A thermodynamical approach to a ten dimensional inflationary universe

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ABSTRACT

The inflationary phase of the evolution of the ten dimensional universe is considered. The form of the stress-energy tensor of the matter in the very early universe is determined by making use of some thermodynamical arguments. In this way, the Einstein field equations are written and some inflationary cosmological solution is found to these equations in which, while the actual dimensions are exponentially expanding, the others are contracting.

January 2006

1 Introduction

The inflationary universe theory could explain some problems namely, the singularity problem, the flatness problem, the horizon problem, the problems of homogeneity and galaxy formation and the uniqueness problem [1]. In the exponentially expanding universe, the main structure is very complicated. For this reason, in this paper, the theory is considered from the more different viewpoint.

The main idea of this paper is to think the fundamental thermodynamical quantities (entropy, temperature, energy ,...) as a function of a Lapse function N and to take the Lapse function as unity in the final expressions. The covariant conservation relation of the stress-energy tensor is compared with $dG = \frac{\partial G}{\partial N} dN = 0$, where G is the Gibbs free energy function, where and anywhere N denotes a Lapse function. In this way, the Einstein field equations are written in the ten dimensional universe and some cosmological solution is found to these equations.

2 The field equations, the Gibbs free energy function and some thermodynamical relations

The G Gibbs free energy function is defined as

$$G = E + pV - TS, \tag{2.1}$$

and

$$dG = dE + pdV - SdT, \tag{2.2}$$

where $E = \rho V$ is the total energy of the universe, ρ is the energy density, $V = R^3 s^6$ the total volume of the universe, R is the scale factor of the actual space, s is the scale factor of the internal space, p is the constant pressure of the universe, S is the constant total entropy of the universe.

Furthermore, it is assumed that $T = \gamma E$, where γ is a positive constant. So, $dG = 0$ gives

$$(1 - \gamma S)\dot{\rho} + (3\frac{\dot{R}}{R} + 6\frac{\dot{s}}{s})((1 - \gamma S)\rho + p) = 0. \tag{2.3}$$

We suppose the classical dynamics of the ten dimensional geometry of the very early universe is determined by the Einstein field equations. We will examine a cosmological model for which the spacetime manifold has the topology $M \times C$ and C has the torus topology $S^1 \times \ldots \times S^1$ (6-times). We will look for a metric g in terms of a cosmic time variable t that will enable us to describe any submanifold t=constant as a product manifold. In a coordinate chart (t, x^1, x^2, x^3) for M and $(y^4, \ldots, y^9)$ for C we write

$$g = g_4 + g_6 \tag{2.4}$$

where g_4 is identified with the metric on M, and g_6 with the metric on C. We let

$$g_4 = -dt^2 + \frac{R^2(t)}{(1 + \frac{k}{4}r^2)^2} \sum_{i=1}^{3} dx^i \otimes dx^i \quad , \quad r^2 = \sum_{i=1}^{3} x^i x^i \tag{2.5}$$

and

$$g_6 = \sum_{i=4}^{9} e^\alpha \otimes e^\alpha. \tag{2.6}$$

In this paper, for an inflationary cosmological solution, the value $k = 0$ is taken. For the internal space $C = S^1 \times \ldots \times S^1$ in the above chart, we have

$$e^\alpha = s(t) dy^\alpha \quad , \quad \alpha = 4, \ldots, 9, \tag{2.7}$$

Then the Einstein equations reduce to the following system of differential equations

$$3 \left[P + 5Q + 6(\frac{\dot{R}}{R})(\frac{\dot{s}}{s}) \right] = \kappa f1 \tag{2.8}$$

$$\left[P + 15Q + 12(\frac{\dot{R}}{R})(\frac{\dot{s}}{s}) + 2(\frac{\ddot{R}}{R}) + 6(\frac{\ddot{s}}{s}) \right] = -\kappa f2 \tag{2.9}$$

$$\left[3P + 10Q + 15(\frac{\dot{R}}{R})(\frac{\dot{s}}{s}) + 3(\frac{\ddot{R}}{R}) + 5(\frac{\ddot{s}}{s}) \right] = -\kappa f2 \tag{2.10}$$

where an overdot denotes derivative with respect to time, κ denotes the universal coupling constant and we have set

$$P = (\frac{\dot{R}}{R})^2 \quad , \quad Q = (\frac{\dot{s}}{s})^2.$$

Furthermore due to the above Einstein field equations, the $f1$, $f2$ functions also satisfy the following covariant conservation relation

$$\dot{f}1 + (3\frac{\dot{R}}{R} + 6\frac{\dot{s}}{s})(f1 + f2) = 0. \tag{2.11}$$

The comparison of the above conservation relation with the previous $dG = 0$ gives

$$f1 = (1 - \gamma S)\rho, \tag{2.12}$$
$$f2 = p \tag{2.13}$$

We seek solutions of the type $R = R_o e^{\alpha t}$ and $s = s_o e^{\beta t}$, where R_o and s_o are arbitrary integration constants. Inserting these R and s exponential functions into the above equations, one obtains the following solutions for α and β:

$$\alpha = \frac{2}{3}\sqrt{\kappa(\gamma S - 1)\rho} \tag{2.14}$$
$$\beta = -\frac{1}{3}\sqrt{\kappa(\gamma S - 1)\rho} \tag{2.15}$$
$$p = (1 - \gamma S)\rho \tag{2.16}$$
$$\gamma S > 1 \tag{2.17}$$

3 Concluding remarks

In this paper, the fundamental thermodynamical quantities have been thought as a function of a Lapse function. After the obtaining of higher dimensional Einstein field equations, some exponential inflationary cosmological solution to these equations have been found. $\gamma S = 1$ corresponds to an Einstein static universe. $\gamma S < 1$ corresponds to an oscillationary universe. $\gamma S > 1$ corresponds to an inflationary universe. The case of $\gamma S \approx 1$ can be explain the transition of the universe from an inflationary phase to the Friedmann phase. As an interesting result, it has been understood that the inflation and the contraction parameters (γS) depend on the total entropy of the universe. Furthermore, it is proved that the entropy influences the Einstein field equations.

References

[1] A.D. Linde, (1990), "Inflation and Quantum Cosmology", Academic Press.