Poynting vector. Arithmetic mistake

R. I. Khrapko
Moscow Aviation Institute, 125993, Moscow, Russia

Expressions for a polarized light beam must be corrected to correspond to results of the papers

All physicists, which consider polarized light beam, start from an expression for magnetic potential **A**

$$\mathbf{A} = \exp(ikz - i\omega t)(\alpha\mathbf{x} + \beta\mathbf{y})u(x, y, z)\,, \tag{1}$$

(see, e.g., Loudon [1] (2.1), Allen et al. [2] (2.9), Zambrini et al. [3] (2)). $i(\alpha\beta^* - \alpha^*\beta) = \sigma$ is usually identified with the spin in the *z*-direction (see, e.g., [1] (2.11), [2] p. 299), and

$$|\alpha|^2 + |\beta|^2 = 1 \tag{2}$$

(see, e.g., [3] p. 1046). Eqn (1) leads to

$$\mathbf{E} = \omega\exp(ikz - i\omega t)[i\alpha\mathbf{x} + i\beta\mathbf{y} - \frac{1}{k}\mathbf{z}(\alpha\partial_x + \beta\partial_y)]u\,, \tag{3}$$

$$\mathbf{B} = \exp(ikz - i\omega t)[-ik\beta\mathbf{x} + ik\alpha\mathbf{y} + \mathbf{z}(\beta\partial_x - \alpha\partial_y)]u\,. \tag{4}$$

(see, e.g., [1] (2.17), [2] (2.10), (2.11)). It is cozily to set, accordingly to (2), $\alpha = -i/\sqrt{2}$, $\beta = 1/\sqrt{2}$ for a right-circularly polarized beam. Then we arrive to

$$\mathbf{E} = \omega\exp(ikz - i\omega t)[\mathbf{x} + i\mathbf{y} + \frac{1}{k}\mathbf{z}(i\partial_x - \partial_y)]u/\sqrt{2}\,, \tag{5}$$

$$\mathbf{B} = \exp(ikz - i\omega t)[-ik\mathbf{x} + k\mathbf{y} + \mathbf{z}(\partial_x + i\partial_y)]u/\sqrt{2}\,. \tag{6}$$

These coincide with the Jackson's expressions [4] (p. 350)

$$\mathbf{E} = \exp(ikz - i\omega t)[\mathbf{x} + i\mathbf{y} + \frac{1}{k}\mathbf{z}(i\partial_x - \partial_y)]E_0\,, \quad \mathbf{B} = -i\frac{k}{\omega}\mathbf{E} \tag{7}$$

if $E_0 = \omega u/\sqrt{2}$.

Now let us calculate a time-average flux of energy in the *z*-direction given by the *z*-component of the real part of the complex Poynting vector [4] (6.132)

$$\mathbf{S} = (\mathbf{E}\times\mathbf{B}^*)/2 \tag{8}$$

(we set $\mu = 1$). Inserting (5) and (6) into

$$<\mathbf{S}> = \Re(\mathbf{E}\times\mathbf{B}^*)/2 \tag{9}$$

yields

$$<S_z> = \Re(E_x B_y^* - E_y B_x^*)/2 = \omega k|u|^2/2\,. \tag{10}$$

Meanwhile all authors write a double quantity : $<S_z> = \omega k|u|^2$ (see, e.g., [1] (3.24), [2] (2.16), [5] (4)), [6] (6), (9)).

The authors' mistake is obvious. They ignore the factor $\frac{1}{2}$ in eqn (8). Instead of (9), they write

$$<\mathbf{S}> = (\mathbf{E}^*\times\mathbf{B} + \mathbf{E}\times\mathbf{B}^*)/2 \tag{11}$$

(see, e.g., [1] (3.8), [2] (2.5), [6] (6), (9)).

$<S_z> = \omega k|u|^2$ is a handy expression. So, if authors intend to use α, β-factors in eqns (1), (3), (4), they have to change $u \to u\sqrt{2}$ in these eqns.

I submit these Comments to PRA as a response to a report of Referee G of my submission **LG10066 "Calculation of absorbed spin contradicts electrodynamics and an experiment".** The Referee considers that I must explain which expression for the fields is the correct one and why other authors use the wrong expression for the Poynting vector.

---

[1] R. Loudon, Phys. Rev. A68, 013806 (2003)
[2] L. Allen, M. J. Padgett, M. Babiker, *Progress in Optics XXXIX* (Elsevier, Amsterdam, 1999).
[3] R. Zambrini, S. M. Barnett, J. Mod. Optics **52** 1045 (2005)
[4] J D Jackson, *Classical Electrodynamics* (Wiley, 1999).
[5] L. Allen, M. J. Padgett, Opt. Commun. 184, 67 (2000).
[6] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, Phys. Rev. A45, 8185 (1992)