

Study on 4D/TSC Condensation Motion by Non-Linear Langevin Equation**

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[Abstract] Tetrahedral symmetric condensate (TSC) with 4 deuterons and 4 electrons has been proposed as a seed of clean 4D fusion with ⁴He product in condensed matter. To solve molecular dynamics motion of 4D/TSC condensation, a nonlinear Langevin equation was formulated with a Coulombic main condensation force term under Platonic symmetry, 6 balancing forces by quantum mechanical electron clouds of dde*(2,2) EQPET molecules on 6 faces of TSC cube and a random quantum mechanical fluctuation term $f(t)$ for d-d distance. Gaussian wave functions for d-d pairs and their ground state energies were first obtained by variational method, for D₂ and EQPET molecules. Then same sigma-value was used for time-dependent Gaussian wave functions of d-d pairs of TSC system to calculate the ensemble-averaged $\langle f(t) \rangle$ for changes of Coulomb energy and force of distorted TSC system deviated from the ideal double Platonic symmetry. Molecular dynamics calculation with TSC Langevin equation by the Verlet time-step method was then done. We obtained mean relative final-stage d-d kinetic energy 13.68 keV with -130.4 keV deep trapping TSC potential at R_{dd} -minimum = 25 fm and time-to-TSC-minimum = 1.4007 fs. Mean kinetic energy of electron of a "d-e-d-e" EQPET molecule of TSC system was estimated as 57.6 keV at R_{dd} = 25 fm. These time-dependent trapping potential for d-d pair of TSC can be approximated by HMEQPET potentials with the empirical relation of $m=4.36 \times 10^4/R_{dd}$, (R_{dd} in fm unit), continuously as a change of condensation time or $R_{dd}(t)$. Barrier factors for fusion reactions as a function of $R_{dd}(t)$ and 4D fusion rate per TSC generation were calculated using these HMEQPET potentials and Fermi's golden rule. We found that 4D/TSC got to the TSC-minimum state with 10 fm-20 fm radius in 1.4007 fs and 4D fusion rate was 100 % per 4D/TSC generation-condensation. Thus we concluded that ⁴He production rate by 4D/TSC was equal to two times of 4D/TSC generation rate in condensed matter (e.g., PdDx).

[Keywords] tetrahedral symmetric condensate, molecular dynamics, Langevin equation, very fast condensation, 100% 4D fusion, ⁴He generation, barrier factor, heavy Cooper pair potential,

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1. INTRODUCTION

In our previous work reported in Proceedings of ICCF13¹⁾, we introduced a non-linear Langevin equation to study molecular dynamics motion of 4D/TSC condensation and gave the first step results. This paper describes the extended detail of the study on 4D/TSC condensation motion and resulting fusion rates producing almost purely ⁴He with 23.8 MeV per ⁴He. We will show in this paper that 4D fusion takes place 100% per TSC generation.

About the concept of Tetrahedral Symmetric Condensate (TSC) and theoretical EQPET models with numerical analyses by our past studies are reviewed in references²⁻¹³⁾.

In our past studies on 4D/TSC or 4H/TSC models²⁻¹³⁾, Coulomb barrier shielding is conceived to attain automatically by strong condensation motion to the central focal point keeping averaged charge neutrality of the system (focal point is T-site in one of our models for PdDx lattice dynamics; See **Fig.1**). Barrier penetration probability calculation for 2d pair or 4d cluster can be approximated by the EQPET formalism in average. And empirical treatment was applied for multi-body strong interaction between many deuterons under the Platonic symmetry. After the molecular dynamics calculation of 4D/TSC condensation motion, we have extended the EQPET model to the HMEQPET model (described in this paper) to treat time-dependent (equivalently R_{dd} - d-d distance- dependent) d-d pair trapping potential of squeezing TSC. The HMEQPET model utilizes the concept of heavy mass Cooper pair $e^*(m,2)$ to approximate the time-dependent d-d trapping potential continuously – hence we use non-integer values for m (mass number of electronic quasi-particle) according to the change of R_{dd} with time. Using approximate trapping potentials by HMEQPET, we can systematically calculate barrier factors (quantum mechanical tunneling probability of d-d pair and 4d cluster through HMEQPET potentials). Using time-integrated barrier factors and Fermi's golden rule for nuclear fusion rate⁸⁾, time-integrated 4D fusion yield per 4D/TSC condensation was calculated to be 100 % 4D fusion per TSC generation. Using same formalisms, we calculated 2D fusion rates for D₂ molecule, muonic-dd-molecule and $dde^*(2,2)$ Cooper pair dd molecule to result in quite reasonable values.

To initiate TSC formation, three models are speculated in our previous papers (modeled with assumptions)^{2,5,6,8,10)}. One is the TSC formation in regular PdD lattice in D-flow from O-site to T-site by D-lattice-phonon excitation (See Fig.1). The second model is the TSC formation by the collision-combination process between trapped D₂ molecule, being lost freedom of rotation, and incoming D₂ molecule on surface of metal-D systems. The third idea is the random formation of TSC by combination of two “bosonized” molecules of $dde^*(2,2)$ in the near surface of Fermi-level gap of PdDx and CaO (or other insulator layer with low work function); generation of Cooper pair near on Fermi surface may play key role in this case (our paper to Proc. JCF7¹⁰⁾). In every model, we have required the “bosonization” of electron-pair to make $dde^*(m,Z)$ molecular size diminished. Diminished TSC cluster or $dde^*(m,Z)$ molecule can

move to a focal point (T-site, for instance in PdDx lattice) in the gap of tight lattice structure for further condensation motion. We need further detailed modeling for these TSC generation process in/on condensed matter. In this work, we start from the assumption that 4D/TSC at $t=0$ state exists and we try to solve the TSC condensation motion by molecular dynamics calculation with non-linear Langevin equation.

The basics of quantum mechanical approach for studying deuteron-clusters (D_2 , D_2^+ , D_3^+ , 4d/TSC, 6D/OSC) dynamics by Langevin equation are given in our latest paper¹⁷⁾. A historical summary on condensed cluster fusion models is given in our latest paper¹⁸⁾.

Tetrahedral Condensation of Deuterons in PdDx

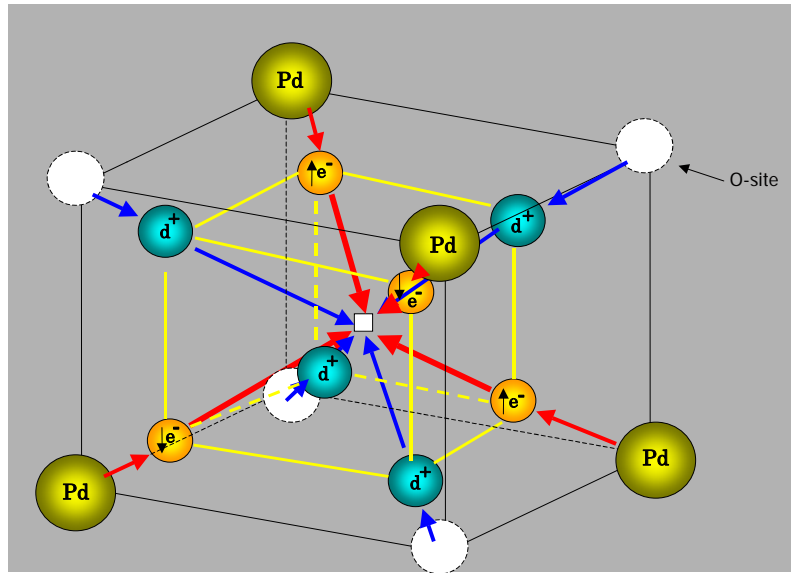


Fig.1: A model of 4D/TSC ($t=0$) formation in PdDx lattice, by phonon excitation of deuterons oscillating as harmonic oscillators at O-sites. 4 deuterons+ 4 1s-electrons may form a TSC cube around the central T-site, transiently with small probability which we need to estimate⁸⁾.

2. NON-LINEAR LANGEVIN EQUATION FOR TSC MOTION

We treat the TSC squeezing motion from TSC ($t=0$; just formed) to TSC-min (minimum size), as illustrated in **Fig.2**. The electron wave function at $t=0$ is set to a vector wave function of 6 wings, each of which is an electron wave function of D_2 molecule as given in reference¹¹⁾, on 6 faces of TSC cube (see **Fig.3**). On each face of TSC cube, we have a “dede molecular like” arrangement. Therefore, TSC($t=0$) system can be regarded as a combination of two

orthogonally coupled D_2 -type molecules having 6 “d-ed-e”-molecular-like faces which are confined by main Coulombic condensation force $-8.38/R_{de}^2$ as derived in our reference¹¹⁾. In Fig.3, the feature of quantum mechanical electron cloud of 4D/TSC($t=0$) is copied from our previous works^{7,8,11)}. Four “electron balls” of “bosonized electron pair” sit at vertexes of cube, alternatively with four deuterons, to form an orthogonally coupled two regular tetrahedrons of deuterons and electron-balls (Double Platonic Symmetry). A dede-molecular-like system has an electron torus of “bosonized” electron bond. By condensation of 4D/TSC, every dede-system on 6 faces diminishes and its d-d pair trapping potential is, in our final stage of this work, given by approximate HMEQPET potential as a function of time (or function of d-d distance $R_{dd}(t)$). Electron kinetic energy of torus bond in a dede-system at $t=0$ is the same with that for D_2 molecule (18 eV), and increases with time-elapse to reach 57.6 keV at $R_{dd}=20.5$ fm of TSC-min state, as we show in this work. However, double Platonic symmetry (DPS) was kept until when 4D/TSC got to the TSC-min state, according to the results by the present molecular dynamics calculation with non-linear Langevin equation.

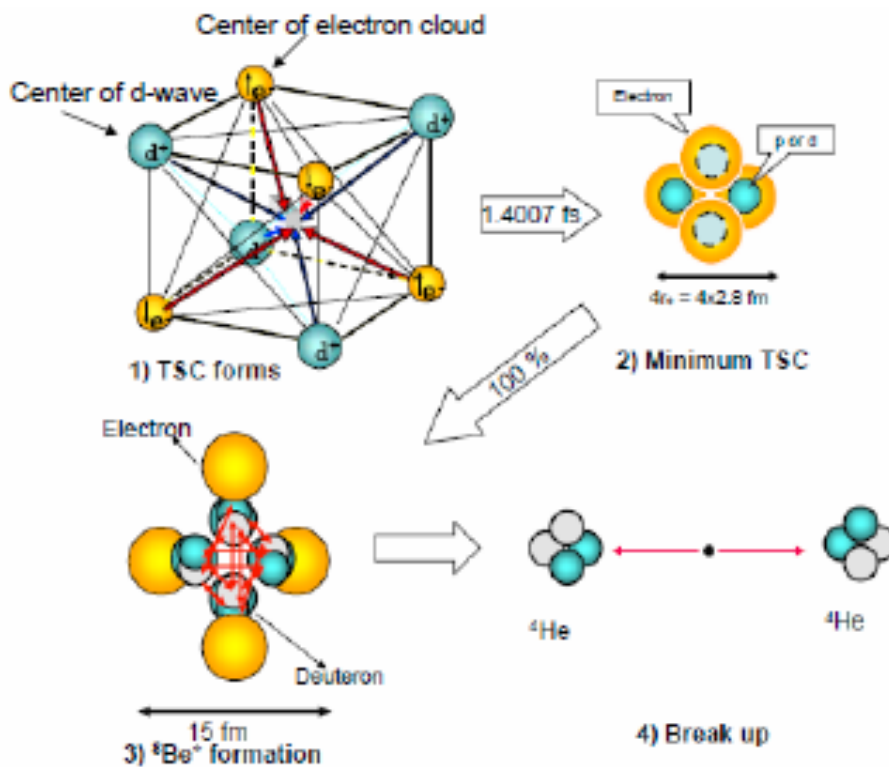


Fig.2 Illustration of 4D/TSC condensation motion to TSC-min state, 4D fusion to form ${}^8\text{Be}^*$ intermediate compound nucleus and break-up to two ${}^4\text{He}$ particles.

Feature of QM Electron Cloud

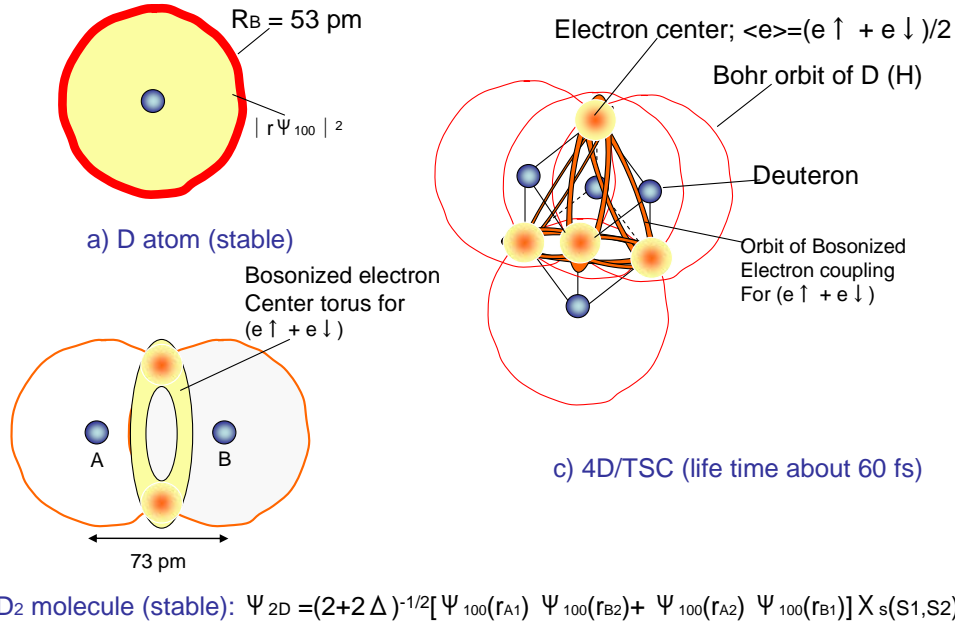


Fig.3 Feature of quantum mechanical electron cloud for 4D/TSC ($t=0$) c), compared with charge-weight distribution of quantum mechanical electron clouds for D-atom and D₂ molecule¹¹.

In our previous works^{1, 17}), we introduced the following non-linear Langevin equation for TSC motion, with distance in pm unit and energy in keV unit.

$$6m_d \frac{d^2 R_{dd}(t)}{dt^2} = -BA \times \frac{11.85}{[R_{dd}(t)]^2} - 6 \frac{\partial V_s(R_{dd}(t); m, Z)}{\partial R_{dd}(t)} + f(t) \quad (1)$$

The first term of right side is the acceleration force to diminish d-d distance, by the main condensation force of TSC as we derived in our previous work¹¹) for a DPS condition of TSC. The factor BA is usually 1.0. We later use adjusted BA factor (smaller than 1.0) for averaged treating of quantum mechanical random fluctuation term of force $f(t)$. The second term of right side of Eq.(1) is the friction (or “deceleration” force) term by quantum mechanical electron clouds of 6 d-e-d-e systems on 6 faces of TSC cube. This is the balancing force (acceleration force for $R_{dd} > R_{min}$, but mostly deceleration force to d-d distance diminishment for $R_{dd} < R_{min}$) forcing d-e-d-e system to get back to the original d-e-d-e state (with 74.1 pm d-d distance). This frictional force is given by the following derivative of EQPET potential,

$$\frac{\partial V_s(R_{dd}; m, Z)}{\partial R_{dd}} = -\frac{1.44}{R_{dd}^2} + \frac{(J'+K')(1+\Delta) + (J+K)\Delta'}{(1+\Delta)^2} \quad (2)$$

, detail function of which is given in Appendix.

The third term of right side of Eq.(1) is due to random fluctuation force. We model that this fluctuation source is generated by the quantum mechanical uncertainty of deuteron positions of TSC. We treat the formulation of $f(t)$ and its ensemble average $\langle f(t) \rangle$ in the next Section 3.

3. QM FLUCTUATION OF DEUTERON POSITIONS

3.1 Gaussian Wave Functions

To treat easily the wave function of d-d pair trapped by the shielded Coulomb potential $V_s(R_{dd}; m, Z)$, we adopt here squared wave function of Gaussian form, as illustrated in **Fig.4**.

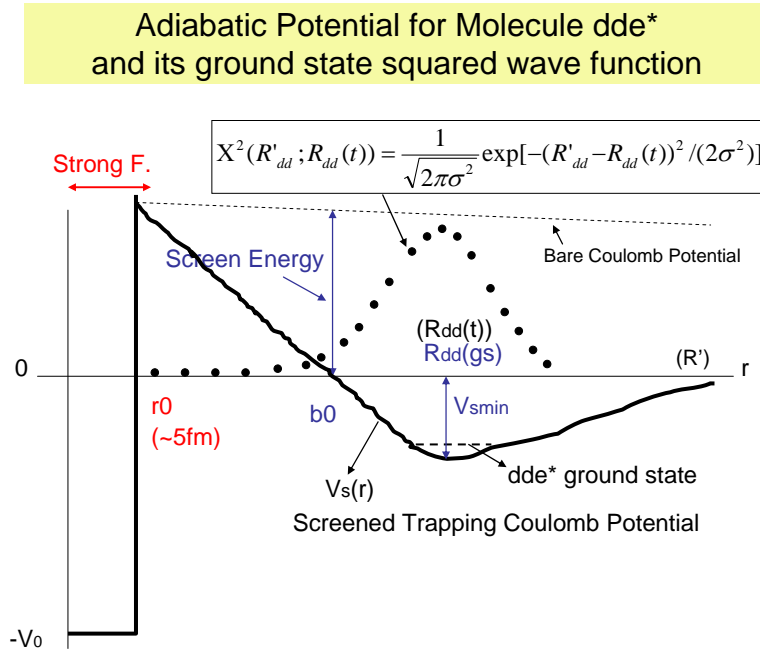


Fig.4: Screened tapping Coulomb potential of EQPET dde* molecule and Gaussian squared wave function for d-d pair. R_{min} value is at V_{smin} . $R_{gs}=R_{dd}(gs)$, ground state d-d distance, and ground state energy E_{gs} are calculated by GWF2 code, as explained later, based on variational principle. Strong interaction for nuclear fusion works only for $R_{dd} \leq r_0$ (5 fm), so that we need to estimate barrier factor from R_{min} to r_0 and $\langle W \rangle$ value for fusion rate formula by Fermi's golden rule^{8,17}.

The squared wave function of Gaussian form for time dependent calculation is:

$$X^2(R'_{dd}; R_{dd}(t)) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp[-(R'_{dd} - R_{dd}(t))^2 / (2\sigma^2)] \quad (3)$$

Unknown parameter σ was determined as explained in the following, based on the variational principle.

$$X(R_{dd}) = \frac{1}{(2\pi)^{1/4} \sqrt{\sigma}} \exp[-(R_{dd} - R_{gs}(m, Z))^2 / (4\sigma^2)] \quad (4)$$

First, we need to make survey study on appropriate Gaussian wave functions (Eq.(4)) for known D_2 molecule and $dd\mu$ (muonic dd) molecule, so as to find appropriate σ -formula, associating proper R_{gs} and E_{gs} values. We survey system total energy to find energy-minimum condition by the following variational method.

$$\delta \langle X(R_{dd}) | H | X(R_{dd}) \rangle = \frac{\partial \langle X(R_{dd}) | H | X(R_{dd}) \rangle}{\partial \sigma} \delta \sigma + \frac{\partial \langle X(R_{dd}) | H | X(R_{dd}) \rangle}{\partial R_{gs}} \delta R_{gs} = 0 \quad (5)$$

, where H is the system Hamiltonian with reduced mass μ (1 for d-d and 1.2 for d-d),

$$H = -\frac{\hbar^2}{2\mu} \nabla^2 + V_s(R_{dd}; m, Z) \quad (6)$$

We look for the conditions as,

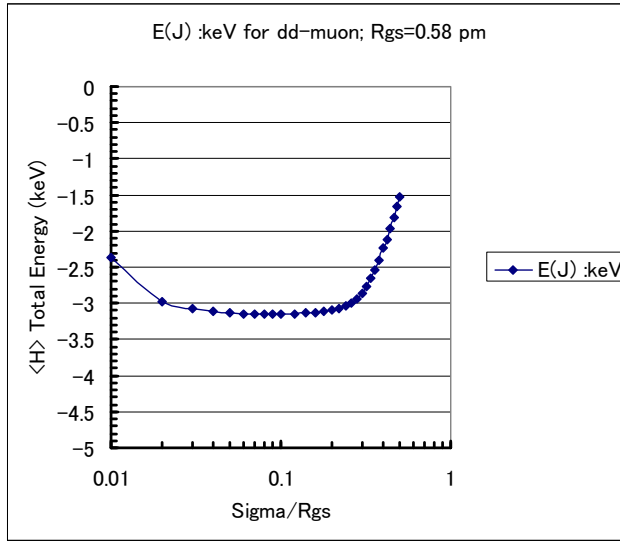
$$\frac{\partial}{\partial \sigma} \langle X(R_{dd}) | H | X(R_{dd}) \rangle = 0 \quad (7)$$

$$\frac{\partial}{\partial R_{gs}} \langle X(R_{dd}) | H | X(R_{dd}) \rangle = 0 \quad (8)$$

We made a simple computer code GWF2 for this variational calculation to find R_{gs} , E_{gs} and σ -formula.

First we show the results for $dd\mu$ (muonic dd) molecule, for which we have a good reference data¹⁴. **Fig.5** shows the survey for σ -formula.

Selection of Gaussian Wave Function for dd-muon



Maximum Possible σ -value is about $0.3R_{gs}$

Fig.5: Search for appropriate σ -formula, in the case of ddp (muonic dd) molecule. We have chosen $\sigma/R_{gs} = 0.3$ as appropriate value, in comparison with known wave function by Hale-Talley¹⁴⁾.

In **Fig.6**, we show a result of search for R_{gs} and E_{gs} for muonic dd molecule. Obtained Gaussian wave function was shown in **Fig.7**, compared with Hale-Talley results¹⁴⁾ in **Fig.8**. We can say that we have realized a practically useful Gaussian wave function for muonic-dd molecule. Here, we note that the difference between dd and dt molecules is the difference in reduced masses; 1 for dd and 1.2 for dt. This difference does not make significant difference in calculated wave functions and potentials.

Then we extended the variational search for D_2 molecule and $dde^*(2,2)$ EQPET molecule. We obtained the same value, $\sigma/R_{gs} = 0.3$ for every case, which we decided to use for time-dependent molecular dynamics calculations for TSC condensation motion. Gaussian wave functions and trapping potentials are shown in **Figs. 9 and 10**, for D_2 and $dde^*(2,2)$ molecules.

We also note that Gaussian wave function is an approximate one, and is useful for analyses around mean values (expected values). However, accuracy of probability in tails of Gaussian wave function is so poor that we can not use it for estimating barrier factor of fusion reaction. We need to introduce the other form of method for barrier factor calculation. We will introduce the HMEQPET method in Section 5, for the purpose.

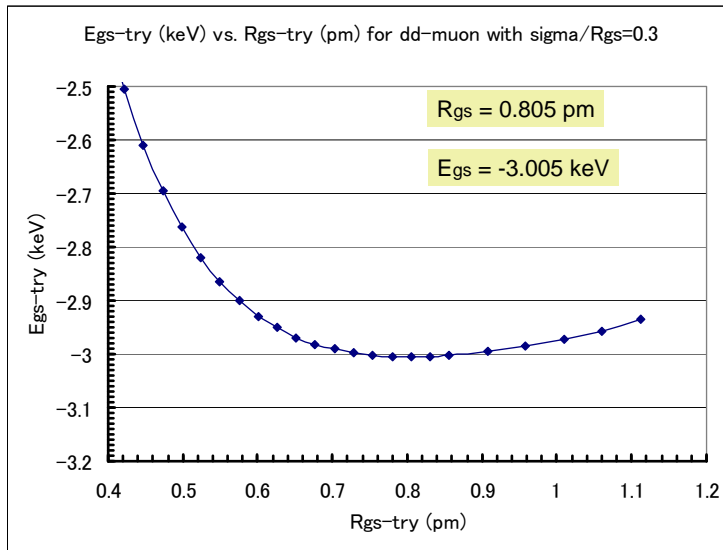


Fig.6: Search for R_{gs} and E_{gs} values with Gaussian wave function of $dd\mu$ molecule.

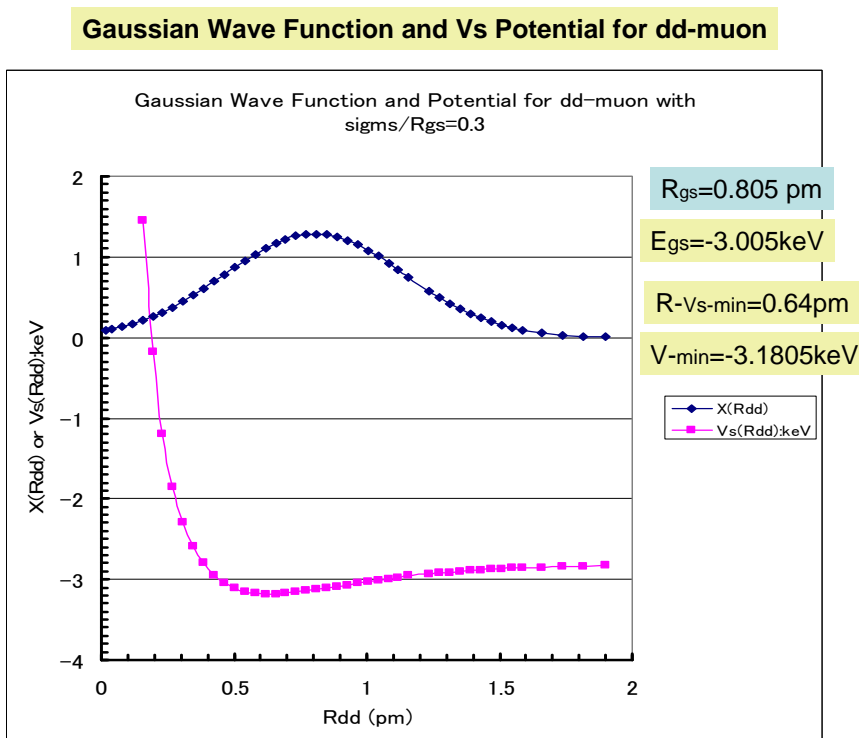


Fig.7: Gaussian wave function and shielded trapping Coulomb potential for $dd\mu$ molecule, calculated by GWF2 Code.

After Hale-Tally, Proc. ICCF4, Trans. Fusion Technology, 26,4T (1994)448

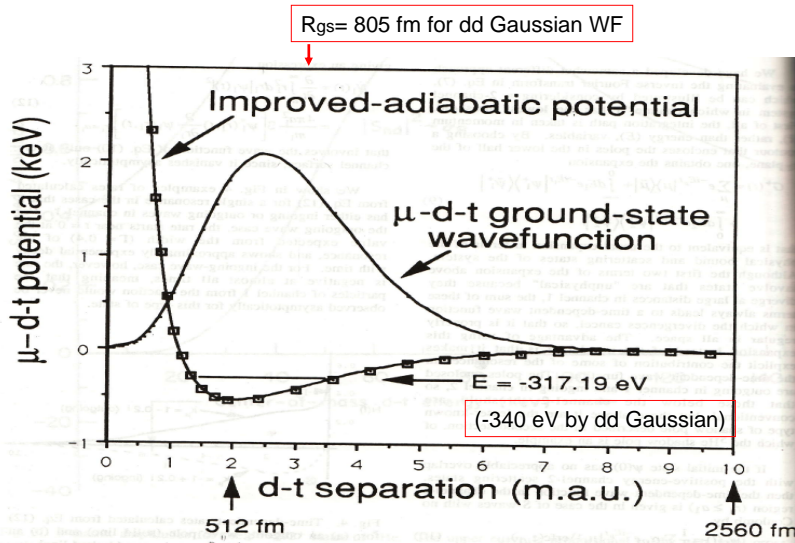


Fig.8: Wave function and trapping Coulomb potential for muonic-dt molecule¹⁴, compared with parameters for muonic-dd molecule by GWF2 Code; Hale and Talley adopted “improved potential” by biasing about + 2.8 keV for original shielded trapping Coulomb potential.

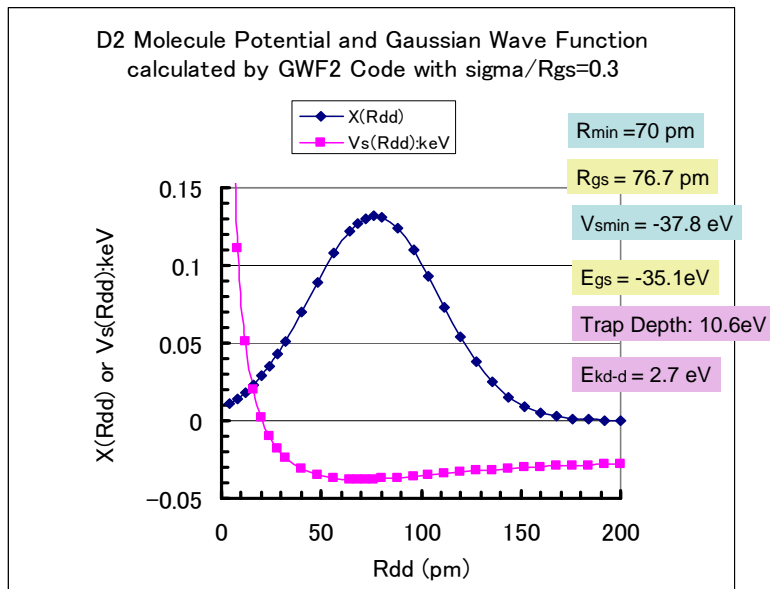


Fig.9: Gaussian wave function and shielded trapping Coulomb potential for D₂ molecule, calculated by GWF2 Code; Ground state d-d distance $R_{gs} = 76.7 \text{ pm}$, which is slightly larger than exact value 74.1 pm. Approximate Gaussian wave function gives in general a slightly larger R_{gs} value. Mean kinetic energy of d-d pair at ground state is 2.7eV, de Broglie wave length of which is reflected in the Gaussian wave function.

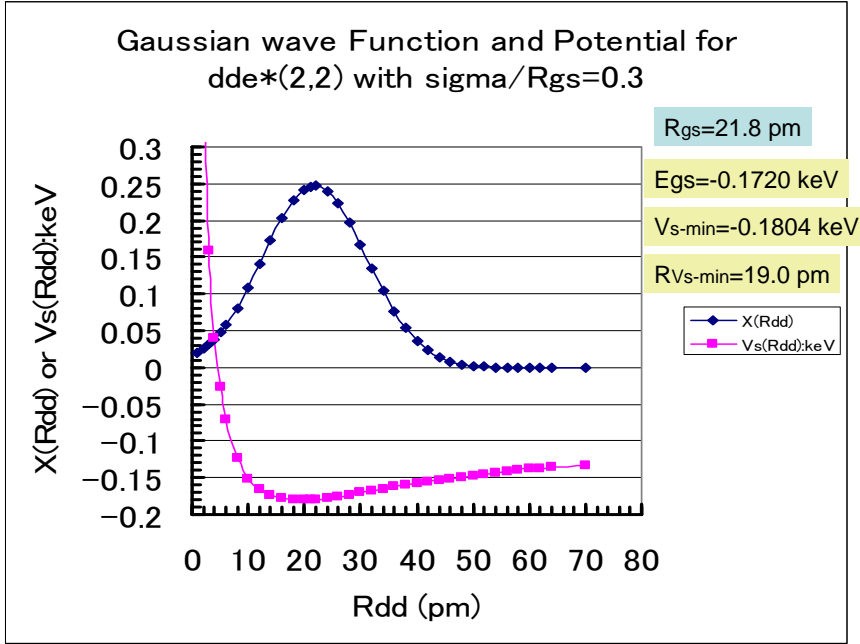


Fig.10: Gaussian wave function and shielded trapping Coulomb potential for Cooper-pair dd molecule, $dde^*(2,2)$; we will use these data for estimating 2D (d-d) fusion rate in condensed matter.

Distortion of System Coulomb Energy by d-d Distance QM Fluctuation

Following the uncertainty weighted by Gaussian wave function, d-d distance of 4D/TSC will fluctuate and distortion of double Platonic arrangement (DPS) takes place. We have estimated approximately the changes of total system Coulomb energy and main TSC condensation force by considering simple balance of geometrical Coulombic forces between deuterons and electrons, as illustrated in **Fig.11**.

Distortion of system Coulombic force is obtained by elementary geometry practice (we omit explanation¹⁷⁾) as,

$$-\frac{\partial \Delta E_c(R'_{dd})}{\partial R_{dd}} = 6.60 \frac{[\Delta R_{dd}]^2}{[R_{dd}(t)]^4} \quad (9)$$

where energy is given in keV unit and R_{dd} in pm unit.

Now the random fluctuation source term of Langevin equation (Eq.(1)) is given as,

$$f(t) = \left[-\frac{\partial \Delta E_c(R'_{dd})}{\partial R_{dd}} \right] \text{mod} [X^2(R'_{dd}; R_{dd}(t))] \quad (10)$$

with

Distortion of Double Platonic Arrangement

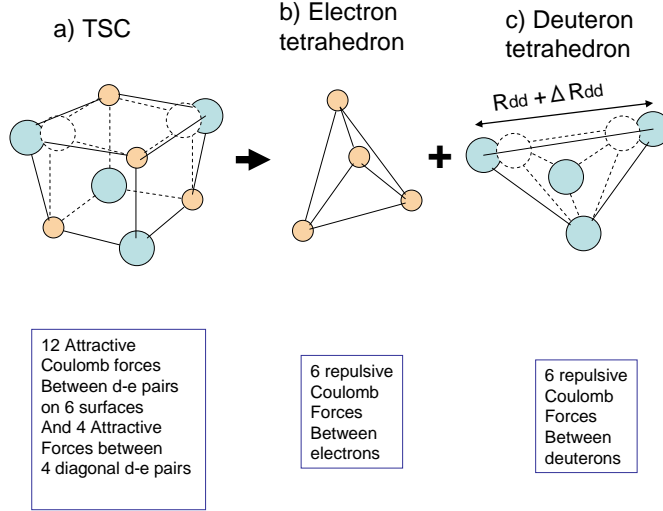


Fig.11: Distortion of Platonic Symmetry of 4D/TSC by the QM fluctuation of d-d distance.

$$\Delta R_{dd} = R'_{dd} - R_{dd}(t) \quad (11)$$

Here we use generation of random Gaussian probability with mod-function for Monte-Carlo calculation. Since we have two d-e-d-e molecular systems, namely up- and down- D_2 -type quasi-molecules in TSC, total fluctuation becomes as,

$$f(t) \Rightarrow f_{up}(t) + f_{down}(t) \quad (12)$$

Using the squared wave function of Eq.(3), we obtain a formula for ensemble-averaged fluctuation force $\langle f(t) \rangle$ as follows;

$$\langle f(t) \rangle = 2 \times 6.6 \int_0^\infty \frac{[R' - R_{dd}(t)]^2}{[R_{dd}(t)]^4} X^2(R') dR' \quad (13)$$

$$\langle f(t) \rangle = \frac{13.2\sigma^2}{\sqrt{\pi}[R_{dd}(t)]^4} \int_0^\infty \sqrt{x} \exp(-x) dx = 13.2 \frac{\sigma^2}{[R_{dd}(t)]^4} \quad (14)$$

with

$$\sigma = 0.372 R_{dd}(t) \quad (15)$$

Here we have decided to adopt a little larger σ -value ($0.373R_{dd}(t)$ was used) than $0.3R_{dd}(t)$ to do conservative estimation of fluctuation effect. Substituting Eq.(15) into Eq.(14), we obtain,

$$\langle f(t) \rangle \approx \frac{1.83}{[R_{dd}(t)]^2} \quad (16)$$

Now the Langevin equation for expected values (simply setting $\langle R_{dd}(t) \rangle = R_{dd}(t)$) becomes as,

$$6m_d \frac{d^2 R_{dd}(t)}{dt^2} = -\frac{11.85}{[R_{dd}(t)]^2} - 6 \frac{\partial V_s(R_{dd}(t); m, Z)}{\partial R_{dd}(t)} + \frac{1.83}{[R_{dd}(t)]^2} \quad (17)$$

As a result, $\langle f(t) \rangle$ makes about 15 % positive bias (deceleration force) to main condensation force term (first term of right side of Eq.(17)), and we can merge this bias into the first term using BA factor (BA=0.873 in this case) in Eq.(1).

4. NUMERICAL SOLUTION BY VERLET METHOD

Equation (17) is highly non-linear, but we can solve numerically by the Verlet's time-step method¹⁵⁾, using the following formulas,

$$G(r, t) = \frac{1.975}{m_d [R(0) - r(t)]^2} + \frac{1}{m_d} \frac{\partial V_s(R_{dd}; m, Z)}{\partial R_{dd}} \quad (18)$$

$$R_{dd}(t) = R(0) - r(t) \quad (19)$$

$$\frac{d^2 r(t)}{dt^2} = G(r, t) \quad (20)$$

$$r(t + \Delta t) = r(t) + v(t)\Delta t + \frac{1}{2} G(r, t)(\Delta t)^2 \quad (21)$$

$$v(t + \Delta t) = v(t) + \frac{\Delta t}{2} [G(r, t + \Delta t) + G(r, t)] \quad (22)$$

By using,

$$m_d = E/c^2 = 2.096 \times 10^{-5} [(keV)/(fs)^2 / (pm)^2] \quad (23)$$

$$G(r,t) = \frac{0.94 \times 10^5}{[R(0) - r(t)]^2} + 4.77 \times 10^4 \frac{\partial V_s(R_{dd}; m, Z)}{\partial R_{dd}} \quad (24)$$

, for BA=1.0.

A simple computer code TSC Langevin was made to carry out numerical computation for 4D/TSC condensation motion. Since the Equation (17) is highly non-linear, we need to be careful about the adiabatic choice of time mesh Δt . Time mesh may start with 0.01 fs, and we need to make stepwise smaller setting, e.g., 0.001 fs for intermediate range and finally very small time-mesh of 0.00001 fs (10^{-20} s). An example of results for BA=0.873 is shown in **Fig.12**, which is the standard result for 4D/TSC condensation motion. Last stage of condensation took place in very short time interval as about 2×10^{-20} s.

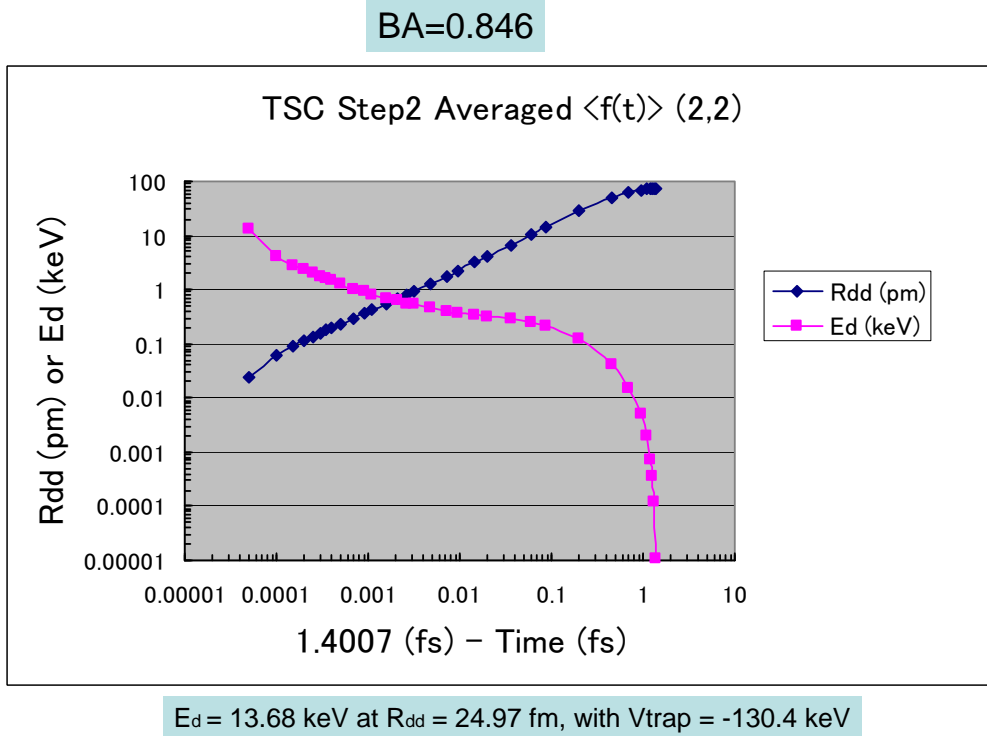


Fig.12: Results of TSC Langevin calculation for 4D/TSC condensation motion. TSC finishes condensation in 1.4007 fs (and 100% breaks up by 4D fusion as explained later). At TSC-min state around $R_{dd}=25$ fm, relative kinetic energy of d-d pair becomes 13.68 keV and d-d pair is trapped within deep trapping potential with -130.4 keV depth (we explain later). For balancing force, $V_s(R_{dd};2,2)$ potential is used in this case.

As an another trial, we used $V_s(R_{dd};1,1)$ potential for balancing force term and got results as shown in **Fig.13**. TSC finished condensation in 3.02095 fs in this case. Trend of acceleration in

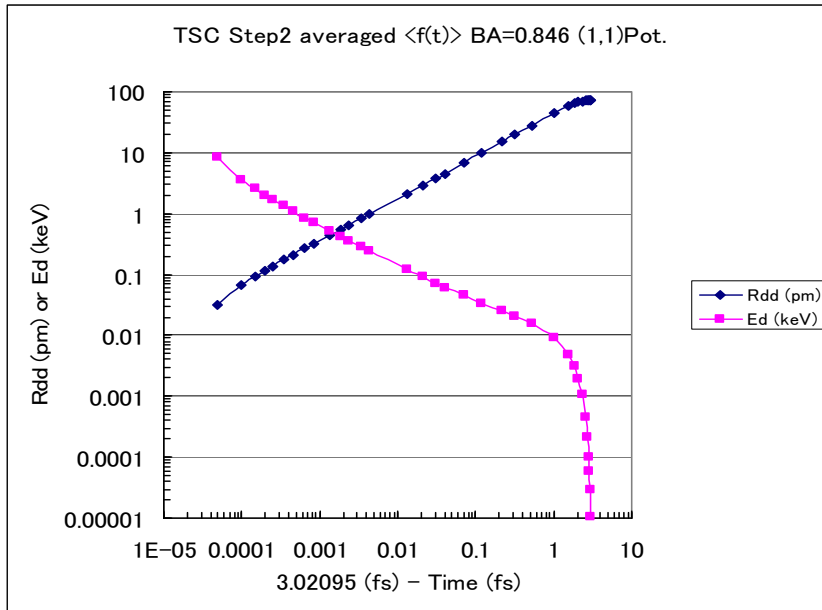
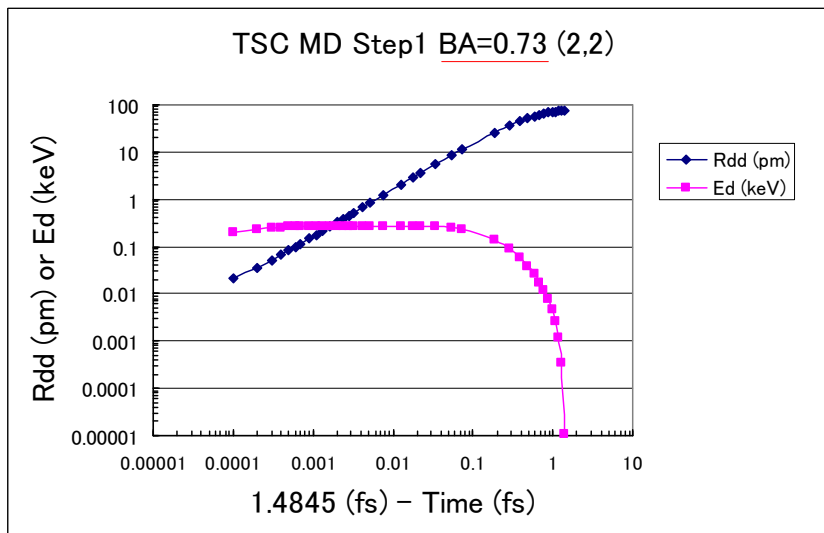


Fig.13: Results of TSC Langevin calculation using $V_s(R_{dd};1,1)$ potential for balancing term.

By Choosing $BA < 1.0$, we roughly estimate averaged effect of $\langle f(t) \rangle$



In this case; **acceleration for $R_{dd} > 1\text{pm}$, and deceleration for $R_{dd} < 1\text{pm}$**

Fig.14: A trial calculation for large friction term ($BA=0.73$) by random fluctuation.

This is virtual case.

the intermediate stage is steeper than the case of $V_s(R_{dd};2,2)$ potential, but situation to get to TSC-min state with similar size is the same.

When the friction term of Langevin equation becomes large, we may have a constant velocity motion, as is the case of rain drop in the sky. A trial calculation using $BA=0.73$ is shown in **Fig.14**. In this virtual case, condensation motion moves with almost constant velocity (kinetic energy) for $R_{dd} < 10$ pm until when TSC-min state comes.

5. HMEQPET METHOD

Now we move to formulate time-dependent d-d trapping potential of TSC condensation motion. The ensemble-averaged Langevin equation is rewritten as,

$$6m_d \frac{d^2 \langle R_{dd} \rangle}{dt^2} = -\frac{11.85}{\langle R_{dd} \rangle^2} - 6 \frac{\partial V_s(\langle R_{dd} \rangle; m, Z)}{\partial \langle R_{dd} \rangle} + 6.6 \left\langle \frac{(R' - R_{dd})^2}{R_{dd}^4} \right\rangle \quad (25)$$

We speculate the total d-d trapping potential form, assuming that $\langle R_{dd} \rangle$ can be simply replaced with $R_{dd}(t)$ for the first and second term of right side and the last term can be integrated with respect to R' , to get,

$$V_{tsc}(R'; R_{dd}(t)) = -\frac{11.85}{R_{dd}(t)} + 6V_s(R_{dd}(t); m, Z) + 2.2 \frac{|R' - R_{dd}(t)|^3}{[R_{dd}(t)]^4} \quad (26)$$

Calculated potential curve by Eq.(26) is shown in **Fig.15**, for the final stage of TSC condensation.

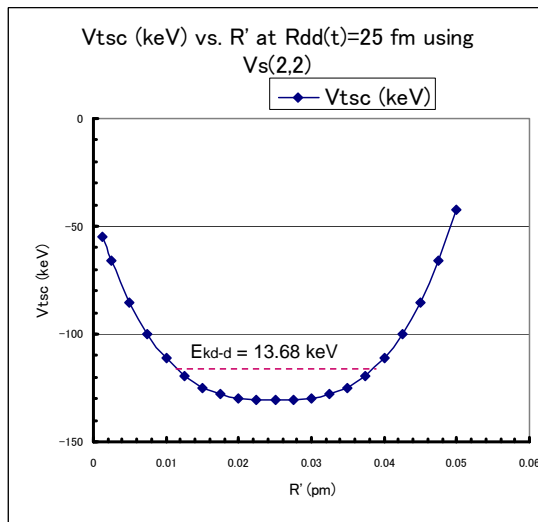


Fig.15: Estimated approximate d-d trapping potential of 4D/TSC at $R_{dd}(t)=25$ fm.

Potential depth ($V_{tsc-min}$)= -130.4 keV and mean kinetic energy of d-d pair 13.68 keV are obtained. This is very fast transient state with about 10^{-20} s of adiabatic time interval. However we can regard this state as an adiabatic state with 13.68 keV ground state energy with V_{tsc} ($R_{dd};R$) trapping potential, for that short time interval.

We can calculate corresponding kinetic energy of electron in ‘dede’ system of TSC-min state, using semi-classical geometry model¹⁷⁾ as shown in **Fig.16**.

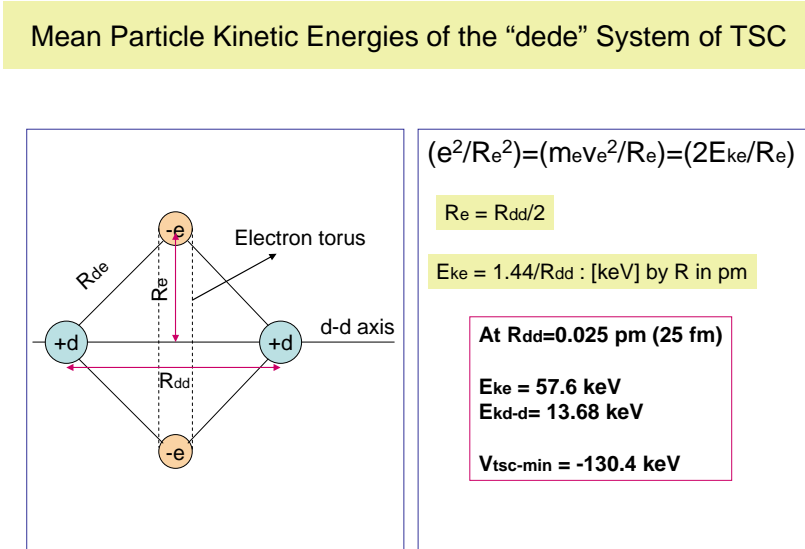


Fig.16: Estimation of mean kinetic energy of “d-e-d-e” system of diminished 4D/TSC. Here the position of “-e” is for the center of quantum mechanical charge distribution (cloud) of electron.

We obtained mean electron kinetic energy $E_{ke} = 57.6 \text{ keV}$ of “bosonized” electron torus (see Fig.3 c) for d-e-d-e system at $R_{dd} = 25 \text{ fm}$ that is at the final stage of TSC condensation motion. At the starting $t = 0$ state, mean electron kinetic energy was 18 eV (16.2 eV in our previous study¹¹⁾ is underestimation). Despite of such high kinetic energy at the TSC-min state, electrons are trapped tightly with deep potential ($-130.4 - 57.6 \text{ keV} = -187.6 \text{ keV}$) in “d-e-d-e” system of a face of 4D/TSC system. We can draw similar “adiabatic” state for d-d distances of $R_{dd} > 25 \text{ fm}$. Thus we can understand that more enhanced mean kinetic energy of electrons trapped in time-dependent TSC potential makes the effective electron wave length shorter to match the Heisenberg Uncertainty Principle (HUP), in the d-e-d-e system: this seems the essence of TSC condensation motion. The ultimate condensation is possible only under the double Platonic symmetry. Namely, we need always four deuterons in the condensing system. This commonly the reason why 4D/TSC can make super screening of mutual Coulomb barrier and condense ultimately into the strong interaction range of nuclear

fusion (4D fusion is therefore predominant there).

Unfortunately, thus obtained trapping potentials and corresponding Gaussian wave functions can not be used for barrier factors (tunneling probabilities for fusion reaction) calculation, because the accuracies in small and large R_{dd} values are poor. Gaussian wave function is useful to estimate deviation from mean (expected) values, $\langle R_{dd} \rangle$, R_{gs} and E_{gs} , but not useful to interpolation to r_0 position where nuclear strong interaction works.

To obtain more useful approximate potential for continuously varying $R_{dd}(t)$ value, we have introduced the concept of Heavy Mass Cooper Pair $e^*(m,2)$ to form virtual $dde^*(m,2)$ molecule. If we can establish one-to-one relation between m and $R_{dd}(t)$, we can replace all $V_{tsc}(R_{dd}(t))$ potentials with $V_s(R_{dd}(t); m,2)$ potentials. First, we have made survey calculations for $V_s(R; m,2)$ as a function of mass number (relative to electron mass) m , as results are shown in **Table-1**. In this table, we have eventually found tight empirical relations as,

$$b_0(m,2) = 0.206R_{gs}(m,2) \quad (27)$$

$$m = 9000/b_0(m,2) \quad (28)$$

Here b_0 and R_{gs} are in fm unit. Using $R_{gs} = 805$ fm for muonic-dd molecule, we obtain $m = 54$ for muonic-dd molecule. So, $V_s(R_{dd}; 54,2)$ potential can be used instead of $V_s(R_{dd}; 207,1)$ potential of dd-muonic molecule. Similarly, $m = 200-500$ range can correspond to V_{tsc} potentials in the final stage of TSC condensation.

Table-1: Main parameters of HMEQPET potentials calculated by GWF2 Code; mean d-d kinetic energy is given as $E_{gs} - V_{s-min}$.

| Molecule | b_0 (pm) | R_{min} (pm) | V_{s-min} (keV) | $V_{s-min} - V(\infty)$ (keV) | R_{gs} (pm) | E_{gs} (keV) |
|----------------|---------------|-------------------|----------------------|----------------------------------|------------------|-------------------|
| D ₂ | 22 | 70 | -0.03782 | -0.0106 | 76.69 | -0.03514 |
| dde*(2,2) | 4.5 | 19.3 | -0.1804 | -0.0716 | 21.82 | -0.17027 |
| dde*(5,2) | 1.9 | 7.6 | -0.4509 | -0.1789 | 8.72 | -0.43007 |
| dde*(10,2) | 0.90 | 3.8 | -0.9019 | -0.3579 | 4.36 | -0.86012 |
| dde*(20,2) | 0.45 | 1.9 | -1.8039 | -0.7159 | 2.18 | -1.7202 |
| dde*(50,2) | 0.18 | 0.76 | -4.5097 | -1.7894 | 0.873 | -4.3003 |
| dde*(100,2) | 0.09 | 0.38 | -9.0194 | -3.5790 | 0.436 | -8.5998 |
| dde*(200,2) | 0.045 | 0.19 | -18.039 | -7.1590 | 0.218 | -17.196 |
| dde*(500,2) | 0.018 | 0.076 | -45.097 | -17.89 | 0.0873 | -42.968 |
| dde*(1000,2) | 0.009 | 0.038 | -90.194 | -35.79 | 0.0436 | -85.858 |
| dde*(2000,2) | 0.0045 | 0.019 | -180.39 | -71.59 | 0.0218 | -171.406 |

Using these HMEQPET potentials, we can approximately calculate barrier factors for time-dependent R_{dd} values, continuously. We can use there, real number (non integer) for m value, virtually.

In our previous study of time-dependent EQPET analysis, we assumed three steps of adiabatic potentials⁷⁾, which was difficulty in the method, but we can make smooth calculation of barrier factors continuously changing HMEQPET potentials, in this work.

6. 4D FUSION RATES AND DISCUSSIONS

Using HMEQPET potentials as a function of $R_{dd}(t)$ with the results of TSC Langevin calculation, we can easily calculate barrier factors for d-d pair and 4d cluster.

First we calculate Gamow integral,

$$\Gamma_{dd}(m, Z) = 0.218\sqrt{\mu} \int_{r_0}^{b_0(m, Z)} \sqrt{V_s(R; m, Z) - E_d} dR \quad (29)$$

with R_{dd} in fm unit and,

$$m = 4.37 \times 10^4 / R_{dd}(t) \quad (30)$$

For the calculation of Gamow integral of steady dde* molecule, $b_0(m, Z)$ should be redefined as the lower crossing distance of potential on the level of E_{gs} . Since E_{gs} is negative value, we need to bias V_s potential in Eq.(29) with $-E_{gs}$ to have positive value in root operator. Approximate calculation can be however done by regarding $b_0(m, Z) = R_{gs}$, instead of using the biased V_s . We applied this approximation in the present calculation of Gamow integrals.

Barrier factor is calculated by,

$$P_{nd}(m, Z) = \exp(-n\Gamma_{dd}(m, Z)) \quad (31)$$

Using Fermi's golden rule⁸⁾, fusion rate is obtained by,

$$\lambda_{nd} = \frac{2}{\hbar} \langle W \rangle P_{nd}(r_0) = 3.04 \times 10^{21} P_{nd}(r_0) \langle W \rangle \quad (32)$$

Here $\langle W \rangle$ value is in MeV unit.

Calculated barrier factors for typical R_{dd} values are shown in **Table-2**.

In Table-2, approximate barrier factors are given according to known D_2 -molecule and muonic dd

molecule, and also for EQPET dde*(2,2) molecule, respectively.

The imaginary part of optical potential $\langle W \rangle$ for effective interaction, namely fusion, is estimated by the empirical rule for PEF (pion exchange force number) values⁸⁾, and given in **Table-3**. Fusion rates are shown in **Table-4**. Here we used the following relations between astrophysical S-value S_{nd} , T-matrix for the effective interaction Hamiltonian of nuclear fusion T_n and pion-exchange number PEF, in $\langle W \rangle$ estimation⁸⁾. Practical function of PEF based on the one pion exchange potential is given in our previous paper¹⁷⁾.

$$S_{nd} \propto T_n^2 \propto (PEF)^{10} \quad (33)$$

$$T_n = \langle W \rangle \propto (PEF)^5 \quad (34)$$

And we used known S-values at $E_d=0$ and $\langle W \rangle$ values for DD¹⁶⁾ and DT reactions, as reference values.

Table-2: Calculated barrier factors for 2d-pair and 4d-cluster under TSC condensation. Time runs from bottom to top with one-to-one relation between time and R_{dd} .

| $R_{dd}=R_{gs}$ (pm) | P_{2d} ; 2D Barrier Factor | P_{4d} ; 4D Barrier Factor |
|------------------------------|------------------------------|------------------------------|
| 0.0206 (TSC-min) | 4.44E-2 | 1.98E-3 |
| 0.0412 | 1.06E-2 | 1.12E-4 |
| 0.103 | 1.43E-3 | 2.05E-6 |
| 0.206 | 3.35E-5 | 1.12E-9 |
| 0.412 | 9.40E-7 | 2.16E-13 |
| 0.805 (μ dd): muonic dd | 1.00E-9 | 1.00E-18 |
| 1.03 | 9.69E-11 | 9.40E-21 |
| 2.06 | 6.89E-15 | 4.75E-29 |
| 4.12 | 9.38E-21 | 8.79E-41 |
| 10.3 | 2.16E-32 | 4.67E-64 |
| 21.8 (dde*(2,2)) | 1.30E-46 | 1.69E-92 |
| 74.1 (D2 molecule) | 1.00E-85 | 1.00E-170 |

Table-3: estimation of $\langle W \rangle$ values

| Cluster | $\langle W \rangle$ (MeV) |
|---------|---------------------------|
| DD | 0.008 |
| DT | 0.115 |
| 3D | 1.93 |
| 4D | 62.0 |

Table-4: Fusion rates calculated by Fermi's golden rule (using Eq.(32))

| Molecule | R _{dd} =R _{gs} (pm) | P _{nd} ; B-Factor | <W> (MeV) | λ _{2d} (f/s) | λ _{4d} (f/s) |
|-----------------|--|----------------------------|-----------|-----------------------|-----------------------|
| D ₂ | 74.1 | 1.0E-85 | 0.008 | 2.4E-66 | |
| dde*(2,2) | 21.8 | 1.3E-46 | 0.008 | 3.16E-27 | |
| μ _{dd} | 0.805 | 1.0E-9 | 0.008 | 2.4E+10 | |
| 4D/TSC-min | 0.021 | 1.98E-3 | 62 | | 3.7E+20 |

Calculated fusion rate for dd-muon molecule, 2.4×10^{10} f/s/pair well corresponds to fusion rate of dt-muon by Hale-Talley¹⁴, $(1.08-1.29) \times 10^{12}$ f/s/pair, considering the dt fusion cross section is 200 times of dd fusion in low energy region. Value for D₂ molecule looks also appropriate.

Calculated 4D fusion rate 3.7×10^{20} f/s/cluster is very large, however this is value for virtual steady molecule state of 4D/TSC-min. In reality, 4D/TSC-min state can exist only for about 2×10^{-20} s in the final stage of condensation. Now we need to derive formulas for fusion yield for transient 4D/TSC condensation.

Microscopic fusion yield η_{4d} per 4D/TSC is defined as, using condensation time t_c (1.4007 fs),

$$\eta_{4d} = 1 - \exp\left(-\int_0^{t_c} \lambda_{4d}(t) dt\right) \quad (35)$$

$$\lambda_{4d}(t) = 3.04 \times 10^{21} \langle W \rangle P_{4d}(r_0; R_{dd}(t)) = 1.88 \times 10^{23} P_{4d}(r_0; R_{dd}(t)) \quad (36)$$

$$\int_0^{t_c} \lambda_{4d}(t) dt = 1.88 \times 10^{23} \int_0^{t_c} P_{4d}(r_0; R_{dd}(t)) dt \quad (37)$$

$$\int_0^{t_c} P_{4d}(r_0; R_{dd}(t)) dt = 2.31 \times 10^{-22} \quad (38)$$

We obtain that η_{4d} is very close to 1.0. As a consequence, we get the simplest result:

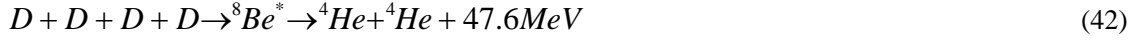
$$\eta_{4d} \cong 1.0 \quad (39)$$

Therefore macroscopic 4D fusion yield becomes equal to 4D/TSC generation rate in condensed matter (e.g., in PdDx under excitation). Defining the TSC generation rate by Q_{tsc} , we define 4D fusion yield Y_{4d} as,

$$Y_{4d} = Q_{tsc} \eta_{4d} \quad (40)$$

$$\text{Practically, } Y_{4d} \approx Q_{tsc} \quad (41)$$

4D fusion and its product are:



Finally we have reached at the simplest conclusion that 4D fusion Yield Y_{4d} (f/s/cc) is equal to 4D/TSC generation rate Q_{tsc} (tsc/s/cc) in condensed matter. Now formulation and estimation of Q_{tsc} becomes essentially important for further theoretical elaboration and guiding experiments.

In the case of muonic-dd fusion rate of 2.4×10^{10} (f/s) with 2×10^{-6} s muon life time, almost 100% dd fusion takes place in 200 ps (2×10^{-10} s). So that, muon goes out after one dd-fusion, will be trapped in another muonic-dd molecule and will make plural chain reactions within muon life time, as well known.

Since 4d fusion yield is 100% for 4D/TSC-min state, we have no 2d fusion rate in the final stage. In the present model calculation, 2D fusion may take place by dde*(2,2) state just after Cooper pair formation. Fusion yield by dde*(2,2) is estimated in analogous way.

$$\eta_{2d} = 1 - \exp(-\lambda_{2d(2,2)}\tau_{(2,2)}) \quad (43)$$

$$\eta_{2d} \approx \lambda_{2d(2,2)}\tau_{(2,2)} \quad (44)$$

Here $\tau_{(2,2)}$ is the life time of dde*(2,2). If we may assume very long life time as 10^4 s (as asserted by X. Z. Li¹⁶⁾), we obtain

$\eta_{2d} = 3.1 \times 10^{-23}$ (f/pair). Even if assuming maximum level of dde*(2,2), or Cooper pair, generation rate of 10^{22} (dde*/s/cc), we get 2D fusion yield of $Y_{2d} = 0.3$ (f/s/cc), so called Jones' level.

However, we may have small probability that 4D/TSC under condensation may deviate far from expected position $\langle R_{dd}(t) \rangle$ and Platonic symmetry would be broken to split into two diminished dde*(m,2) molecules which may cause higher 2D fusion rates. To know quantitative answer to this question, we further need to develop a Monte-Carlo Langevin code to treat random $f(t)$ source for TSC condensation motion. This is one of our future works. Another minor neutron production may come from break up of ${}^8\text{Be}^*$ with 47.6 MeV excited state by 4D fusion going out to minor branch of $n + {}^7\text{Be}$ ($Q = -18.9$ MeV).

7. CONCLUSIONS

- 1) We have introduced a non-linear Langevin equation for the study of molecular dynamics motion of TSC condensation. Developed calculations were very successful to estimate condensation time, barrier factors and fusion rates.
- 2) Molecular dynamics of 4D/TSC condensation was numerically solved by the Verlet's time step method.

- 3) Condensation time of 4D/TSC was very short as 1.4 fs, compared with our primitive analysis¹¹⁾.
- 4) 4D fusion happens in about 2×10^{-20} s in the last stage of condensation with 100 % probability.
- 5) 4D fusion yield and ^4He production yield becomes equal to 4D/TSC generation rate in condensed matter.
- 6) 2D fusion rate is negligibly small.
- 7) The concept of heavy mass Cooper pair and HMEQPET potentials are useful tool to quantify time-dependent d-d trapping potential under condensation of TSC.
- 8) We need elaboration for developing Monte-Carlo Langevin code, quantitative models for 4D/TSC or 4H/TSC formation in circumstances in/on condensed matter, TSC + host metal interaction for transmutation reactions, etc.
- 9) We may conclude that the double Platonic symmetry of 4D (or 4H)/TSC can realize a) super-screening of mutual d-d Coulomb repulsion, b) predominant 4D fusion with ^4He ash and c) apparently hard radiation-less nuclear fusion, in the dynamic clustering process of deuterons in condensed matter.

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[Appendix]

Shielded Coulomb potential, equivalently d-d trapping potential, for dde*(m,Z) EQPET molecule is given in our previous works^{4, 8)} as,

$$V_s(R_{dd}; m, Z) = V_h + \frac{1.44}{R_{dd}} + \frac{J + K}{1 + \Delta} \quad (\text{A-1})$$

with

$$V_h = -0.0136mZ^2[\text{keV}] \quad (\text{A-2})$$

$$J = 0.0272Z^2m\left(-\frac{1}{y} + \left(1 + \frac{1}{y}\right)\exp(-2y)\right) \quad (\text{A-3})$$

$$K = 0.0272Z^2m(1 + y)\exp(-y) \quad (\text{A-4})$$

$$\Delta = \left(1 + y + \frac{y^2}{3}\right)\exp(-y) \quad (\text{A-5})$$

$$y = \frac{Zm}{52.9}R_{dd} \quad (\text{A-6})$$

The derivative of the potential becomes, with R_{dd} in pm unit and energy in keV unit.

$$\frac{\partial V_s(R_{dd}; m, Z)}{\partial R_{dd}} = -\frac{1.44}{R_{dd}^2} + \frac{(J' + K')(1 + \Delta) + (J + K)\Delta'}{(1 + \Delta)^2} \quad (\text{A-7})$$

with

$$J' = 5.14 \times 10^{-4} Z^3 m^2 \left(\frac{1 - \exp(-2y)}{y^2} - 2 \left(1 + \frac{1}{y}\right) \exp(-2y) \right) \quad (\text{A-8})$$

$$K' = -5.14 \times 10^{-4} Z^3 m^2 y \exp(-y) \quad (\text{A-9})$$

$$\Delta' = -6.30 \times 10^{-3} (y + y^3) \exp(-y) \quad (\text{A-11})$$

Here R and r are used in [pm] unit, energy in [keV] unit and time in [fs] unit.