Abstract:

Jiang Chun-Xuan is a Chinese mathematician who claims to have developed new number theoretic tools consisting mostly in the Jiang function $J_n(s \#)$ where $s \# = 2.3.5...p, p < n$ denotes the primorial function to solve fundamental problems in Number Theory such as the Goldbach Conjecture, the Twin Prime Conjecture, the k-tuple Conjecture, and some other 600 basic prime number theorems. The fundamental motivation of Jiang to develop a number theory different from the one we are familiar with (we, number theorists) comes from his recent claim (1997) that the Riemann Hypothesis (RH) which lies at the foundations of all prime number theories, is false, that all calculations done to improve it are misunderstood, and that the entire speculative theory done through it (see Connes, Bombieri, Zagier et al.) are eventually uncertain. Our goal in this paper will be to review Jiang’s achievements from his disproof of RH to his establishment of the new number theory.

1) Jiang 1997 disproof of RH:

The function $\zeta(s)$ of the great mathematician Riemann is defined over the complex number by

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p} \frac{1}{1 - \frac{1}{p^s}}$$

when $\text{Re } s > 1$. This function is claimed by Riemann himself [2] to satisfy the following functional equation

$$\pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) \zeta(s) = \pi^{-\frac{1-s}{2}} \Gamma\left(\frac{1-s}{2}\right) \zeta(1-s)$$

where $\Gamma(s) = \int_{0}^{\infty} e^{-t} t^{s-1} dt$ denotes the Euler Gamma function.

The simplest form of Riemann functional equation is often denoted by

$$\xi(s) = \xi(1-s)$$
In their independent 1896 proofs of the prime number theorem, Hadamard and De La Vallée Poussin stated basically that

$$\zeta(s) \neq 0, s = 1$$

and it is basically evident that $$\zeta(s)$$ has no zero for $$\text{Re } s = 1$$.

Riemann it his epoch-making 1859 paper [2] stated that all the nontrivial roots of his function lie in the critical strip [0,1] and made the following:

**Riemann Hypothesis (RH)**: $$\zeta(s) = 0 \Rightarrow s = \frac{1}{2} + it$$ where one ignores the trivial zeros –2, -4, et al.

RH has become throughout the past decades the most fundamental problem in Analytic Number Theory and prime number theory.

In [1] Jiang defined a new function $$\beta(s)$$ that is the dual of Riemann zeta-function:

$$\beta(s) = \sum_{n=1}^{\infty} \frac{\lambda(n)}{n^s} = \prod_{p} \frac{1}{1 + \frac{1}{p^s}}$$

where $$\lambda(n)$$ denotes the Liouville function and the duality is exhibited by:

$$\zeta(s) \beta(s) = \zeta(2s)$$

and then directly

$$\zeta\left(1 + 2it\right) = \zeta\left(\frac{1}{2} + it\right) \beta\left(\frac{1}{2} + it\right)$$

Jiang mostly proved in [1] that the beta-function is not infinite for real part equal to $$\frac{1}{2}$$ and then, following the fundamental remark of Hadamard and De la Vallée Poussin, Riemann zeta-function cannot have nontrivial zeros in the said critical line $$\text{Re}(s) = \frac{1}{2}$$.

The duality identity above allows us to reformulate interestingly RH in term of the beta function. The beta function exhibits trivial zeros so that each odd negative integer is a trivial zero of the beta function following basically

$$\beta(-k) = \frac{\zeta(-2k)}{\zeta(-k)} = 0, k = 2n + 1, n \in \mathbb{N}$$

and one the other hand $$\beta(s)$$ exhibits nontrivial zeros, being established that $$\gamma_n$$ denoting the imaginary part of the n-th nontrivial zero of the zeta function,
following the Riemann 1859 assertion that all the nontrivial zeros of the zeta function have real parts equal to ½, with the following formalism

\[ \beta \left( \frac{1}{4} + i \gamma_n \right) = \zeta \left( \frac{1}{4} + i \gamma_n \right) = 0 \]

The imaginary parts of the n-th nontrivial zeros of the beta function being denoted by \( \gamma_n \). RH then becomes:

**Riemann Hypothesis\(^*\) (RH\(^*\))**: \( \beta(s) = 0 \Rightarrow s = \frac{1}{4} + i \gamma_n, s \neq -k \) where one ignores the trivial zeros \(-1, -2, -3 \text{ et al.}\).

The new “critical line” parameterised with \( \text{Re} s = \frac{1}{4} \) being the equivalent of the critical line corresponding to the zeta function.

One may easily compute the nontrivial zeros of the beta function from a table of zeros of the zeta function, as recalled above (here for curiosity) \[19\] :

<table>
<thead>
<tr>
<th>( \gamma_n )</th>
<th>( \gamma_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.1347251</td>
<td>7.06736257</td>
</tr>
<tr>
<td>21.0220396</td>
<td>10.5110198</td>
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<td>25.0108576</td>
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<td>28.2231239</td>
</tr>
<tr>
<td>59,347044</td>
<td>29.673522</td>
</tr>
</tbody>
</table>

On the other hand we have clearly that the duality identity recalled above implies the trivial zeros of the zeta function to be the trivial zeros of the beta function, and the nontrivial zeros of the zeta function then become the nontrivial poles of the beta function. Then all \( \gamma_n \) denote both the imaginary parts of the nontrivial zeros of the zeta function and the nontrivial poles of the beta function. RH then becomes expressible in term of the nontrivial poles of the beta function into a third reformulation that keeps it basically unchanged.
**Riemann Hypothesis** (RH) : \( \beta(s) = \infty \Rightarrow s = \frac{1}{2} + i\gamma_n, s \neq -2k \) where one ignores the trivial zeros at -2,-4,-6 et al.

RH \iff all the nontrivial poles of the beta function lie along the “critical line” \( \text{Re} \ s = \frac{1}{2} \).

Jiang starts with an amazing expression for both \( \beta(s) \) and \( \zeta(s) \) which he coins their exponential formulas. These formulas, to the best knowledge of the present author, are not found in other RH books (in none of them) and are sufficient to Jiang to follow his entire disproof. These formula could be introduced through the following entry to make the reader more familiar with them

**Entry 1** :

\[
|\zeta(s)| = \prod_p \frac{1}{\sqrt{1 - \frac{2}{p^s} + \frac{1}{p^{2s}}}}
\]

**Proof**: The proof is obvious

\[
\prod_p \frac{1}{\sqrt{1 - \frac{2}{p^s} + \frac{1}{p^{2s}}}} = \prod_p \frac{1}{\sqrt{(1 - \frac{1}{p^s})^2}} = \prod_p \frac{1}{1 - \frac{1}{p^s}} = |\zeta(s)|
\]

For all \( s \in R, s \succ 1 \).

However a complete proof of them will be far from this easy entry, as it will follow further.

**Theorem 1.1.** :

\[
\zeta(s) = \frac{1}{\prod_p \sqrt{1 - \frac{2\cos(t \log p)}{p^\sigma} + \frac{1}{p^{2\sigma}}}} e^{i \sum_{p} \tan^{-1} \left( \frac{\sin(t \log p)}{p^\sigma - \cos(t \log p)} \right)}
\]

**Theorem 1.2.** :

\[
\beta(s) = \frac{1}{\prod_p \sqrt{1 + \frac{2\cos(t \log p)}{p^\sigma} + \frac{1}{p^{2\sigma}}}} e^{i \sum_{p} \tan^{-1} \left( \frac{\sin(t \log p)}{p^\sigma + \cos(t \log p)} \right)}
\]
**Proof**: Let \( s = \sigma + it \) we have

\[
1 - \frac{1}{p^s} = 1 - e^{-it \ln p} = 1 - \frac{e^{-it \ln p}}{p^\sigma} = 1 - \frac{\cos(t \ln p) - i \sin(t \ln p)}{p^\sigma} = 1 - \frac{\cos(t \ln p)}{p^\sigma} + i \frac{\sin(t \ln p)}{p^\sigma}
\]

Let then \( x = 1 - \frac{\cos(t \ln p)}{p^\sigma}, y = \frac{\sin(t \ln p)}{p^\sigma} \). Then,

\[
x + iy = \sqrt{x^2 + y^2} e^{i \arg(x + iy)} = e^{i \arg(x + iy)} \sqrt{x^2 + y^2}
\]

On the other side

\[
\sqrt{x^2 + y^2} = \sqrt{\left(1 - \frac{\cos(t \ln p)}{p^\sigma}\right)^2 + \frac{\sin^2(t \ln p)}{p^{2\sigma}}} = \sqrt{1 - \frac{2\cos(t \ln p)}{p^\sigma} + \frac{\cos^2(t \ln p)}{p^{2\sigma}} + \frac{\sin^2(t \ln p)}{p^{2\sigma}}}
\]

\[
= \sqrt{1 - \frac{2\cos(t \ln p)}{p^\sigma}} + 1
\]

We have on the other side

\[
\tan(\arg(x + iy)) = \frac{y}{x}
\]

\[
\frac{y}{x} = \frac{\sin(t \ln p)}{p^\sigma - \cos(t \ln p)} \iff \arg(x + iy) = \tan^{-1}\left(\frac{\sin(t \ln p)}{p^\sigma - \cos(t \ln p)}\right)
\]

We finally obtain

\[
\prod_p \left(1 - \frac{1}{p^s}\right) = \prod_p \sqrt{1 - \frac{2\cos(t \ln p)}{p^\sigma} + \frac{1}{p^{2\sigma}}} e^{i \sum_p \tan^{-1}\left(\frac{\sin(t \ln p)}{p^\sigma - \cos(t \ln p)}\right)}
\]

The theorem 1.1. now follows.

However a similar demonstration has to be done in respect to the Jiang beta function to obtain the second identity.

Further considerations about Jiang’s proof are found in [1,3,4].

In [4] it is above all seen that the French mathematician Antoine Balan [5] found a result about RH that is the exact opposite of those obtained by Jiang. Therefore it seems to us that

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4 Jiang himself does not give any proof of these beautiful identities in [1,3] or all his number theory papers and the author himself tried by many attempts to construct a proof of these identities. This one is the one Jiang himself had in mind when disproving RH, following [38].

5 The author has no words to acknowledge Jiang Chun-Xuan for its highly helpful collaboration [38].
the falsity of Jiang’s 1997 statement is best showed by showing that Balan is all right. But however Balan seems to be specialized in number theory, while Jiang is. Balan’s paper, which is without doubts great, is the only one exactly opposed to Jiang’s [1] but has not attracted more attention than [1] in number theoretic circles, being also misunderstood (as said Dan Velleman, doubts have to come simultaneously in direction to Jiang’s and Balan’s paper until one of them will be entirely verified or disproved : after doubts will disappear while verifying one of them is proving great interest for the future of number theory).

Jiang’s papers have been sent worldwide to mathematicians of the stature of Alain Connes, Enrico Bombieri, Terence Tao et al. but these great number theorists did not give any priority to the consideration of Jiang’s work. This is a most comprehendible reaction, given the number going larger and larger of laymen who try to solve the greatest unsolved problems in mathematics without knowing much about the technical literature about their own field of research.

On the other hand one may also imagine how distasteful it should be to mathematician to show them that the greatest mathematical conjecture ever, that seems provide the number theoretical foundations of mathematics. With the time some ultimate beauty has been assigned with the true of RH. To differ from this point of view, Jiang quoted further Iwaniec:

“Analytic number theory is fortunate to have one of the most famous unsolved problems, the Riemann hypothesis. Not so fortunately, this puts us in a defensive position, because outsiders who are unfamiliar with the depth of the problem, in their pursuit for the ultimate truth, tend to judge our abilities rather harshly. In concluding this talk I wish to emphasize my advocacy for analytic number theory by saying again that the theory flourishes with or without the Riemann hypothesis. Actually, many brilliant ideas have evolved while one was trying to avoid the Riemann hypothesis, and results were found which cannot be derived from the Riemann hypothesis. So, do not cry, there is healthy life without the Riemann hypothesis. I can imagine a clever person who proves the Riemann hypothesis, only to be disappointed not to find new important applications. Well, an award of one million dollars should dry the tears; no applications are required.” [6]

However it is largely false to believe that Jiang never reached the attention and priority of geniuses belonging to the mainstream. The most stupefying example of this is given by the case of Don Zagier, surely the world’s greatest specialist about modular and automorphic forms.

Jiang contacted Zagier as it is quoted in Jiang’s huge monograph [3], Foundations of Santilli’s Isonumber Theory with Applications to new cryptograms, Fermat’s Last theorem and Goldbach’s Conjecture, where he compiled his contributions to number theory, including his disproof of RH, his “proofs” of the Goldbach Conjecture and the Twin prime Conjecture, along with the newly established Isonumber Theory (following [12]) and some 600 other claimed prime number theorem. (The pages concerned by the reference to Zagier are 170 and 337).

The correspondence between Jiang and Zagier, as it follows from the data available in [3], started on March 1984, and was intense enough to the impression of Jiang, during a few months (seeming to end on page 337 in [3] the 2 July 1984). As it is now well-known, and

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6 Daniel J. Velleman, private communication, 25/01/2008 : “number theorists are right to doubt the work of both Balan and Jiang until it is in publishable form.”
related in [21], Zagier during his youth manifested a clear tendency to believe RH to be false. He was then opposed to Bombieri, the other world’s greatest specialist about RH, who believed RH to be true. It is interesting to note that Jiang has contacted Zagier at the time at which he doubted that RH is actually true, similarly as Littlewood, and even Alan Turing [22,23,24,25], who tried to found a counter-example, while establishing the basis of the further massive computation of the nontrivial zeros of the zeta function [19].

Zagier nevertheless began to tend to believe RH to be true when considering the spectacular development of the computations of the nontrivial zeros of the zeta function [26]. Jiang’s own later objections to RH tended to point out some errors in the computations of the so called zeros that keep Number Theory into a coherent space of calculations but with a non-appropriate formalism (because Jiang claims the error of calculations is satisfied by the error of RH). Zagier wrote his famous article “The first 50 million prime numbers”, and founded the theory of toroidal automorphic forms from which RH for finite fields has recently been proved [27].

In order to follow the mainstream prime conception, Jiang argues that:

“The distribution of prime number does not involve Quantum chaos, randomness et al. There is order in the sequence of prime numbers.” [7]

This view has been received with enthusiasm by the great philosopher Stein Johansen in [8].

Moreover Jiang’s works seems to move along with the development of Hadronic Mechanics pioneered by Ruggero Maria Santilli, as seen in [3] and particularly in Isonumber Theory. (If Jiang’s work is right then it is the foundation of Isonumber Theory. In particular Santilli himself claimed in [3] :

“I would like to express my utmost appreciation to Professor Chun-Xuan Jiang for having understood the significance of the new iso-, geno-, hyper-numbers and their isoduals I identified for a resolution of the above problems. The significance of the new numbers had escaped other scholars in number theory in the past two decades since their original formulation. I would like also to congratulate Professor Jiang for the simply monumental work he has done in this monograph [e.g. Foundations of Santilli’s Isonumber Theory with Applications to new cryptograms, Fermat’s Last theorem and Goldbach’s Conjecture], work that, to my best knowledge, has no prior occurrence in the history of number theory in regard to joint novelty, dimension, diversification, articulation and implications. I have no doubt that Professor Jiang’s monograph creates a new era in number theory which encompasses and includes as particular case all preceding work in the field.”

More recently Indian number theorist Tribikram Pati claimed to have disproved RH in [9] By showing that RH is equivalent to :

7 Santilli is in fact by far the main publisher of Jiang’s papers as the editor in chief of the Hadronic Journal the Algebras, Groups and Geometries, etc. Knowing about Jiang’s disproof of RH, he decided to publish it in this last journal (the great seismologist Chen I Wan [34,35] making Santilli aware of the Jiang Chun-Xuan phenomenon in China). In 2002 Santilli rewrote Jiang’s submitted RH paper and suggested him to investigate the Jiang function [33].
The proof given by Pati of this equivalence follow a succession of lemmas in [9] which are almost all taken from the so called “RH-bible” [20]. A decisive theorem found in [20] could be used in the future to improve or disprove Jiang’s claim that the zeta function does not admit any zero on the critical line, because it states that:

**Theorem 2.1:** Assume RH. Then for $t \gg \tau \gg 0$

$$\left|\zeta\left(\frac{1}{2}+it\right)\right| \leq C' e^{\frac{\ln t}{\ln \ln t}}$$

Where $C'$ and $C$ are two positive real constants such that $C' > 1$.

**Proof:** It is found in Titchmarsh [20]. Theorem 14.14 (A).

**Remark:** Jiang’s claims that $\left|\zeta\left(\frac{1}{2}+it\right)\right| \neq 0, \forall t$ is equivalent to:

$$C'' e^{\chi(t)} \leq \left|\zeta\left(\frac{1}{2}+it\right)\right| \leq C' e^{\frac{\ln t}{\ln \ln t}}$$

Where $C'' > 0$ and $\chi(t)$ is a function of $t$ that would be required to be similar to those taken by Titchmarsh and Pati on the right side. One should also reformulate the theorem 1 in term of the beta function:

**Entry 3:** Assume $RH \iff RH^* \iff RH^{**}$ then

$$\left|\beta\left(\frac{1}{2}+it\right)\right| \geq \left|\zeta(1+it)\right| C^{-1} e^{-\frac{\ln t}{\ln \ln t}}$$

**Proof:** It is obvious:

$$\left|\zeta\left(\frac{1}{2}+it\right)\right| = \left|\frac{\zeta(1+it)}{\beta\left(\frac{1}{2}+it\right)}\right| \iff \left|\beta\left(\frac{1}{2}+it\right)\right|^{-1} \leq \left|\zeta(1+it)\right| C' e^{\frac{\ln t}{\ln \ln t}}$$

Since $\left|\zeta(1+it)\right|$ this imply that $\left|\beta\left(\frac{1}{2}+it\right)\right| \geq 0$ as expected.

But the fundamental lemma in Pati’s claimed proof is introduced by Pati himself and states that:
**Theorem 2.2:** If \( a(t) \geq 0 \) and \( k(t) \geq 0 \) for \( t > \tau > 0 \) and there exist \( \theta > 0 \) and both \( a \) and \( k \) tend to be infinite as \( t \) tend to be infinite such that:

\[
\lim_{{t \to +\infty}} \frac{a(t)}{k(t)} = +\infty
\]

and for \( t > \tau, b(t) = 1 + \frac{\theta}{k(t)} \) then for all \( t \) greater or equal to \( \tau_0 \), a sufficiently larger constant than \( \tau \) then directly:

\[
a(t) + b(t) < a(t)b(t)
\]

The proof of this theorem is done by Pati in [9], and is rather easy (and now well-known by some number theorists).

Pati obtains the paradox above from the Theorem 1 by defining:

\[
a(t) = \frac{1}{C} \left( C' \ln t \ln \ln \ln t \ln \ln t + C'' \ln t \left( \frac{C' \ln t \ln \ln \ln t}{\ln \ln t} + \theta \right) \right)
\]

\[
b(t) = \frac{C' \ln t \ln \ln \ln t}{\ln \ln t} + \theta
\]

\[
k(t) = \frac{C' \ln t \ln \ln \ln t}{\ln \ln t}
\]

Where \( C, C', C'', \theta \) denote fixed positive constants. It is rather easy as it is seen in [9], that by using Pati notations it follows that:

\[
b(t) = 1 + \frac{\theta}{k(t)}, \lim_{{t \to +\infty}} a(t) = +\infty, \lim_{{t \to +\infty}} k(t) = +\infty, \lim_{{t \to +\infty}} \frac{a(t)}{k(t)} = +\infty,
\]

so that Pati claims he is able to apply the theorem [9].

\[
\ln \left( \frac{\zeta \left( \frac{1}{2} + it \right)}{C_z \exp \left( \frac{C_2 \ln \ln \ln t + C_2 \ln \ln t + \theta}{\ln \ln t} \right) e^{C_4 \ln t + \theta}} \right) < C_3 \frac{\ln t \ln \ln \ln t}{\ln \ln t} b(t) + C_4 \ln t = C_3 \frac{\ln t \ln \ln \ln t}{\ln \ln t} + C_4 \ln t + \theta
\]

\[
\Rightarrow \ln e^{C_2 \exp \left( \frac{C_2 \ln \ln \ln t + C_2 \ln \ln t + \theta}{\ln \ln t} \right) + C_4 \ln t + \theta} < C_3 \frac{\ln t \ln \ln \ln t}{\ln \ln t} + C_4 \ln t + \theta
\]
Since Pati’s is based on theorem 1 which is implied by RH, Pati claims RH would imply this paradox and therefore RH is false. Pati’s works, moreover, seems to us manifest a tendency to improve or mimic Jiang’s disproof. Pati manifested furthermore interests in Jiang’s works and in reading [1,4,10].

Resulting correspondence with Schadeck and Pati, Jiang get in 2008 the idea to write a fundamental paper [11]: *Riemann 1859 Paper Is False*, which is not yet published and rejected by some number theorist belonging or not to the mainstream.

In this most astonishing paper (the most impressive he has ever written) Jiang claims that the functional equation stated by Riemann is respected by a function $\zeta(s)$ that is not the same that $\zeta(s)$.

In [11] one explicitly founds that

$$\pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right)n^{-s} = \int_0^\infty x^{\frac{s-1}{2}} e^{-x^{\frac{s}{2}} \pi x} dx = \int_0^\infty x^{\frac{s-1}{2}} \left(\frac{\theta(x) - 1}{2}\right) dx$$

Where $\theta(x) := \sum_{n=\infty}^{\infty} e^{-\pi^2 x}$ is the Jacobi theta function whose functional equation is:

$$x^2 \theta(x) = \theta(x^{-1})$$

where the variable has to be taken positive. From it, which is a most BASIC well-known by all number theorists and even all real mathematicians Jiang claims that he obtains;

$$\overline{\zeta}(s) = \frac{\pi^{\frac{s}{2}}}{\Gamma\left(\frac{s}{2}\right)} \left\{ \frac{1}{s(s-1)} + \int_1^\infty \left(\frac{x^{\frac{s-1}{2}}}{x^{\frac{1}{2}}} + x^{\frac{s-1}{2}}\right) \cdot \left(\frac{\theta(x) - 1}{2}\right) dx \right\}$$

$$\Leftrightarrow \pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) \overline{\zeta}(s) = \pi^{-\frac{1-s}{2}} \Gamma\left(\frac{1-s}{2}\right) \overline{\zeta}(1-s)$$

Then the properties of these newly formed function are:

1. $\overline{\zeta}(s)$ has no zero for $\sigma > 1$;
2. The only pole of $\overline{\zeta}(s)$ is at $s = 1$; it has residue 1 and is simple;
3. $\overline{\zeta}(s)$ has trivial zeros at $s = -2, -4, \ldots$ but $\zeta(s)$ has no zeros;
4. The nontrivial zeros lie inside the region $0 \leq \sigma \leq 1$ and are symmetric about both the vertical line $\sigma = \frac{1}{2}$

and from them Jiang claims that RH is expressed only in term of the new function which we call here the *pseudozeta-function* and then refers to his disproof in [1] and says:
He finishes his considerations about RH and the Riemann 1859 paper by giving brief courses about the new number theory he suggests to the upcoming generation of mathematicians. However the considerations about Riemann 1859 paper haven’t attracted any attention from scholars; it is without doubt the most obscure step in Jiang’s works, which imply some further utmost objections. The new Number Theory developed by Jiang has nevertheless the advantage to manifest:

1. A good connection and a great compatibility (perhaps the greatest) with Santilli’s isomathematics which are Lie-admissible mathematics (see [12] for more information because it should take hundreds of pages to introduce it, what we cannot doing here for evident need of brevity).
2. The Prime distribution manifests order rather than randomness (see discussion about that in [40] by Tao).
3. A great are of applications including ISOCRYPTOGRAPHY which may constitute the greatest cryptographic system in the World (see [3] and inspect the impressive last chapter).
5. and so on (the list cannot here be exhaustive and it is recommended to the interested reader to inspect [3] for some more details).

Just like prime number theory and analytic number theory are roughly the study of Riemann zeta function, one has clearly to say that Jiang Number Theory (the new JNT) is EXCLUSIVELY the study of the class of functions \( J_n(p\#) \) with respect to the index integer \( n \).

Then one has to start with Santilli’s basic rules of isomathematics found in [3], [12] and a larger and larger literature, where we start by recalling the Santilli isounit

\[
\hat{I} = \frac{1}{T} \neq 1, \hat{I} > 0
\]

with related Santilli isonumbers

\[
\hat{A} = \hat{I} \times A
\]

and Santilli isoproduct

\[
\hat{A} \times B = A \times T \times B
\]

and finally the fundamental identity define for the Santilli isounit through the Santilli isoproduct:

\[
\hat{I} \times A = T^{-1} \hat{T} A = A \hat{I} = A T T^{-1} = A
\]

etc.
More details are found in Jiang [3] and Santilli [12].

**Definition 2.1 (Jiang [3]):**

\( \hat{d} \) isodivides \( \hat{n} \) and we write \( \hat{d} \mid \hat{n} \) when \( \hat{n} = \hat{c} \times \hat{d} \) for some \( \hat{c} \).

Similarly, \( \hat{d} \) does not isodivide \( \hat{n} \) and we write \( \hat{d} \nmid \hat{n} \) when \( \hat{n} \neq \hat{c} \times \hat{d} \) for some \( \hat{c} \).

One notes that isodivisibility is similar to conventional divisibility with respect to Santilli isonumbers and Santilli isoproduct.

From isodivisibility Jiang defines (and it is rather instinctive to define) isocongruences by the following:

**Definition 2.1 (Jiang [3]):**

Given isointegers \( \hat{a}, \hat{b}, \hat{m} \) with \( \hat{m} > 0 \). We say that \( \hat{a} \) is isocongruent to \( \hat{b} \) module \( \hat{m} \) and we write

\[ \hat{a} \equiv \hat{b} \text{ (mod } \hat{m}) \]

when \( \hat{a} \mid \hat{n} \).

The isocongruence, just as the isodivisibility, satisfies all axioms of the conventional congruence (resp. the conventional divisibility). Here the term conventional would refer to what is commonly coined *conventional mathematics*, namely *unitary* pre-Santilli mathematics (the author proposed the term *unimathematics* because the unit is always equal to 1 : as says Rowlands quoted in [8] “the 1 is already loaded”).

Then in [3] Jiang investigates a large number of isoequations consisting into isocongruences and defines the Jiang function \( J_n(p) \) through the following theorem:

**Theorem 2.13 (Jiang [3]):**

The equation

\[ \sum_{i=1}^{n} \hat{x}_i \equiv \hat{A} \text{ (mod } \hat{p}) \]

where \( \hat{p} \in \hat{P} = \hat{IP}, P = \{ p = \text{prime} \} \) has exactly \( J_n(p) + (-1)^n \) solutions if \( \hat{d} \mid \hat{n} \) and has exactly \( J_n(p) \) when \( \hat{d} \nmid \hat{n} \).

**Proof:** It is not found in [3] but the author could explain what it should be.

We know that the congruence equation \( \sum_{i=1}^{n} x_i \equiv A \text{ (mod } p) \) has the same solutions than

\[ Ax_1 + \sum_{i=2}^{n} x_i \equiv 1 \text{ (mod } p) \]  [32]. Thus it has the same number of solutions.
Then

**Definition A :**

(Fundamental definition in Jiang Number Theory)

\[ J_n(p) := \frac{(p-1)^n - (-1)^n}{p} \]

Moreover the Jiang functions \( J_n(p) \) is often obtained at the very beginning in [3] to count the number of solutions of such basic isoequations that involve insocongruences and isoprimes and isodivisibility.

An other most general example as in theorem 2.13, that involves multivalued functions of Santilli isointegers is:

**Theorem 2.13 (Jiang [3]) :**

The equation

\[ f(\bar{x}_1, \ldots, \bar{x}_n) \equiv A \pmod{p} \]

has \( W_n \) solutions and then \( W_n = J_n \left( 1 + O(1) \right) \).

Hundreds of such theorems which are basically obtained by lifting the unitary ones into “unimathematics” are found in [3].

The function \( J_n(p) \) is extended to \( J_n(p\#) \) by the definition B still found in [3] by

**Definition B :**

(Extended fundamental definition in Jiang Number Theory)

\[ J_n(N\#) := \prod_{3 \leq p \leq N} \left( \frac{(p-1)^n - (-1)^n}{p} \right) \prod_{p \mid N} \left( 1 + \frac{(-1)^n p}{(p-1)^n - (-1)^n} \right) \]

where \( N \) denotes a Santilli isointeger.

The most basic property of Jiang function is that \( J_n(p\#) \neq 0 \).

Jiang claims that he get the idea to define his function in 1997 by making use the basic definitions of Euler’s totient function undefined explicitly but most useful in Arithmetic and usable through a list of simple properties such as:

1. \( \phi(p) = p - 1 \) when \( p \) is a prime.
2. \( \phi(p^A) = p^A \left(1 - \frac{1}{p}\right) \), p a prime, that looks like the product expression of Riemann zeta function.

3. \( \phi(n) \) is ALWAYS EVEN for all \( n \geq 3 \).

4. \( \phi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right) \)

Usually Euler \( \phi(n) \) function is taken to count the number of integers prime to a given integer. One has firstly to note the resemblance between \( \phi(p) \) through its 4\(^{th} \) property and the Definition B of Jiang function.

Moreover, given the formal definition of what the number theorists now call the twin prime constant \( \Pi_2 \)

\[
\Pi_2 = \prod_{3 \leq p} \left(1 - \frac{1}{(1 - p)^2}\right)
\]

one should think that it is deeply connected to Jiang function and also related to Riemann zeta-function, as the author could have shown it in a paper that would perhaps be published after the present year.

We would claim here that

**Definition C:**

(non-formal given the mathematical knowledge of our time)

<table>
<thead>
<tr>
<th>Euler’s ( \phi(n) ) function is the arithmetical pattern(^8 ) of Jiang’s ( J_n(p#) ) function throughout Santilli’s Isomathematics, that is, Jiang’s ( J_n(p#) ) function is naturally generated into Isomathematics through Euler’s ( \phi(n) ) function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jiang often says in [3] that Jiang’s ( J_n(p#) ) function is a generalization in fact of Euler’s ( \phi(n) ) function, that it counts the number of solutions of basic isoequations from which its definition follows, just as Euler’s ( \phi(n) ) function counts the numbers of integers that are prime to a given integer and less than itself.</td>
</tr>
<tr>
<td>Jiang himself says :</td>
</tr>
<tr>
<td>“Let p# = 30, Euler function ( \phi(30) = \prod_{p \leq \pi} (p-1) = 8 ), We have ( (30, j) = 1 ) [where ( (a, b) ) denotes the greatest common divisor or gcd], where ( j = 1, 7, 11, 13, 17, 19, 23, 29 ). We have 8 equations, ( p(j) = 30i + j, j = 1, 7, 11, 13, 17, 19, 23, 29 ). Every has infinitely many prime solutions</td>
</tr>
</tbody>
</table>

\(^8\) However there is many reason for which the mathematical concept of pattern has to be useful for the future of mathematics and also for which this remark is in a way useful. It is well-known that mathematical structures are best and most generally studied under the concept of category and modern category theory [41]. Let us realise that categories are all built from numbers and let an hypercategory denotes any sequence of category constituting Mathematics or a mathematical theory (or an algebra). A pattern generate hypercategories.
We study twin primes $p_2 = p_1 + 2, J_2(30) = \prod_{p < p_i} (p-2) = 3$. We have 3 twin primes sub equations: $p_2 = p_{11} + 2 = p_{13}, p_2 = p_{17} + 2 = p_{19}, p_2 = p_{29} + 2 = p_1$. Every has infinitely many twin primes solutions.

We study $p_3 = p_2 + p_1 + 1, 8^2 = 64$. We have 64 equations,

$$J_3(30) = \prod_{p < p_i} ((p-1)^2 - \chi(p)) = \prod_{p < p_i} (p^2 - 3p + 3) = 39.$$  

We have 39 sub equations: $p_3 = p_1 + p_{11} + 1$ write as $p_3 = 1 + 11 + 1, 
11 + 1 + 1, 1 + 17 + 1, 1 + 29 + 1, 29 + 1 + 1, 11 + 11 + 1, 11 + 17 + 1, 17 + 11 + 1, 11 + 19 + 1, 19 + 11 + 1, 11 + 29 + 1, 29 + 11 + 1, 13 + 17 + 1, 17 + 13 + 1, 13 + 23 + 1, 23 + 13 + 1, 13 + 29 + 1, 29 + 13 + 1, 17 + 19 + 1, 19 + 17 + 1, 17 + 23 + 1, 23 + 17 + 1, 17 + 29 + 1, 29 + 17 + 1, 19 + 23 + 1, 23 + 19 + 1, 19 + 29 + 1, 29 + 19 + 1, 23 + 23 + 1, 23 + 29 + 1, 29 + 23 + 1, 29 + 29 + 1.$

Every has infinitely many prime solutions.

We study $p_4 = p_3 + p_2 + p_1 + 2$. Just as $8^3 = 512$, we have 512 equations.

$$J_4(30) = \prod_{p < p_i} ((p-1)^3 - \chi(p)) = \prod_{p < p_i} (p^3 - 4p^2 + 6p - 4) = 255.$$  

We have 255 sub equations of $p_4 = p_3 + p_2 + p_1 + 2, p_4 = 1 + 7 + 7 + 2 = 17, ..., every has infinitely many prime solutions.” [13]

The function $J_n(p \#)$ of Jiang is shown in [3] to exhibit a lot of amazing functional properties which are exhaustively:

1. $J_n \left(2^m\right) = \phi^{n-1} \left(2^m\right) = 2^{(n-1)m-1}$
2. $J_n(1) = J_1(p \#) = J_n(2) = 1$
3. $J_n(ab) = J_n(a)J_n(b), (a, b) = 1$
4. $J_n \left(\xi^m\right) = \omega^{(n-1)m-1} J_n \left(\xi\right)$
5. $J_n(\xi) = \sum_{k=1}^{\phi(\xi)} J_{n-1} \left(\xi_k \equiv p_k \#ight)$
6. $J_n(ab) = \frac{d^{n-1} J_n(a)J_n(b)}{J_n(d)}, (a, b) = d$
7. \[ \left( \frac{p-1}{p} \right)^n \left( \frac{1}{n} \right)^{-1} = \left( \frac{(p-1)^{n-1}}{p} \right)^{n-2} (p-2) \]

8. \( J_n ((k-2)\#) \geq J_n ((k-1)\#) \)

9. \[ \frac{J_n(\zeta^m)(\zeta^{m})^{k-1}}{\phi^{n+k-2}(\xi^m)} = \frac{J_n(\zeta^m)\zeta^{k-1}}{\phi^{n+k-2}(\xi^m)} \]

10. \( (p-1)^{n-1} = \frac{(p-1)^n}{p} + \frac{(p-1)^{n-1}}{p} = J_n(p) + J_{n-1}(p) \)

11. \( a/b \Rightarrow J_n(a)/J_n(b), n > 1 \)

However, these 11 properties seem to set up Jiang’s function as the most amazing function or “analytical toy” ever built.

Unfortunately, no demonstrations of the 11 magic properties are known. Perhaps Jiang himself will be able to give us them, because they are best needed to make conventional number theorists interested about his contributions and to improve some of his statements.

From the most general \( J_n(p\#) \) Jiang defines a series of particular functions such as:

**Definition A.1**: \( J_1(p) = J_1(p\#) = 1 \)

**Definition B.1**: Let \( \zeta := p\# \)

\[ J_2(\zeta) = \prod_{3 \leq p \leq \zeta} (p-2) \prod_{p \mid \zeta} \frac{p-1}{p-2} \]

\[ J_2(\zeta) \neq 0 \]

\[ J_2(\zeta) = \prod_{5 \leq p \leq \zeta} (p-4) \]

\[ J_2(\zeta) = 6 \prod_{11 \leq p \leq \zeta} (p-5) \]

\[ J_2(\zeta) = \prod_{7 \leq p \leq \zeta} (p-6) \]

\[ J_2(\zeta) = 2 \prod_{11 \leq p \leq \zeta} (p-7) \]
Note the most resemblance with the twin prime constant through its simplest expression. Dozens of different expressions are found in Jiang [3].

**Definition B.2:**

\[
J_3(\xi) = \prod_{3 \leq p \leq \xi} p^2 - 3p + 3 - (p-2) \left( \frac{-\hat{b}}{p} \right), \quad \hat{p} = \hat{I}p, \hat{\hat{b}} = \hat{\hat{b}}
\]

\[
J_4(\xi \equiv N) = \frac{1}{2} \prod_{3 \leq p \leq N} \left( 1 - \frac{1}{(p-1)^2} \right) \prod_{p / N} \frac{p-1}{p-2} \frac{N^2}{\log^3 N} (1 + O(1))
\]

\[
J_5(\xi) = \phi(\xi) \prod_{3 \leq p \leq \xi} (p-2) \prod_{p / \xi} \frac{p-1}{p-2}
\]

**Definition B.3:**

\[
J_4(\xi) = \prod_{3 \leq p \leq \xi} \left( \frac{(p-1)^4 - 1}{p} - \frac{(p-1)^3 + 1}{p} \right)
\]

**Definition B.4:**

\[
J_5(\xi) = \prod_{3 \leq p \leq \xi} \left( (p-1)^4 - ((p-2)^3 - (p-2)^2 + p) \right)
\]

**Definition B.5:**

\[
J_6(\xi) = \prod_{3 \leq p \leq \xi} \left( (p-1)^5 - ((p-2)^4 - (p-2)^3 + (p-2)^2 - p + 3) \right)
\]

Note that the definition of the twin prime constant clearly appears in the right side of the second expression of the definition B.2.

An infinitude of such functions can be built to raise number theoretic problems. The most useful are by far those presented in definitions. B.1/B.2/B.3.

Using them Jiang claims to have proved the Goldbach Conjecture and the Twin Prime Conjecture. Here one reproduces his claimed proofs from [3]:

---

17
An improvement of this proof is given further in [3], and show an interesting connection between Jiang’s function and a class of generalised zeta functions:

**Remark 6.23. [3], p266 :**

Euler’s proof that there must exist an infinitude of prime numbers relates the zeta function to prime numbers through the well-known *Euler product form* which now is the pattern of the so called L-functions, and relates then interestingly the zeta function to Euler’s totient function:

\[
\zeta(1) = \lim_{n \to \infty} \frac{n^#}{\phi(n^#)} = \infty
\]

Following Euler’s remarkable remark Jiang is able to prove the Twin Prime Conjecture by setting:

\[
\chi(2) = 1, \chi(p) = 2, p \nRightarrow 2
\]

and then

\[
\lim_{n \to \infty} \frac{n^#}{J_2(n^#)} = \prod_{p} \frac{1}{1 - \chi(p) / p} = \sum_{n=1}^{\infty} \frac{\chi(n)}{n} = \zeta_2(1)
\]

where one defines immediately

\[
\zeta_2(s) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s}
\]

with the following direct improvement of Jiang’s proof

\[
\zeta_2(1) = \zeta(1) = \infty
\]

implying the existence of an infinitude of 2-tuples of primes.

The second one is taken “binary” and also very simple:
At the beginning of 2008, Jiang contacted the great British number theorist Martin Huxley, who is, to the best knowledge, the first in the top academic institutes to become interested in Jiang’s works.

Huxley then told Jiang [17]:

“To say that someone else’s work is actually wrong, you have to be extremely certain that your own calculations are correct, and that you have actually read and understood their work.

(…)

If you have got a new method, the Jiang Function, which solves the famous problems, then bring it into the open and write a full explanation and send it to a Mathematics journal, Annals of Maths or the Proceedings of the London Math. Soc. or the Duke Math. Journal or suchlike. If it works, then most people will be happy to forget about the Riemann Hypothesis completely and use your method instead. If you don’t explain your method, then everybody else is entitled to be as rude about you as you are about them, or what is even worse, to ignore you completely, which is what I myself am likely to do, as I am sent more papers than I have time to study anyway.”

Objection to JNT:

1. The Riemann Hypothesis might be true: Balan’s objection [5]:

Had we \( \frac{1}{2} \leq s_0 \leq 1 \) and \( \zeta(s_0) = 0 \), then by recalling the so-called prime zeta function:

\[
\zeta_p(s) := \sum_{p} \frac{1}{p^s}
\]

9 One could recall that Huxley, one of the greatest analytic number theorist of our time, proved an important result in prime gaps [39]: \( p_{n+1} - p_n < p_n^{\frac{7}{12}} \). Huxley further replied to Jiang: “Thank you for sending me the preprint of your Disproof. Obviously I have not had time to study it carefully. I have a couple of immediate comments…” [17].
one would have $\zeta_p'(s_0) = \infty$ because of the well-known identity bridging the zeta function to the prime zeta function:

$$\zeta_p(s) = \sum_{n=1}^{\infty} \frac{\mu(n)}{n} \ln \left( \zeta(ns) \right)$$

and then

$$\zeta'_{\frac{s_0}{2}} \frac{\zeta_{\frac{s_0}{2}}}{\zeta_{\frac{s_0}{2}}} = \zeta_p' \left( \frac{s_0}{2} \right) + \zeta_p' \left( s_0 \right) + \ldots$$

and

$$\beta' \left( \frac{s_0}{2} \right) = -\zeta_p' \left( \frac{s_0}{2} \right) + \zeta_p' \left( s_0 \right) + \ldots$$

The first equality seems to show that $\zeta_p' \left( \frac{s_0}{2} \right) = \infty$ by a “recurrence” (in Balan’s own words) supposition that the zeros of the zeta function lie on the critical line when $\text{Im}(s) < \text{Im}(s_0)$.

Then it is found in [5] that

$$\beta \left( \frac{s_0}{2} \right) = 0 \Rightarrow \beta(s) \neq \frac{\zeta(2s)}{\zeta(s)}$$

This is absurd. Balan thus claims

$$\beta \left( \frac{s_0}{2} \right) = \frac{\zeta(s_0)}{\zeta \left( \frac{s_0}{2} \right)} = 0$$

“RH IS TRUE”. [5]

Finally, according to Balan in [5] he is able to set a generalization of the functional equation for the zeta function:

$$\Gamma_{\pi} \left( s \right) \sum_{n=1}^{\infty} \frac{1}{(n^2 + an)^s} = \Gamma_{\pi} \left( \frac{1}{2} - s \right) \sum_{k=1}^{\infty} \frac{\cos(\pi ka)}{k^{1-2s}}$$

\[10\] No proof of this last result is directly given in [5], but the identity seems sound, being a good objection to JNT.
2. The Jiang function is probably not the most useful and intuitive generalization of Euler’s totient function.

One has on the other side Jordan’s totient function (references are found in [18]):

\[ \overline{j}_k := n^k \prod_{p \mid n} \frac{1}{1 - \frac{1}{p^k}} \]

where interestingly by defining

\[ j_k(n) := \frac{\overline{j}_k}{n^k} \]

one clearly sees that \( j_k(n) \to \zeta(k) \) as \( n \to \infty \).

Moreover Jordan’s totient function counts the number of k-tuples \((u_0, u_1, \ldots, u_k)\) that are prime with n. Broader and broader generalizations of the totient concept are found in [31].

3. There is another reason for RH to be true, based on rigorous consideration of the computed nontrivial zeros of the zeta function through the counting function \( N(T) \) of Riemann-Von-Mangoldt. However it could remain possible to construct a quantum system from the nontrivial zeros of the zeta function from which a proof or at least an argument in favour of RH might follow.

The Hilbert-Polyà approach seems quite promising. In 2006, the Chinese mathematician Sze Kui Ng from the Baptist University of Hong-Kong pointed out several arguments in favour of RH as seen in [4,29] and also pointed out that the foundation of Jiang’s critics of the calculation of the nontrivial zeros of the zeta function aren’t based rigorously enough [30].

4. JNT has no connections to other fields of number theory such as transcendental number theory, which remain as fundamental as prime number theory, while analytic number theory through the current theory of L-functions permits such connections and even implies them. Moreover JNT seems to be a “personal theory of Jiang” which does not seem to be able to provide a complete understanding of the concept of number.

**Reply to the objections:**

1. Had RH been true, JNT would not have been disproved, because Jiang’s claims that RH is not true is not taken as a foundation of JNT but only as a *motivation* and an urgent reason to see further.

And moreover Balan’s proof is much less intuitive that Jiang’s. The calculations of the nontrivial zeros of the zeta function, being improved, would not contradict JNT fundamentally.
2. However Jordan’s function is interestingly intuitively identified to the zeta function for greater and greater values, and a bridging between the zeta function and the Jiang function has to be shown in the future. But this definition, seems to us to be the expression of Euler’s totient function the nearest to the Jiang function. The Jiang function remains the most useful tool to prove the Twin Prime Conjecture and the k-tuple conjecture, just as Jiang gives a proof of the Prime Number Theorem using Euler’s totient function in [3]:

Following it we are able to set the Jordan’s totient function as a quest to the Jiang function which has direct applications in improving the k-tuple Conjectures.

<table>
<thead>
<tr>
<th>Theorem 6.24</th>
<th>We have the following equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log \prod_{k&lt;p \leq N} \left(1 - \frac{k}{p}\right) \approx - \sum_{k&lt;p \leq N} \frac{k}{p}.$</td>
<td>$\prod_{k&lt;p \leq N} \left(1 - \frac{k}{p}\right) \approx \frac{c_p}{\log^k N}.$ (6.1)</td>
</tr>
<tr>
<td>$- \sum_{k&lt;p \leq N} \frac{1}{p} \approx - \log \log N.$</td>
<td>$\phi(\omega) \approx \frac{c_1}{\log N}.$ (6.2)</td>
</tr>
<tr>
<td>From (6.1) and (6.2) we have</td>
<td>From (6.3) we have</td>
</tr>
<tr>
<td>$\prod_{k&lt;p \leq N} \left(1 - \frac{k}{p}\right) \approx \frac{c_p}{\log^k N}.$</td>
<td>$\phi(\omega) = \prod_{2 \leq p \leq N} \left(1 - \frac{1}{p}\right) \approx \frac{c_1}{\log N}.$ (6.3)</td>
</tr>
<tr>
<td>From (6.3) we have</td>
<td>From (6.4) we have the prime number theorem</td>
</tr>
<tr>
<td>$\phi(\omega) \approx \frac{c_1}{\log N}.$</td>
<td>$\pi(N) \approx N.$ (6.4)</td>
</tr>
<tr>
<td>In virtue of the results compiled above one is able to create a series of generalisation of the zeta function through the series of Jiang’s $J_n(n#)$ functions for all integers index n. All of them relate to Jiang’s function and are useful to investigate all n-tuple of prime:</td>
<td></td>
</tr>
</tbody>
</table>

**Definition 1.1.**

$$\zeta_n(1) := \lim_{n \to \infty} \frac{n\#}{J_n(n\#)}$$

$$\zeta_n(s) := \sum_{n=1}^{\infty} \frac{J_n(n)}{n^s}$$
\( \chi_n(n) \) being a **Jiang character** defined by a simple recurrence formula (or program) over the \( n \) running through all the natural integers.

Jiang himself extends his remark 6.24 by showing a similar result as those given in [3], pp. 220-221 with the hope to prove the existence of an infinitude of 3-tuples of primes. He then defines the function

\[
\zeta_3(s) = \sum_{n=1}^{\infty} \frac{\chi_3(n)}{n^s} = \prod_p \frac{1}{1 - \chi_3(p)}
\]

And gives the following condition to define the Jiang character \( \chi_3(n) \):

\[
\chi_3(2) = 1, \chi_3(3) = 2, \chi_3(n) = 3, n > 3
\]

One is then able to give a definition for all Jiang characters:

\[
\chi_n(2) = 1, \chi_n(3) = 2, ..., \chi_n(a) = a - 1, ..., \chi_n(m) = n, m > n
\]

We should also notice that each zeta function of order \( n \) \( \zeta_n(s) \) might be assigned with a respective dual or conjugate that keep the Jiang character basically unchanged. We thus define the beta function of order \( n \):

\[
\beta_m(s) = \sum_{n=1}^{\infty} \frac{\chi_m(n)\lambda(n)}{n^s} = \prod_p \frac{1}{1 + \chi_m(p)/p^s}
\]

**Definition 1.2.** We notice that Jiang defines the Jiang function in all his papers and high-level article only for prime numbers and primorial numbers. We define the Jiang function extended to all natural integers \( N \) as it follows:

\[
J_n(N) := \prod_{p \mid N \setminus p \neq 2} \left( \frac{(p-1)^n - (-1)^n}{p} \right) \prod_{p \mid N \setminus (p-1)(-1)^n} \left( 1 + \frac{(-1)^n p}{(p-1)^n - (-1)^n} \right)
\]

**Remark:** We notice that the Jiang function \( J_n(p\#) \) may be written as a product formula for the Jiang functions for primes \( J_n(p) \) as it follows:

\[
J_n(N) := \prod_{p \mid N \setminus p \neq 2} J_n(p) \prod_{p \mid N} \left( 1 + \frac{(-1)^n p}{(p-1)^n - (-1)^n} \right)
\]

We then define the **Jiang totient function** \( \Phi_n(N) \) defined over all the natural integers as it follows:
\[
\Phi_n(N) := \prod_{p \mid N} \left( 1 + \frac{(-1)^n p}{(p - 1)^n - (-1)^n} \right) = \prod_{p \mid N} \left( 1 + \frac{(-1)^n}{J_n(p)} \right)
\]

From this one now will be able to write the Jiang function extended to all natural integers as it follows:

\[
J_n(N) := \Phi_n(N) \prod_{p \mid N, p > 2} J_n(p)
\]

It follows easily that the Jiang totient function justifies its appellation because of its relations to Euler’s totient function, and mostly to Jordan’s totient function through the function \( J_n(n) \) because \( \Phi_n(N) \) might be defined simply as

\[
\Phi_n(N) = \prod_{p \mid N} \left( 1 - \frac{1}{p^n} \left( \frac{(-p)^{n+1}}{(p - 1)^{n+1} - (-1)^{n+1}} \right) \right)
\]

\( \Phi_n(N) \) is then a \( n \)-deformed Jordan’s totient function, when generalised through

\[
\Phi_{n,k}(N) = \prod_{p \mid N} \left( 1 - \frac{1}{p^{n+k}} \left( \frac{(-p)^{n+k+1}}{(p - 1)^{n+k+1} - (-1)^{n+k+1}} \right) \right)
\]

with the property that

\[
\Phi_{0,k}(N) = \prod_{p \mid N} \left( 1 + \frac{1}{p^k} \right)
\]

Which appears to be the dual function to \( \overline{J}_k(n) \) defined as \( D_k(n) \) following the duality identity:

\[
D_k(n) = \frac{\overline{J}_{2k}(n)}{\overline{J}_k(n)}
\]

Moreover this particular connection to the Jordan’s totient function is certainly not the unique and the best one among the multitude that are easily conceivable.

3. A further and a last reply to the objections to JNT would be drawn through the quotation of a most remarkable identity found in Havil [36] bridging the Euler’s totient function to the deepest transcendental or presumed irrational constants such as \( \pi \), \( e \) and \( \gamma \).

\[
\lim_{n \to \infty} \frac{1}{n} \ln n \prod_{p \mid n} \left( 1 + \frac{1}{p} \right) = \frac{6e^\gamma}{\pi^2}
\]
The prime number theorem \( \pi(x) \approx \frac{x}{\ln x} \) allows us to approximate the n-th prime number as
\( p_n \approx n \ln n \). On the other side we have
\[
\lim_{n \to \infty} \Phi_{0,1}(n) = \lim_{n \to \infty} D_1(n) = \prod_p \left(1 + \frac{1}{p}\right)
\]
and therefore one obtains:
\[
\lim_{N \to \infty} \frac{\Phi_{0,1}(N)}{NP_N} \to \frac{6e^\gamma}{\pi^2}
\]
This is just an amazing identity that should inspire number theorists to be interested in JNT in the near future, and to connect JNT to all other fields of number theory.

In the hope that Jiang’s work, which, even if it is false, constitutes a formidable attempt to raise the largest number of deepest number theoretic problems, will receive an echo into the circles of mathematicians, the author would concludes his course on Jiang’s works by a list of challenges for the future.

### 2) Challenges for the future:

1. To extend Jiang’s foundations of Santilli’s isonumber theory to genomathematics and hypermathematics [12].

2. To establish the number theoretical foundations of informatics through computability theory which seems implicitly connected to the isotopic formalism found in [12].

3. To discover the exact and complete order behind the distribution of primes, Santilli isoprimes, Santilli genoprimes, Santilli hyperprimes and their respective isoduals.

4. Hypernumbers = sequences of ordinary numbers
sequences of bits = programs
what about infinite sequences? What about Number theoretic aspect of the building of computer programs? What is infinity and what are thoughts that are compressed into programs? What is the link between programming and LIFE (since, as seen in [3,12] hypermathematics has been built to represent consistently biological systems).

5. To extend informatics to Hadronic Mathematics to which the best introduction seems to the author to be found in Santilli’s latest work as of 2008 [12] with related software, programs and programming.

6. To extend the formal definition of pattern distastefully evoked in this paper to all mathematical concepts and/or structures.\(^{11}\)

To concentrate all upcoming ideas useful to solve these problems the author is trying to generalize Information Theory into Hadronic Information Theory (HIT) with an appropriate hypermathematical formalism and number theoretic foundations.

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\(^{11}\) A pattern, as it should be noticed, is a special case of category into the area of category theories [41,42]. Algebras are generated through units under its elements and laws; by extrapolation hypercategories are generated by patterns which are at least the generalization of units into categories theories [41,42].
The author would define for instance HIT as the *semantic embedding of Hypernumber Theory* thus needing the rigorous establishment as recalled in the *Challenges* above of the new *Santilli Hypernumber Theory (SHT)* just as the Jiang Santilli isonumber theory in which Jiang Number Theory (the great JNT) has its kingdom.

A series of papers is to appear about HIT, following [15] and [16], in which information will be understood as programs which are themselves understood as sequences of numbers which themselves appear to be Santilli hypernumbers. But the establishment of Santilli’s Hypernumber Theory should take a long time.

The starting definition from SHT to HIT will be the definition of PATTERN.

We have seen in [12] that the pattern for isomathematics is the Santilli isounit and the pattern for Jiang function is Euler totient function.

\[
\text{IS JIANG FUNCTION: } J_{n}(p\#) := \prod_{3 \leq p \leq n} \left( \frac{(p-1)^n - (-1)^n}{p} \right) \prod_{p \equiv \xi \pmod{p}} 1 + \frac{(-1)^n p}{(p-1)^n - (-1)^n} \]

THE UNIVERSAL PATTERN FOR SHT AND HIT?

Further improvements of JNT are needed to set this rigorously. To the best knowledge of the author, the greatest steps done for instance to define patterns of mathematical theories and the Universe as a music of particles or a system are found in Johansen [8] and Rowlands [14].

**Acknowledgements:**

I cannot find words to express my gratitude and esteem toward Professors Jiang Chun-Xuan, Chen I Wan, Stein Johansen, Tribikram Pati, Sze Kui Ng and several others for their helpful conversations and feedback.

Laurent Schadeck.
Appendix:

One should assert without doubt that real number theorists won’t be interested in the new tool developed by Jiang, the Jiang function $J_n(p)$ and the most useful of them, the function $J_2(p)$ until there will be serious computations of the value of these astonishing functions through advanced programs on computers that will tend (as well as it was done to make all number theorists interested in the zeta function and Euler’s totient function) to show astonishing properties of these functions.\(^{12}\)

In fact the Jiang functions behave in such a way that they appear to be highly compatible with easy programs exhibiting experimental laws, and to constitute crucial concept for the future of the architecture of computers and programming, as well as Euler’s totient function.

Particularly, we now all that

$$\phi(p) = p - 1$$

A particularly simple abstract computation of the function $J_2(p)$ shows us first that

$$J_2(p) = p - 2$$

Moreover an interesting property seems to be exhibited by the function $J_2(p)$ for all value of $p$: the following tables show us that the values $p_n$ for which $J_2(p_n) = p_{n+k}$ is a prime and which are less than N, tend to be equal to $\frac{N}{\ln(N)} \approx \pi(N)^{13}$.

Moreover, the graph of the most useful functions $J_2(p)$ and $J_3(p)$ are presented below, along with the graph corresponding to the function $f(n) = p_n$ according to 65536 values.

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<th>$p_n$</th>
<th>$n$</th>
<th>$J_2(p)$</th>
<th>$S(n) := #{J_2(p_n) = p_{n+k}}$</th>
<th>$\sum_{i=1}^n S(i)$</th>
<th>$\frac{n}{\ln(n)} \approx \pi(n)$</th>
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\(^{12}\) Experimental number theory is the princess of mathematics, in a way, as it is pointed out by Chaitin [43] and Jonathan Borwein among several other scholars [44].

\(^{13}\) This seems interestingly to show us that there is actually infinitely many twin prime because $\frac{N}{\ln(N)} \rightarrow \infty$ as $N \rightarrow \infty$; this seems the best improvement of Jiang’s proof.
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Moreover we should notice that \[ \lim_{n \to \infty} \frac{J_n(p)}{p^{n-1}} = 1 \] provide the best polynomial approximation for large primes.

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References:


[8] Stein Johansen, *Initiation to Hadronic Philosophy, the philosophy underlying Hadronic Mechanics*, Lecture at 18th workshop on hadronic mechanics, University of Karlstad, Sweden, 22/07/05.


