at X^3_l can be mapped to $X^3_l \subset H$ if tangent space contains also $M_\pm \subset M^2$ or $M^2 \subset M^4 \subset M^8$ itself having interpretation as preferred hyper-complex plane. This condition is not satisfied by all

surfaces X^3_l as is clear from the fact that the inverse map involves local E^4 translation. The requirements that the distribution of hyper-quaternionic planes containing M^2 corresponds to a distribution of 4-D tangent planes should fix the E^4 translation to a high degree.

4. A natural requirement is that the image of $X^3_l \subset H$ in M^8 is light-like. The condition that the determinant of induced metric vanishes gives an additional condition reducing the number of free parameters by one. This condition cannot be formulated as a condition on CP_2 coordinate characterizing the hyper-quaternionic plane. Since M^4 projections are same for the two representations, this condition is satisfied if the contributions from CP_2 and E^4 and projections to the induced metric are identical: $s_{kl}\partial_\alpha s^k \partial_\beta s^l = e_{kl}\partial_\alpha e^k \partial_\beta e^l$. This condition means that only a subset of light-like surfaces of M^8 are realized physically. One might argue that this is as it must be since the volume of E^4 is infinite and that of CP_2 finite: only an infinitesimal portion of all possible light-like 3-surfaces in M^8 can can have H counterparts. The conclusion would be that number theoretical compactification is 4-D isometry between $X^4 \subset H$ and $X^4 \subset M^8$ at X^3_l. This unproven conjecture is unavoidable.

5. $M^2 \subset T(X^4(X^3_l))$ condition fixes $T(X^4(X^3_l))$ in the generic case by extending the tangent space of X^3_l, and the construction of configuration space spinor structure fixes boundary conditions completely by additional conditions necessary when X^3_l corresponds to a light-like 3 surfaces defining wormhole throat at which the signature of induced metric changes. What is especially beautiful that only the data in $T(X^4(X^3_l))$ at X^3_l is needed to calculate the vacuum functional of the theory as Dirac determinant: the only remaining conjecture (strictly speaking un-necessary but realistic looking) is that this determinant gives exponent of Kähler action for the preferred extremal and there are excellent hopes for this by the structure of the basic construction.

The basic criticism relates to the condition that light-like 3-surfaces are mapped to light-like 3-surfaces guaranteed by the condition that $M^8 - H$ duality is isometry at X^3_l.

4.4.4 Strong form of $M^8 - H$ duality

The proposed picture is the minimal one. One can of course ask whether the original much stronger conjecture that the preferred extrema of Kähler action correspond to hyper-quaternionic surfaces could make sense in some form. One can also wonder whether one could allow the choice of the plane M^2 of non-physical polarization to be local so that one would have $M^2(x) \subset M^4 \subset M^4 \times E^4$, where M^4 is fixed hyper-quaternionic sub-space of M^8 and identifiable as M^4 factor of H.

1. If M^2 is same for all points of X^3_l, the inverse map $X^3_l \subset H \rightarrow X^3_l \subset M^8$ is fixed apart from possible non-uniquencess related to the local translation in E^4 from the condition that hyper-quaternionic planes represent light-like tangent 4-planes of light-like 3-surfaces. The question is whether not only X^3_l but entire four-surface $X^4(X^3_l)$ could be mapped to the tangent space of M^8. By selecting suitably the local E^4 translation one might hope of achieving the achieving this. The conjecture would be that the preferred extrema of Kähler action are those for which the distribution integrates to a distribution of tangent planes.

2. There is however a problem. What is known about extremals of Kähler action is not consistent with the assumption that fixed M^2 of $M_\pm \subset M^2$is contained in the tangent space of X^4. This suggests that one should relax the condition that $M^2 \subset M^4 \subset M^8$ is a fixed hyper-complex plane associated with the tangent space or normal space X^4 and allow M^2 to vary from point to point so that one would have $M^2 = M^2(x)$. In $M^8 \rightarrow H$ direction the justification comes from the observation (to be discussed below) that it is possible to uniquely fix the convention assigning CP_2 point to a hyper-quaternionic plane containing varying hyper-complex plane $M^2(x) \subset M^4$.

   Number theoretic compactification fixes naturally $M^4 \subset M^8$ so that it applies to any $M^2(x) \subset M^4$. Under this condition the selection is parameterized by an element of $SO(3)/SO(2) = S^2$. Note that M^4 projection of X^4 would be at least 2-dimensional in hyper-quaternionic case. In co-hyper-quaternionic case E^4 projection would be at least 2-D. $SO(2)$ would act as a number theoretic gauge symmetry and the $SO(3)$ valued chiral field would approach to constant at X^3_l invariant under global $SO(2)$ in the case that one keeps the assumption that M^2 is fixed ad X^3_l.

3. This picture requires a generalization of the map assigning to hyper-quaternionic plane a point of CP_2 so that this map is defined for all possible choices of $M^2 \subset M^4$. Since the $SO(3)$ rotation of the hyper-quaternionic unit defining M^2 rotates different choices parameterized by S^2 to each other, a natural assumption is that the hyper-quaternionic planes related by $SO(3)$ rotation correspond to the same point of CP_2. Denoting by M^2 the standard representative of M^2, this means that for the map $M^8 \to H$ one must perform $SO(3)$ rotation of hyper-quaternionic plane taking $M^2(x)$ to M^2 and map the rotated plane to CP_2 point. In $M^8 \to H$ case one must first map the point of CP_2 to hyper-quaternionic plane and rotate this plane by a rotation taking $M^2(x)$ to M^2.

4. In this framework local M^2 can vary also at the surfaces X_l^3, which considerably relaxes the boundary conditions at wormhole throats and light-like boundaries and allows much more general variety of light-like 3-surfaces since the basic requirement is that M^4 projection is at least 1-dimensional. The physical interpretation would be that a local choice of the plane of non-physical polarizations is possible everywhere in $X^4(X_l^3)$. This does not seem to be in any obvious conflict with physical intuition.

These observation provide support for the conjecture that (classical) $S^2 = SO(3)/SO(2)$ conformal field theory might be relevant for (classical) TGD.

1. General coordinate invariance suggests that the theory should allow a formulation using any light-like 3-surface X^3 inside $X^4(X_l^3)$ besides X_l^3 identified as union of wormhole throats and boundary components. For these surfaces the element $g(x) \in SO(3)$ would vary also at partonic 2-surfaces X^2 defined as intersections of $\delta CD \times CP_2$ and X^3 (here CD denotes causal diamond defined as intersection of future and past directed light-cones). Hence one could have $S^2 = SO(3)/SO(2)$ conformal field theory at X^2 (regarded as quantum fluctuating so that also $g(x)$ varies) generalizing to WZW model for light-like surfaces X^3.

2. The presence of E^4 factor would extend this theory to a classical $E^4 \times S^2$ WZW model bringing in mind string model with 6-D Euclidian target space extended to a model of light-like 3-surfaces. A further extension to X^4 would be needed to integrate the WZW models associated with 3-surfaces to a full 4-D description. General Coordinate Invariance however suggests that X_l^3 description is enough for practical purposes.

3. The choices of $M^2(x)$ in the interior of X_l^3 is dictated by dynamics and the first optimistic conjecture is that a classical solution of $SO(3)/SO(2)$ Wess-Zumino-Witten model obtained by coupling $SO(3)$ valued field to a covariantly constant $SO(2)$ gauge potential characterizes the choice of $M^2(x)$ in the interior of $M^8 \supset X^4(X_l^3) \subset H$ and thus also partially the structure of the preferred extremal. Second optimistic conjecture is that the Kähler action involving also E^4 degrees of freedom allows to assign light-like 3-surface to light-like 3-surface.

4. The best that one can hope is that $M^8 - H$ duality could allow to transform the extremely non-linear classical dynamics of TGD to a generalization of WZW-type model. The basic problem is to understand how to characterize the dynamics of CP_2 projection at each point.

In H picture there are two basic types of vacuum extremals: CP_2 type extremals representing elementary particles and vacuum extremals having CP_2 projection which is at most 2-dimensional Lagrange manifold and representing say hadron. Vacuum extremals can appear only as limiting cases of preferred extremals which are non-vacuum extremals. Since vacuum extremals have so decisive role in TGD, it is natural to requires that this notion makes sense also in M^8 picture. In particular, the notion of vacuum extremal makes sense in M^8.

This requires that Kähler form exist in M^8. E^4 indeed allows full S^2 of covariantly constant Kähler forms representing quaternionic imaginary units so that one can identify Kähler form and construct Kähler action. The obvious conjecture is that hyper-quaternionic space-time surface is extremal of this Kähler action and that the values of Kähler actions in M^8 and H are identical. The elegant manner to achieve this, as well as the mapping of vacuum extremals to vacuum extremals and the mapping of light-like 3-surfaces to light-like 3-surfaces is to assume that $M^8 - H$ duality is Kähler isometry so that induced Kähler forms are identical.

This picture contains many speculative elements and some words of warning are in order.

1. Light-likeness conjecture would boil down to the hypothesis that $M^8 - H$ correspondence is Kähler isometry so that the metric and Kähler form of X^4 induced from M^8 and H would be identical. This would guarantee also that Kähler actions for the preferred extremal are identical. This conjecture is beautiful but strong.

2. The slicing of $X^4(X_l^3)$ by light-like 3-surfaces is very strong condition on the classical dynamics of Kähler action and does not make sense for pieces of CP_2 type vacuum extremals.

Minkowskian-Euclidian $\leftrightarrow$ associative–co-associative

The 8-dimensionality of M^8 allows to consider both associativity (hyper-quaternionicity) of the tangent space and associativity of the normal space- let us call this co-assosiativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \rightarrow k$ duality.

Are the known extremals of Kähler action consistent with the strong form of $M^8 - H$ duality

It is interesting to check whether the known extremals of Kähler action [33] are consistent with strong form of $M^8 - H$ duality assuming that M^2 or its light-like ray is contained in $T(X^4)$ or normal space.

1. CP_2 type vacuum extremals correspond cannot be hyper-quaternionic surfaces but co-hyper-quaternionicity is natural for them. In the same manner canonically imbedded M^4 can be only hyper-quaternionic.

2. String like objects are associative since tangent space obviously contains $M^2(x)$. Objects of form $M^1 \times X^3 \subset M^4 \times CP_2$ do not have M^2 either in their tangent space or normal space in H. So that the map from $H \rightarrow M^8$ is not well defined. There are no known extremals of Kähler action of this type. The replacement of M^1 random light-like curve however gives vacuum extremal with vanishing volume, which need not mean physical triviality since fundamental objects of the theory are light-like 3-surfaces.

3. For canonically imbedded CP_2 the assignment of $M^2(x)$ to normal space is possible but the choice of $M^2(x) \subset N(CP_2)$ is completely arbitrary. For a generic CP_2 type vacuum extremals M^4 projection is a random light-like curve in $M^4 = M^1 \times E^3$ and $M^2(x)$ can be defined uniquely by the normal vector $n \in E^3$ for the local plane defined by the tangent vector dx^μ/dt and acceleration vector d^2x^μ/dt^2 assignable to the orbit.

4. Consider next massless extremals. Let us fix the coordinates of X^4 as $(t, z, x, y) = (m^0, m^2, m^1, m^2)$. For simplest massless extremals CP_2 coordinates are arbitrary functions of variables $u = k \cdot m = t - z$ and $v = \epsilon \cdot m = x$, where $k = (1, 1, 0, 0)$ is light-like vector of M^4 and $\epsilon = (0, 0, 1, 0)$ a polarization vector orthogonal to it. Obviously, the extremals defines a decomposition $M^4 = M^2 \times E^2$. Tangent space is spanned by the four H-vectors $\nabla_\alpha h^k$ with M^4 part given by $\nabla_\alpha m^k = \delta^k_\alpha$ and CP_2 part by $\nabla_\alpha s^k = \partial_u s^k k_\alpha + \partial_v s^k \epsilon_\alpha$.

   The normal space cannot contain M^4 vectors since the M^4 projection of the extremal is M^4. To realize hyper-quaternionic representation one should be able to from these vector two vectors of M^2, which means linear combinations of tangent vectors for which CP_2 part vanishes. The vector $\partial_t h^k - \partial_z h^k$ has vanishing CP_2 part and corresponds to M^4 vector $(1, -1, 0, 0)$ fix assigns to each point the plane M^2. To obtain M^2 one would need $(1,1,0,0)$ too but this is not possible. The vector $\partial_y h^k$ is M^4 vector orthogonal to ϵ but M^2 would require also $(1,0,0,0)$. The proposed generalization of massless extremals allows the light-like line $M_\pm$ to depend on point of M^4 [33], and leads to the introduction of Hamilton-Jacobi coordinates involving a local decomposition of M^4 to $M^2(x)$ and its orthogonal complement with light-like coordinate lines having interpretation as curved light rays. $M^2(x) \subset T(X^4)$ assumption fails fails also for vacuum extremals of form $X^1 \times X^3 \subset M^4 \times CP_2$, where X^1 is light-like random curve. In the latter case, vacuum property follows from the vanishing of the determinant of the induced metric.

5. The deformations of string like objects to magnetic flux quanta are basic conjectural extremals of Kähler action and the proposed picture supports this conjecture. In hyper-quaternionic case the assumption that local 4-D plane of X^3 defined by modified gamma matrices contains $M^2(x)$ but that $T(X^3)$ does not contain it, is very strong. It states that $T(X^4)$ at each point can be regarded as a product $M^2(x) \times T^2$, $T^2 \subset T(CP_2)$, so that hyper-quaternionic X^4 would be a collection of Cartesian products of infinitesimal 2-D planes $M^2(x) \subset M^4$ and $T^2(x) \subset CP_2$. The extremals in question could be seen as local variants of string like objects $X^2 \times Y^2 \subset M^4 \times CP_2$, where X^2 is minimal surface and Y^2 holomorphic surface of CP_2. One can say that X^2 is replaced by a collection of infinitesimal pieces of $M^2(x)$ and Y^2 with similar pieces of homologically non-trivial geodesic sphere $S^2(x)$ of CP_2, and the Cartesian products of these pieces are glued together to form a continuous surface defining an extremal of Kähler action. Field equations would pose conditions on how $M^2(x)$ and $S^2(x)$ can depend on x. This description applies to magnetic flux quanta, which are the most important must-be extremals of Kähler action.

Geometric interpretation of strong $M^8 - H$ duality

In the proposed framework $M^8 - H$ duality would have a purely geometric meaning and there would nothing magical in it.

1. $X^4(X^3_l) \subset H$ could be seen a curve representing the orbit of a light-like 3-surface defining a 4-D surface. The question is how to determine the notion of tangent vector for the orbit of X^3_l. Intuitively tangent vector is a one-dimensional arrow tangential to the curve at point X^3_l. The identification of the hyper-quaternionic surface $X^4(X^3_l) \subset M^8$ as tangent vector conforms with this intuition.

2. One could argue that M^8 representation of space-time surface is kind of chart of the real space-time surface obtained by replacing real curve by its tangent line. If so, one cannot avoid the question under which conditions this kind of chart is faithful. An alternative interpretation is that a representation making possible to realize number theoretical universality is in question.

3. An interesting question is whether $X^4(X^3_l)$ as orbit of light-like 3-surface is analogous to a geodesic line -possibly light-like- so that its tangent vector would be parallel translated in the sense that $X^4(X^3)$ for any light-like surface at the orbit is same as $X^4(X^3_l)$. This would give justification for the possibility to interpret space-time surfaces as a geodesic of configuration space: this is one of the first -and practically forgotten- speculations inspired by the construction of configuration space geometry. The light-likeness of the geodesic could correspond at the level of X^4 the possibility to decompose the tangent space to a direct sum of two light-like spaces and 2-D transversal space producing the foliation of X^4 to light-like 3-surfaces X^3_l along light-like curves.

4. M^8-H duality would assign to X^3_l classical orbit and its tangent vector at X^3_l as a generalization of Bohr orbit. This picture differs from the wave particle duality of wave mechanics stating that once the position of particle is known its momentum is completely unknown. The outcome is however the same: for X^3_l corresponding to wormhole throats and light-like boundaries of X^4, canonical momentum densities in the normal direction vanish identically by conservation laws and one can say that the the analog of (q,p) phase space as the space carrying wave functions is replaced with the analog of subspace consisting of points $(q,0)$. The dual description in M^8 would not be analogous to wave functions in momentum space space but to those in the space of unique tangents of curves at their initial points.

The Kähler and spinor structures of M^8

If one introduces M^8 as dual of H, one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in H are also extremals of M^8 Kähler action with same value of Kähler action. As found, this leads to the conclusion that the$M^8 - H$ duality is Kähler isometry. Coupling of spinors to Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in H should have full M^8 dual.

There are strong physical constraints on M^8 dual and they could kill the hypothesis. The basic constraint to the spinor structure of M^8 is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different H-chiralities and parity breaking.

1. By the flatness of the metric of E^4 its spinor connection is trivial. E^4 however allows full S^2 of covariantly constant Kähler forms so that one can accommodate free independent Abelian gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units.

2. One should be able to distinguish between quarks and leptons also in M^8, which suggests that one introduce spinor structure and Kähler structure in E^4. The Kähler structure of E^4 is unique apart form $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of S^2 representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of H.

3. Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and Z^0 contains both axial and vector parts. The free Kähler forms could thus allow to produce M^8 counterparts of these gauge potentials possessing same couplings as their H counterparts. This picture would produce parity breaking in M^8 picture correctly.

4. Only the charged parts of classical electro-weak gauge fields would be absent. This would conform with the standard thinking that charged classical fields are not important. The predicted classical W fields is one of the basic distinctions between TGD and standard model and in this framework. A further prediction is that this distinction becomes visible only in situations, where H picture is necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of E^4 partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

5. Also super-symmetries of quantum TGD crucial for the construction of configuration space geometry force this picture. In the absence of coupling to Kähler gauge potential all constant spinor fields and their conjugates would generate super-symmetries so that M^8 would allow $N = 8$ super-symmetry. The introduction of the coupling to Kähler gauge potential in turn means that all covariantly constant spinor fields are lost. Only the representation of all three neutral parts of electro-weak gauge potentials in terms of three independent Kähler gauge potentials allows right-handed neutrino as the only super-symmetry generator as in the case of H.

6. The $SO(3)$ element characterizing $M^2(x)$ is fixed apart from a local $SO(2)$ transformation, which suggests an additional $U(1)$ gauge field associated with $SO(2)$ gauge invariance and representable as Kähler form corresponding to a quaternionic unit of E^4. A possible identification of this gauge field would be as a part of electro-weak gauge field.

M^8 dual of configuration space geometry and spinor structure?

If one introduces M^8 spinor structure and preferred extremals of M^8 Kähler action, one cannot avoid the question whether it is possible or useful to formulate the notion of configuration space geometry and spinor structure for light-like 3-surfaces in M^8 using the exponent of Kähler action as vacuum functional.

1. The isometries of the configuration space in M^8 and H formulations would correspond to symplectic transformation of $\delta M^4_\pm \times E^4$ and $\delta M^4_\pm \times CP_2$ and the Hamiltonians involved would belong to the representations of $SO(4)$ and $SU(3)$ with 2-dimensional Cartan sub-algebras. In H picture color group would be the familiar $SU(3)$ but in M^8 picture it would be $SO(4)$. Color confinement in both $SU(3)$ and $SO(4)$ sense could allow these two pictures without any inconsistency.

2. For $M^4 \times CP_2$ the two spin states of covariantly constant right handed neutrino and antineutrino spinors generate super-symmetries. This super-symmetry plays an important role in the proposed construction of configuration space geometry. As found, this symmetry would be present also in M^8 formulation so that the construction of M^8 geometry should reduce more or less to the replacement of CP_2 Hamiltonians in representations of $SU(3)$ with E^4 Hamiltonians in representations of $SO(4)$. These Hamiltonians can be taken to be proportional to functions of E^4 radius which is $SO(4)$ invariant and these functions bring in additional degree of freedom.

3. The construction of Dirac determinant identified as a vacuum functional can be done also in M^8 picture and the conjecture is that the result is same as in the case of H. In this framework the construction is much simpler due to the flatness of E^4. In particular, the generalized eigen modes of the Dirac operator $D_K(Y^3_l)$ restricted to the X^3_l correspond to a situation in which one has fermion in induced Maxwell field mimicking the neutral part of electro-weak gauge field in H as far as couplings are considered. Induced Kähler field would be same as in H. Eigen modes are localized to regions inside which the Kähler magnetic field is non-vanishing and apart from the fact that the metric is the effective metric defined in terms of canonical momentum densities via the formula $\hat{\Gamma}^\alpha = \partial L_K/\partial h^k_\alpha \Gamma_k$ for effective gamma matrices. This in fact, forces the localization of modes implying that their number is finite so that Dirac determinant is a product over finite number eigenvalues. It is clear that M^8 picture could dramatically simplify the construction of configuration space geometry.

4. The eigenvalue spectra of the transversal parts of D_K operators in M^8 and H should identical. This motivates the question whether it is possible to achieve a complete correspondence between H and M^8 pictures also at the level of spinor fields at X^3 by performing a gauge transformation eliminating the classical W gauge boson field altogether at X^3_l and whether this allows to transform the modified Dirac equation in H to that in M^8 when restricted to X^3_l. That something like this might be achieved is supported by the fact that in Coulombic gauge the component of gauge potential in the light-like direction vanishes so that the situation is effectively 2-dimensional and holonomy group is Abelian.

Why $M^8 - H$ duality is useful?

Skeptic could of course argue that $M^8 - H$ duality produces only an inflation of unproven conjectures. There are however strong reasons for $M^8 - H$ duality: both theoretical and physical.

1. The map of $X^3_l \subset H \to X^3_l \subset M^8$ and corresponding map of space-time surfaces would allow to realize number theoretical universality. $M^8 = M^4 \times E^4$ allows linear coordinates as natural coordinates in which one can say what it means that the point of imbedding space is rational/algebraic. The point of $X^4 \subset H$ is algebraic if it is mapped to an algebraic point of M^8 in number theoretic compactification. This of course restricts the symmetry groups to their rational/algebraic variants but this does not have practical meaning. Number theoretical compactication could in fact be motivated by the number theoretical universality.

2. $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action since the Kähler form in E^4 has constant components. If the spinor connection in E^4 is combination of the three Kähler forms mimicking neutral part of electro-weak gauge potential, the eigenvalue spectrum for the modified Dirac operator would correspond to that for a fermion in $U(1)$ magnetic field defined by an Abelian magnetic field whereas in $M^4 \times CP_2$ picture $U(2)_{ew}$ magnetic fields would be present.

3. $M^8 - H$ duality provides insights to low energy hadron physics. M^8 description might work when H-description fails. For instance, perturbative QCD which corresponds to H-description

fails at low energies whereas M^8 description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong $SU(2)_L \times SU(2)_R$ and by flatness of E^4 spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in CP_2. One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin.

4.4.5 $M^8 - H$ duality and low energy hadron physics

The description of $M^8 - H$ at the configuration space level can be applied to gain a view about color confinement and its dual for electro-weak interactions at short distance limit. The basic idea is that $SO(4)$ and $SU(3)$ provide provide dual descriptions of quark color using E^4 and CP_2 partial waves and low energy hadron physics corresponds to a situation in which M^8 picture provides the perturbative approach whereas H picture works at high energies. The basic prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics and this is indeed the case.

Consider color confinement at the long length scale limit in terms of $M^8 - H$ duality.

1. At high energy limit only lowest color triplet color partial waves for quarks dominate so that QCD description becomes appropriate whereas very higher color partial waves for quarks and gluons are expected to appear at the confinement limit. Since configuration space degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD.

2. The success of $SO(4)$ sigma model in the description of low lying hadrons would directly relate to the fact that this group labels also the E^4 Hamiltonians in M^8 picture. Strong $SO(4)$ quantum numbers can be identified as orbital counterparts of right and left handed electro-weak isospin coinciding with strong isospin for lowest quarks. In sigma model pion and sigma boson form the components of E^4 valued vector field or equivalently collection of four E^4 Hamiltonians corresponding to spherical E^4 coordinates. Pion corresponds to S^3 valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the E^4 radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted.

3. The generalization of sigma model would assign to quarks E^4 partial waves belonging to the representations of $SO(4)$. The model would involve also 6 $SO(4)$ gluons and their $SO(4)$ partial waves. At the low energy limit only lowest representations would be be important whereas at higher energies higher partial waves would be excited and the description based on CP_2 partial waves would become more appropriate.

4. The low energy quark model would rely on quarks moving $SO(4)$ color partial waves. Left *resp.* right handed quarks could correspond to $SU(2)_L$ *resp.* $SU(2)_R$ triplets so that spin statistics problem would be solved in the same manner as in the standard quark model.

5. Family replication phenomenon is described in TGD framework the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [17].

To my opinion these observations are intriguing enough to motivate a concrete attempt to construct low energy hadron physics in terms of $SO(4)$ gauge theory.

4.4.6 The notion of number theoretical braid

Braids -not necessary number theoretical- provide a realization discretization as a space-time correlate for the finite measurement resolution. The notion of braid was inspired by the idea about quantum TGD as almost topological quantum field theory. Although the original form of this idea has been buried, the notion of braid has survived: in the decomposition of space-time sheets to string world sheets, the ends of strings define representatives for braid strands at light-like 3-surfaces.

The notion of number theoretic universality inspired the much more restrictive notion of number theoretic braid requiring that the points in the intersection of the braid with the partonic 2-surface correspond to rational or at most algebraic points of H in preferred coordinates fixed by symmetry considerations. The challenge has been to find a unique identification of the number theoretic braid or at least of the end points of the braid. The following consideration suggest that the number theoretic braids are not a useful notion in the generic case but make sense and are needed in the intersection of real and p-adic worlds which is in crucial role in TGD based vision about living matter [38].

It is only the braiding that matters in topological quantum field theories used to classify braids. Hence braid should require only the fixing of the end points of the braids at the intersection of the braid at the light-like boundaries of CDs and the braiding equivalence class of the braid itself. Therefore it is enough is to specify the topology of the braid and the end points of the braid in accordance with the attribute "number theoretic". Of course, the condition that all points of the strand of the number theoretic braid are algebraic is impossible to satisfy.

The situation in which the equations defining X^2 make sense both in real sense and p-adic sense using appropriate algebraic extension of p-adic number field is central in the TGD based vision about living matter [38]. The reason is that in this case the notion of number entanglement theoretic entropy having negative values makes sense and entanglement becomes information carrying. This motivates the identification of life as something in the intersection of real and p-adic worlds. In this situation the identification of the ends of the number theoretic braid as points belonging to the intersection of real and p-adic worlds is natural. These points -call them briefly algebraic points- belong to the algebraic extension of rationals needed to define the algebraic extension of p-adic numbers. This definition however makes sense also when the equations defining the partonic 2-surfaces fail to make sense in both real and p-adic sense. In the generic case the set of points satisfying the conditions is discrete. For instance, according to Fermat's theorem the set of rational points satisfying $X^n+Y^n = Z^n$ reduces to the point $(0,0,0)$ for $n = 3,4,....$ Hence the constraint might be quite enough in the intersection of real and p-adic worlds where the choice of the algebraic extension is unique.

One can however criticize this proposal.

1. One must fix the the number of points of the braid and outside the intersection and the non-uniquencess of the algebraic extension makes the situation problematic. Physical intuition suggests that the points of braid define carriers of quantum numbers assignable to second quantized induced spinor fields so that the total number of fermions antifermions would define the number of braids. In the intersection the highly non-trivial implication is that this number cannot exceed the number of algebraic points.

2. In the generic case one expects that even the smallest deformation of the partonic 2-surface can change the number of algebraic points and also the character of the algebraic extension of rational numbers needed. The restriction to rational points is not expected to help in the generic case. If the notion of number theoretical braid is meant to be practical, must be able to decompose WCW to open sets inside which the numbers of algebraic points of braid at its ends are constant. For real topology this is expected to be impossible and it does not make sense to use p-adic topology for WCW whose points do not allow interpretation as p-adic partonic surfaces.

3. In the intersection of real and p-adic worlds which corresponds to a discrete subset of WCW, the situation is different. Since the coefficients of polynomials involved with the definition of the partonic 2-surface must be rational or at most algebraic, continuous deformations are not possible so that one avoids the problem.

4. This forces to ask the reason why for the number theoretic braids. In the generic case they seem to produce only troubles. In the intersection of real and p-adic worlds they could however allow the construction of the elements of M-matrix describing quantum transitions changing p-adic

to real surfaces and vice versa as realizations of intentions and generation of cognitions. In this the case it is natural that only the data from the intersection of the two worlds are used. In [38] I have sketched the idea about number theoretic quantum field theory as a description of intentional action and cognition.

There is also the the problem of fixing the interior points of the braid modulo deformations not affecting the topology of the braid.

1. Infinite number of non-equivalent braidings are possible. Should one allow all possible braidings for a fixed light-like 3-surface and say that their existence is what makes the dynamics essentially three-dimensional even in the topological sense? In this case there would be no problems with the condition that the points at both ends of braid are algebraic.

2. Or should one try to characterize the braiding uniquely for a given partonic 2-surfaces and corresponding 4-D tangent space distributions? The slicing of the space-time sheet by partonic 2-surfaces and string word sheets suggests that the ends of string world sheets could define the braid strands in the generic context when there is no algebraicity condition involved. This could be taken as a very natural manner to fix the topology of braid but leave the freedom to choose the representative for the braid. In the intersection of real and p-adic worlds there is no good reason for the end points of strands in this case to be algebraic at both ends of the string world sheet. One can however start from the braid defined by the end points of string world sheets, restrict the end points to be algebraic at the end with a smaller number of algebraic points and and then perform a topologically non-trivial deformation of the braid so that also the points at the other end are algebraic? Non-trivial deformations need not be possible for all possible choices of algebraic braid points at the other end of braid and different choices of the set of algebraic points would give rise to different braidings. A further constraint is that only the algebraic points at which one has assign fermion or antifermion are used so that the number of braid points is not always maximal.

3. One can also ask whether one should perform the gauge fixing for the strands of the number theoretic braid using algebraic functions making sense both in real and p-adic context. This question does not seem terribly relevant since since it is only the topology of the braid that matters.

4.4.7 Connection with string model and Equivalence Principle at space-time level

Coset construction allows to generalize Equivalence Principle and understand it at quantum level. This is however not quite enough: a precise understanding of Equivalence Principle is required also at the classical level. Also the mechanism selecting via stationary phase approximation a preferred extremal of Kähler action providing a correlation between quantum numbers of the particle and geometry of the preferred extremals is still poorly understood.

Is stringy action principle coded by the geometry of preferred extremals?

It seems very difficult to deduce Equivalence Principle as an identity of gravitational and inertial masses identified as Noether charges associated with corresponding action principles. Since string model is an excellent theory of quantum gravitation, one can consider a less direct approach in which one tries to deduce a connection between classical TGD and string model and hope that the bridge from string model to General Relativity is easier to build. Number theoretical compactification gives good hopes that this kind of connection exists.

1. Number theoretic compactification implies that the preferred extremals of Kähler action have the property that one can assign to each point of M^4 projection $P_{M^4}(X^4(X^3_l))$ of the preferred extremal $M^2(x)$ identified as the plane of non-physical polarizations and also as the plane in which local massless four-momentum lies.

2. If the distribution of the planes $M^2(x)$ is integrable, one can slice $P_{M^4}(X^4(X^3_l))$ to string world-sheets. The intersection of string world sheets with $X^3 \subset \delta M^4_\pm \times CP_2$ corresponds to a light-like

curve having tangent in local tangent space $M^2(x)$ at light-cone boundary. This is the first candidate for the definition of number theoretic braid. Second definition assumes M^2 to be fixed at δCD: in this case the slicing is parameterized by the sphere S^2 defined by the light rays of $\delta M^4_\pm$.

3. One can assign to the string world sheet -call it Y^2 - the standard area action

$$S_G(Y^2) = \int_{Y^2} T\sqrt{g_2}d^2y \ , \quad (4.4.1)$$

where g_2 is either the induced metric or only its M^4 part. The latter option looks more natural since M^4 projection is considered. T is string tension.

4. The naivest guess would be $T = 1/\hbar G$ apart from some numerical constant but one must be very cautious here since $T = 1/L_p^2$ apart from a numerical constant is also a good candidate if one accepts the basic argument identifying G in terms of p-adic length L_p and Kähler action for two pieces of CP_2 type vacuum extremals representing propagating graviton. The formula reads $G = L_p^2 exp(-2aS_K(CP_2))$, $a \leq 1$ [20, 29]. The interaction strength which would be L_p^2 without the presence of CP_2 type vacuum extremals is reduced by the exponential factor coming from the exponent of Kähler function of configuration space.

5. One would have string model in either $CD \times CP_2$ or $CD \subset M^4$ with the constraint that stringy world sheet belongs to $X^4(X_l^3)$. For the extremals of $S_G(Y^2)$ gravitational four-momentum defined as Noether charge is conserved. The extremal property of string world sheet need not however be consistent with the preferred extremal property. This constraint might bring in coupling of gravitons to matter. The natural guess is that graviton corresponds to a string connecting wormhole contacts. The strings could also represent formation of gravitational bound states when they connect wormhole contacts separated by a large distance. The energy of the string is roughly $E \sim \hbar TL$ and for $T = 1/\hbar G$ gives $E \sim L/G$. Macroscopic strings are not allowed except as models of black holes. The identification $T \sim 1/L_p^2$ gives $E \sim \hbar L/L_p^2$, which does not favor long strings for large values of $\hbar$. The identification $G_p = L_p^2/\hbar_0$ gives $T = 1/\hbar G_p$ and $E \sim \hbar_0 L/L_p^2$, which makes sense and allows strings with length not much longer than p-adic length scale. Quantization - that is the presence of configuration space degrees of freedom- would bring in massless gravitons as deformations of string whereas strings would carry the gravitational mass.

6. The exponent $exp(iS_G)$ can appear as a phase factor in the definition of quantum states for preferred extremals. S_G is not however enough. One can assign also to the points of number theoretic braid action describing the interaction of a point like current Qdx^μ/ds with induced gauge potentials A_μ. The corresponding contribution to the action is

$$S_{braid} = \int_{braid} iTr(Q\frac{dx^\mu}{ds}A_\mu)dx \ . \quad (4.4.2)$$

In stationary phase approximation subject to the additional constraint that a preferred extremal of Kähler action is in question one obtains the desired correlation between the geometry of preferred extremal and the quantum numbers of elementary particle. This interaction term carries information only about the charges of elementary particle. It is quite possible that the interaction term is more complex: for instance, it could contain spin dependent terms (Stern-Gerlach experiment).

7. The constraint coming from preferred extremal property of Kähler action can be expressed in terms of Lagrange multipliers

$$S_c = \int_{Y^2} \lambda^k D_\alpha(\frac{\partial L_K}{\partial_\alpha h^k})\sqrt{g_2}d^2y \ . \quad (4.4.3)$$

8. The action exponential reads as

$$exp(iS_G + S_{braid} + S_c) \ . \tag{4.4.4}$$

The resulting field equations couple stringy M^4 degrees of freedom to the second variation of Kähler action with respect to M^4 coordinates and involve third derivatives of M^4 coordinates at the right hand side. If the second variation of Kähler action with respect to M^4 coordinates vanishes, free string results. This is trivially the case if a vacuum extremal of Kähler action is in question.

9. An interesting question is whether the preferred extremal property boils down to the condition that the second variation of Kähler action with respect to M^4 coordinates or actually all coordinates vanishes so that gravitonic string is free. As a matter fact, the stronger condition is required that the Noether currents associated with the modified Dirac action are conserved. The physical interpretation would be in terms of quantum criticality which is the basic conjecture about the dynamics of quantum TGD. This is clear from the fact that in 1-D system criticality means that the potential $V(x) = ax+bx^2+..$ has $b = 0$. In field theory criticality corresponds to the vanishing of the term $m^2\phi^2/2$ so that massless situation corresponds to massless theory and criticality and long range correlations. For more than one dynamical variable there is a hierarchy of criticalities corresponding to the gradual reduction of the rank of the matrix of the matrix defined by the second derivatives of $V(x)$ and this gives rise to a classification of criticalities. Maximum criticality would correspond to the total vanishing of this matrix. In infinite-D case this hierarchy is infinite.

What does the equality of gravitational and inertial masses mean?

Consider next the question in what form Equivalence Principle could be realized in this framework.

1. Coset construction inspires the conjecture that gravitational and inertial four-momenta are identical. Also some milder form of it would make sense. What is clear is that the construction of preferred extremal involving the distribution of $M^2(x)$ implies that conserved four-momentum associated with Kähler action can be expressed formally as stringy four-momentum. The integral of the conserved inertial momentum current over X^3 indeed reduces to an integral over the curve defining string as one integrates over other two degrees of freedom. It would not be surprising if a stringy expression for four-momentum would result but with string tension depending on the point of string and possibly also on the component of four-momentum. If the dependence of string tension on the point of string and on the choice of the stringy world sheet is slow, the interpretation could be in terms of coupling constant evolution associated with the stringy coordinates. An alternative interpretation is that string tension corresponds to a scalar field. A quite reasonable option is that for given X^3_l T defines a scalar field and that the observed T corresponds to the average value of T over deformations of X^3_l.

2. The minimum option is that Kähler mass is equal to the sum gravitational masses assignable to strings connecting points of wormhole throat or two different wormhole throats. This hypothesis makes sense even for wormhole contacts having size of order Planck length.

3. The condition that gravitational mass equals to the inertial mass (rest energy) assigned to Kähler action is the most obvious condition that one can imagine. The breaking of Poincare invariance to Lorentz invariance with respect to the tip of CD supports this form of Equivalence Principle. This would predict the value of the ratio of the parameter R^2T and p-adic length scale hypothesis would allow only discrete values for this parameter. $p \simeq 2^k$ following from the quantization of the temporal distance $T(n)$ between the tips of CD as $T(n) = 2^nT_0$ would suggest string tension $T_n = 2^nR^2$ apart from a numerical factor. $G_p \propto 2^nR^2/\hbar_0$ would emerge as a prediction of the theory. G can be seen either as a prediction or RG invariant input parameter fixed by quantum criticality. The arguments related to p-adic coupling constant evolution suggest $R^2/\hbar_0G = 3\times 2^{23}$ [20, 29].

4. The scalar field property of string tension should be consistent with the vacuum degeneracy of Kähler action. For instance, for the vacuum extremals of Kähler action stringy action is non-vanishing. The simplest possibility is that one includes the integral of the scalar $J^{\mu\nu}J_{\mu\nu}$ over the degrees transversal to M^2 to the stringy action so that string tension vanishes for vacuum extremals. This would be nothing but dimensional reduction of 4-D theory to a 2-D theory using the slicing of $X^4(X_l^3)$ to partonic 2-surfaces and stringy word sheets. For cosmic strings Kähler action reduces to stringy action with string tension $T \propto 1/g_K^2 R^2$ apart from a numerical constant. If one wants consistency with $T \propto 1/L_p^2$, one must have $T \propto 1/g_K^2 2^n R^2$ for the cosmic strings deformed to Kähler magnetic flux tubes. This looks rather plausible if the thickness of deformed string in M^4 degrees of freedom is given by p-adic length scale.

4.5 Does modified Dirac action define the fundamental action principle?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The Dirac determinant associated with the modified Dirac action is an excellent candidate in this respect.

The original working hypothesis was that Dirac determinant defines the vacuum functional of the theory having interpretation as the exponent of Kähler function of world of classical worlds (WCW) expressible and that Kähler function reduces to Kähler action for a preferred extremal of Kähler action.

4.5.1 What are the basic equations of quantum TGD?

A good place to start is to as what might the basic equations of quantum TGD. There are two kinds of equations at the level of space-time surfaces.

1. Purely classical equations define the dynamics of the space-time sheets as preferred extremals of Kähler action. Preferred extremals are quantum critical in the sense that second variation vanishes for critical deformations representing zero modes. This condition guarantees that corresponding fermionic currents are conserved. There is infinite hierarchy of these currents and they define fermionic counterparts for zero modes. Space-time sheets can be also regarded as hyper-quaternionic surfaces. What these statements precisely mean has become clear only during this year. A rigorous proof for the equivalence of these two identifications is still lacking.

2. The purely quantal equations are associated with the representations of various super-conformal algebras and with the modified Dirac equation. The requirement that there are deformations of the space-time surface -actually infinite number of them- giving rise to conserved fermionic charges implies quantum criticality at the level of Kähler action in the sense of critical deformations. The precise form of the modified Dirac equation is not however completely fixed without further input. Quantal equations involve also generalized Feynman rules for M-matrix generalizing S-matrix to a "complex square root" of density matrix and defined by time-like entanglement coefficients between positive and negative energy parts of zero energy states is certainly the basic goal of quantum TGD.

3. The notion of weak electric-magnetic duality generalizing the notion of electric-magnetic duality [47, 41] leads to a detailed understanding of how TGD reduces to almost topological quantum field theory [47, 41]. If Kähler current defines Beltrami flow [55] it is possible to find a gauge in which Coulomb contribution to Kähler action vanishes so that it reduces to Chern-Simons term. If light-like 3-surfaces and ends of space-time surface are extremals of Chern-Simons action also effective 2-dimensionality is realized. The condition that the theory reduces to almost topological QFT and the hydrodynamical character of field equations leads to a detailed ansatz for the general solution of field equations and also for the solutions of the modified Dirac equation relying on the notion of Beltrami flow for which the flow parameter associated with the flow lines defined by a conserved current extends to a global coordinate. This makes the theory is in well-defined sense completely integrable. Direct connection with massless theories

emerges: every conserved Beltrami currents corresponds to a pair of scalar functions with the first one satisfying massless d'Alembert equation in the induced metric. The orthogonality of the gradients of these functions allows interpretation in terms of polarization and momentum directions. The Beltrami flow property can be also seen as one aspect of quantum criticality since the conserved currents associated with critical deformations define this kind of pairs.

4. The hierarchy of Planck constants provides also a fresh view to the quantum criticality. The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. It however seems that TGD actually predicts this hierarchy of covering spaces. The extreme non-linearity of the field equations defined by Kähler action means that the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is 1-to-many. This leads naturally to the introduction of the covering space of $CD\times CP_2$, where CD denotes causal diamond defined as intersection of future and past directed light-cones.

At the level of WCW there is the generalization of the Dirac equation which can be regarded as a purely classical Dirac equation. The modified Dirac operators associated with quarks and leptons carry fermion number but the Dirac equations are well-defined. An orthogonal basis of solutions of these Dirac operators define in zero energy ontology a basis of zero energy states. The M-matrices defining entanglement between positive and negative energy parts of the zero energy state define what can be regarded as analogs of thermal S-matrices. The M-matrices associated with the solution basis of the WCW Dirac equation define by their orthogonality unitary U-matrix between zero energy states. This matrix finds the proper interpretation in TGD inspired theory of consciousness. WCW Dirac equation as the analog of super-Virasoro conditions for the "gamma fields" of superstring models defining super counterparts of Virasoro generators was the main focus during earlier period of quantum TGD but has not received so much attention lately and will not be discussed in this chapter.

4.5.2 Quantum criticality and modified Dirac action

The precise mathematical formulation of quantum criticality has remained one of the basic challenges of quantum TGD. The question leading to a considerable progress in the problem was simple: Under what conditions the modified Dirac action allows to assign conserved fermionic currents with the deformations of the space-time surface? The answer was equally simple: These currents exists only if these deformations correspond to vanishing second variations of Kähler action - which is what criticality is. The vacuum degeneracy of Kähler action strongly suggests that the number of critical deformations is always infinite and that these deformations define an infinite inclusion hierarchy of super-conformal algebras. This inclusion hierarchy would correspond to a fractal hierarchy of breakings of super-conformal symmetry generalizing the symmetry breaking hierarchies of gauge theories. These super-conformal inclusion hierarchies would realize the inclusion hierarchies for hyper-finite factors of type II_1.

Quantum criticality and fermionic representation of conserved charges associated with second variations of Kähler action

It is rather obvious that TGD allows a huge generalizations of conformal symmetries. The development of the understanding of conservation laws has been slow. Modified Dirac action provides excellent candidates for quantum counterparts of Noether charges. Unfortunately, the isometry charges vanish for Cartan algebras. The only manner to obtain non-trivial isometry charges is to add a direct coupling to the charges in Cartan algebra as will be found later. This addition involves Chern-Simons Dirac action so that the original intuition guided by almost TQFT idea was not wrong after all.

1. Conservation of the fermionic current requires the vanishing of the second variation of Kähler action

1. The modified Dirac action assigns to a deformation of the space-time surface a conserved charge expressible as bilinears of fermionic oscillator operators only if the first variation of the modified

Dirac action under this deformation vanishes. The vanishing of the first variation for the modified Dirac action is equivalent with the vanishing of the second variation for the Kähler action. This can be seen by the explicit calculation of the second variation of the modified Dirac action and by performing partial integration for the terms containing derivatives of Ψ and $\overline{\Psi}$ to give a total divergence representing the difference of the charge at upper and lower boundaries of the causal diamond plus a four-dimensional integral of the divergence term defined as the integral of the quantity

$$\begin{aligned} \Delta S_D &= \overline{\Psi}\Gamma^k D_\alpha J^\alpha_k \Psi \ , \\ J^\alpha_k &= \frac{\partial^2 L_K}{\partial h^k_\alpha \partial h^l_\beta}\delta h^k_\beta + \frac{\partial^2 L_K}{\partial h^k_\alpha \partial h^l}\delta h^l \ . \end{aligned} \tag{4.5.1}$$

Here h^k_β denote partial derivative of the imbedding space coordinate with respect to space-time coordinates. This term must vanish:

$$D_\alpha J^\alpha_k = 0 \ .$$

The condition states the vanishing of the second variation of Kähler action. This can of course occur only for preferred deformations of X^4. One could consider the possibility that these deformations vanish at light-like 3-surfaces or at the boundaries of CD. Note that covariant divergence is in question so that J^α_k does not define conserved classical charge in the general case.

2. It is essential that the modified Dirac equation holds true so that the modified Dirac action vanishes: this is needed to cancel the contribution to the second variation coming from the determinant of the induced metric. The condition that the modified Dirac equation is satisfied for the deformed space-time surface requires that also Ψ suffers a transformation determined by the deformation. This gives

$$\delta\Psi = -\frac{1}{D}\times \Gamma^k J^\alpha_k \Psi \ . \tag{4.5.2}$$

Here $1/D$ is the inverse of the modified Dirac operator defining the counterpart of the fermionic propagator.

3. The fermionic conserved currents associated with the deformations are obtained from the standard conserved fermion current

$$J^\alpha = \overline{\Psi}\Gamma^\alpha\Psi \ . \tag{4.5.3}$$

Note that this current is conserved only if the space-time surface is extremal of Kähler action: this is also needed to guarantee Hermiticity and same form for the modified Dirac equation for Ψ and its conjugate as well as absence of mass term essential for super-conformal invariance [56, 57]. Note also that ordinary divergence rather only covariant divergence of the current vanishes.

The conserved currents are expressible as sums of three terms. The first term is obtained by replacing modified gamma matrices with their increments in the deformation keeping Ψ and its conjugate constant. Second term is obtained by replacing Ψ with its increment $\delta\Psi$. The third term is obtained by performing same operation for $\delta\overline{\Psi}$.

$$J^\alpha = \overline{\Psi}\Gamma^k J_k^\alpha \Psi + \overline{\Psi}\hat{\Gamma}^\alpha \delta\Psi + \delta\overline{\Psi}\hat{\Gamma}^\alpha\Psi \ . \quad (4.5.4)$$

These currents provide a representation for the algebra defined by the conserved charges analogous to a fermionic representation of Kac-Moody algebra [53].

4. Also conserved super charges corresponding to super-conformal invariance are obtained. The first class of super currents are obtained by replacing Ψ or $\overline{\Psi}$ right-handed neutrino spinor or its conjugate in the expression for the conserved fermion current and performing the above procedure giving two terms since nothing happens to the covariantly constant right handed-neutrino spinor. Second class of conserved currents is defined by the solutions of the modified Dirac equation interpreted as c-number fields replacing Ψ or $\overline{\Psi}$ and the same procedure gives three terms appearing in the super current.

5. The existence of vanishing of second variations is analogous to criticality in systems defined by a potential function for which the rank of the matrix defined by second derivatives of the potential function vanishes at criticality. Quantum criticality becomes the prerequisite for the existence of quantum theory since fermionic anti-commutation relations in principle can be fixed from the condition that the algebra in question is equivalent with the algebra formed by the vector fields defining the deformations of the space-time surface defining second variations. Quantum criticality in this sense would also select preferred extremals of Kähler action as analogs of Bohr orbits and the the spectrum of preferred extremals would be more or less equivalent with the expected existence of infinite-dimensional symmetry algebras.

2. About the general structure of the algebra of conserved charges

Some general comments about the structure of the algebra of conserved charges are in order.

1. Any Cartan algebra of the isometry group $P \times SU(3)$ (there are two types of them for P corresponding to linear and cylindrical Minkowski coordinates) defines critical deformations (one could require that the isometries respect the geometry of CD). The corresponding charges are conserved but vanish since the corresponding conjugate coordinates are cyclic for the Kähler metric and Kähler form so that the conserved current is proportional to the gradient of a Killing vector field which is constant in these coordinates. Therefore one cannot represent isometry charges as fermionic bilinears. Four-momentum and color quantum numbers are defined for Kähler action as classical conserved quantities but this is probably not enough. This can be seen as a problem.

   (a) Four-momentum and color Cartan algebra emerge naturally in the representations of super-conformal algebras. In the case of color algebra the charges in the complement of the Cartan algebra can be constructed in standard manner as extension of those for the Cartan algebra using free field representation of Kac-Moody algebras. In string theories four-momentum appears linearly in bosonic Kac-Moody generators and in Sugawara construction [48] of super Virasoro generators as bilinears of bosonic Kac-Moody generators and fermionic super Kac-Moody generators [53]. Also now quantized transversal parts for M^4 coordinates could define a second quantized field having interpretation as an operator acting on spinor fields of WCW. The angle coordinates conjugate to color isospin and hyper charge take the role of M^4 coordinates in case of CP_2.

   (b) Somehow one should be able to feed the information about the super-conformal representation of the isometry charges to the modified Dirac action by adding to it a term coupling fermionic current to the Cartan charges in general coordinate invariant and isometry invariant manner. As will be shown later, this is possible. The interpretation is as measurement interaction guaranteeing also the stringy character of the fermionic propagators. The values of the couplings involved are fixed by the condition of quantum criticality assumed in the sense that Kähler function of WCW suffers only a $U(1)$ gauge transformation $K \to K + f + \overline{f}$, where f is a holomorphic function of WCW coordinates depending also on zero modes.

(c) The simplest addition involves the modified gamma matrices defined by a Chern-Simon term at the light-like wormhole throats and is sum of Chern-Simons Dirac action and corresponding coupling term linear in Cartan charges assignable to the partonic 2-surfaces at the ends of the throats. Hence the modified Dirac equation in the interior of the space-time sheet is not affected and nothing changes as far as quantum criticality in interior is considered.

2. The action defined by four-volume gives a first glimpse about what one can expect. In this case modified gamma matrices reduce to the induced gamma matrices. Second variations satisfy d'Alembert type equation in the induced metric so that the analogs of massless fields are in question. Mass term is present only if some dimensions are compact. The vanishing of excitations at light-like boundaries is a natural boundary condition and might well imply that the solution spectrum could be empty. Hence it is quite possible that four-volume action leads to a trivial theory.

3. For the vacuum extremals of Kähler action the situation is different. There exists an infinite number of second variations and the classical non-determinism suggests that deformations vanishing at the light-like boundaries exist. For the canonical imbedding of M^4 the equation for second variations is trivially satisfied. If the CP_2 projection of the vacuum extremal is one-dimensional, the second variation contains a on-vanishing term and an equation analogous to massless d'Alembert equation for the increments of CP_2 coordinates is obtained. Also for the vacuum extremals of Kähler action with 2-D CP_2 projection all terms involving induced Kähler form vanish and the field equations reduce to d'Alembert type equations for CP_2 coordinates. A possible interpretation is as the classical analog of Higgs field. For the deformations of non-vacuum extremals this would suggest the presence of terms analogous to mass terms: these kind of terms indeed appear and are proportional to δs^k. M^4 degrees of freedom decouple completely and one obtains QFT type situation.

4. The physical expectation is that at least for the vacuum extremals the critical manifold is infinite-dimensional. The notion of finite measurement resolution suggests infinite hierarchies of inclusions of hyper-finite factors of type II_1 possibly having interpretation in terms of inclusions of the super conformal algebras defined by the critical deformations.

5. The properties of Kähler action give support for this expectation. The critical manifold is infinite-dimensional in the case of vacuum extremals. Canonical imbedding of M^4 would correspond to maximal criticality analogous to that encountered at the tip of the cusp catastrophe. The natural guess would be that as one deforms the vacuum extremal the previously critical degrees of freedom are transformed to non-critical ones. The dimension of the critical manifold could remain infinite for all preferred extremals of the Kähler action. For instance, for cosmic string like objects any complex manifold of CP_2 defines cosmic string like objects so that there is a huge degeneracy is expected also now. For CP_2 type vacuum extremals M^4 projection is arbitrary light-like curve so that also now infinite degeneracy is expected for the deformations.

3. Critical super algebra and zero modes

The relationship of the critical super-algebra to configuration space geometry is interesting.

1. The vanishing of the second variation plus the identification of Kähler function as a Kähler action for preferred extremals means that the critical variations are orthogonal to all deformations of the space-time surface with respect to the configuration space metric and thus correspond to zero modes. This conforms with the fact that configuration space metric vanishes identically for canonically imbedded M^4. Zero modes do not seem to correspond to gauge degrees of freedom so that the super-conformal algebra associated with the zero modes has genuine physical content.

2. Since the action of X^4 local Hamiltonians of $\delta M^4_\times CP_2$ corresponds to the action in quantum fluctuating degrees of freedom, critical deformations cannot correspond to this kind of Hamiltonians.

3. The notion of finite measurement resolution suggests that the degrees of freedom which are below measurement resolution correspond to vanishing gauge charges. The sub-algebras of critical super-conformal algebra for which charges annihilate physical states could correspond to this kind of gauge algebras.

4. The conserved super charges associated with the vanishing second variations cannot give configuration space metric as their anti-commutator. This would also lead to a conflict with the effective 2-dimensionality stating that the configuration space line-element is expressible as sum of contribution coming from partonic 2-surfaces as also with fermionic anti-commutation relations.

4. Connection with quantum criticality

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical. Basic example of criticality is bifurcation diagram for cusp catastrophe. For some mysterious reason I failed to realize that quantum criticality realized as the vanishing of the second variation makes possible a more or less unique identification of preferred extremals and considered alternative identifications such as absolute minimization of Kähler action which is just the opposite of criticality. Both the super-symmetry of D_K and conservation Dirac Noether currents for modified Dirac action have thus a connection with quantum criticality.

1. Finite-dimensional critical systems defined by a potential function $V(x^1, x^2, ..)$ are characterized by the matrix defined by the second derivatives of the potential function and the rank of system classifies the levels in the hierarchy of criticalities. Maximal criticality corresponds to the complete vanishing of this matrix. Thom's catastrophe theory classifies these hierarchies, when the numbers of behavior and control variables are small (smaller than 5). In the recent case the situation is infinite-dimensional and the criticality conditions give additional field equations as existence of vanishing second variations of Kähler action.

2. The vacuum degeneracy of Kähler action allows to expect that this kind infinite hierarchy of criticalities is realized. For a general vacuum extremal with at most 2-D CP_2 projection the matrix defined by the second variation vanishes because $J_{\alpha\beta} = 0$ vanishes and also the matrix $(J^\alpha_{\ k} + J_k^{\ \alpha})(J^\beta_{\ l} + J_l^{\ \beta})$ vanishes by the antisymmetry $J^\alpha_{\ k} = -J_k^{\ \alpha}$. Recall that the formulation of Equivalence Principle in string picture demonstrated that the reduction of stringy dynamics to that for free strings requires that second variation with respect to M^4 coordinates vanish. This condition would guarantee the conservation of fermionic Noether currents defining gravitational four-momentum and other Poincare quantum numbers but not those for gravitational color quantum numbers. Encouragingly, the action of CP_2 type vacuum extremals having random light-like curve as M^4 projection have vanishing second variation with respect to M^4 coordinates (this follows from the vanishing of Kähler energy momentum tensor, second fundamental form, and Kähler gauge current). In this case however the momentum is vanishing.

3. Conserved bosonic and fermionic Noether charges would characterize quantum criticality. In particular, the isometries of the imbedding space define conserved currents represented in terms of the fermionic oscillator operators if the second variations defined by the infinitesimal isometries vanish for the modified Dirac action. For vacuum extremals the dimension of the critical manifold is infinite: maybe there is hierarchy of quantum criticalities for which this dimension decreases step by step but remains always infinite. This hierarchy could closely relate to the hierarchy of inclusions of hyper-finite factors of type II_1. Also the conserved charges associated with Super-symplectic and Super Kac-Moody algebras would require infinite-dimensional critical manifold defined by the spectrum of second variations.

4. Phase transitions are characterized by the symmetries of the phases involved with the transitions, and it is natural to expect that dynamical symmetries characterize the hierarchy of quantum criticalities. The notion of finite quantum measurement resolution based on the hierarchy of Jones inclusions indeed suggests the existence of a hierarchy of dynamical gauge symmetries characterized by gauge groups in ADE hierarchy [20] with degrees of freedom below the measurement resolution identified as gauge degrees of freedom.

5. A breakthrough in understanding of the criticality was the discovery that the realization that the hierarchy of singular coverings of $CD \times CP_2$ needed to realize the hierarchy of Planck constants could correspond directly to a similar hierarchy of coverings forced by the factor that classical canonical momentum densities correspond to several values of the time derivatives of the imbedding space coordinates led to a considerable progress if the understanding of the relationship between criticality and hierarchy of Planck constants [34, 40]. Therefore the problem which led to the geometrization program of quantum TGD, also allowed to reduce the hierarchy of Planck constants introduced on basis of experimental evidence to the basic quantum TGD. One can say that the 3-surfaces at the ends of CD *resp.* wormhole throats are critical in the sense that they are unstable against splitting to n_b *resp.* n_a surfaces so that one obtains space-time surfaces which can be regarded as surfaces in $n_a \times n_b$ fold covering of $CD \times CP_2$. This allows to understand why Planck constant is effectively replaced with $n_a n_b \hbar_0$ and explains charge fractionization.

Preferred extremal property as classical correlate for quantum criticality, holography, and quantum classical correspondence

The Noether currents assignable to the modified Dirac equation are conserved only if the first variation of the modified Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X^3_l)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago! The question whether these extremals correspond to absolute minima remains however open.

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X^3_l)$ vanishing at the intersections of $X^4(X^3_l)$ wth the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X^3_l with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of configuration space represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the configuration space metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to

the holographic correspondence $X^2 \to X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X^3_l)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X^3_l involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

5. There is a possible connection with the notion of self-organized criticality [49] introduced to explain the behavior of systems like sand piles. Self-organization in these systems tends to lead "to the edge". The challenge is to understand how system ends up to a critical state, which by definition is unstable. Mechanisms for this have been discovered and based on phase transitions occurring in a wide range of parameters so that critical point extends to a critical manifold. In TGD Universe quantum criticality suggests a universal mechanism of this kind. The criticality for the preferred extremals of Kähler action would mean that classically all systems are critical in well-defined sense and the question is only about the degree of criticality. Evolution could be seen as a process leading gradually to increasingly critical systems. One must however distinguish between the criticality associated with the preferred extremals of Kähler action and the criticality caused by the spin glass like energy landscape like structure for the space of the maxima of Kähler function.

4.5.3 Handful of problems with a common resolution

Theory building could be compared to pattern recognition or to a solving a crossword puzzle. It is essential to make trials, even if one is aware that they are probably wrong. When stares long enough to the letters which do not quite fit, one suddenly realizes what one particular crossword must actually be and it is soon clear what those other crosswords are. In the following I describe an example in which this analogy is rather concrete. Let us begin by listing the problems.

1. The condition that modified Dirac action allows conserved charges leads to the condition that the symmetries in question give rise to vanishing second variations of Kähler action. The interpretation is as quantum criticality and there are good arguments suggesting that the critical symmetries define an infinite-dimensional super-conformal algebra forming an inclusion hierarchy related to a sequence of symmetry breakings closely related to a hierarchy of inclusions of hyper-finite factors of types II_1 and III_1. This means an enormous generalization of the symmetry breaking patterns of gauge theories.

   There is however a problem. For the translations of M^4 and color hyper charge and isospin (more generally, any Cartan algebra of $P \times SU(3)$) the resulting fermionic charges vanish. The trial for the crossword in absence of nothing better would be the following argument. By the abelianity of these charges the vanishing of quantal representation of four-momentum and color Cartan charges is not a problem and that classical representation of these charges or their super-conformal representation is enough.

2. Modified Dirac equation is satisfied in the interior of space-time surface always. This means that one does not obtain off-mass shell propagation at all in 4-D sense. Effective 2-dimensionality suggests that off mass shell propagation takes place along wormhole throats. The reduction to almost topological QFT with Kähler function reducing to Chern-Simonst type action implied by the weak form of electric-magnetic duality and a proper gauge choice for the induced Kähler gauge potential implies effective 3-dimensionality at classical level. This inspires the question whether Chern-Simons type action resulting from an instanton term could define the modified gamma matrices appearing in the 3-D modified Dirac action associated with wormhole throats and the ends of the space-time sheet at the boundaries of CD.

   The assumption that modified Dirac equation is satisfied also at the ends and wormhole throats would realize effective 2-dimensionality as conditions on the boundary values of the 4-D Dirac equation but would would not allow off mass shell propagation. Therefore one could argue that effective 2-dimensionality in this sense holds true only for incoming and outgoing particles.

The reduction of Kähler action to Chern-Simons term together with effective 2-dimensionality suggests that Kähler function corresponds to an extremum of this action with a constraint term due to the weak form of electric-magnetic duality. Without this term the extrema of Chern-Simons action have 2-D CP_2 projection not consistent with the weak form of electric-magnetic duality. The extrema are not maxima of Kähler function: they are obtained by varying with respect to tangent space data of the partonic 2-surfaces. Lagrange multiplier term induces also to the modified gamma matrices a contribution which is of the same general form as for any general coordinate invariant action.

3. Quantum classical correspondence requires that the geometry of the space-time sheet should correlate with the quantum numbers characterizing positive (negative) energy part of the quantum state. One could argue that by multiplying WCW spinor field by a suitable phase factor depending on the charges of the state, the correspondence follows from stationary phase approximation. This crossword looks unconvincing. A more precise connection between quantum and classical is required.

4. In quantum measurement theory classical macroscopic variables identified as degrees of freedom assignable to the interior of the space-time sheet correlate with quantum numbers. Stern Gerlach experiment is an excellent example of the situation. The generalization of the imbedding space concept by replacing it with a book like structure implies that imbedding space geometry at given page and for given causal diamond (CD) carries information about the choice of the quantization axes (preferred plane M^2 of M^4 *resp.* geodesic sphere of CP_2 associated with singular covering/factor space of CD *resp.* CP_2). This is a big step but not enough. Modified Dirac action as such does not seem to provide any hint about how to achieve this correspondence. One could even wonder whether dissipative processes or at least the breaking of T and CP characterizing the outcome of quantum jump sequence should have space-time correlate. How to achieve this?

Each of these problems makes one suspect that something is lacking from the modified Dirac action: there should exist an elegant manner to feed information about quantum numbers of the state to the modified Dirac action in turn determining vacuum functional as an exponent Kähler function identified as Kähler action for the preferred extremal assumed to be dictated by by quantum criticality and equivalently by hyper-quaternionicity.

This observation leads to what might be the correct question. Could a general coordinate invariant and Poincare invariant modification of the modified Dirac action consistent with the vacuum degeneracy of Kähler action allow to achieve this information flow somehow? In the following one manner to achieve this modification is discussed. It must be however emphasized that I have considered many alternatives and the one discussed below finds its justification only from the fact that it is the simplest one found hitherto.

The identification of the measurement interaction term

The idea is simple: add to the modified Dirac action a term which is analogous to the Dirac action in $M^4 \times CP_2$. One can consider two options according to whether the term is assigned with interior or with a 3-D light-like 3-surface and last years have been continual argumentation about which option is the correct one.

1. The additional term would be essentially the analog of the ordinary Dirac action at the imbedding space level.

$$\begin{aligned}
S_{int} &= \sum_A Q_A \int \overline{\Psi} g^{AB} j_{B\alpha} \hat{\Gamma}^{\alpha} \Psi \sqrt{g} d^4x \ , \\
g_{AB} &= j^k_A h_{kl} j^l_B \ , \ g^{AB} g_{BC} = \delta^A_C \ , \\
j_{B\alpha} &= j^k_B h_{kl} \partial_\alpha h^l \ .
\end{aligned} \tag{4.5.5}$$

The sum is over isometry charges Q_A interpreted as quantal charges and j^{Ak} denotes the Killing vector field of the isometry. g^{AB} is the inverse of the tensor g_{AB} defined by the local inner products of Killing vectors fields in M^4 and CP_2. The space-time projections of the Killing vector fields $j_{B\alpha}$ have interpretation as classical color gauge potentials in the case of $SU(3)$. In M^4 degrees of freedom and for Cartan algebra of $SU(3)$ $j_{B\alpha}$ reduce to the gradients of linear M^4 coordinates in case of translations. Modified gamma matrices could be assigned to Kähler action or its instanton term or with Chern-Simons action.

2. The added term containing quantal charges must make sense in the modified Dirac equation. This requires that the physical state is an eigenstate of momentum and color charges. This allows only color hyper-charge and color isospin so that there is no hope of obtaining exactly the stringy formula for the propagator. The modified Dirac operator is given by

$$\begin{aligned} D &= D + D_{int} = \hat{\Gamma}^{\alpha} D_{\alpha} + \hat{\Gamma}^{\alpha} \sum_A Q_A g^{AB} j_{B\alpha} \\ &= \hat{\Gamma}^{\alpha}(D_{\alpha} + \partial_{\alpha}\phi) \ , \ \partial_{\alpha}\phi = \sum_A Q_A g^{AB} j_{B\alpha} \ . \end{aligned} \tag{4.5.6}$$

The conserved fermionic isometry currents are

$$J^{A\alpha} = \sum_B Q_B \overline{\Psi} g^{BC} j^k_C h_{kl} j^l_A \hat{\Gamma}^{\alpha} \Psi = Q_A \overline{\Psi} \hat{\Gamma}^{\alpha} \Psi \ . \tag{4.5.7}$$

Here the sum is restricted to a Cartan sub-algebra of Poincare group and color group.

3. An important restriction is that by four-dimensionality of M^4 and CP_2 the rank of g_{AB} is 4 so that g^{AB} exists only when one considers only four conserved charges. In the case of M^4 this is achieved by a restriction to translation generators $Q_A = p_A$. g_{AB} reduces to Minkowski metric and Killing vector fields are constants. The Cartan sub-algebra could be however replaced by any four commuting charges in the case of Poincare algebra (second one corresponds to time translation plus translation, boost and rotation in given direction). In the case of $SU(3)$ one must restrict the consideration either to $U(2)$ sub-algebra or its complement. $CP_2 = SU(3)/SU(2)$ decomposition would suggest the complement as the correct choice. One can indeed build the generators of $U(2)$ as commutators of the charges in the complement. On the other hand, Cartan algebra is enough in free field construction of Kac-Moody algebras.

4. What is remarkable that for the Cartan algebra of $M^4 \times SU(3)$ the measurement interaction term is equivalent with the addition of gauge part $\partial_{\alpha}\phi$ of the induced Kähler gauge potential A_{α}. This property might hold true for any measurement interaction term. This also suggests that the change in Kähler function is only the transformation $A_{\alpha} \rightarrow A_{\alpha} + \partial_{\alpha}\phi$, $\partial_{\alpha}\phi = \sum_A Q_A g^{AB} j_{B\alpha}$.

5. Recall that the ϕ for $U(1)$ gauge transformations respecting the vanishing of the Coulomb interaction term of Kähler action [34, 40] the current $j^{\alpha}_K \phi$ is conserved, which implies that the change of the Kähler action is trivial. These properties characterize the gauge transformations respecting the gauge in which Coulombic interaction term of the Kähler action vanishes so that Kähler action reduces to 3-dimensional generalized Chern-Simons term if the weak form of electric-magnetic duality holds true guaranteeing among other things that the induced Kähler field is not too singular at the wormhole throats [34, 40]. The scalar function assignable to the measurement interaction terms does not have this property and this is what is expected since it must change the value of the Kähler function and therefore affect the preferred extremal.

Concerning the precise form of the modified Dirac action the basic clue comes from the observation that the measurement interaction term corresponds to the addition of a gauge part to the induced CP_2 Kähler gauge potential A_{α}. The basic question is what part of the action one assigns the measurement interaction term.

1. One could define the measurement interaction term using either the four-dimensional instanton term or its reduction to Chern-Simons terms. The part of Dirac action defined by the instanton term in the interior does not reduce to a 3-D form unless the Dirac equation defined by the instanton term is satisfied : this cannot be true. Hence Chern-Simons term is the only possibility.

   The classical field equations associated with the Chern-Simons term cannot be assumed since they would imply that the CP_2 projection of the wormhole throat and space-like 3-surface are 2-dimensional. This might hold true for space-like 3-surfaces at the ends of CD and incoming and outgoing particles but not for off mass shell particles. This is however not a problem since $D_\alpha \hat{\Gamma}^\alpha_{C-S}$ for the modified gamma matrices for Chern-Simons action does not contain second derivatives. This is due to the topological character of this term. For Kähler action second derivatives are present and this forces extremal property of Kähler action in the modified Dirac Kähler action so that classical physics results as a consistency condition.

2. If one assigns measurement interaction term to both D_K and D_{C-S} the measurement interaction corresponds to a mere gauge transformation for AS_α and is trivial. Therefore it seems that one must choose between D_K or D_{C-S}. At least formally the measurement interaction term associated with D_K is gauge equivalent with its negative D_{C-S}. The addition of the measurement interaction to D_K changes the basis for the 4-D induced spinors by the phase $exp(-iQK\phi)$ and therefore also the basis for the generalized eigenstates of D_{C-S} and this brings in effectively the measurement interaction term affecting the Dirac determinant.

3. The definition of Dirac determinant should be in terms of Chern-Simons action induced by the instanton term and identified as a product of the generalized eigenvalues of this operator. The modified Dirac equation for Ψ is consistent with that for its conjugate if the coefficient of the instanton term is real and one uses the Dirac action $\overline{\Psi}(D^{\rightarrow} - D^{\leftarrow})\Psi$ giving modified Dirac equation as

$$D_{C-S}\Psi + \frac{1}{2}(D_\alpha \hat{\Gamma}^\alpha_{C-S})\Psi \quad = \quad 0 \ . \tag{4.5.8}$$

   As noticed, the divergence of gamma matrices does not contain second derivatives in the case of Chern-Simons action. In the case of Kähler action they occur unless field equations equivalent with the vanishing of the divergence term are satisfied.

   Also the fermionic current is conserved in this case, which conforms with the idea that fermions flow along the light-like 3-surfaces. If one uses the action $\overline{\Psi}D^{\rightarrow}\Psi$, $\overline{\Psi}$ does not satisfy the Dirac equation following from the variational principle and fermion current is not conserved. Also if the Chern-Simons term is imaginary - as a naive idea about dissipation would suggest- the Dirac equation fails to be consistent with the conjugation.

4. Off mass shell states appear in the lines of the generalized Feynman diagrams and for these D_{C-S} cannot annihilate the spinor field. The generalized eigen modes lf D_{C-S} should be such that one obtains the counterpart of Dirac propagator which is purely algebraic and does not therefore depend on the coordinates of the throat. This is satisfied if the generalized eigenvalues are expressible in terms of covariantly constant combinations of gamma matrices and here only M^4 gamma matrices are possible. Therefore the eigenvalue equation reqards as

$$D\Psi \quad = \quad \lambda^k \gamma_k \Psi \ \ , \ \ D = D_{C-S} + D_\alpha \hat{\Gamma}^\alpha_{C-S} \ \ , \ \ D_{C-S} = \hat{\Gamma}^\alpha_{C-S} D_\alpha \ \ . \tag{4.5.9}$$

   Here the covariant derivatives D_α contain the measurement interaction term as an apparent gauge term. Covariant constancy allows to take the square of this equation and one has

$$(D^2 + \left[D, \lambda^k \gamma_k\right])\Psi + \quad = \quad \lambda^k \lambda_k \Psi \ \ . \tag{4.5.10}$$

The commutator term is analogous to magnetic moment interaction. The generalized eigenvalues correspond to $\lambda = \sqrt{\lambda^k \lambda_k}$ and Dirac determinant is defined as a product of the eigenvalues. λ is completely analogous to mass. For incoming lines this mass would vanish so that all incoming particles irrespective their actual quantum numbers would be massless in this sense and the propagator is indeed that for a massless particle. Note that the eigen modes define the boundary values for the solutions of $D_K \Psi = 0$ so that the values of λ indeed define the counterpart of the momentum space.

This transmutation of massive particles to effectively massless ones might make possible the application of the twistor formalism as such in TGD framework [28]. $N = 4$ SUSY is one of the very few gauge theory which might be UV finite but it is definitely unphysical due to the masslessness of the basic quanta. Could the resolution of the interpretational problems be that the four-momenta appearing in this theory do not directly correspond to the observed four-momenta?

Objections

The alert reader has probably raised several critical questions. Doesn't the need to solve λ_k as functions of incoming quantum numbers plus the need to construct the measurement interactions makes the practical application of the theory hopelessly difficult? Could the resulting pseudo-momentum λ_k correspond to the actual four-momentum? Could one drop the measurement interaction term altogether and assume that the quantum classical correspondence is through the identification of the eigenvalues as the four-momenta of the on mass shell particles propagating at the wormhole throats? Could one indeed assume that the momenta have a continuous spectrum and thus do not depend on the boundary conditions at all? Usually the thinking is just the opposite and in the general case would lead to to singular eigen modes.

1. Only the information about four-momentum would be fed into the space-time geometry. TGD however allows much more general measurement interaction terms and it would be very strange if the space-time geometry would not correlate also with the other quantum numbers. Mass formulas would of course contain information also about other quantum numbers so that this claim is not quite justified.

2. Number theoretic considerations and also the construction of octonionic variant of Dirac equation [19, 40] force the conclusion that the spectrum of pseudo four-momentum is restricted to a preferred plane M^2 of M^4 and this excludes the interpretation of λ^k as a genuine four-momentum. It also improves the hopes that the sum over pseudo-momenta does not imply divergences.

3. Dirac determinant would depend on the mass spectrum only and could not be identified as exponent of Kähler function. Note that the original guideline was the dream about stringy propagators. This is achieved for $\lambda_A \lambda^A = n$ in suitable units. This spectrum would of course also imply that Dirac determinant defined in terms of ζ function regularization is independent of the space-time surface and could not be identified with the exponent of Kähler function. One must of course take the identification of exponent of Kähler function as Dirac determinant as an additional conjecture which is not necessary for the calculation of Kähler function if the weak form of electric-magnetic duality is accepted.

4. All particles would behave as massless particles and this would not be consistent with the proposed Feynman diagrammatics inspired by zero energy ontology. Since wormhole throats carry on mass shell particles with positive or negative energy so that the net momentum can be also space-like propagators diverge for massless particles. One might overcome this problem by assuming small thermal mass (from p-adic thermodynamics [4]) and this is indeed assumed to reduce the number of generalized Feynman diagrams contributing to a given reaction to finite number.

Second objection of the skeptic reader relates to the delicacies of $U(1)$ gauge invariance. The modified Dirac action seems to break gauge symmetries and this breaking of gauge symmetry is absolutely essential for the dependence of the Dirac determinant on the quantum numbers. It however seems that this breaking of gauge invariance is only apparent.

1. One must distinguish between genuine $U(1)$ gauge transformations carried out for the induced Kähler gauge potential A_α and apparent gauge transformations of the Kähler gauge potential A_k of $S^2 \times CP_2$ induced by symplectic transformations deforming the space-time surface and affect also induced metric. This delicacy of $U(1)$ gauge symmetry explains also the apparent breaking of $U(1)$ gauge symmetry of Chern-Simons Dirac action due to the presence of explicit terms A_k and A_α.

2. CP_2 Kähler gauge potential is obtained in complex coordinates from Kähler function as $(K_{\xi^i}, K_{\overline{\xi^i}}) = (\partial_{\xi^i}K, -\partial_{\overline{\xi^i}}K)$. Gauge transformations correspond to the additions $K \to K + f + \overline{f}$, where f is a holomorphic function. Kähler gauge potential has a unique gauge in which the Kähler function of CP_2 is $U(2)$ invariant and contains no holomorphic part. Hence A_k is defined in a preferred gauge and is a gauge invariant quantity in this sense. Same applies to S^2 part of the Kähler potential if present.

3. A_α should be also gauge invariant under gauge transformation respecting the vanishing of Coulombic interaction energy. The allowed gauge transformations $A_\alpha \to A_\alpha + \partial_\alpha\phi$ must satisfy $D_\alpha(j^\alpha_K\phi) = 0$. If the scalar function ϕ reduces to constant at the wormhole throats and at the ends of the space-time surface D_{C-S} is gauge invariant. The gauge transformations for which ϕ does not satisfy this condition are identified as representations of critical deformations of space-time surface so that the change of A_α would code for this kind of deformation and indeed affect the modified Dirac operator and Kähler function (the change would be due to the change of zero modes).

Some details about the modified Dirac equation defined by Chern-Simons action

First some general comments about D_{C-S} are in order.

1. Quite generally, there is vacuum avoidance in the sense that Ψ must vanish in the regions where the modified gamma matrices vanish. A physical analogy for the system consider is a charged particle in an external magnetic field. The effective metric defined by the anti-commutators of the modified gamma matrices so that standard intuitions might not help much. What one would naively expect would be analogs of bound states in magnetic field localized into regions inside which the magnetic field is non-vanishing.

2. If only CP_2 Kähler form appears in the Kähler action, the modified Dirac action defined by the Chern-Simons term is non-vanishing only when the dimension of the CP_2 projection of the 3-surface is $D(CP_2) \geq 2$ and the induced Kähler field is non-vanishing. This conforms with the properties of Kähler action. The solutions of the modified Dirac equation with a vanishing eigenvalue λ would naturally correspond to incoming and outgoing particles.

3. $D(CP_2) \leq 2$ is apparently inconsistent with the weak form of electric-magnetic duality requiring $D(CP_2) = 3$. The conclusion is wrong: the variations of Chern-Simons action are subject to the constraint that electric-magnetic duality holds true expressible in terms of Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma}J_{\beta\gamma})\sqrt{g_4}d^3x \quad . \tag{4.5.11}$$

   This gives a constraint force to the field equations and also a dependence on the induced 4-metric so that one has only almost topological QFT. This term also guarantees the M^4 part of WCW Kähler metric is non-trivial. The condition that the ends of space-time sheet and wormhole throats are extrema of Chern-Simons action subject to the electric-magnetic duality constraint is strongly suggested by the effective 2-dimensionality.

4. Electric-magnetic duality constraint gives an additional term to the Dirac action determined by the Lagrange multiplier term. This term gives an additional contribution to the modified gamma matrices having the same general form as coming from Kähler action and Chern-Simons action. In the following this term will not be considered. For the extremals it only affects the modified gamma matrices and leaves the general form of solutions unchanged.

In absence of the constraint from the weak form of electric-magnetic duality the explicit expression of D_{C-S} is given by

$$\begin{aligned}
D &= \hat{\Gamma}^\mu D_\mu + \frac{1}{2} D_\mu \hat{\Gamma}^\mu \ , \\
\hat{\Gamma}^\mu &= \frac{\partial L_{C-S}}{\partial_\mu h^k}\Gamma_k = \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_\mu \ , \\
D_\mu \hat{\Gamma}^\mu &= B^\alpha_K (J_{k\alpha} + \partial_\alpha A_k) \ , \\
B^\alpha_K &= \epsilon^{\alpha\beta\gamma} J_{\beta\gamma} \ , \ J_{k\alpha} = J_{kl}\partial_\alpha s^l \ , \ \hat{\epsilon}^{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma}\sqrt{g_3} \ .
\end{aligned} \tag{4.5.12}$$

Note $\hat{\epsilon}^{\alpha\beta\gamma} =$ does not depend on the induced metric.

The extremals of Chern-Simons action without constraint term satisfy

$$B^\alpha_K (J_{kl} + \partial_l A_k)\partial_\alpha h^l = 0 \ , \ B^\alpha_K = \epsilon^{\alpha\beta\gamma} J_{\beta\gamma} \ . \tag{4.5.13}$$

For a non-vanishing Kähler magnetic field B^α these equations hold true when CP_2 projection is 2-dimensional. This implies a vanishing of Chern-Simons action in absence of the constraint term realizing electric-magnetic duality, which is therefore absolutely essential in order for having a non-vanishing WCW metric.

Consider now the situation in more detail.

1. Suppose that one can assign a global coordinate to the flow lines of the Kähler magnetic field. In this case one might hope that ordinary intuitions about motion in constant magnetic field might be helpful. The repetition of the discussion of [34, 40] leads to the condition $B \wedge dB = 0$ implying that a Beltrami flow for which current flows along the field lines and Lorentz forces vanishes is in question. This need not be the generic case.

2. With this assumption the modified Dirac operator reduces to a one-dimensional Dirac operator

$$D = \hat{\epsilon}^{r\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_r \ . \tag{4.5.14}$$

3. The general solutions of the modified Dirac equation is covariantly constant with respect to the coordinate r:

$$D_r \Psi = 0 \ . \tag{4.5.15}$$

   The solution to this condition can be written immediately in terms of a non-integrable phase factor $Pexp(i\int A_r dr)$, where integration is along curve with constant transversal coordinates. If $\hat{\Gamma^v}$ is light-like vector field also $\hat{\Gamma^v}\Psi_0$ defines a solution of D_{C-S}. This solution corresponds to a zero mode for D_{C-S} and does not contribute to the Dirac determinant. Note that the dependence of these solutions on transversal coordinates of X^3_l is arbitrary.

4. The formal solution associated with a general eigenvalue can be constructed by integrating the eigenvalue equation separately along all coordinate curves. This makes sense if r indeed assigned to light-like curves indeed defines a global coordinate. What is strange that there is no correlation between the behaviors with respect longitudinal coordinate and transversal coordinates. System would be like a collection of totally uncorrelated point like particles reflecting the flow of the current along flux lines. It is difficult to say anything about the spectrum of the generalized eigenvalues in this case: it might be that the boundary conditions at the ends of the flow lines fix the allowed values of λ. Clearly, the Beltrami flow property is what makes this case very special.

A connection with quantum measurement theory

It is encouraging that isometry charges and also other charges could make themselves visible in the geometry of space-time surface as they should by quantum classical correspondence. This suggests an interpretation in terms of quantum measurement theory.

1. The interpretation resolves the problem caused by the fact that the choice of the commuting isometry charges is not unique. Cartan algebra corresponds naturally to the measured observables. For instance, one could choose the Cartan algebra of Poincare group to consist of energy and momentum, angular momentum and boost (velocity) in particular direction as generators of the Cartan algebra of Poincare group. In fact, the choices of a preferred plane $M^2 \subset M^4$ and geodesic sphere $S^2 \subset CP_2$ allowing to fix the measurement sub-algebra to a high degree are implied by the replacement of the imbedding space with a book like structure forced by the hierarchy of Planck constants. Therefore the hierarchy of Planck constants seems to be required by quantum measurement theory. One cannot overemphasize the importance of this connection.

2. One can add similar couplings of the net values of the measured observables to the currents whose existence and conservation is guaranteed by quantum criticality. It is essential that one maps the observables to Cartan algebra coupled to critical current characterizing the observable in question. The coupling should have interpretation as a replacement of the induced Kähler gauge potential with its gauge transform. Quantum classical correspondence encourages the identification of the classical charges associated with Kähler action with quantal Cartan charges. This would support the interpretation in terms of a measurement interaction feeding information to classical space-time physics about the eigenvalues of the observables of the measured system. The resulting field equations remain second order partial differential equations since the second order partial derivatives appear only linearly in the added terms.

3. What about the space-time correlates of electro-weak charges? The earlier proposal explains this correlation in terms of the properties of quantum states: the coupling of electro-weak charges to Chern-Simons term could give the correlation in stationary phase approximation. It would be however very strange if the coupling of electro-weak charges with the geometry of the space-time sheet would not have the same universal description based on quantum measurement theory as isometry charges have.

    (a) The hint as how this description could be achieved comes from a long standing un-answered question motivated by the fact that electro-weak gauge group identifiable as the holonomy group of CP_2 can be identified as $U(2)$ subgroup of color group. Could the electro-weak charges be identified as classical color charges? This might make sense since the color charges have also identification as fermionic charges implied by quantum criticality. Or could electro-weak charges be only represented as classical color charges by mapping them to classical color currents in the measurement interaction term in the modified Dirac action? At least this question might make sense.

    (b) It does not make sense to couple both electro-weak and color charges to the same fermion current. There are also other fundamental fermion currents which are conserved. All the following currents are conserved.

    $$\begin{aligned} J^\alpha &= \overline{\Psi} O \hat{\Gamma}^\alpha \Psi \\ O &\in \{1 \ , \ J \equiv J_{kl}\Sigma^{kl} \ , \ \Sigma_{AB} \ , \ \Sigma_{AB} J\} \ . \end{aligned} \tag{4.5.16}$$

    Here J_{kl} is the covariantly constant CP_2 Kähler form and Σ_{AB} is the (also covariantly) constant sigma matrix of M^4 (flatness is absolutely essential).

    (c) Electromagnetic charge can be expressed as a linear combination of currents corresponding to $O = 1$ and $O = J$ and vectorial isospin current corresponds to J. It is natural to couple of electromagnetic charge to the the projection of Killing vector field of color hyper charge and coupling it to the current defined by $O_{em} = a+bJ$. This allows to interpret the puzzling finding that electromagnetic charge can be identified as anomalous color hyper-charge for induced spinor fields made already during the first years of TGD. There exist no conserved axial isospin currents in accordance with CVC and PCAC hypothesis which belong to the basic stuff of the hadron physics of old days.

(d) Color charges would couple naturally to lepton and quark number current and the $U(1)$ part of electro-weak charges to the $n = 1$ multiple of quark current and $n = 3$ multiple of the lepton current (note that leptons *resp.* quarks correspond to $t = 0$ *resp.* $t = \pm 1$ color partial waves). If electro-weak *resp.* couplings to H-chirality are proportional to 1 *resp.* Γ_9, the fermionic currents assigned to color and electro-weak charges can be regarded as independent. This explains why the possibility of both vectorial and axial couplings in 8-D sense does not imply the doubling of gauge bosons.

(e) There is also an infinite variety of conserved currents obtained as the quantum critical deformations of the basic fermion currents identified above. This would allow in principle to couple an arbitrary number of observables to the geometry of the space-time sheet by mapping them to Cartan algebras of Poincare and color group for a particular conserved quantum critical current. Quantum criticality would therefore make possible classical space-time correlates of observables necessary for quantum measurement theory.

(f) The coupling constants associated with the deformations would appear in the couplings. Quantum criticality ($K \rightarrow K + f + \overline{f}$ condition) should predict the spectrum of these couplings. In the case of momentum the coupling would be proportional to $\sqrt{G/\hbar_0} = kR/\hbar_0$ and $k \sim 2^{11}$ should follow from quantum criticality. p-Adic coupling constant evolution should follow from the dependence on the scale of CD coming as powers of 2.

4. Quantum criticality implies fluctuations in long length and time scales and it is not surprising that quantum criticality is needed to produce a correlation between quantal degrees of freedom and macroscopic degrees of freedom. Note that quantum classical correspondence can be regarded as an abstract form of entanglement induced by the entanglement between quantum charges Q_A and fermion number type charges assignable to zero modes.

5. Space-time sheets can have an arbitrary number of wormhole contacts so that the interpretation in terms of measurement theory coupling short and long length scales suggests that the measurement interaction terms are localizable at the wormhole throats. This would favor Chern-Simons term or possibly instanton term if reducible to Chern-Simons terms. The breaking of CP and T might relate to the fact that state function reductions performed in quantum measurements indeed induce dissipation and breaking of time reversal invariance.

6. The experimental arrangement quite concretely splits the quantum state to a quantum superposition of space-time sheets such that each eigenstate of the measured observables in the superposition corresponds to different space-time sheet already before the realization of state function reduction. This relates interestingly to the question whether state function reduction really occurs or whether only a branching of wave function defined by WCW spinor field takes place as in multiverse interpretation in which different branches correspond to different observers. TGD inspired theory consciousness requires that state function reduction takes place. Maybe multiversalist might be able to find from this picture support for his own beliefs.

7. One can argue that "free will" appears not only at the level of quantum jumps but also as the possibility to select the observables appearing in the modified Dirac action dictating in turn the Kähler function defining the Kähler metric of WCW representing the "laws of physics". This need not to be the case. The choice of CD fixes M^2 and the geodesic sphere S^2: this does not fix completely the choice of the quantization axis but by isometry invariance rotations and color rotations do not affect Kähler function for given CD and for a given type of Cartan algebra. In M^4 degrees of freedom the possibility to select the observables in two manners corresponding to linear and cylindrical Minkowski coordinates could imply that the resulting Kähler functions are different. The corresponding Kähler metrics do not differ if the real parts of the Kähler functions associated with the two choices differ by a term $f(Z) + \overline{f(Z)}$, where Z denotes complex coordinates of WCW, the Kähler metric remains the same. The function f can depend also on zero modes. If this is the case then one can allow in given CD superpositions of WCW spinor fields for which the measurement interactions are different. This condition is expected to pose non-trivial constraints on the measurement action and quantize coupling parameters appearing in it.

New view about gravitational mass and matter antimatter asymmetry

The physical interpretation of the additional term in the modified Dirac action might force quite a radical revision of the ideas about matter and antimatter.

1. The term $p_A \partial_\alpha m^A$ contracted with the fermion current is analogous to a gauge potential coupling to fermion number. Since the additional terms in the modified Dirac operator induce stringy propagation, a natural interpretation of the coupling to the induced spinor fields is in terms of gravitation. One might perhaps say that the measurement of four momentum induces gravitational interaction. Besides momentum components also color charges take the role of gravitational charges. As a matter fact, any observable takes this role via coupling to the projections of Killing vector fields of Cartan algebra. The analogy of color interactions with gravitational interactions is indeed one of the oldest ideas in TGD.

2. The coupling to four-momentum is through fermion number (both quark number and lepton number). For states with a vanishing fermion number isometry charges therefore vanish. In this framework matter antimatter asymmetry would be due to the fact that matter (antimatter) corresponds to positive (negative) energy parts of zero energy states for massive systems so that the contributions to the net gravitational four-momentum are of same sign. Could antimatter be unobservable to us because it resides at negative energy space-time sheets? As a matter fact, I proposed already years ago that gravitational mass is essentially the magnitude of the inertial mass but gave up this idea.

3. Bosons do not couple at all to gravitation if they are purely local bound states of fermion and anti-fermion at the same space-time sheet (say represented by generators of super Kac-Moody algebra). Therefore the only possible identification of gauge bosons is as wormhole contacts. If the fermion and anti-fermion at the opposite throats of the contact correspond to positive and negative energy states the net gravitational energy receives a positive contribution from both sheets. If both correspond to positive (negative) energy the contributions to the net four-momentum have opposite signs. It is not yet clear which identification is the correct one.

4.5.4 Generalized eigenvalues of D_{C-S} and General Coordinate Invariance

The fixing of light-like 3-surface to be the wormhole throat at which the signature of induced metric changes from Minkowskian to Euclidian corresponds to a convenient fixing of gauge. General Coordinate Invariance however requires that any light-like surface Y_l^3 parallel to X_l^3 in the slicing is equally good choice. In particular, it should give rise to same Kähler metric but not necessarily the same exponent of Kähler function identified as the product of the generalized eigenvalues of $D_{C,S}$ at Y_l^3.

General Coordinate Invariance requires that the components of Kähler metric of configuration space defined in terms of Kähler function as

$$G_{k\bar{l}} = \partial_k \partial_{\bar{l}} K = \sum_i \partial_k \partial_{\bar{l}} \lambda_i$$

remain invariant under this flow. Here complex coordinate are of course associated with the configuration space. This is the case if the flow corresponds to the addition of sum of holomorphic function $f(z)$ and its conjugate $\overline{f}(z))$ which is anti-holomorphic function to K. This boils down to the scaling of eigenvalues λ_i by

$$\lambda_i \rightarrow exp(f_i(z) + \overline{f_i(z)})\lambda_i \quad . \tag{4.5.17}$$

If the eigenvalues are interpreted as vacuum conformal weights, general coordinate transformations correspond to a spectral flow scaling the eigenvalues in this manner. This in turn would induce spectral flow of ground state conformal weights if the squares of λ_i correspond to ground state conformal weights.

4.6 Representations for the configuration space gamma matrices in terms of super-symplectic charges at light cone boundary

During years I have considered several variants for the representation of WCW gamma matrices and each of these proposals has had some weakness.

1. One question has been whether the Noether currents assignable to WCW Hamiltonians should play any role in the construction or whether one can use only the generalization of flux Hamiltonians. Magnetic flux Hamiltonians do not refer to the space-time dynamics implying genuine 2-dimensionality, which is a catastrophe. If the sum of the magnetic and electric flux Hamiltonians and the weak form of self duality is assumed effective 2-dimensionality is achieved. The challenge is to identify the super-partners of the flux Hamiltonians and postulate correct anti-commutation relations for the induced spinor fields to achieve anti-commutation to flux Hamiltonians.

2. In the original proposal for WCW gamma matrices the covariantly constant right handed spinors played a key role. This led to interpretational problems with quarks. Are they needed at all or do leptons and quarks define somehow equivalent representations? I discovered only recently a brutally simple but deadly objection against this approach: the resulting WCW gamma matrices do not generate all WCW spinors from Fock vacuum. Therefore all modes of the induced spinor fields must be used.

The latter objection forced to realize that nothing is changed if one replaces the covariantly constant right handed neutrino with the collection of quark spinor modes q_n *resp.* leptonic spinor modes L_n multiplied by the contractions $J_{A+} = j^{Ak}\Gamma_k$ *resp.* its conjugate $J_{A-} = j^{A\overline{k}}\Gamma_{\overline{k}}$. It is essential that only of these contractions is used for a given H-chirality.

1. If the anti-commutator of the spinor fields is or form $J = J_{\alpha\beta}\epsilon^{\alpha\beta}\delta^2(x, y)$ at X^2 for magnetic flux Hamiltonians and appropriate generalization of this fro the sum of magnetic and electric flux Hamiltonians, the "half-Poisson bracket" $\partial_k H_A J^{k\overline{l}}\partial_{\overline{l}} H_B$ from the quark spinor field and its conjugate as anti-commutator from the leptonic spinor field can combine to the full Poisson bracket if the remaining factors are identical.

2. This happens if the quark modes and lepton-like modes are in 1-1 correspondence and the contractions of the eigenmodes resulting in the contraction satisfy $\overline{q}_m\gamma^0 q_n = \overline{L}_m\gamma^0 L_n = \Phi_{mn}$. The resulting Hamiltonians define an X^2-local algebra: that this extension is needed became obvious already earlier. A stronger condition is that the spinors can be expressed in terms of scalar function bases $\{\Phi_m\}$ so that one would have $q_{m,i} = \{\Phi_m\}q_i$ and $L_{m,i} = \{\Phi_m\}L_i$ so that one would assign to the super-currents the local Hamiltonians $\Phi_m H_A$.

3. One could of course still argue that it is questionable to use sum of quark and lepton gamma matrices since this the resulting objects to not have a well defined fermion number and cannot be used to generate physical states from vacuum. How seriously this argument should be taken is not clear to me at this moment. One could of course consider also a scenario in which one divides leptonic (or quark) modes to two classes analogous to quark and lepton modes and uses J_{A_+} *resp.* J_{A_-} for these two classes.

In any case, the recent view is that all modes of the induced spinor fields must be used, that lepton-quark degeneracy is absolutely essential for the construction of WCW geometry, and that the original super-symmetrization of the flux Hamiltonians combined with weak electric-magnetic duality is the correct approach. There are also fermionic Noether charges and their super counterparts implied by the criticality but these can be assigned with zero modes.

This section represents both the earlier version of the construction of configuration gamma matrices and the construction introducing explicitly the notion of finite measurement resolution. The motivation for the latter option is that if the number the generalized eigen modes of modified Dirac operator is finite, strictly local anti-commutation relations fail unless one restricts the set of points included to that corresponding to number theoretic braid. In the following integral expressions for configuration space Hamiltonians and their super-counterparts are derived first. After that the motivations for replacing integrals with sums are discussed and the expressions for Hamiltonians and super Hamiltonians are derived.

4.6.1 Magnetic flux representation of the super-symplectic algebra

In order to derive representation of the configuration space gamma matrices and super charges it is good to restate the basic facts about the magnetic flux representation of the configuration space gamma matrices using the original approach based on 2-dimensional integrals.

4.6.2 Quantization of the modified Dirac action and configuration space geometry

The quantization of the modified Dirac action involves a fusion of various number theoretical ideas. The naive approach would be based on standard canonical quantization of induced spinor fields by posing anti-commutation relations between Ψ and canonical momentum density $\partial L/\partial(\partial_t\Psi)$.

Generalized magnetic and electric fluxes

Isometry invariants are just a special case of fluxes defining natural coordinate variables for the configuration space. Canonical transformations of CP_2 act as $U(1)$ gauge transformations on the Kähler potential of CP_2 (similar conclusion holds at the level of $\delta M^4_+ \times CP_2$).

One can generalize these transformations to local symplectic transformations by allowing the Hamiltonians to be products of the CP_2 Hamiltonians with the real and imaginary parts of the functions $f_{s,n,k}$ defining the Lorentz covariant function basis H_A, $A \equiv (a,s,n,k)$ at the light cone boundary: $H_A = H_a \times f(s,n,k)$, where a labels the Hamiltonians of CP_2.

One can associate to any Hamiltonian H^A of this kind magnetic or electric flux via the following formulas:

$$Q_{m/e}(H_A|X^2) \quad = \quad \int_{X^2} H_A J_{m/e} \quad . \tag{4.6.1}$$

Here the magnetic (electric) flux J_m (J_e) denotes the flux associated with induced Kähler field and its dual which is well-defined since X^2 is part of 4-D space-time surface.

The flux Hamiltonians

$$Q_i(H_A|X^2) \quad = \quad Q_i(H_A|X^2) \quad , \quad A \equiv (a,s,n,k) \tag{4.6.2}$$

provide a representation of WCW Hamiltonians as far as the "kinetic" part of Kähler form is considered.

Anti-commutation relations between oscillator operators associated with same partonic 2-surface

The construction of WCW gamma matrices leads to the anti-commutation relations given by

$$\begin{aligned}
\{\overline{\Psi}(x)\gamma^0,\Psi(x)\} \quad &= \quad [J_e+J_m)\delta^2_{x,y} \quad , \\
J_e \quad &= \quad \int J^{03}\sqrt{g_4} \quad .
\end{aligned} \tag{4.6.3}$$

Kähler magnetic flux $J_m = \epsilon^{\alpha\beta}J_{\alpha\beta}\sqrt{g_2}$ has no dependence on the induced metric.

If the weak- form of the electric-magnetic duality holds true, Kähler electric flux relates to it via the formula

$$J^{03}\sqrt{g_4} = KJ_{12} \quad ,$$

where K is symplectic invariant and identifiable in terms of Kähler coupling strength from classical charge quantization condition for Kähler electric flux. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K gives the condition $K = g_K^2/\hbar = 4\pi\alpha_K$, where g_K is Kähler coupling constant. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant. The arguments leading to the identification $\epsilon \pm 1$ at the opposite boundaries of CD are discussed in [34, 40]. An alternative identification is as $\epsilon = 0$ but predicts that WCW is trivial in M^4 degrees of freedom if Kähler function reduces to Chern-Simons terms.

The general form of the anti-commutation relations is therefore

$$\{\overline{\Psi}(x)\gamma^0, \Psi(x)\} = (1+K)J\delta^2_{x,y} \ . \tag{4.6.4}$$

What is nice that at the limit of vacuum extremals the right hand side vanishes when both J and J^1 vanish so that spinor fields become non-dynamical. One can criticize the non-vanishing of the anti-commutator for vacuum extremals of Kähler action.

For the latter option the fermionic counterparts of local flux Hamiltonians can be written in the form

$$\begin{aligned}
H_{A,\pm,n} &= \epsilon_q(A,\mp,n)H_{A,\pm,q,n} + \epsilon_L(A,\pm)H_{A,\mp,L,n} \quad , \\
H_{A,+,q,n} &= \oint \overline{\Psi} J_+^A q_n d^2x \ , \\
H_{A,-,q,n} &= \oint \overline{q}_n J_-^A \Psi d^2x \ , \\
H_{A,-,L,n} &= \oint \overline{\Psi} J_+^A L_n d^2x \ , \\
H_{A,+,L,n} &= \oint \overline{L}_n J_-^A \Psi d^2x \ , \\
J_+^A &= j^{Ak}\Gamma_k \ , \ J_-^A = j^{A\overline{k}}\Gamma_{\overline{k}} \ .
\end{aligned} \tag{4.6.5}$$

The commutative parameters $\epsilon_q(A,\pm,n)$ *resp.* $\epsilon_L(A,\pm,n)$ are assumed to carry quark *resp.* lepton number opposite to that of $H_{A,\mp,q,n}$ *resp.* $H_{A,\mp,L,n}$ and satisfy $\epsilon_i(A,+,n)\epsilon_i(A,-,n) = 1$. One encounters a hierarchy discrete algebras satisfying this condition in the construction of a symplectic analog of conformal quantum field theory required by the construction of quantum TGD [5]. Associativity condition fixes uniquely the commutative multiplication of these units and analogs of plane waves with discrete momentum are in question.

Suppose that there is a one-one correspondence between quark modes and leptonic modes is satisfied and the label n decomposes as $n = (m,i)$, where n labels a scalar function basis and i labels spinor components. This would give

$$\begin{aligned}
q_n = q_{m,i} &= \Phi_m q_i \ , \\
L_n = L_{m,i} &= \Phi_m L_i \ , \\
\overline{q}_i\gamma^0 q_j &= \overline{L}_i\gamma^0 L_j = g_{ij} \ .
\end{aligned} \tag{4.6.6}$$

Suppose that the inner products g_{ij} are constant. The simplest possibility is $g_{ij} = \delta_{ij}$ Under these assumptions the anti-commutators of the super-symmetric flux Hamiltonians give flux Hamiltonians.

$$\{H_{A,+,n}, H_{A,-,n}\} = g_{ij}\oint \overline{\Phi}_m \Phi_n H_A J d^2x \ . \tag{4.6.7}$$

The product of scalar functions can be expressed as

$$\overline{\Phi}_m\Phi_n = c_{mn}^{\ \ k}\Phi_k \ . \tag{4.6.8}$$

Note that the notion of symplectic QFT [16] led to a scalar function algebra of similar kind consisting of phase factors and there excellent reasons to consider the possibility that there is a deep connection with this approach.

One expects that the symplectic algebra is restricted to a direct sum of symplectic algebras localized to the regions where the induced Kähler form is non-vanishing implying that the algebras associated with different region form to a direct sum. Also the contributions to configuration space metric are direct sums. The symplectic algebras associated with different region can be truncated to finite-dimensional spaces of symplectic algebras associated with the regions in question. As far as coordinatization of the reduced configuration space is considered, these symplectic sub-spaces are enough. These truncated algebras naturally correspond to the hyper-finite factor property of the Clifford algebra of configuration space.

Generalization of WCW Hamiltonians and anti-commutation relations between flux Hamiltonians belonging to different ends of CD

This picture requires a generalization of the view about configuration space Hamiltonians since also the interaction term between the ends of the line is present not taken into account in the previous approach.

1. The proposed representation of WCW Hamiltonians as flux Hamiltonians [22, 43, 18]

$$Q(H_A) \quad = \quad \int H_A J d^2x \ . \tag{4.6.9}$$

   works for the kinetic terms only since J is not expectred to be the same at the ends of the line.

   The assumption that Poisson bracket of WCW Hamiltonians reduces to the level of imbedding space - in other words $\{Q(H_A), Q(H_B\} = Q(\{H_A, H_B\})$ - can be justified. One starts from the representation in terms of say flux Hamiltonians $Q(H_A)$ and *defines* $J_{A,B}$ as $J_{A,B} \equiv Q(\{H_A, H_B\})$. One has $\partial H_A/\partial t_B = \{H_B, H_A\}$, where t_B is the parameter associated with the exponentiation of H_B. The inverse J^{AB} of $J_{A,B} = \partial H_B/\partial t_A$ is expressible as $J^{A,B} = \partial t_A/\partial H_B$. From these formulas one can deduce by using chain rule that the bracket $\{Q(H_A), Q(H_B\} = \partial t_C Q(H_A) J^{CD} \partial t_D Q(H_B)$ of flux Hamiltonians equals to the flux Hamiltonian $Q(\{H_A, H_B\})$.

2. One should be able to assign to WCW Hamiltonians also a part corresponding to the interaction term. The symplectic conjugation associated with the interaction term permutes the WCW coordinates assignable to the ends of the line. One should reduce this apparently non-local symplectic conjugation (if one thinks the ends of line as separate objects) to a non-local symplectic conjugation for $\delta CD \times CP_2$ by identifying the points of lower and upper end of CD related by time reflection and assuming that conjugation corresponds to time reflection. Formally this gives a well defined generalization of the local Poisson brackets between time reflected points at the boundaries of CD. The connection of Hermitian conjugation and time reflection in quantum field theories is is in accordance with this picture.

3. Perhaps the only manner to proceed is to assign to the flux Hamiltonian also a part obtained by the replacement of the flux integral over X^2 with an integral over the projection of X^2 to a sphere S^2 assignable to the light-cone boundary or to a geodesic sphere of CP_2, which come as two varieties corresponding to homologically trivial and non-trivial spheres. The projection is defined as by the geodesic line orthogonal to S^2 and going through the point of X^2. The hierarchy of Planck constants assigns to CD a preferred geodesic sphere of CP_2 as well as a unique sphere S^2 as a sphere for which the radial coordinate r_M or the light-cone boundary defined uniquely is constant: this radial coordinate corresponds to spherical coordinate in the rest system defined by the time-like vector connecting the tips of CD. Either spheres or possibly both of them could be relevant.

   Recall that also the construction of number theoretic braids and symplectic QFT [16] led to the proposal that braid diagrams and symplectic triangulations could be defined in terms of projections of braid strands to one of these spheres. One could also consider a weakening for the condition that the points of the number theoretic braid are algebraic by requiring only that the S^2 coordinates of the projection are algebraic and that these coordinates correspond to the discretization of S^2 in terms of the phase angles associated with θ and ϕ.

This gives for the corresponding contribution of the WCW Hamiltonian the expression

$$Q(H_A)_{int} = (1+K)\int_{S^2_{\pm}} H_A X\delta^2(s_+,s_-)d^2s_{\pm} = (1+K)\int_{P(X^2_+)\cap P(X^2_-)} \frac{\partial(s^1,s^2)}{\partial(x^1_{\pm},x^2_{\pm})} d^2 s_{\pm} \qquad (4.6.10)$$

Here the Poisson brackets between ends of the line using the rules involve delta function $\delta^2(s_+,s_-)$ at S^2 and the resulting Hamiltonians can be expressed as a similar integral of $H_{[A,B]}$ over the upper or lower end since the integral is over the intersection of S^2 projections.

The expression must vanish when the induced Kähler form vanishes for either end. This is achieved by identifying the scalar X in the following manner:

$$\begin{aligned} X &= J^+_{kl} + J^-_{kl} \ , \\ J^{kl}_{\pm} &= \partial_\alpha s^k \partial_\beta s^l J^{\alpha\beta}_{\pm} \ . \end{aligned} \qquad (4.6.11)$$

The tensors are lifts of the induced Kähler form of $X^2_{\pm}$ to S^2 (not CP_2).

4. One could of course ask why these Hamiltonians could not contribute also to the kinetic terms and why the brackets with flux Hamiltonians should vanish. This relate to how one *defines* the Kähler form. It was shown above that in case of flux Hamiltonians the definition of Kähler form as brackets gives the basic formula $\{Q(H_A),Q(H_B)\} = Q(\{H_A,H_B\}$ and same should hold true now. In the recent case $J_{A,B}$ would contain an interaction term defined in terms of flux Hamiltonians and the previous argument should go through also now by identifying Hamiltonians as sums of two contributions and by introducing the doubling of the coordinates t_A.

5. The quantization of the modified Dirac operator must be reconsidered. It would seem that one must add to the super-Hamiltonian completely analogous term obtained by replacing J with $X\partial(s^1,s^2)/\partial(x^1_{\pm},x^2_{\pm})$. Besides the anti-commutation relations defining correct anti-commutators to flux Hamiltonians, one should pose anti-commutation relations consistent with the anti-commutation relations of super Hamiltonians. In these anti-commutation relations $J\delta^2(x,y)$ would be replaced with $X\delta^2(s^+,s^-)$. This would guarantee that the oscillator operators at the ends of the line are not independent and that the resulting Hamiltonian reduces to integral over either end for $H_{[A,B]}$.

4.6.3 Expressions for configuration space super-symplectic generators in finite measurement resolution

The expressions of configuration space Hamiltonians and their super counterparts just discussed were based on 2-dimensional integrals. This is problematic for several reasons.

1. In p-adic context integrals do not makes sense so that this representation fails in p-adic context (for pe-adic numbers see[47]). Sums would be more appropriate if one wants number theoretic universality at the level of basic formulas.

2. The use of sums would also conform with the notion of finite measurement resolution having discretization in terms of intersections of X^2 with number theoretic braids as a space-time correlate.

3. Number theoretic duality suggests a unique realization of the discretization in the sense that only the points of partonic 2-surface X^2 whose $\delta M^4_{\pm}$ projections commute in hyper-octonionic sense and thus belong to the intersections of the projection $P_{M^4}(X^2)$ with radial light-like geodesics $M_{\pm}$ representing intersections of $M^2 \subset M^4 \subset M^8$ with $\delta M^4_{\pm} \times CP_2$ contribute to the configuration space Hamiltonians and super Hamiltonians and therefore to the configuration space metric.

Clearly, finite measurement resolution seems to be an unavoidable aspect of the geometrization of the configuration space as one can expect on basis of the fact that configuration space Clifford algebra provides representation for hyper-finite factors of type II_1 whose inclusions provide a representation for the finite measurement resolution. This means that the infinite-dimensional configuration space can be represented as a finite-dimensional space in arbitrary precise approximation so that also also configuration Clifford algebra and configuration space spinor fields becomes finite-dimensional.

The modification of anti-commutation relations to this case is

$$\{\overline{\Psi}(x_m)\gamma^0, \Psi(x_n)\} = (1+K)J\delta_{x_m,x_n} \ . \tag{4.6.12}$$

Note that the constancy of γ^0 implies a complete symmetry between the two points. The number of points must be the maximal one consistent with the Kronecker delta type anti-commutation relations so that information is not lost.

The question arises about the choice of the points x_m. This choice should general coordinate invariant. The number theoretic vision leads to the notion of number theoretic braid defined as the set of points common to real and p-adic variant of X^2. The points of the number theoretic braid are excellent candidates for points x_n. The p-adic variant exists only if X^2 is defined by rational functions with coefficients which are possibly algebraic and thus make sense both in real and p-adic sense. These points belong to the algebraic extension of rational numbers appearing in the representation of X^2 as an algebraic surface but one can consider quite generally the possibility that the points of the number theoretic braid are rational or in a finite algebraic extension of rationals. What is important that if one restricts the consideration to rational points this criterion makes sense even if X^2 is not algebraic. In the generic case one can expect that the number of these points is finite.

4.6.4 Configuration space geometry and hierarchy of inclusions of hyper-finite factors of II_1

The configuration space metric defined as anti-commutators of the configuration space gamma matrices is extremely degenerate since it effectively corresponds to a quadratic form in N-dimensional space, where N_m is the total number of the eigenmodes of D_K. Since two Hamiltonians whose values and corresponding Killing vector fields co-incide at the points of B are equivalent for given ray $M_\pm$, it is natural to pose a cutoff in the number of Hamiltonians used for the representation of reduced configuration space in given region inside which induced Kähler form is non-vanishing. The natural manner to pose this cutoff is by ordering the representations with respect to dimension and eigenvalue of Casimir operator for the irreducible representations of $SO(3) \times SO(4)$ in case of M^8 and for the representations of $SO(3) \times SU(3)$ in case of H.

This boils down to a hierarchy of approximate representations of the configuration space as Kähler manifold with spinor structure with a truncation of the Clifford algebra to a finite dimensional Clifford algebra. This is in spirit with the proposed interpretation of the inclusion sequence of hyper-finite factors of type II_1 and with the very notion of hyper-finiteness. A surprisingly concrete connection of the configuration space geometry with generalized eigenvalue spectrum of $D_K(X^3)$ and basic quantum physics results. For instance, from the general expression of Kähler metric in terms of Kähler function

$$G_{k\bar{l}} = \partial_k\partial_{\bar{l}}K = \frac{\partial_k\partial_{\bar{l}}exp(K)}{exp(K)} - \frac{\partial_k exp(K)}{exp(K)}\frac{\partial_{\bar{l}}exp(K)}{exp(K)} \ , \tag{4.6.13}$$

and from the expression of $exp(K) = \prod_i \lambda_i$ as the product of of finite number of eigenvalues of $D_K(X^3)$, the expression

$$G_{k\bar{l}} = \sum_i \frac{\partial_k\partial_{\bar{l}}\lambda_i}{\lambda_i} - \frac{\partial_k\lambda_i}{\lambda_i}\frac{\partial_{\bar{l}}\lambda_i}{\lambda_i} \tag{4.6.14}$$

for the configuration space metric follows. Here complex coordinates refer to the complex coordinates of configuration space.

A good candidate for these complex coordinates are the complex coordinates of $S^2 \times S$, $S = CP_2$ or E^4, for the points of B so that a close connection with the geometry of imbedding space is obtained. Once these coordinates have been specified G can be contracted with the Killing vector fields of configuration space isometries defining the coordinates for the truncated configuration space. By studying the behavior of eigenvalue spectrum under small deformations of X_l^3 by symplectic transformations of $\delta CD \times S$ the components of G can be estimated.

4.7 Super-conformal symmetries at space-time and configuration space level

The physical interpretation and detailed mathematical understanding of super-conformal symmetries has developed rather slowly and has involved several side tracks. In the following I try to summarize the basic picture with minimal amount of formulas with the understanding that the statement "Noether charge associated with geometrically realized Kac-Moody symmetry" is enough for the reader to write down the needed formula explicitly.

4.7.1 Configuration space as a union of symmetric spaces

In finite-dimensional context globally symmetric spaces are of form G/H and connection and curvature are independent of the metric, provided it is left invariant under G. The hope is that same holds true in infinite-dimensional context. The most one can hope of obtaining is the decomposition $C(H) = \cup_i G/H_i$ over orbits of G. One could allow also symmetry breaking in the sense that G and H depend on the orbit: $C(H) = \cup_i G_i/H_i$ but it seems that G can be chosen to be same for all orbits. What is essential is that these groups are infinite-dimensional. The basic properties of the coset space decomposition give very strong constraints on the group H, which certainly contains the subgroup of G, whose action reduces to diffeomorphisms of X^3.

Consequences of the decomposition

If the decomposition to a union of coset spaces indeed occurs, the consequences for the calculability of the theory are enormous since it suffices to find metric and curvature tensor for single representative 3-surface on a given orbit (contravariant form of metric gives propagator in perturbative calculation of matrix elements as functional integrals over the configuration space). The representative surface can be chosen to correspond to the maximum of Kähler function on a given orbit and one obtains perturbation theory around this maximum (Kähler function is not isometry invariant).

The task is to identify the infinite-dimensional groups G and H and to understand the zero mode structure of the configuration space. Almost twenty (seven according to long held belief!) years after the discovery of the candidate for the Kähler function defining the metric, it became finally clear that these identifications follow quite nicely from $Diff^4$ invariance and $Diff^4$ degeneracy as well as special properties of the Kähler action.

The guess (not the first one!) would be following. G corresponds to the symplectic transformations of $\delta M^4_\pm \times CP_2$ leaving the induced Kähler form invariant. If G acts as isometries the values of Kähler form at partonic 2-surfaces (remember effective 2-dimensionality) are zero modes and configuration space allows slicing to symplectic orbits of the partonic 2-surface with fixed induced Kähler form. Quantum fluctuating degrees of freedom would correspond to symplectic group and to the fluctuations of the induced metric. The group H dividing G would in turn correspond to the Kac-Moody symmetries respecting light-likeness of X_l^3 and acting in X_l^3 but trivially at the partonic 2-surface X^2. This coset structure was originally discovered via coset construction for super Virasoro algebras of super-symplectic and super Kac-Moody algebras and realizes Equivalence Principle at quantum level.

Configuration space isometries as a subgroup of $Diff(\delta M^4_+ \times CP_2)$

The reduction to light cone boundary leads to the identification of the isometry group as some subgroup of for the group G for the diffeomorphisms of $\delta M^4_+ \times CP_2$. These diffeomorphisms indeed act in a natural manner in δCH, the the space of 3-surfaces in $\delta M^4_+ \times CP_2$. Configuration space is expected to decompose to a union of the coset spaces G/H_i, where H_i corresponds to some subgroup of G containing the transformations of G acting as diffeomorphisms for given X^3. Geometrically the vector fields acting as diffeomorphisms of X^3 are tangential to the 3-surface. H_i could depend on the topology of X^3 and since G does not change the topology of 3-surface each 3-topology defines separate orbit of G. Therefore, the union involves sum over all topologies of X^3 plus possibly other 'zero modes'. Different topologies are naturally glued together since singular 3-surfaces intermediate between two 3-topologies correspond to points common to the two sectors with different topologies.

4.7.2 Isometries of configuration space geometry as symplectic transformations of $\delta M^4_+ \times CP_2$

During last decade I have considered several candidates for the group G of isometries of the configuration space as the sub-algebra of the subalgebra of $Diff(\delta M^4_+ \times CP_2)$. To begin with let us write the general decomposition of $diff(\delta M^4_+ \times CP_2)$:

$$diff(\delta M^4_+ \times CP_2) \quad = \quad S(CP_2) \times diff(\delta M^4_+) \oplus S(\delta M^4_+) \times diff(CP_2) \ . \qquad (4.7.1)$$

Here $S(X)$ denotes the scalar function basis of space X. This Lie-algebra is the direct sum of light cone diffeomorphisms made local with respect to CP_2 and CP_2 diffeomorphisms made local with respect to light cone boundary.

The idea that entire diffeomorphism group would act as isometries looks unrealistic since the theory should be more or less equivalent with topological field theory in this case. Consider now the various candidates for G.

1. The fact that symplectic transformations of CP_2 and M^4_+ diffeomorphisms are dynamical symmetries of the vacuum extremals suggests the possibility that the diffeomorphisms of the light cone boundary and symplectic transformations of CP_2 could leave Kähler function invariant and thus correspond to zero modes. The symplectic transformations of CP_2 localized with respect to light cone boundary acting as symplectic transformations of CP_2 have interpretation as local color transformations and are a good candidate for the isometries. The fact that local color transformations are not even approximate symmetries of Kähler action is not a problem: if they were exact symmetries, Kähler function would be invariant and zero modes would be in question.

2. CP_2 local conformal transformations of the light cone boundary act as isometries of δM^4_+. Besides this there is a huge group of the symplectic symmetries of $\delta M^4_+ \times CP_2$ if light cone boundary is provided with the symplectic structure. Both groups must be considered as candidates for groups of isometries. $\delta M^4_+ \times CP_2$ option exploits fully the special properties of $\delta M^4_+ \times CP_2$, and one can develop simple argument demonstrating that $\delta M^4_+ \times CP_2$ symplectic invariance is the correct option. Also the construction of configuration space gamma matrices as super-symplectic charges supports $\delta M^4_+ \times CP_2$ option.

This picture remained same for a long time. The discovery that Kac-Moody algebra consisting of X^2 local symmetries generated by Hamiltonians of isometry sub-algebra of symplectic algebra forced to challenge this picture and ask whether also X^2-local transformations of symplectic group could be involved.

1. The basic condition is that the X^2 local transformation acts leaves induced Kähler form invariant apart from diffeomorphism. Denote the infinitesimal generator of X^2 local symplecto morphism by $\Phi_A(x)j^{Ak}$, where A labels Hamiltonians in the sum and by j^α the generator of X^2 diffeomorphism.

2. The invariance of $J = \epsilon^{\alpha\beta}J_{\alpha\beta}\sqrt{g_2}$ modulo diffeomorphism under the infinitesimal symplectic transformation gives

$$\{H^A, \Phi_A\} \quad \equiv \quad \partial_\alpha H^A \epsilon^{\alpha\beta}\partial_\beta \Phi_A = \partial_\alpha J j^\alpha \ . \qquad (4.7.2)$$

3. Note that here the Poisson bracket is not defined by $J^\alpha\beta$ but $\epsilon^{\alpha\beta}$ defined by the induced metric. Left hand side reflects the failure of symplectomorphism property due to the dependence of $\Phi_A(x)$ on X^2 coordinate which and comes from the gradients of $\delta M^4 \times CP_2$ coordinates in the expression of the induced Kähler form. Right hand side corresponds to the action of infinitesimal diffeomorphism.

4. Let us assume that one can restrict the consideration to single Hamiltonian so that the transformation is generated by $\Phi(x)H_A$ and that to each $\Phi(x)$ there corresponds a diffeomorphism of X^2, which is a symplectic transformation of X^2 with respect to symplectic form $\epsilon^{\alpha\beta}$ and generated by Hamiltonian $\Psi(x)$. This transforms the invariance condition to

$$\{H^A, \Phi\} \equiv \partial_\alpha H^A \epsilon^{\alpha\beta}\partial_\beta \Phi = \partial_\alpha J \epsilon^{\alpha\beta}\partial_\beta \Psi_A = \{J, \Psi_A\} \ . \tag{4.7.3}$$

This condition can be solved identically by assuming that Φ_A and Ψ are proportional to arbitrary smooth function of J:

$$\Phi = f(J) \ , \quad \Psi_A = -f(J)H_A \ . \tag{4.7.4}$$

Therefore the X^2 local symplectomorphisms of H reduce to symplectic transformations of X^2 with Hamiltonians depending on single coordinate J of X^2. The analogy with conformal invariance for which transformations depend on single coordinate z is obvious. As far as the anti-commutation relations for induced spinor fields are considered this means that $J = consant$ curves behave as points points. For extrema of J appearing as candidates for points of number theoretic braids $J = constant$ curves reduce to points.

5. From the structure of the conditions it is easy to see that the transformations generate a Lie-algebra. For the transformations $\Phi^1_A H^A$ $\Phi^2_A H^A$ the commutator is

$$\Phi_A^{[1,2]} = f_A^{\ BC}\Phi_B\Phi_C \ , \tag{4.7.5}$$

where $f_A^{\ BC}$ are the structure constants for the symplectic algebra of $\delta M^4_\pm \times CP_2$. From this form it is easy to check that Jacobi identifies are satisfied. The commutator has same form as the commutator of gauge algebra generators. BRST gauge symmetry is perhaps the nearest analog of this symmetry. In the case of isometries these transforms realized local color gauge symmetry in TGD sense.

6. If space-time surface allows a slicing to light-like 3-surfaces Y^3_l parallel to X^3_l, these conditions make sense also for the partonic 2-surfaces defined by the intersections of Y^3_l with $\delta M^4_\pm \times CP_2$ and "parallel" to X^2. The local symplectic transformations also generalize to their local variants in X^3_l. Light-likeness of X^3_l means effective metric 2-dimensionality so that 2-D Kähler metric and symplectic form as well as the invariant $J = \epsilon^{\alpha\beta}J_{\alpha\beta}$ exist. A straightforward calculation shows that the the notion of local symplectic transformation makes sense also now and formulas are exactly the same as above.

4.7.3 SUSY algebra defined by the anticommutation relations of fermionic oscillator operators and WCW local Clifford algebra elements as chiral super-fields

Whether TGD allows space-time supersymmetry has been a long-standing question. Majorana spinors appear in $N = 1$ super-symmetric QFTs- in particular minimally super-symmetric standard model (MSSM). Majorana-Weyl spinors appear in M-theory and super string models. An undesirable consequence is chiral anomaly in the case that the numbers of left and right handed spinors are not same. For $D = 11$ and $D = 10$ these anomalies cancel which led to the breakthrough of string models and later to M-theory. The probable reason for considering these dimensions is that standard model does not predict right-handed neutrino (although neutrino mass suggests that right handed neutrino exists) so that the numbers of left and right handed Weyl-spinors are not the same.

In TGD framework the situation is different. Covariantly constant right-handed neutrino spinor acts as a super-symmetry in CP_2. One might think that right-handed neutrino in a well-defined sense disappears from the spectrum as a zero mode so that the number of right and left handed chiralities in $M^4 \times CP_2$ would not be same. For light-like 3-surfaces covariantly constant right-handed neutrino does not however solve the counterpart of Dirac equation for a non-vanishing four-momentum and color quantum numbers of the physical state. Therefore it does not disappear from the spectrum anymore and one expects the same number of right and left handed chiralities.

In TGD framework the separate conservation of baryon and lepton numbers excludes Majorana spinors and also the the Minkowski signature of $M^4 \times CP_2$ makes them impossible. The conclusion that TGD does not allow super-symmetry is however wrong. For $\mathcal{N} = 2N$ Weyl spinors are indeed possible and if the number of right and left handed Weyl spinors is same super-symmetry is possible. In 8-D context right and left-handed fermions correspond to quarks and leptons and since color in TGD framework corresponds to CP_2 partial waves rather than spin like quantum number, also the numbers of quark and lepton-like spinors are same.

The physical picture suggest a new kind of approach to super-symmetry in the sense that the anticommutations of fermionic oscillator operators associated with the modes of the induced spinor fields define a structure analogous to SUSY algebra. This means that $\mathcal{N} = 2N$ SUSY with large N is in question allowing spins higher than two and also large fermion numbers. Recall that $\mathcal{N} \leq 32$ is implied by the absence of spins higher than two and the number of real spinor components is $N = 32$ also in TGD. The situation clearly differs from that encountered in super-string models and SUSYs and the large value of N allows to expect very powerful constraints on dynamics irrespective of the fact that SUSY is broken. Right handed neutrino modes define a sub-algebra for which the SUSY is only slightly broken by the absence of weak interactions and one could also consider a theory containing a large number of $\mathcal{N} = 2$ super-multiplets corresponding to the addition of right-handed neutrinos and antineutrinos at the wormhole throat.

Masslessness condition is essential for super-symmetry and at the fundamental level it could be formulated in terms of modified gamma matrices using octonionic representation and assuming that they span local quaternionic sub-algebra at each point of the space-time sheet. SUSY algebra has standard interpretation with respect to spin and isospin indices only at the partonic 2-surfaces so that the basic algebra should be formulated at these surfaces. Effective 2-dimensionality would require that partonic 2-surfaces can be taken to be ends of any light-like 3-surface Y_l^3 in the slicing of the region surrounding a given wormhole throat.

Super-algebra associated with the modified gamma matrices

Anti-commutation relations for fermionic oscillator operators associated with the induced spinor fields are naturally formulated in terms of the modified gamma matrices. Super-conformal symmetry suggests that the anti-commutation relations for the fermionic oscillator operators at light-like 3-surfaces or at their ends are most naturally formulated as anti-commutation relations for SUSY algebra. The resulting anti-commutation relations would fix the quantum TGD.

$$\begin{aligned} \{a^\dagger_{n\alpha}, a_{n\beta}\} &= D_{mn} D_{\alpha\beta} \ , \\ D &= (p^\mu + \sum_a Q^\mu_a)\hat{\sigma}^\mu \ . \end{aligned} \tag{4.7.6}$$

Here p^μ and Q^μ_a are space-time projections of momentum and color charges in Cartan algebra. Their action is purely algebraic. The anti-commutations are nothing but a generalization of the ordinary equal-time anticommutation relations for fermionic oscillator operators to a manifestly covariant form. The matrix $D_{m,n}$ is expected to reduce to a diagonal form with a proper normalization of the oscillator operators. The experience with extended SUSY algebra suggest that the anti-commutators could contain additional central term proportional to $\delta_{\alpha\beta}$.

One can consider basically two different options concerning the definition of the super-algebra.

1. If the super-algebra is defined at the 3-D ends of the intersection of X^4 with the boundaries of CD, the modified gamma matrices appearing in the operator D appearing in the anti-commutator are associated with Kähler action. If the generalized masslessness condition $D^2 = 0$ holds true -as suggested already earlier- one can hope that no explicit breaking of super-symmetry takes place and elegant description of massive states as effectively massless states

making also possible generalization of twistor is possible. One must however notice that also massive representatives of SUSY exist.

2. SUSY algebra could be also defined at 2-D ends of light-like 3-surfaces.

According to considerations of [47] these options are equivalent for a large class of space-time sheets. If the effective 3-dimensionality realized in the sense that the effective metric defined by the modified gamma matrices is degenerate, propagation takes place along 3-D light-like 3-surfaces. This condition definitely fails for string like objects.

One can realize the local Clifford algebra also by introducing theta parameters in the standard manner and the expressing a collection of local Clifford algebra element with varying values of fermion numbers (function of CD and CP_2 coordinates) as a chiral super-field. The definition of a chiral super field requires the introduction of super-covariant derivatives. Standard form for the anti-commutators of super-covariant derivatives D_α make sense only if they do not affect the modified gamma matrices. This is achieved if p_k acts on the position of the tip of CD (rather than internal coordinates of the space-time sheet). Q_a in turn must act on CP_2 coordinates of the tip.

Super-fields associated with WCW Clifford algebra

WCW local Clifford algebra elements possess definite fermion numbers and it is not physically sensible to super-pose local Clifford algebra elements with different fermion numbers. The extremely elegant formulation of super-symmetric theories in terms of super-fields encourages to ask whether the local Clifford algebra elements could allow expansion in terms of complex theta parameters assigned to various fermionic oscillator operator in order to obtain formal superposition of elements with different fermion numbers. One can also ask whether the notion of chiral super field might make sense.

The obvious question is whether it makes sense to assign super-fields with the modified gamma matrices.

1. Modified gamma matrices are not covariantly constant but this is not a problem since the action of momentum generators and color generators is purely algebraic space-time coordinates.

2. One can define the notion of chiral super-field also at the fundamental level. Chiral super-field would be continuation of the local Clifford algebra of associated with CD to a local Clifford algebra element associated with the union of CDs. This would allow elegant description of cm degrees of freedom, which are the most interesting as far as QFT limit is considered.

3. Kähler function of WCW as a function of complex coordinates could be extended to a chiral super-field defined in quantum fluctuation degrees of freedom. It would depend on zero modes too. Does also the latter dependence allow super-space continuation? Coefficients of powers of theta would correspond to fermionic oscillator operators. Does this function define the propagators of various states associated with light-like 3-surface? Configuration space complex coordinates would correspond to the modes of induced spinor field so that super-symmetry would be realized very concretely.

4.7.4 Identification of Kac-Moody symmetries

The Kac-Moody algebra of symmetries acting as symmetries respecting the light-likeness of 3-surfaces plays a crucial role in the identification of quantum fluctuating configuration space degrees of freedom contributing to the metric.

Identification of Kac-Moody algebra

The generators of bosonic super Kac-Moody algebra leave the light-likeness condition $\sqrt{g_3} = 0$ invariant. This gives the condition

$$\delta g_{\alpha\beta} Cof(g^{\alpha\beta}) = 0 \ , \tag{4.7.7}$$

Here Cof refers to matrix cofactor of $g_{\alpha\beta}$ and summation over indices is understood. The conditions can be satisfied if the symmetries act as combinations of infinitesimal diffeomorphisms $x^\mu \rightarrow x^\mu + \xi^\mu$ of X^3 and of infinitesimal conformal symmetries of the induced metric

$$\delta g_{\alpha\beta} = \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha\xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \tag{4.7.8}$$

Ansatz as an X^3-local conformal transformation of imbedding space

Write δh^k as a super-position of X^3-local infinitesimal diffeomorphisms of the imbedding space generated by vector fields $J^A = j^{A,k}\partial_k$:

$$\delta h^k = c_A(x) j^{A,k} \ . \tag{4.7.9}$$

This gives

$$\begin{aligned} c_A(x) & \left[D_k j^A_l + D_l j^A_k\right] \partial_\alpha h^k \partial_\beta h^l + 2\partial_\alpha c_A h_{kl} j^{A,k}\partial_\beta h^l \\ = & \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha\xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \end{aligned} \tag{4.7.10}$$

If an X^3-local variant of a conformal transformation of the imbedding space is in question, the first term is proportional to the metric since one has

$$D_k j^A_l + D_l j^A_k = 2h_{kl} \ . \tag{4.7.11}$$

The transformations in question includes conformal transformations of $H_\pm$ and isometries of the imbedding space H.

The contribution of the second term must correspond to an infinitesimal diffeomorphism of X^3 reducible to infinitesimal conformal transformation ψ^μ:

$$2\partial_\alpha c_A h_{kl} j^{A,k}\partial_\beta h^l = \xi^\mu\partial_\mu g_{\alpha\beta} + g_{\mu\beta}\partial_\alpha\xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \tag{4.7.12}$$

A rough analysis of the conditions

One could consider a strategy of fixing c_A and solving solving ξ^μ from the differential equations. In order to simplify the situation one could assume that $g_{ir} = g_{rr} = 0$. The possibility to cast the metric in this form is plausible since generic 3-manifold allows coordinates in which the metric is diagonal.

1. The equation for g_{rr} gives

   $$\partial_r c_A h_{kl} j^{Ak}\partial_r h^k = 0 \ . \tag{4.7.13}$$

   The radial derivative of the transformation is orthogonal to X^3. No condition on ξ^α results. If c_A has common multiplicative dependence on $c_A = f(r)d_A$ by a one obtains

   $$d_A h_{kl} j^{Ak}\partial_r h^k = 0 \ . \tag{4.7.14}$$

   so that J^A is orthogonal to the light-like tangent vector $\partial_r h^k$ X^3 which is the counterpart for the condition that Kac-Moody algebra acts in the transversal degrees of freedom only. The condition also states that the components g_{ri} is not changed in the infinitesimal transformation.

   It is possible to choose $f(r)$ freely so that one can perform the choice $f(r) = r^n$ and the notion of radial conformal weight makes sense. The dependence of c_A on transversal coordinates is constrained by the transversality condition only. In particular, a common scale factor having free dependence on the transversal coordinates is possible meaning that X^3- local conformal transformations of H are in question.

2. The equation for g_{ri} gives

$$\partial_r \xi^i = \partial_r c_A h_{kl} j^{Ak} h^{ij} \partial_j h^k \ . \tag{4.7.15}$$

The equation states that g_{ri} are not affected by the symmetry. The radial dependence of ξ^i is fixed by this differential equation. No condition on ξ^r results. These conditions imply that the local gauge transformations are dynamical with the light-like radial coordinate r playing the role of the time variable. One should be able to fix the transformation more or less arbitrarily at the partonic 2-surface X^2.

3. The three independent equations for g_{ij} give

$$\xi^\alpha \partial_\alpha g_{ij} + g_{kj} \partial_i \xi^k + g_{ki} \partial_j \xi^k = \partial_i c_A h_{kl} j^{Ak} \partial_j h^l \ . \tag{4.7.16}$$

These are 3 differential equations for 3 functions ξ^α on 2 independent variables x^i with r appearing as a parameter. Note however that the derivatives of ξ^r do not appear in the equation. At least formally equations are not over-determined so that solutions should exist for arbitrary choices of c_A as functions of X^3 coordinates satisfying the orthogonality conditions. If this is the case, the Kac-Moody algebra can be regarded as a local algebra in X^3 subject to the orthogonality constraint.

This algebra contains as a subalgebra the analog of Kac-Moody algebra for which all c_A except the one associated with time translation and fixed by the orthogonality condition depends on the radial coordinate r only. The larger algebra decomposes into a direct sum of representations of this algebra.

Commutators of infinitesimal symmetries

The commutators of infinitesimal symmetries need not be what one might expect since the vector fields ξ^μ are functionals c_A and of the induced metric and also c_A depends on induced metric via the orthogonality condition. What this means that $j^{A,k}$ in principle acts also to ϕ_B in the commutator $[c_A J^A, c_B J^B]$.

$$\left[c_A J^A, c_B J^B\right] = c_A c_B J^{[A,B]} + J^A \circ c_B J^B - J^B \circ c_A J^A \ , \tag{4.7.17}$$

where $\circ$ is a short hand notation for the change of c_B induced by the effect of the conformal transformation J^A on the induced metric.

Luckily, the conditions in the case $g_{rr} = g_{ir} = 0$ state that the components g_{rr} and g_{ir} of the induced metric are unchanged in the transformation so that the condition for c_A resulting from g_{rr} component of the metric is not affected. Also the conditions coming from $g_{ir} = 0$ remain unchanged. Therefore the commutation relations of local algebra apart from constraint from transversality result.

The commutator algebra of infinitesimal symmetries should also close in some sense. The orthogonality to the light-like tangent vector creates here a problem since the commutator does not obviously satisfy this condition automatically. The problem can be solved by following the recipes of non-covariant quantization of string model.

1. Make a choice of gauge by choosing time translation P^0 in a preferred M^4 coordinate frame to be the preferred generator $J^{A_0} \equiv P^0$, whose coefficient $\Phi_{A_0} \equiv \Psi(P^0)$ is solved from the orthogonality condition. This assumption is analogous with the assumption that time coordinate is non-dynamical in the quantization of strings. The natural basis for the algebra is obtained by allowing only a single generator J^A besides P^0 and putting $d_A = 1$.

2. This prescription must be consistent with the well-defined radial conformal weight for the $J^A \neq P^0$ in the sense that the proportionality of d_A to r^n for $J^A \neq P^0$ must be consistent with commutators. SU(3) part of the algebra is of course not a problem. From the Lorentz vector property of P^k it is clear that the commutators resulting in a repeated commutation have well-defined radial conformal weights only if one restricts $SO(3,1)$ to $SO(3)$ commuting with P^0. Also D could be allowed without losing well-defined radial conformal weights but the argument below excludes it. This picture conforms with the earlier identification of the Kac-Moody algebra.

   Conformal algebra contains besides Poincare algebra and the dilation $D = m^k\partial_{m^k}$ the mutually commuting generators $K^k = (m^r m_r \partial_{m^k} - 2m^k m^l \partial_{m^l})/2$. The commutators involving added generators are

$$\begin{array}{ll} \left[D, K^k\right] = -K^k \ , & \left[D, P^k\right] = P^k \ , \\ \left[K^k, K^l\right] = 0 \ , & \left[K^k, P^l\right] = m^{kl}D - M^{kl} \ . \end{array} \tag{4.7.18}$$

   From the last commutation relation it is clear that the inclusion of K^k would mean loss of well-defined radial conformal weights.

3. The coefficient dm^0/dr of $\Psi(P^0)$ in the equation

$$\Psi(P^0)\frac{dm^0}{dr} = -J^{Ak}h_{kl}\partial_r h^l$$

   is always non-vanishing due to the light-likeness of r. Since P^0 commutes with generators of $SO(3)$ (but not with D so that it is excluded!), one can *define* the commutator of two generators as a commutator of the remaining part and identify $\Psi(P^0)$ from the condition above.

4. Of course, also the more general transformations act as Kac-Moody type symmetries but the interpretation would be that the sub-algebra plays the same role as $SO(3)$ in the case of Lorentz group: that is gives rise to generalized spin degrees of freedom whereas the entire algebra divided by this sub-algebra would define the coset space playing the role of orbital degrees of freedom. In fact, also the Kac-Moody type symmetries for which c_A depends on the transversal coordinates of X^3 would correspond to orbital degrees of freedom. The presence of these orbital degrees of freedom arranging super Kac-Moody representations into infinite multiplets labeled by function basis for X^2 means that the number of degrees of freedom is much larger than in string models.

5. It is possible to replace the preferred time coordinate m^0 with a preferred light-like coordinate. There are good reasons to believe that orbifold singularity for phases of matter involving non-standard value of Planck constant corresponds to a preferred light-ray going through the tip of $\delta M^4_\pm$. Thus it would be natural to assume that the preferred M^4 coordinate varies along this light ray or its dual. The Kac-Moody group $SO(3) \times E^3$ respecting the radial conformal weights would reduce to $SO(2) \times E^2$ as in string models. E^2 would act in tangent plane of $S^2_\pm$ along this ray defining also $SO(2)$ rotation axis.

Hamiltonians

The action of these transformations on Kähler action is well-defined and one can deduce the conserved quantities having identification as configuration space Hamiltonians. Hamiltonians also correspond to closed 2-forms. The condition that the Hamiltonian reduces to a dual of closed 2-form is satisfied because X^2-local conformal transformations of $M^4_\pm \times CP_2$ are in question (X^2-locality does not imply any additional conditions).

The action of Kac-Moody algebra on spinors and fermionic representations of Kac-Moody algebra

One can imagine two interpretations for the action of generalized Kac-Moody transformations on spinors.

1. The basic goal is to deduce the fermionic Noether charge associated with the bosonic Kac-Moody symmetry and this can be done by a standard recipe. The first contribution to the charge comes from the transformation of modified gamma matrices appearing in the modified Dirac action associated with fermions. Second contribution comes from spinor rotation.

2. Both SO(3) and SU(3) rotations have a standard action as spin rotation and electro-weak rotation allowing to define the action of the Kac-Moody algebra J^A on spinors.

How central extension term could emerge?

The central extension term of Kac-Moody algebra could correspond to a symplectic extension which can emerge from the freedom to add a constant term to Hamiltonians as in the case of super-symplectic algebra. The expression of the Hamiltonians as closed forms could allow to understand how the central extension term emerges.

In principle one can construct a representation for the action of Kac-Moody algebra on fermions a representations as a fermionic bilinear and the central extension of Kac-Moody algebra could emerge in this construction just as it appears in Sugawara construction.

About the interpretation of super Kac-Moody symmetries

Also the light like 3-surfaces X_l^3 of H defining elementary particle horizons at which Minkowskian signature of the metric is changed to Euclidian and boundaries of space-time sheets can act as causal determinants, and thus contribute to the configuration space metric. In this case the symmetries correspond to the isometries of the imbedding space localized with respect to the complex coordinate of the 2-surface X^2 determining the light like 3-surface X_l^3 so that Kac-Moody type symmetry results. Also the condition $\sqrt{g_3} = 0$ for the determinant of the induced metric seems to define a conformal symmetry associated with the light like direction.

If is enough to localize only the H-isometries with respect to X_l^3, the purely bosonic part of the Kac-Moody algebra corresponds to the isometry group $M^4 \times SO(3,1) \times SU(3)$. The physical interpretation of these symmetries is not so obvious as one might think. The point is that one can generalize the formulas characterizing the action of infinitesimal isometries on spinor fields of finite-dimensional Kähler manifold to the level of the configuration space. This gives rise to bosonic generators containing also a sigma-matrix term bilinear in fermionic oscillator operators. This representation need not be equivalent with the purely fermionic representations provided by induced Dirac action. Thus one has two groups of local color charges and the challenge is to find a physical interpretation for them.

The following arguments support one possible identification.

1. The hint comes from the fact that $U(2)$ in the decomposition $CP_2 = SU(3)/U(2)$ corresponds in a well-defined sense electro-weak algebra identified as a holonomy algebra of the spinor connection. Hence one could argue that the $U(2)$ generators of either $SU(3)$ algebra might be identifiable as generators of local $U(2)_{ew}$ gauge transformations whereas non-diagonal generators would correspond to Higgs field. This interpretation would conform with the idea that Higgs field is a genuine scalar field rather than a composite of fermions.

2. Since X_l^3-local $SU(3)$ transformations represented by fermionic currents are characterized by central extension they would naturally correspond to the electro-weak gauge algebra and Higgs bosons. This is also consistent with the fact that both leptons and quarks define fermionic Kac Moody currents.

3. The fact that only quarks appear in the gamma matrices of the configuration space supports the view that action of the generators of X_l^3-local color transformations on configuration space spinor fields represents local color transformations. If the action of X_l^3-local $SU(3)$ transformations on configuration space spinor fields has trivial central extension term the identification as a representation of local color symmetries is possible.

The topological explanation of the family replication phenomenon is based on an assignment of 2-dimensional boundary to a 3-surface characterizing the elementary particle. The precise identification of this surface has remained open and one possibility is that the 2-surface X^2 defining the light light-like surface associated with an elementary particle horizon is in question. This assumption would conform with the notion of elementary particle vacuum functionals defined in the zero modes characterizing different conformal equivalences classes for X^2.

The relationship of the Super-Kac Moody symmetry to the standard super-conformal invariance

Super-Kac Moody symmetry can be regarded as $N=4$ complex super-symmetry with complex H-spinor modes of H representing the 4 physical helicities of 8-component leptonic and quark like spinors acting as generators of complex dynamical super-symmetries. The super-symmetries generated by the covariantly constant right handed neutrino appear with *both* M^4 helicities: it however seems that covariantly constant neutrino does not generate any global super-symmetry in the sense of particle-sparticle mass degeneracy. Only righthanded neutrino spinor modes (apart from covariantly constant mode) appear in the expressions of configuration space gamma matrices forming a subalgebra of the full super-algebra.

$N=2$ real super-conformal algebra is generated by the energy momentum tensor $T(z)$, $U(1)$ current $J(z)$, and super generators $G^{\pm}(z)$ carrying $U(1)$ charge. Now $U(1)$ current would correspond to right-handed neutrino number and super generators would involve contraction of covariantly constant neutrino spinor with second quantized induced spinor field. The further facts that $N=2$ algebra is associated naturally with Kähler geometry, that the partition functions associated with $N=2$ super-conformal representations are modular invariant, and that $N=2$ algebra defines so called chiral ring defining a topological quantum field theory [46], lend a further support for the belief that $N=2$ super-conformal algebra acts in super-symplectic degrees of freedom.

The values of c and conformal weights for $N=2$ super-conformal field theories are given by

$$\begin{aligned} c &= \frac{3k}{k+2} \ , \\ \Delta_{l,m}(NS) &= \frac{l(l+2)-m^2}{4(k+2)} \ , \quad l=0,1,...,k \ , \\ q_m &= \frac{m}{k+2} \ , \quad m=-l,-l+2,....,l-2,l \ . \end{aligned} \tag{4.7.19}$$

q_m is the fractional value of the $U(1)$ charge, which would now correspond to a fractional fermion number. For $k=1$ one would have $q=0,1/3,-1/3$, which brings in mind anyons. $\Delta_{l=0,m=0}=0$ state would correspond to a massless state with a vanishing fermion number. Note that $SU(2)_k$ Wess-Zumino model has the same value of c but different conformal weights. More information about conformal algebras can be found from the appendix of [46].

For Ramond representation $L_0-c/24$ or equivalently G_0 must annihilate the massless states. This occurs for $\Delta=c/24$ giving the condition $k=2\left[l(l+2)-m^2\right]$ (note that k must be even and that $(k,l,m)=(4,1,1)$ is the simplest non-trivial solution to the condition). Note the appearance of a fractional vacuum fermion number $q_{vac}=\pm c/12=\pm k/4(k+2)$. I have proposed that NS and Ramond algebras could combine to a larger algebra containing also lepto-quark type generators but this not necessary.

The conformal algebra defined as a direct sum of Ramond and NS $N=4$ complex sub-algebras associated with quarks and leptons might further extend to a larger algebra if lepto-quark generators acting effectively as half odd-integer Virasoro generators can be allowed. The algebra would contain spin and electro-weak spin as fermionic indices. Poincare and color Kac-Moody generators would act as symplectically extended isometry generators on configuration space Hamiltonians expressible in terms of Hamiltonians of $X^3_l\times CP_2$. Electro-weak and color Kac-Moody currents have conformal weight $h=1$ whereas T and G have conformal weights $h=2$ and $h=3/2$.

The experience with $N=4$ complex super-conformal invariance suggests that the extended algebra requires the inclusion of also second quantized induced spinor fields with $h=1/2$ and their super-partners with $h=0$ and realized as fermion-antifermion bilinears. Since G and Ψ are labeled by 2×4 spinor indices, super-partners would correspond to $2\times(3+1)=8$ massless electro-weak gauge boson states with polarization included. Their inclusion would make the theory highly predictive since induced spinor and electro-weak fields are the fundamental fields in TGD.

4.7.5 Coset space structure for configuration space as a symmetric space

The key ingredient in the theory of symmetric spaces is that the Lie-algebra of G has the following decomposition

$$\begin{aligned} & g = h + t \ , \\ & [h,h] \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ . \end{aligned}$$

In present case this has highly nontrivial consequences. The commutator of *any* two infinitesimal generators generating nontrivial deformation of 3-surface belongs to h and thus vanishing norm in the configuration space metric at the point which is left invariant by H. In fact, this same condition follows from Ricci flatness requirement and guarantees also that G acts as isometries of the configuration space. This generalization is supported by the properties of the unitary representations of Lorentz group at the light cone boundary and by number theoretical considerations.

The algebras suggesting themselves as candidates are symplectic algebra of $\delta M^{\pm} \times CP_2$ and Kac-Moody algebra mapping light-like 3-surfaces to light-like 3-surfaces to be discussed in the next section.

The identification of the precise form of the coset space structure is however somewhat delicate.

1. The essential point is that both symplectic and Kac-Moody algebras allow representation in terms of X_l^3-local Hamiltonians. The general expression for the Hamilton of Kac-Moody algebra is

$$H = \sum \Phi_A(x) H^A \ . \tag{4.7.20}$$

Here H^A are Hamiltonians of $SO(3) \times SU(3)$ acting in $\delta X_l^3 \times CP_2$. For symplectic algebra any Hamiltonian is allowed. If x corresponds to any point of X_l^3, one must assume a slicing of the causal diamond CD by translates of $\delta M_{\pm}^4$.

2. For symplectic generators the dependence of form on r^{Δ} on light-like coordinate of $\delta X_l^3 \times CP_2$ is allowed. Δ is complex parameter whose modulus squared is interpreted as conformal weight. Δ is identified as analogous quantum number labeling the modes of induced spinor field.

3. One can wonder whether the choices of the $r_M = constant$ sphere S^2 is the only choice. The Hamiltonin-Jacobi coordinate for $X^4_{X_l^3}$ suggest an alternative choice as E^2 in the decomposition of $M^4 = M^2(x) \times E^2(x)$ required by number theoretical compactification and present for known extremals of Kähler action with Minkowskian signature of induced metric. In this case $SO(3)$ would be replaced with $SO(2)$. It however seems that the radial light-like coordinate u of $X^4(X_l^3)$ would remain the same since any other curve along light-like boundary would be space-like.

4. The vector fields for representing Kac-Moody algebra must vanish at the partonic 2-surface $X^2 \subset \delta M_{\pm}^4 \times CP_2$. The corresponding vector field must vanish at each point of X^2:

$$j^k = \sum \Phi_A(x) J^{kl} H_l^A = 0 \ . \tag{4.7.21}$$

This means that the vector field corresponds to $SO(2) \times U(2)$ defining the isotropy group of the point of $S^2 \times CP_2$.

This expression could be deduced from the idea that the surfaces X^2 are analogous to origin of CP_2 at which $U(2)$ vector fields vanish. Configuration space at X^2 could be also regarded as the analog of the origin of local $S^2 \times CP_2$. This interpretation is in accordance with the original idea which however was given up in the lack of proper realization. The same picture can be deduced from braiding in which case the Kac-Moody algebra corresponds to local $SO(2) \times U(2)$ for each point of the braid at X^2. The condition that Kac-Moody generators with positive conformal weight annihilate physical states could be interpreted by stating effective 2-dimensionality in the sense that the deformations of X_l^3 preserving its light-likeness do not affect the physics. Note however that Kac-Moody type Virasoro generators do not annihilate physical states.

5. Kac-Moody algebra generator must leave induced Kähler form invariant at X^2. This is of course trivial since the action leaves each point invariant. The conditions of Cartan decomposition are satisfied. The commutators of the Kac-Moody vector fields with symplectic generators are non-vanishing since the action of symplectic generator on Kac-Moody generator restricted to X^2 gives a non-vanishing result belonging to the symplectic algebra. Also the commutators of Kac-Moody generators are Kac-Moody generators.

4.7.6 The relationship between super-symplectic and Super Kac-Moody algebras, Equivalence Principle, and justification of p-adic thermodynamics

The relationship between super-symplectic algebra (SS) acting at light-cone boundary and Super Kac-Moody algebra (SKM) acting on light-like 3-surfaces has remained somewhat enigmatic due to the lack of physical insights. This is not the only problem. The question to precisely what extent Equivalence Principle (EP) remains true in TGD framework and what might be the precise mathematical realization of EP is waiting for an answer. Also the justification of p-adic thermodynamics for the scaling generator L_0 of Virasoro algebra -in obvious conflict with the basic wisdom that this generator should annihilate physical states- is lacking. It seems that these three problems could have a common solution.

New vision about the relationship between SSV and $SKMV$

Consider now the new vision about the relationship between SSV and $SKMV$.

1. The isometries of H assignable with SKM are also symplectic transformations [22] (note that I have used the attribute "canonical" instead of "symplectic" previously). Hence might consider the possibility that SKM could be identified as a subalgebra of SS. If this makes sense, a generalization of the coset construction obtained by replacing finite-dimensional Lie group with infinite-dimensional symplectic group suggests itself. The differences of SSV and $SKMV$ elements would annihilate physical states and commute/anticommute with $SKMV$. Also the generators O_n, $n > 0$, for both algebras would annihilate the physical states so that the differences of the elements would annihilate automatically physical states for $n > 0$.

2. The super-generator G_0 contains the Dirac operator D of H. If the action of SSV and $SKMV$ Dirac operators on physical states are identical then cm of degrees of freedom disappear from the differences $G_0(SCV) - G_0(SKMV)$ and $L_0(SCV) - L_0(SKMV)$. One could interpret the identical action of the Dirac operators as the long sought-for precise realization of Equivalence Principle (EP) in TGD framework. EP would state that the total inertial four-momentum and color quantum numbers assignable to SS (imbedding space level) are equal to the gravitational four-momentum and color quantum numbers assignable to SKM (space-time level). Note that since super-symplectic transformations correspond to the isometries of the "world of classical worlds" the assignment of the attribute "inertial" to them is natural.

Consistency with p-adic thermodynamics

The consistency with p-adic thermodynamics provides a strong reality test and has been already used as a constraint in attempts to understand the super-conformal symmetries in partonic level.

1. In physical states the p-adic thermal expectation value of the SKM and SS conformal weights would be non-vanishing and identical and mass squared could be identified equivalently either as the expectation value of SKM or SS scaling generator L_0. There would be no need to give up Super Virasoro conditions for $SCV - SKMV$.

2. There is consistency with p-adic mass calculations for hadrons [17] since the non-perturbative SS contributions and perturbative SKM contributions to the mass correspond to space-time sheets labeled by different p-adic primes. The earlier statement that SS is responsible for the dominating non-perturbative contributions to the hadron mass transforms to a statement reflecting $SS - SKM$ duality. The perturbative quark contributions to hadron masses can be calculated

most conveniently by using p-adic thermodynamics for SKM whereas non-perturbative contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SS. Also the proposal that the exotic analogs of baryons resulting when baryon looses its valence quarks [31] remains intact in this framework.

3. The results of p-adic mass calculations depend crucially on the number N of tensor factors contributing to the Super-Virasoro algebra. The required number is $N = 5$ and during years I have proposed several explanations for this number. It seems that holonomic contributions that is electro-weak and spin contributions must be regarded as contributions separate from those coming from isometries. SKM algebras in electro-weak degrees and spin degrees of of freedom, would give 2+1=3 tensor factors corresponding to $U(2)_{ew} \times SU(2)$. $SU(3)$ and $SO(3)$ (or $SO(2) \subset SO(3)$ leaving the intersection of light-like ray with S^2 invariant) would give 2 additional tensor factors. Altogether one would indeed have 5 tensor factors.

There are some further questions which pop up in mind immediately.

1. Why mass squared corresponds to the thermal expectation value of the net conformal weight? This option is forced among other things by Lorentz invariance but it is not possible to provide a really satisfactory answer to this question yet. In the coset construction there is no reason to require that the mass squared equals to the integer value conformal weight for SKM algebra. This allows the possibility that mass squared has same value for states with different values of SKM conformal weights appearing in the thermal state and equals to the average of the conformal weight.

2. The coefficient of proportionality can be however deduced from the observation that the mass squared values for CP_2 Dirac operator correspond to definite values of conformal weight in p-adic mass calculations. It is indeed possible to assign to partonic 2-surface X^2 CP_2 partial waves correlating strongly with the net electro-weak quantum numbers of the parton so that the assignment of ground state conformal weight to CP_2 partial waves makes sense.

3. In the case of M^4 degrees of freedom it is strictly speaking not possible to talk about momentum eigen states since translations take parton out of δH_+. This would suggests that 4-momentum must be assigned with the tip of the light-cone containing the particle but this is not consistent with zero energy ontology. Hence it seems that one must restrict the translations of X^3_l to time like translations in the direction of geometric future at $\delta M^4_+ \times CP_2$. The decomposition of the partonic 3-surface X^3_l to regions $X^3_{l,i}$ carrying non-vanishing induced Kähler form and the possibility to assign $M^2(x) \subset M^4$ to the tangent space of $X^4(X^3_l)$ at points of X^3_l suggests that the points of number theoretic braid to which oscillator operators can be assigned can carry four-momentum in the plane defined by $M^2(x)$. One could assume that the four-momenta assigned with points in given region X^3_i are collinear but even this restriction is not necessary.

4. The additivity of conformal weight means additivity of mass squared at parton level and this has been indeed used in p-adic mass calculations. This implies the conditions

$$(\sum_i p_i)^2 = \sum_i m_i^2 \tag{4.7.22}$$

The assumption $p_i^2 = m_i^2$ makes sense only for massless partons moving collinearly. In the QCD based model of hadrons only longitudinal momenta and transverse momentum squared are used as labels of parton states, which together with the presence of preferred plane M^2 would suggest that one has

$$\begin{aligned} p^2_{i,||} &= m_i^2 \ , \\ -\sum_i p^2_{i,\perp} + 2\sum_{i,j} p_i \cdot p_j &= 0 \ . \end{aligned} \tag{4.7.23}$$

The masses would be reduced in bound states: $m_i^2 \rightarrow m_i^2 - (p_T^2)_i$. This could explain why massive quarks can behave as nearly massless quarks inside hadrons.

How it is possible to have negative conformal weights for ground states?

p-Adic mass calculations require negative conformal weights for ground states [19]. The only elegant solution of the problems caused by this requirement seems to be p-adic: the conformal weights are positive in the real sense but as p-adic numbers their dominating part is negative integer (in the real sense), which can be compensated by the conformal weights of Super Virasoro generators.

1. If $\pm\lambda_i^2$ as such corresponds to a ground state conformal weight and if λ_i is real the ground state conformal weight positive in the real sense. In complex case (instanton term) the most natural formula is $h = \pm|\lambda|^2$.

2. The first option is based on the understanding of conformal excitations in terms of CP breaking instanton term added to the modified Dirac operator. In this case the conformal weights are identified as $h = n - |\lambda_k|^2$ and the minus sign comes from the Euclidian signature of the effective metric for the modified Dirac operator. Ground state conformal weight would be non-vanishing for non-zero modes of $D(X_l^3)$. Massless bosons produce difficulties unless one has $h = |\lambda_i(1) - \lambda_i(2)|^2$, where $i = 1, 2$ refers to the two wormhole throats. In this case the difference can vanish and its non-vanishing would be due to the symmetric breaking. This scenario is assumed in p-adic mass calculations. Fermions are predicted to be always massive since zero modes of $D(X^2)$ represent super gauge degrees of freedom.

3. In the context of p-adic thermodynamics a loop hole opens allowing λ_i to be real. In spirit of rational physics suppose that one has in natural units $h = \lambda_i^2 = xp^2 - n$, where x is integer. This number is positive and large in the real sense. In p-adic sense the dominating part of this number is $-n$ and can be compensated by the net conformal weight n of Super Virasoro generators acting on the ground state. xp^2 represents the small Higgs contribution to the mass squared proportional to $(xp^2)_R \simeq x/p^2$ ($_R$ refers to canonical identification). By the basic features of the canonical identification $p > x \simeq p$ should hold true for gauge bosons for which Higgs contribution dominates. For fermions x should be small since p-adic mass calculations are consistent with the vanishing of Higgs contribution to the fermion mass. This would lead to the earlier conclusion that xp^2 and hence B_K is large for bosons and small for fermions and that the size of fermionic (bosonic) wormhole throat is large (small). This kind of picture is consistent with the p-adic modular arithmetics and suggests by the cutoff for conformal weights implied by the fact that both the number of fermionic oscillator operators and the number of points of number theoretic braid are finite. This solution is however tricky and does not conform with number theoretical universality.

4.7.7 Comparison of TGD and stringy views about super-conformal symmetries

The best manner to represent TGD based view about conformal symmetries is by comparison with the conformal symmetries of super string models.

Basic differences between the realization of super conformal symmetries in TGD and in super-string models

The realization super-symmetries in TGD framework differs from that in string models in several fundamental aspects.

1. In TGD framework super-symmetry generators acting as configuration space gamma matrices carry either lepton or quark number. Majorana condition required by the hermiticity of super generators which is crucial for super string models would be in conflict with the conservation of baryon and lepton numbers and is avoided. This is made possible by the realization of bosonic generators represented as Hamiltonians of symplectic transformations rather than vector fields generating them. This kind of representation applies also in Kac-Moody sector since the local

transversal isometries localized in X_l^3 and respecting light-likeness condition can be regarded as X^2 local symplectic transformations, whose Hamiltonians generate also isometries. The fermionic representations of super-symplectic and super Kac-Moody generators can be identified as Noether charges in standard manner.

2. Super-symmetry generators can be identified as configuration space gamma matrices carrying quark and lepton numbers and the notion of super-space is not needed at all. Therefore no super-variant of geometry is needed. The distinction between Ramond and N-S representations important for $N = 1$ super-conformal symmetry and allowing only ground state weight 0 an 1/2 disappears. Indeed, for $N = 2$ super-conformal symmetry it is already possible to generate spectral flow transforming these Ramond and N-S representations to each other (G_n is not Hermitian anymore). This means that the interpretation of λ_i^2 (λ_i is generalized eigenvalue of $D_K(X^2)$) as ground state conformal weight does not lead to difficulties.

3. Kac-Moody and symplectic algebras generate larger algebra obtained by making symplectic algebra X^2 local. This realization of super symmetries is what distinguishes between TGD and super string models and leads to a totally different physical interpretation of super-conformal symmetries. What makes spinor field mode a generator of gauge super-symmetry is that is c-number and not an eigenmode of $D_K(X^2)$ and thus represents non-dynamical degrees of freedom. If the number of eigen modes of $D_K(X^2)$ is indeed finite means that most of spinor field modes represent super gauge degrees of freedom. One must be here somewhat cautious since bound state in the Coulomb potential associated with electric part of induced electro-weak gauge field might give rise to an infinite number of bound states which eigenvalues converging to a fixed eigenvalue (as in the case of hydrogen atom).

4. The finite number of spinor modes means that the representations of super-conformal algebras reduces to finite-dimensional ones in TGD framework and the notion of number theoretic braid indeed implies this. The physical interpretation is in terms of finite measurement resolution.

Basic super-conformal symmetries

The identification of explicit representations of super conformal algebras was for a long time plagued by the lack of appropriate formalism. The modified Dirac operator D_K associated with Kähler action resolves this problem if one accepts the implications of number theoretic compactification supported by what is known about preferred extremals of Kähler action and one can identify the charges associated with symplectic and Kac-Moody algebra as Noether charges. Fermionic generators can in turn be identified from the condition that they anticommute to X^2 local Hamiltonians of corresponding bosonic transformations. In case of Super Virasoro algebra Sugaware construction allows to construct super generators G.

1. Covariantly constant right handed neutrino is the fundamental generator of dynamical super conformal symmetries and appears in both leptonic and quark-like realizations of gamma matrices. Γ matrices have also Super Kac-Moody counterparts and reduce in special case to symplectic ones. Also super currents whose anti-commutators give products of corresponding Hamiltonians can be defined so that both ordinary product and Poisson bracket give rise to quark and lepton like realizations of super-symmetries. Besides this there are also electric and magnetic representations of the gamma matrices.

2. The zero modes of $D_K(X^2)$ which do not depend on the light-like radial coordinate of X_l^3 define super conformal symmetries for which any c-number spinor field generates super conformal symmetry. These symmetries are pure gauge symmetries but also them can be parameterized by Hamiltonians and by functions depending only on the coordinates of the transverse section X^2 so that one obtains also now both function algebra and symplectic algebra localized with respect to X^2. Similar picture applies in both super-symplectic and super Kac-Moody sector. In particular, one can deduce canonical expressions for the super currents associated with these super symmetries. Since all charge states are possible for the generators of these super symmetries, these super symmetries naturally correspond to those assignable to electro-weak degrees of freedom.

3. The notion of X^2 local super-symmetry makes sense if the choice of coordinates x for X^2 is specified by the inherent properties of X^2 so that same coordinates x apply for all surfaces obtained as deformations of X^2. The regions, where induced Kähler form is non-vanishing define good candidates for coordinate patches. The Hamilton-Jacobi coordinates associated with the decomposition of M^4 are a natural choice. Also geodesic coordinates can be considered. The redundancy related to rotations of coordinate axis around origin can be reduced by choosing second axis so that it connects the origin to nearest point of the number theoretic braid.

4. The diffeomorphisms of light-like coordinate of $\delta M^4_\pm$ and X^3_l playing the role of conformal transformations. One can construct fermionic representations of as Noether charges associated with modified Dirac action. The problem is however that that super-generators cannot be derived in this manner so that these transformations cannot be regarded as symplectic transformations. The manner to circumvent the difficulty is to construct fermionic super charges Γ_A as gamma matrices for both super symplectic and super Kac-Moody algebras in terms of generators $j^{Ak}\Gamma_k$ and corresponding Kac-Moody algebra elements T^A as fermionic super charges. From these operators super generators G can be constructed by the standard Sugawara construction allowing to interpret operators $G = T^A\Gamma_A$ as Dirac operators at the level of configuration space. By coset construction the actions of super-symplectic and super Kac-Moody Dirac operators are identical. Internal consistency requires that the Virasoro generators obtained as anticommutator $L = \{G, G^\dagger\}$ are equal to the Virasoro generators derived as fermionic Noether charges.

Finite measurement resolution and cutoff in the spectrum of conformal weights

The basic properties of Kähler action imply that the number generalized eigenvalues λ_i of $D_K(X^2)$ is finite. The interpretation is that the notion of finite measurement resolution is coded by Kähler action to space-time dynamics. This has also implications for the representations of super-conformal algebras.

1. The fermionic representations of various super-algebras involve only finite number of oscillator operators. Hence some kind of cutoff in the number of states reflecting the finiteness of the measurement resolution is unavoidable. A cutoff reduce integers as labels of the generators of super-conformal algebras to a finite number of integers. Finite field $G(p, 1)$ for some prime p would be a natural candidate. Since p-adic integers modulo p are in question the cutoff could relate closely to effective p-adicity and p-adic length scale-hypothesis.

2. The interpretation of the eigenvalues of the modified Dirac operator as ground state conformal weights raises the question how to represent states with conformal weights $n + \lambda_i^2$, $n > 0$. The notion of number theoretic braid allows to circumvent the difficulty. Since canonical anti-commutation relations fail, one must replace the integral representations of super-conformal generators with discrete sums over the points of number theoretic braid, the resulting representations of super-conformal algebras must reduce to representation of finite-dimensional algebras. The cutoff on conformal weight must result from the fact that the higher Virasoro generators are expressible in terms of lower ones. The cutoff is not a problem since $n < 3$ cutoff for conformal weights gives an excellent accuracy in p-adic mass calculations. A not-very-educated guess but the only one that one can imagine is that for $p \simeq 2^k$, $n_{max} = k$ defines the cutoff on allowed conformal weights.

What are the counter parts of stringy conformal fields in TGD framework?

The experience with string models would suggest the conformal symmetries associated with the complex coordinates of X^2 as a candidate for conformal super-symmetries. One can imagine two counterparts of the stringy coordinate z in TGD framework.

1. Super-symplectic and super Kac-Moody symmetries are local with respect to X^2 in the sense that the coefficients of generators depend on the invariant $J = \epsilon^{\alpha\beta}J_{\alpha\beta}\sqrt{g_2}$ rather than being completely free [22]. Thus the real variable J replaces complex coordinate and effective 1-dimensionality holds true also now but in different sense than for conformal field theories.

2. The slicing of X^2 by string world sheets Y^2 and partonic 2-surfaces X^2 implied by number theoretical compactification implies string-parton duality and involves the super conformal fermionic gauge symmetries associated with the coordinates u and w in the dual dimensional reductions to stringy and partonic dynamics. These coordinates define the natural analogs of stringy coordinate.

3. An further identification for TGD parts of conformal fields is inspired by $M^8 - H$ duality. Conformal fields would be fields in configuration space. The counterpart of z coordinate could be the hyper-octonionic M^8 coordinate m appearing as argument in the Laurent series of configuration space Clifford algebra elements. m would characterize the position of the tip of CD and the fractal hierarchy of CDs within CDs would give a hierarchy of Clifford algebras and thus inclusions of hyper-finite factors of type II_1. Reduction to hyper-quaternionic field -that is field in M^4 center of mass degrees of freedom- would be needed to obtained associativity. The arguments m at various level might correspond to arguments of N-point function in quantum field theory.

Generalized coset representation

X^2 local super-symplectic algebra as super Kac-Moody algebra as sub-algebra. Since X^2 locality corresponds to a full 2-D gauge invariance, one can conclude that SKM is in well defined sense sub-algebra of super-symplectic algebra so that generalized coset construction makes sense and generalizes Equivalence Principle in the sense that not only four-momenta but all analogous quantum numbers associated with SKM and SS algebras are identical.

1. In this framework the ground state conformal weights associated with both super-symplectic and super Kac-Moody algebras can be identified as squares of the eigenvalues λ_i of $D_K(X^2)$. This identification together with p-adic mass thermodynamics predicts that λ_i^2 gives to mass squared a contribution analogous to the square of Higgs vacuum expectation. This identification would resolve the long-standing problem of identifying the values of these ground state conformal weights for super-conformal algebras and give a direct connection with Higgs mechanism.

2. The identification of SKM as a sub-algebra of super-symplectic algebra becomes more convincing if the light-like coordinate r allows lifting to a light-like coordinate of H. This is achieved if r is identified as coordinate associated with a light-like curve whose tangent at point $x \in X_l^3$ is light-like vector in $M^2(x) \subset T(X^4(X^3)$. With this interpretation of SKM algebra as sub-algebra of super-symplectic algebra becomes natural.

3. The existence of a lifting of SS and SKM algebras to entire H would solve the problems. The lifting problem is obviously non-trivial only in M^4 degrees of freedom. Suppose that the existence of an integrable distribution of planes $M^2(x)$ and their orthogonal complements $E^2(x)$ belonging to the tangent space of M^4 projection $P_{M^4}(X^4(X^3))$ characterizes the preferred extremals with Minkowskian signature of induced metric. In this case the lifting of the super-symplectic and super Kac-Moody algebras to entire H is possible. The local degrees of freedom contributing to the configuration space metric would belong to the integrable distribution of orthogonal complements $E^2(x)$ of $M^2(x)$ having physical interpretation as planes of physical polarizations.

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Chapter 5

Does the Modified Dirac Equation Define the Fundamental Action Principle?

5.1 Introduction

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The Dirac determinant associated with the modified Dirac action is an excellent candidate in this respect.

The original working hypothesis was that Dirac determinant defines the vacuum functional of the theory having interpretation as the exponent of Kähler function of world of classical worlds (WCW) and that Kähler function reduces to Kähler action for a preferred extremal of Kähler action.

Two alternative choices represented themselves as candidates for the modified Dirac action: either the 3-D Chern-Simons Dirac action or 4-D Kähler action with imaginary measurement interaction term added. Quite recently it became clear that the addition of a measurement interaction term to either Chern-Simons action or Kähler action resolves a bundle of conceptual problems. The question which option is correct is not completely settled yet although it seems that the measurement interaction term assigned to Chern-Simons-Dirac action creates more problems that it solves.

5.1.1 What are the basic equations of quantum TGD?

A good place to start is to as what might the basic equations of quantum TGD. There are two kinds of equations at the level of space-time surfaces.

1. Purely classical equations define the dynamics of the space-time sheets as preferred extremals of Kähler action. Preferred extremals are quantum critical in the sense that second variation vanishes for critical deformations representing zero modes. This condition guarantees that corresponding fermionic currents are conserved. There is infinite hierarchy of these currents and they define fermionic counterparts for zero modes. Space-time sheets can be also regarded as hyper-quaternionic surfaces. What these statements precisely mean has become clear only during this year. A rigorous proof for the equivalence of these two identifications is still lacking.

2. The purely quantal equations are associated with the representations of various super-conformal algebras and with the modified Dirac equation. The requirement that there are deformations of the space-time surface -actually infinite number of them- giving rise to conserved fermionic charges implies quantum criticality at the level of Kähler action in the sense of critical deformations. The precise form of the modified Dirac equation is not however completely fixed without further input. Quantal equations involve also generalized Feynman rules for M-matrix generalizing S-matrix to a "complex square root" of density matrix and defined by time-like entanglement coefficients between positive and negative energy parts of zero energy states is certainly the basic goal of quantum TGD.

3. The notion of weak electric-magnetic duality leads to a detailed understanding of how TGD reduces to almost topological quantum field theory. If Kähler current defines Beltrami flow it is possible to find a gauge in which Coulomb contribution to Kähler action vanishes so that it reduces to Chern-Simons term. If light-like 3-surfaces and ends of space-time surface are extremals of Chern-Simons action also effective 2-dimensionality is realized. The condition that the theory reduces to almost topological QFT and the hydrodynamical character of field equations leads to a detailed ansatz for the general solution of field equations and also for the solutions of the modified Dirac equation relying on the notion of Beltrami flow for which the flow parameter associated with the flow lines defined by a conserved current extends to a global coordinate. This makes the theory is in well-defined sense completely integrable. Direct connection with massless theories emerges: every conserved Beltrami currents corresponds to a pair of scalar functions with the first one satisfying massless d'Alembert equation in the induced metric. The orthogonality of the gradients of these functions allows interpretation in terms of polarization and momentum directions. The Beltrami flow property can be also seen as one aspect of quantum criticality since the conserved currents associated with critical deformations define this kind of pairs.

4. The hierarchy of Planck constants provides also a fresh view to the quantum criticality. The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. It however seems that TGD actually predicts this hierarchy of covering spaces. The extreme non-linearity of the field equations defined by Kähler action means that the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is 1-to-many. This leads naturally to the introduction of the covering space of $CD\times CP_2$, where CD denotes causal diamond defined as intersection of future and past directed light-cones.

At the level of WCW there is the generalization of the Dirac equation which can be regarded as a purely classical Dirac equation. The modified Dirac operators associated with quarks and leptons carry fermion number but the Dirac equations are well-defined. An orthogonal basis of solutions of these Dirac operators define in zero energy ontology a basis of zero energy states. The M-matrices defining entanglement between positive and negative energy parts of the zero energy state define what can be regarded as analogs of thermal S-matrices. The M-matrices associated with the solution basis of the WCW Dirac equation define by their orthogonality unitary U-matrix between zero energy states. This matrix finds the proper interpretation in TGD inspired theory of consciousness. WCW Dirac equation as the analog of super-Virasoro conditions for the "gamma fields" of superstring models defining super counterparts of Virasoro generators was the main focus during earlier period of quantum TGD but has not received so much attention lately and will not be discussed in this chapter.

Quantum classical correspondence requires a coupling between quantum and classical and this coupling should also give rise to a generalization of quantum measurement theory. The big question mark is how to realize this coupling. The addition of a measurement interaction term to the modified Dirac action turned out to do the job [18, 30] and solves a handful of problems of quantum TGD and unifies various visions about the physics predicted by quantum TGD.

5.1.2 Modified Dirac equation for induced classical spinor fields

The basic vision is that WCW geometry reduces to the second quantization of induced spinor fields. This means that WCW gamma matrices are linear combinations of fermionic oscillator operators and the vacuum functional of the theory is identifiable as Dirac determinant. An unproven conjecture is that this determinant equals to the exponent of Kähler action for its preferred extremal.

The motivation for the modified Dirac action came from the observation that the counterpart of the ordinary Dirac equation is internally consistent only if the space-time surfaces are minimal surfaces. One can however assign to any general coordinate invariant action principle for space-time surfaces a unique modified Dirac action, which is internally consistent and super-symmetric. Space-time geometry must carry information about conserved quantum charges assignable to partonic 2-surfaces and it took considerable to to realize that this is achieved via a measurement interaction terms linear in conserved charges. It took still some time to conclude that Kähler action with a measurement interaction term is is required in order the code information about quantum numbers to the space-time geometry.

Preferred extremals as critical extremals

The study of the modified Dirac equation leads to a detailed view about criticality. Quantum criticality [64] fixes the values of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current except in the case of Cartan algebra of isometries. Quantum criticality allows to fix the values of couplings appearing in the measurement interaction by using the condition $K \rightarrow K + f + \overline{f}$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs). The discovery that the hierarchy of Planck constants realized in terms of singular covering spaces of $CD \times CP_2$ can be understood in terms of the extremely non-linear dynamics of Kähler action implying 1-to-many correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates led to a further very concrete understanding of the criticality at space-time level and its relationship to zero energy ontology [34].

Inclusion of the measurement interaction term

One can pose several conditions on the measurement interaction term of Dirac action. The term should be linear in the measured charges which must commute and act on their eigenstates. The effective 2-dimensionality requires that the measurement interaction term is 3-dimensional and this allows only the Dirac action associated with the generalized Chern-Simons action [56]. Measurement interaction term must define fermionic 3-D propagators along wormhole throats. This is necessary because 4-D Dirac equation is satisfied always and cannot define the fermionic propagator. For Chern-Simons term off mass shell propagation is possible since 3-D Chern-Simons Dirac equation need not to be satisfied.

1. The basic vision is that the addition of the measurement interaction term induces a $U(1)$ gauge transformation $K \rightarrow K + f + \overline{f}$ of the Kähler function of WCW. Here f is holomorphic function of WCW ("world of classical worlds") complex coordinates and arbitrary function of zero mode coordinates. Altough WCW Kähler metric is not affected, Kähler function changes and this means that preferred extremal changes also and therefore codes information about the values of the measured observables.

2. The measurement interaction is assumed to be linear in the measured charges which must commute and therefore belong to the Cartan algebra. Cartan algebra plays a key role not only at quantum level but also at the level of space-time geometry since quantum critical conserved currents vanish for the Cartan algebra of isometries and the measurement interaction terms giving rise to conserved currents are possible only for Cartan algebras. Furthermore, modified Dirac equation makes sense only for the eigen states of Cartan algebra generators. The hierarchy of Planck constants realized in terms of the book like structure of the generalized imbedding space assigns to each CD (causal diamond) preferred Cartan algebra: in case of Poincare algebra there are two of them corresponding to linear and cylindrical M^4 coordinates. The origin of the hierarchy of Planck constants can be now understood from the basic quantum TGD and it relates directly with criticality [34].

3. The values of Cartan charges are feeded to 3-D Chern-Simons Dirac action via the measurement interaction term. Measurement interaction term corresponds to a term resulting from the $U(1)$ transformation ϕ of the CP_2 Kähler potential. Since this term is assigned only with the Chern-Simons Dirac action, it does not reduce to a mere gauge transformation with a trivial effect. This picture is consistent with the reduction of TGD to almost topological QFT [54] implied by electric-magnetic duality and the vanishing of the Coulomb interaction term in Kähler action [34].

4. One can require that the propagating states are generalized eigenstates of the modified Dirac equation. The generalized eigenvalues are of form $D_{C-S}\Psi = \lambda^k \gamma_k \Psi$, where only the covariantly constant M^4 gamma matrices can appear. λ^k is completely analogous to four-momentum and the propagator is formally massless propagator so that ordinary twistor formalism should apply.

The identification with actual four-momentum does not however make sense. This suggests that also massless gauge theories could make sense if the four-momenta do not correspond to the actual four-momenta.

CP breaking and matter-antimatter asymmetry

Chern-Simons Dirac action used to defined measurement interaction term breaks CP and T symmetries and therefore provides a first principle description for the breaking of these symmetries. CP breaking could also reflect to the discretization of the relative coordinate between the tips of the CD. One could label the positions of the lower tip of CD by M^4 and the relative positions of the upper tip by a discrete space consisting of discrete variants of hyperboloids with proper time coordinate coming as powers of 2. This CP and T breaking would be apparent and due to the fixing the rest system to the observer assigned with the "lower" boundary of CD serving as a role of medium forcing the CP breaking at the level sub-CDs. One can of course argue that the CP breaking induced by Chern-Simons action gives the special role for the "lower" boundary of CD. In fact, the breaking of Lorentz invariance at the level of CD (but not at the level of WCW) could even make possible a spontaneous breaking of CPT symmetry.

What one should still continue to be worried about?

The construction of WCW spinor structure in terms of induced spinor fields has been continual shifting between various options. 3-D or 4-D modified Dirac action at the fundamental level? Does the idea about TGD as almost topological QFT make sense or not? Is the identification of Kähler function as Dirac determinant really needed? Does it even make sense?

The reduction to almost topological QFT based on weak electric-magnetic duality gives the explicit form of the WCW Kähler function and one understand how the measurement interaction term affects it. This is of utmost importance for the construction of quantum TGD since WCW Kähler metric becomes directly calculable. The progress in some aspects however forces always to challenge the basic assumptions so that there is no hope about the end of endless confusiomn.

1. The basic idea has been that a correlation between 4-D geometry of the space-time sheet and quantum numbers would be achieved by the identification of the exponent of Kähler function as a Dirac determinant. The effect of the measurement interaction to the Kähler function is however induced by the same gauge transformation of the induced Kähler gauge potential appearing in Chern-Simons action as appears in Chern-Simons Dirac action. Therefore Dirac determinant is not needed to calculate the Kähler function and one can ask whether the identification of Kähler function as a Dirac determinant has any practical value.

2. One can still worry whether the measurement interaction is really needed. The propagator reduces formally to massless Dirac propator in which the analog of four-momentum is expressible in terms of quantum numbers propagating in the line. This would be a fantastic news for a believer in the twistor program since also massive case and virtual momenta could be treated. One could however argue that the road involving minimum amount of calculations is the safest one: why not to identify the four-momentum with the physical four-momentum and try to resolve the resulting problems?

3. The teasing hen-egg question still remains.
   Does the 4-D Kähler-Dirac action with Chern-Simons term define preferred extremals giving Kähler function as a Kähler action reducing to Chern-Simons term? In this case the induced spinor fields in the interior of space-time surface would be present and one would have a symmetry in the sense that one could use the restriction of the induced spinor field and Chern-Simons action for any light-like 3-surface to constructe the quantum theory.
   Or should 3-D Chern-Simons-Dirac action be interpreted as the Kähler function of WCW to which one directly assigns the modified Dirac action making possible to construct the spinor structure of WCW and does electric-magnetic duality make possible the assignment of preferred extremal of Kähler action to a given 3-surface. In this case the induced spinor fields in the interior of space-time surface would not be needed at all and wormhole throats and ends of the space-time surface would play a special role as carriers of spinorial shock waves.

5.1.3 Identification of configuration space gamma matrices as super Hamiltonians

The basic super-algebra corresponds to the fermionic oscillator operators and can be regarded as a generalization $\mathcal{N}$ super algebras by replacing $\mathcal{N}$ with the number of solutions of the modified Dirac equation which can be infinite. This leads to QFT SUSY limit of TGD different in many respects crucially from standard SUSYs.

Configuration space gamma matrices identified as super generators of super-symplectic and are expressible in terms of these oscillator operators. Super-symplectic and super charges are assumed to be expressible as integrals over 2-dimensional partonic surfaces X^2 and interior degrees of freedom of X^4 can be regarded as zero modes representing classical variables in one-one correspondence with quantal degrees of freedom at X_l^3 as indeed required by quantum measurement theory.

5.2 Modified Dirac equation

In the following the problems of the ordinary Dirac action are discussed and the notion of modified Dirac action is introduced.

5.2.1 Problems associated with the ordinary Dirac action

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or some other general principle selecting preferred extremals as Bohr orbits [22, 20].

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the configuration space geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of the configuration space geometry so that there is internal inconsistency.

5.2.2 Super-symmetry forces modified Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$\begin{aligned} D_\alpha T^\alpha_k &= 0 \ , \\ T^\alpha_k &= \frac{\partial}{\partial h^k_{\ \alpha}} L_K \ . \end{aligned} \tag{5.2.1}$$

If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l \Psi \ , \\ D_\alpha J^{\alpha k} &= 0 \ . \end{aligned} \tag{5.2.2}$$

having a vanishing divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^{\alpha} = \overline{\nu_R} T^{\alpha}_l \Gamma^l \Psi \tag{5.2.3}$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_{\alpha} J^{\alpha k} = \overline{\nu_R} \Gamma^k T^{\alpha}_l \Gamma^l D_{\alpha} \Psi \ . \tag{5.2.4}$$

The requirement that this current vanishes is guaranteed if one assumes that modified Dirac equation

$$\begin{aligned} \hat{\Gamma}^{\alpha} D_{\alpha} \Psi &= 0 \ , \\ \hat{\Gamma}^{\alpha} &= T^{\alpha}_l \Gamma^l \ . \end{aligned} \tag{5.2.5}$$

This equation must be derivable from a modified Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi} \hat{\Gamma}^{\alpha} D_{\alpha} \Psi \ . \tag{5.2.6}$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with effective induced gamma matrices and the requirement

$$D_{\mu} \hat{\Gamma}^{\mu} = 0 \tag{5.2.7}$$

guaranteing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

5.2.3 How can one avoid minimal surface property?

These observations suggest how to avoid the emergence of the minimal surface property as a consequence of field equations. It is not induced metric which appears in field equations. Rather, the effective metric appearing in the field equations is defined by the anti-commutators of $\hat{\gamma}_{\mu}$

$$\hat{g}_{\mu\nu} = \{\hat{\Gamma}_{\mu}, \hat{\Gamma}_{\nu}\} = 2T^k_{\mu} T_{\nu k} \ . \tag{5.2.8}$$

Here the index raising and lowering is however performed by using the induced metric so that the problems resulting from the non-invertibility of the effective metric are avoided. It is this dynamically generated effective metric which must appear in the number theoretic formulation of the theory.

Field equations state that space-time surface is minimal surface with respect to the effective metric. Note that a priori the choice of the bosonic action principle is arbitrary. The requirement that effective metric defined by energy momentum tensor has only non-diagonal components except in the case of non-light-like coordinates, is satisfied for the known solutions of field equations.

5.2.4 Does the modified Dirac action define the fundamental action principle?

There is quite fundamental and elegant interpretation of the modified Dirac action as a fundamental action principle discussed also in [20]. In this approach vacuum functional can be defined as the Grassmannian functional integral associated with the exponent of the modified Dirac action. This definition is invariant with respect to the scalings of the Dirac action so that theory contains no free parameters.

An alternative definition is as a Dirac determinant which might be calculated in TGD framework without applying the poorly defined functional integral. There are good reasons to expect that the Dirac determinant exponent of Kähler function for a preferred Bohr orbit like extremal of the Kähler action with the value of Kähler coupling strength coming out as a prediction. Hence the dynamics of the modified Dirac action at light-like partonic 3-surfaces X_l^3, even when restricted to almost-topological dynamics induced by Chern-Simons action, would dictate the dynamics at the interior of the space-time sheet.

The knowledge of the symplectic currents and super-currents, together with the anti-commutation relations stating that the fermionic super-currents S_A and S_B associated with Hamiltonians H_A and H_B anti-commute to a bosonic current $H_{[A,B]}$, allows in principle to deduce the anti-commutation relations satisfied by the induced spinor field. In fact, these conditions replace the usual anti-commutation relations used to quantize free spinor field. Since the normal ordering of the Dirac action would give Kähler action,

Kähler coupling strength would be determined completely by the anti-commutation relations of the super-symplectic algebra. Kähler coupling strength would be dynamical and the selection of preferred extremals of Kähler action would be more or less equivalent with quantum criticality because criticality corresponds to conformal invariance and the hyper-quaternionic version of the super-conformal invariance results only for the extrema of Kähler action. p-Adic (or possibly more general) coupling constant evolution and quantum criticality would come out as a prediction whereas in the case that Kähler action is introduced as primary object, the value of Kähler coupling strength must be fixed by quantum criticality hypothesis.

The mixing of the M^4 chiralities of the imbedding space spinors serves as a signal for particle massivation and breaking of super-conformal symmetry. The induced gamma matrices for the space-time surfaces which are deformations of M^4 indeed contain a small contribution from CP_2 gamma matrices: this implies a mixing of M^4 chiralities even for the modified Dirac action so that there is no need to introduce this mixing by hand.

5.2.5 Which Dirac action?

Which modified Dirac action should one choose? The four-dimensional modified Dirac action associated with Kähler action or 3-D Dirac action associated with Chern-Simons (Chern-Simons) action? Or something else? To express the number of proposed answers to this question requires the fingers of both hands.

1. The first guess inspired by TGD as almost-TQFT (for topological QFTs see [43]) was that Chern-Simons action is enough. The idea was that one starts from Chern-Simons action and end up with Kähler function defined by a preferred extremal of Kähler action defining Kähler function with exponent of Kähler function identified as a Dirac determinant. The difficulties related to the definition of the Dirac determinant however forced to give up this approach. It must be emphasized that Dirac determinant is a potentially problematic notion quite generally.

2. TGD reduces to almost topological QFT if the Kähler action for the preferred extremals reduces to Chern-Simons action associated with the ends of the space-time surface and with the light-like wormhole throats at which the signature of the induced metric changes. The assumption that Kähler current defines so called Beltrami flow indeed allows to find a gauge for Kähler gauge potential in which the Coulomb term in Kähler action vanishes and the boundary term reduces to generalized Chern-Simons action if the weak form of electric-magnetic duality holds true. Both Dirac-Kähler action and Dirac-Chern-Simons action are needed in the fermionic sector. It must be emphasized that the identification of the exponent of Kähler function as a Dirac determinant is not necessary anymore although it could make sense.

One ends up with a very detailed ansatz for preferred extremals and one should rigorously show that these surfaces are critical in the sense of having infinite number of deformations giving a vanishing second variation of Kähler action. One should also demonstrate that preferred extremals are hyper-quaternionic- a property forced by number theoretical vision [20].

3. Quantum classical correspondence requires the coding of the quantum numbers of positive and negative energy parts of the zero energy state to the space-time geometry. Chern-Simons term with a measurement interaction term allows to obtain a coupling between the space-time geometry and the quantum numbers of super-conformal representations and stringy propagators. It took some time to realize that measurement interaction term can be regarded as a gauge part of the Kähler gauge potential. Since it is however present only for the Chern-Simons part of the action it affects the physics. For instance, the value of Kähler function identified both as Dirac determinant and directly as Chern-Simons term is affected and therefore also the preferred extremal is affected.

4. Kähler Dirac operator D_K annihilates the induced spinor fields in the interior of space-time surface. At the ends of the space-time surface induces spinors are generalized eigenstates of $D_{C-S}+Q$, where Q represents the measurement interaction which effectively reduces to a gauge part in Kähler gauge potential. The generalized eigenvalue is the analog of the action of ordinary Dirac operator $p_k\gamma^k$ but the pseudo-momentum λ_k replacing p_k is not the real momentum. For instance, by number theoretic considerations this pseudo-momentum is in preferred plane M^2 of M^4 assigned to a given CD in zero energy ontology. λ_k is quantized and Dirac propagator defined by Chern-Simons Dirac action reduces effectively to a massless propagator suggesting that twistor formalism can be applied almost as such.

5. An interesting conjecture is that any light-like 3-surfaces in the slicing of space-time surface by light-like 3-surfaces "parallel" to wormhole throat is physically equivalent and the Kähler functions obtained as Dirac determinants differ only by a gauge transformation for Kähler function: $K \rightarrow K + f + \overline{f}$, where f is holomorphic function of complex coordinates of WCW and a priori arbitrary function of zero modes. Also measurement interaction term induces only this kind of gauge change but replaces the space-time surface with a new one.

5.3 Quantum criticality and modified Dirac action

The precise mathematical formulation of quantum criticality has remained one of the basic challenges of quantum TGD. The question leading to a considerable progress in the problem was simple: Under what conditions the modified Dirac action allows to assign conserved fermionic currents with the deformations of the space-time surface? The answer was equally simple: These currents exists only if these deformations correspond to vanishing second variations of Kähler action - which is what criticality is. The vacuum degeneracy of Kähler action strongly suggests that the number of critical deformations is always infinite and that these deformations define an infinite inclusion hierarchy of super-conformal algebras. This inclusion hierarchy would correspond to a fractal hierarchy of breakings of super-conformal symmetry generalizing the symmetry breaking hierarchies of gauge theories. These super-conformal inclusion hierarchies would realize the inclusion hierarchies for hyper-finite factors of type II_1.

5.3.1 Quantum criticality and fermionic representation of conserved charges associated with second variations of Kähler action

It is rather obvious that TGD allows a huge generalizations of conformal symmetries. The development of the understanding of conservation laws has been slow. In the approach based on D_{C-S} the non-conservation of gauge charges posed the basic problem and led to the introduction of the gauge part A_a of Kähler gauge potential. Modified Dirac action provides excellent candidates for quantum counterparts of Noether charges. Unfortunately, the isometry charges vanish for Cartan algebras. The only manner to obtain non-trivial isometry charges is to add a direct coupling to the charges in Cartan algebra as will be found later.

Conservation of the fermionic current requires the vanishing of the second variation of Kähler action

1. The modified Dirac action assigns to a deformation of the space-time surface a conserved charge expressible as bilinears of fermionic oscillator operators only if the first variation of the modified Dirac action under this deformation vanishes. The vanishing of the first variation for the modified Dirac action is equivalent with the vanishing of the second variation for the Kähler action. This can be seen by the explicit calculation of the second variation of the modified Dirac action and by performing partial integration for the terms containing derivatives of Ψ and $\overline{\Psi}$ to give a total divergence representing the difference of the charge at upper and lower boundaries of the causal diamond plus a four-dimensional integral of the divergence term defined as the integral of the quantity

$$\begin{aligned} \Delta S_D &= \overline{\Psi}\Gamma^k D_\alpha J_k^\alpha \Psi \ , \\ J_k^\alpha &= \frac{\partial^2 L_K}{\partial h_\alpha^k \partial h_\beta^l}\delta h_\beta^k + \frac{\partial^2 L_K}{\partial h_\alpha^k \partial h^l}\delta h^l \ . \end{aligned} \tag{5.3.1}$$

   Here h_β^k denote partial derivative of the imbedding space coordinate with respect to space-time coordinates. This term must vanish:

$$D_\alpha J_k^\alpha = 0 \ .$$

   The condition states the vanishing of the second variation of Kähler action. This can of course occur only for preferred deformations of X^4. One could consider the possibility that these deformations vanish at light-like 3-surfaces or at the boundaries of CD. Note that covariant divergence is in question so that J_k^α does not define conserved classical charge in the general case.

2. It is essential that the modified Dirac equation holds true so that the modified Dirac action vanishes: this is needed to cancel the contribution to the second variation coming from the determinant of the induced metric. The condition that the modified Dirac equation is satisfied for the deformed space-time surface requires that also Ψ suffers a transformation determined by the deformation. This gives

$$\delta\Psi = -\frac{1}{D} \times \Gamma^k J_k^\alpha \Psi \ . \tag{5.3.2}$$

   Here $1/D$ is the inverse of the modified Dirac operator defining the counterpart of the fermionic propagator.

3. The fermionic conserved currents associated with the deformations are obtained from the standard conserved fermion current

$$J^\alpha = \overline{\Psi}\Gamma^\alpha\Psi \ . \tag{5.3.3}$$

   Note that this current is conserved only if the space-time surface is extremal of Kähler action: this is also needed to guarantee Hermiticity and same form for the modified Dirac equation for Ψ and its conjugate as well as absence of mass term essential for super-conformal invariance. Note also that ordinary divergence rather only covariant divergence of the current vanishes.

   The conserved currents are expressible as sums of three terms. The first term is obtained by replacing modified gamma matrices with their increments in the deformation keeping Ψ and its conjugate constant. Second term is obtained by replacing Ψ with its increment $\delta\Psi$. The third term is obtained by performing same operation for $\delta\overline{\Psi}$.

$$J^{\alpha} = \overline{\Psi}\Gamma^k J_k^{\alpha}\Psi + \overline{\Psi}\hat{\Gamma}^{\alpha}\delta\Psi + \delta\overline{\Psi}\hat{\Gamma}^{\alpha}\Psi \ . \tag{5.3.4}$$

These currents provide a representation for the algebra defined by the conserved charges analogous to a fermionic representation of Kac-Moody algebra.

4. Also conserved super charges corresponding to super-conformal invariance are obtained. The first class of super currents are obtained by replacing Ψ or $\overline{\Psi}$ right handed neutrino spinor or its conjugate in the expression for the conserved fermion current and performing the above procedure giving two terms since nothing happens to the covariantly constant right handed-neutrino spinor. Second class of conserved currents is defined by the solutions of the modified Dirac equation interpreted as c-number fields replacing Ψ or $\overline{\Psi}$ and the same procedure gives three terms appearing in the super current.

5. The existence of vanishing of second variations is analogous to criticality in systems defined by a potential function for which the rank of the matrix defined by second derivatives of the potential function vanishes at criticality. Quantum criticality becomes the prerequisite for the existence of quantum theory since fermionic anti-commutation relations in principle can be fixed from the condition that the algebra in question is equivalent with the algebra formed by the vector fields defining the deformations of the space-time surface defining second variations. Quantum criticality in this sense would also select preferred extremals of Kähler action as analogs of Bohr orbits and the the spectrum of preferred extremals would be more or less equivalent with the expected existence of infinite-dimensional symmetry algebras.

About the general structure of the algebra of conserved charges

Some general comments about the structure of the algebra of conserved charges are in order.

1. Any Cartan algebra of the isometry group $P\times SU(3)$ (there are two types of them for P corresponding to linear and cylindrical Minkowski coordinates) defines critical deformations (one could require that the isometries respect the geometry of CD). The corresponding charges are conserved but vanish since the corresponding conjugate coordinates are cyclic for the Kähler metric and Kähler form so that the conserved current is proportional to the gradient of a Killing vector field which is constant in these coordinates. Therefore one cannot represent isometry charges as fermionic bilinears. Four-momentum and color quantum numbers are defined for Kähler action as classical conserved quantities but this is probably not enough. This can be seen as a problem.

   (a) Four-momentum and color Cartan algebra emerge naturally in the representations of super-conformal algebras. In the case of color algebra the charges in the complement of the Cartan algebra can be constructed in standard manner as extension of those for the Cartan algebra using free field representation of Kac-Moody algebras. In string theories four-momentum appears linearly in bosonic Kac-Moody generators and in the Sugawara representation [48] of super Virasoro generators as bilinears of bosonic Kac-Moody generators and fermionic super Kac-Moody generators. Also now quantized transversal parts for M^4 coordinates could define a second quantized field having interpretation as an operator acting on spinor fields of WCW. The angle coordinates conjugate to color isospin and hypercharge take the role of M^4 coordinates in case of CP_2.

   (b) Somehow one should be able to feed the information about the super-conformal representation of the isometry charges to the modified Dirac action by adding to it a term coupling fermionic current to the Cartan charges in general coordinate invariant and isometry invariant manner. As will be shown later, this is possible. The interpretation is as measurement interaction guaranteing also the stringy character of the fermionic propagators. The values of the couplings involved are fixed by the condition of quantum criticality assumed in the sense that Kähler function of WCW suffers only a $U(1)$ gauge transformation $K \rightarrow K + f + \overline{f}$, where f is a holomorphic function of WCW coordinates depending also on zero modes.

(c) The simplest addition involves the modified gamma matrices defined by a Chern-Simon term at the light-like wormhole throats and is sum of Chern-Simons Dirac action and corresponding coupling term linear in Cartan charges assignable to the partonic 2-surfaces at the ends of the throats. Hence the modified Dirac equation in the interior of the space-time sheet is not affected and nothing changes as far as quantum criticality in interior is considered.

2. The action defined by four-volume gives a first glimpse about what one can expect. In this case modified gamma matrices reduce to the induced gamma matrices. Second variations satisfy d'Alembert type equation in the induced metric so that the analogs of massless fields are in question. Mass term is present only if some dimensions are compact. The vanishing of excitations at light-like boundaries is a natural boundary condition and might well imply that the solution spectrum could be empty. Hence it is quite possible that four-volume action leads to a trivial theory.

3. For the vacuum extremals of Kähler action the situation is different. There exists an infinite number of second variations and the classical non-determinism suggests that deformations vanishing at the light-like boundaries exist. For the canonical imbedding of M^4 the equation for second variations is trivially satisfied. If the CP_2 projection of the vacuum extremal is one-dimensional, the second variation contains a on-vanishing term and an equation analogous to massless d'Alembert equation for the increments of CP_2 coordinates is obtained. Also for the vacuum extremals of Kähler action with 2-D CP_2 projection all terms involving induced Kähler form vanish and the field equations reduce to d'Alembert type equations for CP_2 coordinates. A possible interpretation is as the classical analog of Higgs field. For the deformations of non-vacuum extremals this would suggest the presence of terms analogous to mass terms: these kind of terms indeed appear and are proportional to δs_k. M^4 degrees of freedom decouple completely and one obtains QFT type situation.

4. The physical expectation is that at least for the vacuum extremals the critical manifold is infinite-dimensional. The notion of finite measurement resolution suggests infinite hierarchies of inclusions of hyper-finite factors of type II_1 possibly having interpretation in terms of inclusions of the super conformal algebras defined by the critical deformations.

5. The properties of Kähler action give support for this expectation. The critical manifold is infinite-dimensional in the case of vacuum extremals. Canonical imbedding of M^4 would correspond to maximal criticality analogous to that encountered at the tip of the cusp catastrophe. The natural guess would be that as one deforms the vacuum extremal the previously critical degrees of freedom are transformed to non-critical ones. The dimension of the critical manifold could remain infinite for all preferred extremals of the Kähler action. For instance, for cosmic string like objects any complex manifold of CP_2 defines cosmic string like objects so that there is a huge degeneracy is expected also now. For CP_2 type vacuum extremals M^4 projection is arbitrary light-like curve so that also now infinite degeneracy is expected for the deformations.

Critical super algebra and zero modes

The relationship of the critical super-algebra to configuration space geometry is interesting.

1. The vanishing of the second variation plus the identification of Kähler function as a Kähler action for preferred extremals means that the critical variations are orthogonal to all deformations of the space-time surface with respect to the configuration space metric and thus correspond to zero modes. This conforms with the fact that configuration space metric vanishes identically for canonically imbedded M^4. Zero modes do not seem to correspond to gauge degrees of freedom so that the super-conformal algebra associated with the zero modes has genuine physical content.

2. Since the action of X^4 local Hamiltonians of $\delta M^4_{\times} CP_2$ corresponds to the action in quantum fluctuating degrees of freedom, critical deformations cannot correspond to this kind of Hamiltonians.

3. The notion of finite measurement resolution suggests that the degrees of freedom which are below measurement resolution correspond to vanishing gauge charges. The sub-algebras of critical super-conformal algebra for which charges annihilate physical states could correspond to this kind of gauge algebras.

4. The conserved super charges associated with the vanishing second variations cannot give configuration space metric as their anti-commutator. This would also lead to a conflict with the effective 2-dimensionality stating that the configuration space line-element is expressible as sum of contribution coming from partonic 2-surfaces as also with fermionic anti-commutation relations.

Connection with quantum criticality

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical. Basic example of criticality is bifurcation diagram for cusp catastrophe. For some mysterious reason I failed to realize that quantum criticality realized as the vanishing of the second variation makes possible a more or less unique identification of preferred extremals and considered alternative identifications such as absolute minimization of Kähler action which is just the opposite of criticality. Both the super-symmetry of D_K and conservation Dirac Noether currents for modified Dirac action have thus a connection with quantum criticality.

1. Finite-dimensional critical systems defined by a potential function $V(x^1, x^2, ..)$ are characterized by the matrix defined by the second derivatives of the potential function and the rank of system classifies the levels in the hierarchy of criticalities. Maximal criticality corresponds to the complete vanishing of this matrix. Thom's catastrophe theory classifies these hierarchies, when the numbers of behavior and control variables are small (smaller than 5). In the recent case the situation is infinite-dimensional and the criticality conditions give additional field equations as existence of vanishing second variations of Kähler action.

2. The vacuum degeneracy of Kähler action allows to expect that this kind infinite hierarchy of criticalities is realized. For a general vacuum extremal with at most 2-D CP_2 projection the matrix defined by the second variation vanishes because $J_{\alpha\beta} = 0$ vanishes and also the matrix $(J^{\alpha}_{\ k} + J_k^{\ \alpha})(J^{\beta}_{\ l} + J_l^{\ \beta})$ vanishes by the antisymmetry $J^{\alpha}_{\ k} = -J_k^{\ \alpha}$. Recall that the formulation of Equivalence Principle in string picture demonstrated that the reduction of stringy dynamics to that for free strings requires that second variation with respect to M^4 coordinates vanish. This condition would guarantee the conservation of fermionic Noether currents defining gravitational four-momentum and other Poincare quantum numbers but not those for gravitational color quantum numbers. Encouragingly, the action of CP_2 type vacuum extremals having random light-like curve as M^4 projection have vanishing second variation with respect to M^4 coordinates (this follows from the vanishing of Kähler energy momentum tensor, second fundamental form, and Kähler gauge current). In this case however the momentum is vanishing.

3. Conserved bosonic and fermionic Noether charges would characterize quantum criticality. In particular, the isometries of the imbedding space define conserved currents represented in terms of the fermionic oscillator operators if the second variations defined by the infinitesimal isometries vanish for the modified Dirac action. For vacuum extremals the dimension of the critical manifold is infinite: maybe there is hierarchy of quantum criticalities for which this dimension decreases step by step but remains always infinite. This hierarchy could closely relate to the hierarchy of inclusions of hyper-finite factors of type II_1. Also the conserved charges associated with super-symplectic and Super Kac-Moody algebras would require infinite-dimensional critical manifold defined by the spectrum of second variations.

4. Phase transitions are characterized by the symmetries of the phases involved with the transitions, and it is natural to expect that dynamical symmetries characterize the hierarchy of quantum criticalities. The notion of finite quantum measurement resolution based on the hierarchy of Jones inclusions indeed suggests the existence of a hierarchy of dynamical gauge symmetries characterized by gauge groups in ADE hierarchy [20] with degrees of freedom below the measurement resolution identified as gauge degrees of freedom.

5. Does this criticality have anything to do with the criticality against the phase transitions changing the value of Planck constant? If the geodesic sphere S^2_I for which induced Kähler form vanishes corresponds to the back of the CP_2 book (as one expects), this could be the case. The homologically non-trivial geodesic sphere S^12_{II} is as far as possible from vacuum extremals. If it corresponds to the back of CP_2 book, cosmic strings would be quantum critical with respect to phase transition changing Planck constant. They cannot however correspond to preferred extremals.

5.3.2 Preferred extremal property as classical correlate for quantum criticality, holography, and quantum classical correspondence

The Noether currents assignable to the modified Dirac equation are conserved only if the first variation of the modified Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X^3_l)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago! The question whether these extremals correspond to absolute minima remains however open.

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X^3_l)$ vanishing at the intersections of $X^4(X^3_l)$ wth the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X^3_l with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of configuration space represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the configuration space metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \rightarrow X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X^3_l)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X^3_l involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

5. There is a possible connection with the notion of self-organized criticality [49] introduced to explain the behavior of systems like sand piles. Self-organization in these systems tends to lead "to the edge". The challenge is to understand how system ends up to a critical state, which by definition is unstable. Mechanisms for this have been discovered and based on phase transitions occurring in a wide range of parameters so that critical point extends to a critical manifold. In TGD Universe quantum criticality suggests a universal mechanism of this kind. The criticality for the preferred extremals of Kähler action would mean that classically all systems are critical in well-defined sense and the question is only about the degree of criticality. Evolution could be seen as a process leading gradually to increasingly critical systems. One must however distinguish between the criticality associated with the preferred extremals of Kähler action and the criticality caused by the spin glass like energy landscape like structure for the space of the maxima of Kähler function.

5.4 Handful of problems with a common resolution

Theory building could be compared to pattern recognition or to a solving a crossword puzzle. It is essential to make trials, even if one is aware that they are probably wrong. When stares long enough to the letters which do not quite fit, one suddenly realizes what one particular crossword must actually be and it is soon clear what those other crosswords are. In the following I describe an example in which this analogy is rather concrete. Let us begin by listing the problems.

1. The condition that modified Dirac action allows conserved charges leads to the condition that the symmetries in question give rise to vanishing second variations of Kähler action. The interpretation is as quantum criticality and there are good arguments suggesting that the critical symmetries define an infinite-dimensional super-conformal algebra forming an inclusion hierarchy related to a sequence of symmetry breakings closely related to a hierarchy of inclusions of hyper-finite factors of types II_1 and III_1. This means an enormous generalization of the symmetry breaking patterns of gauge theories.

   There is however a problem. For the translations of M^4 and color hyper charge and isospin (more generally, any Cartan algebra of $P \times SU(3)$) the resulting fermionic charges vanish. The trial for the crossword in absence of nothing better would be the following argument. By the abelianity of these charges the vanishing of quantal representation of four-momentum and color Cartan charges is not a problem and that classical representation of these charges or their super-conformal representation is enough.

2. Modified Dirac equation is satisfied in the interior of space-time surface always. This means that one does not obtain off-mass shell propagation at all in 4-D sense. Effective 2-dimensionality suggests that off mass shell propagation takes place along wormhole throats. The reduction to almost topological QFT with Kähler function reducing to Chern-Simonst type action implied by the weak form of electric-magnetic duality and a proper gauge choice for the induced Kähler gauge potential implies effective 3-dimensionality at classical level. This inspires the question whether Chern-Simons type action resulting from an instanton term could define the modified gamma matrices appearing in the 3-D modified Dirac action associated with wormhole throats and ends of the space-time sheet at the boundaries of CD. The assumption that modified Dirac equation is satisfied also at the ends and wormhole throats would realize effective 2-dimensionality as conditions on the boundary values of the 4-D Dirac equation but would would not allow off mass shell propagation. Therefore one could argue that effective 2-dimensionality holds true only for incoming and outgoing particles. The reduction of Kähler action to generalized Chern-Simons term means that the maxima of Kähler function should correspond to extrema of this action. The presence of also the Chern-Simons term corresponding to $J + J_1$ would give these extrema.

3. Quantum classical correspondence requires that the geometry of the space-time sheet should correlate with the quantum numbers characterizing positive (negative) energy part of the quantum state. One could argue that by multiplying WCW spinor field by a suitable phase factor depending on the charges of the state, the correspondence follows from stationary phase approx-

imation. This crossword looks unconvincing. A more precise connection between quantum and classical is required.

4. In quantum measurement theory classical macroscopic variables identified as degrees of freedom assignable to the interior of the space-time sheet correlate with quantum numbers. Stern Gerlach experiment is an excellent example of the situation. The generalization of the imbedding space concept by replacing it with a book like structure implies that imbedding space geometry at given page and for given causal diamond (CD) carries information about the choice of the quantization axes (preferred plane M^2 of M^4 *resp.* geodesic sphere of CP_2 associated with singular covering/factor space of CD *resp.* CP_2). This is a big step but not enough. Modified Dirac action as such does not seem to provide any hint about how to achieve this correspondence. One could even wonder whether dissipative processes or at least the breaking of T and CP characterizing the outcome of quantum jump sequence should have space-time correlate. How to achieve this?

Each of these problems makes one suspect that something is lacking from the modified Dirac action: there should exist an elegant manner to feed information about quantum numbers of the state to the modified Dirac action in turn determining vacuum functional as an exponent Kähler function identified as Kähler action for the preferred extremal assumed to be dictated by by quantum criticality and equivalently by hyper-quaternionicity.

This observation leads to what might be the correct question. Could a general coordinate invariant and Poincare invariant modification of the modified Dirac action consistent with the vacuum degeneracy of Kähler action allow to achieve this information flow somehow? In the following one manner to achieve this modification is discussed. It must be however emphasized that I have considered many alternatives and the one discussed below finds its justification only from the fact that it is the simplest one found hitherto.

5.4.1 The identification of the measurement interaction term

The idea is simple: add to the modified Dirac action a term which is analogous to the Dirac action in $M^4 \times CP_2$. One can consider two options according to whether the term is assigned with interior or with a 3-D light-like 3-surface and last years have been continual argumentation about which option is the correct one.

1. The additional term would be essentially the analog of the ordinary Dirac action at the imbedding space level.

$$\begin{aligned} S_{int} &= \sum_A Q_A \int \overline{\Psi} g^{AB} j_{B\alpha} \hat{\Gamma}^\alpha \Psi \sqrt{g} d^4x \ , \\ g_{AB} &= j_A^k h_{kl} j_B^l \ , \ g^{AB} g_{BC} = \delta^A_C \ , \\ j_{B\alpha} &= j_B^k h_{kl} \partial_\alpha h^l \ . \end{aligned} \quad (5.4.1)$$

The sum is over isometry charges Q_A interpreted as quantal charges and j^{Ak} denotes the Killing vector field of the isometry. g^{AB} is the inverse of the tensor g_{AB} defined by the local inner products of Killing vectors fields in M^4 and CP_2. The space-time projections of the Killing vector fields $j_{B\alpha}$ have interpretation as classical color gauge potentials in the case of $SU(3)$. In M^4 degrees of freedom and for Cartan algebra of $SU(3)$ $j_{B\alpha}$ reduce to the gradients of linear M^4 coordinates in case of translations. Modified gamma matrices could be assigned to Kähler action or its instanton term or with Chern-Simons action.

2. The added term containing quantal charges must make sense in the modified Dirac equation. This requires that the physical state is an eigenstate of momentum and color charges. This allows only color hyper-charge and color isospin so that there is no hope of obtaining exactly the stringy formula for the propagator. The modified Dirac operator is given by

$$\begin{aligned} D_{tot} &= D + D_{int} = \hat{\Gamma}^{\alpha} D_{\alpha} + \hat{\Gamma}^{\alpha} \sum_{A} Q_{A} g^{AB} j_{B\alpha} \\ &= \hat{\Gamma}^{\alpha}(D_{\alpha} + \partial_{\alpha}\phi) \ , \ \partial_{\alpha}\phi = \sum_{A} Q_{A} g^{AB} j_{B\alpha} \ . \end{aligned} \tag{5.4.2}$$

The conserved fermionic isometry currents are

$$J^{A\alpha} = \sum_{B} Q_{B} \overline{\Psi} g^{BC} j^{k}_{C} h_{kl} j^{l}_{A} \hat{\Gamma}^{\alpha} \Psi = Q_{A} \overline{\Psi} \hat{\Gamma}^{\alpha} \Psi \ . \tag{5.4.3}$$

Here the sum is restricted to a Cartan sub-algebra of Poincare group and color group.

3. An important restriction is that by four-dimensionality of M^4 and CP_2 the rank of g_{AB} is 4 so that g^{AB} exists only when one considers only four conserved charges. In the case of M^4 this is achieved by a restriction to translation generators $Q_A = p_A$. g_{AB} reduces to Minkowski metric and Killing vector fields are constants. The Cartan sub-algebra could be however replaced by any four commuting charges in the case of Poincare algebra (second one corresponds to time translation plus translation, boost and rotation in given direction). In the case of $SU(3)$ one must restrict the consideration either to $U(2)$ sub-algebra or its complement. $CP_2 = SU(3)/SU(2)$ decomposition would suggest the complement as the correct choice. One can indeed build the generators of $U(2)$ as commutators of the charges in the complement. On the other hand, Cartan algebra is enough in free field construction of Kac-Moody algebras.

4. What is remarkable that for the Cartan algebra of $M^4 \times SU(3)$ the measurement interaction term is equivalent with the addition of gauge part $\partial_\alpha\phi$ of the induced Kähler gauge potential A_α. This property might hold true for any measurement interaction term. This also suggests that the change in Kähler function is only the transformation $A_\alpha \rightarrow A_\alpha + \partial_\alpha\phi$, $\partial_\alpha\phi = \sum_A Q_A g^{AB} j_{B\alpha}$.

5. Recall that the ϕ for $U(1)$ gauge transformations respecting the vanishing of the Coulomb interaction term of Kähler action [34] the current $j^{\alpha}_{K}\phi$ is conserved, which implies that the change of the Kähler action is trivial. These properties characterize the gauge transformations respecting the gauge in which Coulombic interaction term of the Kähler action vanishes so that Kähler action reduces to 3-dimensional generalized Chern-Simons term if the weak form of electric-magnetic duality holds true guaranteing among other things that the induced Kähler field is not too singular at the wormhole throats [34]. The scalar function assignable to the measurement interaction terms does not have this property and this is what is expected since it must change the value of the Kähler function and therefore affect the preferred extremal.

   The reduction to 3-D form however gives a non-trivial WCW metric in M^4 degrees of freedom only if one replaces CP_2 Kähler form J with the sum $J + J_1$, where J_1 is the Kähler form of the $r_M = constant$ sphere so that the time-like line connecting the tips of CD carries monopole charge [34]. This enriches dramatically the vacuum sector of the theory giving better hopes about a realistic description of gravitation in long length scales. The basic non-vacuum extremals of Kähler action are not lost.

Concerning the precise form of the modified Dirac action the basic clue comes from the observation that the measurement interaction term corresponds to the addition of a gauge part to the induced CP_2 Kähler gauge potential A_α. The basic question is what part of the action one assigns the measurement interaction term.

1. One could define the measurement interaction term using either the four-dimensional instanton term or its reduction to Chern-Simons terms. The part of Dirac action defined by the instanton term in the interior does not reduce to a 3-D form unless the Dirac equation defined by the instanton term is satisfied : this cannot be true. Hence Chern-Simons term is the only possibility.

The classical field equations associated with the Chern-Simons term cannot be assumed since they would imply that the CP_2 projection of the wormhole throat and space-like 3-surface are 2-dimensional. This might hold true for space-like 3-surfaces at the ends of CD and incoming and outgoing particles but not for off mass shell particles. This is however not a problem since $D_\alpha \hat{\Gamma}^\alpha_{C-S}$ for the modified gamma matrices for Chern-Simons action does not contain second derivatives. This is due to the topological character of this term. For Kähler action second derivatives are present and this forces extremal property of Kähler action in the modified Dirac Kähler action so that classical physics resuts as a consistency condition.

2. If one assigns measurement interaction term to both D_K and D_{C-S} the measurement interaction corresponds to a mere gauge transformation for AS_α and is trivial. Therefore it seems that one must choose between D_K or D_{C-S}. At least formally the measurement interaction term associated with D_K is gauge equivalent with its negative D_{C-S}. The addition of the measurement interaction to D_K changes the basis for the 4-D induced spinors by the phase $exp(-iQK\phi)$ and therefore also the basis for the generalized eigenstates of D_{C-S} and this brings in effectively the measurement interaction term affecting the Dirac determinant.

3. The definition of Dirac determinant should be in terms of Chern-Simons action induced by the instanton term and identified as a product of the generalized eigenvalues of this operator. The modified Dirac equation for Ψ is consistent with that for its conjugate if the coefficient of the instanton term is real and one uses the Dirac action $\overline{\Psi}(D^{\rightarrow} - D^{\leftarrow})\Psi$ giving modified Dirac equation as

$$D_{C-S}\Psi + \frac{1}{2}(D_\alpha \hat{\Gamma}^\alpha_{C-S})\Psi = 0 \ . \tag{5.4.4}$$

As noticed, the divergence of gamma matrices does not contain second derivatives in the case of Chern-Simons action. In the case of Kähler action they occur unless field equations equivalent with the vanishing of the divergence term are satisfied.

Also the fermionic current is conserved in this case, which conforms with the idea that fermions flow along the light-like 3-surfaces. If one uses the action $\overline{\Psi}D^{\rightarrow}\Psi$, $\overline{\Psi}$ does not satisfy the Dirac equation following from the variational principle and fermion current is not conserved. Also if the Chern-Simons term is imaginary - as a naive idea about dissipation would suggest- the Dirac equation fails to be consistent with the conjugation.

4. Off mass shell states appear in the lines of the generalized Feynman diagrams and for these D_{C-S} cannot annihilate the spinor field. The generalized eigenmodes lf D_{C-S} should be such that one obtains the counterpart of Dirac propagator which is purely algebraic and does not therefore depend on the coordinates of the throat. This is satisfied if the generalized eigenvalues are expressible in terms of covariantly constant combinations of gamma matrices and here only M^4 gamma matrices are possible. Therefore the eigenvalue equation reqads as

$$D\Psi = \lambda^k\gamma_k\Psi \ , \ D = D_{C-S} + D_\alpha\hat{\Gamma}^\alpha_{C-S} \ , \ D_{C-S} = \hat{\Gamma}^\alpha_{C-S}D_\alpha \ . \tag{5.4.5}$$

Here the covariant derivatives D_α contain the measurement interaction term as an apparent gauge term. Covariant constancy allows to take the square of this equation and one has

$$(D^2 + [D, \lambda^k\gamma_k])\Psi = \lambda^k\lambda_k\Psi \ . \tag{5.4.6}$$

The commutator term is analogous to magnetic moment interaction. The generalized eigenvalues correspond to $\lambda = \sqrt{\lambda^k\lambda_k}$ and Dirac determinant is defined as a product of the eigenvalues.

λ is completely analogous to mass. For incoming lines this mass would vanish so that all incoming particles irrespective their actual quantum numbers would be massless in this sense and the propagator is indeed that for a massless particle. Note that the eigen-modes define the boundary values for the solutions of $D_K\Psi = 0$ so that the values of λ indeed define the counterpart of the momentum space.

This transmutation of massive particles to effectively massless ones might make possible the application of the twistor formalism as such in TGD framework [28]. $N = 4$ SUSY is one of the very few gauge theory which might be UV finite but it is definitely unphysical due to the masslessness of the basic quanta. Could the resolution of the interpretational problems be that the four-momenta appearing in this theory do not directly correspond to the observed four-momenta?

5.4.2 Objections

The alert reader has probably raised several critical questions. Doesn't the need to solve λ_k as functions of incoming quantum numbers plus the need to construct the measurement interactions makes the practical application of the theory hopelessly difficult? Could the resulting pseudo-momentum λ_k correspond to the actual four-momentum? Could one drop the measurement interaction term altogether and assume that the quantum classical correspondence is through the identification of the eigenvalues as the four-momenta of the on mass shell particles propagating at the wormhole throats? Could one indeed assume that the momenta have a continuous spectrum and thus do not depend on the boundary conditions at all? Usually the thinking is just the opposite and in the general case would lead to to singular eigen modes.

1. Only the information about four-momentum would be feeded into the space-time geometry. TGD however allows much more general measurement interaction terms and it would be very strange if the space-time geometry would not correlate also with the other quantum numbers. Mass formulas would of course contain information also about other quantum numbers so that this claim is not quite justified.

2. Number theoretic considerations and also the construction of octonionic variant of Dirac equation [19] force the conclusion that the spectrum of pseudo four-momentum is restricted to a preferred plane M^2 of M^4 and this excludes the interpretation of λ^k as a genuine four-momentum. It also improves the hopes that the sum over pseudo-momenta does not imply divergences.

3. Dirac determinant would depend on the mass spectrum only and could not be identified as exponent of Kähler function. Note that the original guideline was the dream about stringy propagators. This is achieved for $\lambda_A\lambda^A = n$ in suitable units. This spectrum would of coure also imply that Dirac determinant defined in terms of ζ function regularization is independent of the space-time surface and could not be identified with the exponent of Kähler function. One must of course take the identification of exponent of Kähler function as Dirac determinant as an additional conjecture which is not necessary for the calculation of Kähler function if the weak form of electric-magnetic duality is accepted.

4. All particles would behave as massless particles and this would not be consistent with the proposed Feynman diagrammatics inspired by zero energy ontology. Since wormhole throats carry on mass shell particles with positive or negative energy so that the net momentum can be also space-like propagators diverge for massless particles. One might overcome this problem by assuming small thermal mass (from p-adic thermodynamics [4]) and this is indeed assumed to reduce the number of generalized Feynman diagrams contributing to a given reaction to finite number.

Second objection of the skeptic reader relates to the delicacies of $U(1)$ gauge invariance. The modified Dirac action seems to break gauge symmetries and this breaking of gauge symmetry is absolutely essential for the dependence of the Dirac determinant on the quantum numbers. It however seems that this breaking of gauge invariance is only apparent.

1. One must distinguish between genuine $U(1)$ gauge transformations carried out for the induced Kähler gauge potential A_α and apparent gauge transformations of the Kähler gauge potential

A_k of $S^2 \times CP_2$ induced by symplectic transformations deforming the space-time surface and affect also induced metric. This delicacy of $U(1)$ gauge symmetry explains also the apparent breaking of $U(1)$ gauge symmetry of Chern-Simons Dirac action due to the presence of explicit terms A_k and A_α.

2. CP_2 Kähler gauge potential is obtained in complex coordinates from Kähler function as $(K_{\xi^i}, K_{\overline{\xi^i}}) = (\partial_{\xi^i}K, -\partial_{\overline{\xi^i}}K)$. Gauge transformations correspond to the additions $K \rightarrow K+f+\overline{f}$, where f is a holomorphic function. Kähler gauge potential has a unique gauge in which the Kähler function of CP_2 is $U(2)$ invariant and contains no holomorphic part. Hence A_k is defined in a preferred gauge and is a gauge invariant quantity in this sense. Same applies to S^2 part of the Kähler potential.

3. A_α should be also gauge invariant under gauge transformation respecting the vanishing of Coulombic interaction energy. The allowed gauge transformations $A_\alpha \rightarrow A_\alpha + \partial_\alpha \phi$ must satisfy $D_\alpha(j^\alpha_K \phi) = 0$. If the scalar function ϕ reduces to constant at the wormhole throats and at the ends of the space-time surface D_{C-S} is gauge invariant. The gauge transformations for which ϕ does not satisfy this condition are identified as representations of critical deformations of space-time surface so that the change of A_α would code for this kind of deformation and indeed affect the modified Dirac operator and Kähler function (the change would be due to the change of zero modes).

5.4.3 Some details about the modified Dirac equation defined by Chern-Simons action

First some general comments about D_{C-S} are in order.

1. Quite generally, there is vacuum avoidance in the sense that Ψ must vanish in the regions where the modified gamma matrices vanish. A physical analogy for the system consider is a charged particle in an external magnetic field. The effective metric defined by the anticommutators of the modified gamma matrices so that standard intuitions might not help much. What one would naively expect would be analogs of bound states in magnetic field localized into regions inside which the magnetic field is nonvanishing.

2. If only CP_2 Kähler form appears in the Kähler action, the modified Dirac action defined by the Chern-Simons term is non-vanishing only when the dimension of the CP_2 projection of the 3-surface is $D(CP_2) \geq 2$ and the induced Kähler field is non-vanishing. This conforms with the properties of Kähler action. The solutions of the modified Dirac equation with a vanishing eigenvalue λ would naturally correspond to incoming and outgoing particles. $D(CP_2) \leq 2$ is inconsistent with the weak form of electric-magnetic duality unless one allows the presence of also S^2 symplectic form J_1 in the conditions (the value of Planck constant would be infinite [34]). The extrema of Chern-Simons action have $D(CP_2) \leq 2$ and vanishing Chern-Simons density so that they would naturally represent on mass shell particles appearing as incoming and outgoing particles. This conforms with the interpretation of the basic extremals as free particles (massless extremals and cosmic strings with 2-D CP_2 projection). One could say that CP breaking is not present for free particles but unavoidably accompanies the propagator lines.

3. If a reduction to almost topological QFT is assumed [34], a realistic WCW metric requires the replacement of J with $J + J_1$, where J_1 is S^2 Kähler form. An analogous replacement must be carried out also for the Chern-Simons term. In this case on can have a non-vanishing Ψ also for 1-dimensional CP_2 projection. On the other hand, one can have also 3-D CP_2 projection for vacuum regions and Ψ must vanish in these regions.

The explicit expression of D_{C-S} is given by

$$\begin{aligned} D &= \hat{\Gamma}^\mu D_\mu + \frac{1}{2} D_\mu \hat{\Gamma}^\mu \ , \\ \hat{\Gamma}^\mu &= \frac{\partial L_{C-S}}{\partial_\mu h^k}\Gamma_k = \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_\mu \ , \\ D_\mu \hat{\Gamma}^\mu &= B_K^\alpha (J_{k\alpha} + \partial_\alpha A_k) \ , \\ B_K^\alpha &= \epsilon^{\alpha\beta\gamma} J_{\beta\gamma} \ , \ J_{k\alpha} = J_{kl}\partial_\alpha s^l \ , \ \hat{\epsilon}^{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma}\sqrt{g_3} \ . \end{aligned} \tag{5.4.7}$$

Note $\hat{\epsilon}^{\alpha\beta\gamma} =$ does not depend on the induced metric.

The extremals of Chern-Simons action satisfy

$$B_K^\alpha (J_{kl} + \partial_l A_k)\partial_\alpha h^l = 0 \ , \ B_K^\alpha = \epsilon^{\alpha\beta\gamma} J_{\beta\gamma} \ . \tag{5.4.8}$$

For non-vanishing Kähler magnetic field B^α these equations hold true when CP_2 projection is 2-dimensional and S^2 projection is 1-dimensional or vice versa. This implies a vanishing of Chern-Simons action for both options. Consider for the simplicity the case when S^2 projection is 1-dimensional.

1. Suppose that one can assign a global coordinate to the flow lines of the Kähler magnetic field. In this case one might hope that ordinary intuitions about motion in constant magnetic field might be helpful. The repetition of the discussion of [34] leads to the condition $B \wedge dB = 0$ implying that a Beltrami flow for which current flows along the field lines and Lorentz forces vanishes is in question. This need not be the generic case.

2. With this assumption the modified Dirac operator reduces to a one-dimensional Dirac operator

$$D = \hat{\epsilon}^{r\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_r \ . \tag{5.4.9}$$

3. The general solutions of the modified Dirac equation is covariantly constant with respect to the coordinate r:

$$D_r \Psi = 0 \ . \tag{5.4.10}$$

   The solution to this condition can be written immediately in terms of a non-integrable phase factor $Pexp(i\int A_r dr)$, where integration is along curve with constant transversal coordinates. If $\hat{\Gamma^v}$ is light-like vector field also $\hat{\Gamma^v}\Psi_0$ defines a solution of D_{C-S}. This solution corresponds to a zero mode for D_{C-S} and does not contribute to the Dirac determinant. Note that the dependence of these solutions on transversal coordinates of X_l^3 is arbitrary.

4. The formal solution associated with a general eigenvalue can be constructed by integrating the eigenvalue equation separately along all coordinate curves. This makes sense if r indeed assigned to light-like curves indeed defines a global coordinate. What is strange that there is no correlation between the behaviors with respect longitudinal coordinate and transversal coordinates. System would be like a collection of totally uncorrelated point like particles reflecting the flow of the current along flux lines. It is difficult to say anything about the spectrum of the generalized eigenvalues in this case: it might be that the boundary conditions at the ends of the flow lines fix the allowed values of λ. Clearly, the Beltrami flow property is what makes this case very special.

5.4.4 A connection with quantum measurement theory

It is encouraging that isometry charges and also other charges could make themselves visible in the geometry of space-time surface as they should by quantum classical correspondence. This suggests an interpretation in terms of quantum measurement theory.

1. The interpretation resolves the problem caused by the fact that the choice of the commuting isometry charges is not unique. Cartan algebra corresponds naturally to the measured observables. For instance, one could choose the Cartan algebra of Poincare group to consist of energy and momentum, angular momentum and boost (velocity) in particular direction as generators of the Cartan algebra of Poincare group. In fact, the choices of a preferred plane $M^2 \subset M^4$ and geodesic sphere $S^2 \subset CP_2$ allowing to fix the measurement sub-algebra to a high degree are implied by the replacement of the imbedding space with a book like structure forced by the hierarchy of Planck constants. Therefore the hierarchy of Planck constants seems to be required by quantum measurement theory. One cannot overemphasize the importance of this connection.

2. One can add similar couplings of the net values of the measured observables to the currents whose existence and conservation is guaranteed by quantum criticality. It is essential that one maps the observables to Cartan algebra coupled to critical current characterizing the observable in question. The coupling should have interpretation as a replacement of the induced Kähler gauge potential with its gauge transform. Quantum classical correspondence encourages the identification of the classical charges associated with Kähler action with quantal Cartan charges. This would support the interpretation in terms of a measurement interaction feeding information to classical space-time physics about the eigenvalues of the observables of the measured system. The resulting field equations remain second order partial differential equations since the second order partial derivatives appear only linearly in the added terms.

3. What about the space-time correlates of electro-weak charges? The earlier proposal explains this correlation in terms of the properties of quantum states: the coupling of electro-weak charges to Chern-Simons term could give the correlation in stationary phase approximation. It would be however very strange if the coupling of electro-weak charges with the geometry of the space-time sheet would not have the same universal description based on quantum measurement theory as isometry charges have.

   (a) The hint as how this description could be achieved comes from a long standing un-answered question motivated by the fact that electro-weak gauge group identifiable as the holonomy group of CP_2 can be identified as $U(2)$ subgroup of color group. Could the electro-weak charges be identified as classical color charges? This might make sense since the color charges have also identification as fermionic charges implied by quantum criticality. Or could electro-weak charges be only represented as classical color charges by mapping them to classical color currents in the measurement interaction term in the modified Dirac action? At least this question might make sense.

   (b) It does not make sense to couple both electro-weak and color charges to the same fermion current. There are also other fundamental fermion currents which are conserved. All the following currents are conserved.

$$\begin{aligned} J^{\alpha} &= \overline{\Psi} O \hat{\Gamma}^{\alpha} \Psi \\ O &\in \{1 \ , \ J \equiv J_{kl}\Sigma^{kl} \ , \ \Sigma_{AB} \ , \ \Sigma_{AB}J\} \ . \end{aligned} \tag{5.4.11}$$

   Here J_{kl} is the covariantly constant CP_2 Kähler form and Σ_{AB} is the (also covariantly) constant sigma matrix of M^4 (flatness is absolutely essential).

   (c) Electromagnetic charge can be expressed as a linear combination of currents corresponding to $O = 1$ and $O = J$ and vectorial isospin current corresponds to J. It is natural to couple of electromagnetic charge to the the projection of Killing vector field of color hyper charge and coupling it to the current defined by $O_{em} = a+bJ$. This allows to interpret the puzzling finding that electromagnetic charge can be identified as anomalous color hyper-charge for induced spinor fields made already during the first years of TGD. There exist no conserved axial isospin currents in accordance with CVC and PCAC hypothesis which belong to the basic stuff of the hadron physics of old days.

(d) Color charges would couple naturally to lepton and quark number current and the $U(1)$ part of electro-weak charges to the $n = 1$ multiple of quark current and $n = 3$ multiple of the lepton current (note that leptons *resp.* quarks correspond to $t = 0$ *resp.* $t = \pm 1$ color partial waves). If electro-weak *resp.* couplings to H-chirality are proportional to 1 *resp.* Γ_9, the fermionic currents assigned to color and electro-weak charges can be regarded as independent. This explains why the possibility of both vectorial and axial couplings in 8-D sense does not imply the doubling of gauge bosons.

(e) There is also an infinite variety of conserved currents obtained as the quantum critical deformations of the basic fermion currents identified above. This would allow in principle to couple an arbitrary number of observables to the geometry of the space-time sheet by mapping them to Cartan algebras of Poincare and color group for a particular conserved quantum critical current. Quantum criticality would therefore make possible classical space-time correlates of observables necessary for quantum measurement theory.

(f) The coupling constants associated with the deformations would appear in the couplings. Quantum criticality ($K \rightarrow K + f + \overline{f}$ condition) should predict the spectrum of these couplings. In the case of momentum the coupling would be proportional to $\sqrt{G/\hbar_0} = kR/\hbar_0$ and $k \sim 2^{11}$ should follow from quantum criticality. p-Adic coupling constant evolution should follow from the dependence on the scale of CD coming as powers of 2.

4. Quantum criticality implies fluctuations in long length and time scales and it is not surprising that quantum criticality is needed to produce a correlation between quantal degrees of freedom and macroscopic degrees of freedom. Note that quantum classical correspondence can be regarded as an abstract form of entanglement induced by the entanglement between quantum charges Q_A and fermion number type charges assignable to zero modes.

5. Space-time sheets can have an arbitrary number of wormhole contacts so that the interpretation in terms of measurement theory coupling short and long length scales suggests that the measurement interaction terms are localizable at the wormhole throats. This would favor Chern-Simons term or possibly instanton term if reducible to Chern-Simons terms. The breaking of CP and T might relate to the fact that state function reductions performed in quantum measurements indeed induce dissipation and breaking of time reversal invariance.

6. The experimental arrangement quite concretely splits the quantum state to a quantum superposition of space-time sheets such that each eigenstate of the measured observables in the superposition corresponds to different space-time sheet already before the realization of state function reduction. This relates interestingly to the question whether state function reduction really occurs or whether only a branching of wave function defined by WCW spinor field takes place as in multiverse interpretation in which different branches correspond to different observers. TGD inspired theory consciousness requires that state function reduction takes place. Maybe multiversalist might be able to find from this picture support for his own beliefs.

7. One can argue that "free will" appears not only at the level of quantum jumps but also as the possibility to select the observables appearing in the modified Dirac action dictating in turn the Kähler function defining the Kähler metric of WCW representing the "laws of physics". This need not to be the case. The choice of CD fixes M^2 and the geodesic sphere S^2: this does not fix completely the choice of the quantization axis but by isometry invariance rotations and color rotations do not affect Kähler function for given CD and for a given type of Cartan algebra. In M^4 degrees of freedom the possibility to select the observables in two manners corresponding to linear and cylindrical Minkowski coordinates could imply that the resulting Kähler functions are different. The corresponding Kähler metrics do not differ if the real parts of the Kähler functions associated with the two choices differ by a term $f(Z) + \overline{f(Z)}$, where Z denotes complex coordinates of WCW, the Kähler metric remains the same. The function f can depend also on zero modes. If this is the case then one can allow in given CD superpositions of WCW spinor fields for which the measurement interactions are different. This condition is expected to pose non-trivial constraints on the measurement action and quantize coupling parameters appearing in it.

5.4.5 New view about gravitational mass and matter antimatter asymmetry

The physical interpretation of the additional term in the modified Dirac action might force quite a radical revision of the ideas about matter and antimatter.

1. The term $p_A\partial_\alpha m^A$ contracted with the fermion current is analogous to a gauge potential coupling to fermion number. Since the additional terms in the modified Dirac operator induce stringy propagation, a natural interpretation of the coupling to the induced spinor fields is in terms of gravitation. One might perhaps say that the measurement of four momentum induces gravitational interaction. Besides momentum components also color charges take the role of gravitational charges. As a matter fact, any observable takes this role via coupling to the projections of Killing vector fields of Cartan algebra. The analogy of color interactions with gravitational interactions is indeed one of the oldest ideas in TGD.

2. The coupling to four-momentum is through fermion number (both quark number and lepton number). For states with a vanishing fermion number isometry charges therefore vanish. In this framework matter antimatter asymmetry would be due to the fact that matter (antimatter) corresponds to positive (negative) energy parts of zero energy states for massive systems so that the contributions to the net gravitational four-momentum are of same sign. Could antimatter be unobservable to us because it resides at negative energy space-time sheets? As a matter fact, I proposed already years ago that gravitational mass is essentially the magnitude of the inertial mass but gave up this idea.

3. Bosons do not couple at all to gravitation if they are purely local bound states of fermion and anti-fermion at the same space-time sheet (say represented by generators of super Kac-Moody algebra). Therefore the only possible identification of gauge bosons is as wormhole contacts. If the fermion and anti-fermion at the opposite throats of the contact correspond to positive and negative energy states the net gravitational energy receives a positive contribution from both sheets. If both correspond to positive (negative) energy the contributions to the net four-momentum have opposite signs. It is not yet clear which identification is the correct one.

5.4.6 Generalized eigenvalues of D_{C-S} and General Coordinate Invariance

The fixing of light-like 3-surface to be the wormhole throat at which the signature of induced metric changes from Minkowskian to Euclidian corresponds to a convenient fixing of gauge. General Coordinate Invariance however requires that any light-like surface Y^3_l parallel to X^3_l in the slicing is equally good choice. In particular, it should give rise to same Kähler metric but not necessarily the same exponent of Kähler function identified as the product of the generalized eigenvalues of $D_{C,S}$ at Y^3_l.

General Coordinate Invariance requires that the components of Kähler metric of configuration space defined in terms of Kähler function as

$$G_{k\bar{l}} = \partial_k\partial_{\bar{l}}K = \sum_i \partial_k\partial_{\bar{l}}\lambda_i$$

remain invariant under this flow. Here complex coordinate are of course associated with the configuration space. This is the case if the flow corresponds to the addition of sum of holomorphic function $f(z)$ and its conjugate $\overline{f}(z))$ which is anti-holomorphic function to K. This boils down to the scaling of eigenvalues λ_i by

$$\lambda_i \rightarrow exp(f_i(z) + \overline{f_i(z)})\lambda_i \quad . \tag{5.4.12}$$

If the eigenvalues are interpreted as vacuum conformal weights, general coordinate transformations correspond to a spectral flow scaling the eigenvalues in this manner. This in turn would induce spectral flow of ground state conformal weights if the squares of λ_i correspond to ground state conformal weights.

5.5 Quaternions, octonions, and modified Dirac equation

Classical number fields define one vision about quantum TGD. This vision about quantum TGD has evolved gradually and involves several speculative ideas.

1. The hard core of the vision is that space-time surfaces as preferred extremals of Kähler action can be identified as what I have called hyper-quaternionic surfaces of M^8 or $M^4 \times CP_2$. This requires only the mapping of the modified gamma matrices to octonions or to a basis of subspace of complexified octonions. This means also the mapping of spinors to octonionic spinors. There is no need to assume that imbedding space-coordinates are octonionic.

2. I have considered also the idea that quantum TGD might emerge from the mere associativity.

   (a) Consider Clifford algebra of WCW. Treat "vibrational" degrees of freedom in terms second quantized spinor fields and add center of mass degrees of freedom by replacing 8-D gamma matrices with their octonionic counterparts - which can be constructed as tensor products of octonions providing alternative representation for the basis of 7-D Euclidian gamma matrix algebra - and of 2-D sigma matrices. Spinor components correspond to tensor products of octonions with 2-spinors: different spin states for these spinors correspond to leptons and baryons.

   (b) Construct a local Clifford algebra by considering Clifford algebra elements depending on point of M^8 or H. The octonionic 8-D Clifford algebra and its local variant are non-accociative. Associative sub-algebra of 8-D Clifford algebra is obtained by restricting the elements so any quaternionic 4-plane. Doing the same for the local algebra means restriction of the Clifford algebra valued functions to any 4-D hyper-quaternionic sub-manifold of M^8 or H which means that the gamma matrices span complexified quaternionic algebra at each point of space-time surface. Also spinors must be quaternionic.

   (c) The assignment of the 4-D gamma matrix sub-algebra at each point of space-time surface can be done in many manners. If the gamma matrices correspond to the tangent space of space-time surface, one obtains just induced gamma matrices and the standard definition of quaternionic sub-manifold. In this case induced 4-volume is taken as the action principle. If Kähler action defines the space-time dynamics, the modified gamma matrices do not span the tangent space in general.

   (d) An important additional element is involved. If the M^4 projection of the space-time surface contains a preferred subspace M^2 at each point, the quaternionic planes are labeled by points of CP_2 and one can equivalently regard the surfaces of M^8 as surfaces of $M^4 \times CP_2$ (number-theoretical "compactification"). This generalizes: M^2 can be replaced with a distribution of planes of M^4 which integrates to a 2-D surface of M^4 (for instance, for string like objects this is necessarily true). The presence of the preferred local plane M^2 corresponds to the fact that octonionic spin matrices Σ_{AB} span 14-D Lie-algebra of $G_2 \subset SO(7)$ rather than that 28-D Lie-algebra of $SO(7,1)$ whereas octonionic imaginary units provide 7-D fundamental representation of G_2. Also spinors must be quaternionic and this is achieved if they are created by the Clifford algebra defined by induced gamma matrices from two preferred spinors defined by real and preferred imaginary octonionic unit. Therefore the preferred plane $M^3 \subset M^4$ and its local variant has direct counterpart at the level of induced gamma matrices and spinors.

   (e) This framework implies the basic structures of TGD and therefore leads to the notion of world of classical worlds (WCW) and from this one ends up with the notion WCW spinor field and WCW Clifford algebra and also hyper-finite factors of type II_1 and III_1. Note that M^8 is exceptional: in other dimensions there is no reason for the restriction of the local Clifford algebra to lower-dimensional sub-manifold to obtain associative algebra.

The above line of ideas leads naturally to (hyper-)quaternionic sub-manifolds and to basic quantum TGD (note that the "hyper" is un-necessary if one accepts just the notion of quaternionic sub-manifold formulated in terms of modified gamma matrices). One can pose some further questions.

1. Quantum TGD reduces basically to the second quantization of the induced spinor fields. Could it be that the theory is integrable only for 4-D hyper-quaternionic space-time surfaces in M^8

(equivalently in $M^4 \times CP_2$) in the sense than one can solve the modified Dirac equation exactly only in these cases?

2. The construction of quantum TGD -including the construction of vacuum functional as exponent of Kähler function reducing to Kähler action for a preferred extremal - should reduce to the modified Dirac equation defined by Kähler action. Could it be that the modified Dirac equation can be solved exactly only for Kähler action.

3. Is it possible to solve the modified Dirac equation for the octonionic gamma matrices and octonionic spinors and map the solution as such to the real context by replacing gamma matrices and sigma matrices with their standard counterparts? Could the associativity conditions for octospinors and modified Dirac equation allow to pin down the form of solutions to such a high degree that the solution can be constructed explicitly?

4. Octonionic gamma matrices provide also a non-associative representation for 8-D version of Pauli sigma matrices and encourage the identification of 8-D twistors as pairs of octonionic spinors conjectured to be highly relevant also for quantum TGD. Does the quaternionicity condition imply that octo-twistors reduce to something closely related to ordinary twistors as the fact that 2-D sigma matrices provide a matrix representation of quaternions suggests?

In the following I will try to answer these questions by developing a detailed view about the octonionic counterpart of the modified Dirac equation and proposing explicit solution ansätze for the modes of the modified Dirac equation.

5.5.1 The replacement of $SO(7,1)$ with G_2

The basic implication of octonionization is the replacement of $SO(7,1)$ as the structure group of spinor connection with G_2. This has some rather unexpected consequences.

Octonionic representation of 8-D gamma matrices

Consider first the representation of 8-D gamma matrices in terms of tensor products of 7-D gamma matrices and 2-D Pauli sigma matrices.

1. The gamma matrices are given by

$$\gamma^0 = 1 \times \sigma_1 \ \ , \ \ \gamma^i = \gamma^i \otimes \sigma_2 \ \ , i = 1,..,7 \ . \tag{5.5.1}$$

7-D gamma matrices in turn can be expressed in terms of 6-D gamma matrices by expressing γ^7 as

$$\gamma^{7)}_{i+1} = \gamma^{6)}_i , i = 1,...,6 \ \ , \ \ \gamma^{7)}_1 = \gamma^{6)}_7 = \prod_{i=1}^{6} \gamma^{6)}_i \ . \tag{5.5.2}$$

2. The octonionic representation is obtained as

$$\gamma_0 = 1 \times \sigma_1 \ \ , \ \ \gamma_i = e_i \otimes \sigma_2 \ . \tag{5.5.3}$$

where e_i are the octonionic units. $e_i^2 = -1$ guarantees that the M^4 signature of the metric comes out correctly. Note that $\gamma_7 = \prod \gamma_i$ is the counterpart for choosing the preferred octonionic unit and plane M^2.

3. The octonionic sigma matrices are obtained as commutators of gamma matrices:

$$\Sigma_{0i} = e_i \times \sigma_3 \ , \ \ \Sigma_{ij} = f_{ij}{}^k e_k \otimes 1 \ . \qquad (5.5.4)$$

These matrices span G_2 algebra having dimension 14 and rank 2 and having imaginary octonion units and their conjugates as the fundamental representation and its conjugate. The Cartan algebra for the sigma matrices can be chosen to be Σ_{01} and Σ_{23} and belong to a quaternionic sub-algebra.

4. The lower dimension of the G_2 algebra means that some combinations of sigma matrices vanish. All left or right handed generators of the algebra are mapped to zero: this explains why the dimension is halved from 28 to 14. From the octonionic triangle expressing the multiplication rules for octonion units [44] one finds $e_4e_5 = e_1$ and $e_6e_7 = -e_1$ and analogous expressions for the cyclic permutations of e_4, e_5, e_6, e_7. From the expression of the left handed sigma matrix $I_L^3 = \sigma_{23} + \sigma^{30}$ representing left handed weak isospin (see the Appendix about the geometry of CP_2 [39]) one can conclude that this particular sigma matrix and left handed sigma matrices in general are mapped to zero. The quaternionic sub-algebra $SU(2)_L \times SU(2)_R$ is mapped to that for the rotation group $SO(3)$ since in the case of Lorentz group one cannot speak of a decomposition to left and right handed subgroups. The elements of the complement of the quaternionic sub-algebra are expressible in terms of Σ_{ij} in the quaternionic sub-algebra.

Some physical implications of $SO(7,1) \rightarrow G_2$ reduction

This has interesting physical implications if one believes that the octonionic description is equivalent with the standard one.

1. If $SU(2)_L$ is mapped to zero only the right-handed parts of electro-weak gauge field survive octonization. The right handed part is neutral containing only photon and Z^0 so that the gauge field becomes Abelian. Z^0 and photon fields become proportional to each other ($Z^0 \rightarrow sin^2(\theta_W)\gamma$) so that classical Z^0 field disappears from the dynamics, and one would obtain just electrodynamics. This might provide a deeper reason for why electrodynamics is an excellent description of low energy physics and of classical physics. This is consistent with the fact that CP_2 coordinates define 4 field degrees of freedom so that single Abelian gauge field should be enough to describe classical physics. This would remove also the interpretational problems caused by the transitions changing the charge state of fermion induced by the classical W boson fields.

   Also the realization of $M^8 - H$ duality led to the conclusion M^8 spinor connection should have only neutral components. The isospin matrix associated with the electromagnetic charge is $e_1 \times 1$ and represents the preferred imaginary octonionic unit so that that the image of the electro-weak gauge algebra respects associativity condition. An open question is whether octonionization is part of M^8-H duality or defines a completely independent duality. The objection is that information is lost in the mapping so that it becomes questionable whether the same solutions to the modified Dirac equation can work as a solution for ordinary Clifford algebra.

2. If $SU(2)_R$ were mapped to zero only left handed parts of the gauge fields would remain. All classical gauge fields would remain in the spectrum so that information would not be lost. The identification of the electro-weak gauge fields as three covariantly constant quaternionic units would be possible in the case of M^8 allowing Hyper-Kähler structure [42], which has been speculated to be a hidden symmetry of quantum TGD at the level of WCW. This option would lead to difficulties with associativity since the action of the charged gauge potentials would lead out from the local quaternionic subspace defined by the octonionic spinor.

3. The gauge potentials and gauge fields defined by CP_2 spinor connection are mapped to fields in $SO(2) \subset SU(2) \times U(1)$ in quaternionic sub-algebra which in a well-defined sense corresponds to M^4 degrees of freedom! Since the resulting interactions are of gravitational character, one might say that electro-weak interactions are mapped to manifestly gravitational interactions. Since $SU(2)$ corresponds to rotational group one cannot say that spinor connection would give rise only to left or right handed couplings, which would be obviously a disaster.

Octo-spinors and their relation to ordinary imbedding space spinors

Octo-spinors are identified as octonion valued 2-spinors with basis

$$\begin{aligned} \Psi_{L,i} &= e_i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ , \\ \Psi_{q,i} &= e_i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ . \end{aligned} \tag{5.5.5}$$

One obtains quark and lepton spinors and conjugation for the spinors transforms quarks to leptons. Note that octospinors can be seen as 2-dimensional spinors with components which have values in the space of complexified octonions.

The leptonic spinor corresponding to real unit and preferred imaginary unit e_1 corresponds naturally to the two spin states of the right handed neutrino. In quark sector this would mean that right handed U quark corresponds to the real unit. The octonions decompose as $1+1+3+\overline{3}$ as representations of $SU(3) \subset G_2$. The concrete representations are given by

$$\begin{array}{ll} \{1 \pm ie_1\} \ , & e_R \text{ and } \nu_R \text{ with spin } 1/2 \ , \\ \{e_2 \pm ie_3\} \ , & e_R \text{ and } \nu_L \text{ with spin -}1/2 \ , \\ \{e_4 \pm ie_5\} & e_L \text{ and } \nu_L \text{ with spin } 1/2 \ , \\ \{e_6 \pm ie_7\} & e_L \text{ and } \nu_L \text{ with spin } 1/2 \ . \end{array} \tag{5.5.6}$$

Instead of spin one could consider helicity. All these spinors are eigenstates of e_1 (and thus of the corresponding sigma matrix) with opposite values for the sign factor $\epsilon = \pm$. The interpretation is in terms of vectorial isospin. States with $\epsilon = 1$ can be interpreted as charged leptons and D type quarks and those with $\epsilon = -1$ as neutrinos and U type quarks. The interpretation would be that the states with vanishing color isospin correspond to right handed fermions and the states with non-vanishing SU(3) isospin (to be not confused with QCD color isospin) and those with non-vanishing SU(3) isospin to left handed fermions. The only difference between quarks and leptons is that the induced Kähler gauge potentials couple to them differently.

The importance of this identification is that it allows a unique map of the candidates for the solutions of the octonionic modified Dirac equation to those of ordinary one. There are some delicacies involved due to the possibility to chose the preferred unit e_1 so that the preferred subspace M^2 can corresponds to a sub-manifold $M^2 \subset M^4$.

5.5.2 Octonionic counterpart of the modified Dirac equation

The solution ansatz for the octonionic counterpart of the modified Dirac equation discussed below makes sense also for ordinary modified Dirac equation which raises the hope that the same ansatz, and even same solution could provide a solution in both cases.

The general structure of the modified Dirac equation

In accordance with quantum holography and the notion of generalized Feynman diagram, the modified Dirac equation involves two equations which must be consistent with each other.

1. There is 3-dimensional generalized eigenvalue equation for which the modified gamma matrices are defined by Chern-Simons action defined by the sum $J_{tot} = J + J_1$ of Kähler forms of S^2 and CP_2 [18, 47].

$$\begin{aligned} D_3\Psi &= [D_{C-S} + Q_{C-S}]\Psi = \lambda^k \gamma_k \Psi \ , \\ Q_{C-S} &= Q_\alpha \hat{\Gamma}^\alpha_{C-S} \ , \ \ Q_\alpha = Q_A g^{AB} j_{B\alpha} \ . \end{aligned} \tag{5.5.7}$$

The gamma matrices γ_k are M^4 gamma matrices in standard Minkowski coordinates and thus constant. Given eigenvalue λ_k defines pseudo momentum which is some function of the genuine momenta p_k and other quantum numbers via the boundary conditions associated with the generalized eigenvalue equation.

The charges Q_A correspond to real four-momentum and charges in color Cartan algebra. The term Q can be rather general since it provides a representation for the measurement interaction by mapping observables to Cartan algebra of isometry group and to the infinite hierarchy of conserved currents implied by quantum criticality. The operator O characterizes the quantum critical conserved current. The surface Y_l^3 can be chosen to be any light-like 3-surface "parallel" to the wormhole throat in the slicing of X^4: this means an additional symmetry. Formally the measurement interaction term can be regarded as an addition of a gauge term to the Kähler gauge potential associated with the Kähler form J_{tot} of $S^2 \times CP_2$.

The square of the equation gives the spinor analog of d'Alembert equation and generalized eigenvalue as the analog of mass squared. The propagator associated with the wormhole throats is formally massless Dirac propagator so that standard twistor formalism applies also without the octonionic representation of the gamma matrices although the physical particles propagating along the opposite wormhole throats are massive on mass shell particles with both signs of energy [47].

2. Second equation is the 4-D modified Dirac equation defined by Kähler action.

$$D_K\Psi = 0 . \quad (5.5.8)$$

The dimensional reduction of this operator to a sum corresponding to $D_{K,3}$ acting on light-like 3-surfaces and 1-D operator $D_{K,1}$ acting on the coordinate labeling the 3-D light-like 3-surfaces in the slicing would allow to assign eigenvalues to $D_{K,3}$ as analogs of energy eigenvalues for ordinary Schrödinger equation. One proposal has been that Dirac determinant could be identified as the product of these eigen values. Another and more plausible identification is as the product of pseudo masses assignable to D_3 defined by Chern-Simons action [56]. It must be however made clear that the identification of the exponent of the Kähler function to Chern-Simons term makes the identification as Dirac determinant un-necessary.

3. There are two options depending on whether one requires that the eigenvalue equation applies only on the wormhole throats and at the ends of the space-time surface or for all 3-surfaces in the slicing of the space-time surface by light-like 3-surfaces. In the latter case the condition that the pseudo four-momentum is same for all the light-like 3-surfaces in the slicing gives a consistency condition stating that the commutator of the two Dirac operators vanishes for the solutions in the case of preferred extremals, which depend on the momentum and color quantum numbers also:

$$[D_K, D_3]\,\Psi = 0 . \quad (5.5.9)$$

This condition is quite strong and there is no deep reason for it since λ_k does not correspond to the physical conserved momentum so that its spectrum could depend on the light-like 3-surface in the slicing. On the other hand, if the eigenvalues of D_3 belong to the preferred hyper-complex plane M^2, D_3 effectively reduces to a 2-dimensional algebraic Dirac operator $\lambda^k\gamma_k$ commuting with D_K: the values of λ^k cannot depend on slice since this would mean that D_K does not commute with D_3.

About the hyper-octonionic variant of the modified Dirac equation

What gives excellent hopes that the octonionic variant of modified Dirac equation could lead to a provide precise information about the solution spectrum of modified Dirac equation is the condition that everything in the equation should be associative. Hence the terms which are by there nature non-associative should vanish automatically.

1. The first implication is that the besides octonionic gamma matrices also octonionic spinors should belong to the local quaternionic plane at each point of the space-time surface. Spinors are also generated by quaternionic Clifford algebra from two preferred spinors defining a preferred plane in the space of spinors. Hence spinorial dynamics seems to mimic very closely the space-time dynamics and one might even hope that the solutions of the modified Dirac action could be seen as maps of the space-time surface to surfaces of the spinor space. The reduction to quaternionic sub-algebra suggest that some variant of ordinary twistors emerges in this manner in matrix representation.

2. The octonionic sigma matrices span G_2 where as ordinary sigma matrices define $SO(7,1)$. On the other hand, the holonomies are identical in the two cases if right-handed charge matrices are mapped to zero so that there are indeed hopes that the solutions of the octonionic Dirac equation cannot be mapped to those of ordinary Dirac equation. If left-handed charge matrices are mapped to zero, the resulting theory is essentially the analog of electrodynamics coupled to gravitation at classical level but it is not clear whether this physically acceptable. It is not clear whether associativity condition leaves only this option under consideration.

3. The solution ansatz to the modified Dirac equation is expected to be of the form $\Psi = D_K(\Psi_0 u_0 + \Psi_1 u_1)$, where u_0 and u_1 are constant spinors representing real unit and the preferred unit e_1. Hence constant spinors associated with right handed electron and neutrino and right-handed d and u quark would appear in Ψ and Ψ_i could correspond to scalar coefficients of spinors with different charge. This ansatz would reduce the modified Dirac equation to $D_K^2 \Psi_i = 0$ since there are no charged couplings present. The reduction of a d'Alembert type equation for single scalar function coupling to $U(1)$ gauge potential and $U(1)$ "gravitation" would obviously mean a dramatic simplification raising hopes about integrable theory.

4. The condition $D_K^2 \Psi = 0$ involves products of three octonions and involves derivatives of the modified gamma matrices which might belong to the complement of the quaternionic sub-space. The restriction of Ψ to the preferred hyper-complex plane M^2 simplifies the situation dramatically but $(D_K^2) D_K \Psi = D_K (D_K^2) \Psi = 0$ could still fail. The problem is that the action of D_K is not algebraic so that one cannot treat reduce the associativity condition to $(AA)A = A(AA)$.

5.5.3 Could the notion of octo-twistor make sense?

Twistors have led to dramatic successes in the understanding of Feynman diagrammatics of gauge theories, $N = 4$ SUSYs, and $N = 8$ supergravity [52, 50, 57]. This motivated the question whether they might be applied in TGD framework too [28] - at least in the description of the QFT limit. The basic problem of the twistor program is how to overcome the difficulties caused by particle massivation and TGD framework suggests possible clues in this respect.

1. In TGD it is natural to regard particles as massless particles in 8-D sense and to introduce 8-D counterpart of twistors by relying on the geometric picture in which twistors correspond to a pair of spinors characterizing light-like momentum ray and a point of M^8 through which the ray traverses. Twistors would consist of a pair of spinors and quark and lepton spinors define the natural candidate for the spinors in question. This approach would allow to handle massive on-mass-shell states but cannot cope with virtual momenta massive in 8-D sense.

2. The emergence of pseudo momentum λ_k from the generalized eigenvalue equation for D_{C-S} suggest a dramatically simpler solution to the problem. Since propagators are effectively massless propagators for pseudo momenta, which are functions of physical on shell momenta (with both signs of energy in zero energy ontology) and of other quantum numbers, twistor formalism can be applied in its standard form. An attractive assumption is that also λ^k are conserved in the vertices but a good argument justifying this is lacking. One can ask whether also $N = 4$ SUSY, $N = 8$ super-gravity, and even QCD could have similar interpretation.

This picture should apply also in the case of octotwistors with minor modifications and one might hope that octotwistors could provide new insights about what happens in the real case.

1. In the case of ordinary Clifford algebra unit matrix and six-dimensional gamma matrices γ_i, $i = 1, ..., 6$ and $\gamma_7 = \prod_i \gamma_i$ would define the variant of Pauli sigma matrices as $\sigma_0 = 1$, $\sigma_k = \gamma_k$, $k = 1, .., 7$ The problem is that masslessness condition does not correspond to the vanishing of the determinant for the matrix $p_k\sigma^k$.

2. In the case of octo-twistors Pauli sigma matrices σ^k would correspond to hyper-octonion units $\{\sigma_0, \sigma_k\} = \{1, ie^k\}$ and one could assign to $p_k\sigma^k$ a matrix by the linear map defined by the multiplication with $P = p_k\sigma^k$. The matrix is of form $P_{mn} = p^k f_{kmn}$, where f_{kmn} are the structure constants characterizing multiplication by hyper-octonion. The norm squared for octonion is the fourth root for the determinant of this matrix. Since $p_k\sigma^k$ maps its octonionic conjugate to zero so that the determinant must vanish (as is easy to see directly by reducing the situation to that for hyper-complex numbers by considering the hyper-complex plane defined by P).

3. Associativity condition for the octotwistors requires that the gamma matrix basis appearing in the generalized eigenvalue equation for Chern-Simons Dirac operator must differs by a local G_2 rotation from the standard hyper-quaternionic gamma matrix for M^4 so that it is always in the local hyper-quaternionic plane. This suggests that octo-twistor can be mapped to an ordinary twistor by mapping the basis of hyper-quaternions to Pauli sigma matrices. A stronger condition guaranteing the commutativity of D_3 with $\lambda^k\gamma_k$ is that λ_k belongs to a preferred hyper-complex plane M^2 assignable to a given CD. Also the two spinors should belong to this plane for the proposed solution ansatz for the modified Dirac equation. Quaternionization would also allow to assign momentum to the spinors in standard manner.

   The spectrum of pseudo-momenta would be 2-dimensional (continuum at worst) and this should certainly improve dramatically the convergence properties for the sum over the non-conserved pseudo-momenta in propagators which in the worst possible of worlds might destroy the manifest finiteness of the theory based on the generalized Feynman diagrams with the throats of wormholes carrying always on mass shell momenta. This effective 2-dimensionality should apply also in the real case and would have no catastrophic consequences since pseudo momenta are in question.

   As a matter fact, the assumption the decomposition of quark momenta to longitudinal and transversal parts in perturbative QCD might have interpretation in terms of pseudo-momenta if they are conserved.

4. $M^8 - H$ duality suggests a possible interpretation of the pseudo-momenta as M^8 momenta which by purely number theoretical reasons must be commutative and thus belong to M^2 hyper-complex plane. One ends up with the similar outcome as one constructs a representation for the quantum states defined by WCW spinor fields as superpositions of real units constructed as ratios of infinite hyper-octonionic integers with precisely defined number theoretic anatomy and transformation properties under standard model symmetries having number theoretic interpretation [19].

5.6 Weak form electric-magnetic duality and its implications

The notion of electric-magnetic duality [51] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [22]. What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.

3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects in electro-weak scale: this should become manifest at LHC energies.

4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found. The solution ansatz applies also to the extremals of Chern-Simons action and and to the conserved currents associated with the modified Dirac equation defined as contractions of the modified gamma matrices between the solutions of the modified Dirac equation. The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

6. The general solution ansatz works for induced Kähler Dirac equation and Chern-Simons Dirac equation and reduces them to ordinary differential equations along flow lines. The induced spinor fields are simply constant along flow lines of indued spinor field for Dirac equation in suitable gauge. Also the generalized eigen modes of the modified Chern-Simons Dirac operator can be deduced explicitly if the throats and the ends of space-time surface at the boundaries of CD are extremals of Chern-Simons action. Chern-Simons Dirac equation reduces to ordinary differential equations along flow lines and one can deduce the general form of the spectrum and the explicit representation of the Dirac determinant in terms of geometric quantities characterizing the 3-surface (eigenvalues are inversely proportional to the lengths of strands of the flow lines in the effective metric defined by the modified gamma matrices).

5.6.1 Could a weak form of electric-magnetic duality hold true?

Holography means that the initial data at the partonic 2-surfaces should fix the configuration space metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} \ . \tag{5.6.1}$$

A more general form of this duality is suggested by the considerations of [34] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [56] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta} J_{\gamma\delta}\sqrt{g_4} \ . \tag{5.6.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the configuration space metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m \quad = \quad (1+K)J_{12} \ , \tag{5.6.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial configuration space metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [39] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_ZF_Z}{\hbar} = 2R_{03} \ . \end{aligned} \tag{5.6.4}$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J \quad = \quad \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \tag{5.6.5}$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \quad (5.6.6)$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi}\times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \quad (5.6.7)$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the modified Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [25] supports this interpretation.

3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \to \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

The weak form of electric-magnetic duality has surprisingly strong implications for basic view about quantum TGD as following considerations show.

5.6.2 Magnetic confinement, the short range of weak forces, and color confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L\overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L\nu_R$. $\nu_L\overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hyper-charge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics.

For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four Gaussian Mersennes corresponding to $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [65].

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [23]. The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_\pm$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

2. The addition of the particles $X^\pm$ replaces generalized Feynman diagrams with the analogs of

stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

3. How should one describe the bound state formed by the fermion and $X^{\pm}$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [38]. If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [31].

Should $J + J_1$ appear in Kähler action?

The presence of the S^2 Kähler form J_1 in weak form of electric-magnetic duality was originally suggested by an erratic argument about the reduction to almost topological QFT to be described in the next subsection. In any case this argument raises the question whether one could replace J with $J + J_1$ in the Kähler action. This would not affect the basic non-vacuum extremals but would modify the vacuum degeneracy of the Kähler action. Canonically imbedded M^4 would become a monopole configuration with an infinite magnetic energy and Kähler action due to the monopole singularity at the line connecting tips of the CD. Action and energy can be made small by drilling a small hole around origin. This is however not consistent with the weak form of electro-weak duality. Amusingly, the modified Dirac equation reduces to ordinary massless Dirac equation in M^4.

This extremal can be transformed to a vacuum extremal by assuming that the solution is also a CP_2 magnetic monopole with opposite contribution to the magnetic charge so that $J + J_1 = 0$ holds true. This is achieved if one can regard space-time surface as a map $M^4 \to CP_2$ reducing to a map $(\Theta, \Phi) = (\theta, \pm\phi)$ with the sign chosen by properly projecting the homologically non-trivial $r_M = constant$ spheres of CD to the homologically non-trivial geodesic sphere of CP_2. Symplectic transformations of $S^2 \times CP_2$ produce new vacuum extremals of this kind. Using Darboux coordinates in which one has $J = \sum_{k=1,2} P_k dQ^k$ and assuming that (P_1, Q_1) corresponds to the CP_2 image of S^2, one can take Q_2 to be arbitrary function of P^2 which in turn is an arbitrary function of of M^4 coordinates to obtain even more general vacuum extremals with 3-D CP_2 projection. Therefore the spectrum of vacuum extremals, which is very relevant for the TGD based description of gravitation in long length scales because it allows to satisfy Einstein's equations as an additional condition, looks much richer than for the original option, and it is natural to ask whether this option might make sense.

An objection is that J_1 is a radial monopole field and this breaks Lorentz invariance to $SO(3)$. Lorentz invariance is broken to $SO(3)$ for a given CD also by the presence of the preferred time direction defined by the time-like line connecting the tips of the CD becoming carrying the monopole charge but is compensated since Lorentz boosts of CDs are possible. Could one consider similar compensation also now? Certainly the extremely small breaking of Lorentz invariance and the vanishing of the monopole charge for the vacuum extremals is all that is needed at the space-time level. No new gauge fields would be introduced since only the Kähler field part of photon and Z^0 boson would receive an additional contribution.

The ultimate fate of the modification depends on whether it is consistent with the general relativistic description of gravitation. Since a breaking of spherical symmetry is involved, it is not at all clear whether one can find vacuum extremals which represent small deformations of the Reissner-Nordström metric and Robertson-Walker metric. The argument below shows that this option does not allow the imbedding of small deformations of physically plausible space-time metrics as vacuum extremals.

The basic vacuum extremal whose deformations should give vacuum extremals allowing interpretation as solutions of Einstein's equations is given by a map $M^4 \to CP_2$ projecting the r_M constant spheres S^2 of M^2 to the homologically non-trivial geodesic sphere of CP_2. The winding number of this map is -1 in order to achieve vanishing of the induced Kähler form $J + J_1$. For instance, the following two canonical forms of the map are possible

$$\begin{aligned} (\Theta, \Psi) &= (\theta_M, -\phi_M) \ , \\ (\Theta, \Psi) &= (\pi - \theta_M, \phi_M) \ . \end{aligned} \tag{5.6.8}$$

Here (Θ, Ψ) refers to the geodesic sphere of CP_2 and (θ_M, ϕ_M) to the sphere of M^4.
The resulting space-time surface is not flat and Einstein tensor is non-vanishing. More complex metrics can be constructed from this metric by a deformation making the CP_2 projection 3-dimensional.

Using the expression of the CP_2 line element in Eguchi-Hanson coordinates [41]

$$\frac{ds^2}{R^2} = \frac{dr^2}{F^2} + \frac{r^2}{F}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + fracr^2 4F sin^2\Theta d\Phi^2) \tag{5.6.9}$$

and s the relationship $r = tan(\Theta)$, one obtains following expression for the CP_2 metric

$$\frac{ds^2}{R^2} = d\theta_M^2 + sin^2(\theta_M)\left[(d\phi_M + cos(\theta)d\Phi)^2 + \frac{1}{4}(d\theta^2 + sin^2(\theta)d\Phi^2\right] \ . \tag{5.6.10}$$

The resulting metric is obtained from the metric of S^2 by replacing $d\phi^2$ which 3-D line element. The factor $sin^2(\theta_M)$ implies that the induced metric becomes singular at North and South poles of S^2. In particular, the gravitational potential is proportional to $sin^2(\theta_M)$ so that gravitational force in the radial direction vanishes at equators. It is very difficult to imagine any manner to produce a small deformation of Reissner-Nordstrm metric or Robertson-Walker metric. Hence it seems that the vacuum extremals produce by $J + J_1$ option are not physical.

5.6.3 Could Quantum TGD reduce to almost topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the modified Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^\alpha A_\alpha$ plus and integral of the boundary term $J^{n\beta}A_\beta\sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta}A_\beta\sqrt{g_4}$ over the ends of the 3-surface.

2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K\epsilon^{n\beta\gamma\delta}J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K\epsilon^{0\beta\gamma\delta}J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the

replacement $\hbar_0 \rightarrow r\hbar_0$ would effectively describe this. Boundary conditions would however give $1/r$ factor so that $\hbar$ would disappear from the Kähler function! The original attempt to realize quantum TGD as an almost topological QFT was in terms of Chern-Simons action but was given up. It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [33]) so that the Coulombic term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

2. The original naive conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma}J_{\beta\gamma})\sqrt{g_4}d^3x \quad . \tag{5.6.11}$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.

3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta}K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^\alpha \partial_\alpha \phi = -j^\alpha A_\alpha \quad . \tag{5.6.12}$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j_K^\alpha$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j^K_\beta \partial_\gamma j^K_\delta = 0 \ . \tag{5.6.13}$$

j_K is a four-dimensional counterpart of Beltrami field [55] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [33]. The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^\alpha \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j_K^\alpha \phi$ and $j_I^\alpha \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \to A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha \phi) = 0 \ . \tag{5.6.14}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q^e_\phi = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the imons type term is trivial if the total weighted Kähler magnetic flux $Q^m_\phi = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function. Kähler function is identified as a Dirac determinant for Chern-Simons Dirac action and the spectrum of this operator should not be invariant under these gauge transformations if this picture is correct. This is is achieved if the gauge transformation is carried only in the Dirac action corresponding to the Chern-Simons term: this assumption is motivated by the breaking of time reversal invariance induced by quantum measurements. The modification of Kähler action can be guessed to correspond just to the Chern-Simons contribution from the instanton term.

7. A reasonable looking guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q^m_ϕ and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

5.6.4 A general solution ansatz based on almost topological QFT property

The basic vision behind the ansatz is the reduction of quantum TGD to almost topological field theory. This requires that the flow parameters associated with the flow lines of isometry currents and Kähler current extend to global coordinates. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when weak electro-weak duality is applied as boundary conditions. The strongest form of the hydrodynamical interpretation requires that all conserved currents are parallel to Kähler current. In the more general case one would have several hydrodynamic flows. Also the braidings (several of them for the most general ansatz) assigned with the light-like 3-surfaces are naturally defined by the flow lines of conserved currents. The independent behavior of particles at different flow lines can be seen as a realization of the complete integrability of the theory. In free quantum field theories on mass shell Fourier components are in a similar role but the geometric interpretation in terms of flow is of course lacking. This picture should generalize also to the solution of the modified Dirac equation.

Basic field equations

Consider first the equations at general level.

1. The breaking of the Poincare symmetry due to the presence of monopole field occurs and leads to the isometry group $T \times SO(3) \times SU(3)$ corresponding to time translations, rotations, and color group. The Cartan algebra is four-dimensional and field equations reduce to the conservation laws of energy E, angular momentum J, color isospin I_3, and color hypercharge Y.

2. Quite generally, one can write the field equations as conservation laws for I, J, I_3, and Y.

$$D_\alpha \left[D_\beta (J^{\alpha\beta} H_A) - j^\alpha_K H^A + T^{\alpha\beta} j^l_A h_{kl} \partial_\beta h^l \right] = 0 \ . \tag{5.6.15}$$

The first term gives a contraction of the symmetric Ricci tensor with antisymmetric Kähler form and vanishes so that one has

$$D_\alpha \left[j^\alpha_K H^A - T^{\alpha\beta} j^k_A h_{kl} \partial_\beta h^l \right] = 0 \ . \tag{5.6.16}$$

For energy one has $H_A = 1$ and energy current associated with the flow lines is proportional to the Kähler current. Its divergence vanishes identically.

3. One can express the divergence of the term involving energy momentum tensor as as sum of terms involving $j_K^\alpha J_{\alpha\beta}$ and contraction of second fundamental form with energy momentum tensor so that one obtains

$$j_K^\alpha D_\alpha H^A \quad = \quad j_K^\alpha J_\alpha{}^\beta j_\beta^A + T^{\alpha\beta} H_{\alpha\beta}^k j_k^A \quad . \tag{5.6.17}$$

Hydrodynamical solution ansatz

The characteristic feature of the solution ansatz would be the reduction of the dynamics to hydrodynamics analogous to that for a continuous distribution of particles initially at the end of X^3 of the light-like 3-surface moving along flow lines defined by currents j_A satisfying the integrability condition $j_A \wedge dj_A = 0$. Field theory would reduce effectively to particle mechanics along flow lines with conserved charges defined by various isometry currents. The strongest condition is that all isometry currents j_A and also Kähler current j_K are proportional to the same current j. The more general option corresponds to multi-hydrodynamics.

Conserved currents are analogous to hydrodynamical currents in the sense that the flow parameter along flow lines extends to a global space-time coordinate. The conserved current is proportional to the gradient $\nabla\Phi$ of the coordinate varying along the flow lines: $J = \Psi\nabla\Phi$ and by a proper choice of Ψ one can allow to have conservation. The initial values of Ψ and Φ can be selected freely along the flow lines beginning from either the end of the space-time surface or from wormhole throats.

If one requires hydrodynamics also for Chern-Simons action (effective 2-dimensionality is required for preferred extremals), the initial values of scalar functions can be chosen freely only at the partonic 2-surfaces. The freedom to chose the initial values of the charges conserved along flow lines at the partonic 2-surfaces means the existence of an infinite number of conserved charges so that the theory would be integrable and even in two different coordinate directions. The basic difference as compared to ordinary conservation laws is that the conserved currents are parallel and their flow parameter extends to a global coordinate.

1. The most general assumption is that the conserved isometry currents

$$J_A^\alpha \quad = \quad j_K^\alpha H^A - T^{\alpha\beta} j_A^k h_{kl} \partial_\beta h^l \tag{5.6.18}$$

and Kähler current are integrable in the sense that $J_A \wedge J_A = 0$ and $j_K \wedge j_K = 0$ hold true. One could imagine the possibility that the currents are not parallel.

2. The integrability condition $dJ_A \wedge J_A = 0$ is satisfied if one one has

$$J_A \quad = \quad \Psi_A d\Phi_A \quad . \tag{5.6.19}$$

The conservation of J_A gives

$$d * (\Psi_A d\Phi_A) \quad = \quad 0 \quad . \tag{5.6.20}$$

This would mean separate hydrodynamics for each of the currents involved. In principle there is not need to assume any further conditions and one can imagine infinite basis of scalar function pairs (Ψ_A, Φ_A) since criticality implies infinite number deformations implying conserved Noether currents.

3. The conservation condition reduces to d'Alembert equation in the induced metric if one assumes that $\nabla\Psi_A$ is orthogonal with every $d\Phi_A$.

$$d * d\Phi_A \quad = \quad 0 \ , \ \ d\Psi_A \cdot d\Phi_A = 0 \ . \tag{5.6.21}$$

   Taking $x = \Phi_A$ as a coordinate the orthogonality condition states $g^{xj}\partial_j\Psi_A = 0$ and in the general case one cannot solve the condition by simply assuming that Ψ_A depends on the coordinates transversal to Φ_A only. These conditions bring in mind $p \cdot p = 0$ and $p \cdot e$ condition for massless modes of Maxwell field having fixed momentum and polarization. $d\Phi_A$ would correspond to p and $d\Psi_A$ to polarization. The condition that each isometry current corresponds its own pair (Ψ_A, Φ_A) would mean that each isometry current corresponds to independent light-like momentum and polarization. Ordinary free quantum field theory would support this view whereas hydrodynamics and QFT limit of TGD would support single flow.

These are the most general hydrodynamical conditions that one can assume. One can consider also more restricted scenarios.

1. The strongest ansatz is inspired by the hydrodynamical picture in which all conserved isometry charges flow along same flow lines so that one would have

$$J_A \quad = \quad \Psi_A d\Phi \ . \tag{5.6.22}$$

   In this case same Φ would satisfy simultaneously the d'Alembert type equations.

$$d * d\Phi \quad = \quad 0 \ , \ \ d\Psi_A \cdot d\Phi = 0. \tag{5.6.23}$$

   This would mean that the massless modes associated with isometry currents move in parallel manner but can have different polarizations. The spinor modes associated with light-light like 3-surfaces carry parallel four-momenta, which suggest that this option is correct. This allows a very general family of solutions and one can have a complete 3-dimensional basis of functions Ψ_A with gradient orthogonal to $d\Phi$.

2. Isometry invariance under $T \times SO(3) \times SU(3)$ allows to consider the possibility that one has

$$J_A \quad = \quad k_A\Psi_A d\Phi_{G(A)} \ , \ \ d * (d\Phi_G(A)) = 0 \ , \ \ d\Psi_A \cdot d\Phi_G(A)) = 0 \ . \tag{5.6.24}$$

   where $G(A)$ is T for energy current, $SO(3)$ for angular momentum currents and $SU(3)$ for color currents. Energy would thus flow along its own flux lines, angular momentum along its own flow lines, and color quantum numbers along their own flow lines. For instance, color currents would differ from each other only by a numerical constant. The replacement of Ψ_A with $\Psi_{G(A)}$ would be too strong a condition since Killing vector fields are not related by a constant factor.

To sum up, the most general option is that each conserved current J_A defines its own integrable flow lines defined by the scalar function pair (Ψ_A, Φ_A). A complete basis of scalar functions satisfying the d'Alembert type equation guaranteeing current conservation could be imagined with restrictions coming from the effective 2-dimensionality reducing the scalar function basis effectively to the partonic 2-surface. The diametrically opposite option corresponds to the basis obtained by assuming that only single Φ is involved.

The proposed solution ansatz can be compared to the earlier ansatz [34] stating that Kähler current is topologized in the sense that for $D(CP_2) = 3$ it is proportional to the identically conserved instanton current (so that 4-D Lorentz force vanishes) and vanishes for $D(CP_2) = 4$ (Maxwell phase). This hypothesis requires that instanton current is Beltrami field for $D(CP_2) = 3$. In the recent case the assumption that also instanton current satisfies the Beltrami hypothesis in strong sense (single function Φ) generalizes the topologization hypothesis for $D(CP_2) = 3$. As a matter fact, the topologization hypothesis applies to isometry currents also for $D(CP_2) = 4$ although instanton current is not conserved anymore.

Can one require the extremal property in the case of Chern-Simons action?

Effective 2-dimensionality is achieved if the ends and wormhole throats are extremals of Chern-Simons action. The strongest condition would be that space-time surfaces allow orthogonal slicings by 3-surfaces which are extremals of Chern-Simons action.

Also in this case one can require that the flow parameter associated with the flow lines of the isometry currents extends to a global coordinate. Kähler magnetic field $B = *J$ defines a conserved current so that all conserved currents would flow along the field lines of B and one would have 3-D Beltrami flow. Note that in magnetohydrodynamics the standard assumption is that currents flow along the field lines of the magnetic field.

For wormhole throats light-likeness causes some complications since the induced metric is degenerate and the contravariant metric must be restricted to the complement of the light-like direction. This means that d'Alembert equation reduces to 2-dimensional Laplace equation. For space-like 3-surfaces one obtains the counterpart of Laplace equation with partonic 2-surfaces serving as sources. The interpretation in terms of analogs of Coulomb potentials created by 2-D charge distributions would be natural.

5.6.5 Hydrodynamic picture in fermionic sector

Super-symmetry inspires the conjecture that the hydrodynamical picture applies also to the solutions of the modified Dirac equation.

4-dimensional modified Dirac equation and hydrodynamical picture

Consider first the solutions of of the induced spinor field in the interior of space-time surface.

1. The local inner products of the modes of the induced spinor fields define conserved currents

$$\begin{aligned} D_\alpha J^\alpha_{mn} &= 0 \ , \\ J^\alpha_{mn} &= \overline{u}_m \hat{\Gamma}^\alpha u_n \ , \\ \hat{\Gamma}^\alpha &= \frac{\partial L_K}{\partial(\partial_\alpha h^k)}\Gamma_k \ . \end{aligned} \tag{5.6.25}$$

   The conjecture is that the flow parameters of also these currents extend to a global coordinate so that one would have in the completely general case the condition

$$\begin{aligned} J^\alpha_{mn} &= \Phi_{mn} d\Psi_{mn} \ , \\ d*(d\Phi_{mn}) &= 0 \ , \ \nabla\Psi_{mn}\cdot\Phi_{mn} = 0 \ . \end{aligned} \tag{5.6.26}$$

   The condition $\Phi_{mn} = \Phi$ would mean that the massless modes propagate in parallel manner and along the flow lines of Kähler current. The conservation condition along the flow line implies tht the current component J_{mn} is constant along it. Everything would reduce to initial values at the ends of the space-time sheet boundaries of CD and 3-D modified Dirac equation would reduce everything to initial values at partonic 2-surfaces.

2. One might hope that the conservation of these super currents for all modes is equivalent with the modified Dirac equation. The modes u_n appearing in Ψ in quantized theory would be kind of "square roots" of the basis Φ_{mn} and the challenge would be to deduce the modes from the conservation laws.

3. The quantization of the induced spinor field in 4-D sense would be fixed by those at 3-D space-like ends by the fact that the oscillator operators are carried along the flow lines as such so that the anti-commutator of the induced spinor field at the opposite ends of the flow lines at the light-like boundaries of CD is in principle fixed by the anti-commutations at the either end. The anti-commutations at 3-D surfaces cannot be fixed freely since one has 3-D Chern-Simons flow reducing the anti-commutations to those at partonic 2-surfaces.

The following argument suggests that induced spinor fields are in a suitable gauge simply constant along the flow lines of the Kähler current just as massless spinor modes are constant along the geodesic in the direction of momentum.

1. The modified gamma matrices are of form $T^\alpha_k \Gamma^k$, $T^\alpha_k = \partial L_K/\partial(\partial_\alpha h^k)$. The H-vectors T^α_k can be expressed as linear combinations of a subset of Killing vector fields j^k_A spanning the tangent space of H. For CP_2 the natural choice are the 4 Lie-algebra generators in the complement of $U(2)$ sub-algebra. For CD one can used generator time translation and three generators of rotation group SO(3). The completeness of the basis defined by the subset of Killing vector fields gives completeness relation $h^k_l = j^{Ak} j_{Ak}$. This implies $T^{\alpha k} = T^{\alpha k} j^A_k j^k_A = T^{\alpha A} j^k_A$. One can defined gamma matrices Γ_A as $\Gamma_k j^k_A$ to get $T^\alpha_k \Gamma^k = T^{\alpha A}\Gamma_A$.

2. This together with the condition that all isometry currents are proportional to the Kähler current (or if this vanishes to same conserved current- say energy current) satisfying Beltrami flow property implies that one can reduce the modified Dirac equation to an ordinary differential equation along flow lines. The quantities T^{tA} are constant along the flow lines and one obtains

$$T^{tA} j_A D_t \Psi = 0 . \tag{5.6.27}$$

By choosing the gauge suitably the spinors are just constant along flow lines so that the spinor basis reduces by effective 2-dimensionality to a complete spinor basis at partonic 2-surfaces.

Generalized eigen modes for the modified Chern-Simons Dirac equation and hydrodynamical picture

Hydrodynamical picture helps to understand also the construction of generalized eigen modes of 3-D Chern-Simons Dirac equation.

The general form of generalized eigenvalue equation for Chern-Simons Dirac action

Consider first the the general form and interpretation of the generalized eigenvalue equation assigned with the modified Dirac equation for Chern-Simons action [18]. This is of course only an approximation since an additional contribution to the modified gamma matrices from the Lagrangian multiplier term guaranteing the weak form of electric-magnetic duality must be included.

1. The modified Dirac equation for Ψ is consistent with that for its conjugate if the coefficient of the instanton term is real and one uses the Dirac action $\overline{\Psi}(D^{\rightarrow} - D^{\leftarrow})\Psi$ giving modified Dirac equation as

$$D_{C-S}\Psi + \frac{1}{2}(D_\alpha \hat{\Gamma}^\alpha_{C-S})\Psi = 0 . \tag{5.6.28}$$

As noticed, the divergence $D_\alpha \hat{\Gamma}^\alpha_{C-S}$ does not contain second derivatives in the case of Chern-Simons action. In the case of Kähler action they occur unless field equations equivalent with the vanishing of the divergence term are satisfied. The extremals of Chern-Simons action provide a natural manner to define effective 2-dimensionality.

Also the fermionic current is conserved in this case, which conforms with the idea that fermions flow along the light-like 3-surfaces. If one uses the action $\overline{\Psi} D^{\rightarrow}\Psi$, $\overline{\Psi}$ does not satisfy the Dirac equation following from the variational principle and fermion current is not conserved.

2. The generalized eigen modes of D_{C-S} should be such that one obtains the counterpart of Dirac propagator which is purely algebraic and does not therefore depend on the coordinates of the throat. This is satisfied if the generalized eigenvalues are expressible in terms of covariantly constant combinations of gamma matrices and here only M^4 gamma matrices are possible. Therefore the eigenvalue equation would read as

$$D\Psi = \lambda^k \gamma_k \Psi \ , \ D = D_{C-S} + \frac{1}{2} D_\alpha \hat{\Gamma}^\alpha_{C-S} \ , \ D_{C-S} = \hat{\Gamma}^\alpha_{C-S} D_\alpha \ . \tag{5.6.29}$$

Here the covariant derivatives D_α contain the measurement interaction term as an apparent gauge term. For extremals one has

$$D = D_{C-S} \ . \tag{5.6.30}$$

Covariant constancy allows to take the square of this equation and one has

$$(D^2 + \left[D, \lambda^k \gamma_k\right])\Psi = \lambda^k \lambda_k \Psi \ . \tag{5.6.31}$$

The commutator term is analogous to magnetic moment interaction.

3. The generalized eigenvalues correspond to $\lambda = \sqrt{\lambda^k \lambda_k}$ and Dirac determinant is defined as a product of the eigenvalues and conjecture to give the exponent of Kähler action reducing to Chern-Simons term. λ is completely analogous to mass. λ_k cannot be however interpreted as ordinary four-momentum: for instance, number theoretic arguments suggest that λ_k must be restricted to the preferred plane $M^2 \subset M^4$ interpreted as a commuting hyper-complex plane of complexified quaternions. For incoming lines this mass would vanish so that all incoming particles irrespective their actual quantum numbers would be massless in this sense and the propagator is indeed that for a massless particle. Note that the eigen-modes define the boundary values for the solutions of $D_K \Psi = 0$ so that the values of λ indeed define the counterpart of the momentum space.

This transmutation of massive particles to effectively massless ones might make possible the application of the twistor formalism as such in TGD framework [28]. $N = 4$ SUSY is one of the very few gauge theory which might be UV finite but it is definitely unphysical due to the masslessness of the basic quanta. Could the resolution of the interpretational problems be that the four-momenta appearing in this theory do not directly correspond to the observed four-momenta?

2. Inclusion of the constraint term

As already noticed one must include also the constraint term due to the weak form of electric-magnetic duality and this changes somewhat the above simple picture.

1. At the 3-dimensional ends of the space-time sheet and at wormhole throats the 3-dimensionality allows to introduce a coordinate varying along the flow lines of Kähler magnetic field $B = *J$. In this case the integrability conditions state that the flow is Beltrami flow. Note that the value of B^α along the flow line defining magnetic flux appearing in anti-commutation relations is constant. This suggests that the generalized eigenvalue equation for the Chern-Simons action reduces to a collection of ordinary apparently independent differential equations associated with the flow lines beginning from the partonic 2-surface. This indeed happens when the CP_2 projection is 2-dimensional. In this case it however seems that the basis u_n is not of much help.

2. The conclusion is wrong: the variations of Chern-Simons action are subject to the constraint that electric-magnetic duality holds true expressible in terms of Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma}J_{\beta\gamma})\sqrt{g_4}d^3x \quad . \tag{5.6.32}$$

   This gives a constraint force to the field equations and also a dependence on the induced 4-metric so that one has only almost topological QFT. This term also guarantees the M^4 part of WCW Kähler metric is non-trivial. The condition that the ends of space-time sheet and wormhole throats are extrema of Chern-Simons action subject to the electric-magnetic duality constraint is strongly suggested by the effective 2-dimensionality. Without the constraint term Chern-Simons action would vanish for its extremals so that Kähler function would be identically zero.

   This term implies also an additional contribution to the modified gamma matrices besides the contribution coming from Chern-Simons action so tht the first guess for the modified Dirac operator would not be quite correct. This contribution is of exactly of the same general form as the contribution for any general general coordinate invariant action. The dependence of the induced metric on M^4 degrees of freedom guarantees that also M^4 gamma matrices are present. In the following this term will not be considered.

3. When the contribution of the constraint term to the modifield gamma matrices is neglected, the explicit expression of the modified Dirac operator D_{C-S} associated with the Chern-Simons term is given by

$$\begin{aligned}
D &= \hat{\Gamma}^\mu D_\mu + \frac{1}{2}D_\mu\hat{\Gamma}^\mu \quad , \\
\hat{\Gamma}^\mu &= \frac{\partial L_{C-S}}{\partial_\mu h^k}\Gamma_k = \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_\mu \quad , \\
D_\mu\hat{\Gamma}^\mu &= B_K^\alpha(J_{k\alpha} + \partial_\alpha A_k) \quad , \\
B_K^\alpha &= \epsilon^{\alpha\beta\gamma}J_{\beta\gamma} \quad , \quad J_{k\alpha} = J_{kl}\partial_\alpha s^l \quad , \quad \hat{\epsilon}^{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma}\sqrt{g_3} \quad .
\end{aligned} \tag{5.6.33}$$

   For the extremals of Chern-Simons action one has $D_\alpha\hat{\Gamma}^\alpha = 0$. Analogous condition holds true when the constraing contriabution to the modified gamma matrices is added.

3. Generalized eigenvalue equation for Chern-Simons Dirac action

Consider now the Chern-Simons Dirac equation in more detail assuming that the inclusion of the constraint contribution to the modified gamma matrices does not induce any complications. Assume also extremal property for Chern-Simons action with constraint term and Beltrami flow property.

1. For the extremals the Chern-Simons Dirac operator (constraint term not included) reduces to a one-dimensional Dirac operator

$$D_{C-S} = \hat{\epsilon}^{r\alpha\beta}\left[2J_{k\alpha}A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_r \quad . \tag{5.6.34}$$

Constraint term implies only a modification of the modified gamma matrices but the form of the operator remains otherwise same when extrema are in question so that one has $D_\alpha \hat{\Gamma^\alpha} = 0$.

2. For the extremals of Chern-Simons action the general solution of the modified Chern-Simons Dirac equation ($\lambda^k = 0$) is covariantly constant with respect to the coordinate r:

$$D_r\Psi = 0 \ . \qquad (5.6.35)$$

The solution to this condition can be written immediately in terms of a non-integrable phase factor $Pexp(i\int A_r dr)$, where integration is along curve with constant transversal coordinates. If $\hat{\Gamma^v}$ is light-like vector field also $\hat{\Gamma^v}\Psi_0$ defines a solution of D_{C-S}. This solution corresponds to a zero mode for D_{C-S} and does not contribute to the Dirac determinant (suggested to give rise to the exponent of Kähler function identified as Kähler action). Note that the dependence of these solutions on transversal coordinates of X^3_l is arbitrary which conforms with the hydrodynamic picture. The solutions of Chern-Simons-Dirac are obtained by similar integration procedure also when extremals are not in question.

The formal solution associated with a general eigenvalue λ can be constructed by integrating the eigenvalue equation separately along all coordinate curves. This makes sense if r indeed assigned to possibly light-like flow lines of B^α or more general Beltrami field possible induced by the constraint term. There are very strong consistency conditions coming from the conditions that Ψ in the interior is constant along the flow lines of Kähler current and continuous at the ends and throats (call them collectively boundaries), where Ψ has a non-trivial variation along the flow lines of B^α.

1. This makes sense only if the flow lines of the Kähler current are transversal to the boundaries so that the spinor modes at boundaries dictate the modes of the spinor field in the interior. Effective 2-dimensionality means that the spinor modes in the interior can be calculated either by starting from the throats or from the ends so that the data at either upper of lower partonic 2-surfaces dictates everything in accordance with zero energy ontology.

2. This gives an infinite number of commuting diagrams stating that the flow-line time evolution along flow lines along wormhole throats from lower partonic 2-surface to the upper one is equivalent with the flow-line time evolution along the lower end of space-time surface to interior, then along interior to the upper end of the space-time surface and then back to the upper partonic 2-surface. If the space-time surface allows a slicing by partonic 2-surfaces these conditions can be assumed for any pair of partonic 2-surfaces connected by Chern-Simons flow evolution.

3. Since the time evolution along interior keeps the spinor field as constant in the proper gauge and since the flow evolutions at the lower and upper ends are in a reverse direction, there is a strong atemptation to assume that the spinor field at the ends of the of the flow lines of Kähler magnetic field are identical apart from a gauge transformation. This leads to a particle-in-box quantizaton of the values of the pseudo-mass (periodic boundary conditions). These conditions will be assumed in the sequel.

These assumptions lead to the following picture about the generalized eigen modes.

1. By choosing the gauge so that covariant derivative reduces to ordinary derivative and using the constancy of $\hat{\Gamma}^r$, the solution of the generalized eigenvalue equation can be written as

$$\begin{aligned} \Psi &= exp(iL(r)\hat{\Gamma^r}\lambda^k\Gamma_k)\Psi_0 \ , \\ L(r) &= \int_0^r \frac{1}{\sqrt{\hat{g}^{rr}}}dr \ . \end{aligned} \qquad (5.6.36)$$

$L(r)$ can be regarded as the along flux line as defined by the effective metric defined by modified gamma matrices. If λ_k is linear combination of Γ^0 and Γ^{r_M} it anti-commutes with Γ^r which contains only CP_2 gamma matrices so that the pseudo-momentum is a priori arbitrary.

2. When the constraint term taking care of the electric-magnetric duality is included, also M^4 gamma matrices are present. If they are in the orthogonal complement of a preferred plane $M^2 \subset M^4$, anti-commutativity is achieved. This assumption cannot be fully justified yet but conforms with the general physical vision. There is an obvious analogy with the condition that polarizations are in a plane orthogonal to M^2. The condition indeed states that only transversal deformations define quantum fluctuating WCW degrees of freedom contributing to the WCW Kähler metric. In M^8-H duality the preferred plane M^2 is interpreted as a hyper-complex plane belonging to the tangent space of the space-time surface and defines the plane of non-physical polarizations. Also a generalization of this plane to an integrable distribution of planes $M^2(x)$ has been proposed and one must consider also now the possibility of a varying plane $M^2(x)$ for the pseudo-momenta. The scalar function Φ appearing in the general solution ansatz for the field equations satisfies massless d'Alembert equation and its gradient defines a local light-like direction at space-time-level and hence a 2-D plane of the tangent space. Maybe the projection of this plane to M^4 could define the preferred M^2. The minimum condition is that these planes are defined only at the ends of space-time surface and at wormhole throats.

3. If one accepts this hypothesis, one can write

$$\begin{aligned} \Psi &= \left[cos(L(r)\lambda) + isin(\lambda(r))\hat{\Gamma^r}\lambda^k\Gamma_k)\right]\Psi_0 \ , \\ \lambda &= \sqrt{\lambda_k\lambda^k} \ . \end{aligned} \tag{5.6.37}$$

4. Boundary conditions should fix the spectrum of masses. If the the flow lines of Kähler current coincide with the flow lines of Kähler magnetic field or more general Beltrami current at wormhole throats one ends up with difficulties since the induced spinor fields must be constant along flow lines and only trivial eigenvalues are possible. Hence it seems that the two Beltrami fields must be transversal. This requires that at the partonic 2-surfaces the value of the induced spinor mode in the interior coincides with its value at the throat. Since the induced spinor fields in interior are constant along flow lines, one must have

$$exp(i\lambda L(_{max})) = 1 \ . \tag{5.6.38}$$

This implies that one has essentially particle in a box with size defined by the effective metric

$$\lambda_n = \frac{n2\pi}{L(r_{max})} \ . \tag{5.6.39}$$

5. This condition cannot however hold true simultaneously for all points of the partonic 2-surfaces since $L(r_{max})$ depends on the point of the surface. In the most general case one can consider only a subset consisting of the points for which the values of $L(r_{max})$ are rational multiples of the value of $L(r_{max})$ at one of the points -call it L_0. This implies the notion of number theoretical braid. Induced spinor fields are localized to the points of the braid defined by the flow lines of the Kähler magnetic field (or equivalently, any conserved current- this resolves the longstanding issue about the identification of number theoretical braids). The number of the included points depends on measurement resolution characterized somehow by the number rationals which are allowed. Only finite number of harmonics and sub-harmonics of L_0 are possible so that for integer multiples the number of points is finite. If $n_{max}L_0$ and L_0/n_{min} are the largest and smallest lengths involved, one can argue that the rationals n_{max}/n, $n = 1, ..., n_{max}$ and n/n_{min}, $n = 1, ..., n_{min}$ are the natural ones.

6. One can consider also algebraic extensions for which L_0 is scaled from its reference value by an algebraic number so that the mass scale m must be scaled up in similar manner. The spectrum comes also now in integer multiples. p-Adic mass calculations predicts mass scales to the inverses of square roots of prime and this raises the expectation that $\sqrt{n}$ harmonics and sub-harmonics of L_0 might be necessary. Notice however that pseudo-momentum spectrum is in question so that this argument is on shaky grounds.

There is also the question about the allowed values of (λ_0, λ_3) for a given value of λ. This issue will be discussed in the next section devoted to the attempt to calculate the Dirac determinant assignable to this spectrum: suffice it to say that integer valued spectrum is the first guess implying that the pseudo-momenta satisfy $n_0^2 - n_3^2 = n^2$ and therefore correspond to Pythagorean triangles. What is remarkable that the notion of number theoretic braid pops up automatically from the Beltrami flow hypothesis.

5.7 How to define Dirac determinant?

The basic challenge is to define Dirac determinant hoped to give rise to the exponent of Kähler action associated with the preferred extremal. The reduction to almost topological QFT gives this kind of expression in terms of Chern-Simons action and one might hope of obtaining even more concrete expression from the Chern-Simons Dirac determinant. The calculation of the previous section allowed to calculate the most general spectrum of the modified Dirac operator. If the number of the eigenvalues is infinite as the naive expectation is then Dirac determinant diverges if calculated as the product of the eigenvalues and one must calculate it by using some kind of regularization procedure. Zeta function regularization is the natural manner to do this.

The following arguments however lead to a concrete vision how the regularization could be avoided and a connection with infinite primes. In fact, the manifestly finite option and the option involving zeta function regularization give Kähler functions differing only by a scaling factor and only the manifestly finite option satisfies number theoretical constraints coming from p-adicization. An explicit expression for the Dirac determinant in terms of geometric data of the orbit of the partonic 2-surface emerges.

Arithmetic quantum field theory defined by infinite emerges naturally. The lines of the generalized Feynman graphs are characterized by infinite primes and the selection rules correlating the geometries of the lines of the generalized Feynman graphs corresponds to the conservation of the sum of number theoretic momenta $log(p_i)$ assignable to sub-braids corresponding to different primes p_i assignable to the orbit of parton. This conforms with the vision that infinite primes indeed characterize the geometry of light-like 3-surfaces and therefore also of space-time sheets. The eigenvalues of the modified Dirac operator are proportional $1/\sqrt{p_i}$ where p_i are the primes appearing in the definition of the p-adic prime and the interpretation as analogs of Higgs vacuum expectation values makes sense and is consistent with p-adic length scale hypothesis and p-adic mass calculations. It must be emphasized that all this is essentially due to single basic hypothesis, namely the reduction of quantum TGD to almost topological QFT guaranteed by the Beltrami ansatz for field equations and by the weak form of electric-magnetic duality.

5.7.1 Dirac determinant when the number of eigenvalues is infinite

At first sight the general spectrum looks the only reasonable possibility but if the eigenvalues correlate with the geometry of the partonic surface as quantum classical correspondence suggests, this conclusion might be wrong. The original hope was the number of eigenvalues would be finite so that also determinant would be finite automatically. There were some justifications for this hope in the definition of Dirac determinant based on the dimensional reduction of D_K as $D_K = D_{K,3} + D_1$ and the identification of the generalized eigenvalues as those assigned to $D_{K,3}$ as analogs of energy eigenvalues assignable to the light-like 3-surface. It will be found that number theoretic input could allow to achieve a manifest finiteness in the case of D_{C-S} and that this option is the only possible one if number theoretic universality is required.

If there are no constraints on the eigenvalue spectrum of D_{C-S} for a given partonic orbit, the naive definition of the determinant gives an infinite result and one must define Dirac determinant using ζ function regularization implying that Kähler function reduces to the derivative of the zeta function $\zeta_D(s)$ -call it Dirac Zeta- associated with the eigenvalue spectrum.

Consider now the situation when the number of eigenvalues is infinite.

1. In this kind of situation zeta function regularization is the standard manner to define the Dirac determinant. What one does is to assign zeta function to the spectrum- let us call it Dirac zeta function and denote by $\zeta_D(s)$- as

$$\zeta_D(s) = \sum_k \lambda_k^{-s} \quad . \tag{5.7.1}$$

If the eigenvalue λ_k has degeneracy g_k it appears g_k times in the sum. In the case of harmonic oscillator one obtains Riemann zeta for which sum representation converges only for $Re(s) \geq 1$. Riemann zeta can be however analytically continued to the entire complex plane and the idea is that this can be done also in the more general case.

2. By the basic conjecture Kähler function corresponds to the logarithm of the Dirac determinant and equals to the sum of the logarithms of the eigenvalues

$$K = log(\prod \lambda_k) = -\frac{d\zeta_D}{ds}_{|s=0} \quad . \tag{5.7.2}$$

The expression on the left hand side diverges if taken as such but the expression on the right had side based on the analytical continuation of the zeta function is completely well-defined and finite quantity. Note that the replacement of eigenvalues λ_k by their powers λ_k^n -or equivalently the increase of the degeneracy by a factor n - brings in only a factor n to K: $K \rightarrow nK$.

3. Dirac determinant involves in the minimal situation only the integer multiples of pseudo-mass scale $\lambda = 2\pi/L_{min}$. One can consider also rational and even algebraic multiples $qL_{min} < L_{max}$, $q \geq 1$, of L_{min} so that one would have several integer spectra simultaneously corresponding to different braids. Here L_{min} and L_{max} are the extrema of the braid strand length determined in terms of the effective metric as $L = \int (\hat{g}^{rr})^{-1/2} dr$. The question what multiples are involved will be needed later.

4. Each rational or algebraic multiple of L_{min} gives to the zeta function a contribution which is of same form so that one has

$$\zeta_D = \sum_q \zeta(log(qx)s) \ , \ x = \frac{L_{min}}{R} \ , \ 1 \leq q < \frac{L_{max}}{L_{min}} \quad . \tag{5.7.3}$$

Kähler function can be expressed as

$$K = \sum_n log(\lambda_n) = -\frac{d\zeta_D(s)}{ds} = -\sum_q log(qx)\frac{d\zeta(s)}{ds}_{|s=0} \ , \ x = \frac{L_{min}}{R} \quad . \tag{5.7.4}$$

What is remarkable that the number theoretical details of ζ_D determine only the overall scaling factor of Kähler function and thus the value of Kähler coupling strength, which would be purely number theoretically determined if the hypothesis about the role of infinite primes is correct. Also the value of R is irrelevant since it does not affect the Kähler metric.

5. The dependence of Kähler function on WCW degrees of freedom would be coded completely by the dependence of the length scales qL_{min} on the complex coordinates of WCW: note that this dependence is different for each scale. This is reminiscent of the coding of the shape of the drum (or more generally - manifold) by the spectrum of its eigen frequencies. Now Kähler geometry would code for the dependence of the spectrum on the shape of the drum defined by the partonic 2-surface and the 4-D tangent space distribution associated with it.

What happens at the limit of vacuum extremals serves as a test for the identification of Kähler function as Dirac determinant. The weak form of electric magnetic duality implies that all components of the induced Kähler field vanish simultaneously if Kähler magnetic field cancels. In the modified Chern-Simons Dirac equation one obtains $L = \int (\hat{g}^{rr})^{-1/2} dr$. The modified gamma matrix $\hat{\Gamma}^r$ approaches a finite limit when Kähler magnetic field vanishes

$$\hat{\Gamma}^r = \epsilon^{r\beta\gamma}(2J_{\beta k}A_\gamma + J_{\beta\gamma}A_k)\Gamma^k \to 2\epsilon^{r\beta\gamma}J_{\beta k}\Gamma^k \quad . \tag{5.7.5}$$

The relevant component of the effective metric is $\hat{g}^{rr}$ and is given by

$$\hat{g}^{rr} = (\hat{\Gamma}^r)^2 = 4\epsilon^{r\beta\gamma}\epsilon^{r\mu\nu}J_{\beta k}J_\mu{}^k A_\gamma A_\nu \quad . \tag{5.7.6}$$

The limit is non-vanishing in general and therefore the eigenvalues remain finite also at this limit as also the parameter $L_{min} = \int (\hat{g}^{rr})^{-1/2} dr$ defining the minimum of the length of the braid strand defined by Kähler magnetic flux line in the effective metric unless $\hat{g}^{rr}$ goes to zero everywhere inside the partonic surface. Chern-Simons action and Kähler action vanish for vacuum extremals so that in this case one could require that Dirac determinant approaches to unity in a properly chosen gauge. Dirac determinant should approach to unit for vacuum extremals indeed approaches to unity since there are no finite eigenvalues at the limit $\hat{g}^{rr} = 0$.

5.7.2 Hyper-octonionic primes

Before detailed discussion of the hyper-octonionic option it is good to consider the basic properties of hyper-octonionic primes.

1. Hyper-octonionic primes are of form

$$\Pi_p = (n_0, n_3, n_1, n_2, ..., n_7) \ , \ \Pi_p^2 = n_0^2 - \sum_i n_i^2 = p \ \text{or} \ p^2 \quad . \tag{5.7.7}$$

2. Hyper-octonionic primes have a standard representation as hyper-complex primes. The Minkowski norm squared factorizes into a product as

$$n_0^2 - n_3^2 = (n_0 + n_3)(n_0 - n_3) \quad . \tag{5.7.8}$$

If one has $n_3 \neq 0$, the prime property implies $n_0 - n_3 = 1$ so that one obtains $n_0 = n_3 + 1$ and $2n_3 + 1 = p$ giving

$$(n_0, n_3) = ((p+1)/2, (p-1)/2) \quad . \tag{5.7.9}$$

Note that one has $(p+1)/2$ odd for $p \ mod \ 4 = 1)$ and $(p+1)/2$ even for $p \ mod \ 4 = 3)$. The difference $n_0 - n_3 = 1$ characterizes prime property.

If n_3 vanishes the prime prime property implies equivalence with ordinary prime and one has $n_3^2 = p^2$. These hyper-octonionic primes represent particles at rest.

3. The action of a discrete subgroup $G(p)$ of the octonionic automorphism group G_2 generates form hyper-complex primes with $n_3 \neq 0$ further hyper-octonionic primes $\Pi(p,k)$ corresponding to the same value of n_0 and p and for these the integer valued projection to M^2 satisfies $n_0^2 - n_3^2 = n > p$. It is also possible to have a state representing the system at rest with $(n_0, n_3) = ((p+1)/2, 0)$ so that the pseudo-mass varies in the range $[\sqrt{p}, (p+1)/2]$. The subgroup $G(n_0, n_3) \subset SU(3)$ leaving invariant the projection (n_0, n_3) generates the hyper-octonionic primes corresponding to the same value of mass for hyper-octonionic primes with same Minkowskian length p and pseudo-mass $\lambda = n \geq \sqrt{p}$.

4. One obtains two kinds of primes corresponding to the lengths of pseudo-momenta equal to p or $\sqrt{p}$. The first kind of particles are always at rest whereas the second kind of particles can be brought at rest only if one interprets the pseudo-momentum as M^2 projection. This brings in mind the secondary p-adic length scales assigned to causal diamonds (CDs) and the primary p-adic lengths scales assigned to particles.

If the M^2 projections of hyper-octonionic primes with length $\sqrt{p}$ characterize the allowed basic momenta, ζ_D is sum of zeta functions associated with various projections which must be in the limits dictated by the geometry of the orbit of the partonic surface giving upper and lower bounds L_{max} and L_{min} on the length L. L_{min} is scaled up to $\sqrt{n_0^2 - n_3^2} L_{min}$ for a given projection (n_0, n_3). In general a given M^2 projection (n_0, n_3) corresponds to several hyper-octonionic primes since $SU(3)$ rotations give a new hyper-octonionic prime with the same M^2 projection. This leads to an inconsistency unless one has a good explanation for why some basic momentum can appear several times. One might argue that the spinor mode is degenerate due to the possibility to perform discrete color rotations of the state. For hyper complex representatives there is no such problem and it seems favored. In any case, one can look how the degeneracy factors for given projection can be calculated.

1. To calculate the degeneracy factor $D(n$ associated with given pseudo-mass value $\lambda = n$ one must find all hyper-octonionic primes Π, which can have projection in M^2 with length n and sum up the degeneracy factors $D(n,p)$ associated with them:

$$\begin{aligned}
D(n) &= \sum_p D(n,p) \ , \\
D(n,p) &= \sum_{n_0^2 - n_3^2 = p} D(p, n_0, n_3) \ , \\
n_0^2 - n_3^2 &= n \ , \ \Pi_p^2(n_0, n_3) = n_0^2 - n_3^2 - \sum_i n_i^2 = n - \sum_i n_i^2 = p \ .
\end{aligned} \tag{5.7.10}$$

2. The condition $n_0^2 - n_3^2 = n$ allows only Pythagorean triangles and one must find the discrete subgroup $G(n_0, n_3) \subset SU(3)$ producing hyper-octonions with integer valued components with length p and components (n_0, n_3). The points at the orbit satisfy the condition

$$\sum n_i^2 = p - n \ . \tag{5.7.11}$$

The degeneracy factor $D(p, n_0, n_3)$ associated with given mass value n is the number of elements of in the coset space $G(n_0, n_3, p)/H(n_0, n_3, p)$, where $H(n_0, n_3, p)$ is the isotropy group of given hyper-octonionic prime obtained in this manner. For $n_0^2 - n_3^2 = p^2$ $D(n_0, n_3, p)$ obviously equals to unity.

5.7.3 Three basic options for the pseudo-momentum spectrum

The calculation of the scaling factor of the Kähler function requires the knowledge of the degeneracies of the mass squared eigen values. There are three options to consider.

First option: all pseudo-momenta are allowed

If the degeneracy for pseudo-momenta in M^2 is same for all mass values- and formally characterizable by a number N telling how many 2-D pseudo-momenta reside on mass shell $n_0^2 - n_3^2 = m^2$. In this case zeta function would be proportional to a sum of Riemann Zetas with scaled arguments corresponding to scalings of the basic mass m to m/q.

$$\zeta_D(s) = N\sum_q \zeta(log(qx)s) \ , \ x = \frac{L_{min}}{R} \ . \tag{5.7.12}$$

This option provides no idea about the possible values of $1 \leq q \leq L_{max}/L_{min}$. The number N is given by the integral of relativistic density of states $\int dk/2\sqrt{k^2+m^2}$ over the hyperbola and is logarithmically divergent so that the normalization factor N of the Kähler function would be infinite.

Second option: All integer valued pseudomomenta are allowed

Second option is inspired by number theoretic vision and assumes integer valued components for the momenta using $m_{max} = 2\pi/L_{min}$ as mass unit. p-Adicization motivates also the assumption that momentum components using m_{max} as mass scale are integers. This would restrict the choice of the number theoretical braids.

Integer valuedness together with masses coming as integer multiples of m_{max} implies $(\lambda_0, \lambda_3) = (n_0, n_3)$ with on mass shell condition $n_0^2 - n_3^2 = n^2$. Note that the condition is invariant under scaling. These integers correspond to Pythagorean triangles plus the degenerate situation with $n_3 = 0$. There exists a finite number of pairs (n_0, n_3) satisfying this condition as one finds by expressing n_0 as $n_0 = n_3 + k$ giving $2n_3k + k^2 = p^2$ giving $n_3 < n^2/2, n_0 < n^2/2 + 1$. This would be enough to have a finite degeneracy $D(n) \geq 1$ for a given value of mass squared and ζ_D would be well defined. ζ_D would be a modification of Riemann zeta given by

$$\begin{aligned} \zeta_D &= \sum_q \zeta_1(log(qx)s) \ , \ x = \frac{L_{min}}{R} \ , \\ \zeta_1(s) &= \sum g_n n^{-s} \ , \ g_n \geq 1 \ . \end{aligned} \tag{5.7.13}$$

For generalized Feynman diagrams this option allows conservation of pseudo-momentum and for loops no divergences are possible since the integral over two-dimensional virtual momenta is replaced with a sum over discrete mass shells containing only a finite number of points. This option looks thus attractive but requires a regularization. On the other hand, the appearance of a zeta function having a strong resemblance with Riemann zeta could explain the finding that Riemann zeta is closely related to the description of critical systems. This point will be discussed later.

Third option: Infinite primes code for the allowed mass scales

According to the proposal of [19, 40] the hyper-complex parts of hyper-octonionic primes appearing in their infinite counterparts correspond to the M^2 projections of real four-momenta. This hypothesis suggests a very detailed map between infinite primes and standard model quantum numbers and predicts a universal mass spectrum [19]. Since pseudo-momenta are automatically restricted to the plane M^2, one cannot avoid the question whether they could actually correspond to the hyper-octonionic primes defining the infinite prime. These interpretations need not of course exclude each other. This option allows several variants and at this stage it is not possible to exclude any of these options.

1. One must choose between two alternatives for which pseudo-momentum corresponds to hyper-complex prime serving as a canonical representative of a hyper-octonionic prime or a projection of hyper-octonionic prime to M^2.

2. One must decide whether one allows a) only the momenta corresponding to hyper-complex primes, b) also their powers (p-adic fractality), or c) all their integer multiples ("Riemann option").

One must also decide what hyper-octonionic primes are allowed.

1. The first guess is that all hyper-complex/hyper-octonionic primes defining length scale $\sqrt{p}L_{min} \leq L_{max}$ or $pL_{min} \leq L_{max}$ are allowed. p-Adic fractality suggests that also the higher p-adic length scales $p^{n/2}L_{min} < L_{max}$ and $p^nL_{min} < L_{max}$, $n \geq 1$, are possible.

   It can however happen that no primes are allowed by this criterion. This would mean vanishing Kähler function which is of course also possible since Kähler action can vanish (for instance, for massless extremals). It seems therefore safer to allow also the scale corresponding to the trivial prime $(n_0, n_3) = (1, 0)$ (1 is formally prime because it is not divisible by any prime different from 1) so that at least L_{min} is possible. This option also allows only rather small primes unless the partonic 2-surface contains vacuum regions in which case L_{max} is infinite: in this case all primes would be allowed and the exponent of Kähler function would vanish.

2. The hypothesis that only the hyper-complex or hyper-octonionic primes appearing in the infinite hyper-octonionic prime are possible looks more reasonable since large values of p would be possible and could be identified in terms of the p-adic length scale hypothesis. All hyper-octonionic primes appearing in infinite prime would be possible and the geometry of the orbit of the partonic 2-surface would define an infinite prime. This would also give a concrete physical interpretation for the earlier hypothesis that hyper-octonionic primes appearing in the infinite prime characterize partonic 2-surfaces geometrically. One can also identify the fermionic and purely bosonic primes appearing in the infinite prime as braid strands carrying fermion number and purely bosonic quantum numbers. This option will be assumed in the following.

5.7.4 Expression for the Dirac determinant for various options

The expressions for the Dirac determinant for various options can be deduced in a straightforward manner. Numerically Riemann option and manifestly finite option do not differ much but their number theoretic properties are totally different.

Riemann option

All integer multiples of these basic pseudo-momenta would be allowed for Riemann option so that ζ_D would be sum of Riemann zetas with arguments scaled by the basic pseudo-masses coming as inverses of the basic length scales for braid strands. For the option involving only hyper-complex primes the formula for ζ_D reads as

$$\begin{gathered}
\zeta_D = \zeta(log(x_{min}s)) + \textstyle\sum_{i,n} \zeta(log(x_{i,n}s)) + \sum_{i,n} \zeta(log(y_{i,n}s)) \ , \\
x_{i,n} = p_i^{n/2} x_{min} \leq x_{max} \ , \ p_i \geq 3 \ , \ y_{i,n} = p_i^n x_{min} \leq x_{max} \ . \ p_i \geq 2 \ ,
\end{gathered} \tag{5.7.14}$$

L_{max} *resp.* L_{min} is the maximal *resp.* minimal length $L = \int (\hat{g}^{rr})^{-1/2} dr$ for the braid strand defined by the flux line of the Kähler magnetic field in the effective metric. The contributions correspond to the effective hyper-complex prime $p_1 = (1, 0)$ and hyper-complex primes with Minkowski lengths $\sqrt{p}$ ($p \geq 3$) and p, $p \geq 2$. If also higher p-adic length scales $L_n = p^{n/2}L_{min} < L_{max}$ and $L_n = p^nL_{min} < L_{max}$, $n > 1$, are allowed there is no further restriction on the summation. For the restricted option only L_n, $n = 0, 2$ is allowed.

The expressions for the Kähler function and its exponent reads as

$$\begin{aligned}
K &= k(log(x_{min}) + \sum_i log(x_i) + \sum_i log(y_i) \ , \\
exp(K) &= (\frac{1}{x_{min}})^k \times \prod_i (\frac{1}{x_i})^k \times \prod_i (\frac{1}{y_i})^k \ , \\
x_i &\leq x_{max} \ , \ y_i \leq x_{max} \ , \ k = -\frac{d\zeta(s)}{ds}_{|s=0} = \frac{1}{2}log(2\pi) \simeq .9184 \ .
\end{aligned} \tag{5.7.15}$$

From the point of view of p-adicization program the appearance of strongly transcendental numbers in the normalization factor of ζ_D is not a well-come property.

If the scaling of the WCW Kähler metric by $1/k$ is a legitimate procedure it would allow to get rid of the transcendental scaling factor k and this scaling would cancel also the transcendental from the exponent of Kähler function. The scaling is not however consistent with the view that Kähler coupling strength determines the normalization of the WCW metric.

This formula generalizes in a rather obvious manner to the cases when one allows M^2 projections of hyper-octonionic primes.

Manifestly finite options

The options for which one does not allow summation over all integer multiples of the basic momenta characterized by the canonical representatives of hyper-complex primes or their projections to M^2 are manifestly finite. They differ from the Riemann option only in that the normalization factor $k = \simeq .9184$ defined by the derivative Riemann Zeta at origin is replaced with $k = 1$. This would mean manifest finiteness of ζ_D. Kähler function and its exponent are given by

$$\begin{aligned} K &= k(log(x_{min}) + \sum_i log(x_i) + \sum_i log(y_i) \ , \ x_i \leq x_{max} \ , \ y_i \leq x_{max} \ , \\ exp(K) &= \frac{1}{x_{min}} \times \prod_i \frac{1}{x_i} \times \prod_i \frac{1}{y_i} \ . \end{aligned} \tag{5.7.16}$$

Numerically the Kähler functions do not differ much since their ratio is .9184. Number theoretically these functions are however completely different. The resulting dependence involves only square roots of primes and is an algebraic function of the lengths p_i and rational function of x_{min}. p-Adicization program would require rational values of the lengths x_{min} in the intersection of the real and p-adic worlds if one allows algebraic extension containing the square roots of the primes involved. Note that in p-adic context this algebraic extension involves two additional square roots for $p > 2$ if one does not want square root of p. Whether one should allow for R_p also extension based on $\sqrt{p}$ is not quite clear. This would give 8-D extension.

For the more general option allowing all projections of hyper-complex primes to M^2 the general form of Kähler function is same. Instead of pseudo-masses coming as primes and their square roots one has pseudomasses coming as square roots of some integers $n \leq p$ or $n \leq p^2$ for each p. In this case the conservation laws are not so strong.

Note that in the case of vacuum extremals $x_{min} = \infty$ holds true so that there are no primes satisfying the condition and Kähler function vanishes as it indeed should.

More concrete picture about the option based on infinite primes

The identification of pseudo-momenta in terms of infinite primes suggests a rather concrete connection between number theory and physics.

1. One could assign the finite hyper-octonionic primes Π_i making the infinite prime to the sub-braids identified as Kähler magnetic flux lines with the same length L in the effective metric. The primes assigned to the finite part of the infinite prime correspond to single fermion and some number of bosons. The primes assigned to the infinite part correspond to purely bosonic states assignable to the purely bosonic braid strands. Purely bosonic state would correspond to the action of a WCW Hamiltonian to the state.

   This correspondence can be expanded to include all quantum numbers by using the pair of infinite primes corresponding to the "vacuum primes" $X \pm 1$, where X is the product of all finite primes [19]. The only difference with respect to the earlier proposal is that physical momenta would be replaced by pseudo-momenta.

2. Different primes p_i appearing in the infinite prime would correspond to their own sub-braids. For each sub-braid there is a N-fold degeneracy of the generalized eigen modes corresponding

to the number N of braid strands so that many particle states are possible as required by the braid picture.

3. The correspondence of infinite primes with the hierarchy of Planck constants could allow to understand the fermion-many boson states and many boson states assigned with a given finite prime in terms of many-particle states assigned to n_a and n_b-sheeted singular covering spaces of CD and CP_2 assignable to the two infinite primes. This interpretation requires that only single p-adic prime p_i is realized as quantum state meaning that quantum measurement always selects a particular p-adic prime p_i (and corresponding sub-braid) characterizing the p-adicity of the quantum state. This selection of number field behind p-adic physics responsible for cognition looks very plausible.

4. The correspondence between pairs of infinite primes and quantum states [19] allows to interpret color quantum numbers in terms of the states associated with the representations of a finite subgroup of $SU(3)$ transforming hyper-octonionic primes to each other and preserving the M^2 pseudo-momentum. Same applies to $SO(3)$. The most natural interpretation is in terms of wave functions in the space of discrete $SU(3)$ and $SO(3)$ transforms of the partonic 2-surface. The dependence of the pseudo-masses on these quantum numbers is natural so that the projection hypothesis finds support from this interpretation.

5. The infinite prime characterizing the orbit of the partonic 2-surface would thus code which multiples of the basic mass $2\pi/L_{min}$ are possible. Either the M^2 projections of hyper-octonionic primes or their hyper-complex canonical representatives would fix the basic M^2 pseudo-momenta for the corresponding number theoretic braid associated. In the reverse direction the knowledge of the light-like 3-surface, the CD and CP_2 coverings, and the number of the allowed discrete $SU(3)$ and $SU(2)$ rotations of the partonic 2-surface would dictate the infinite prime assignable to the orbit of the partonic 2-surface.

One would also like to understand whether there is some kind of conservation laws associated with the pseudo-momenta at vertices. The arithmetic QFT assignable to infinite primes would indeed predict this kind of conservation laws.

1. For the manifestly finite option the ordinary conservation of pseudo-momentum conservation at vertices is not possible since the addition of pseudo-momenta does not respect the condition $n_0 - n_3 = 1$. In fact, this difference in the sum of hyper-complex prime momenta tells how many momenta are present. If one applies the conservation law to the sum of the pseudo-momenta corresponding to different primes and corresponding braids, one can have reactions in which the number of primes involved is conserved. This would give the selection rule $\sum_1^N p_i = \sum_1^N p_f$. These reactions have interpretation in terms of the geometry of the 3-surface representing the line of the generalized Feynman diagram.

2. Infinite primes define an arithmetic quantum field theory in which the total momentum defined as $\sum n_i log(p_i)$ is a conserved quantity. As matter fact, each prime p_i would define a separately conserved momentum so that there would be an infinite number of conservation laws. If the sum $\sum_i log(p_i)$ is conserved in the vertex , the primes p_i associated with the incoming particle are shared with the outgoing particles so that also the total momentum is conserved. This looks the most plausible option and would give very powerful number theoretical selection rules at vertices since the collection of primes associated with incoming line would be union of the collections associated with the outgoing lines and also total pseudo-momentum would be conserved.

3. For the both Riemann zeta option and manifestly finite options the arithmetic QFT associated with infinite primes would be realized at the level of pseudo-momenta meaning very strong selection rules at vertices coding for how the geometries of the partonic lines entering the vertex correlate. WCW integration would reduce for the lines of Feynman diagram to a sum over light-like 3-surfaces characterized by (x_{min}, x_{max}) with a suitable weighting factor and the exponent of Kähler function would give an exponential damping as a function of x_{min}.

Which option to choose?

One should be able to make two choices. One must select between hyper-complex representations and the projections of hyper-octonionic primes and between the manifestly finite options and the one producing Riemann zeta?

Hyper-complex option seems to be slightly favored over the projection option.

1. The appearance of the scales $\sqrt{p_i}x_{min}$ and possibly also their p^n multiples brings in mind p-adic length scales coming as $\sqrt{p}^n$ multiples of CP_2 length scale. The scales $p_i x_{min}$ associated with hyper-complex primes reducing to ordinary primes in turn bring in mind the size scales assignable to CDs. The hierarchy of Planck constants implies also $\hbar/\hbar_0 = \sqrt{n_a n_b}$ multiples of these length scales but mass scales would not depend on n_a and n_b [21]. For large values of p the pseudo-momenta are almost light-like for hyper-complex option whereas the projection option allows also states at rest.

2. Hyper-complex option predicts that only the p-adic pseudo-mass scales appear in the partition function and is thus favored by the p-adic length scale hypothesis. Projection option predicts also the possibility of the mass scales (not all of them) coming as $1/\sqrt{n}$. These mass scales are however not predicted by the hierarchy of Planck constants.

3. The same pseudo-mass scale can appear several times for the projection option. This degeneracy corresponds to the orbit of the hyper-complex prime under the subgroup of $SU(3)$ respecting integer property. Similar statement holds true in the case of $SO(3)$: these groups are assigned to the two infinite primes characterizing parton. The natural assignment of this degeneracy is to the discrete color rotational and rotational degrees associated with the partonic 2-surface itself rather than spinor modes at fixed partonic 2-surface. That the pseudo-mass would depend on color and angular momentum quantum numbers would make sense.

Consider next the arguments in favor of the manifestly finite option.

1. The manifestly finite option is admittedly more elegant than the one based on Riemann zeta and also guarantees that no additional loop summations over pseudo-momenta are present. The strongest support for the manifestly finite option comes from number theoretical universality.

2. One could however argue that the restriction of the pseudo-momenta to a finite number is not consistent with the modified Dirac-Chern-Simons equation. Quantum classical correspondence however implies correlation between the geometry of the partonic orbits and the pseudo-momenta and the summation over all prime valued pseudo-momenta is present but with a weighting factor coming from Kähler function implying exponential suppression.

The Riemann zeta option could be also defended.

1. The numerical difference of the normalization factors of the Kähler function is however only about 8 per cent and quantum field theorists might interpret the replacement the length scales x_i and y_i with x_i^d and y_i^d, $d \simeq .9184$, in terms of an anomalous dimension of these length scales. Could one say that radiative corrections mean the scaling of the original preferred coordinates so that one could still have consistency with number theoretic universality?

2. Riemann zeta with a non-vanishing argument could have also other applications in quantum TGD. Riemann zeta has interpretation as a partition function and the zeros of partition functions have interpretation in terms of phase transitions. The quantum criticality of TGD indeed corresponds to a phase transition point. There is also experimental evidence that the distribution of zeros of zeta corresponds to the distribution of energies of quantum critical systems in the sense that the energies correspond to the imaginary parts of the zeros of zeta [46].

   The first explanation would be in terms of the analogs of the harmonic oscillator coherent states with integer multiple of the basic momentum taking the role of occupation number of harmonic oscillator and the zeros $s = 1/2+iy$ of ζ defining the values of the complex coherence parameters. TGD inspired strategy for the proof of Riemann hypothesis indeed leads to the identification of the zeros as coherence parameters rather than energies as in the case of Hilbert-Polya hypothesis

[16] and the vanishing of the zeta at zero has interpretation as orthogonality of the state with respect to the state defined by a vanishing coherence parameter interpreted as a tachyon. One should demonstrate that the energies of quantum states can correspond to the imaginary parts of the coherence parameters.

Second interpretation could be in terms of quantum critical zero energy states for which the "complex square root of density matrix" defines time-like entanglement coefficients of M-matrix. The complex square roots of the probabilities defined by the coefficient of harmonic oscillator states (perhaps identifiable in terms of the multiples of pseudo-momentum) in the coherent state defined by the zero of ζ would define the M-matrix in this situation. Energy would correspond also now to the imaginary part of the coherence parameter. The norm of the state would be completely well-defined.

Representation of configuration Kähler metric in terms of eigenvalues of D_{C-S}

A surprisingly concrete connection of the configuration space metric in terms of generalized eigenvalue spectrum of D_{C-S} results. From the general expression of Kähler metric in terms of Kähler function

$$G_{k\bar{l}} = \partial_k\partial_{\bar{l}}K = \frac{\partial_k\partial_{\bar{l}}exp(K)}{exp(K)} - \frac{\partial_k exp(K)}{exp(K)}\frac{\partial_{\bar{l}}exp(K)}{exp(K)} \ , \tag{5.7.17}$$

and from the expression of $exp(K) = \prod_i \lambda_i$ as the product of of finite number of eigenvalues of D_{C-S} , the expression

$$G_{k\bar{l}} = \sum_i \frac{\partial_k\partial_{\bar{l}}\lambda_i}{\lambda_i} - \frac{\partial_k\lambda_i}{\lambda_i}\frac{\partial_{\bar{l}}\lambda_i}{\lambda_i} \tag{5.7.18}$$

for the configuration space metric follows. Here complex coordinates refer to the complex coordinates of configuration space. Hence the knowledge of the eigenvalue spectrum of $D_{C-S}(X^3)$ as function of some complex coordinates of configuration space allows to deduce the metric to arbitrary accuracy. If the above arguments are correct the calculation reduces to the calculation of the derivatives of $log(\sqrt{p}L_{min}/R)$, where L_{min} is the length of the Kähler magnetic flux line between partonic 2-surfaces with respect to the effective metric defined by the anti-commutators of the modified gamma matrices. Note that these length scales have different dependence on WCW coordinates so that one cannot reduce everything to L_{min}. Therefore one would have explicit representation of the basic building brick of WCW Kähler metric in terms of the geometric data associated with the orbit of the partonic 2-surface.

The formula for the Kähler action of CP_2 type vacuum extremals is consistent with the Dirac determinant formula

The first killer test for the formula of Kähler function in terms of the Dirac determinant based on infinite prime hypothesis is provided by the action of CP_2 type vacuum extremals. One of the first attempts to make quantitative predictions in TGD framework was the prediction for the gravitational constant. The argument went as follows.

1. For dimensional reasons gravitational constant must be proportional to p-adic length scale squared, where p characterizes the space-time sheet of the graviton. It must be also proportional to the square of the vacuum function for the graviton representing a line of generalized Feynman diagram and thus to the exponent $exp(-2K)$ of Kähler action for topologically condensed CP_2 type vacuum extremals with very long projection. If topological condensation does not reduce much of the volume of CP_2 type vacuum extremal, the action is just Kähler action for CP_2 itself. This gives

$$\hbar_0 G = L_p^2 exp(2L_K(CP_2) = pR^2 exp(2L_K(CP_2) \ . \tag{5.7.19}$$

2. Using as input the constraint $\alpha_K \simeq \alpha_{em} \sim 1/137$ for Kähler coupling strengths coming from the comparison of the TGD prediction for the rotation velocity of distant galaxies around galactic nucleus and the p-adic mass calculation for the electron mass, one obtained the result

$$exp(2L_K(CP_2) = \frac{1}{p \times \prod_{p_i \leq 23} p_i} . \tag{5.7.20}$$

The product contains the product of all primes smaller than 24 ($p_i \in \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$). The expression for the Kähler function would be just of the form predicted by the Dirac determinant formula with L_{min} replaced with CP_2 length scale. As a matter fact, this was the first indication that particles are characterized by several p-adic primes but that only one of them is "active". As explained, the number theoretical state function reduction explains this.

3. The same formula for the gravitational constant would result for any prime p but the value of Kähler coupling strength would depend on prime p logarithmically for this option. I indeed proposed that this formula fixes the discrete evolution of the Kähler coupling strength as function of p-adic prime from the condition that gravitational constant is renormalization group invariant quantity but gave up this hypothesis later. It is wisest to keep an agnostic attitude to this issue.

4. I also made numerous brave attempts to deduce an explicit formula for Kähler coupling strength. The general form of the formula is

$$\frac{1}{\alpha_K} = klog(K^2), \quad K^2 = p \times 2 \times 3 \times 5.. \times 23 . \tag{5.7.21}$$

The problem is the exact value of k cannot be known precisely and the guesses for is value depend on what one means with number theoretical universality. Should Kähler action be a rational number? Or is it Kähler function which is rational number (it is for the Dirac determinant option in this particular case). Is Kähler coupling strength $g_K^2/4\pi$ or g_K^2 a rational number? Some of the guesses were $k = \pi/4$ and $k = 137/107$. The facts that the value of Kähler action for the line of a generalized diagram is not exactly CP_2 action and the value of α_K is not known precisely makes these kind of attempts hopeless in absence of additional ideas.

Also other elementary particles -in particular exchanged bosons- should involve the exponent of Kähler action for CP_2 type vacuum extremal. Since the values of gauge couplings are gigantic as compared to the expression of the gravitational constant the value of Kähler action must be rather small form them. CP_2 type vacuum extremals must be short in the sense that L_{min} in the effective metric is very short. Note however that the p-adic prime characterizing the particle according to p-adic mass calculations would be large also now. One can of course ask whether this p-adic prime characterizes the gravitational space-time sheets associated with the particle and not the particle itself. The assignment of p-adic mass calculations with thermodynamics at gravitational space-time sheets of the particle would be indeed natural. The value of α_K would depend on p in logarithmic manner for this option. The topological condensation of could also eat a lot of CP_2 volume for them.

Eigenvalues of D_{C-S} as vacuum expectations of Higgs field?

Infinite prime hypothesis implies the analog of p-adic length scale hypothesis but since pseudo-momenta are in question, this need not correspond to the p-adic length scale hypothesis for the actual masses justified by p-adic thermodynamics. Note also that L_{min} does not correspond to CP_2 length scale. This is actually not a problem since the effective metric is not M^4 metric and one can quite well consider the possibility that L_{min} corresponds to CP_2 length scale in the the induced metric. The reason is that light-like 3- surface is in question the distance along the Kähler magnetic flux line reduces essentially to a distance along the partonic 2-surface having size scale of order CP_2 length for the partonic 2-surfaces identified as wormhole throats. Therefore infinite prime can code for genuine p-adic length scales associated with the light-like 3-surface and quantum states would correspond by number theoretical state function reduction hypothesis to single ordinary prime.

Support for this identification comes also from the expression of gravitational constant deduced from p-adic length scale hypothesis. The result is that gravitational constant is assumed to be proportional to have the expression $G = L_p^2 exp(-2S_K(CP_2))$, where p characterizes graviton or the space-time sheet mediating gravitational interaction and exponent gives Kähler action for CP_2 type vacuum extremal representing graviton. The argument allows to identify the p-adic prime $p = M_{127}$ associated with electron (largest Mersenne prime which does not correspond to super-astronomical length scale) as the p-adic prime characterizing also graviton. The exponent of Kähler action is proportional to $1/p$ which conforms with the general expression for Kähler function. I have considered several identifications of the numerical factor and one of them has been as product of primes $2 \leq p \leq 23$ assuming that somehow the primes $\{2, ..., 23, p\}$ characterize graviton. This guess is indeed consistent with the prediction of the infinite-prime hypothesis.

The first guess inspired by the p-adic mass calculations is that the squares λ_i^2 of the eigenvalues of D_{C-S} could correspond to the conformal weights of ground states. Another natural physical interpretation of λ is as an analog of the Higgs vacuum expectation. The instability of the Higgs=0 phase would corresponds to the fact that $\lambda = 0$ mode is not localized to any region in which ew magnetic field or induced Kähler field is non-vanishing. By the previous argument one would have order of magnitude estimate $h_0 = \sqrt{2\pi/L_{min}}$.

1. The vacuum expectation value of Higgs is only proportional to the scale of λ. Indeed, Higgs and gauge bosons as elementary particles correspond to wormhole contacts carrying fermion and anti-fermion at the two wormhole throats and must be distinguished from the space-time correlate of its vacuum expectation as something proportional to λ. For free fermions the vacuum expectation value of Higgs does not seem to be even possible since free fermions do not correspond to wormhole contacts between two space-time sheets but possess only single wormhole throat (p-adic mass calculations are consistent with this). If fermion suffers topological condensation as indeed assumed to do in interaction region, a wormhole contact is generated and makes possible the generation of Higgs vacuum expectation value.

2. Physical considerations suggest that the vacuum expectation of Higgs field corresponds to a particular eigenvalue λ_i of modified Chern-Simons Dirac operator so that the eigenvalues λ_i would define TGD counterparts for the minima of Higgs potential. For the minimal option one has only a finite number of pseudo-mass eigenvalues inversely proportional $\sqrt{p}$ so that the identification as a Higgs vacuum expectation is consistent with the p-adic length scale hypothesis. Since the vacuum expectation of Higgs corresponds to a condensate of wormhole contacts giving rise to a coherent state, the vacuum expectation cannot be present for topologically condensed CP_2 type vacuum extremals representing fermions since only single wormhole throat is involved. This raises a hen-egg question about whether Higgs contributes to the mass or whether Higgs is only a correlate for massivation having description using more profound concepts. From TGD point of view the most elegant option is that Higgs does not give rise to mass but Higgs vacuum expectation value accompanies bosonic states and is naturally proportional to λ_i. With this interpretation λ_i could give a contribution to both fermionic and bosonic masses.

3. If the coset construction for super-symplectic and super Kac-Moody algebra implying Equivalence Principle is accepted, one encounters what looks like a problem. p-Adic mass calculations require negative ground state conformal weight compensated by Super Virasoro generators in order to obtain massless states. The tachyonicity of the ground states would mean a close analogy with both string models and Higgs mechanism. λ_i^2 is very natural candidate for the ground state conformal weights identified but would have wrong sign. Therefore it seems that λ_i^2 can define only a deviation of the ground state conformal weight from negative value and is positive.

4. In accordance with this λ_i^2 would give constant contribution to the ground state conformal weight. What contributes to the thermal mass squared is the deviation of the ground state conformal weight from half-odd integer since the negative integer part of the total conformal weight can be compensated by applying Virasoro generators to the ground state. The first guess motivated by cyclotron energy analogy is that the lowest conformal weights are of form $h_c = -n/2 + \lambda_i^2$ where the negative contribution comes from Super Virasoro representation. The

negative integer part of the net conformal weight can be canceled using Super Virasoro generators but Δh_c would give to mass squared a contribution analogous to Higgs contribution. The mapping of the real ground state conformal weight to a p-adic number by canonical identification involves some delicacies.

5. p-Adic mass calculations are consistent with the assumption that Higgs type contribution is vanishing (that is small) for fermions and dominates for gauge bosons. This requires that the deviation of λ_i^2 with smallest magnitude from half-odd integer value in the case of fermions is considerably smaller than in the case of gauge bosons in the scale defined by p-adic mass scale $1/L(k)$ in question. Somehow this difference could relate to the fact that bosons correspond to pairs of wormhole throats.

Is there a connection between p-adic thermodynamics, hierarchy of Planck constants, and infinite primes

The following observations suggest that there might be an intrinsic connection between p-adic thermodynamics, hierarchy of Planck constants, and infinite primes.

1. p-Adic thermodynamics [19] is based on string mass formula in which mass squared is proportional to conformal weight having values which are integers apart from the contribution of the conformal weight of vacuum which can be non-integer valued. The thermal expectation in p-adic thermodynamics is obtained by replacing the Boltzman weight $exp(-E/T)$ of ordinary thermodynamics with p-adic conformal weight p^{n/T_p}, where n is the value of conformal weight and $1/T_p = m$ is integer values inverse p-adic temperature. Apart from the ground state contribution and scale factor p-adic mass squared is essentially the expectation value

$$\langle n \rangle = \frac{\sum_n g(n) n p^{\frac{n}{T_p}}}{\sum_n g(n) p^{\frac{n}{T_p}}} . \qquad (5.7.22)$$

$g(n)$ denotes the degeneracy of a state with given conformal weight and depends only on the number of tensor factors in the representations of Virasoro or Super-Virasoro algebra. p-Adic mass squared is mapped to its real counterpart by canonical identification $\sum x_n p^n \to \sum x_n p^{-n}$. The real counterpart of p-adic thermodynamics is obtained by the replacement $p^{-\frac{n}{T_p}}$ and gives under certain additional assumptions in an excellent accuracy the same results as the p-adic thermodynamics.

2. An intriguing observation is that one could interpret p-adic and real thermodynamics for mass squared also in terms of number theoretic thermodynamics for the number theoretic momentum $log(p^n) = nlog(p)$. The expectation value for this differs from the expression for $\langle n \rangle$ only by the factor $log(p)$.

3. In the proposed characterization of the partonic orbits in terms of infinite primes the primes appearing in infinite prime are identified as p-adic primes. For minimal option the p-adic prime characterizes $\sqrt{p}$- or p- multiple of the minimum length L_{min} of braid strand in the effective metric defined by modified Chern-Simons gamma matrice. One can consider also $(\sqrt{p})^n$ and p^n (p-adic fractality)- and even integer multiples of L_{min} if they are below L_{max}. If light-like 3-surface contains vacuum regions arbitrary large p:s are possible since for these one has $L_{min} \to \infty$. Number theoretic state function reduction implies that only single p can be realized -one might say "is active"- for a given quantum state. The powers p_i^n appearing in the infinite prime have interpretation as many particle states with total number theoretic momentum $n_i log(p)_i$. For the finite part of infinite prime one has one fermion and $n_i - 1$ bosons and for the bosonic part n_i bosons. The arithmetic QFT associated with infinite primes - in particular the conservation of the number theoretic momentum $\sum n_i log(p_i)$ - would naturally describe the correlations between the geometries of light-like 3-surfaces representing the incoming lines of the vertex of generalized Feynman diagram. As a matter fact, the momenta associated with different primes are separately conserved so that one has infinite number of conservation laws.

4. One must assign two infinite primes to given partonic two surface so that one has for a given prime p two integers n_+ and n_-. Also the hierarchy of Planck constants assigns to a given page of the Big Book two integers and one has $\hbar = n_a n_b \hbar_0$. If one has $n_a = n_+$ and $n_b = n_-$ then the reactions in which given initial number theoretic momenta $n_{\pm,i} log(p_i)$ is shared between final states would have concrete interpretation in terms of the integers n_a, n_b characterizing the coverings of incoming and outgoing lines.

   Note that one can also consider the possibility that the hierarchy of Planck constants emerges from the basic quantum TGD. Basically due to the vacuum degeneracy of Kähler action the canonical momentum densities correspond to several values of the time derivatives of the imbedding space oordinates so that for a given partonic 2-surface there are several space-time sheets with same conserved quantities defined by isometry currents and Kähler current. This forces the introduction of N-fold covering of $CD \times CP_2$ in order to describe the situation. The splitting of the partonic 2-surface into N pieces implies a charge fractionization during its travel to the upper end of CD. One can also develop an argument suggesting that the coverings factorize to coverings of CD and CP_2 so that the number of the sheets of the covering is $N = n_a n_b$ [34].

These observations make one wonder whether there could be a connection between p-adic thermodynamics, hierarchy of Planck constants, and infinite primes.

1. Suppose that one accepts the identification $n_a = n_+$ and $n_b = n_-$. Could one perform a further identification of these integers as non-negative conformal weights characterizing physical states so that conservation of the number theoretic momentum for a given p-adic prime would correspond to the conservation of conformal weight. In p-adic thermodynamics this conformal weight is sum of conformal weights of 5 tensor factors of Super-Virasoro algebra. The number must be indeed five and one could assign them to the factors of the symmetry group. One factor for color symmetries and two factors of electro-weak $SU(2)_L \times U(1)$ are certainly present. The remaining two factors could correspond to transversal degrees of freedom assignable to string like objects but one can imagine also other identifications [19].

2. If this interpretation is correct, a given conformal weight $n = n_a = n_+$ (say) would correspond to all possible distributions of five conformal weights n_i, $i = 1, ..., 5$ between the n_a sheets of covering of CD satisfying $\sum_{i=1}^{5} n_i = n_a = n_+$. Single sheet of covering would carry only unit conformal weight so that one would have the analog of fractionization also now and a possible interpretation would be in terms of the instability of states with conformal weight $n > 1$. Conformal thermodynamics would also mean thermodynamics in the space of states determined by infinite primes and in the space of coverings.

3. The conformal weight assignable to the CD would naturally correspond to mass squared but there is also the conformal weight assignable to CP_2 and one can wonder what its interpretation might be. Could it correspond to the expectation of pseudo mass squared characterizing the generalized eigenstates of the modified Dirac operator? Note that one should allow in the spectrum also the powers of hyper-complex primes up to some maximum power $p^{n_{max}/2} \leq L_{max}/L_{min}$ so that Dirac determinant would be non-vanishing and Kähler function finite. From the point of conformal invariance this is indeed natural.

5.8 Number theoretic braids and global view about anti-commutations of induced spinor fields

The anti-commutations of the induced spinor fields are reasonably well understood locally. The basic objects are 3-dimensional light-like 3-surfaces. These surfaces can be however seen as random light-like orbits of partonic 2-D partonic surface and the effective 2-dimensionality means that partonic 2-surfaces plus there 4-D tangent space take the role of fundamental dynamical objects. This is expressed concretely by the condition that the ends of the space-time surface and wormhole throats are extremals of Chern-Simons action. For $J + J_1$ option this allows also 3-D CP_2 projections. Conformal invariance would in turn make the 2-D partons 1-D objects (analogous to Euclidian strings) and braids, which can be regarded as the ends of string world sheets with Minkowskian signature, in turn would discretize these Euclidian strings. It must be however noticed that the status of Euclidian strings is uncertain.

Somehow these views should be unifiable into a more global view about the situation allowing to understand the reduction of effective dimension of the system as one goes to short scales.

1. The notions of measurement resolution and braid concept indeed provides the needed physical insights in this respect. The precise definition of the notion of braid and its number theoretic counterpart remains however open and one can imagine several alternatives.

2. Electric-magnetic duality and the ideas stimulated by it led to a further progress. The braid concept emerges automatically from the reduction of Chern-Simons Dirac equation to separate ordinary differential equations at the flux lines of the Kähler magnetic field. Boundary conditions at the ends of the light-like 3-surface allow only solutions which are concentrated on braids for which the strands have same length in the effective metric defined by the modified gamma matrices and a connection with p-adic length scale hypothesis, hierarchy of Planck constants, and infinite primes emerges. Number theoretic braids correspond to braids for which the length is rational or at most algebraic number. Possible additional conditions on the coordinates of X^2 can be of course considered but already the quantization of lengths is enough to guarantee that the exponent of Kähler function identified as Dirac determinant rational function and for rational braid lengths a simple algebraic number involving only square roots of primes making sense also p-adically.

   The identification of flows defining the braids has been one of the key issues since quite a large variety of candidates can be imagined. The integrability of this flow by Beltrami condition implies for that there is huge variety of topologically equivalent flows and resolves also the identification issue: all isometry currents and Kähler current and perhaps also the instanton current define one and same flow.

5.8.1 Quantization of the modified Dirac action and configuration space geometry

The quantization of the modified Dirac action involves a fusion of various number theoretical ideas. The naive approach would be based on standard canonical quantization of induced spinor fields by posing anti-commutation relations between Ψ and canonical momentum density $\partial L/\partial(\partial_t\Psi)$.

One can imagine two alternative forms of the anti-commutation relations.

1. The standard canonical anti-commutation relations for the induced the spinor fields would be given by

$$\{\overline{\Psi}\hat{\Gamma}^0(x),\Psi(y)\} = \delta^2_{x,y} \ . \tag{5.8.1}$$

   The factor that $\hat{\Gamma}^0(x)$ corresponds to the canonical momentum density associated with Kähler action. The discrete variant of the anti-commutation relations applying in the case of non-stringy space-time sheets is

$$\{\overline{\Psi}\hat{\Gamma}^0(x_i),\Psi(x_j)\} = \delta_{i,j} \ . \tag{5.8.2}$$

   where x_i and x_j label the points of the number theoretic braid. These anticommutations are are inconsistent at the limit of vacuum extremal and also extremely non-linear in the imbedding space coordinates.

2. The construction of WCW gamma matrices leads to a nonsingular form of anti-commutation relations given by

$$\{\overline{\Psi}(x)\gamma^0, \Psi(x)\} = (1+K)J\delta_{x,y} \ . \quad (5.8.3)$$

Here J denotes the Kähler magnetic flux J_m and Kähler electric flux relates to via the formula $J_e = KJ_m$, where K is symplectic invariant. What is nice that at the limit of vacuum extremals the right hand side vanishes so that spinor fields become non-dynamical. Therefore this optin- actually the original one- seems to be the only reasonable choice.

For the latter option the super counterparts of local flux Hamiltonians can be written in the form

$$\begin{aligned}
H_{A,+,n} &= H_{A,+,q,n} + H_{A,+,L,n} \ , \ H_{A,-,n} = H_{A,-,q,n} + H_{A,-,L,n} \ , \\
H_{A,+,q,n} &= \oint \overline{\Psi} J_+^A q_n d^2x \ , \\
H_{A,-,q,n} &= \oint \overline{q}_n J_-^A \Psi d^2x \ , \\
H_{A,-,L,n} &= \oint \overline{\Psi} J_+^A L_n d^2x \ , \\
H_{A,+,L,n} &= \oint \overline{L}_n J_-^A \Psi d^2x \ , \\
J_+^A &= j^{Ak}\Gamma_k \ , \ J_-^A = j^{A\overline{k}}\Gamma_{\overline{k}} \ .
\end{aligned} \quad (5.8.4)$$

Suppose that there is a one-one correspondence between quark modes and leptonic modes is satisfied and the label n decomposes as $n = (m,i)$, where n labels a scalar function basis and i labels spinor components. This would give

$$\begin{aligned}
q_n = q_{m,i} &= \Phi_m q_i \ , \\
L_n = L_{m,i} &= \Phi_m L_i \ , \\
\overline{q}_i\gamma^0 q_j &= \overline{L}_i \gamma^0 L_j = g_{ij} \ .
\end{aligned} \quad (5.8.5)$$

Suppose that the inner products g_{ij} are constant. The simplest possibility is $g_{ij} = \delta_{ij}$ Under these assumptions the anticommutators of the super-symmetric flux Hamiltonians give flux Hamiltonians.

$$\{H_{A,+,n}, H_{A,-,n}\} = g_{ij} \oint (1+K)\overline{\Phi}_m \Phi_n H_A J d^2x \ . \quad (5.8.6)$$

The product of scalar functions can be expressed as

$$\overline{\Phi}_m \Phi_n = c_{mn}^{\ \ k} \Phi_k \ . \quad (5.8.7)$$

Note that the notion of symplectic QFT led to a scalar function algebra of similar kind consisting of phase factors and there excellent reasons to consider the possibility that there is a deep connection with this approach.

One expects that the symplectic algebra is restricted to a direct sum of symplectic algebras localized to the regions where the induced Kähler form is non-vanishing implying that the algebras associated with different region form to a direct sum. Also the contributions to configuration space metric are direct sums. The symplectic algebras associated with different region can be truncated to finite-dimensional spaces of symplectic algebras $S^2 \times S$ associated with the regions in question. As far as coordinatization of the reduced configuration space is considered, these symplectic sub-spaces are enough. These truncated algebras naturally correspond to the hyper-finite factor property of the Clifford algebra of configuration space.

5.8.2 Expressions for configuration space super-symplectic generators in finite measurement resolution

The expressions of configuration space Hamiltonians and their super counterparts just discussed were based on 2-dimensional integrals. This is problematic for several reasons.

1. In p-adic context integrals do not makes sense so that this representation fails in p-adic context. Sums would be more appropriate if one wants number theoretic universality at the level of basic formulas.

2. The use of sums would also conform with the notion of finite measurement resolution having discretization in terms of intersections of X^2 with number theoretic braids as a space-time correlate.

3. Number theoretic duality suggests a unique realization of the discretization in the sense that only the points of partonic 2-surface X^2 whose $\delta M^4_\pm$ projections commute in hyper-octonionic sense and thus belong to the intersections of the projection $P_{M^4}(X^2)$ with radial light-like geodesics $M_\pm$ representing intersections of $M^2 \subset M^4 \subset M^8$ with $\delta M^4_\pm \times CP_2$ contribute to the configuration space Hamiltonians and super Hamiltonians and therefore to the configuration space metric.

Clearly, finite measurement resolution seems to be an unavoidable aspect of the geometrization of the configuration space as one can expect on basis of the fact that configuration space Clifford algebra provides representation for hyper-finite factors of type II_1 whose inclusions provide a representation for the finite measurement resolution. This means that the infinite-dimensional configuration space can be represented as a finite-dimensional space in arbitrary precise approximation so that also also configuration Clifford algebra and configuration space spinor fields becomes finite-dimensional.

The modification of anti-commutation relations to this case is

$$\{\overline{\Psi}(x_m)\gamma^0, \Psi(x_n)\} = (1+K)J\delta_{x_m,x_n} \ . \quad (5.8.8)$$

Note that the constancy of γ^0 implies a complete symmetry between the two points. The number of points must be the maximal one consistent with the Kronecker delta type anti-commutation relations so that information is not lost.

The question arises about the choice of the points x_m. This choice should ge general coordinate invariant. The following ideas have been considered.

1. The number theoretic vision leads to the notion of number theoretic braid defined as the set of points common to real and p-adic variant of X^2. The points of the number theoretic braid are excellent candidates for points x_n. The p-adic variant of X^2 exists only if X^2 is defined by rational functions with coefficients which are possibly algebraic and thus make sense both in real and p-adic sense. These points belong to the algebraic extension of rational numbers appearing in the representation of X^2 as an algebraic surface but one can consider quite generally the possibility that the points of the number theoretic braid are rational or in a finite algebraic extension of rationals. If one restricts the consideration to rational points this criterion makes sense even if X^2 is not algebraic. In the generic case one can expect that the number of these points is finite. The objection is that this definition is not general coordinate invariant. One can however identify preferred coordinates since CP_2 and the sphere S^2 associated with light-cone boundary are symmetric spaces.

2. An alternative identification emerged from the solution of the Chern-Simons Dirac equation. Since the number of generalized eigenmodes of D_{C-S} is finite for a given braid, the local anti-commutation relations cannot be satisfied unless they are restricted to a finite subset of points of X^2 and the condition that the lengths of the braid strands in the effective metric are same fixes the braid uniquely. Number theoretic braids are obtained when the length is rational or algebraic number. This identification of number theoretic braid is enough for p-adicization since it guarateens that the exponent of Kähler function is rational function of braid lengths labelled by a finite number of primes. The interpretation of the braids as orbits of the ends of string

world sheets is possible and the braiding of the string ends brings in topological QFT aspect. Of utmost importance is that this identification is also general coordinate invariant and guarantees that WCW Kähler function is rational function of general coordinate invariant variables and thus allows p-adicization.

This option leaves no room for the choice of the ends of braid strands unless one allows the length scale identified as the minimum length of the braid strand to vary in certain limits. This variation might only induce a gauge transformation $K \rightarrow K+f+\overline{f}$ of the WCW Kähler function. One can of consider the possibility that the braid points as points of the imbedding space are rational or algebraic numbers in some preferred coordinates of H in the intersection of the real and p-adic worlds so that the original hypothesis would make sense in this intersection only.

For the latter option the number of solutions of the Chern-Simons Dirac equation for given spinorial quantum numbers is the number of braid points so that the number of fermionic oscillator operators for a given mode is same as the number of braid points and the anticommutation relations have a unique solution.

Symplectic fusion algebra [17] might also be important element in quantization. The relationship between symplectic fusion algebra and its conjugate has not been characterized and one can consider the possibility that the algebra generators satisfy the conditions $e_m\overline{e}_n = \delta_{m,n}$. If induced spinor field at points of number theoretic braid defining the symplectic fusion algebra is multiplied by e_m then the anti-commutation relations reduce automatically to a form in which anti-commutators at same point are involved. This would reduce the number of conditions to $8N_B$ from $8N_B^2$. The notion of finite measurement resolution could be used to defend this option.

5.8.3 QFT description of particle reactions at the level of braids

The overall view conforms with zero energy ontology in which hierarchy of causal diamonds (CDs) within CDs gives rise to a hierarchy of generalized Feynman diagrams and geometric description of the radiative corrections. Each sub-CD gives also rise to to zero energy states and thus particle reactions in its own time scale so that improvement of the time resolution brings in also new physics as it does also in reality.

The natural question is what happens to the braids at vertices.

1. The vision based on infinite primes led to the conclusion that the selection rules of arithmetic quantum field theory based on the conservation of the total number theoretic momentum $P = \sum n_i log(p_i)$ dictate the selection rules at the vertixes. For given p_i the momentum $n_i log(p_i)$ can be shared between the outgoing lines and this allows several combinations of infinite primes in outgoing lines having interpretations in terms of singular coverings of CD and CP_2.

2. What happens then to the braid strands? If the bosons and fermions with given p_i are shared between several outgoing particles, does this require that the braid strands replicate? Or is their number preserved if one regards each braid strand as having n_a *resp.* n_b copies at the sheets of the corresponding coverings? This is required by the conservation of number theoretic momentum if one accepts the connection between the hierarchy of Planck constants and infinite primes.

3. The question raised already earlier is whether DNA replication could have a counterpart at the level of fundamental physics. The interpretation of the incoming lines of generalized Feynman diagram as representations of topological quantum computations and the virtual particle lines as representations of quantum communications would support this picture. The no-cloning theorem [53] would hold true since exact copies of quantum states would not be possible by the conservation of the number theoretical momentum. One could however say that the bosonic occupation number n_i means the presence of n_i-fold copy of same piece of information so that the sharing of information by sharing the pages of the singular covering associated with n_i would be possible in the limits posed by the values of n_i. Note again that the identification $n_i = n_a$ or $n_i = n_b$ (two infinite primes characterize the quantum state) makes sense only if only one of the p-adic primes associated with the 3-surface is realized as a physical state since the identification forces the selection of the covering. The quantum model for DNA based on hierarchy of Planck constants [37] inspires the question whether DNA replication could be actually accompanied by

its proposed counterpart at the fundamental level defining the fundamental information transfer process.

4. The localization of the quantum numbers to braid strands suggests that braid ends of a given braid continue to one particular line or more generally, are shared between several lines. This condition is quite strong since without additional quantization conditions the ends of the braids of outgoing particles do not co-incide with the ends of the incoming braid. These kind of quantization conditions would conform with the generalized Bohr orbit property of light-like 3-surfaces.

5. Without these quantization conditions one meets the challenge of calculating the anticommutators of fermionic oscillator operators associated with non-co-inciding points of the incoming and outgoing braids. This raises the question whether one should regard the quantizations of induced spinor fields based on the L_{min} as one possible gauge only and allow the variation of L_{min} in some limits. If these quantizations are equivalent, the fermionic oscillator operators would be unitarily related. How to deduce this unitary transformation would be the non-trivial problem and it seems that the simpler picture is much more attractive.

This picture means that particle reactions occur at several levels which brings in mind a kind of universal mimicry inspired by Universe as a Universal Computer hypothesis. Particle reactions in QFT sense correspond to the reactions for the number theoretic braids inside partons. This level seems to be the simplest one to describe mathematically. At parton level particle reactions correspond to generalized Feynman diagrams obtained by gluing partonic 3-surfaces along their ends at vertices. Particle reactions are realized also at the level of 4-D space-time surfaces. One might hope that this multiple realization could code the dynamics already at the simple level of single partonic 3-surface.

5.8.4 How do generalized braid diagrams relate to the perturbation theory?

The association of generalized braid diagrams characterized by infinite primes to the incoming and outgoing partonic legs and internal lines of the generalized Feynman diagrams forces to ask whether the generalized braid diagrams could give rise to a counterpart of perturbation theoretical formalism via the functional integral over configuration space degrees of freedom.

The basic question is how the functional integral over configuration space degrees of freedom relates to the generalized braid diagrams.

1. If one believes in perturbation theoretic approach, the basic conjecture motivated also number theoretically is that radiative corrections in this sense sum up to zero for critical values of Kähler coupling strength and Kähler function codes radiative corrections to classical physics via the dependence of the scale of M^4 metric on Planck constant. Cancelation could occur only for critical values of Kähler coupling strength α_K: for general values of α_K the cancellation would require separate vanishing of each term in the sum and does not occur.

   In perturbative approach the expression of Kähler function as Chern-Simons action could be used and propagator would correspond to the inverse of the 1-1 part of the second variation of the Chern-Simons action with respect to complex WCW coordinates evaluated allowing only the extrema of Chern-Simons action for the ends of space-time surface and for wormhole throats. One would have perturbation theory for a sum over maxima of Kähler function. From the expression of the Kähler function as Dirac determinant the maxima would correspond to the local minima of $L_p = \sqrt{p}L_{min}$ for a given infinite prime. The connection between Chern-Simons representation and Dirac determinant representation of Kähler function would be obviously highly desirable.

2. The possibility to define WCW functional integral in terms of harmonic analysis for infinite-dimensional spaces leads to a non-perturbative approach to functional integration allowing also a generalization the p-adic context [20]. In this approach there is no need to make additional assumptions.

For both cases the assignment of the collection of braids characterized by pairs of infinite primes allows to organize the generalized Feynman diagrams into a sum of generalized Feynman diagrams and for each diagram type the exponent of Kähler function - if given by the Dirac determinant- would be simply the product $\prod_i L_{p_i}^{-1}$, $L_p = \sqrt{p}L_{min}$. One should perform a sum over different infinite primes in the internal lines subject to the conservation of the total number theoretic momenta. The conservation of the incoming number theoretic momentum would allow only a finite number of configurations for the intermediate lines. For the approach based on harmonic analysis the expression of the Kähler function in terms of the Dirac determinant would be optimal since it is manifestly algebraic function.

Both approaches involve a perturbative summation in the sense of introducing sub-CDs with time scales coming as 2^{-n} powers of the time scale of CD defining the infrared cutoff.

1. The addition of zero energy insertions corresponding to sub-CDs as radiative corrections allows to improve measurement resolution. Hence a connection with QFT type Feyman diagram expansion would be obtained and Connes tensor product would have a practical computational realization.

2. The time scale resolution defined by the temporal distance between the tips of the causal diamond defined by the future and past light-cones applies to the addition of zero energy sub-states and one obtains a direct connection with p-adic length scale evolution of coupling constants since the time scales in question naturally come as negative powers of two. More precisely, p-adic primes near power of two are very natural since the coupling constant evolution comes in powers of two of fundamental 2-adic length scale.

5.8.5 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

The condition $T_n = 2^n T_0$ would assign to the hierarchy of CDs as hierarchy of time scales coming as octaves. A weaker condition would be $T_p = pT_0$, p prime, and would assign all secondary p-adic time scales to the size scale hierarchy of CDs.

One can wonder how this picture relates to the earlier hypothesis that p-adic length coupling constant evolution. Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks like an attractive idea but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3. For $T_p = pT_0$ the above argument is not enough for p-adic length

scale hypothesis and p-adic length scale hypothesis might be seen as an outcome of a process analogous to natural selection. Resonance like effect favoring octaves of a fundamental frequency might be in question. In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of configuration space.

5.9 How to define generalized Feynman diagrams?

S-matrix codes to a high degree the predictions of quantum theories. The longstanding challenge of TGD has been to construct or at least demonstrate the mathematical existence of S-matrix- or actually M-matrix which generalizes this notion in zero energy ontology (ZEO) [5]. This work has led to the notion of generalized Feynman diagram and the challenge is to give a precise mathematical meaning for this object. The attempt to understand the counterpart of twistors in TGD framework [28] has inspired several key ideas in this respect but it turned out that twistors themselves need not be absolutely necessary in TGD framework.

1. The notion of generalized Feyman diagram defined by replacing lines of ordinary Feynman diagram with light-like 3-surfaces (elementary particle sized wormhole contacts with throats carrying quantum numbers) and vertices identified as their 2-D ends - I call them partonic 2-surfaces is central. Speaking somewhat loosely, generalized Feynman diagrams (plus background space-time sheets) define the "world of classical worlds" (WCW). These diagrams involve the analogs of stringy diagrams but the interpretation is different: the analogs of stringy loop diagrams have interpretation in terms of particle propagating via two different routes simultaneously (as in the classical double slit experiment) rather than as a decay of particle to two particles. For stringy diagrams the counterparts of vertices are singular as manifolds whereas the entire diagrams are smooth. For generalized Feynman diagrams vertices are smooth but entire diagrams represent singular manifolds just like ordinary Feynman diagrams do. String like objects however emerge in TGD and even ordinary elementary particles are predicted to be magnetic flux tubes of length of order weak gauge boson Compton length with monopoles at their ends as shown in accompanying article. This stringy character should become visible at LHC energies.

2. Zero energy ontology (ZEO) and causal diamonds (intersections of future and past directed lightcones) is second key ingredient. The crucial observation is that in ZEO it is possible to identify off mass shell particles as pairs of on mass shell particles at throats of wormhole contact since both positive and negative signs of energy are possible. The propagator defined by modified Dirac action does not diverge (except for incoming lines) although the fermions at throats are on mass shell. In other words, the generalized eigenvalue of the modified Dirac operator containing a term linear in momentum is non-vanishing and propagator reduces to $G = i/\lambda\gamma$, where γ is so called modified gamma matrix in the direction of stringy coordinate [18]. This means opening of the black box of the off mass shell particle-something which for some reason has not occurred to anyone fighting with the divergences of quantum field theories.

3. A powerful constraint is number theoretic universality requiring the existence of Feynman amplitudes in all number fields when one allows suitable algebraic extensions: roots of unity are certainly required in order to realize p-adic counter parts of plane waves. Also imbedding space, partonic 2-surfaces and WCW must exist in all number fields and their extensions. These constraints are enormously powerful and the attempts to realize this vision have dominated quantum TGD for last two decades.

4. Representation of 8-D gamma matrices in terms of octonionic units and 2-D sigma matrices is a further important element as far as twistors are considered [28]. Modified gamma matrices at space-time surfaces are quaternionic/associative and allow a genuine matrix representation. As a matter fact, TGD and WCW can be formulated as study of associative local sub-algebras of the local Clifford algebra of 8-D imbedding space parameterized by quaternionic space-time surfaces. Central conjecture is that quaternionic 4-surfaces correspond to preferred extremals of Kähler action [18] identified as critical ones (second variation of Kähler action vanishes for infinite number of deformations defining super-conformal algebra) and allow a slicing to string worldsheets parametrized by points of partonic 2-surfaces.

5. As far as twistors are considered, the first key element is the reduction of the octonionic twistor structure to quaternionic one at space-time surfaces and giving effectively 4-D spinor and twistor structure for quaternionic surfaces.

Quite recently quite a dramatic progress took place in this approach [28].

1. The progress was stimulated by the simple observation that on mass shell property puts enormously strong kinematic restrictions on the loop integrations. With mild restrictions on the number of parallel fermion lines appearing in vertices (there can be several since fermionic oscillator operator algebra defining SUSY algebra generates the parton states)- all loops are manifestly finite and if particles has always mass -say small p-adic thermal mass also in case of massless particles and due to IR cutoff due to the presence largest CD- the number of diagrams is finite. Unitarity reduces to Cutkosky rules [58] automatically satisfied as in the case of ordinary Feynman diagrams.

2. Ironically, twistors which stimulated all these development do not seem to be absolutely necessary in this approach although they are of course possible. Situation changes if one does not assume small p-adically thermal mass due to the presence of massless particles and one must sum infinite number of diagrams. Here a potential problem is whether the infinite sum respects the algebraic extension in question.

This is about fermionic and momentum space aspects of Feynman diagrams but not yet about the functional (not path-) integral over small deformations of the partonic 2-surfaces. The basic challenges are following.

1. One should perform the functional integral over WCW degrees of freedom for fixed values of on mass shell momenta appearing in the internal lines. After this one must perform integral or summation over loop momenta. Note that the order is important since the space-time surface assigned to the line carries information about the quantum numbers associated with the line by quantum classical correspondence realized in terms of modified Dirac operator.

2. One must define the functional integral also in the p-adic context. p-Adic Fourier analysis relying on algebraic continuation raises hopes in this respect. p-Adicity suggests strongly that the loop momenta are discretized and ZEO predicts this kind of discretization naturally.

It indeed seems that the functional integrals over WCW could be carried out at general level both in real and p-adic context. This is due to the symmetric space property (maximal number of isometries) of WCW required by the mere mathematical existence of Kähler geometry [34] in infinite-dimensional context already in the case of much simpler loop spaces [45].

1. The p-adic generalization of Fourier analysis allows to algebraize integration- the horrible looking technical challenge of p-adic physics- for symmetric spaces for functions allowing the analog of discrete Fourier decomposion. Symmetric space property is indeed essential also for the existence of Kähler geometry for infinite-D spaces as was learned already from the case of loop spaces. Plane waves and exponential functions expressible as roots of unity and powers of p multiplied by the direct analogs of corresponding exponent functions are the basic building bricks and key functions in harmonic analysis in symmetric spaces. The physically unavoidable finite measurement resolution corresponds to algebraically unavoidable finite algebraic dimension of algebraic extension of p-adics (at least some roots of unity are needed). The cutoff in roots of unity is very reminiscent to that occurring for the representations of quantum groups and is certainly very closely related to these as also to the inclusions of hyper-finite factors of type II¡sub¿1¡/sub¿ defining the finite measurement resolution.

2. WCW geometrization reduces to that for a single line of the generalized Feynman diagram defining the basic building brick for WCW. Kähler function decomposes to a sum of "kinetic" terms associated with its ends and interaction term associated with the line itself. p-Adicization boils down to the condition that Kähler function, matrix elements of Kähler form, WCW Hamiltonians and their super counterparts, are rational functions of complex WCW coordinates just as they are for those symmetric spaces that I know of. This allows straightforward continuation to p-adic context.

3. As far as diagrams are considered, everything is manifestly finite as the general arguments (non-locality of Kähler function as functional of 3-surface) developed two decades ago indeed allow to expect. General conditions on the holomorphy properties of the generalized eigenvalues λ of the modified Dirac operator can be deduced from the conditions that propagator decomposes to a sum of products of harmonics associated with the ends of the line and that similar decomposition takes place for exponent of Kähler action identified as Dirac determinant. This guarantees that the convolutions of propagators and vertices give rise to products of harmonic functions which can be Glebsch-Gordanized to harmonics and only the singlet contributes to the WCW integral in given vertex. The still unproven central conjecture is that Dirac determinant equals the exponent of Kähler function.

In the following this vision about generalized Feynman diagrams is discussed in more detail.

5.9.1 Questions

The goal is a proposal for how to perform the integral over WCW for generalized Feynman digrams and the best manner to proceed to to this goal is by making questions.

What does finite measurement resolution mean?

The first question is what finite measurement resolution means.

1. One expects that the algebraic continuation makes sense only for a finite measurement resolution in which case one obtains only finite sums of what one might hope to be algebraic functions. The finiteness of the algebraic extension would be in fact equivalent with the finite measurement resolution.

2. Finite measurement resolution means a discretization in terms of number theoretic braids. p-Adicization condition suggests that that one must allow only the number theoretic braids. For these the ends of braid at boundary of CD are algebraic points of the imbedding space. This would be true at least in the intersection of real and p-adic worlds.

3. The question is whether one can localize the points of the braid. The necessity to use momentum eigenstates to achieve quantum classical correspondence in the modified Dirac action [18] suggests however a delocalization of braid points, that is wave function in space of braid points. In real context one could allow all possible choices for braid points but in p-adic context only algebraic points are possible if one wants to replace integrals with sums. This implies finite measurement resolution analogous to that in lattice. This is also the only possibility in the intersection of real and p-adic worlds.

   A non-trivial prediction giving a strong correlation between the geometry of the partonic 2-surface and quantum numbers is that the total number $n_F + n_{\overline{F}}$ of fermions and antifermions is bounded above by the number n_{alg} of algebraic points for a given partonic 2-surface: $n_F + n_{\overline{F}} \leq n_{alg}$. Outside the intersection of real and p-adic worlds the problematic aspect of this definition is that small deformations of the partonic 2-surface can radically change the number of algebraic points unless one assumes that the finite measurement resolution means restriction of WCW to a sub-space of algebraic partonic surfaces.

4. One has also a discretization of loop momenta if one assumes that virtual particle momentum corresponds to ZEO defining rest frame for it and from the discretization of the relative position of the second tip of CD at the hyperboloid isometric with mass shell. Only the number of braid points and their momenta would matter, not their positions. The measurement interaction term in the modified Dirac action gives coupling to the space-time geometry and Kähler function through generalized eigenvalues of the modified Dirac operator with measurement interaction term linear in momentum and in the color quantum numbers assignable to fermions [18].

How to define integration in WCW degrees of freedom?

The basic question is how to define the integration over WCW degrees of freedom.

1. What comes mind first is Gaussian perturbation theory around the maxima of Kähler function. Gaussian and metric determinants cancel each other and only algebraic expressions remain. Finiteness is not a problem since the Kähler function is non-local functional of 3-surface so that no local interaction vertices are present. One should however assume the vanishing of loops required also by algebraic universality and this assumption look unrealistic when one considers more general functional integrals than that of vacuum functional since free field theory is not in question. The construction of the inverse of the WCW metric defining the propagator is also a very difficult challenge. Duistermaat-Hecke theorem states that something like this known as localization might be possible and one can also argue that something analogous to localization results from a generalization of mean value theorem.

2. Symmetric space property is more promising since it might reduce the integrations to group theory using the generalization of Fourier analysis for group representations so that there would be no need for perturbation theory in the proposed sense. In finite measurement resolution the symmetric spaces involved would be finite-dimensional. Symmetric space structure of WCW could also allow to define p-adic integration in terms of p-adic Fourier analysis for symmetric spaces. Essentially algebraic continuation of the integration from the real case would be in question with additional constraints coming from the fact that only phase factors corresponding to finite algebraic extensions of rationals are used. Cutoff would emerge automatically from the cutoff for the dimension of the algebraic extension.

How to define generalized Feynman diagrams?

Integration in symmetric spaces could serve as a model at the level of WCW and allow both the understanding of WCW integration and p-adicization as algebraic continuation. In order to get a more realistic view about the problem one must define more precisely what the calculation of the generalized Feynman diagrams means.

1. WCW integration must be carried out separately for all values of the momenta associated with the internal lines. The reason is that the spectrum of eigenvalues λ_i of the modified Dirac operator D depends on the momentum of line and momentum conservation in vertices translates to a correlation of the spectra of D at internal lines.

2. For tree diagrams algebraic continuation to the p-adic context if the expression involves only the replacement of the generalized eigenvalues of D as functions of momenta with their p-adic counterparts besides vertices. If these functions are algebraically universal and expressible in terms of harmonics of symmetric space , there should be no problems.

3. If loops are involved, one must integrate/sum over loop momenta. In p-adic context difficulties are encountered if the spectrum of the momenta is continuous. The integration over on mass shell loop momenta is analogous to the integration over sub-CDs, which suggests that internal line corresponds to a $sub-CD$ in which it is at rest. There are excellent reasons to believe that the moduli space for the positions of the upper tip is a discrete subset of hyperboloid of future light-cone. If this is the case, the loop integration indeed reduces to a sum over discrete positions of the tip. p-Adization would thus give a further good reason why for zero energy ontology.

4. Propagator is expressible in terms of the inverse of generalized eigenvalue and there is a sum over these for each propagator line. At vertices one has products of WCW harmonics assignable to the incoming lines. The product must have vanishing quantum numbers associated with the phase angle variables of WCW. Non-trivial quantum numbers of the WCW harmonic correspond to WCW quantum numbers assignable to excitations of ordinary elementary particles. WCW harmonics are products of functions depending on the "radial" coordinates and phase factors and the integral over the angles leaves the product of the first ones analogous to Legendre polynomials $P_{l,m}$, These functions are expected to be rational functions or at least algebraic functions involving only square roots.

5. In ordinary QFT incoming and outgoing lines correspond to propagator poles. In the recent case this would mean that the generalized eigenvalues $\lambda = 0$ characterize them. Internal lines coming

as pairs of throats of wormhole contacts would be on mass shell with respect to momentum but off shell with respect to λ.

5.9.2 Generalized Feynman diagrams at fermionic and momentum space level

Negative energy ontology has already led to the idea of interpreting the virtual particles as pairs of positive and negative energy wormhole throats. Hitherto I have taken it as granted that ordinary Feynman diagrammatics generalizes more or less as such. It is however far from clear what really happens in the verties of the generalized Feynmann diagrams. The safest approach relies on the requirement that unitarity realized in terms of Cutkosky rules in ordinary Feynman diagrammatics allows a generalization. This requires loop diagrams. In particular, photon-photon scattering can take place only via a fermionic square loop so that it seems that loops must be present at least in the topological sense.

One must be however ready for the possibility that something unexpectedly simple might emerge. For instance, the vision about algebraic physics allows naturally only finite sums for diagrams and does not favor infinite perturbative expansions. Hence the true believer on algebraic physics might dream about finite number of diagrams for a given reaction type. For simplicity generalized Feynman diagrams without the complications brought by the magnetic confinement since by the previous arguments the generalization need not bring in anything essentially new.

The basic idea of duality in early hadronic models was that the lines of the dual diagram representing particles are only re-arranged in the vertices. This however does not allow to get rid of off mass shell momenta. Zero energy ontology encourages to consider a stronger form of this principle in the sense that the virtual momenta of particles could correspond to pairs of on mass shell momenta of particles. If also interacting fermions are pairs of positive and negative energy throats in the interaction region the idea about reducing the construction of Feynman diagrams to some kind of lego rules might work.

Virtual particles as pairs of on mass shell particles in ZEO

The first thing is to try to define more precisely what generalized Feynman diagrams are. The direct generalization of Feynman diagrams implies that both wormhole throats and wormhole contacts join at vertices.

1. A simple intuitive picture about what happens is provided by diagrams obtained by replacing the points of Feynman diagrams (wormhole contacts) with short lines and imagining that the throats correspond to the ends of the line. At vertices where the lines meet the incoming on mass shell quantum numbers would sum up to zero. This approach leads to a straightforward generalization of Feynman diagrams with virtual particles replaced with pairs of on mass shell throat states of type $++$, $--$, and $+-$. Incoming lines correspond to $++$ type lines and outgoing ones to $--$ type lines. The first two line pairs allow only time like net momenta whereas $+-$ line pairs allow also space-like virtual momenta. The sign assigned to a given throat is dictated by the the sign of the on mass shell momentum on the line. The condition that Cutkosky rules generalize as such requires $++$ and $--$ type virtual lines since the cut of the diagram in Cutkosky rules corresponds to on mass shell outgoing or incoming states and must therefore correspond to $++$ or $--$ type lines.

2. The basic difference as compared to the ordinary Feynman diagrammatics is that loop integrals are integrals over mass shell momenta and that all throats carry on mass shell momenta. In each vertex of the loop mass incoming on mass shell momenta must sum up to on mass shell momentum. These constraints improve the behavior of loop integrals dramatically and give excellent hopes about finiteness. It does not however seem that only a finite number of diagrams contribute to the scattering amplitude besides tree diagrams. The point is that if a the reactions $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$,, where N_i denote particle numbers, are possible in a common kinematical region for N_2-particle states then also the diagrams $N_1 \rightarrow N_2 \rightarrow N_2 \rightarrow N_3$ are possible. The virtual states N_2 include all all states in the intersection of kinematically allow regions for $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$. Hence the dream about finite number possible diagrams is

not fulfilled if one allows massless particles. If all particles are massive then the particle number N_2 for given N_1 is limited from above and the dream is realized.

3. For instance, loops are not possible in the massless case or are highly singular (bringing in mind twistor diagrams) since the conservation laws at vertices imply that the momenta are parallel. In the massive case and allowing mass spectrum the situation is not so simple. As a first example one can consider a loop with three vertices and thus three internal lines. Three on mass shell conditions are present so that the four-momentum can vary in 1-D subspace only. For a loop involving four vertices there are four internal lines and four mass shell conditions so that loop integrals would reduce to discrete sums. Loops involving more than four vertices are expected to be impossible.

4. The proposed replacement of the elementary fermions with bound states of elementary fermions and monopoles $X_\pm$ brings in the analog of stringy diagrammatics. The 2-particle wave functions in the momentum degrees of freedom of fermiona and $X_\pm$ migh allow more flexibility and allow more loops. Note however that there are excellent hopes about the finiteness of the theory also in this case.

Loop integrals are manifestly finite

One can make also more detailed observations about loops.

1. The simplest situation is obtained if only 3-vertices are allowed. In this case conservation of momentum however allows only collinear momenta although the signs of energy need not be the same. Particle creation and annihilation is possible and momentum exchange is possible but is always light-like in the massless case. The scattering matrices of supersymmetric YM theories would suggest something less trivial and this raises the question whether something is missing. Magnetic monopoles are an essential element of also these theories as also massivation and symmetry breaking and this encourages to think that the formation of massive states as fermion $X_\pm$ pairs is needed. Of course, in TGD framework one has also high mass excitations of the massless states making the scattering matrix non-trivial.

2. In YM theories on mass shell lines would be singular. In TGD framework this is not the case since the propagator is defined as the inverse of the 3-D dimensional reduction of the modified Dirac operator D containing also coupling to four-momentum (this is required by quantum classical correspondence and guarantees stringy propagators),

$$\begin{aligned} D &= i\hat{\Gamma}^\alpha p_\alpha + \hat{\Gamma}^\alpha D_\alpha \ , \\ p_\alpha &= p_k \partial_\alpha h^k \ . \end{aligned} \tag{5.9.1}$$

The propagator does not diverge for on mass shell massless momenta and the propagator lines are well-defined. This is of course of essential importance also in general case. Only for the incoming lines one can consider the possibility that 3-D Dirac operator annihilates the induced spinor fields. All lines correspond to generalized eigenstates of the propagator in the sense that one has $D_3\Psi = \lambda\gamma\Psi$, where γ is modified gamma matrix in the direction of the stringy coordinate emanating from light-like surface and D_3 is the 3-dimensional dimensional reduction of the 4-D modified Dirac operator. The eigenvalue λ is analogous to energy. Note that the eigenvalue spectrum depends on 4-momentum as a parameter.

3. Massless incoming momenta can decay to massless momenta with both signs of energy. The integration measure $d^2k/2E$ reduces to dx/x where $x \geq 0$ is the scaling factor of massless momentum. Only light-like momentum exchanges are however possible and scattering matrix is essentially trivial. The loop integrals are finite apart from the possible delicacies related to poles since the loop integrands for given massless wormhole contact are proportional to dx/x^3 for large values of x.

4. Irrrespective of whether the particles are massless or not, the divergences are obtained only if one allows too high vertices as self energy loops for which the number of momentum degrees of freedom is $3N-4$ for N-vertex. The construction of SUSY limit of TGD in [23] led to the conclusion that the parallelly propagating N fermions for given wormhole throat correspond to a product of N fermion propagators with same four-momentum so that for fermions and ordinary bosons one has the standard behavior but for $N>2$ non-standard so that these excitations are not seen as ordinary particles. Higher vertices are finite only if the total number N_F of fermions propagating in the loop satisfies $N_F > 3N-4$. For instance, a 4-vertex from which $N=2$ states emanate is finite.

Taking into account magnetic confinement

What has been said above is not quite enough. The weak form of electric-magnetic duality [51] leads to the picture about elementary particles as pairs of magnetic monopoles inspiring the notions of weak confinement based on magnetic monopole force. Also color confinement would have magnetic counterpart. This means that elementary particles would behave like string like objects in weak boson length scale. Therefore one must also consider the stringy case with wormhole throats replaced with fermion-$X_\pm$ pairs ($X_\pm$ is electromagnetically neutral and $\pm$ refers to the sign of the weak isospin opposite to that of fermion) and their super partners.

1. The simplest assumption in the stringy case is that fermion-$X_\pm$ pairs behave as coherent objects, that is scatter elastically. In more general case only their higher excitations identifiable in terms of stringy degrees of freedom would be created in vertices. The massivation of these states makes possible non-collinear vertices. An open question is how the massivation fermion-$X_\pm$ pairs relates to the existing TGD based description of massivation in terms of Higgs mechanism and modified Dirac operator.

2. Mass renormalization could come from self energy loops with negative energy lines as also vertex normalization. By very general arguments supersymmetry implies the cancellation of the self energy loops but would allow non-trivial vertex renormalization [23].

3. If only 3-vertices are allowed, the loops containing only positive energy lines are possible if on mass shell fermion-$X_\pm$ pair (or its superpartner) can decay to a pair of positive energy pair particles of same kind. Whether this is possible depends on the masses involved. For ordinary particles these decays are not kinematically possible below intermediate boson mass scale (the decays $F_1 \to F_2 + \gamma$ are forbidden kinematically or by the absence of flavor changing neutral currents whereas intermediate gauge bosons can decay to on mass shell fermion-antifermion pair).

4. The introduction of IR cutoff for 3-momentum in the rest system associated with the largest CD (causal diamond) looks natural as scale parameter of coupling constant evolution and p-adic length scale hypothesis favors the inverse of the size scale of CD coming in powers of two. This parameter would define the momentum resolution as a discrete parameter of the p-adic coupling constant evolution. This scale does not have any counterpart in standard physics. For electron, d quark, and u quark the proper time distance between the tips of CD corresponds to frequency of 10 Hz, 1280 Hz, and 160 Hz: all these frequencies define fundamental bio-rhythms [36].

These considerations have left completely untouched one important aspect of generalized Feynman diagrams: the necessity to perform a functional integral over the deformations of the partonic 2-surfaces at the ends of the lines- that is integration over WCW. Number theoretical universality requires that WCW and these integrals make sense also p-adically and in the following these aspects of generalized Feynman diagrams are discussed.

5.9.3 How to define integration and p-adic Fourier analysis, integral calculus, and p-adic counterparts of geometric objects?

p-Adic differential calculus exists and obeys essentially the same rules as ordinary differental calculus. The only difference from real context is the existence of p-adic pseudoconstants: any function which depends on finite number of pinary digits has vanishing p-adic derivative. This implies non-determinism of p-adic differerential equations. One can defined p-adic integral functions using the fact that indefinite integral is the inverse of differentiation. The basis problem with the definite integrals is that p-adic numbers are not well-ordered so that the crucial ordering of the points of real axis in definite integral is not unique. Also p-adic Fourier analysis is problematic since direct counterparts of ep(ix) and trigonometric functions are not periodic. Also exp(-x) fails to converse exponentially since it has p-adic norm equal to 1. Note also that these functions exists only when the p-adic norm of x is smaller than 1.

The following considerations support the view that the p-adic variant of a geometric objects, integration and p-adic Fourier analysis exists but only when one considers highly symmetric geometric objects such as symmetric spaces. This is wellcome news from the point of view of physics. At the level of space-time surfaces this is problematic. The field equations associated with Kähler action and modified Dirac equation make sense. Kähler action defined as integral over p-adic space-time surface fails to exist. If however the Kähler function identified as Kähler for a preferred extremal of Kähler action is rational or algebraic function of preferred complex coordinates of WCW with ratonal coefficients, its p-adic continuation is expected to exist.

Circle with rotational symmetries and its hyperbolic counterparts

Consider first circle with emphasis on symmetries and Fourier analysis.

1. In this case angle coordinate ϕ is the natural coordinate. It however does not make sense as such p-adically and one must consider either trigonometric functions or the phase $exp(i\phi)$ instead. If one wants to do Fourier analysis on circle one must introduce roots $U_{n,N} = exp(in2\pi/N)$ of unity. This means discretization of the circle. Introducing all roots $U_{n,p} = exp(i2\pi n/p)$, such that p divides N, one can represent all $U_{k,n}$ up to $n = N$. Integration is naturally replaced with sum by using discrete Fourier analysis on circle. Note that the roots of unity can be expressed as products of powers of roots of unity $exp(in2\pi/p^k)$, where p^k divides N.

2. There is a number theoretical delicacy involved. By Fermat's theorem $a^{p-1}\ mod\ p = 1$ for $a = 1, ...p-1$ for a given p-adic prime so that for any integer M divisible by a factor of $p-1$ the M:th roots of unity exist as ordinary p-adic numbers. The problem disappears if these values of M are excluded from the discretization for a given value of the p-adic prime. The manner to achieve this is to assume that N contains no divisors of $p-1$ and is consistent with the notion of finite measurement resolution. For instance, $N = p^n$ is an especially natural choice guaranteing this.

3. The p-adic integral defined as a Fourier sum does not reduce to a mere discretization of the real integral. In the real case the Fourier coefficients must approach to zero as the wave vector $k = n2\pi/N$ increases. In the p-adic case the condition consistent with the notion of finite measurement resolution for angles is that the p-adic valued Fourier coefficients approach to zero as n increases. This guarantees the p-adic convergence of the discrete approximation of the integral for large values of N as n increases. The map of p-adic Fourier coefficients to real ones by canonical identification could be used to relate p-adic and real variants of the function to each other.

This finding would suggests that p-adic geometries -in particular the p-adic counterpart of CP_2, are discrete. Variables which have the character of a radial coordinate are in natural manner p-adically continuous whereas phase angles are naturally discrete and described in terms of algebraic extensions. The conclusion is disappoing since one can quite well argue that the discrete structures can be regarded as real. Is there any manner to escape this conclusion?

1. Exponential function $exp(ix)$ exists p-adically for $|x|_p \leq 1/p$ but is not periodic. It provides representation of p-adic variant of circle as group $U(1)$. One obtains actually a hierarchy of groups $U(1)_{p,n}$ corresponding to $|x|_p \leq 1/p^n$. One could consider a generalization of phases as products $Exp_p(N, n2\pi/N + x) = exp(in2\pi n/N)exp(ix)$ of roots of unity and exponent functions with an imaginary exponent. This would assign to each root of unity p-adic continuum interpreted as the analog of the interval between two subsequent roots of unity at circle. The hierarchies

of measurement resolutions coming as $2\pi/p^n$ would be naturally accompanied by increasingly smaller p-adic groups $U(1)_{p,n}$.

2. p-Adic integration would involve summation plus possibly also an integration over each p-adic variant of discretization interval. The summation over the roots of unity implies that the integral of $\int exp(inx)dx$ would appear for $n = 0$. Whatever the value of this integral is, it is compensated by a normalization factor guaranteing orthonormality.

3. If one interprets the p-adic coordinate as p-adic integer without the identification of points differing by a multiple of n as different points the question whether one should require p-adic continuity arises. Continuity is obtained if $U_n(x + mp^m) = U_n(x)$ for large values of m. This is obtained if one has $n = p^k$. In the spherical geometry this condition is not needed and would mean quantization of angular momentum as $L = p^k$, which does not look natural. If representations of translation group are considered the condition is natural and conforms with the spirit of the p-adic length scale hypothesis.

The hyperbolic counterpart of circle corresponds to the orbit of point under Lorentz group in two 2-D Minkowski space. Plane waves are replaced with exponentially decaying functions of the coordinate η replacing phase angle. Ordinary exponent function $exp(x)$ has unit p-adic norm when it exists so that it is not a suitable choice. The powers p^n existing for p-adic integers however approach to zero for large values of $x = n$. This forces discretization of η or rather the hyperbolic phase as powers of p^x, $x = n$. Also now one could introduce products of $Exp_p(nlog(p) + z) = p^n exp(x)$ to achieve a p-adic continuum. Also now the integral over the discretization interval is compensated by orthonormalization and can be forgotten. The integral of exponential function would reduce to a sum $\int Exp_p dx = \sum_k p^k = 1/(1-p)$. One can also introduce finite-dimensional but non-algebraic extensions of p-adic numbers allowing e and its roots $e^{1/n}$ since e^p exists p-adically.

Plane with translational and rotational symmetries

Consider first the situation by taking translational symmetries as a starting point. In this case Cartesian coordinates are natural and Fourier analysis based on plane waves is what one wants to define. As in the previous case, this can be done using roots of unity and one can also introduce p-adic continuum by using the p-adic variant of the exponent function. This would effectively reduce the plane to a box. As already noticed, in this case the quantization of wave vectors as multiples of $1/p^k$ is required by continuity.

One can take also rotational symmetries as a starting point. In this case cylindrical coordinates (ρ, ϕ) are natural.

1. Radial coordinate can have arbitrary values. If one wants to keep the connection $\rho = \sqrt{x^2 + y^2}$ with the Cartesian picture square root allowing extension is natural. Also the values of radial coordinate proportional to odd power of p are problematic since one should introduce $\sqrt{p}$: is this extension internally consistent? Does this mean that the points $\rho \propto p^{2n+1}$ are excluded so that the plane decomposes to annuli?

2. As already found, angular momentum eigen states can be described in terms of roots of unity and one could obtain continuum by allowing also phases defined by p-adic exponent functions.

3. In radial direction one should define the p-adic variants for the integrals of Bessel functions and they indeed might make sense by algebraic continuation if one consistently defines all functions as Fourier expansions. Delta-function renormalization causes technical problems for a continuum of radial wave vectors. One could avoid the problem by using expontentially decaying variants of Bessel function in the regions far from origin, and here the already proposed description of the hyperbolic counterparts of plane waves is suggestive.

4. One could try to understand the situation also using Cartesian coordinates. In the case of sphere this is achieved by introducing two coordinate patches with Cartesian coordinates. Pythagorean phases are rational phases (orthogonal triangles for which all sides are integer valued) and form a dense set on circle. Complex rationals (orthogonal triangles with integer valued short sides) define a more general dense subset of circle. In both cases it is difficult to imagine a discretized

version of integration over angles since discretization with constant angle increrement is not possible.

The case of sphere and more general symmetric space

In the case of sphere spherical coordinates are favored by symmetry considerations. For spherical coordinates $sin(\theta)$ is analogous to the radial coordinate of plane. Legedre polynomials expressible as polynomials of $sin(\theta)$ and $cos(\theta)$ are expressible in terms of phases and the integration measure $sin^2(\theta)d\theta d\phi$ reduces the integral of S^2 to summation. As before one can introduce also p-adic continuum. Algebraic cutoffs in both angular momentum l and m appear naturally. Similar cutoffs appear in the representations of quantum groups and there are good reasons to expect that these phenomena are correlated.

Exponent of Kähler function appears in the integration over configuration space. From the expression of Kähler gauge potential given by $A_\alpha = J_\alpha{}^\theta \partial_\theta K$ one obtains using $A_\alpha = cos(\theta)\delta_{\alpha,\phi}$ and $J_{\theta\phi} = sin(\theta)$ the expression $exp(K) = sin(\theta)$. Hence the exponent of Kähler function is expressible in terms of spherical harmonics.

The completion of the discretized sphere to a p-adic continuum- and in fact any symmetric space- could be performed purely group theoretically.

1. Exponential map maps the elements of the Lie-algebra to elements of Lie-group. This recipe generalizes to arbitrary symmetric space G/H by using the Cartan decomposition $g = t + h$, $[h, h] \subset h$,$[h, t] \subset t$,$[t, t] \subset h$. The exponentiation of t maps t to G/H in this case. The exponential map has a p-adic generalization obtained by considering Lie algebra with coefficients with p-adic norm smaller than one so that the p-adic exponent function exists. As a matter fact, one obtains a hierarchy of Lie-algebras corresponding to the upper bounds of the p-adic norm coming as p^{-k} and this hierarchy naturally corresponds to the hierarchy of angle resolutions coming as $2\pi/p^k$. By introducing finite-dimensional transcendental extensions containing roots of e one obtains also a hierarchy of p-adic Lie-algebras associated with transcendental extensions.

2. In particular, one can exponentiate the complement of the $SO(2)$ sub-algebra of $SO(3)$ Lie-algebra in p-adic sense to obtain a p-adic completion of the discrete sphere. Each point of the discretized sphere would correspond to a p-adic continuous variant of sphere as a symmetric space. Similar construction applies in the case of CP_2. Quite generally, a kind of fractal or holographic symmetric space is obtained from a discrete variant of the symmetric space by replacing its points with the p-adic symmetric space.

3. In the N-fold discretization of the coordinates of M-dimensional space t one $(N-1)^M$ discretization volumes which is the number of points with non-vanishing t-coordinates. It would be nice if one could map the p-adic discretization volumes with non-vanishing t-coordinates to their positive valued real counterparts by applying canonical identification. By group invariance it is enough to show that this works for a discretization volume assignable to the origin. Since the p-adic numbers with norm smaller than one are mapped to the real unit interval, the p-adic Lie algebra is mapped to the unit cell of the discretization lattice of the real variant of t. Hence by a proper normalization this mapping is possible.

The above considerations suggest that the hierarchies of measurement resolutions coming as $\Delta\phi = 2\pi/p^n$ are in a preferred role. One must be however cautious in order to avoid too strong assumptions. The following arguments however support this identification.

1. The vision about p-adicization characterizes finite measurement resolution for angle measurement in the most general case as $\Delta\phi = 2\pi M/N$, where M and N are positive integers having no common factors. The powers of the phases $exp(i2\pi M/N)$ define identical Fourier basis irrespective of the value of M unless one allows only the powers $exp(i2\pi kM/N)$ for which $kM < N$ holds true: in the latter case the measurement resolutions with different values of M correspond to different numbers of Fourier components. Otherwise themeasurement ersolution is just $\Delta\phi = 2\pi/p^n$. If one regards N as an ordinary integer, one must have $N = p^n$ by the p-adic continuity requirement.

2. One can also interpret N as a p-adic integer and assume that state function reduction selects one particular prime (no superposition of quantum states with different p-adic topologies). For $N = p^n M$, where M is not divisible by p, one can express $1/M$ as a p-adic integer $1/M = \sum_{k\geq 0} M_k p^k$, which is infinite as a real integer but effectively reduces to a finite integer $K(p) = \sum_{k=0}^{N-1} M_k p^k$. As a root of unity the entire phase $exp(i2\pi M/N)$ is equivalent with $exp(i2\pi R/p^n)$, $R = K(p)M$ mod p^n. The phase would non-trivial only for p-adic primes appearing as factors in N. The corresponding measurement resolution would be $\Delta\phi = R2\pi/N$. One could assign to a given measurement resolution all the p-adic primes appearing as factors in N so that the notion of multi-p p-adicity would make sense. One can also consider the identification of the measurement resolution as $\Delta\phi = |N/M|_p = 2\pi/p^k$. This interpretation is supported by the approach based on infinite primes [19].

What about integrals over partonic 2-surfaces and space-time surface?

One can of course ask whether also the integrals over partonic 2-surfaces and space-time surface could be p-adicized by using the proposed method of discretization. Consider first the p-adic counterparts of the integrals over the partonic 2-surface X^2.

1. WCW Hamiltonians and Kähler form are expressible using flux Hamiltonians defined in terms of X^2 integrals of JH_A, where H_A is $\delta CD \times CP_2$ Hamiltonian, which is a rational function of the preferred coordinates defined by the exponentials of the coordinates of the sub-space t in the appropriate Cartan algebra decomposition. The flux factor $J = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ is scalar and does not actually depend on the induced metric.

2. The notion of finite measurement resolution would suggest that the discretization of X^2 is somehow induced by the discretization of $\delta CD \times CP_2$. The coordinates of X^2 could be taken to be the coordinates of the projection of X^2 to the sphere S^2 associated with $\delta M^4_\pm$ or to the homologically non-trivial geodesic sphere of CP_2 so that the discretization of the integral would reduce to that for S^2 and to a sum over points of S^2.

3. To obtain an algebraic number as an outcome of the summation, one must pose additional conditions guaranteing that both H_A and J are algebraic numbers at the points of discretization (recall that roots of unity are involved). Assume for definiteness that S^2 is $r_M = constant$ sphere. If the remaining preferred coordinates are functions of the preferred S^2 coordinates mapping phases to phases at discretization points, one obtains the desired outcome. These conditions are rather strong and mean that the various angles defining CP_2 coordinates -at least the two cyclic angle coordinates- are integer multiples of those assignable to S^2 at the points of discretization. This would be achieved if the preferred complex coordinates of CP_2 are powers of the preferred complex coordinate of S^2 at these points. One could say that X^2 is algebraically continued from a rational surface in the discretized variant of $\delta CD \times CP_2$. Furthermore, if the measurement resolutions come as $2\pi/p^n$ as p-adic continuity actually requires and if they correspond to the p-adic group $G_{p,n}$ for which group parameters satisfy $|t|_p \leq p^{-n}$, one can precisely characterize how a p-adic prime characterizes the real partonic 2-surface. This would be a fulfilment of one of the oldest dreams related to the p-adic vision.

A even more ambitious dream would be that even the integral of the Kähler action for preferred extremals could be defined using a similar procedure. The conjectured slicing of Minkowskian space-time sheets by string world sheets and partonic 2-surfaces encourages these hopes.

1. One could introduce local coordinates of H at both ends of CD by introducing a continuous slicing of $M^4 \times CP_2$ by the translates of $\delta M^4_\pm \times CP_2$ in the direction of the time-like vector connecting the tips of CD. As space-time coordinates one could select four of the eight coordinates defining this slicing. For instance, for the regions of the space-time sheet representable as maps $M^4 \to CP_2$ one could use the preferred M^4 time coordinate, the radial coordinate of δM^4_+, and the angle coordinates of $r_M = constant$ sphere.

2. Kähler action density should have algebraic values and this would require the strengthening of the proposed conditions for X^2 to apply to the entire slicing meaning that the discretized space-time surface is a rational surface in the discretized $CD \times CP_2$. If this condition applies to

the entire space-time surface it would effectively mean the discretization of the classical physics to the level of finite geometries. This seems quite strong implication but is consistent with the preferred extremal property implying the generalized Bohr rules. The reduction of Kähler action to 3-dimensional boundary terms is implied by rather general arguments. In this case only the effective algebraization of the 3-surfaces at the ends of CD and of wormhole throats is needed [34]. By effective 2-dimensionality these surfaces cannot be chosen freely.

3. If Kähler function and WCW Hamiltonians are rational functions, this kind of additional conditions are not necessary. It could be that the integrals of defining Kähler action flux Hamiltonians make sense only in the intersection of real and p-adic worlds assumed to be relevant for the physics of living systems.

Tentative conclusions

These findings suggest following conclusions.

1. Exponent functions play a key role in the proposed p-adicization. This is not an accident since exponent functions play a fundamental role in group theory and p-adic variants of real geometries exist only under symmetries- possibly maximal possible symmetries- since otherwise the notion of Fourier analysis making possible integration does not exist. The inner product defined in terms of integration reduce for functions representable in Fourier basis to sums and can be carried out by using orthogonality conditions. Convolution involving integration reduces to a product for Fourier components. In the case of imbedding space and WCW these conditions are satisfied but for space-time surfaces this is not possible.

2. There are several manners to choose the Cartan algebra already in the case of sphere. In the case of plane one can consider either translations or rotations and this leads to different p-adic variants of plane. Also the realization of the hierarchy of Planck constants leads to the conclusion that the extended imbedding space and therefore also WCW contains sectors corresponding to different choices of quantization axes meaning that quantum measurement has a direct geometric correlate.

3. The above described 2-D examples represent symplectic geometries for which one has natural decomposition of coordinates to canonical pairs of cyclic coordinate (phase angle) and corresponding canonical conjugate coordinate. p-Adicization depends on whether the conjugate corresponds to an angle or noncompact coordinate. In both cases it is however possible to define integration. For instance, in the case of CP_2 one would have two canonically conjugate pairs and one can define the p-adic counterparts of CP_2 partial waves by generalizing the procedure applied to spherical harmonics. Products of functions expressible using partial waves can be decomposed by tensor product decomposition to spherical harmonics and can be integrated. In particular inner products can be defined as integrals. The Hamiltonians generating isometries are rational functions of phases: this inspires the hope that also WCW Hamiltonians also rational functions of preferred WCW coordinates and thus allow p-adic variants.

4. Discretization by introducing algebraic extensions is unavoidable in the p-adicization of geometrical objects but one can have p-adic continuum as the analog of the discretization interval and in the function basis expressible in terms of phase factors and p-adic counterparts of exponent functions. This would give a precise meaning for the p-adic counterparts of the imbedding space and WCW if the latter is a symmetric space allowing coordinatization in terms of phase angles and conjugate coordinates.

5. The intersection of p-adic and real worlds would be unique and correspond to the points defining the discretization.

5.9.4 Harmonic analysis in WCW as a manner to calculate WCW functional integrals

Previous examples suggest that symmetric space property, Kähler and symplectic structure and the use of symplectic coordinates consisting of canonically conjugate pairs of phase angles and corresponding "radial" coordinates are essential for WCW integration and p-adicization. Kähler function, the components of the metric, and therefore also metric determinant and Kähler function depend on the "radial" coordinates only and the possible generalization involves the identification the counterparts of the "radial" coordinates in the case of WCW.

Conditions guaranteing the reduction to harmonic analysis

The basic idea is that harmonic analysis in symmetric space allows to calculate the functional integral over WCW.

1. Each propagator line corresponds to a symmetric space defined as a coset space G/H of the symplectic group and Kac-Moody group and one might hope that the proposed p-adicization works for it- at least when one considers the hierarchy of measurement resolutions forced by the finiteness of algebraic extensions. This coset space is as a manifold Cartesian product $(G/H) \times (G/H)$ of symmetric spaces G/H associated with ends of the line. Kähler metric contains also an interaction term between the factors of the Cartesian product so that Kähler function can be said to reduce to a sum of "kinetic" terms and interaction term.

2. Effective 2-dimensionality and ZEO allow to treat the ends of the propagator line independently. This means an enormous simplification. Each line contributes besides propagator a piece to the exponent of Kähler action identifiable as interaction term in action and depending on the propagator momentum. This contribution should be expressible in terms of generalized spherical harmonics. Essentially a sum over the products of pairs of harmonics associated with the ends of the line multiplied by coefficients analogous to $1/(p^2-m^2)$ in the case of the ordinary propagator would be in question. The optimal situation is that the pairs are harmonics and their conjugates appear so that one has invariance under G analogous to momentum conservation for the lines of ordinary Feynman diagrams.

3. Momentum conservation correlates the eigenvalue spectra of the modified Dirac operator D at propagator lines [18]. G-invariance at vertex dictates the vertex as the singlet part of the product of WCW harmonics associated with the vertex and one sums over the harmonics for each internal line. p-Adicization means only the algebraic continuation to real formulas to p-adic context.

4. The exponent of Kähler function depends on both ends of the line and this means that the geometries at the ends are correlated in the sense that that Kähler form contains interaction terms between the line ends. It is however not quite clear whether it contains separate "kinetic" or self interaction terms assignable to the line ends. For Kähler function the kinetic and interaction terms should have the following general expressions as functions of complex WCW coordinates:

$$\begin{aligned} K_{kin,i} &= \sum_n f_{i,n}(Z_i)\overline{f_{i,n}(Z_i)} + c.c \ , \\ K_{int} &= \sum_n g_{1,n}(Z_1)\overline{g_{2,n}(Z_2)} + c.c \ , i = 1,2 \ . \end{aligned} \quad (5.9.2)$$

   Here $K_{kin,i}$ define "kinetic" terms and K_{int} defines interaction term. One would have what might be called holomorphic factorization suggesting a connection with conformal field theories.

   Symmetric space property -that is isometry invariance- suggests that one has

$$f_{i,n} = f_{2,n} \equiv f_n \ , \ g_{1,n} = g_{2,n} \equiv g_n \quad (5.9.3)$$

   such that the products are invariant under the group H appearing in G/H and therefore have opposite H quantum numbers. The exponent of Kähler function does not factorize although the terms in its Taylor expansion factorize to products whose factors are products of holomorphic and antihilomorphic functions.

5. If one assumes that the exponent of Kähler function reduces to a product of eigenvalues of the modified Dirac operator eigenvalues must have the decomposition

$$\lambda_k = \prod_{i=1,2} exp\left[\sum_n c_{k,n} g_n(Z_i)\overline{g_n(Z_i)} + c.c\right] \times exp\left[\sum_n d_{k,n} g_n(Z_1)\overline{g_n(Z_2)} + c.c\right] \quad (5.9.4)$$

Hence also the eigenvalues coming from the Dirac propagators have also expansion in terms of G/H harmonics so that in principle WCW integration would reduce to Fourier analysis in symmetric space.

Generalization of WCW Hamiltonians

This picture requires a generalization of the view about configuration space Hamiltonians since also the interaction term between the ends of the line is present not taken into account in the previous approach.

1. The proposed representation of WCW Hamiltonians as flux Hamiltonians [22, 18]

$$\begin{aligned} Q(H_A) &= \int H_A(1+K)Jd^2x \ , \\ J &= \epsilon^{\alpha\beta}J_{\alpha\beta} \ , \ J^{03}\sqrt{g_4} = KJ_{12} \ . \end{aligned} \quad (5.9.5)$$

works for the kinetic terms only since J cannot be the same at the ends of the line. The formula defining K assumes weak form of self-duality (^{03} refers to the coordinates in the complement of X^2 tangent plane in the 4-D tangent plane). K is assumed to be symplectic invariant and constant for given X^2. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K gives the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant. Within experimental uncertainties one has $\alpha_K = g_K^/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

The assumption that Poisson bracket of WCW Hamiltonians reduces to the level of imbedding space - in other words $\{Q(H_A), Q(H_B\} = Q(\{H_A, H_B\})$ - can be justified. One starts from the representation in terms of say flux Hamiltonians $Q(H_A)$ and *defines* $J_{A,B}$ as $J_{A,B} \equiv Q(\{H_A, H_B\})$. One has $\partial H_A/\partial t_B = \{H_B, H_A\}$, where t_B is the parameter associated with the exponentiation of H_B. The inverse J^{AB} of $J_{A,B} = \partial H_B/\partial t_A$ is expressible as $J^{A,B} = \partial t_A/\partial H_B$. From these formulas one can deduce by using chain rule that the bracket $\{Q(H_A), Q(H_B\} = \partial t_C Q(H_A) J^{CD} \partial t_D Q(H_B)$ of flux Hamiltonians equals to the flux Hamiltonian $Q(\{H_A, H_B\})$.

2. One should be able to assign to WCW Hamiltonians also a part corresponding to the interaction term. The symplectic conjugation associated with the interaction term permutes the WCW coordinates assignable to the ends of the line. One should reduce this apparently non-local symplectic conjugation (if one thinks the ends of line as separate objects) to a non-local symplectic conjugation for $\delta CD \times CP_2$ by identifying the points of lower and upper end of CD related by time reflection and assuming that conjugation corresponds to time reflection. Formally this gives a well defined generalization of the local Poisson brackets between time reflected points at the boundaries of CD. The connection of Hermitian conjugation and time reflection in quantum field theories is is in accordance with this picture.

3. The only manner to proceed is to assign to the flux Hamiltonian also a part obtained by the replacement of the flux integral over X^2 with an integral over the projection of X^2 to a sphere S^2 assignable to the light-cone boundary or to a geodesic sphere of CP_2, which come as two varieties corresponding to homologically trivial and non-trivial spheres. The projection is defined as by the geodesic line orthogonal to S^2 and going through the point of X^2. The hierarchy of

Planck constants assigns to CD a preferred geodesic sphere of CP_2 as well as a unique sphere S^2 as a sphere for which the radial coordinate r_M or the light-cone boundary defined uniquely is constant: this radial coordinate corresponds to spherical coordinate in the rest system defined by the time-like vector connecting the tips of CD. Either spheres or possibly both of them could be relevant.

Recall that also the construction of number theoretic braids and symplectic QFT [16] led to the proposal that braid diagrams and symplectic triangulations could be defined in terms of projections of braid strands to one of these spheres. One could also consider a weakening for the condition that the points of the number theoretic braid are algebraic by requiring only that the S^2 coordinates of the projection are algebraic and that these coordinates correspond to the discretization of S^2 in terms of the phase angles associated with θ and ϕ.

This gives for the corresponding contribution of the WCW Hamiltonian the expression

$$Q(H_A)_{int} = \int_{S^2_\pm} H_A X \delta^2(s_+, s_-) d^2 s_\pm = \int_{P(X^2_+)\cap P(X^2_-)} \frac{\partial(s^1, s^2)}{\partial(x^1_\pm, x^2_\pm)} d^2 x_\pm \ . \tag{5.9.6}$$

Here the Poisson brackets between ends of the line using the rules involve delta function $\delta^2(s_+, s_-)$ at S^2 and the resulting Hamiltonians can be expressed as a similar integral of $H_{[A,B]}$ over the upper or lower end since the integral is over the intersection of S^2 projections.

The expression must vanish when the induced Kähler form vanishes for either end. This is achieved by identifying the scalar X in the following manner:

$$\begin{aligned} X &= J^{kl}_+ J^-_{kl} \ , \\ J^{kl}_\pm &= (1 + K_\pm)\partial_\alpha s^k \partial_\beta s^l J^{\alpha\beta}_\pm \ . \end{aligned} \tag{5.9.7}$$

The tensors are lifts of the induced Kähler form of $X^2_\pm$ to S^2 (not CP_2).

4. One could of course ask why these Hamiltonians could not contribute also to the kinetic terms and why the brackets with flux Hamiltonians should vanish. This relate to how one *defines* the Kähler form. It was shown above that in case of flux Hamiltonians the definition of Kähler form as brackets gives the basic formula $\{Q(H_A), Q(H_B)\} = Q(\{H_A, H_B\}$ and same should hold true now. In the recent case $J_{A,B}$ would contain an interaction term defined in terms of flux Hamiltonians and the previous argument should go through also now by identifying Hamiltonians as sums of two contributions and by introducing the doubling of the coordinates t_A.

5. The quantization of the modified Dirac operator must be reconsidered. It would seem that one must add to the super-Hamiltonian completely analogous term obtained by replacing $(1 + K)J$ with $X\partial(s^1, s^2)/\partial(x^1_\pm, x^2_\pm)$. Besides the anticommutation relations defining correct anticommutators to flux Hamiltonians, one should pose anticommutation relations consistent with the anticommutation relations of super Hamiltonians. In these anticommutation relations $(1 + K)J\delta^2(x, y)$ would be replaced with $X\delta^2(s^+, s^-)$. This would guarantee that the oscillator operators at the ends of the line are not independent and that the resulting Hamiltonian reduces to integral over either end for $H_{[A,B]}$.

6. In the case of CP_2 the Hamiltonians generating isometries are rational functions. This should hold true also now so that p-adic variants of Hamiltonians as functions in WCW would make sense. This in turn would imply that the components of the WCW Kähler form are rational functions. Also the exponentiation of Hamiltonians make sense p-adically if one allows the exponents of group parameters to be functions $Exp_p(t)$.

Does the expansion in terms of partial harmonics converge?

The individual terms in the partial wave expansion seem to be finite but it is not at all clear whether the expansion in powers of K actually converges.

1. In the proposed scenario one performs the expansion of the vacuum functional $exp(K)$ in powers of K and therefore in negative powers of α_K. In principle an infinite number of terms can be present. This is analogous to the perturbative expansion based on using magnetic monopoles as basic objects whereas the expansion using the contravariant Kähler metric as a propagator would be in positive powers of α_K and analogous to the expansion in terms of magnetically bound states of wormhole throats with vanishing net value of magnetic charge. At this moment one can only suggest various approaches to how one could understand the situation.

2. Weak form of self-duality and magnetic confinement could change the sitution. Performing the perturbation around magnetic flux tubes together with the assumed slicing of the space-time sheet by stringy world sheets and partonic 2-surfaces could mean that the perturbation corresponds to the action assignable to the electric part of Kähler form proportional to α_K by the weak self-duality. Hence by $K = 4\pi\alpha_K$ relating Kähler electric field to Kähler magnetic field the expansion would come in powers of a term containing sum of terms proportional to α_K^0 and α_K. This would leave to the scattering amplitudes the exponents of Kähler function at the maximum of Kähler function so that the non-analytic dependence on α_K would not disappear.

A further reason to be worried about is that the expansion containing infinite number of terms proportional to α_K^0 could fail to converge.

1. This could be also seen as a reason for why magnetic singlets are unavoidable except perhaps for $\hbar < \hbar_0$. By the holomorphic factorization the powers of the interaction part of Kähler action in powers of $1/\alpha_K$ would naturally correspond to increasing and opposite net values of the quantum numbers assignable to the WCW phase coordinates at the ends of the propagator line. The magnetic bound states could have similar expansion in powers of α_K as pairs of states with arbitrarily high but opposite values of quantum numbers. In the functional integral these quantum numbers would compensate each other. The functional integral would leave only an expansion containing powers of α_K starting from some finite possibly negative (unless one assumes the weak form of self-duality) power. Various gauge coupling strengths are expected to be proportional to α_K and these expansions should reduce to those in powers of α_K.

2. Since the number of terms in the fermionic propagator expansion is finite, one might hope on basis of super-symmetry that the same is true in the case of the functional integral expansion. By the holomorpic factorization the expansion in powers of K means the appearance of terms with increasingly higher quantum numbers. Quantum number conservation at vertices would leave only a finite number of terms to tree diagrams. In the case of loop diagrams pairs of particles with opposite and arbitrarily high values of quantum numbers could be generated at the vertex and magnetic confinement might be necessary to guarantee the convergence. Also super-symmetry could imply cancellations in loops.

Could one do without flux Hamiltonians?

The fact that the Kähler functions associated with the propagator lines can be regarded as interaction terms inspires the question whether the Kähler function could contain only the interaction terms so that Kähler form and Kähler metric would have components only between the ends of the lines.

1. The basic objection is that flux Hamiltonians too beautiful objects to be left without any role in the theory. One could also argue that the WCW metric would not be positive definite if only the non-diagonal interaction term is present. The simplest example is Hermitian 2×2-matrix with vanishing diagonal for which eigenvalues are real but of opposite sign.

2. One could of course argue that the expansions of $exp(K)$ and λ_k give in the general powers $(f_n\overline{f_n})^m$ analogous to diverging tadpole diagrams of quantum field theories due to local interaction vertices. These terms do not produce divergences now but the possibility that the

exponential series of this kind of terms could diverge cannot be excluded. The absence of the kinetic terms would allow to get rid of these terms and might be argued to be the symmetric space counterpart for the vanishing of loops in WCW integral.

3. In zero energy ontology this idea does not look completely non-sensical since physical states are pairs of positive and negative energy states. Note also that in quantum theory only creation operators are used to create positive energy states. The manifest non-locality of the interaction terms and absence of the counterparts of kinetic terms would provide a trivial manner to get rid of infinities due to the presence of local interactions. The safest option is however to keep both terms.

Summary

The discussion suggests that one must treat the entire Feynman graph as single geometric object with Kähler geometry in which the symmetric space is defined as product of what could be regarded as analogs of symmetric spaces with interaction terms of the metric coming from the propagator lines. The exponent of Kähler function would be the product of exponents associated with all lines and contributions to lines depend on quantum numbers (momentum and color quantum numbers) propagating in line via the coupling to the modified Dirac operator. The conformal factorization would allow the reduction of integrations to Fourier analysis in symmetric space. What is of decisive importance is that the entire Feynman diagrammatics at WCW level would reduce to the construction of WCW geometry for a single propagator line as a function of quantum numbers propagating on the line.

5.10 Could the notion of hyperdeterminant be useful in TGD framework?

The vanishing of ordinary determinant tells that a group of linear equations possesses non-trivial solutions. Hyperdeterminant [59] generalizes this notion to a situation in which one has homogenous multilinear equations. The notion has applications to the description of quantum entanglement and has stimulated interest in physics blogs [60, 61]. Hyperdeterminant applies to hyper-matrices with n matrix indices defined for an n-fold tensor power of vector space - or more generally - for a tensor product of vector spaces with varying dimensions. Hyper determinant is an n-linear function of the arguments in the tensor factors with the property that all partial derivatives of the hyper determinant vanish at the point, which corresponds to a non-trivial solution of the equation. A simple example is potential function of n arguments linear in each argument.

5.10.1 About the definition of hyperdeterminant

Hyperdeterminant was discovered by Cayley for a tensor power of 2-dimensional vector space V_2 (n-linear case for n-fold tensor power of 2-dimensional linear space) and he gave an explicit formula for the hyperdeterminant in this case. For $n = 3$ the definition is following.

$$A^1_{i_3j_3} = \frac{1}{2}\epsilon^{i_1j_1}\epsilon^{i_2j_2}\epsilon^{i_3j_3}A_{i_1i_2i_3}A_{j_1j_2j_3} \quad .$$

In more general case one must take tensor product of $k = 2$ hyper-matrices and perform the contractions of indices belonging to the two groups in by using n 2-D permutations symbols.

$$det(A) = \frac{1}{2^n}(\prod_{a=1}^{n}\epsilon^{i^a_kj^a_k})A_{i^a_1i^a_2...i^a_n}A_{j^a_1j^a_2...j^a_n} \quad .$$

The first guess is that the definition for V_k, $k > 2$ is essentially identical: one takes the tensor product of k hyper-matrices and performs the contractions using k-dimensional permutation symbols.

Under some conditions one can define hyperdeterminant also when one has a tensor product of linear spaces with different dimensions. The condition is that the largest vector space dimension in the product does not exceed the sum of other dimensions.

5.10.2 Could hyperdeterminant be useful in the description of criticality of Kähler action?

Why the notion of hyperdeterminant- or rather its infinite-dimensional generalization- might be interesting in TGD framework relates to the quantum criticality of TGD stating that TGD Universe involves a fractal hierarchy of criticalities: phase transitions inside phase transitions inside... At classical level the lowest order criticality means that the extremal of Kähler action possesses non-trivial second variations for which the action is not affected. The system is critical. In QFT context one speaks about zero modes. The vanishing of the so called Gaussian (of functional) determinant associated with second variations is the condition for the existence of critical deformations. In QFT context this situation corresponds to the presence of zero modes.

The simplest physical model for a critical system is cusp catastrophe defined by a potential function $V(x)$ which is fourth order polynomial. At the edges of cusp two extrema of potential function stable and unstable extrema co-incide and the rank of the matrix defined by the potential function vanishes. This means vanishing of its determinant. At the tip of the cusp the also the third derivative vanishes of potential function vanishes. This situation is however not describable in terms of hyperdeterminant since it is genuinely non-linear rather than only multilinear.

In a complete analogy, one can consider also the vanishing of n:th variations in TGD framework as higher order criticality so that the vanishing of hyperdeterminant might serve as a criterion for the higher order critical point and occurrence of phase transition.

1. The field equations are formally multilinear equations for variables which correspond to imbedding space coordinates at different space-time points. The generic form of the variational equations is

$$\int \frac{\delta^n S}{\delta h^{k_1}(x_1)\delta h^{k_2}(x_2)...\delta h^{k_n}(x_n)}\delta h^{k_2}(x_2)...\delta h^{k_n}(x_n)\prod_{i=2}^{n} d^4x_k = 0 \quad .$$

   Here the partial derivatives are replaced with functional derivatives. On basis of the formula one has formally an n-linear situation. This is however an illusion in the generic case. For a local action the equations reduce to local partial differential equations involving higher order derivatives and field equations involve products of field variables and their various partial derivatives at single point so that one has a genuinely non-linear situation in absence of special symmetries.

2. If one has multi-linearity, the tensor product is formally an infinite tensor power of 8-D (or actually 4-D by General Coordinate Invariance) linear tangent spaces of H associated with the space-time points. A less formal representation is in terms of some discrete basis for the deformations allowing also linear ordering of the basis functions. One might hope in some basis vanishing diagonal terms in all orders and multilinearity.

3. When one uses discretization, the equations stating the vanishing of the second variation couple nearest neighbour points given as infinite-D matrix with non-vanishing elements at diagonal and in a band along diagonal. For higher variations one obtains similar matrix along a diagonal of infinite cube and the width of the band increases by two units as n increases by 1 unit. One might perhaps say that the range of long range correlations increases as n increases. The vanishing of the elements at the diagonal- not necessarily in this representation- is necessary in order to achieve multi-linear situation.

5.10.3 Could the field equations for higher variations be multilinear?

The question is whether for some highly symmetric actions- say Kähler action for preferred extremals- the notion of functional (or Gaussian) determinant could have a generalization to hyperdeterminant allowing to concisely express whether the solutions allow deformations for which the action is not affected.

1. In standard field theory framework this notion need not be of much use but in TGD framework, where Kähler action has infinite-dimensional vacuum degeneracy, the situation is quite different. Vacuum degeneracy means that every space-time surface with at most 2-D CP_2 projection which

is so called Lagrangian manifold is vacuum extremal. Physically this correspond to Kähler gauge potential, which is pure gauge and implies spin glass degeneracy. This dynamical and local $U(1)$ symmetry of vacua is induced by symplectic transformations of CP_2 and has nothing to do with U(1) gauge invariance. For non-vacua it corresponds to isometries of "world of classical worlds". In particular, for M^4 imbedded in canonical manner to $M^4 \times CP_2$ fourth order variation is the first non-vanishing variation. The static mechanical analogy is potential function which is fourth order polynomial. Dynamical analogy is action for which both kinetic and potential terms are fourth order polynomials.

2. The vacuum degeneracy is responsible for much of new physics and mathematics related to TGD. Vacuum degeneracy and the consequent complete failure of canonical quantization and path integral approach forced the vision about physics as geometry of "World of Classical Worlds" (WCW) meaning a generalization of Einstein's geometrization of physics program. 4-D spin glass degeneracy is of the physical implications and among other things allows to have a failure of the standard form of classical determinism as a space-time correlate of quantum non-determinism. There are reasons to hope that also the hierarchy of Planck constants reduces to the 1-to-many correspondence between canonical momentum densities and time derivatives of imbedding space-coordinates. Quantum criticality and its classical counterparts is a further implication of the vacuum degeneracy and has provided a lot of insights to the world according to TGD. Therefore it would be nice if the generalization of the hyperdeterminant could provide new insights to quantum criticality.

5.10.4 Multilinearity, integrability, and cancellation of infinities

The multilinearity in the general sense would have a very interesting physical interpretation. One can consider the variations of both Kähler action and Kähler function defined as Kähler action for a preferred extremal.

1. Multilinearity would mean multi-linearization of field equations in some discrete basis for deformations- say the one defined by second variations. Dynamics would be only apparently non-linear. One might perhaps say that the theory is integrable- perhaps even in the usual sense. The basic idea behind quantum criticality is indeed the existence of infinite number of conserved currents assignable to the second variations hoped to give rise to an integrable theory. In fact, the possibility -or more or less the fact - that also higher variations can vanish for more restricted configurations would imply further conserved currents.

2. Second implication would be the vanishing of local divergences. These divergences result in QFT from purely local interaction terms with degree higher than two. Even mass insertion which is second order produces divergences. If diagonal terms are absent from Kähler function, also these divergences are absent in the functional integral. The main idea behind the notion of Kähler function is that it is a non-local functional of 3-surface although Kähler action is a local functional of space-time sheet serving as the analog of Bohr orbit through 3-surface. As one varies the 3-surface, one obtains a 3-surface (light-like wormhole throats with degenerate four-metric) which is also an extremal of Chern-Simons action satisfying weak form of electric magnetic duality.

3. The weak of electric magnetic duality together with the Beltrami property for conserved currents associated with isometries and for Kähler current and corresponding instanton current imply that the Coulomb term in Kähler action vanishes and it reduces to Chern-Simons term at 3-D light-like wormhole throats plus Lagrange multiplier term taking care that the weak electric magnetic duality is satisfied. This contributes a constraint force to field equations so that the theory does not reduce to topological QFT but to what could be called almost topological QFT.

4. Chern-Simons term is a local functional of 3-surface and one argue that the dangerous locality creeps in via the electric-magnetic duality after all. By using the so called Darboux coordates (P_i, Q_i) for CP_2 Chern-Simons action reduces to a third order polynomial proportional to $\epsilon^{ijk} P_i dP_j dQ_k$ so that one indeed has multilinearity rather than non-linearity. The Lagrangian multiplier term however breaks strict locality and also contributes to higher functional derivatives of Kähler function and is potentially dangerous. It contains information about the preferred

extremals via the normal derivatives associated with the Kähler electric field in normal direction and it higher derivatives.

5. One has however good hopes about multilinearity of higher variations Kähler function and of Kähler action for preferred extremals on basis of general arguments related to the symmetric space property of WCW. As a matter fact, effective two-dimensionality seems to guarantee genuine non-locality. Recall that effective two-dimensionality is implied by the strong form of General Coordinate Invariance stating that the basic geometric objects can be taken to be either light-like 3-surfaces or space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond. This implies that partonic 2-surfaces defining the intersections of these surfaces plus their 4-D tangent space-data code for physics. By effective 2-dimensionality Chern-Simons action is a non-local functional of data about partonic 2-surface and its tangent space. Hence the n:th variation of 3-surface and space-time surface reduces to a non-local functional of n:th variation of the partonic 2-surface and its tangent space data. This is just what genuine multilinearity means.

5.10.5 Hyperdeterminant and entanglement

A highly interesting application of hyperdeterminants is to the description of quantum entanglement- in particular to the entanglement of n qubits in quantum computation. For pure states the matrix describing entanglement between two systems has minimum rank for pure states and thus vanishing determinant. Hyper-matrix and hyperdeterminant emerge naturally when one speaks about entanglement between n quantum systems. The vanishing of hyperdeterminant means that the state is not maximally non-pure.

For the called hyper-finite factor defined by second quantized induced spinor fields one has very formally infinite tensor product of 8-D H-spinor space. By induced spinor equation the dimension effectively reduces to four. Similar formal $8 \rightarrow 4$ reduction occurs by General Coordinate Invariance for the n:th variations. Quantum classical correspondence states that many-fermion states have correlates at the level of space-time geometry. The very naive question inspired also by supersymmetry is whether the vanishing of n-particle hyperdeterminant for the fermionic entanglement has as a space-time correlate n:th order criticality. If so, one could say that the non-locality with all its beautiful consequences is forced by quantum classical correspondence!

5.10.6 Could multilinear Higgs potentials be interesting?

It seems that hyperdeterminant has quite limited applications to finite-dimensional case. The simplest situation corresponds to a potential function $V(x_1, .., x_n)$. In this case one obtains also partial derivatives up to n:th order for single variable and one has genuine non-linearity rather than multilinearity. This spoils the possibility to apply the notion of hyperdeterminant to tell whether critical deformations are possible unless the potential function is multilinear function of its arguments. An interesting idea is that Higgs potential of this form. In this case the extrema allow scalings of the coordinates x_i. In 3-D case 3-linear function of 6 coordinates coming as doublets (x_i, y_i) , $i = 1, 2, 3$ and characterized by a matrix $A_{i_1 i_2 i_3}$, where i_k is two-valued index, would provide an example of this kind of Higgs potential.

The physical interpretation of Kähler Dirac equation is not at all straightforward. The following arguments inspired by effective 2-dimensionality suggest that the modified gamma matrices and corresponding effective metric could allow dual gravitational description of the physics associated with wormhole throats. This applies in particular to condensed matter physics.

Three Dirac equations

To begin with, Dirac equation appears in three forms in TGD.

1. The Dirac equation in world of classical worlds codes for the super Virasoro conditions for the super Kac-Moody and similar representations formed by the states of wormhole contacts forming the counterpart of string like objects (throats correspond to the ends of the string. This Dirac generalizes the Dirac of 8-D imbedding space by bringing in vibrational degrees of freedom. This

Dirac equation should gives as its solutions zero energy states and corresponding M-matrices generalizing S-matrix and their collection defining the unitary U-matrix whose natural application appears in consciousness theory as a coder of what Penrose calls U-process.

2. There is generalized eigenvalue equation for Chern-Simons Dirac operator at light-like wormhole throats. The generalized eigenvalue is $p^k\gamma_k$. The interpretation of pseudo-momentum p^k has been a problem but twistor Grassmannian approach suggests strongly that it can be interpreted as the counterpart of equally mysterious region momentum appearing in momentum twistor Grassmannian approach to $\mathcal{N} = 4$ SYM. The pseudo-/region momentum p is quantized (this does not spoil the basics of Grasssmannian residues integral approach) and $1/p^k\gamma_k$ defines propagator in lines of generalized Feynman diagrams. The Yangian symmetry discovered generalizes in a very straightforward manner and leads alsoto the realization that TGD could allow also a twistorial formulation in terms of product $CP_3 \times CP_3$ of two twistor spaces [29]. General arguments lead to a proposal for explicit form for the solutions of field equation represented identified as holomorphic 6-surfaces in this space subject to additional partial different equations for homogenenous functions of projective twistor coordinates suggesting strongly the quantal interpretation as analogs of partial waves. Therefore quantum-classical correspondence would be realize in beatiful manner.

3. There is Kähler Dirac equation in the interior of space-time. In this equation the gamma matrices are replaced with modified gamma matrices defined by the contractions of canonical momentum currents $T^\alpha_k = \partial L/\partial_\alpha h^k$ with imbedding space gamma matrices Γ_k. This replacement is required by internal consistency and by super-conformal symmetries.

Could Kähler Dirac equation provide a first principle justification for the light-hearted use of effective mass and the analog of Dirac equation in condensed manner physics? This would conform with the holographic philosophy. Partonic 2-surfaces with tangent space data and their light-like orbits would give hologram like representation of physics and the interior of space-time the 4-D representation of physics. Holography would have in the recent situation interpretation also as quantum classical correspondence between representations of physics in terms of quantized spinor fields at the light-like 3-surfaces on one hand and in terms of classical fields on the other hand.

The resulting dispersion relation for the square of the Kähler-Dirac operator assuming that induced like metric, Kähler field, etc. are very slowly varying contains quadratic and linear terms in momentum components plus a term corresponding to magnetic moment coupling. In general massive dispersion relation is obtained as is also clear from the fact that Kähler Dirac gamma matrices are combinations of M^4 and CP_2 gammas so that modified Dirac mixes different M^4 chiralities (basic signal for massivation). If one takes into account the dependence of the induced geometric quantities on space-time point dispersion relations become non-local.

Does energy metric provided the gravitational dual for condensed matter systems?

The modified gamma matrices define an effective metric via their anticommutators which are quadratic in components of energy momentum tensor (canonical momentum densities). This effective metric vanishes for vacuum extremals. Note that the use of modified gamma matrices guarantees among other things internal consistency and super-conformal symmetries of the theory. The physical interpretation has remained obscure hitherto although corresponding effective metric for Chern-Simons Dirac action has now a clear physical interpretation.

If the above argument is on the right track, this effective metric should have applications in condensed matter theory. In fact, energy metric has a natural interpretation in terms of effective light velocities which depend on direction of propagation. One can diagonalize the energy metric $g_e^{\alpha\beta}$ (contravariant form results from the anticommutators) and one can denote its eigenvalues by (v_0, v_i) in the case that the signature of the effective metric is $(1, -1, -1, -1)$. The 3-vector v_i/v_0has interpretation as components of effective light velocity in various directions as becomes clear by thinking the d'Alembert equation for the energy metric. This velocity field could be interpreted as that of hydrodynamic flow. The study of the extremals of Kähler action shows that if this flow is actually Beltrami flow so that the flow parameter associated with the flow lines extends to global coordinate, Kähler action reduces to a 3-D Chern-Simons action and one obtains effective topological QFT. The conserved fermion current $\overline{\Psi}\Gamma_e^\alpha\Psi$ has interpretation as incompressible hydrodynamical flow.

This would give also a nice analogy with AdS/CFT correspondence allowing to describe various kinds of physical systems in terms of higher-dimensional gravitation and black holes are introduced quite routinely to describe condensed matter systems. In TGD framework one would have an analogous situation but with 10-D space-time replaced with the interior of 4-D space-time and the boundary of AdS representing Minkowski space with the light-like 3-surfaces carrying matter. The effective gravitation would correspond to the "energy metric". One can associate with it curvature tensor, Ricci tensor and Einstein tensor using standard formulas and identify effective energy momentum tensor associated as Einstein tensor with effective Newton's constant appearing as constant of proportionality. Note however that the besides ordinary metric and "energy" metric one would have also the induced classical gauge fields having purely geometric interpretation and action would be Kähler action. This 4-D holography would provide a precise, dramatically simpler, and also a very concrete dual description. This cannot be said about model of graphene based on the introduction of 10-dimensional black holes, branes, and strings chosen in more or less ad hoc manner.

This raises questions. Does this give a general dual gravitational description of dissipative effects in terms of the "energy" metric and induced gauge fields? Does one obtain the counterparts of black holes? Do the general theorems of general relativity about the irreversible evolution leading to black holes generalize to describe analogous fate of condensed matter systems caused by dissipation? Can one describe non-equilibrium thermodynamics and self-organization in this manner?

One might argue that the incompressible Beltrami flow defined by the dynamics of the preferred extremals is dissipationless and viscosity must therefore vanish locally. The failure of complete non-determinism of Kähler action however means generation of entropy since the knowledge about the state decreases gradually. This in turn should have a phenomenological local description in terms of viscosity which characterizes the transfer of energy to shorter scales and eventually to radiation. The deeper description should be non-local and basically topological and might lead to quantization rules. For instance, one can imagine the quantization of the ratio η/s of the viscosity to entropy density as multiples of a basic unit defined by its lower bound (note that this would be analogous to Quantum Hall effect). For the first M-theory inspired derivation of the lower bound of η/s [63]. The lower bound for η/s is satisfied in good approximation by what should have been QCD plasma but found to be something different (RHIC and the first evidence for new physics from LHC [31]).

An encouraring sign comes from the observation that for so called massless extremals representing classically arbitrarily shaped pulses of radiation propagating without dissipation and dispersion along single direction the canonical momentum currents are light-like. The effective contravariant metric vanishes identically so that fermions cannot propate in the interior of massless extremals! This is of course the case also for vacuum extremals. Massless extremals are purely bosonic and represent bosonic radiation. Many-sheeted space-time decomposes into matter containing regions and radiation containing regions. Note that when wormhole contact (particle) is glued to a massless extremal, it is deformed so that CP_2 projection becomes 4-D guaranteing that the weak form of electric magnetic duality can be satisfied. Therefore massless extremals can be seen as asymptotic regions. Perhaps one could say that dissipation corresponds to a decoherence process creating space-time sheets consisting of matter and radiation. Those containing matter might be even seen as analogs blackholes as far as energy metric is considered.

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Chapter 6

Miscellaneous topics

6.1 Introduction

As the title tells, this chapter contains topics which do not fit naturally under any umbrella, but which I feel might be of some relevance. Basically TGD inspired comments to the work of the people not terribly relevant to quantum TGD itself are in question. For few years ago Witten's approach to 3-D quantum gravitation raised a considerable interest and this inspired the comparison of this approach with quantum TGD in which light-like 3-surfaces are in a key role. Few years later the entropic gravity of Verlinde stimulated a lot of fuss in blogs and it is interesting to point out how the formal thermodynamical structure (or actually its "square root") emerges in the fundamental formulation of TGD. Lisi's E_8 theory was a further blog favorite and some comments about its failures and possible manners to cure them are discussed. It is also shown ho how E_8 can be seed as being replaced with the Kac-Moody algebra associated standard model symmetry group in TGD framework.

6.2 Light-like 3-surfaces as vacuum solutions of 3-D vacuum Einstein equations and Witten's approach to quantum gravitation

There is an interesting relationship to the recent yet unpublished work of Witten related to 3-D quantum blackholes [26], which allows to get additional perspective.

1. The motivation of Witten is to find an exact quantum theory for blackholes in 3-D case. Witten proposes that the quantum theory for 3-D AdS_3 blackhole with a negative cosmological constant can be reduced by AdS_3/CFT_2 correspondence to a 2-D conformal field theory at the 2-D boundary of AdS_3 analogous to blackhole horizon. This conformal field theory would be a Chern-Simons theory associated with the isometry group $SO(1,2)\times SO(1,2)$ of AdS_3. Witten restricts the consideration to $\Lambda < 0$ solutions because $\Lambda = 0$ does not allow black-hole solutions and Witten believes that $\Lambda > 0$ solutions are non-perturbatively unstable.

2. This conformal theory would have the so called monster group [25, 26] as the group of its discrete hidden symmetries. The primary fields of the corresponding conformal field theory would form representations of this group. The existence of this kind of conformal theory has been demonstrated already [27]. In particular, it has been shown that this theory does not allow massless states. On the other hand, for the 3-D vacuum Einstein equations the vanishing of the Einstein tensor requires the vanishing of curvature tensor, which means that gravitational radiation is not possible. Hence AdS_3 theory in Witten's sense might define this conformal field theory.

6.2.1 Similarities with TGD

Witten's construction has obviously a strong structural similarity to TGD.

1. Chern-Simons action for the induced Kähler form - or equivalently, for the induced classical color gauge field proportional to Kähler form and having Abelian holonomy - corresponds to the Chern-Simons action in Witten's theory. Note however that in the recent formulation of Quantum TGD Kähler action and corresponding instanton density $J \wedge J$ define real and imaginary parts of complexified Kähler action. The imaginary part of the complexified Kähler function does not contribute to the configuration space metric but gives first principle description of anyons and purely topological degrees of freedom.

2. Light-like 3-surfaces can be regard as 3-D solutions of vacuum Einstein equations. Due to the effective 2-dimensionality of the induced metric Einstein tensor vanishes identically and vacuum Einstein equations are satisfied for $\Lambda = 0$. One can say that light-like partonic 3-surfaces correspond to empty space solutions of Einstein equations. Even more, partonic 3-surfaces are very much analogous to 3-D black-holes if one identifies the counterpart of black-hole horizon with the intersection of $\delta M^4_{\pm} \times CP_2$ with the partonic 2-surface.

3. For light-like 3-surfaces curvature tensor is non-vanishing which raises the question whether one obtains gravitons in this case. The fact that time direction does not contribute to the metric means that propagating waves are not possible so that no 3-D gravitational radiation is obtained. There is analog for this result at quantum level. If partonic fermions are assumed to be free fields as is done in the recent formulation of quantum TGD, gravitons can be obtained only as parton-antiparton bound states connected by flux tubes and are therefore genuinely stringy objects. Hence it is not possible to speak about 3-D gravitons as single parton states.

4. Vacuum Einstein equations can be regarded as gauge fixing allowing to eliminate partially the gauge degeneracy due to the general coordinate invariance. Additional super conformal symmetries are however present and have an identification in terms of additional symmetries related to the fact that space-time surfaces correspond to preferred extremals of Kähler action whose existence was concluded before the discovery of the formulation in terms of light-like 3-surfaces.

6.2.2 Differences from TGD

There are also interesting differences.

1. According to Witten, his theory has no obvious generalization to 4-D black-holes whereas 3-D light-like determinants define the generalization of blackhole horizons which are also light-like 3-surfaces in the induced metric. In particular, light-like 3-surfaces define a 4-D quantum holography.

2. Partonic 3-surfaces are dynamical unlike AdS_3 and the analog of Witten's theory results by freezing the vibrational degrees of freedom in TGD framework.

3. The very notion of light-likeness involves the induced metric implying that the theory is almost-topological but not quite. This small but important distinction indeed guarantees that the theory is physically interesting.

4. In Witten's theory the gauge group corresponds to the isometry group $SO(1,2) \times SO(1,2)$ of AdS_3. The group of isometries of light-like 3-surface is something much much mightier. It corresponds to the conformal transformations of 2-dimensional section of the 3-surfaces made local with respect to the radial light-like coordinate in such a manner that radial scaling compensates the conformal scaling of the metric produced by the conformal transformation.

   The direct TGD counterpart of the Witten's gauge group would be thus infinite-dimensional and essentially same as the group of 2-D conformal transformations. Presumably this can be interpreted in terms of the extension of conformal invariance implied by the presence of ordinary conformal symmetries associated with 2-D cross section plus "conformal" symmetries with respect to the radial light-like coordinate. This raises the question about the possibility to formulate quantum TGD as something analogous to string field theory using using Chern-Simons action for this infinite-dimensional group.

5. Monster group does not have any special role in TGD framework. However, all finite groups and - as it seems - also compact groups can appear as groups of dynamical symmetries at the partonic level in the general framework provided by the inclusions of hyper-finite factors of type II_1 [20, 16]. Compact groups and their quantum counterparts would closely relate to a hierarchy of Jones inclusions associated with the TGD based quantum measurement theory with finite measurement resolution defined by inclusion as well as to the generalization of the imbedding space related to the hierarchy of Planck constants [20]. Discrete groups would correspond to the number theoretical braids providing representations of Galois groups for extensions of rationals realized as braidings [17].

6. To make it clear, I am not suggesting that AdS_3/CFT_2 correspondence should have a TGD counterpart. If it had, a reduction of TGD to a closed string theory would take place. The almost-topological QFT character of TGD excludes this on general grounds. More concretely, the dynamics would be effectively 2-dimensional if the radial superconformal algebras associated with the light-like coordinate would act as pure gauge symmetries. Concrete manifestations of the genuine 3-D character are following.

   (a) Generalized super-conformal representations decompose into infinite direct sums of stringy super-conformal representations.

   (b) In p-adic thermodynamics explaining successfully particle massivation radial conformal symmetries act as dynamical symmetries crucial for the particle massivation interpreted as a generation of a thermal conformal weight.

   (c) The maxima of Kähler function defining Kähler geometry in the world of classical worlds correspond to special light-like 3-surfaces analogous to bottoms of valleys in spin glass energy landscape meaning that there is infinite number of different 3-D light-like surfaces associated with given 2-D partonic configuration each giving rise to different background affecting the dynamics in quantum fluctuating degrees of freedom. This is the analogy of landscape in TGD framework but with a direct physical interpretation in say living matter.

As noticed, Witten's theory is essentially for 2-D fundamental objects. It is good to sum up what is needed to get a theory for 3-D fundamental objects in TGD framework from an approach similar to Witten's in many respects. This connection is obtained if one brings in 4-D holography, replaces 3-metrics with light-like 3-surfaces (light-likeness constraint is possible by 4-D general coordinate invariance), and accepts the new view about S-matrix implied by the zero energy ontology [16].

1. Light-like 3-surfaces can be regarded as solutions vacuum Einstein equations with vanishing cosmological constant (Witten considers solutions with non-vanishing cosmological constant). The effective 2-D character of the induced metric is what makes this possible.

2. Zero energy ontology is also an essential element: quantum states of 3-D theory in zero energy ontology correspond to generalized S-matrices [16]: **Matrix** or M-matrix might be a proper term. **Matrix** is a "complex square root" of density matrix -matrix valued generalization of Schrodinger amplitude - defining time like entanglement coefficients. Its "phase" is unitary matrix and might be rather universal. **Matrix** is a functor from the category of Feyman cobordisms and matrices have groupoid like structure [16]. Without this generalization theory would reduce to a theory for 2-D fundamental objects.

3. Theory becomes genuinely 4-D because S-matrix is not universal anymore but characterizes zero energy states.

4. 4-D holography is obtained via the Kähler metric of the world of classical worlds assigning to light-like 3-surface a preferred extremal of Kähler action as the analog of Bohr orbit containing 3-D light-like surfaces as sub-manifolds (analogs of black hole horizons and light-like boundaries). Interiors of 4-D space-time sheets corresponds to zero modes of the metric and to the classical variables of quantum measurement theory (quantum classical correspondence). The conjecture is that Dirac determinant for the modified Dirac action associated with partonic 3-surfaces defines the vacuum functional as the exponent of Kähler function with Kähler coupling strength fixed completely as the analog of critical temperature so that everything reduces to almost topological QFT [18].

5. The counterpart of the ordinary unitary S-matrix in mathematical sense is between zero energy states. I call it U-matrix [16]. As such it would have nothing to do with particle reactions but it is quite possible and also natural that M-matrix would serve as building block of U-matrix so that also U-matrix would be experimentally measurable quantity. It is crucial for understanding consciousness via *moment of consciousness as quantum jump* identification.

6.3 Entropic gravity and TGD

Eric Verlinde has posted an interesting eprint titled *On the Origin of Gravity and the Laws of Newton* to arXiv.org [30]. Lubos has commented the article [31]. What Linde heuristically derives is Newton's $F = ma$ and gravitational force $F = GMm/R^2$ from thermodynamical considerations plus something else which I try to clarify (at least to myself!) in the following.

6.3.1 Verlinde's argument for $F = ma$

The idea is to deduce Newton's $F = ma$ and gravitational force from thermodynamics by assuming that space-time emerges in some sense. There are however various assumptions involved which more or less impy that both special and general relativity has been feeded in besides quantum theory and thermodynamics.

1. Time translation invariance is required in order to have the notions of conserved energy and thermodynamics. This assumption requires not only require time but also symmetry with respect to time translations. This is quite a powerful assumption and time translation symmetry not hold true in General Relativity- this was actually the basic motivation for quantum TGD.

2. Holography is assumed. Information stored on surfaces, or screens and discretization is assumed. Again this means in practice the assumption of space-time since otherwise the notion of holography does not make sense. One could of course say that one considers the situation in the already emerged region of space-time but this idea does not look very convincing to me.

   Comment: In TGD framework holography is an essential piece of theory: light-like 3-surfaces code for the physics and space-time sheets are analogous to Bohr orbits fixed by the light-like 3-surfaces defining the generalized Feynman diagrams.

3. The first law of thermodynamics in the form

$$dE = TdS - Fdx \quad .$$

   Here F denotes generalized force and x some coordinate variable. In usual thermodynamics pressure P would appear in the role of F and volume V in the role of x. Also chemical potential and particle number form a similar pair. If energy is conserved for the motion one has

$$Fdx = TdS \quad .$$

   This equation is basic thermodynamics and is used to deduce Newton's equations.

After this some quantum tricks -a rather standard game with Uncertainty Principle and quantization when nothing concrete is available- are needed to obtain F=ma which as such does not involve $\hbar$ nor Boltzmann constant k_B. What is needed are thermal expression for acceleration and force and identifying these one obtains F=ma.

1. The condition $\Delta S = 2\pi k_B$ states that entropy is quantized with a unit of 2π appearing as a unit. log(2) would be more natural unit if bit is the unit of information.

2. The identification $\Delta x = \hbar/mc$ involves Uncertainty principle for momentum and position. The presence of light velocity c in the formula means that Minkowski space and Special Relativity creeps in. At this stage I would not speak about emergence of space-time anymore.

   This gives

$$F = T\frac{\Delta S}{\Delta x} = T\frac{2\pi mck_B}{\hbar} \quad .$$

F has been exressed in terms of thermal parameters and mass.

3. Next one feeds in something from General Relativity to obtain expression for acceleration in terms of thermal parameters. Unruh effect means that in an accelerted motion system measures temperate proportional to acceleration:

$$k_BT = \frac{\hbar a}{2\pi} \quad .$$

This quantum effect is known as Unruh effect. This temperature is extremely low for accelerations encountered in everyday life - something like 10^{-16} K for free fall near Earth's surface.

Using this expression for T in previous equation one obtains the desired $F = ma$, which would thus have a thermodynamical interpretation. At this stage I have even less motivations for talking about emergence of space-time. Essentially the basic conceptual framework of Special and General Relativities, of wave mechanics and of thermodynamics are introduced by the formulas containing the basic parameters involved.

6.3.2 Verlinde's argument for $F = GMm/R^2$

The next challenge is to derive gravitational force from thermodynamic consideration. Now holography with a very specially chosen screen is needed.

Comment: In TGD framework light-like 3-surfaces (or equivalently their space-like duals) represent the holographic screens and in principle there is a slicing of space-time surface by equivalent screens. Also Verlinde introduces a slicing of space-time surfaces by holographic screens identified as surfaces for which gravitational potential is constant. Also I have considered this kind of identification.

1. The number of bits for the information represented on the holographic screen is assumed to be proportional to area.

$$N = \frac{A}{G\hbar} \quad .$$

This means bringing in blackhole thermodynamics and general relativity since the notion of area requires geometry.

Comment: In TGD framework the counterpart for the finite number of bits is finite measurement resolution meaning that the 2-dimensional partonic surface is effectively replaced with a set of points carrying fermion or antifermion number or possibly purely bosonic symmetry generator. The orbits of these points define braid giving a connection with topological QFTs for knots, links and braids and also with topological quantum computation.

2. It is assumed that the area of horizon corresponds to the area $A = 4\pi R^2$ for the sphere with radius which R which is the distance between the masses. This means a very special choice of the holographic screen. Entropy obviously depends very sensitively on R.

Comment: In TGD framework the counterpart of the area would be the symplectic area of partonic 2-surfaces. This is invariant under symplectic transformations of light-cone boundary. These "partonic" 2-surfaces can have macroscopic size and the counterpart for blackhole horizon is one example of this kind of surface. Anyonic phases are second example of a phase assigned with a macroscopic partonic 2-surface.

3. Special relativity is brought in via the bomb formula

$$E = mc^2 \quad .$$

One introduces also other expression for the rest energy. Thermodynamics gives for non-relativistic thermal energy the expression

$$E = \frac{1}{2} N k_B T \ .$$

This thermal energy is identified with the rest mass. This identification looks to me completely ad hoc and I think that kind of holographic duality is assumed to justify it. The interpretation is that the points/bits on the holographic screen behave as particles in thermodynamical equilibrium and represent the mass inside the spherical screen. What are these particles on the screen? Do they correspond to gravitational flux?

Comment: In TGD framework p-adic thermodynamics replaces Higgs mechanism and identify particle's mass squared as thermal conformal weight. In this sense inertia has thermal origin in TGD framework. Gravitational flux is mediated by flux tubes with gigantic value of gravitational Planck constant and the intersections of the flux tubes with sphere could be TGD counterparts for the points of the screen in TGD. These 2-D intersections of flux tubes should be in thermal equilibrium at Unruh temperature. The light-like 3-surfaces indeed contain the particles so that the matter at this surface represents the system. Since all light-like 3-surfaces in the slicing are equivalent means that one can choose the reresentation of the system rather freely .

4. Eliminating the rest energy E from these two formulas one obtains $NT = 2mc^2$ and using the expression for N in terms of area identified as that of a sphere with radius equal to the distance R between the two masses, one obtains the standard form for gravitational force.

It is difficult to say whether the outcome is something genuinely new or just something resulting unavoidably by feeding in basic formulas from general thermodynamics, special relativity, and general relativity and using holography principle in highly questionable and ad hoc manner.

6.3.3 In TGD quantum classical correspondence predicts that thermodynamics has space-time correlates

From TGD point of view entropic gravity is a misconception. On basis of quantum classical correspondence - the basic guiding principle of quantum TGD - one expects that all quantal notions have space-time correlates. If thermodynamics is a genuine part of quantum theory, also temperature and entropy should have the space-time correlates and the analog of Verlinde's formula could exist. Even more, the generalization of this formula is expected to make sense for all interactions.

Zero energy ontology makes thermodynamics an integral part of quantum theory.

1. In zero energy ontology quantum states become zero energy states consisting of pairs of the positive and negative energy states with opposite conserved quantum numbers and interpreted in the usual ontology as physical events. These states are located at opposite light-like boundaries of causal diamond (CD) defined as the intersection of future and past directed light-cones. There is a fractal hierarchy of them. M-matrix generalizing S-matrix defines time-like entanglement coefficients between positive and negative energy states. M-matrix is essentially a "complex" square root of density matrix expressible as positive square root of diagonalized density matrix and unitary S-matrix. Thermodynamics reduces to quantum physics and should have correlate at the level of space-time geometry. The failure of the classical determinism in standard sense of the word makes this possible in quantum TGD (special properties of Kähler action (Maxwell action for induced Kahler form of CP_2) due to its vacuum degeneracy analogous to gauge degeneracy). Zero energy ontology allows also to speak about coherent states of bosons, say of Cooper pairs of fermions- without problems with conservation laws and the undeniable existence of these states supports zero energy ontology.

2. Quantum classical correspondence is very strong requirement. For instance, it requires also that electrons traveling via several routes in double slit experiment have classical correlates. They have. The light-like 3-surfaces describing electrons can branch and the induced spinor fields at them "branch" also and interfere again. Same branching occurs also for photons so that electrodynamics has hydrodynamical aspect too emphasize in recent empirical report about knotted light beams. This picture explains the findings of Afshar challenging the Copenhagen interpretation.

These diagrams could be seen as generalizations of stringy diagrams but do not describe particle decays in TGD framework. In TGD framework stringy diagrams are replaced with a direct generalization of Feynman diagrams in which the ends of 3-D lightlike lines meet along 2-D partonic surfaces at their ends. The mathematical description of vertices becomes much simpler since the 2-D manifolds describing vertices are not singular unlike the 1-D manifolds associated with string diagrams ("eyeglass" in fusion of closed strings).

3. If entropy has a space-time correlate then also first and second law should have such and Verlinde's argument that gravitational force attraction follows from first law assuming energy correlation might identify this correlate. This of course applies only to the classical gravitation. Also other classical forces should allow analogous interpretation as space-time correlates for something quantal.

6.3.4 The simplest identification of thermodynamical correlates in TGD framework

The first questions that pop up are following. Inertial mass emerges from p-adic thermodynamics as thermal conformal weight. Could the first law for p-adic thermodynamics, which allows to calculate particle masses in terms of thermal conformal weights, allow to deduce also other classical forces? One could think that by adding to the Hamiltonian defining partition function chemical potential terms characterizing charge conservation it might be possible to obtain also other forces.

In fact, the situation might be much simpler. The basic structure of quantum TGD allows a very natural thermodynamical interpretation.

1. The basic structure of quantum TGD suggests a thermodynamic interpretation. The basic observation is that the vacuum functional identified as the exponent of Kähler function is analogous to a square root of partition function and Kähler coupling strength is analogous to critical temperature. Kähler function identified as Kähler action for a preferred extremal appears in the role of Hamiltonian. Preferred extremal property realizes holography identifying space-time surface as analog of Bohr orbit. One can interpret the exponent of Kähler function as the density of states in the world of classical worlds so that Kähler function would be analogous to entropy density. Ensemble entropy is average of Kähler function involving functional integral over the world of classical worlds. This exponent is the counterpart for the quantity Ω appearing in Verlinde's basic formula.

2. The addition of a measurement interaction term to the modified Dirac action gives rise to a coupling to conserved charges. Vacuum functional is identified as Dirac determinant and this addition is visible as an addition of an interaction term to Kähler function. The interaction gives rise to forces coupling to various charges at classical level for quantum states with fixed quantum numbers for positive energy part of the state. These terms are analogous to chemical potential terms in thermodynamics fixing the average values of various charges or particle numbers. In ordinary non-relativistic thermodynamics energy is in a special role. In the recent case there is a complete quantum number democracy very natural in a framework with coordinate invariance and with four-momentum assigned with the isometries of the 8-D imbedding space. In Verlinde's formula there is exponential factor $exp(-E/T - Fx)$ analogous to the measurement interaction term. In TGD however conserved charges multiplied by chemical potentials defining generalized forces appear in the exponent.

3. This gives an analog of thermodynamics in the world of classical worlds (WCW) for fixed values of quantum numbers of the positive energy part of state. For zero energy states one however has also additional thermodynamics- or rather its square root. This thermodynamics is for the conserved quantum numbers whose averages are fixed. For general zero energy states one has sum over state pairs labelled by momenta and various other quantum numbers labelling the positive energy part of the state. The coefficients of the conserved quantities of the measurement interaction term linear in conserved quantum numbers define the analogs of temperature and various chemical potentials. The field equations defined by Kähler function and chemical potential terms have thermodynamical interpretation and give coupling to conserved charges and also to their thermal averages. What is important is that temperature and various chemical potentials assigned to

positive and negative energy parts of the state allow a complete geometrization in a general coordinate invariant manner and allow explicit expressions in terms of functions expressible in terms of the induced geometry.

4. The explicit expressions must be deduced from Dirac determinant defining exponent of Kähler function plus measurement interaction term, in which the conserved isometry charges of Cartan algebra (necessarily!) appearing in the exponent are contracted with the analogs of chemical potentials. One make two rather detailed educated guesses for the chemical potentials. For modified Dirac action the measurement interaction term is 4-dimensional. For the Kähler action one can imagine two candidates for the measurement interaction term. For the first option the term is 4-dimensional and for the second one 3-dimensional.

6.3.5 Some details related to the measurement interaction term

As noticed, one can imagine two options for the measurement interaction term defining the chemical potentials in terms of the space-time geometry.

1. For both options the M^4 part of the interaction term is proportional to $n(M^4)G/R$ and CP_2 part to a dimensionless constant $n(CP_2)$, and the condition that there is no dependence of $\hbar$ excludes the dependence on the dimensionless constant $G\hbar/R^2$.

2. One can consider two different forms of the measurement interaction part in Kähler function. For the first option the conserved Kähler current replaces fermion current in the modified Dirac action and defines a 4-dimensional interaction term highly analogous to that assigned with the modified Dirac action. The source term induced to the field equations corresponds to the variation of

$$[\frac{G}{R} \times n(M^4) p_{q,A} g^{AB}(M^4) j_{A\alpha} + n(CP_2) Q_{q,A} g^{AB} J_{A\alpha}(CP_2)] J^\alpha \quad .$$

   Here J^α is Kähler current.

3. For the second option the measurement interaction term in Kähler action is sum over contractions of quantum Cartan charges with corresponding classical Noether charges giving the sum of the term

$$[\frac{G}{R} \times n(M^4) p_{q,A} p^{cl,A} + n(CP_2) Q_{q,A} Q^{cl,A}]$$

   from both ends of the space-time sheet. For a general space-time sheet the classical charges are different at its ends so that the variation gives non-trivial boundary conditions equating the normal (time-like) component of the canonical momentum current with the contraction of the variation of classical Noether charges contracted with quantum charges. By the extremal property the measurement interaction terms at the ends of the space-time sheet cancel each other so that the effect on Kähler function is only via the boundary conditions in accordance with zero energy ontology. For this option the thermodynamics for conserved charges is visible at space-time level only via the appearence of the average quantal charges and universal chemical potentials.

4. The vanishing of Kähler gauge current *resp.* classical Noether charges for the first *resp.* second option would suggest an interpretation in terms of infinite temperature limit. The fact that momenta and color charges are in completely symmetric position suggests however the vanishing of chemical potentials. One can in fact fix the value of the temperature to say $T = R/G$ without loss of information and code thermodynamics in terms of the chemical potentials alone.

   The vanishing of the measurement interaction term occurs for the vacuum extremals. For CP_2 type vacuum extemals with Euclidian signature of the induced metric interpretation in terms of vanishing chemical potentials is more natural. For vacuum extremals with Minkowskian signature of the induced metric fluctuations and consequently classical non-determinism are maximal so that the interpretation in terms of high temperature phase cannot be excluded. One must however notice that CP_2 projection for vacuum extremals is 2-dimensional whereas high temperature limit would suggest 4-D projection so that the interpretation in terms of vanishing chemical potentials is more natural also now.

To sum up, TGD suggests two thermodynamical interpretations. p-Adic thermodynamics gives inertial mass squared as thermal conformal weight and also the basic formulation of quantum TGD allows thermodynamical interpretation. The thermodynamical structure of quantum TGD has of course been guiding principle for two decades. In particular, quantum criticality as the counterpart of thermal criticality has been extremely useful guide line and led to a breakthrough in the understanding of the modified Dirac equation during the last year. Also p-adic thermodynamics has been in the scene for more than 15 years and makes TGD a theory able to make precise quantitative predictions. Some conclusions drawn from Verlinde's argument is that gravitation is entropic interaction, that gravitons do not exist, and that string models and theories introducing higher-dimensional space-time are a failure. TGD view is different. Only a generalization of string model allowing to realize space-time as surface is needed and this requires fixed 8-D imbedding space. Gravitons also exist and only classical gravitation as well as other classical interactions code for thermodynamical information by quantum classical correspondence. In any case, it is encouraging that also colleagues might be finally beginning to get on the right track although the path from Verlinde's arguments to quantum TGD as it is now will be desperately long and tortuous if colleagues continually refuse to receive the helping hand.

6.4 E_8 theory of Garrett Lisi and TGD

Recently (towards end of the year 2007) there has been a lot of fuss about the E_8 theory proposed by Garrett Lisi [24] in physics blogs, in media, and even New Scientist [28] wrote about the topic. There are serious objections against Lisi's theory and it is interesting to find whether the theory could be modified so that it would survive the basic objections. Although it seems that Lisi's theory cannot be saved, one achieves further insights about HO-H duality. Number theoretical spontaneous compactification can be formulated in terms of the Kac-Moody algebra assignable to Poincare group and standard model gauge group having also rank 8. The representation can be constructed in standard manner using quantized M^8 coordinates at partonic 2-surfaces. Also E_8 representations are in principle possible and the question concerns their physical interpretation.

6.4.1 Objections against Lisi's theory

The basic claim of Lisi is that one can understand the particle spectrum of standard model in terms of the adjoint representation of a noncompact version E_8 group [29]. There are several objections against E_8 gauge theory interpretation of Lisi.

1. Statistics does not allow to put fermions and bosons in the same gauge multiplet. Also the identification of graviton as a part of a gauge multiplet seems very strange if not wrong since there are no roots corresponding to a spin 2 two state.

2. Gauge couplings come out wrong for fermions and one must replace YM action with an ad hoc action.

3. Poincare invariance is a problem. There is no clear relationship with the space-time geometry so that the interpretation of spin as E_8 quantum numbers is not really justified.

4. Finite-dimensional representations of non-compact E_8 are non-unitary. Non-compact gauge groups are however not possible since one would need unitary infinite-dimensional representations which would change the physical interpretation completely. Note that also Lorentz group has only infinite-D unitary representations and only the extension to Poincare group allows to have fields transforming according to finite-D representations.

5. The prediction of three fermion families is nice but one can question the whole idea of putting particles with mass scales differing by a factor of order 10^{12} (top and neutrinos) into same multiplet. For some reason colleagues stubbornly continue to see fundamental gauge symmetries where there seems to be no such symmetry. Accepting the existence of a hierarchy of mass scales

seems to be impossible for a theoretical physicist in main main stream although fractals have been here for decades.

6. Also some exotic particles not present in standard model are predicted: these carry weak hyper charge and color (6-plet representation) and are arranged in three families.

6.4.2 Three attempts to save Lisi's theory

To my opinion, the shortcomings of E_8 theory as a gauge theory are fatal but the possibility to put gauge bosons and fermions of the standard model to E_8 multiplets is intriguing and motivatse the question whether the model could be somehow saved by replacing gauge theory with a theory based on extended fundamental objects possessing conformal invariance.

1. In TGD framework H-HO duality allows to consider Super-Kac Moody algebra with rank 8 with Cartan algebra assigned with the quantized coordinates of partonic 2-surface in 8-D Minkowski space M^8 (identifiable as hyper-octonions HO). The standard construction for the representations of simply laced Kac-Moody algebras allows quite a number of possibilities concerning the choice of Kac-Moody algebra and the non-compact E_8 would be the maximal choice.

2. The first attempt to rescue the situation would be the identification of the weird spin 1/2 bosons in terms of supersymmetry involving addition of righthanded neutrino to the state giving it spin 1. This options does not seem to work.

3. The construction of representations of non-simply laced Kac-Moody algebras (performed by Goddard and Olive at eighties [23]) leads naturally to the introduction of fermionic fields for algebras of type B, C, and F: I do not know whether the construction has been made for G_2. E_6, E_7, and E_8 are however simply laced Lie groups with single root length 2 so that one does not obtain fermions in this manner.

4. The third resuscitation attempt is based on fractional statistics. Since the partonic 2-surfaces are 2-dimensional and because one has a hierarchy of Planck constants, one can have also fractional statistics. Spin 1/2 gauge bosons could perhaps be interpreted as anyonic gauge bosons meaning that particle exchange as permutation is replaced with braiding homotopy. If so, E_8 would not describe standard model particles and the possibility of states transforming according to its representations would reflect the ability of TGD to emulate any gauge or Kac-Moody symmetry.

The standard construction for simply laced Kac-Moody algebras might be generalized considerably to allow also more general algebras and fractionization of spin and other quantum numbers would suggest fractionization of roots. In stringy picture the symmetry group would be reduced considerably since longitudinal degrees of freedom (time and one spatial direction) are non-physical. This would suggest a symmetry breaking to $SO(1,1) \times E_6$ representations with ground states created by tachyonic Lie allebra generators and carrying mass squared 2 in suitable units. In TGD framework the tachyonic conformal weight can be compensated by super-symplectic conformal weight so that massless states getting their masses via Higgs mechanism and p-adic thermodynamics would be obtained.

6.4.3 Could super-symmetry rescue the situation?

E_8 is unique among Lie algebras in that its adjoint rather than fundamental representation has the smallest dimension. One can decompose the 240 roots of E_8 to 112 roots for which two components of SO(7,1) root vector are $\pm$ 1 and to 128 vectors for which all components are $\pm$ 1/2 such that the sum of components is even. The latter roots Lisi assigns to fermionic states. This is not consistent with spin and statistics although SO(3,1) spin is half-integer in M^8 picture.

The first idea which comes in mind is that these states correspond to super-partners of the ordinary fermions. In TGD framework they might be obtained by just adding covariantly constant right-handed neutrino or antineutrino state to a given particle state. The simplest option is that fermionic super-partners are complex scalar fields and sbosons are spin 1/2 fermions. It however seems that the super-conformal symmetries associated with the right-handed neutrino are strictly local in the sense that global super-generators vanish. This would mean that super-conformal super-symmetries change the color and angular momentum quantum numbers of states. This is a pity if indeed true since super-symmetry could be broken by different p-adic mass scale for super partners so that no explicit breaking would be needed.

6.4.4 Could Kac Moody variant of E_8 make sense in TGD?

One can leave gauge theory framework and consider stringy picture and its generalization in TGD framework obtained by replacing string orbits with 3-D light-like surfaces allowing a generalization of conformal symmetries.

H-HO duality is one of the speculative aspects of TGD. The duality states that one can either regard imbedding space as $H = M^4 \times CP_2$ or as 8-D Minkowski space M^8 identifiable as the space HO of hyper-octonions which is a subspace of complexified octonions. Spontaneous compactification for M^8 described as a phenomenon occurring at the level of Kac-Moody algebra would relate HO-picture to H-picture which is definitely the fundamental picture. For instance, standard model symmetries have purely number theoretic meaning in the resulting picture.

The question is whether the non-compact E_8 could be replaced with the corresponding Kac Moody algebra and act as a stringy symmetry. Note that this would be by no means anything new. The Kac-Moody analogs of E_{10} and E_{11} algebras appear in M-theory speculations. Very little is known about these algebras. Already $E < sub > n < /sub >$, $n > 8$ is infinite-dimensional as an analog of Lie algebra. The following argument shows that E_8 representations do not work in TGD context unless one allows anyonic statistics.

1. In TGD framework space-time dimension is D=8. The speculative hypothesis of HO-H duality inspired by string model dualities states that the descriptions based on the two choices of imbedding space are dual. One can start from 8-D Cartan algebra defined by quantized M^8 coordinates regarded as fields at string orbit just as in string model. A natural constraint is that the symmetries act as isometries or holonomies of the effectively compactified M^8. The article "The Octonions" [22] of John Baez discusses exceptional Lie groups and shows that compact form of E_8 appears as isometry group of 16-dimensional octo-octonionic projective plane $E_8/(Spin(16)/Z_2)$: the analog of CP_2 for complexified octonions. There is no 8-D space allowing E_8 as an isometry group. Only SO(1,7) can be realized as the maximal Lorentz group with 8-D translational invariance.

2. In HO picture some Kac Moody algebra with rank 8 acting on quantized M^8 coordinates defining stringy fields is natural. The charged generators of this algebra are constructible using the standard recipe involving operators creating coherent states and their conjugates obtained as operator counterparts of plane waves with momenta replaced by roots of the simply laced algebra in question and by normal ordering.

3. Poincare group has 4-D maximal Cartan algebra and this means that only 4 Euclidian dimensions remain. Lorentz generators can be constructed in standard manner in terms of Kac-Moody generators as Noether currents.

4. The natural Kac-Moody counterpart for spontaneous compactification to CP_2 would be that these dimensions give rise to the generators of electro-weak gauge group identifiable as a product of isometry and holonomy groups of CP_2 in the dual H-picture based on $M^4 \times CP_2$. Note that in this picture electro-weak symmetries would act geometrically in E^4 whereas in CP_2 picture they would act only as holonomies.

Could one weaken the assumption that Kac-Moody generators act as symmetries and that spin-statistics relation would be satisfied?

1. The hierarchy of Planck constants relying on the generalization of the notion of imbedding space breaks Poincare symmetry to Lorentz symmetry for a given sector of the world of classical worlds for which one considers light-like 3-surfaces inside future and past directed light cones. Translational invariance is obtained from the wave function for the position of the tip of the light cone in M^4. In this kind of situation one could consider even E_8 symmetry as a dynamical symmetry.

2. The hierarchy of Planck constants involves a hierarchy of groups and fractional statistics at the partonic 2-surface with rotations interpreted as braiding homotopies. The fractionization of spin allows anyonic statistics and could allow bosons with anyonic half-odd integer spin. Also more general fractional spins are possible so that one can consider also more general algebras than Kac-Moody algebras by allowing roots to have more general values. Quantum versions of Kac-Moody algebras would be in question. This picture would be consistent with the view that TGD can emulate any gauge algebra with 8-D Cartan algebra and Kac-Moody algebra dynamically. This vision was originally inspired by the study of the inclusions of hyper-finite factors of type II¡sub¿1¡/sub¿. Even higher dimensional Kac-Moody algebras are predicted to be possible.

3. It must be emphasized that these considerations relate in TGD framework to Super-Kac Moody algebra only. The so called super-symplectic algebra is the second quintessential part of the story. In particular, color is not spin-like quantum number for quarks and quark color corresponds to color partial waves in the world of classical worlds or more concretely, to the rotational degrees of freedom in CP_2 analogous to ordinary rotational degrees of freedom of rigid body. Arbitrarily high color partial waves are possible and also leptons can move in triality zero color partial waves and there is a considerable experimental evidence for color octet excitations of electron and muon but put under the rug.

6.4.5 Can one interpret three fermion families in terms of E_8 in TGD framework?

The prediction of three fermion generations by E_8 picture must be taken very seriously. In TGD three fermion generations correspond to three lowest genera $g = 0, 1, 2$ (handle number) for which all 2-surfaces have Z_2 as global conformal symmetry (hyper-ellipticity [21, 19]). One can assign to the three genera a dynamical SU(3) symmetry. Theye are related by SU(3) triality which brings in mind the triality symmetry acting on fermion generations in E_8 model. SU(3) octet and singlet bosons correspond to pairs of light-like 3-surfaces defining the throats of a wormhole contact and since their genera can be different one has color singlet and octet bosons. Singlet corresponds to ordinary bosons. Color octet bosons must be heavy since they define neutral currents between fermion families.

The three E_8 anyonic boson families cannot represent family replication since these symmetries are not local conformal symmetries: it obviously does not make sense to assign a handle number to a given point of partonic 2-surface! Also bosonic octet would be missing in E_8 picture.

One could of course say that in E_8 picture based on fractional statistics, anyonic gauge bosons can mimic the dynamical symmetry associated with the family replication. This is in spirit with the idea that TGD Universe is able to emulate practically any gauge - or Kac-Moody symmetry and that TGD Universe is busily mimicking also itself.

To sum up, the rank 8 Kac-Moody algebra - emerging naturally if one takes HO-H duality seriously - corresponds very naturally to Kac-Moody representations in terms of free stringy fields for Poincare-, color-, and electro-weak symmetries. One can however consider the possibility of anyonic symmetries and the emergence of non-compact version of E_8 as a dynamical symmetry, and TGD suggests much more general dynamical symmetries if TGD Universe is able to act as the physics analog of the Universal Turing machine.

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Chapter 1

Appendix

A-1 Basic properties of CP_2 and elementary facts about p-adic numbers

A-1.1 CP_2 as a manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-1.1}$$

Here λ is any non-zero complex number. Note that CP_2 can be also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three oordinate charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1, 2$ the coordinates of Eguchi and Freund [45] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it \ , \\ \xi^2 &= x + iy \ . \end{aligned} \tag{A-1.2}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi + \Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi - \Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \tag{A-1.3}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connected, with Euler number Euler number 3, Pontryagin number 3 and second $b = 1$.

A-1.2 Metric and Kähler structure of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \rightarrow exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i \bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5.

The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \tag{A-1.4}$$

where the Hermitian, in fact Kähler metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2\partial_a\partial_{\bar{b}}K \ , \tag{A-1.5}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= log(F) \ , \\ F &= 1+r^2 \ . \end{aligned} \tag{A-1.6}$$

The Kähler function for S^2 has the same form. It gives the S^2 metric $dzd\bar{z}/(1+r^2)^2$ related to its standard form in spherical coordinates by the coordinate transformation $(r,\phi) = (tan(\theta/2),\phi)$.

The representation of the CP_2 metric is deducible from S^5 metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2+r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2+\sigma_2^2)}{F} \ , \tag{A-1.7}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-1.8}$$

R denotes the radius of the geodesic circle of CP_2. The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2\sum_A e_k^A e_l^A \ , \tag{A-1.9}$$

are given by

$$\begin{aligned} e^0 &= \tfrac{dr}{F} \ , \quad & e^1 &= \tfrac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \tfrac{r\sigma_2}{\sqrt{F}} \ , \quad & e^3 &= \tfrac{r\sigma_3}{F} \ . \end{aligned} \tag{A-1.10}$$

The explicit representations of vierbein vectors are given by

$$\begin{aligned} e^0 &= \tfrac{dr}{F} \ , & e^1 &= \tfrac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\ e^2 &= \tfrac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} \ , & e^3 &= \tfrac{r(d\Psi + cos\Theta d\Phi)}{2F} \ . \end{aligned} \tag{A-1.11}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \tag{A-1.12}$$

The vierbein connection satisfying the defining relation

$$de^A = -V^A_B \wedge e^B \ , \tag{A-1.13}$$

is given by

$$\begin{array}{lllllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r-\frac{1}{r})e^3 \ , & V_{12} & = & (2r+\frac{1}{r})e^3 \ . \end{array} \tag{A-1.14}$$

The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{lllllll} R_{01} & = & e^0\wedge e^1 - e^2 \wedge e^3 \ , & R_{23} & = & e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} & = & e^0\wedge e^2 - e^3 \wedge e^1 \ , & R_{31} & = & -e^0 \wedge e^2 + e^3 \wedge e^1 \ , \\ R_{03} & = & 4e^0\wedge e^3 + 2e^1 \wedge e^2 \ , & R_{12} & = & 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{array} \tag{A-1.15}$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$J = -ig_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \tag{A-1.16}$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$J^k_{\ r}J^{rl} = -s^{kl} \ . \tag{A-1.17}$$

The form J is integer valued and by its covariant constancy satisfies free Maxwell equations. Hence it can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling). Locally one has therefore

$$J = dB \ , \tag{A-1.18}$$

where B is the so called Kähler potential, which is not defined globally since J describes homological magnetic monopole.

It should be noticed that the magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$\begin{aligned} B &= 2re^3 \ , \\ J &= 2(e^0\wedge e^3 + e^1 \wedge e^2) = \frac{r}{F^2}dr\wedge(d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F}sin\Theta d\Theta d\Phi \ . \end{aligned} \tag{A-1.19}$$

The vierbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1,1).

Useful coordinates for CP_2 are the so called canonical coordinates in which Kähler potential and Kähler form have very simple expressions

$$\begin{aligned} B &= \sum_{k=1,2} P_k dQ_k \ , \\ J &= \sum_{k=1,2} dP_k \wedge dQ_k \ . \end{aligned} \tag{A-1.20}$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$
\begin{aligned}
P_1 &= -\frac{1}{1+r^2} \ , \\
P_2 &= \frac{r^2 cos\Theta}{2(1+r^2)} \ , \\
Q_1 &= \Psi \ , \\
Q_2 &= \Phi \ .
\end{aligned} \tag{A-1.21}
$$

A-1.3 Spinors in CP_2

CP_2 doesn't allow spinor structure in the conventional sense [17]. However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0,1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallel propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the non-allowed -1- factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential $exp(i2\Phi)$, where Φ is the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

A-1.4 Geodesic sub-manifolds of CP_2

Geodesic sub-manifolds are defined as sub-manifolds having common geodesic lines with the imbedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [16] a general characterization of the geodesic sub-manifolds for an arbitrary symmetric space G/H is given. Geodesic sub-manifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$
[X,[Y,Z]] \in t \ \text{ for } \ X,Y,Z \in t \ . \tag{A-1.22}
$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic sub-manifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S_I^2 \ : \ \xi^1 = \bar{\xi}^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S_{II}^2 \ : \ \xi^1 = \xi^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Phi = 0) \ .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S_I^2. S_{II}^2 is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-2 CP_2 geometry and standard model symmetries

A-2.1 Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [21] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e=\pm 1$, CP_2-chirality l,r and M^4-chirality L,R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{A-2.1}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5\times\gamma_5$, $1\times\gamma_5$ and $\gamma_5\times 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e=\pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4) = SU(2)_L\times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+1_+ + n_-1_-) \ . \tag{A-2.2}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{aligned} V_{01} &= -\frac{e^1}{r} \ , & V_{23} &= \frac{e^1}{r} \ , \\ V_{02} &= -\frac{e^2}{r} \ , & V_{31} &= \frac{e^2}{r} \ , \\ V_{03} &= (r-\frac{1}{r})e^3 \ , & V_{12} &= (2r+\frac{1}{r})e^3 \ , \end{aligned} \tag{A-2.3}$$

and

$$B = 2re^3 \ , \tag{A-2.4}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \tag{A-2.5}$$

where one have defined

$$\begin{aligned} I_L^1 &= \frac{(\Sigma_{01}-\Sigma_{23})}{2} \ , \\ I_L^2 &= \frac{(\Sigma_{02}-\Sigma_{13})}{2} \ . \end{aligned} \tag{A-2.6}$$

A_{ch} is clearly left handed so that one can perform the identification

$$W^{\pm} = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{A-2.7}$$

where $W^{\pm}$ denotes the charged intermediate vector boson.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$\begin{aligned} X &= re^3 \ , \\ Y &= \frac{e^3}{r} \ , \end{aligned} \tag{A-2.8}$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$\begin{aligned} \bar{\gamma} &= aX + bY \ , \\ \bar{Z}^0 &= cX + dY \ , \end{aligned} \tag{A-2.9}$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$\begin{aligned} A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\ &+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ . \end{aligned} \tag{A-2.10}$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5\Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \tag{A-2.11}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-2.12}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$\begin{aligned} Q_{em} &= \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} \ , \\ I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ . \end{aligned} \tag{A-2.13}$$

The fields γ and Z^0 are defined via the relations

$$\begin{aligned} \gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\ Z^0 &= 4(a+b)\bar{Z}^0 = 4(X - Y) \ . \end{aligned} \tag{A-2.14}$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \tag{A-2.15}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of Weinberg angle is a dynamical problem. The angle is completely fixed once the YM action is fixed by requiring that action contains no cross term of type γZ^0. Pure symmetry non-broken electro-weak YM action leads to a definite value for the Weinberg angle. One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+1_+ + n_-1_-) \ , \tag{A-2.16}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \end{aligned} \tag{A-2.17}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-2.18}$$

Evaluating the expressions above one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{A-2.19}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \tag{A-2.20}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} \ , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) \ , \end{aligned} \tag{A-2.21}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 cross term (this coefficient must vanish) the expression

$$\begin{aligned} X &= -\frac{K}{2g^2} + \frac{fp}{18} \ , \\ K &= Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] \ , \end{aligned} \tag{A-2.22}$$

In the general case the value of the coefficient K is given by

$$K = \sum_i \left[-\frac{(18+2n_i^2)sin^2\theta_W}{9}\right] \ , \tag{A-2.23}$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9\sum_i 1}{(fg^2 + 2\sum_i(18+n_i^2))} \ . \tag{A-2.24}$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9}{(\frac{fg^2}{2} + 28)} \ . \tag{A-2.25}$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is quite close to the typical value 9/24 of GUTs [19].

A-2.2 Discrete symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:
a) Symmetries must be realized as purely geometric transformations.
b) Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [22].

The action of the reflection P on spinors of is given by

$$\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0\Psi \ . \tag{A-2.26}$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) \ , \\ \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ . \end{aligned} \tag{A-2.27}$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \Psi^\dagger\gamma^2\gamma^0\otimes 1 \ . \end{aligned} \qquad \text{(A-2.28)}$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-3 Basic facts about induced gauge fields

Since the classical gauge fields are closely related in TGD framework, it is not possible to have space-time sheets carrying only single kind of gauge field. For instance, em fields are accompanied by Z^0 fields for extremals of Kähler action. Weak forces is however absent unless the space-time sheets contains topologically condensed exotic weakly charged particles responding to this force. Same applies to classical color forces. The fact that these long range fields are present forces to assume that there exists a hierarchy of scaled up variants of standard model physics identifiable in terms of dark matter.

Classical em fields are always accompanied by Z^0 field and some components of color gauge field. For extremals having homologically non-trivial sphere as a CP_2 projection em and Z^0 fields are the only non-vanishing electroweak gauge fields. For homologically trivial sphere only W fields are non-vanishing. Color rotations does not affect the situation.

For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy by 2-dimensionality of the CP_2 projection. Color gauge field has $U(1)$ holonomy for all space-time surfaces and quantum classical correspondence suggest a weak form of color confinement meaning that physical states correspond to color neutral members of color multiplets.

A-3.1 Induced gauge fields for space-times for which CP_2 projection is a geodesic sphere

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP_2 projection, only vacuum extremals and space-time surfaces for which CP_2 projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. This can be verified by explicit examples.

$r = \infty$ surface gives rise to a homologically non-trivial geodesic sphere for which e_0 and e_3 vanish imply the vanishing of W field. For space-time sheets for which CP_2 projection is $r = \infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \ .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

$Im(\xi^1) = Im(\xi^2) = 0$ corresponds to homologically trivial geodesic sphere. A more general representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case e^1 and e^3 vanish so that the induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP_2 projection color rotations and weak symmetries commute.

A-3.2 Space-time surfaces with vanishing em, Z^0, or Kähler fields

In the following the induced gauge fields are studied for general space-time surface without assuming the extremal property. In fact, extremal property reduces the study to the study of vacuum extremals and surfaces having geodesic sphere as a CP_2 projection and in this sense the following arguments are somewhat obsolete in their generality.

Space-times with vanishing em, Z^0, or Kähler fields

The following considerations apply to a more general situation in which the homologically trivial geodesic sphere and extremal property are not assumed. It must be emphasized that this case is possible in TGD framework only for a vanishing Kähler field.

Using spherical coordinates (r, Θ, Ψ, Φ) for CP_2, the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta)d\Phi) + \frac{r^2}{2F} sin(\Theta)d\Theta \wedge d\Phi \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-3.1}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3+2p)\frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta)d\Phi) + (3+p)\frac{r^2}{2F} sin(\Theta)d\Theta \wedge d\Phi \ , \\ p &= sin^2(\Theta_W) \ , \end{aligned} \tag{A-3.2}$$

where Θ_W denotes Weinberg angle.

a) The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi \ , \\ (3+2p)\frac{1}{r^2 F}(d(r^2)/d\Theta)(k + cos(\Theta)) &+ (3+p)sin(\Theta) = 0 \ , \end{aligned} \tag{A-3.3}$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D\left[|\frac{(k+u}{C}| \right]^{\epsilon} \ , \\ u &\equiv cos(\Theta) \ , \ C = k + cos(\Theta_0) \ , \ D = \frac{r_0^2}{1+r_0^2} \ , \ \epsilon = \frac{3+p}{3+2p} \ , \end{aligned} \tag{A-3.4}$$

where C and D are integration constants. $0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u + k| = [(1 + r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \leq k+1 \ ,$$

where $sign(x)$ denotes the sign of x.

The expressions for Kähler form and Z^0 field are given by

$$\begin{aligned} J &= -\frac{p}{3+2p} X du \wedge d\Phi \ , \\ Z^0 &= -\frac{6}{p} J \ . \end{aligned} \tag{A-3.5}$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

b) The vanishing of Z^0 fields is achieved by the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically. Also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F} du \wedge d\Phi$ is useful.

c) The vanishing Kähler field corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$
\begin{aligned}
Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u} du \wedge d\Phi = (k+u) du \wedge d\Phi \ , \\
r &= \sqrt{\frac{X}{1-X}} \ , \ X = D|k+u| \ , \\
\gamma &= -\frac{p}{2} Z^0 \ .
\end{aligned}
\tag{A-3.6}
$$

For a vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

The effective form of CP_2 metric for surfaces with 2-dimensional CP_2 projection

The effective form of the CP_2 metric for a space-time having vanishing em,Z^0, or Kähler field is of practical value in the case of vacuum extremals and is given by

$$
\begin{aligned}
ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta} d\Theta^2 + s^{eff}_{\Phi\Phi} d\Phi^2] \ , \\
s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\
s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ ,
\end{aligned}
\tag{A-3.7}
$$

and is useful in the construction of vacuum imbedding of, say Schwartchild metric.

Topological quantum numbers

Space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$
\begin{aligned}
\Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\
\Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ .
\end{aligned}
\tag{A-3.8}
$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which space-time surface becomes ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global imbedding for, say a constant magnetic field. Although global imbedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteing the absence of edges are satisfied.

For instance, the vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \tag{A-3.9}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

A-4 p-Adic numbers and TGD

A-4.1 p-Adic number fields

p-Adic numbers (p is prime: 2,3,5,...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [18]. p-Adic numbers are representable as power expansion of the prime number p of form:

$$x = \sum_{k \geq k_0} x(k)p^k, \ x(k) = 0, ..., p-1 \ . \tag{A-4.1}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \ . \tag{A-4.2}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \ , \tag{A-4.3}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x, y) = |x - y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x, z) \leq max\{d(x, y), d(y, z)\} \ . \tag{A-4.4}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x, y) \leq D \ . \tag{A-4.5}$$

This division of the metric space into classes has following properties:

a) Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.

b) Distances of points x and y inside single class are smaller than distances between different classes.

c) Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [20]. The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

A-4.2 Canonical correspondence between p-adic and real numbers

The basic challenge encountered by p-adic physicist is how to map the predictions of the p-adic physics to real numbers. p-Adic probabilities provide a basic example in this respect. Identification via common rationals and canonical identification and its variants have turned out to play a key role in this respect.

Basic form of canonical identification

There exists a natural continuous map $I : R_p \to R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \to x = \sum_{k<N} y_k p^{-k} \quad , \\ y_k &\in \{0, 1, .., p-1\} \quad . \end{aligned} \tag{A-4.6}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique $(1 = 0.999...)$ for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \quad , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \quad . \end{aligned} \tag{A-4.7}$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^{N} x_k p^k \quad , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^k + (x_N - 1)p^N + (p-1)p^{N+1} \sum_{k=0,..} p^k \\ &= y_1 + (x_N - 1)p^N - p^{N+1} \quad , \end{aligned} \tag{A-4.8}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

The topology induced by canonical identification

The topology induced by the canonical identification in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see Fig. A-4.2) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

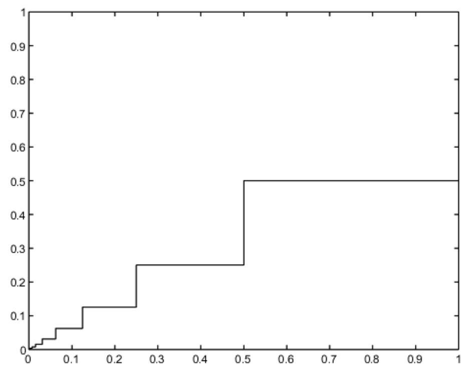

Figure 1: The real norm induced by canonical identification from 2-adic norm.

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common pinary digits. Furthermore, the condition $x +_p y < max\{x, y\}$ holds in general for the p-adic sum of the real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of p. Moreover one has $x \times_p y < x \times y$ in general. The p-Adic negative -1_p associated with p-adic unit 1 is given by $(-1)_p = \sum_k (p-1)p^k$ and defines p-adic negative for each real number x. An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

These results suggest that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |x|_p |y|_R \leq (xy)_R &\leq x_R y_R \ , \end{aligned} \tag{A-4.9}$$

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to the case of $(R_p)^n$ (a linear vector space over the p-adic numbers).

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |\lambda|_p |y|_R \leq (\lambda y)_R &\leq \lambda_R y_R \ , \end{aligned} \tag{A-4.10}$$

where the norm of the vector $x \in T_p^n$ is defined in some manner. The case of Euclidian space suggests the definition

$$(x_R)^2 = (\sum_n x_n^2)_R \ . \tag{A-4.11}$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of p.

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some non-linear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

Modified form of the canonical identification

The original form of the canonical identification is continuous but does not respect symmetries even approximately. This led to a search of variants which would do better in this respect. The modification of the canonical identification applying to rationals only and given by

$$I_Q(q = p^k \times \frac{r}{s}) = p^k \times \frac{I(r)}{I(s)} \tag{A-4.12}$$

is uniquely defined for rationals, maps rationals to rationals, has also a symmetry under exchange of target and domain. This map reduces to a direct identification of rationals for $0 \leq r < p$ and $0 \leq s < p$. It has turned out that it is this map which most naturally appears in the applications. The map is obviously continuous locally since p-adically small modifications of r and s mean small modifications of the real counterparts.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. The predictions for the light elementary particle masses are within extreme accuracy same for I and I_Q but I_Q is theoretically preferred since the real probabilities obtained from p-adic ones by I_Q sum up to one in p-adic thermodynamics.

Generalization of number concept and notion of imbedding space

TGD forces an extension of number concept: roughly a fusion of reals and various p-adic number fields along common rationals is in question. This induces a similar fusion of real and p-adic imbedding spaces. Since finite p-adic numbers correspond always to non-negative reals n-dimensional space R^n must be covered by 2^n copies of the p-adic variant R_p^n of R^n each of which projects to a copy of R_+^n (four quadrants in the case of plane). The common points of p-adic and real imbedding spaces are rational points and most p-adic points are at real infinity.

For a given p-adic space-time sheet most points are literally infinite as real points and the projection to the real imbedding space consists of a discrete set of rational points: the interpretation in terms of the unavoidable discreteness of the physical representations of cognition is natural. Purely local p-adic physics implies real p-adic fractality and thus long range correlations for the real space-time surfaces having enough common points with this projection.

p-Adic fractality means that M^4 projections for the rational points of space-time surface X^4 are related by a direct identification whereas CP_2 coordinates of X^4 at these points are related by I, I_Q or some of its variants implying long range correlates for CP_2 coordinates. Since only a discrete set of points are related in this manner, both real and p-adic field equations can be satisfied and there are no problems with symmetries. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs. Also multi-p-fractality is possible.

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