Propagation of spherical electromagnetic waves in different inertial frames; an alternative to the Lorentz transformation

Teimuraz Bregadze
tebr50@yahoo.com

The application of the Lorentz transformation, which assumes planar character of electromagnetic waves, to the points of the front of a spherical electromagnetic wave gives a distorted picture of the physical reality. The expressions of Doppler effect and aberration, as well as the transformation law of the wave fronts for spherical electromagnetic waves, essentially different from those for planar electromagnetic waves, are obtained. Some consequences are discussed. Particularly, the necessity of introduction of the "natural" units of length (changing in the same way as the lengths of electromagnetic waves) for measuring distances from moving points to the fronts of electromagnetic waves, as well as of the "real" distance covered by a moving point relative to a stationary point containing the number of electromagnetic waves which is equal to the difference in the numbers of waves contained between the front of an electromagnetic wave and the stationary and moving points respectively. A revised equation for the addition of velocities is also obtained.

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The equation of the front of a spherical electromagnetic wave propagating from the origin of a stationary frame is that of a sphere:

$$x^2 + y^2 + z^2 = c^2t^2 , \tag{1}$$

where x, y, z, and t are the space and time coordinates of a point of the wave front, and c is the velocity of light in emptiness.

According to the Lorentz transformation, in the frame with axes coinciding with those of the stationary frame at the instant of the start of emission of the wave ($t=t'=0$) and moving with velocity $v = \beta c$ along axis X, the equation of the same wave front is: $x'^2 + y'^2 + z'^2 = c^2t'^2$, where

$$x' = \frac{x - \beta ct}{\sqrt{1-\beta^2}} ,\ y' = y ,\ z' = z ,\ t' = \frac{ct - \beta x}{c\sqrt{1-\beta^2}} \tag{2}$$

are the space and time coordinates of the same point of the wave front in the moving frame.

In figure 1 the sphere with centre in the origin of the stationary frame, O, represents the wave front in the stationary frame when the clock at that point shows time t; O_1 is the origin of the moving frame; and $|OO_1| = \beta ct$.

If we try to represent the same wave front in the moving frame using equations (2), we will get a spheroid with centre in O' and one of the focuses in O_1, where $|O'O_1| = \frac{\beta ct}{\sqrt{1-\beta^2}}$; the long half axis of the spheroid is $|O'C'| = \frac{ct}{\sqrt{1-\beta^2}}$ and the short half axes are equal to $|O'E'| = ct$.

Indeed, according to equations (2), point $O(0,0,0,t)$ in the stationary frame becomes point $O'\left(-\frac{\beta ct}{\sqrt{1-\beta^2}},0,0,\frac{t}{\sqrt{1-\beta^2}}\right)$ in the moving frame. If we assume that the origin of the moving frame is in point *O'*, then the equation of the wave front in this frame is:

$$(1-\beta^2)x_1'^2 + y'^2 + z'^2 = c^2t^2$$

where $x_1' = x' + \frac{\beta ct}{\sqrt{1-\beta^2}} = \frac{x}{\sqrt{1-\beta^2}}$.

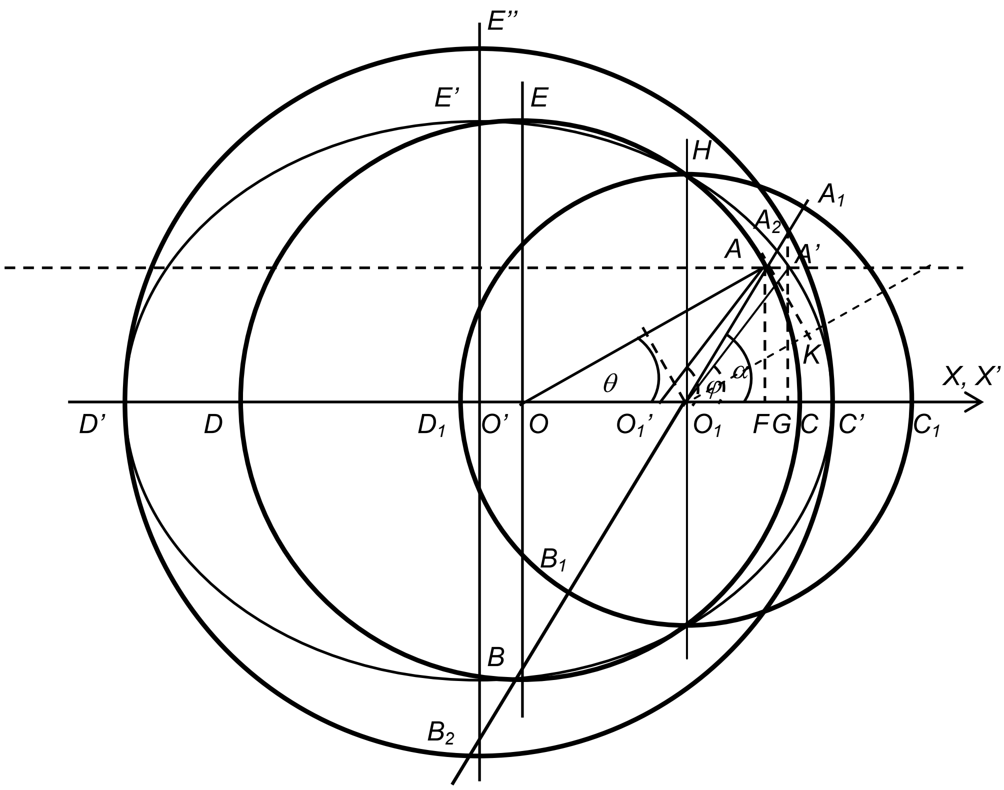

Figure 1. The fronts of electromagnetic waves in stationary and moving inertial frames.

At the same time we have to bear in mind that the time coordinates of the points of the wave front in the moving frame vary along the axis *X'* from $t_1' = t\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}$ at point *C'* to $t_2' = t\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$ at point *D'*.

The design of the Lorentz transformation is to give the picture of the physical reality a moving observer shall have from the point of view of a stationary observer. But the picture that results from this transformation is dramatically different from the one a moving observer really gets in his frame; in other words, a moving observer never gets the picture of the wave front we have described above.

According to the Lorentz transformation, when the moving clock in point *O'* shows time $t' = \frac{t}{\sqrt{1-\beta^2}}$, point *E* of the wave front in the stationary frame becomes point *E'* in the moving frame. But the moving observer actually discovers that at that instant of time in point *O'* the wave front is not in point *E'*, but in point *E"*, which is a point of the sphere with centre in *O'* and radius $R = \frac{ct}{\sqrt{1-\beta^2}}$, because the point from which the emission of the wave started in the moving frame is *O'*.

Let in point *E* of the stationary frame a clock is placed. At the instant when the wave front arrives at that point the clock reads *t*. This fact is an invariant and must be observed in any frame. According to the Lorentz transformation point *E* is point *E'* in the moving frame and the wave front arrives at that point when the clock there shows $t' = \frac{t}{\sqrt{1-\beta^2}}$. That is simply impossible: a clock cannot show different times simultaneously.

The Lorentz transformation assigns unique qualities to stationary frames, radically distinguishing them from the moving ones: time coordinates of the points in stationary frames do not depend on spatial coordinates and in moving ones they do. It is unclear how the differences in the readings of the clocks in different points arise when a stationary frame starts its movement. For example, let each carriage of a long train have its own engine and clock (the carriages, engines and clocks are identical). When the train is at rest at the station all clocks on the train show the same time as the clocks on the station. At some instant of time the engines of all carriages are switched on simultaneously and the train starts moving. Why should some time later the stationary observers at different carriages register different differences between the readings of the moving and their own clocks?

The electromagnetic waves are spherical [1], but the Lorentz transformation requires that those waves be planar. This is because the phase of an electromagnetic wave is to be an invariant, and under the Lorentz transformation the phase which is the same in any inertial frame is that of a plane wave [2]. It is believed that electromagnetic waves may be considered as planar in any small part of space [3]. While deriving the expressions of the Doppler effect and aberration, Einstein emphasizes that the source of electromagnetic waves is “very far from the origin of co-ordinates"; and that "an observer is moving with velocity *v* relatively to an infinitely distant source of light" [4], obviously assuming that at far distances from the source of wave (i.e. at the distances that are much greater than the length of wave) the differences between spherical and plane waves shall become infinitesimal. Even if those arguments were correct, that would only mean that the Lorentz transformation is an approximation to the physical reality, and a more precise theory would give correct results for any spatial dimensions. Below it is shown that the expressions of the Doppler effect and aberration for spherical and plane waves maintain their differences even at infinity. It is not surprising that the application of the Lorentz transformation for the spherical fronts of electromagnetic waves gives distorted picture of the reality. This is, undoubtedly, the weakest point in the special relativity. The search for a better transformation law is necessary.

A correct transformation must maintain the spherical form of electromagnetic waves in any inertial frame. That is the requirement of the constancy of the light velocity in emptiness and the isotropy of space.

Let us get the expression of the Doppler effect for spherical electromagnetic waves first.

It is obvious that in any frame all waves emitted by a source of an electromagnetic wave in emptiness are contained between the source and wave front (at any instant of time, the last wave, or a part of it just emitted, is at the source and the wave first emitted has just arrived at the location of the wave front). In any direction the distance between the source and the front of the emitted wave is equal to the sum of wavelengths contained between them.

Let in figure 1 the source of wave is fixed in the origin O_1 of the frame moving with a constant velocity $v=\beta c$.

If the wave emitted is a spherical one, then all waves emitted in time *t* in the stationary frame, n_0', (which is equal to the number of oscillations within the source of wave) will be contained in the space between the point O_1 and sphere *O*. Assuming that the light velocity is the same in both frames, for some point *A* of the wave front in the stationary frame we have:

$$|O_1A| = ct\sqrt{1-2\beta\cos\theta+\beta^2} = n_0\lambda_0\sqrt{1-2\beta\cos\theta+\beta^2} = n_0'\lambda_\theta\,, \qquad (3)$$

where n_0 is the number of oscillations within the source of wave in the interval of time *t* in the frame in which the source of wave is at rest; λ_0 is the length of the emitted wave in the same frame; λ_θ is the length of the wave emitted in direction $\angle AOO_1 = \theta$ relative to the direction of the movement of the source in the stationary frame.

From (3):

$$\lambda_\theta = \frac{n_0\lambda_0\sqrt{1-2\beta\cos\theta+\beta^2}}{n_0'}. \qquad (4)$$

Sometimes it is more convenient to use angle $\angle AO_1C = \alpha$ instead of angle θ. Angle α is the one at which, from the point of view of the stationary frame, the ray *OA* (or the point *A* of the wave front) propagates in the moving frame connected with the source. If we assume that the ray *OA* propagates through an imaginary moving tube of a very small diameter, then angle α is the slope of the tube as seen from the stationary frame.

$$\sin\alpha = \frac{\sin\theta}{\sqrt{1-2\beta\cos\theta+\beta^2}} \tag{5}$$

So, instead of (4) we can use the following equation:

$$\lambda_\alpha = \frac{n_0\lambda_0\left(\sqrt{1-\beta^2\sin^2\alpha}-\beta\cos\alpha\right)}{n_0'}.$$

If the wave emitted from the moving source in O_1 represents a thin ray of plane wave (i.e. the front of wave lies in a plane perpendicular to the direction of propagation of the ray) propagating in direction OA, then AK is the line of intersection of the plane of the picture with the plane of the wave front at some instant of time t in point O. In this case

$$|O_1K| = ct(1-\beta\cos\theta) = n_0\lambda_0(1-\beta\cos\theta) = n_0'\lambda_\theta,$$

and

$$\lambda_\theta = \frac{n_0\lambda_0(1-\beta\cos\theta)}{n_0'}. \tag{6}$$

From equations (4) and (6) it is evident that in regard to Doppler effect spherical and plane waves maintain their uniqueness at any distances. Hence, the adoption of the plane-wave approximation for electromagnetic waves is not justified and only leads to complications.

We can consider the lengths of electromagnetic waves as natural units of measurement of space intervals. We have good reason to assume that any physical unit of measurement of distances from a moving source to the front of electromagnetic wave undergoes the same kind of changes as the lengths of electromagnetic waves:

$$U_\theta = \frac{n_0U_0\sqrt{1-2\beta\cos\theta+\beta^2}}{n_0'};$$

U_θ is a "natural" unit of length in direction θ; U_0 is a "mathematical" unit, i.e. a hypothetical unit which undergoes no changes. When $\beta\to 0$, then $U_\theta \to U_0$ i.e. "natural" units coincide with "mathematical" units when distances between two stationary points or a stationary point and wave front are measured.

Using angle α, we have:

$$U_\alpha = \frac{n_0U_0\left(\sqrt{1-\beta^2\sin^2\alpha}-\beta\cos\alpha\right)}{n_0'}.$$

In "natural" units the distance between the moving source O_1 and the front of wave in any direction is equal to:

$$|O_1A|' = ct\left(\sqrt{1-\beta^2\sin^2\alpha}-\beta\cos\alpha\right) : \frac{U_\alpha}{U_0} = ct\frac{n_0'}{n_0} = |O_1A_1|.$$

Thus, when measuring distances from a moving source O_1 to the points of the wave front in "natural" units, we get a new sphere with centre in O_1 and radius

$r = R\dfrac{n_0'}{n}$; or vice versa, if we assume that the front of an electromagnetic wave in emptiness maintains its spherical form relative to any point of reference whether stationary of moving, that would mean that any measuring device, including our eyes, uses "natural" units of measurement in the assessment of distances from moving points.

Now we have to prove that $\dfrac{n_0'}{n_0} = \sqrt{1-\beta^2}$, as it is shown in figure 1: $|O_1H| = r = \sqrt{1-\beta^2}ct$.

Indeed, it is obvious that the spheres O and O_1 intersect and for the points of intersection $n_0'\lambda_\alpha = n_0'\lambda_0$, i.e. $\lambda_\alpha = \lambda_0$. Also, $|O_1A| = n_0'\lambda_\alpha$, $|O_1B| = n_0'\lambda_{\pi+\alpha}$, and $|O_1A||O_1B| = r^2$, where $r = |O_1A_1| = |O_1B_1| = n_0'\lambda_0$; so, $\lambda_\alpha\lambda_{\pi+\alpha} = \lambda_0^2$.

Thus, for the points of intersection of the spheres: $\lambda_\alpha = \lambda_{\pi+\alpha} = \lambda_0$; hence $\alpha = \pm\pi/2$ and $\lambda_{\pm\pi/2} = \lambda_0$.

From $n_0'\lambda_{\pi/2} = n_0\lambda_0\sqrt{1-\beta^2}$ follows: $n_0' = n_0\sqrt{1-\beta^2}$ and $r = R\sqrt{1-\beta^2}$.

Thus,
$$\lambda_\theta = \frac{\lambda_0\sqrt{1-2\beta\cos\theta+\beta^2}}{\sqrt{1-\beta^2}}$$

The same expression was obtained in [5].

It is obvious that $n_0 = t/\tau_0$ and $n_0' = t'/\tau_0 = t/\tau_0'$, where t and t' are the readings of the clocks at the origins of the stationary and moving frames O and O_1 respectively, and τ_0 and τ_0' are the periods of oscillation within the source of wave in its rest frame and in the stationary frame respectively.

Thus $t' = t\sqrt{1-\beta^2}$ and $\tau_0' = \tau_0/\sqrt{1-\beta^2}$; i.e. in a moving point time flows $1/\sqrt{1-\beta^2}$ times slower than in a stationary frame, or, what is the same, any period of time (e.g. the time of a complete revolution of a hand of a clock) is $1/\sqrt{1-\beta^2}$ times longer in a moving point than in a stationary point.

It is interesting that in figure 1:
$$|A_1B_1| = |AB|\sqrt{\frac{1-\beta^2}{1-\beta^2\sin^2\alpha}},$$
which coincides with the equation of the Fitzgerald-Lorentz contraction, i.e. this expression is the same for spherical and planar electromagnetic waves.

This simple geometrical fact, which is described in [6], points also to the necessity of introduction of "natural" units.

From the point of view of the stationary frame, the sphere of radius R with centre in stationary point O is transformed into the sphere of radius $r = R\sqrt{1-\beta^2}$ with centre in moving point O_1. This means that points A and A_1 in figure 1 are the same points considered relative different points of reference. So, if in point A of the stationary frame a clock is placed, which reads $t = \tau$, then the same clock is placed in point A_1 of the moving frame as well and its reading must be $t' = \tau\sqrt{1-\beta^2}$. But, as already mentioned above, the same clock cannot show different times. That leaves the only possible solution: the reading must be zero (or in other words, whatever the reading may be, we have to count time from that point). Thus, time coordinates of the points of the front of electromagnetic wave emitted from the origin of the frame at t=0 must be zero. (If an electromagnetic wave is emitted from the origin of the frame at $t = t_0$, then the time coordinates of the points of the wave front is $t' = t_0$ too.) Then the readings of the clocks in the origins of the stationary and moving frames are $t = \tau$ and $t' = \tau\sqrt{1-\beta^2}$ respectively. The clocks beyond those spheres show "negative" time (this means that the impulses that arrive at those points have been issued earlier than the readings of the clocks at the origins of the frames).

Let at $t = t' = 0$ a moving clock *O'* is in point *O* of the stationary frame and an electromagnetic wave starts emitting from the same point. After some time *t* the moving clock arrives at point *A* and the wave front arrives at point *C*. In figure 2(a) we can see the picture in "mathematical" units: $|OA| = \beta ct$, $|OC| = ct$, $|AC| = |O'C'| = (1-\beta)ct$.

But the measurement of the distance from the moving source to the front of an electromagnetic wave must give the result in "natural" units; otherwise the light velocity would not be independent from the velocity of the source. Indeed, from the point of view of a stationary observer, if $\frac{|OC|}{t} = c$, then $\frac{|O'C|}{t\sqrt{1-\beta^2}} \neq c$.

The "natural" unit in direction $O'C$ is $U = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}U_0$; so, in those units $|O'C'| = \frac{|AC|}{U/U_0} = ct\sqrt{1-\beta^2}$. In this case (figure 2(b)) the point of wave front in direction $O'C$ is split into two points: *C* and C' (relative to points *O* and O' respectively). This is the picture we should get allowing for the fact of constancy of light velocity.

In the reality the points *C* and C' are the same point and we get the picture represented in figure 2(c), where $|A_1A|$ is equal to $|CC'|$ in figure 2(b), or the picture represented in figure 2(d), where $|O_1O|$ is equal to $|CC'|$ in figure 2(b).

Distances $|OA|$ and $|OO'|$ in figure 2(a) are incommensurable, because they are to be expressed in different units. That means that it is impossible for both ends of those segments to coincide. In figure 2(c) the ends of those segments coincide in point *O*, and in figure 2(d) the other ends of those segments coincide in point *A*. Figure 2(c) may be called the viewpoint of the observer at point *O* and figure 2(d) the viewpoint of the observer at point *A*.

A stationary clock at point A_1 in figure 2(c) (or at point *A* in figure 2(d)) shall show $t_1 = t - \frac{|OA_1|}{c} = t\sqrt{1-\beta^2}$, which coincides exactly with the reading of the moving clock O'. So, by comparing the readings of the clocks, it is impossible to tell which one of the points A_1 and O' (or *A* and O' in figure 2(d)) is moving.

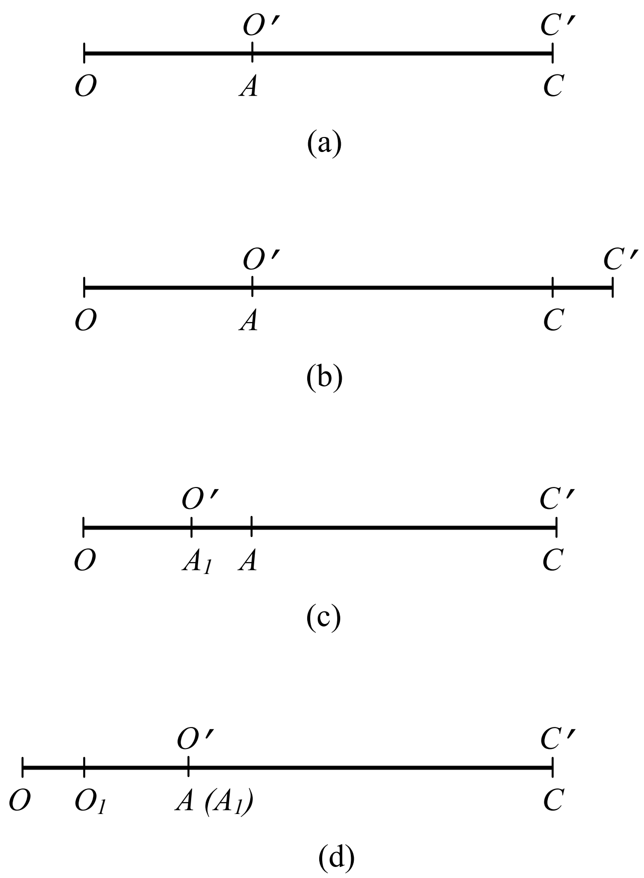

Figure 2. An illustration of the effect of "natural" units.
(a) All distances are in "mathematical" units; (b) distance *O'C'* is in "natural" units; (c) the real picture for the observer in point *O*; (d) the real picture for the observer in point *A*.

Thus, for the observer in point *O* (figure 2(c)) the moving clock O' is in point A_1 and not in point *A*; for the observer in point *A* (figure 2(d)) the moving clock is in point *A*, but at the beginning the clock was in point O_1 and not in point *O*. This may seem unbelievable and needing some additional commentaries.

When measuring the distance from some stationary point to a moving point by means of a ruler, we are substituting the distance between those two points by the distance between two stationary points of the ruler. We cannot be sure that this is a legitimate operation. The same is true for the measurements using radar: the time the radar impulse requires for covering the way from the radar to a point (stationary or moving) is equal to that from the point to radar, which is tantamount to measuring a distance between two stationary points. Direct measurement with a ruler or radar gives not the "real", but the "mathematical" value of the distance to a moving point.

If a direct measurement of the distance to a moving point gives $|OO'| = L = \beta ct$, then the "real" distance is:

$$L' = ct - ct\sqrt{1-\beta^2} = \frac{1-\sqrt{1-\beta^2}}{\beta}L = \frac{L\beta}{1+\sqrt{1-\beta^2}}.$$

When $\beta = 1, L' = L$.

Thus,

$$\frac{L'}{c} = t - t', \qquad (7)$$

i.e. the difference in readings of two synchronized clocks is equal to the time required for light for covering the "real" distance between those clocks.

It must be noted that there is nothing new in this simple mathematical fact which follows from the special relativity as well.

Eq. (7) is true for any velocities (even for the zero one!), as well as for any path of a moving point. In the case of zero velocity, i.e. if both clocks are stationary ones, *L'* is equal to the "real" length of the path which we consider a communicating signal will make from one point to another.

Let *O* and *A* be two marks on a ruler and $|OA| = L$. Then if in point *O* in figure 2(c) the clock shows *t*, the synchronized clock in point *A* shows $t_A = t - \frac{L}{c}$. That means: if a moving clock O' arrives from point *O* at point *A* with velocity $v \approx c$, the difference in the readings of the clocks in *O* and O' will be $\Delta t = t - t_A = t\left(1-\sqrt{1-\beta^2}\right) \approx \frac{L}{c}$. If the speed of the moving clock *v*<*c*, the difference in the readings of the same clocks will be: $\Delta t = t - t_{A_1} = t\left(1-\sqrt{1-\beta^2}\right) = \frac{L_1}{c} < \frac{L}{c}$. That is because in this case the distance covered by the moving clock is not $|OA| = L$, but $|OA_1| = L_1 < L$ (we do not have to mix up the length of the ruler with the distance covered by the clock). For the observer in point *A* (figure 2(d)), the clock arriving at *A*, as well as his own clock, shows some time t_1, and $t = \frac{t_1}{\sqrt{1-\beta^2}}$ time ego the clock was at point O_1; if $v \approx c$, then point O_1 almost coincides with point *O* (the left mark on the ruler); if *v*<*c*, then the distance covered by clock is $|O_1A| = L_1 < L$. Point O_1 does not coincide with the left mark on the ruler.

In figures 3(c) and 3(d), we can see that the real distance covered by the clock is L_1 only, although, according to the ruler, the distance between the first and last points of the clock's travelling is *L*.

This is an effect of the use of different units of measurement, but we can look at this in another way as well: instead of the movement of the clock in the frame of the ruler, let us consider the movement of both of them in some preferable frame, during which the clock O' moving to the right from the left mark of the ruler, *O*, and the right mark, *A*, of the ruler moving to the left, arrive at point A_1 simultaneously (figure 2(c)). In reality no introduction of a preferable frame is necessary, because this is only an illusion caused by the use of different units of length.

The fact that the movement of the ruler is an illusion may be illustrated by the following example: let us imagine that the earth does not rotate around its axis and a jet is flying with velocity *v* along the earth's equator. Since the speed of the jet *v*<*c*, the real distance covered by it during its circumnavigation will be less than the length of the equator which in this case represents a ruler. This fact is due to the use of different units of measurement for the distances to stationary and moving points. We may explain this fact also by assuming that the earth is rotating with adequate velocity in opposite direction in a preferable frame, but this rotation will be undetectable for an observer on the earth's surface; i.e. this rotation is only an illusory one.

To be more convincing in our reasoning let us use again the method of counting waves. The number of the waves emitted from a stationary source of electromagnetic waves in point *O* (figure 2(a)) in time *t* is n_0. If there is no moving point O' in *A*, those n_0 waves are distributed between the source in point *O* and the point of wave front *C* in the following way: βn_0 waves are located between *O* and *A*, and $(1-\beta)n_0$ waves between *A* and *C*. If now instead of stationary point *A* we consider moving point O', then between O' and the point of wave front *C* there will be $n_0' = \sqrt{1-\beta^2}\, n_0$ waves. Indeed, from the point of view of the stationary frame, $|AC| = (1-\beta)n_0\lambda_0 = n_0'\lambda_0 \dfrac{\sqrt{1-\beta}}{\sqrt{1+\beta}}$, from which $n_0' = \sqrt{1-\beta^2}\, n_0$. The remaining $\left(1-\sqrt{1-\beta^2}\right)n_0$ waves are located between points *O* and O'. Since between *O* and *A*, and between *O* and O' different numbers of waves are contained, the distances $|OA|$ and $|OO'|$ cannot be equal to each other. It is obvious that the distances $|OA_1|$ and $|OO'|$ in figure 2(c) contain the same numbers of waves. Thus, the point O' is in point A_1 of the stationary frame and not in point *A* as we obtain by means of measurement in "mathematical" units.

On the other hand we cannot exclude the possibility that some kind of measurement may give a "real" distance: $|OO'|_r = ct_1\left(1-\sqrt{1-\beta^2}\right)$; then the mathematical distance is $|OO'|_m = \beta c t_1$, where $t_1 = \dfrac{t}{\sqrt{1-\beta^2}}$.

We have to expect that direct measurements with a ruler or radar will always give "mathematical" values, while some indirect measurements (the ones which cannot be reduced to measurements with a ruler or radar) may give "real" values.

In D. Frisch and H. Smith experiment [7] there is no direct measurement of the distances covered by muons. So, we cannot be sure whether the height of Mt. Washington, *H,* is a "mathematical" distance covered by muons or a "real" one.

In the latter case, the distance covered by muons in "mathematical" units would be $H_1 = \dfrac{H\beta}{1-\sqrt{1-\beta^2}}$. The time necessary for covering this distance by muons in the stationary and their rest frames are $t = \dfrac{H}{c\left(1-\sqrt{1-\beta^2}\right)}$ and $t' = \dfrac{H\sqrt{1-\beta^2}}{c\left(1-\sqrt{1-\beta^2}\right)}$ respectively. According to the experimental data $H \approx 1910$ m, $\beta \approx 0.995$, original number of muons $n_0 \approx 568$, and their half lifetime $\tau_0 \approx 1.5\text{x}10^{-6}$ sec. Those values yield $t' = 0.708\text{x}10^{-6}$ sec. and the number of remaining muons, $n \approx 410$. This value is much closer to the experimental value of 412 as against the value of $n \approx 422$ obtained by the authors in their calculations, but this one particular case, of course, does not prove anything. Only statistical analysis of series of similar experiments can give statistically significant answer whether our suggestion is true or not.

Now let us find out what the front of the electromagnetic wave is like in a moving frame. Let in Fig. 1 the number of waves emitted from the origin of the stationary frame, *O*, during time period of *t* is n_0 , then $|OA| = n_0\lambda_0$. If this array of n_0 waves is watched from the moving frame, then for a spherical wave point *A* becomes some point A_2 on the line *OA*:

$$|O_1A_2| = n_0\lambda_\theta = n_0\frac{|O_1A|}{n_0'} = \frac{ct\sqrt{1-2\beta\cos\theta+\beta^2}}{\sqrt{1-\beta^2}}.$$

The set of points A_2 for all possible directions forms the surface of a sphere with centre in *O'* where $|O'O_1| = \dfrac{\beta ct}{\sqrt{1-\beta^2}}$, and radius $R' = |O'A_2| = \dfrac{ct}{\sqrt{1-\beta^2}}$. Thus, we have got what we were looking for: the wave front in the moving frame at the instant of time when the clock in point *O* shows time *t*.

In other words this means that in moving frames, while describing the propagation of spherical electromagnetic waves, the units of time and space intervals are contracted in $1/\sqrt{1-\beta^2}$ times and this contraction occurs uniformly in all directions, which are radically different from the predictions of the Lorentz transformation.

Now we can write new expressions for the transformation of the fronts of spherical electromagnetic waves. If the origin of the moving frame is in point O_1, then:

$$x' = \frac{x-\beta ct}{\sqrt{1-\beta^2}},\; y' = \frac{y}{\sqrt{1-\beta^2}},\; z' = \frac{z}{\sqrt{1-\beta^2}},\; t' = t = 0;$$

t' and *t* are time coordinates of the points of the wave front.

If we assume that the origin of the moving frame is in point *O'*, then:

$$x_1' = \frac{x}{\sqrt{1-\beta^2}},\; y' = \frac{y}{\sqrt{1-\beta^2}},\; z' = \frac{z}{\sqrt{1-\beta^2}},\; t' = t = 0; \qquad (8)$$

We have to add here the following relations as well:

$$t_{O_1} = t_O\sqrt{1-\beta^2},\; t_{O'} = \frac{t_o}{\sqrt{1-\beta^2}}, \qquad (9)$$

were t_O, t_{O_1} and $t_{O'}$ are the readings of the clocks in points *O*, O_1 and *O'* (the centres of the spheres in Fig. 1) respectively.

Equations (8) and (9) are the counterparts of equations (1) for spherical electromagnetic wave.

In the case of a plane wave:

$$n_0\lambda_\theta = n_0\frac{|O_1F|}{n_0'} = \frac{ct(1-\beta\cos\theta)}{\sqrt{1-\beta^2}} = |O_1A'|.$$

To find the location of point A', let us notice that in direction X (the direction of movement of the frame) it is impossible to make distinction between the spherical and plane waves. Thus, X components of those waves must be the same in any frame, i.e. x coordinate of point A' must be the same as that of point A_2. So point A' lies on the segment $|A_2G|$ (figure 1); then $|A'G| = |AF|$ and, from $\Delta OAO_1'$, $\sin\varphi = \dfrac{\sin\theta\sqrt{1-\beta^2}}{1-\beta\cos\theta}$, where $\varphi = \angle A'O_1G$ ($(\varphi - \theta)$ is the angle of aberration for plane electromagnetic waves), as follows from the Lorentz transformation.

In the case of spherical waves angles φ and α coincide. Thus, according to new transformation law the slope of the above-mentioned tube is the same in the stationary and moving frames and the angle of aberration is $(\alpha - \theta)$. The relation between those angles is given by (5).

In figure 1 we can see that using plane wave approximation instead of spherical wave approximation for electromagnetic waves, we are substituting point A by point K. That is the size of the error we make using the Lorentz transformation.

Let three clocks – clock 1, clock 2, and clock 3 – be placed in O, O' and A points respectively. At the beginning, when all of them show zero time they coincide in point O of the stationary frame. Clocks 2 and 3 are moving relative stationary clock 1 with velocities $v_1 = \beta_1 c$ and $v = \beta c$ respectively, θ is the angle between those velocities.

Fig. 3 represents the picture after some time t_1 by the stationary clock 1. $|OO'| = \beta_1 c t_1$ and $|OA| = \beta c t_1$; clock 2 shows $t_2 = t_1\sqrt{1-\beta_1^2}$ and clock 3 – $t_3 = t_1\sqrt{1-\beta^2}$.

In the rest frame of clock 2, when clock 3 shows $t_3 = t_1\sqrt{1-\beta^2}$, clock 2 must show

$$t_2' = t_1 \frac{\sqrt{1-\beta^2}}{\sqrt{1-\beta_2^2}}, \tag{10}$$

because, as we have seen, it is necessary that $t_2'^2 = (\beta_2 t_2')^2 + t_3^2$.

In this frame the distance between clocks 2 and 3 is not $|O'A|$, but $|O_1'A|$, where

$$|O_1'A| = \frac{|O'A|}{\sqrt{1-\beta_1^2}} = c\beta_2 t_2'. \tag{11}$$

That is because in the rest frame of clock 2 the unit of length is $1/\sqrt{1-\beta_1^2}$ times shorter than in the rest frame of clock 1, and any distance, in particular the distance between clocks 2 and 3, must be the same times larger.

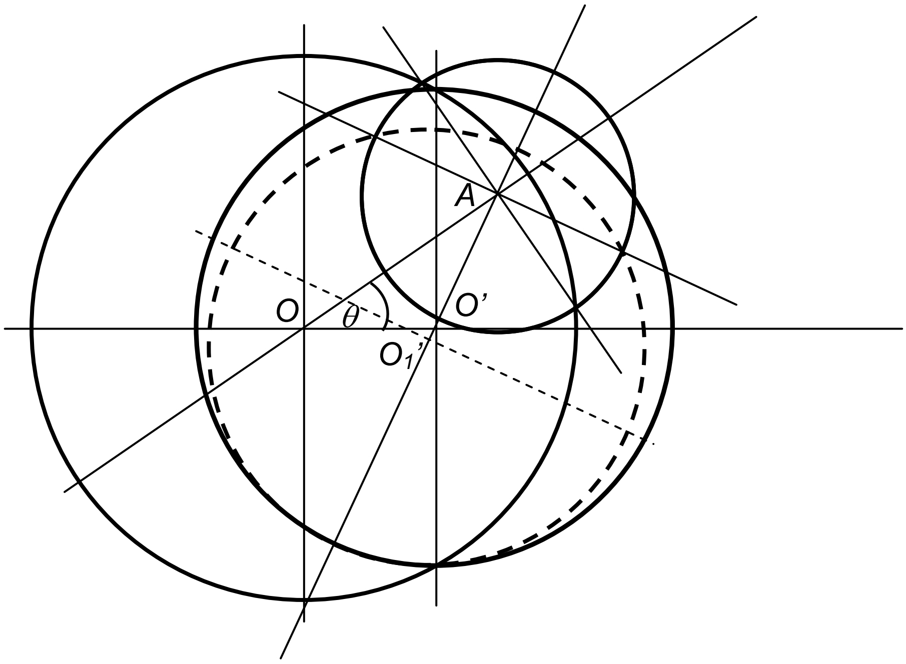

Figure 3. An illustration of the law of addition of velocities.

In Fig. 3

$$|O'A|^2 = \left(\beta^2 + \beta_1{}^2 - 2\beta\beta_1 \cos\theta\right)c^2 t_1{}^2 . \tag{12}$$

From (10), (11) and (12):

$$\beta_2{}^2 = \frac{\beta^2 + \beta_1{}^2 - 2\beta\beta_1 \cos\theta}{1 - 2\beta\beta_1 \cos\theta + \beta^2 \beta_1{}^2} .$$

Let in Fig. 1 a Michelson-Morley device is oriented in direction α relative to axis X along which the earth is moving. The length of each arm of the device (a one-way distance covered by the light in each direction in the rest frame of the devise) is R. From the point of view of a stationary frame in which the device (together with the earth) is moving with velocity β along the axis X one arm of the device is O_1A and the other - O_1B. In "natural" units those arms (O_1A_1 and O_1B_1 respectively) are equal and the light emitted from point O at $t = t' = 0$ arrives at both ends of the device simultaneously not only in the stationary, but also in the moving frame, after the periods of time $t = \tau$ and $t' = \tau\sqrt{1-\beta^2}$ respectively. Thus, in the Michelson-Morley experiment not only the total times of the round trips of light impulses in opposite directions are equal in any frame, but also the times of the one-way back and forth travels of those impulses.

Now let us have a brief overview.

The use of the Lorentz transformation, which assumes the planar character of electromagnetic waves, in the cases of the fronts of spherical electromagnetic waves, as it was to be expected, leads to a distorted picture of physical reality.

The new transformation developed in this paper maintains spherical form of electromagnetic waves in all inertial frames. According to this transformation, contrary to the predictions of the Lorentz transformation, the units of time and space intervals in moving frames are contracted in $1/\sqrt{1-\beta^2}$ times and this contraction occurs uniformly in all directions. It seems evident that such transformation won't have any effect on the mathematical expressions of physical laws.

The expressions of Doppler effect and aberration for spherical electromagnetic waves derived here differ from the similar expressions for planar electromagnetic waves and those differences persist up to infinity.

Proceeding from the fact of constancy of light velocity in emptiness and isotropy of space, we have to assume that the units of length undergo the same kind of changes as the lengths of electromagnetic waves, and "natural" units of length are to be introduced. "Natural" units coincide with "mathematical" ones when the distances between two stationary points or between a stationary point and the wave front are measured. Using "natural" units we get that the times of back and forth travels of an electromagnetic impulse during its round trip must be equal in any frame.

In order to avoid logical difficulties it is necessary to assign to the points of the front of an electromagnetic wave, emitted from the origin of a frame at t=0, zero time coordinates; otherwise the arrival of an electromagnetic impulse at some point of a stationary frame at some instant of time in that point, which must be an invariant, will occur at different times in different frames (this simple fact has been strangely overlooked by the physicists for so many years).

In order to maintain the correct balance of the numbers of emitted electromagnetic waves the "real" distances covered by moving points are to be introduced. A "real" distance between a stationary and a moving point contains the number of waves which is equal to the difference in the numbers of waves contained between the front of the wave and the stationary and moving points respectively. In other words, the difference between the readings of the synchronized stationary and moving clocks is equal to the time necessary for the light for covering the "real" distance between those clocks.

A correction to the expression of the law of addition of velocities is made.

It is encouraging that the new approach is much simpler and seems to be a better approximation to the reality. A thorough experimental check-up of all theoretical results of the relativistic theory is absolutely necessary.

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