Riemann Paper (1859) Is False

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Abstract

In 1859 Riemann defined the zeta function $\zeta(s)$. From Gamma function he derived the zeta function with Gamma function $\bar{\zeta}(s)$. $\bar{\zeta}(s)$ and $\zeta(s)$ are the two different functions. It is false that $\bar{\zeta}(s)$ replaces $\zeta(s)$. Therefore Riemann hypothesis (RH) is false. The Jiang function $J_n(\omega)$ can replace RH.

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In 1859 Riemann defined the Riemann zeta function (RZF) [1]

$$\zeta(s)=\prod_{P}(1-P^{-s})^{-1}=\sum_{n=1}^{\infty}\frac{1}{n^{s}}\ , \tag{1}$$

where $s=\sigma+ti, i=\sqrt{-1}$, σ and t are real, P ranges over all primes. RZF is the function of the complex variable s with $\sigma\geq 0, t\neq 0$, which is absolutely convergent.

In 1896 J. Hadamard and de la Vallee Poussin proved independently [2]

$$\zeta(1+ti)\neq 0\,. \tag{2}$$

In 1998 Jiang proved [3]

$$\zeta(s)\neq 0\,, \tag{3}$$

where $0\leq\sigma\leq 1$.

Riemann paper (1859) is false [1]. We define Gamma function [1, 2]

$$\Gamma\left(\frac{s}{2}\right)=\int_{0}^{\infty}e^{-t}t^{\frac{s}{2}-1}dt\ . \tag{4}$$

For $\sigma>0$. On setting $t=n^{2}\pi x$, we observe that

$$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)n^{-s}=\int_{0}^{\infty}x^{\frac{s}{2}-1}e^{-n^{2}\pi x}dx\ . \tag{5}$$

Hence, with some care on exchanging summation and integration, for $\sigma>1$,

$$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\overline{\zeta}(s)=\int_{0}^{\infty}x^{\frac{s}{2}-1}\left(\sum_{n=1}^{\infty}e^{-n^{2}\pi x}\right)dx$$

$$=\int_{0}^{\infty}x^{\frac{s}{2}-1}\left(\frac{\vartheta(x)-1}{2}\right)dx\,, \tag{6}$$

where $\overline{\zeta}(s)$ is called Riemann zeta function with gamma function.

$$\vartheta(x):=\sum_{n=-\infty}^{\infty}e^{-n^{2}\pi x}\,, \tag{7}$$

is the Jacobi theta function. The functional equation for $\vartheta(x)$ is

$$x^{\frac{1}{2}}\vartheta(x)=\vartheta(x^{-1}), \qquad (8)$$

and is valid for $x>0$.

Finally, using the functional equation of $\vartheta(x)$, we obtain

$$\overline{\zeta}(s)=\frac{\pi^{\frac{s}{2}}}{\Gamma\left(\frac{s}{2}\right)}\left\{\frac{1}{s(s-1)}+\int_1^{\infty}(x^{\frac{s}{2}-1}+x^{-\frac{s}{2}-\frac{1}{2}})\cdot(\frac{\vartheta(x)-1}{2})dx\right\}. \qquad (9)$$

From (9) we obtain the functional equation

$$\pi^{-\frac{S}{2}}\Gamma\left(\frac{s}{2}\right)\overline{\zeta}(s)=\pi^{-\frac{1-s}{2}}\Gamma\left(\frac{1-s}{2}\right)\overline{\zeta}(1-s). \qquad (10)$$

The function $\overline{\zeta}(s)$ satisfies the following:

1. $\overline{\zeta}(s)$ has no zero for $\sigma>1$;
2. The only pole of $\overline{\zeta}(s)$ is at $s=1$, it has residue 1 and is simple;
3. $\overline{\zeta}(s)$ has trivial zeros at $s=-2,-4,...$ but $\zeta(s)$ has no zeros;
4. The nontrivial zeros lie inside the region $0\le\sigma\le1$ and are symmetric about both the vertical line $\sigma=1/2$.

The strip $0\le\sigma\le1$ is called the critical strip and the vertical line $\sigma=1/2$ is called the critical line.

Conjecture (The Riemann Hypothesis). All nontrivial zeros of $\overline{\zeta}(s)$ lie on the critical line $\sigma=1/2$, which is false. [3]

$\overline{\zeta}(s)$ and $\zeta(s)$ are the two different functions. It is false that $\overline{\zeta}(s)$ replaces $\zeta(s)$, Pati proved that is not all complex zeros of $\overline{\zeta}(s)$ lie on the critical line: $\sigma=1/2$ [4].

Schadeck pointed out that the falsity of RH implies the falsity of RH for finite fields [5, 6]. RH is not directly related to prime theory. Using RH mathematicians prove many prime theorems which is false. In 1994 Jiang discovered Jiang function $J_n(\omega)$ which can replace RH, if $J_n(\omega)\ne0$ then the prime equation has infinitely many prime solutions; and if $J_n(\omega)=0$ then the prime equation has finitely many prime solutions. By using $J_n(\omega)$ Jiang proves about 600 prime theorems including the Goldbach's theorem, twin prime theorem and theorems on arithmetic progressions in primes [7, 8].

In the same way we have a general formula involving $\overline{\zeta}(s)$

$$\int_0^\infty x^{s-1}\sum_{n=1}^\infty F(nx)dx=\sum_{n=1}^\infty \int_0^\infty x^{s-1}F(nx)dx$$

$$=\sum_{n=1}^\infty \frac{1}{n^s}\int_0^\infty y^{s-1}F(y)dy=\overline{\zeta}(s)\int_0^\infty y^{s-1}F(y)dy, \qquad (11)$$

where $F(y)$ is arbitrary.

From (11) we obtain many zeta functions $\overline{\zeta}(s)$ which are not directly related to the number theory.Using Jiang function we prove the following theorems.

Primes Represented by $P_1^n+mP_2^n$ [9]

（1）Let $n=3$ and $m=2$. We have

$$P_3=P_1^3+2P_2^3.$$

We have Jiang function

$$J_3(\omega)=\prod_{3\le P}(P^2-3P+3-\chi(P))\neq 0,$$

Where $\chi(P)=2P-1$ if $2^{\frac{P-1}{3}}\equiv 1$ (mod P); $\chi(P)=-P+2$ if $2^{\frac{P-1}{3}}\not\equiv 1$ (mod P); $\chi(P)=1$ otherwise.

Since $J_n(\omega)\neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.

We have the best asymptotic formula

$$\pi_2(N,3)=\left|\{P_1,P_2:P_1,P_2\le N, P_1^3+2P_2^3=P_3\text{ prime}\}\right|$$

$$\sim\frac{J_3(\omega)\omega}{6\Phi^3(\omega)}\frac{N^2}{\log^3 N}=\frac{1}{3}\prod_{3\le P}\frac{P(P^2-3P+3-\chi(P))}{(P-1)^3}\frac{N^2}{\log^3 N}.$$

where $\omega=\prod_{2\le P}P$ is called primorial, $\Phi(\omega)=\prod_{2\le P}(P-1)$.

It is the simplest theorem which is called the Heath-Brown problem [10].

（2）Let $n=P_0$ be an odd prime, $2\mid m$ and $m\neq\pm b^{P_0}$.

we have

$$P_3 = P_1^{P_0} + mP_2^{P_0}$$

We have

$$J_3(\omega) = \prod_{3 \le P} (P^2 - 3P + 3 - \chi(P)) \neq 0 ,$$

where $\chi(P) = -P + 2$ if $P | m$; $\chi(P) = (P_0 - 1)P - P_0 + 2$ if $m^{\frac{P-1}{P_0}} \equiv 1$ (mod P); $\chi(P) = -P + 2$ if $m^{\frac{P-1}{P_0}} \not\equiv 1$(mod P); $\chi(P) = 1$ otherwise.

Since $J_n(\omega) \neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.

We have

$$\pi_2(N,3) \sim \frac{J_3(\omega)\omega}{2P_0\Phi^3(\omega)} \frac{N^2}{\log^3 N}. \qquad .$$

The Polynomial $P_1^n + (P_2 + 1)^2$ **Captures Its Primes** [9]

（1）Let $n = 4$, We have

$$P_3 = P_1^4 + (P_2 + 1)^2 ,$$

We have Jiang function

$$J_3(\omega) = \prod_{3 \le P} (P^2 - 3P + 3 - \chi(P)) \neq 0 ,$$

Where $\chi(P) = P$ if $P \equiv 1$ (mod 4); $\chi(P) = P - 4$ if $P \equiv 1$ （mod 8）; $\chi(P) = -P + 2$ otherwise.

Since $J_n(\omega) \neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.

We have the best asymptotic formula

$$\pi_2(N,3) = \left| \{P_1, P_2 : P_1, P_2 \le N, P_1^4 + (P_2 + 1)^2 = P_3 \text{ prime}\} \right|$$

$$\sim \frac{J_3(\omega)\omega}{8\Phi^3(\omega)} \frac{N^2}{\log^3 N}.$$

It is the simplest theorem which is called Friedlander-Iwaniec problem [11].

（2）Let $n = 4m$, We have

$$P_3 = P_1^{4m} + (P_2 + 1)^2 ,$$

where $m = 1, 2, 3, \cdots$.

We have Jiang function

$$J_3(\omega) = \prod_{3 \le P \le P_i} (P^2 - 3P + 3 - \chi(P)) \ne 0 ,$$

where $\chi(P) = P - 4m$ if $8m \mid (P-1)$; $\chi(P) = P - 4$ if $8 \mid (P-1)$; $\chi(P) = P$ if $4 \mid (P-1)$; $\chi(P) = -P + 2$ otherwise.

Since $J_3(\omega) \ne 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime. It is a generalization of Euler proof for the existence of infinitely many primes.

We have the best asymptotic formula

$$\pi_2(N,3) \sim \frac{J_3(\omega)\omega}{8m\Phi^3(\omega)} \frac{N^2}{\log^3 N} .$$

（3）Let $n = 2b$. We have

$$P_3 = P_1^{2b} + (P_2 + 1)^2 ,$$

where b is an odd.

We have Jiang function

$$J_3(\omega) = \prod_{3 \le P} (P^2 - 3P + 3 - \chi(P)) \ne 0 ,$$

where $\chi(P) = P - 2b$ if $4b \mid (P-1)$; $\chi(P) = P - 2$ if $4 \mid (P-1)$; $\chi(P) = -P + 2$ otherwise.

We have the best asymptotic formula

$$\pi_2(N,3) \sim \frac{J_3(\omega)\omega}{4b\Phi^3(\omega)} \frac{N^2}{\log^3 N} .$$

（4）Let $n = P_0$, We have

$$P_3 = P_1^{P_0} + (P_2 + 1)^2 ,$$

where P_0 is an odd prime.

We have Jiang function

$$J_3(\omega) = \prod_{3 \le P} (P^2 - 3P + 3 - \chi(P)) \ne 0 ,$$

where $\chi(P)=P_0+1$ if $P_0\,|\,(P-1)$; $\chi(P)=0$ otherwise.
Since $J_3(\omega)\neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.
We have the best asymptotic formula

$$\pi_2(N,3)\sim \frac{J_3(\omega)\omega}{2P_0\Phi^3(\omega)}\frac{N^2}{\log^3 N}.$$

The Jiang function $J_n(\omega)$ is closely related to the prime distribution. Using $J_n(\omega)$ we are able to tackle almost all the prime problems in the prime distribution.

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