

The Math of Hua Luogeng – 华罗庚 – Another Game in Town

by Frank D. (Tony) Smith, Jr. - June 2009

Edward Witten said (*Nature* 438 (22/29 December 2005) 1085):

“... Albert Einstein famously devoted the latter part of his life to seeking a ... ‘unified field theory’ ... String theory is the only known generalization of relativistic quantum field theory that makes sense. ... string theory ... may well be the only way to reconcile gravity and quantum mechanics ...”.

Steven Weinberg (in a 22 Feb 2009 *Telegraph* article by Graham Farmelo) is quoted as saying that “... for a truly fundamental theory of physics ... “strings are the only game in town” ...”.

Could Witten and Weinberg both be Wrong ?

Setting up Another Game in Town requires solving Three Tasks:

- 1 - Construct an EPR Physics Model that is consistent with Experimental Tests of Einstein-Podolsky-Rosen Reality;
- 2 - Use that EPR Physics Model to construct a Local Lagrangian that gives Gravity and the Standard Model with calculable Force Strengths and Particle Masses – such calculations will be seen to require the Math of Hua Luogeng;
- 3 - Quantize the Classical Local Lagrangian structure to get a Global Algebraic Quantum Field Theory (AQFT) which, as Bert Schroer said in hep-th/9608083, has “... emphasis on locality and ... insistence of separating local ... properties ... resid[ing] in ... the algebraic structure of local observables ... from global properties ... enter[ing] through ... states and ... representation spaces of local observables.

1 - An EPR Physics Model

Joy Christian in arXiv 0904.4259 “Disproofs of Bell, GHZ, and Hardy Type Theorems and the Illusion of Entanglement” says: “... a [geometrically] correct local-realistic framework ... provides exact, deterministic, and local underpinnings for at least the Bell, GHZ-3, GHZ-4, and Hardy states. ... The alleged non-localities of these states ... result from misidentified [geometries] of the EPR elements of reality. ...

The correlations are ... the classical correlations among the points of a 3 or 7-sphere ... S^3 and S^7 ... are ... parallelizable ...

The correlations ... can be seen most transparently in the elegant language of Clifford algebra ...”.

To go beyond the interesting but not completely physically realistic Bell, GHZ-3, GHZ-4, and Hardy states, we must consider more complicated spaces than S^3 and S^7 , but still require that they be parallelizable and be related to Clifford algebra structure.

As Martin Cederwall said in hep-th/9310115: “... The only simply connected compact parallelizable manifolds are the Lie groups [including $S^3 = SU(2)$] and S^7 ...”.

We know that $S^3 = SU(2) = Spin(4) / SU(2)$ so that it has global symmetry of $Spin(4)$ transformations and that 6-dimensional $Spin(4)$ is the grade-2 part of the 16-dimensional $Cl(4)$ Clifford algebra with graded structure $16 = 1 + 4 + 6 + 4 + 1$ (where grades are 0,1,2, ...).

We also know that $S^7 = Spin(8) / Spin(7)$ so that it has global symmetry of $Spin(8)$ transformations and that 28-dimensional $Spin(8)$ is the grade-2 part of the 256-dimensional $Cl(8)$ Clifford algebra with graded structure $256 = 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1$.

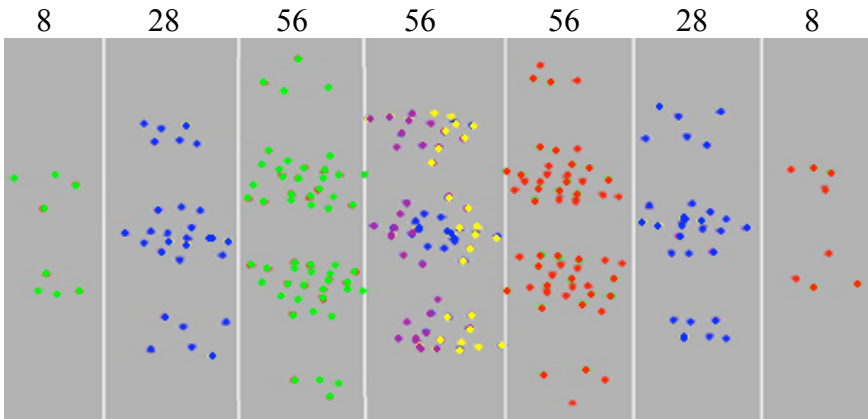
To get a Clifford algebra related parallelizable Lie group large enough to represent a realistic physics model, take the tensor product $Cl(8) \otimes Cl(8)$ which by the 8-periodicity property of Real Clifford algebras is $256 \times 256 = 65,536$ -dimensional $Cl(16)$ with graded structure $(1 \times 1) + (1 \times 8 + 8 \times 1) + (1 \times 28 + 28 \times 1 + 8 \times 8) + \dots = 1 + 16 + 120 + \dots$ whose $28 + 28 + 64 = 120$ -dimensional grade-2 part is $Spin(16)$ and whose spinor representation has $256 = 128 + 128$ dimensions.

$Spin(16)$ has $Cl(16)$ Clifford algebra structure and is a Lie group, and therefore parallelizable, but it has grade-2 bivector bosonic structure and so can only represent physical things like gauge bosons and vector spacetime, and cannot represent physical things like fermions with spinor structure.

However, if we add one of the two 128-dimensional $Cl(16)$ half-spinor representations to the bosonic adjoint 120-dimensional representation of $Spin(16)$, we get the $120 + 128 = 248$ -dimensional exceptional Lie group E_8 which Garrett Lisi in arXiv 0711.0770 used to try to describe a complete realistic physics model unifying Gravity and the Standard Model. Garrett Lisi acknowledges some flaws in his model of 0711.0770, but here we will use a variant of his approach that unifies Gravity and the Standard Model without such flaws.

248-dimensional E_8 has a 7-grading (due to Thomas Larsson)
 $8 + 28 + 56 + 64 + 56 + 28 + 8$
 (where grades are -3,-2,-1,0,1,2,3)

If 8 of the 64 central grade-0 elements are assigned to an 8-dimensional Cartan subalgebra of E_8 , the remaining $248 - 8 = 240$ elements are the 240 Root Vectors of E_8 which have a graded structure



that indicates a physical interpretation for each of them:

The 128 odd-grade Root Vectors of E8 come from the 128 half-spinors and are shown as

$$0 + 0 + 0 + 0 + 56 + 0 + 8 = 64 \text{ red}$$

The 8 grade +3 represent the basic state of 8 fermion particles.

$$8 + 0 + 56 + 0 + 0 + 0 + 0 = 64 \text{ green}$$

The 8 grade -3 represent the basic state of 8 fermion antiparticles.

The $120 - 8 = 112$ even-grade Root Vectors of E8 come from the 112 Root Vectors of Spin(16) and are shown as

$$0 + 28 + 0 + 8 + 0 + 28 + 0 = 64 \text{ blue}$$

The 8 grade 0 represent 8 basic spacetime dimensions.

$$0 + 0 + 0 + 24 + 0 + 0 + 0 = 24 \text{ purple}$$

The 24 represent the Root Vectors of a gauge group Spin(8).

$$0 + 0 + 0 + 24 + 0 + 0 + 0 = 24 \text{ gold}$$

The 24 represent the Root Vectors of another gauge group Spin(8).

2 - A Local Lagrangian that gives General Relativity and the Standard Model with Calculable Force Strengths and Particle Masses

Use the E8 physical interpretations to construct a Lagrangian
by
integration over 8-dim spacetime base manifold
of
curvature terms from the two Spin(8) gauge groups
and
a Dirac fermion particle-antiparticle term.

This differs from conventional Gravity plus Standard Model in four respects:

- 1 - 8-dimensional spacetime
- 2 - two Spin(8) gauge groups
- 3 - no Higgs
- 4 - 1 generation of fermions

These differences can be reconciled as follows:

Introduce (freezing out at lower-than-Planck energies) a preferred Quaternionic 4-dim subspace of the original (high-energy) 8-dim spacetime,
thus forming an 8-dim Kaluza-Klein spacetime $M_4 \times CP^2$
where M_4 is 4-dim physical spacetime and CP^2 is a 4-dim internal symmetry space.

Let the first Spin(8) gauge group act on the M_4 physical spacetime through the SU(3) subgroup of its U(4) subgroup.

Meinhard E. Mayer said (*Hadronic Journal* 4 (1981) 108-152):

“... each point of ... the ... fibre bundle ... E consists of a four-dimensional spacetime point x [in M_4] to which is attached

the homogeneous space $G / H [SU(3) / U(2) = CP^2] \dots$
the components of the curvature lying in the homogeneous space
 $G / H [= SU(3) / U(2)]$ could be reinterpreted as Higgs scalars
(with respect to spacetime $[M^4]$)

...

the Yang-Mills action reduces to
a Yang-Mills action for the h-components $[u(2) \text{ components }]$ of
the curvature over $M [M^4]$

and

a quartic functional for the “Higgs scalars”, which not only
reproduces the Ginzburg-Landau potential, but also gives the
correct relative sign of the constants, required for the BEHK ...
Brout-Englert-Higgs-Kibble ... mechanism to work. ...”.

So, freezing out of a Kaluza-Klein $M^4 \times CP^2$ spacetime plus internal
symmetry space produces a classical Lagrangian for the
 $SU(3) \times U(2) = SU(3) \times SU(2) \times U(1)$ Standard Model
Including a BEHK Higgs mechanism.

Let the second Spin(8) gauge group act on the M^4 physical
spacetime through its Conformal Subgroup $U(2,2) = Spin(2,4)$.

Rabindra Mohapatra said (section 14.6 of Unification and Supersymmetry, 2
nd edition, Springer-Verlag 1992):

“... gravitational theory can emerge from the gauging of conformal
symmetry ... we start with a Lagrangian invariant under full local
conformal symmetry and fix conformal and scale gauge to obtain
the usual action for gravity. ...”.

See also MacDowell and Mansouri (Phys. Rev. Lett. 38 (1977) 739) and
Chamseddine and West (Nucl. Phys. B 129 (1977) 39).

At this stage, we have reconciled the first 3 of the 4 differences
between our E8 Physics Model and conventional Gravity plus the
Standard Model. As to the fourth, the existence of 3 generations of

fermions, note that the 8 first generation fermion particles and the 8 first generation antiparticles can each be represented by the 8 basis elements of the Octonions O ,

and that the second and third generations can be represented by Pairs of Octonions OxO and Triples of Octonions $OxOxO$, respectively.

When the unitary Octonionic 8-dim spacetime is reduced to the Kaluza-Klein $M4 \times CP2$, there are 3 possibilities for a fermion propagator from point A to point B:

- 1 – A and B are both in $M4$, so its path can be represented by the single O ;
- 2 – Either A or B, but not both, is in $CP2$, so its path must be augmented by one projection from $CP2$ to $M4$, which projection can be represented by a second O , giving a second generation OxO ;
- 3 – Both A and B are in $CP2$, so its path must be augmented by two projections from $CP2$ to $M4$, which projections can be represented by a second O and a third O , giving a third generation $OxOxO$.

Therefore, all four differences have been reconciled, and our classical Lagrangian $E8$ Physics Model describes Gravity as well as the Standard Model with a BEHK Higgs mechanism.

However, for our classical Lagrangian $E8$ Physics Model to be said to be complete and realistic, it must allow us to calculate such things as Force Strengths and Particle Masses that are consistent with experimental and observational results.

This requires the use of the Math of Hua Luogeng, about whom Wang Yuan ("Hua Loo-Keng", translated by Peter Shiu, Springer 1999) said: "... A mathematician has to be judged by his research accomplishments and not by the number of university degrees earned. In Hua's case there are many of the former and none of the latter ... Teacher Hua only had a junior middle-school education ...

Hua visited the Soviet Union ...[in]... 1946 ... [Later]... in ... 1946 ... Hua Loo-Keng went to the Institute for Advanced Study at Princeton ... In the spring of 1948, Hua Loo-Keng was appointed full professor at the University of Illinois in Urbana. ... Hua's decision to return to China ... in 1950 ... was based on his belief ... that the Chinese Communist Party and the Chinese Government ... would ... support ... his wish ... for ... mathematics in China ... to arrive at international level ... Besides this, he saw the racial prejudice in the United States ...[and]... isolationist policy being implemented against the Chinese Communist Party and all its work ...

In 1935, E. Cartan proved that, under analytic mapping, there are precisely six types of irreducible homogeneous symmetric bounded domains. Of these there are two types of exceptional domains with dimensions 16 and ...[27]... and the remaining four types are called classical domains ... $R_I = \dots m \times n$ matrix ... $R_{II} = \dots$ symmetric matrices ... $R_{III} = \dots$ skew-symmetric matrix ... $R_{IV} [= \text{a homogeneous space of } 2 \times n \text{ real matrices }] \dots$ Classical domains can thus be regarded as the generalisation of the unit disc in the plane to higher dimensions [They are the subject of Hua's book]... Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains [Institute of Mathematics, Academia Sinica, No. 4 in the A-series, Russian translation 1959, English translation 1963] ... Now let R be a bounded and simply connected domain whose points ...[are]... made up of n complex variables, so that the corresponding Euclidean space has real dimension $2n$. Suppose that L is a part of the boundary for R satisfying the following condition: Every analytic function in R takes its maximum modulus on L and that, for each point x in L , there is an analytic function $f(z)$ in R which takes its maximum modulus at x . We then call L the characteristic manifold of R [also known as the Shilov Boundary of R], and it is a uniquely determined compact manifold. Generally speaking, L is only a part of the boundary for R ... The following three kernels [and volumes of related geometric structures] can be computed: ... Bergman kernel ... Cauchy kernel ... Poisson kernel ...

Hua also generalized the notion of a bilinear (fractional linear) transformation in one complex variable to that for several complex variables ...

Hua ... seemed to have been the only person working on the subject ... but the work ... influenced ... the work on the theory of functions of several complex variables by the Russian mathematician I. I. Pyateskij-Shapiro ...”.

Hua’s calculated volumes related to the kernels and Shilov boundaries are the key to calculation of Force Strengths and Particle Masses. For example, the Lagrangian term for each of the Forces is integrated over the M4 physical spacetime base manifold, but each of the Four Forces sees M4 in terms of its own symmetry, consequently with its own measure which measure is proportional to Hua-calculated volumes. Since M4 was formed by a freezing out of a Quaternionic structure, M4 is a 4-dimensional manifold with Quaternionic structure and therefore can be seen as one of Joseph Wolf’s 4 equivalence classes:

for Electromagnetism: $T4 = U(1)^4$

for Weak Force: $S2 \times S2 = SU(2) / U(1) \times SU(2) / U(1)$

for Color Force: $CP2 = SU(3) / U(2)$

for Gravity: $S4 = Spin(5) / Spin(4) = Sp(2) / Sp(1) \times Sp(1)$

When we also take into account the relevant volumes related to the curvature term in the Lagrangian for each force, and the masses involved for forces with gauge bosons related to mass, the calculations produce results that are reasonably close to experimental observation. Full calculations of Force Strengths, the Dark Energy : Dark Matter : Ordinary Matter Ratio, Particle Masses, and Kobayashi-Maskawa Parameters, and discussion of details oversimplified here (such as signature and particle/spacetime polarizations beyond the basic level, etc.) are in my free pdf book “E8 and $Cl(16) = Cl(8) \times Cl(8)$ ” which is available at <http://www.tony5m17h.net/E8physicsbook.pdf> and <http://www.valdostamuseum.org/hamsmith/E8physicsbook.pdf> and <http://vixra.org/abs/0907.0006>

3 - A Global Algebraic Quantum Field Theory (AQFT)

Since the E8 classical Lagrangian is Local, it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing E8 Global Time.

Mathematically, this is done by embedding E8 into Cl(16) and using a copy of Cl(16) to represent each Local Lagrangian Region.

A Global Structure is then formed by taking the tensor products of the copies of Cl(16). Due to Real Clifford Algebra 8-periodicity, $Cl(16) = Cl(8) \times Cl(8)$ and any Real Clifford Algebra, no matter how large, can be embedded in a tensor product of factors of Cl(8), and therefore of $Cl(8) \times Cl(8) = Cl(16)$.

Just as the completion of the union of all tensor products of 2×2 complex Clifford algebra matrices produces the usual Hyperfinite III von Neumann factor that describes creation and annihilation operators on the fermionic Fock space over $C^{(2n)}$ (see John Baez's Week 175),

we can take the completion of the union of all tensor products of $Cl(16) = Cl(8) \times Cl(8)$ to produce a generalized Hyperfinite III von Neumann factor that gives a natural Algebraic Quantum Field Theory structure for our E8 Physics model,

thus

making it a complete realistic theory that satisfies Einstein's criteria (quoted by Wilczek in the winter 2002 issue of *Deadalus*) :

“... a theorem which at present can not be based upon anything more than upon a faith in the simplicity, i.e., intelligibility, of nature: there are no arbitrary constants ...

that is to say,

nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally completely determined constants occur ...”.

EPR (Christian) => E8 (Lisi) + Cl(16) (Lull)

(Mayer) => M4xCP2 (Batakis) + Higgs + Standard Model

(MacDowell-Mansouri) => Conformal Gravity

(Segal) => Dark Energy : DM : OM

(HUA) => Force Strengths + Particle Masses = EXPERIMENTAL TESTS

Feynman:

The whole purpose of physics is to find a number, with decimal points, etc!

Otherwise you haven't done anything.

(Segal-Connes) => Clifford Real-Periodic HyperFinite Factor AQFT