The "Missing" Measurements of the Gravitational Constant

Maurizio Michelini
ENEA- Casaccia Research Centre , Rome , Italy ; e-mail : 317michyz28@libero.it

The G measurements are made with torsion balance in "vacuum" to the aim of eliminating the air convection disturbance. Nevertheless, the accuracy of the measured G values appears unsatisfying. In 2000 J.Luo and Z.K.Hu first denounced the presence of some unknown systematic problem in high vacuum G measurement. In this work a new systematic effect is analysed which arises in calm air from the non-zero balance of the overall momentum discharged by the air molecules on the test mass in the vacuum chamber. This effect is normally negligible, but the disturbing force becomes comparable to the gravitational force when the chamber pressure drops to about 10^{-5} bar , at which the molecule mean free path equals the thickness of the meatus facing the test mass. At the epoch of Heyl's measurement at 1 millibar (1927), the technology of vacuum pumps reaching void levels up to 10^{-9} bar was developed, but this chance was not used. The recent G measurements used high vacuum techniques up to 10^{-10} bar and 10^{-11} bar, so the effect of the air meatus results very little. What happened to the "missing" measurements made at vacuum pressures in the "forbidden" interval between millibar and nanobar ? As a matter of fact, we were not able to find the related papers in the literature. This lack appears embarrassing in absence of an adequate physical explanation.

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Everyone knows the simple experience of two flat microscopy glasses which cannot be separated from each other when their surfaces touch. Obviously this effect is due to the pressure of external air whose molecules are unable to penetrate between the corrugations of the polished surfaces, so within the small meatus there is a considerable air depression. The mean free path of the air molecules at normal pressure is about 10^{-7} metres, that is of the same order of magnitude of the polished surface corrugations. In general, the molecules are not able to *freely* penetrate within a meatus whose thickness is reduced to about 1 mean free path .

When we consider the meatus facing the gravitational test mass of a torsion balance placed in a vacuum chamber, the little air depression within the meatus originates a little force on the test mass, which adds to the gravitational force. The disturbing force is often negligible, but when the pressure within the vacuum chamber is reduced beyond a millibar (for instance to avoid other disturbances due to air convection) the meatus optical thickness further reduces, so the above condition about 1 mean free path may occur. It appears necessary to investigate this phenomenon to obtain a quantitative prediction of the disturbing drawing force arising in the G measurements.

The torsion balance apparatus was first used by Cavendish in 1798 in a simple form which permitted him to reach an unexpected accuracy. In the following two centuries the torsion balance was used by several experimenters (Boys, Eotvos, Heyl, etc.) who substantially improved the technique, but the level of accuracy did not show a dramatic enhancement.

Several methods were devised in the XX century to measure G. In a Conference organised at London in 1998 - two centuries after Cavendish - by C.C.Speake and T.J.Quinn [1] a variety of papers described the methods of measurement and their potential accuracy related to the disturbances and systematic errors. Many experiments were described. For instance: a torsion balance where the gravitational torque is balanced by an electrostatic torque produced by an electrometer; a torsion-strip balance where the fibre is substituted by a strip; a dynamic method based on a rotating torsion pendulum with angular acceleration feedback; a *free fall* method where the determination of G depends on changes in acceleration of the falling object, etc.

Notwithstanding the technological improvement, up to now the gravitational constant is the less accurately known among the most important constants in physics. The uncertainty has been recognised to depend on various experimental factors.

To eliminate the air thermal convection on the test mass, in 1897 K.F. Braun made a torsion balance measurement after extracting the air from the ampoule. The level of vacuum obtained with his technique is not known.

In 1905 W. Gaede invented the rotary pumps reaching the void level of 10^{-6} bar. Subsequently Gaede developed the molecular drag pumps (1915) using Hg vapour. In 1923 the mercury was substituted by refined or synthetic oil, which enabled to reach void levels around 10^{-9} bar.

In 1927 Heyl [2] made a benchmark measurement with a heavy torsion balance to the aim of establishing a firm value of G. Although the high vacuum technology was available, he adopted a chamber pressure equal to 1-2 millibar. The molecule mean free path at 1 millibar is about 10^{-4} metres, a quantity much smaller than the thickness of the meatus. From our present investigation it appears that the air pressure effect does not alter the accuracy of the classical G measurements performed at pressures higher than some millibars. But this fact was unknown at the epoch. In any case the choice of high vacuum was compelling against the air convection disturbance.

After 1958 the development of turbomolecular pumps and the improved molecular drag pumps made available an ultra-high-vacuum up to 10^{-13} bar. Also this spectacular jumping was apparently disregarded by the G experimenters. In 1987 G.T. Gillies published an Index of measurements [3] containing over 200 experiments, which does not reveal the choice of pressures between the millibar and the nanobar. At the end of the ninety the indecisive measured values of G became publicly discussed.

A status of the recent G measurements was published in 2000 by J.Luo and Z.K.Hu [4] in which the presence of some unknown systematic effect was first denounced: "This situation, with a disagreement far in access to the estimate, suggests the presence of unknown systematic problems".

In 2003 R.Kritzer [5] concluded that "the large spread in G measurements compared to small error estimates, indicates that there are large systematic errors in various results".

Among the last experiments, some of them used new sophisticated methods with technologies coupled to very low pressures within the test chamber. This fact shows a new attention to the problems of possible unknown air effects.

J.H. Gundlach and S.M.Merkowitz [6] made a measurement where a flat pendulum is suspended by a torsion fiber without torque since the accelerated rotation of the attracting masses equals the gravitational acceleration of the pendulum.
To minimise the air dynamic effect, the pressure was lowered to 10^{-7} Torr ($p_o \approx 10^{-10}$ bar). At this pressure the classical mean free path $l = m/\sigma\, \delta_o$ (valid within a large homogeneous medium) is of the order of 1000 metres. Hence within the vacuum chamber the lack of flux homogeneity is everywhere present .

Another accurate measurement was performed in 2002 by

M.L.Gershteyn et al. [7] in which the pendulum feels a unique drawing mass fixed at different distances from the test mass. The change of the oscillation period determines G. To minimise the air disturbance, the pressure in the vacuum chamber was lowered to 10^{-6} Pascal (i.e. $p_o = 10^{-11}$ bar). The reason for such a dramatic lowering is not discussed. The authors revealed the presence of a variation of G with the orientation (regard to the fixed stars) amounting to 0.054%. Incidentally, the anisotropy of G agrees with the gravitational-inertial theory discussed in ref. 19.

In 2004 a new torsion balance configuration with four attracting spheres located within the vacuum chamber ($p_o = 1.5 \times 10^{-10}$ bar) was described by Z.K.Hu and J.Luo [8]. The four masses are aligned and each test mass oscillates between a pair of attracting masses. Each test mass determines with the adjacent spheres a small meatus (estimated about 4 mm) and a large meatus (about 16 mm). During the experiment the authors found the presence of an abnormal period of the torsion pendulum, which resulted independent of the material wire, test mass, torsion beam and could not be explained with external magnetic or electric fields. Adopting a magnetic damper system, the abnormal mode was suppressed, but the variance of the fundamental period of the pendulum introduced an uncertainty as large as 1400 ppm, testifying the presence of a systematic disturbance in determining G.

We applied to this problem the analysis carried out in this paper. From the air density of the vacuum chamber, we calculate the optical thickness of the small meatus and the related air depression (eq.5), which substituted in eq(7) gives upon the test mass a disturbing force rising up to $F(p_o) \approx 10^{-14}$ newton, equivalent to about 10^{-4} times the gravitational force, which alters the pendulum period. This fact agrees with the author conclusions [8] that the torsion balance configuration would have an inherent accuracy of about 10 ppm in determining G, but the uncertainty in the fundamental period reduces this accuracy to 1400 ppm.

The presence of an abnormal disturbance was previously described (1998) by Z.K.Hu, J.Luo, X.H. Fu et al. [9] in dealing with the time-of-swing method. They found the presence of "important non-linear effects in the motion of the pendulum itself, independent of any defect in the detector, caused by the finite amplitude of the swing*"*. Their configuration consisted in a torsion balance with heavy masses external to the vacuum chamber, where the pressure was lowered to $p_o = 2 \times 10^{-10}$ bar. The test mass, diameter about 19 mm, was suspended within a stainless vacuum tube placed between two heavy masses distant 60 mm apart. Since the test mass oscillates up to 8 mm from the centre of the vacuum tube, the optical thickness of the small meatus can be deduced. The smaller this thickness, the greater the disturbing force $F(p_o)$. Repeating the analysis carried out for the preceding experiment, we found a force $F(p_o)$ which represents a little fraction of the gravitational force, due to the heavy attractor masses.

Comparing with many measurements made in the era of high vacuum technology [10,11,12,13,14,15,16,17,18] we observed that the experimenters did never report "vacuum" pressures between millibar and nanobar. The reason for this avoidance does not appear to have been discussed.

The scattering of molecules hitting a smooth surface does not generally follow the optical reflection because they may interact with single atoms/molecules of the lattice. As it happens when two free particles come in collision, these molecules can be scattered in all directions. Conversely, the molecules hitting the surface from a nearly parallel direction interact softly with the field of the atomic lattice. In fact the nearly parallel molecules, whose little momentum $\boldsymbol{q} = m\text{v}$ makes an angle $\alpha \approx \pi/2$ with the vertical axis, receive from the field a small vertical momentum $\Delta q \approx 2\, m\text{v} \cos\alpha$ which redirects the molecules along a nearly optical reflection. Since the momentum *hv/c* of the UV rays is comparable to the momentum of air molecules at normal temperature, the air molecules hitting a polished surface show a phenomenon analogous to the limiting angle of optical reflection presented by photons incident the surface between two media.

To resume: after scattering on a flat surface a fraction of the nearly orthogonal molecules becomes quasi-parallel. As a consequence an isotropic flux ϕ_o of molecules hitting a smooth surface, after scattering becomes a non-isotropic flux $\psi_w(\alpha)$. This condition may be described by the relationship

$$(1) \qquad \psi_w(\alpha) \cong \phi_o\,(1 - \Delta_1 \cos\alpha + \Delta_2 \sin\alpha)$$

where ϕ_o the parameters Δ_1, Δ_2 satisfy the total flux condition $\int_0^{\pi/2} \sin\alpha\, \psi_w(\alpha)\, d\alpha = \phi_o$. Moreover we assume (similarly with the photon scattering on surgaces) that about χ percent of the nearly orthogonal molecules become quasi-parallel after scattering on the wall. Applying these two conditions one obtains the figures $\Delta_1 \cong 1.46\chi$, $\Delta_2 = 2\Delta_1/\pi \cong 0.928\chi$, where χ may range between 10% down to 0,01% for smoothed glass walls.

This physical condition makes easy to understand the molecular flux depression within the meatus around the moving mass. This phenomenon becomes particularly evident at low air pressures. For instance when the pressure is about 0.23 millibar, 99.99% molecules hitting the test mass come from scattering with other molecules within the meatus, whereas 0.01% molecules come directly from the scattering on the chamber wall. To feel a flux depression in the meatus it is necessary that the molecules coming from wall-scattering be about a half of the total. Within an air meatus of thickness s this happens when the optical thickness $\Sigma s = s\sigma\,\delta_o/m \approx 10^7 s\, \delta_o$ equals a mean free path, i.e. when the air density equals $\delta_o \approx 10^{-7}/s$. For usual torsion balances the critical vacuum pressure which maximise the flux depression is around $p_o \approx (1\div3) \times 10^{-5}$ bar.

The ancient G measurements adopted a torsion balance at atmospheric pressure, so the meatus effect took place between the test mass and the attracting sphere. This happens also to G measurements in vacuum when the heavy masses are comprised within the chamber. But in general the G measurements in vacuum are made with the heavy masses outside the chamber. In this case we define "meatus" the air comprised between the test mass and the adjacent wall of the vacuum chamber (fig.1). At a pressure of the order of some millibar the molecular flux upon the moving mass is highly uniform, so the sum of every momentum discharged by molecules on the sphere is null for any practical purpose.

However, when the pressure in the chamber is further reduced, the molecular flux begins to show a little depression in the meatus. The flux depression in the circular meatus may be expressed along the radial direction x

$$(2) \qquad \phi(x) \cong \phi_m\,(1 + k\,x^2)$$

where ϕ_m is the minimum figure the flux takes on the axis. Since the flux on the boundary ($x = L$) is the unperturbed flux ϕ_o, then one gets $\phi_m(1 + k\,L^2) = \phi_o$, which shows that k is linked to the flux parameters of the meatus

$$k = (\phi_o/\phi_m - 1)/L^2$$

where $L \cong R\cos\beta$ is the radius of the area of the test mass experiencing the flux depression. The angle β, defined by $\sin\beta = R/R{+}s$ (where R is the radius of the moving mass, s is the minimum thickness of the meatus), plays a fundamental role since it describes the *shadow* of the moving mass on the adjacent chamber wall.

Choosing spherical co-ordinates with the same axis of the meatus and origin (fig.1) in the point B, the monokinetic transport theory gives us the angular flux $\psi_B(\alpha)$ of incident molecules integrating along the meatus thickness $s(\alpha)$ the scattered molecules, to which one must add the uncollided molecules due to the flux $\psi_S(\alpha)$ scattered on the surface of the moving mass

$$\psi_B(\alpha) = \int_o^{s(\alpha)} \Sigma\,\phi(r)\exp(-\underline{\Sigma r})\,dr \;+\; \psi_S(\alpha)\;\exp(-\underline{\Sigma s}(\alpha))$$

where $\Sigma(r) = \sigma\,\delta(r)/m$ is the air macroscopic cross section; $\Sigma\,\phi(r)$ is the density of isotropically scattered molecules; $\underline{\Sigma s}(\alpha)$ is the optical thickness along α of the meatus. This angular flux holds for $\alpha \le \beta$. The above presentation of the problem has only an instructive character denoting the complexity of the problem, because the fluxes $\phi(r)$ and $\psi_S(\alpha)$ are unknown.

Fig.1 – Schematic drawing of a torsion balance comprised in a vacuum chamber.

To solve the problem of calculating the molecular flux within the meatus we adopt the principle of superposition of the effects. Consider the test sphere surrounded by the air in the vacuum chamber at pressure p_o. To obtain the disturbing force $F(p_o)$ on the test mass we must calculate the flux in the point A of the sphere and in the point C diametrically opposite (fig.1). Let's now *remove* the sphere and substitute an equal volume of air at pressure p_o , so the chamber results filled with the uniform molecular flux ϕ_o. Let's calculate the flux incident on both sides of the point A considering a spherical co-ordinates system with origin in this point (fig.1). The angular flux on the right-side of the point A is due to the scattering on the molecules within the sphere volume and to the uncollided molecules coming from the surface of the sphere (point P) where there is the uniform flux ϕ_o

(3) $$\psi'_+(\alpha) = \int_o^{t(\alpha)} \Sigma\,\phi_o\exp(-\Sigma\,r)\,dr \;+\; \phi_o\exp[-\Sigma\,t(\alpha)]$$

where $t(\alpha) = 2R\cos\alpha$ is the distance between the points A and P (fig.1) placed on the (virtual) surface of the removed mass. Let's notice that the first term in eq(3) represents the flux due to the scattering source occupying the sphere volume. When we cancel this source term (for instance reintroducing the test mass), eq(3) gives the flux

$$\psi_{A+}(\alpha) = \phi_o\exp(-2\Sigma R\cos\alpha)\ .$$

On the left-side of the point A the flux comes from scattering on the air within the meatus and from the uncollided molecules coming from the chamber wall

$$\psi_{A-}(\alpha) \;=\; \phi_o\,[1-\exp(-\Sigma z(\alpha))] + \psi_w(\alpha)\exp(-\Sigma z(\alpha))$$

where $z(\alpha)$ is the wall distance and $\psi_w(\alpha)$ is the flux scattered on the chamber wall, as defined by eq(1). Since in general the size of the chamber is much larger than R , one may assume the distance $z(\alpha) \cong s/\cos\alpha$. Subtracting the flux $\psi_{A+}(\alpha)$ from $\psi_{A-}(\alpha)$ gives the actual flux on the point A of the test mass

$$\psi_A(\alpha) \cong \phi_o\;[1-\exp(-2\Sigma R\cos\alpha)] - [\phi_o-\psi_w(\alpha)]\exp(-\Sigma s/\cos\alpha).$$

Now we calculate with the same procedure the incident flux on the point C

$$\psi_C(\alpha) \cong \phi_o\,[1-\exp(-2\Sigma R\cos\alpha)] - [\phi_o-\psi_w(\alpha)]\;\exp(-\Sigma\;(s+2R)/\cos\alpha)].$$

The disturbing force on the moving mass is linked to the different pressures on the points A and C due to the momentum discharged by the molecular flux on these points. The molecular flux shows the following difference across the test mass diameter

(4) $$\Delta\phi_o = (\phi_C - \phi_A) = \phi_o\int_0^{\pi/2}\sin\alpha\;[\psi_C(\alpha) - \psi_A(\alpha)]d\alpha\ .$$

Substituting and putting $w(\alpha) = \psi_w(\alpha)/\phi_o$, one gets the flux difference

$$\Delta\phi_o = \phi_o\int_o^{\pi/2}\sin\alpha\;\;[1-w(\alpha)]\;[\exp(-\Sigma s/\cos\alpha) - \exp(-\Sigma(s+2R)/\cos\alpha)]d\alpha$$

which confirms that the meatus depression depends on the anisotropy of the flux $\psi_w(\alpha)$ scattered on the wall. Through eq(1) we also have

$$w(\alpha) \;=\; 1-\Delta_1\cos\alpha + \Delta_2\sin\alpha$$

which, substituting in the above equation gives the relative air depression

(5) $$\Delta p_o/p_o \;=\; \Delta\phi_o/\phi_o = \Delta_1\,\Gamma(\Sigma s,\,\Sigma R) - \Delta_2\;\Omega\;(\Sigma s,\,\Sigma R)$$

where the functions

$$\Gamma\;(\Sigma s,\,\Sigma R) = \int_o^{\pi/2}\sin\alpha\,\cos\alpha\;[\exp(-\Sigma s/\cos\alpha) - \exp(-\Sigma(s+2R)/\cos\alpha)]d\alpha$$

$$\Omega\;(\Sigma s,\,\Sigma R) = \int_0^{\pi/2}\sin^2\alpha\;[\exp(-\Sigma s/\cos\alpha) - \exp(-\Sigma(s+2R)/\cos\alpha)]\;d\alpha$$

depend on the meatus geometry and on the air density $\delta_o = \Sigma\,m/\sigma$ in the vacuum chamber. These functions do not appear to have been tabulated. Fitting functions have been used for calculations, whose accuracy is not completely satisfying.

To give a quantitative idea, the relative depression $\Delta p_o/p_o$ has been calculated assuming the usual size of a torsion balance. For instance: test mass radius R = 5 mm, meatus thickness s = 4 mm, chamber pressure $p_o \approx 1$ millibar , air density $\cong 10^{-3}$ kg/m^3 , macroscopic scattering cross section $\Sigma \cong 10^4\,m^{-1}$, $\Sigma s \cong 40$. Substituting in eq(5) one obtains $\Delta p_o/p_o \approx 1.5\text{x}10^{-20}$ which shows the high uniformity of the molecular flux within the meatus at 1 millibar.

However the chamber pressure $p_o = 10^{-5}$ bar corresponds to a sensible depression $\Delta p_o/p_o \approx 3.37\text{x}10^{-3}$ which alters the gravitational force when the gravitational masses are aligned.

The disturbing force due to the small depression within the meatus $\Delta p(r) = mv[\phi_o - \phi(r)]$ is defined by

(6) $$F = \int_o^L 2\pi\boldsymbol{mv}\;r\;[\phi_o - \phi(r)]\;dr$$

where $L = R\cos\beta$ is the radius of the circular section of the meatus where $p(L) = p_o$. Substituting the flux distribution given by eq(2) one gets the pressure

$$p(r) = mv\phi_m\,(1+k\,r^2)$$

and the corresponding depression within the meatus

$$p_o - p(r) = \;p_o\,[1-(\phi_m/\phi_o)\;(1+k\,r^2)].$$

Substituting the expression of k by eq(2a) one obtains

$$p_o - p(r) = \;p_o\,(1-\phi_m/\phi_o)\;(1-r^2/L^2)$$

which, substituted in eq(6), gives us the force

(7) $$F = (\pi/2)\;p_o\,L^2\;\;(\Delta p_o/p_o)$$

where the relative depression is given by eq(5). Assuming for smoothed chamber walls a value χ = 0.1% we obtain the disturbing force reported in Table 1. One can notice that in the assumed torsion balance apparatus with light test mass (R = 5 mm) the disturbing force $F(p_o)$ takes a maximum at a pressure $p_o \approx$1pascal =10^{-5} bar which makes the optical thickness of the meatus about equal to 1. This maximum is estimated to rise *near the figure* of the measured gravitational force F_{gr} . Even taking into account the questionable accuracy of the fitting functions, the values of the disturbing force explain *ad abundantiam* why the region of the intermediate pressures between millibar and nanobar was avoided by the experimenters.

Obviously, what is of interest in the measurements is the systematic error due to $F(p_o)$. For instance in the Gershteyn's light torsion balance (where F_{gr} is estimated around 10^{-10} newton) the measurement was made at a pressure p_o =10^{-11} bar, so the disturbing force $F(p_o)$ gives a systematic error $\varepsilon \approx 2x10^{-8}$.

In the Heyl's heavy balance experiment (where the measured F_{gr} was of the order of 10^{-9} newton) the disturbing force $F(p_o)$ at a pressure p_o = 1 millibar gives $\varepsilon \approx 10^{-16}$. However the random error due to the air convection was probably around $\varepsilon \approx 10^{-4}$, that is much larger than the systematic error of the calm air effect.

Table 1 – Calculation of the disturbing force due to the air molecules within the vacuum chamber of a gravitational torsion balance. The assumed geometrical characteristics are : meatus thickness s = 4 mm , moving mass radius *R* = 5mm.

Vacuum pressure p_o (pascal)	*Air density* δ_o (kg/m^3)	*Meatus optical width* Σs (m.f.p.)	*Flux depression in the meatus* $\Delta\phi_o/\phi_o$	*Disturbing force* $F(p_o)$ (newton)
100	10^{-3}	40	$1.4x10^{-22}$	$1.0x10^{-25}$
10	10^{-4}	4	$2.86x10^{-6}$	$7.2x10^{-10}$
1	10^{-5}	0.4	$3.37x10^{-5}$	$8.4x10^{-10}$
0.1	10^{-6}	$4x10^{-2}$	$6.74x10^{-5}$	$1.7x10^{-10}$
10^{-2}	10^{-7}	$4x10^{-3}$	$1.8x10^{-5}$	$4.5x10^{-12}$
10^{-3}	10^{-8}	$4x10^{-4}$	$\approx 4.4x10^{-6}$	$\approx 1.1x10^{-13}$
10^{-4}	10^{-9}	$4x10^{-5}$	$\approx 1.1x10^{-6}$	$\approx 2.8x10^{-15}$
10^{-5}	10^{-10}	$4x10^{-6}$	$\approx 2.8x10^{-7}$	$\approx 7x10^{-17}$
10^{-6}	10^{-11}	$4x10^{-7}$	$\approx 8x10^{-8}$	$\approx 2x10^{-18}$

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