Paradox of the classical Beth optics experiment

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A celebrated Beth's experiment contradicts the angular momentum conservation law in the frame of Maxwell electrodynamics because Beth's birefringent plate experienced a torque without an angular momentum flux in the surrounding space. However, this paradox can be removed by introducing a classical spin tensor.

1. Angular momentum of a circularly polarized light

It has been known for long time that, on the basis of either the wave theory [1, 2] or the quantum theory (by assigning an angular momentum of $\pm h/2\pi$ to a photon), a circularly polarized light should exert a torque on a doubly refracting plate which changes the state of polarization of the light, or on a medium which (maybe partly) absorbs the light.

R. A. Beth explained [3] that the moment of force or torque exerted on a doubly refracting medium by a light wave passing through it arises from the fact that the dielectric constant ε is a tensor. Consequently the electric intensity $\mathbf{E}$ is, in general, not parallel to the electric polarization $\mathbf{P}$ in the medium. The torque per unit volume produced by the action of the electric field on the polarization of the medium is

$$\tau / V = \mathbf{P} \times \mathbf{E} . \quad (1.1)$$

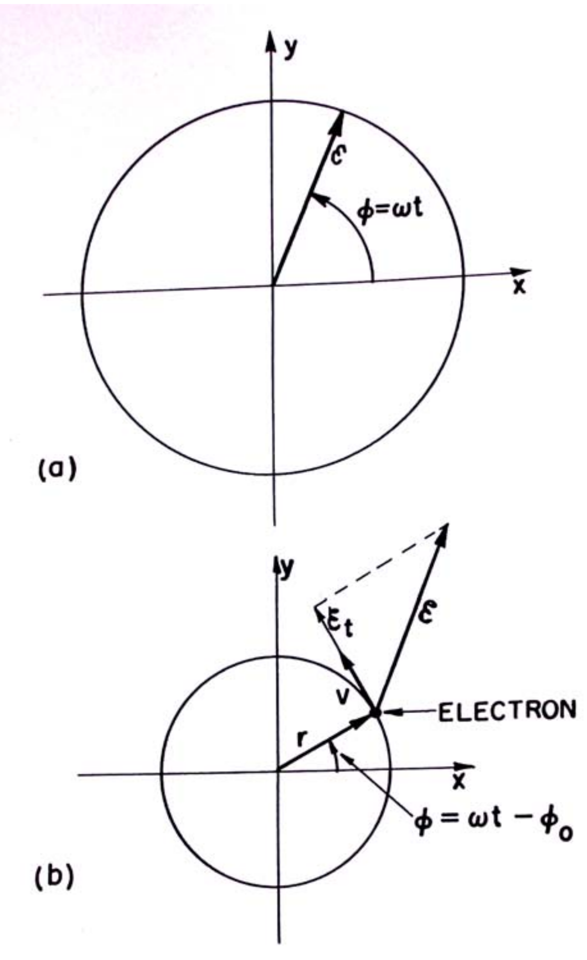

Fig. 17–5. (a) The electric field $\mathcal{E}$ in a circularly polarized light wave. (b) The motion of an electron being driven by the circularly polarized light.

R. Feynman repeated this explanation [4]. We quote him from [4] with insignificant abridgements.

"If we have a beam of light containing a large number of photons all circularly polarized the same way, it will carry angular momentum. If the total energy carried by the beam in a certain time is W, then there are $N = 2\pi W / h\omega$ photons. Each one carries the angular momentum $h/2\pi$, so there is a total angular momentum of

$$J_z = Nh/2\pi = W/\omega . \quad (1.2)$$

Can we provide classically that light which is right circularly polarized carries an angular momentum and energy in proportion $1/\omega$? Here we have a case where we can go from the quantum things to the classical things. Remember what right circularly polarized light is, classically. It's described by an electric field so that the electric vector $\mathbf{E}$ goes in a circle – as drawn in Fig. 17-5(a). Now suppose that such a light shines on a wall which is going to absorb it – or at least some of it – and consider an atom in the wall according to the classical physics. We'll suppose that the atom is isotropic, so the result is that the electron moves in a circle, as shown in Fig. 17-5(b). The electron is displaced at some displacement $\mathbf{r}$ from its equilibrium position at the origin and goes around with some phase lag with respect to the vector $\mathbf{E}$. The relation between $\mathbf{E}$ and $\mathbf{r}$ might be as shown in Fig. 17-5(b). As time goes on, the electric field rotates and the displacement rotates with the same frequency, so their relative orientation stays the same. Now let's look at the work being done on this electron. The rate that energy is being put into this electron is v, its velocity, times the component of $\mathbf{E}$ parallel to the velocity:

$$dW / dt = eE_t v \,. \tag{1.3}$$

But look, there is angular momentum being poured into this electron, because there is always a torque about the origin. The torque is $eE_t r$ which must be equal to the rate of change of angular momentum dJ_z / dt:

$$dJ_z / dt = eE_t r \,. \tag{1.4}$$

Remembering that $v = \omega r$, we have that

$$dJ_z / dW = 1/\omega \,. \tag{1.5}$$

Therefore, if we integrate the total angular momentum which is absorbed, it is proportional to the total energy – the constant of proportionality being $1/\omega$.”

Thus Beth’s and Feynman’s reasoning prove that a circularly polarized plane wave carries angular momentum whose density is proportional to the energy density. Unfortunately, the authors did not give an expression for the angular momentum flux density through the field quantities. At the same time, the scientific community denies angular momentum of plane waves at all.

Heitler wrote [5]:

“In Maxwell’s theory the Poynting vector $\mathbf{E}\times\mathbf{H}$ (divided by c^2) is interpreted as the density of momentum of the field. We can then also define an angular momentum relative to a given point O or to a given axis,

$$\mathbf{J} = \int_V \mathbf{r}\times(\mathbf{E}\times\mathbf{H})dV \tag{1.6}$$

where $\mathbf{r}$ is the distance from O and V is the volume of a transverse slice of the beam [$c = 1$ in this paper].

A plane wave traveling in the *z*-direction and with infinite extension in the *xy*-directions can have no angular momentum about the *z*-axis, because $\mathbf{E}\times\mathbf{H}$ is in the *z*-direction and $[\mathbf{r}\times(\mathbf{E}\times\mathbf{H})]_z = 0$. However, this is no longer the case for a wave with finite extension in the *xy*-plane. Consider a cylindrical wave with its axis in the *z*-direction and traveling in this direction. At the wall of the cylinder $r = R$, say, we let the amplitude drop to zero. It can be shown that the wall of such a wave packet gives a finite contribution to J_z.”

Ohanian wrote [6]:

“In an infinite plane wave, the $\mathbf{E}$ and $\mathbf{H}$ fields are everywhere perpendicular to the wave vector and the energy flow is everywhere parallel to the wave vector. However, in a wave of finite transverse extent, the $\mathbf{E}$ and $\mathbf{H}$ fields have a component parallel to the wave vector (the field lines are closed loops) and the energy flow has components perpendicular to the wave vector. For instance, Fig. 1 shows the time-average transverse energy flow in a circularly polarized wave propagating in the *z*-direction; the wave has a finite extent in the *x* and *y* directions. The circulating energy flow in the wave implies the existence of angular momentum, whose direction is along the direction of propagation.”

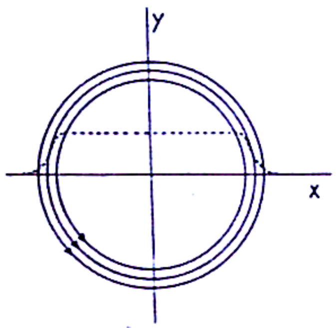

Fig. 1. This pattern of circular flow lines represents the time-average energy flow, or the momentum density, in a circularly polarized electromagnetic wave packet. On a given wave front, say $z = 0$, the fields are assumed to be constant within a circular area and to decrease to zero outside of this area (the dashed line gives the field amplitude as a function of radius). The energy flow has been calculated from an approximate solution of Maxwell’s equations. The picture only shows the flow in the transverse directions. The flow in the longitudinal direction is much larger; the net flow is helical.

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Simmonds and Guttman wrote [7]:

“The electric and magnetic field of a cylindrical beam can have a nonzero *z*-component only within the ‘skin’ region of the wave. Having *z*-component within this region implies the possibility of a nonzero *z*-component of angular momentum within this region. Since the wave is identically zero outside the skin and constant inside the skin region, the skin region is the only one in which the z-component of angular momentum does not vanish.

In Fig. 9.3 we plot an acceptable function $E_0(x, y) = E_0(r)$. We have explicitly made E_0 constant over a large central region of the wave and confined the variation of the function from this constant value to zero to lie within a ‘skin’ of thickness δ which lies a distance R_0 from the axis.”

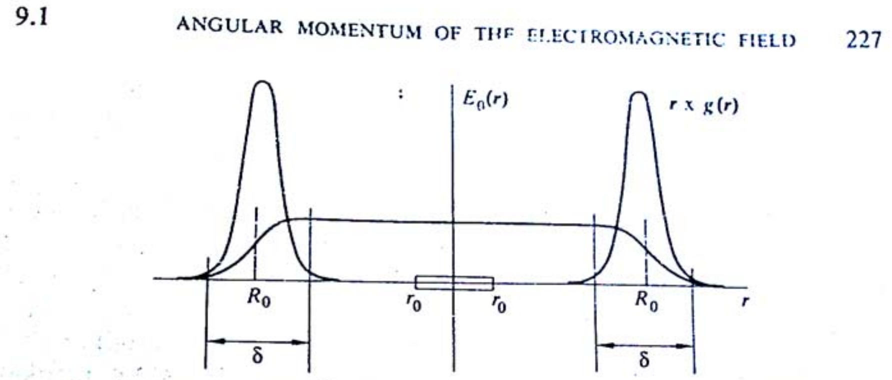

Fig. 9.3. The electric field amplitude and the angular momentum density across a cylindrical beam.

A calculation of the angular momentum $\mathbf{J}$, according to eqn. (1.6), requires an explicit expression for the beam. We use the Jackson’s expressions [8] with $k = \omega$ here,

$$\mathbf{E} = \exp(i\omega z - i\omega t)[\mathbf{x} + i\mathbf{y} + \frac{1}{\omega}\mathbf{z}(i\partial_x - \partial_y)]E_0(x, y), \quad \mathbf{H} = -i\mathbf{E}, \tag{1.7}$$

Transform the integrand of J_z from eqn. (1.6),

$$\Re[x(\mathbf{E}\times\mathbf{H}^*)_y - y(\mathbf{E}\times\mathbf{H}^*)_x]/2 = \Re[x(E_zH_x^* - E_xH_z^*) - y(E_yH_z^* - E_zH_y^*)]/2$$
$$= \Re[x(E_ziE_x^* - E_xiE_z^*) + y(E_ziE_y^* - E_yiE_z^*)]/2 = -\Im[xE_zE_x^* + y\,E_zE_y^*]$$
$$= -\Im[xE_0(i\partial_x - \partial_y)E_0 + y(-iE_0)(i\partial_x - \partial_y)E_0]/\omega = -(x\partial_x + y\partial_y)E_0^2/2\omega. \tag{1.8}$$

Substituting (1.8) into (1.6) and integrating by part yields

$$J_z = \int E_0^2 dV/\omega. \tag{1.9}$$

The power P of the beam is,

$$\mathsf{P} = \int_a (\mathbf{E}\times\mathbf{H})_z da = \int_a \Re(E_xH_y^* - E_yH_x^*)da = \int_a E_0^2 da \tag{1.10}$$

where a is an area of the beam section. If l is a length of the transverse slice of the beam, i.e. $V = la$, the energy of the slice is

$$W = \int E_0^2 dV \tag{1.11}$$

because $c = 1$. So the relation between the total angular momentum J_z and the total energy W,

$$J_z/W = 1/\omega, \tag{1.12}$$

is the same in Beth – Feynman paradigm and in the scientific community paradigm. However, the distribution of the angular momentum is different. According to Beth – Feynman, the angular momentum density is proportional to energy density in a beam or in a plane wave, but, according to the community, the angular momentum is located near the wall of the beam and is absent in the plane wave.

In connection with this difference an important question was raised at the V. L. Ginsburg Moscow Physical Seminar in the spring of 1999. The question was about absorption of a circularly polarized light by a round flat target, which is divided concentrically into an inner disc and a closely fitting outer annulus [8].

If the target absorbs a circularly polarized beam, the annulus absorbs the wall or ‘skin’ of the beam, which carries the angular momentum, according to the community, and the disc absorbs the body of the beam, which has no angular momentum. Since the Poynting vector is perpendicular to the disc, an infinitesimal force

$$d\mathsf{F}^i = T^{ij}da_j \tag{1.13}$$

acting on a surface element da_j of the disc is also perpendicular to the disc (T^{ij} is the Maxwell stress tensor). So, the disc does not perceive a torque when the target absorbs a circularly polarized beam. There are no ponderomotive forces, which are capable to twist the disc. Tangential forces act only on the annulus.

But it is clear that in reality the disc does perceive a torque from the wave, since the disc gets angular momentum, according to Beth – Feynman. The disc will be twisted in contradiction with the community paradigm.

Allen and Padgett [9] attempted to explain the torque acting on the disc within the scope of the paradigm. They mentally decomposed the beam into three beams: the inner beam, the annulus beam, and the remainder. They wrote, “Any form of aperture introduces an intensity gradient, so a field component is induced in the propagation direction and the dilemma is potentially resolved.”

Alas! A small clearance between the inner disc and outer annulus does not aperture a beam and does not induce longitudinal field components. The imaginary decomposition of a wave is not capable to create longitudinal field components and, correspondingly, transverse momentum and torque acting on the disc. Maxwell stress tensor cannot supply the disc with a torque. According to the Maxwell theory, the disc absorbs energy and feels normal pressure only.

Thus the mental experiment shows a weakness of the community paradigm. Does the Beth experiment confirm the formula (1.6)?

2. The Beth experiment

The classical Beth’s experiment [3] was made 70 years ago. A beam of circularly polarized light exerts a torque on a doubly refracting plate, which changes the state of polarization of the light beam. The apparatus used involves a torsional pendulum with about a ten minute period consisting of a round quartz half-wave plate one inch in diameter (M at Fig. 3 from [3]) suspended with its plane horizontal from a quartz fiber about 25 centimeters long. A circularly polarized light beam (power P = 80 mW, $\lambda = 1.2\ \mu$m, $\omega = 1.6 \cdot 10^{15}$ s^{-1}) passes through the half-wave plate from below upwards. Because the plate reverses the handedness of the circular polarization of the beam, according to (1.6) and (1.12), the torque acting on the plate must be

$$\tau = 2P/\omega \qquad (2.1)$$

However, and this is the main point, in order to redouble the torque, the beam is reflected and passes through the plate the second time on the way back. For this, about 4 millimeters above the plate is mounted a fixed quartz quarter-wave plate T (Fig. 3). The *top* side of the upper plate was coated by evaporation with a reflecting layer of aluminum The rotation of the pendulum is observed by a telescope using the small mirror *m* at Fig.3. As a result, the torque exerting on the half-wave plate is 20 dyne cm. This result is in accordance with the formula

$$\tau = 4P/\omega \qquad (2.2)$$

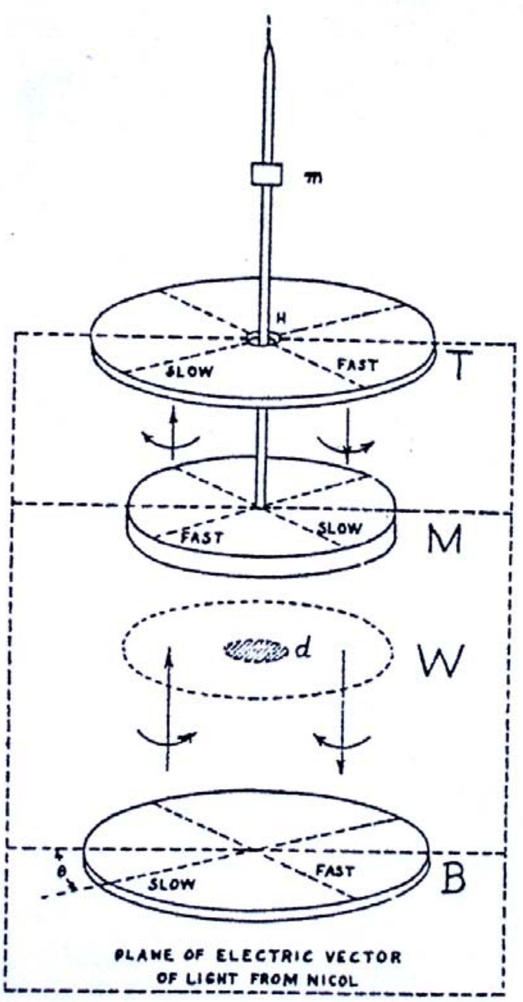

FIG. 3. Wave plate arrangement.

3. The Beth's result is a puzzle

It is evident that the reflected beam cancels the energy flux in the Beth’s apparatus. I.e. the Poynting vector $\mathbf{E} \times \mathbf{H} = 0$ in the experiment. Thus, according to eqn. (1.6), no angular momentum is contained in the double beam. So, no torque must act on the Beth plate according to eqn. (1.6). Why then the plate experiences the torque (2.2)?

We verify our claim $\mathbf{E}\times\mathbf{H}=0$ here. Let us start from the Jackson beam (1.7) with $\omega=1$ for simplicity,

$$\mathbf{E}_1=\exp(iz-it)[\mathbf{x}+i\mathbf{y}+\mathbf{z}(i\partial_x-\partial_y)]E_0\,,\quad \mathbf{H}_1=\exp(iz-it)[-i\mathbf{x}+\mathbf{y}+\mathbf{z}(\partial_x+i\partial_y)]E_0\,. \tag{3.1}$$

Changing the sign of z we get the reflected beam. But the quarter-wave plate T changes the handedness of the circularly polarization of the beam. Thus, the sign of *y* must be change as well. So, the reflected beam is

$$\mathbf{E}_2=\exp(-iz-it)[\mathbf{x}-i\mathbf{y}+\mathbf{z}(-i\partial_x-\partial_y)]E_0\,,\quad \mathbf{H}_2=\exp(-iz-it)[-i\mathbf{x}-\mathbf{y}+\mathbf{z}(-\partial_x+i\partial_y)]E_0\,. \tag{3.2}$$

Adding up the expressions (3.1) and (3.2) we get the total field

$$E_x=\Re[\exp(iz-it)+\exp(-iz-it)]E_0=2E_0\cos z\cos t\,, \tag{3.3}$$

$$E_y=\Re[i\exp(iz-it)-i\exp(-iz-it)]E_0=-2E_0\sin z\cos t\,, \tag{3.4}$$

$$E_z=\Re[\exp(iz-it)(i\partial_x-\partial_y)+\exp(-iz-it)(-i\partial_x-\partial_y)]E_0=-2(\sin z\partial_x+\cos z\partial_y)E_0\cos t\,, \tag{3.5}$$

$$H_x=\Re[-i\exp(iz-it)-i\exp(-iz-it)]E_0=-2E_0\cos z\sin t\,, \tag{3.6}$$

$$H_y=\Re[\exp(iz-it)-\exp(-iz-it)]E_0=2E_0\sin z\sin t\,, \tag{3.7}$$

$$H_z=\Re[\exp(iz-it)(\partial_x+i\partial_y)+\exp(-iz-it)(-\partial_x+i\partial_y)]E_0=2(\sin z\partial_x+\cos z\partial_y)E_0\sin t\,, \tag{3.8}$$

As a result we get

$$\mathbf{E}=2[(\mathbf{x}\cos z-\mathbf{y}\sin z)-\mathbf{z}(\sin z\partial_x+\cos z\partial_y)]E_0\cos t \tag{3.9}$$

$$\mathbf{H}=-2[(\mathbf{x}\cos z-\mathbf{y}\sin z)-\mathbf{z}(\sin z\partial_x+\cos z\partial_y)]E_0\sin t \tag{3.10}$$

The **E** and **H** fields are parallel to each other everywhere. So, the Poynting vector is zero.

4. An explanation of the Beth result

The formula (1.6) predicts the zero result of the Beth experiment because this formula is incorrect. As Ohanian wrote, the existence of angular momentum (1.6) is caused by circulating energy flow in the wave. In other words, eqn. (1.6) represents an orbital angular momentum of electromagnetic field. It is in accordance with the fact that Maxwell electrodynamics does not know spin. Spin is considered as a pure quantum phenomenon. Maxwell electrodynamics knows the energy-momentum tensor $T^{\lambda\mu}$ (Maxwell-Minkowski tensor), but it does not know a spin tensor, or rather, spin tensor of the modern classical electrodynamics is zero. We introduce classical spin into the electrodynamics. We introduce a spin tensor $\Upsilon^{\lambda\mu\nu}$ [10, 11], i.e. we add a spin term to eqn. (1.6):

$$J^{ij}=\int_V 2r^{[i}T^{j]0}dV+\int_V\Upsilon^{ij0}dV\,. \tag{4.1}$$

Energy flux density, i.e. the Poynting vector $T^{0j}=T^{j0}$, is zero, $T^{j0}=0$, in the Beth experiment. So, the first term on the right of eqn. (4.1), i.e. the orbital term, is zero. But spin flows from the beam into the Beth plate, and a torque acts on the plate due to the second term.

The sense of the spin tensor $\Upsilon^{\lambda\mu\nu}$ is as follows. The component Υ^{ij0} is a volume density of spin. This means that

$$dS^{ij}=\Upsilon^{ij0}dV \tag{4.2}$$

is spin of electromagnetic field inside the spatial element dV. The component Υ^{ijk} is a flux density of spin flowing in the direction of the x^k axis. For example,

$$dS_z/dt=dS^{xy}/dt=d\tau^{xy}=\Upsilon^{xyz}da_z \tag{4.3}$$

is *z*-component of spin flux passing through the surface element da_z per unit time, i.e. the torque acting on the element.

The explicit expression for the spin tensor is [10, 11] (see also Supplement)

$$\Upsilon^{\lambda\mu\nu}=A^{[\lambda}\partial^{|\nu|}A^{\mu]}+\Pi^{[\lambda}\partial^{|\nu|}\Pi^{\mu]}, \tag{4.4}$$

where A^λ and Π^λ are magnetic and electric vector potentials which satisfy

$$2\partial_{[\mu}A_{\nu]}=F_{\mu\nu},\quad 2\partial_{[\mu}\Pi_{\nu]}=-e_{\mu\nu\alpha\beta}F^{\alpha\beta} \tag{4.5}$$

where $F^{\alpha\beta} = -F^{\beta\alpha}$, $F_{\mu\nu} = F^{\alpha\beta} g_{\mu\alpha} g_{\nu\beta}$ is the field strength tensor of a free electromagnetic field.

A relation between Π and F can be readily obtained in the vector form as follows. If $\mathrm{div}\mathbf{E} = 0$, then $\mathbf{E} = \mathrm{curl}\Pi$. If also $\partial\mathbf{E}/\partial t = \mathrm{curl}\mathbf{H}$, then

$$\partial\Pi/\partial t = \mathbf{H}. \tag{4.6}$$

This reasoning is analogous to the common one:
If $\mathrm{div}\mathbf{H} = 0$, then $\mathbf{H} = \mathrm{curl}\,\mathbf{A}$. If also $\partial\mathbf{H}/\partial t = -\mathrm{curl}\mathbf{E}$, then

$$\partial\mathbf{A}/\partial t = -\mathbf{E}. \tag{4.7}$$

Now use the spin tensor (4.4) for calculating of the spin flux into Beth plate. Since the orbital term is zero, wall terms, ∂_x, ∂_y, in eqns. (3.9), (3.10) may be neglected, and we have for the fields

$$\mathbf{E} = 2(\mathbf{x}\cos z - \mathbf{y}\sin z)E_0 \cos t, \tag{4.8}$$

$$\mathbf{H} = -2(\mathbf{x}\cos z - \mathbf{y}\sin z)E_0 \sin t, \tag{4.9}$$

$$\mathbf{A} = -\int \mathbf{E}dt = -2(\mathbf{x}\cos z - \mathbf{y}\sin z)E_0 \sin t, \tag{4.10}$$

$$\Pi = \int \mathbf{H}\,dt = 2(\mathbf{x}\cos z - \mathbf{y}\sin z)E_0 \cos t. \tag{4.11}$$

When calculating the spin tensor (4.4) a signature of metric tensor must be taken into account. Because $g^{ij} = -1$, $\partial^i = -\partial_i$. Thus, the spin flux density onto the low side of the Beth plate is

$$\Upsilon^{xyz} = (A^x\partial^z A^y - A^y\partial^z A^x)/2 + (\Pi^x\partial^z\Pi^y - \Pi^y\partial^z\Pi^x)/2 = 2E_0^2(\sin^2 t + \cos^2 t) = 2E_0^2. \tag{4.12}$$

The same calculation for the domain above the plate gives $\Upsilon^{xyz} = -2E_0^2$. This means that S^{xy}-component of the spin moves opposite the *z*-direction, i.e. towards the plate also. In result, the plate receives the spin flux density, or torque density, of $4E_0^2$ with the absence of energy flux! Thus, the torque is

$$\tau = 4\int E_0^2 da, \tag{4.13}$$

and recalling (1.10), we get ($\omega = 1$)

$$\tau = 4\mathrm{P} \tag{4.14}$$

as the Beth experiment shows.

It is remarkable that volume density of spin equals zero, i.e. $\Upsilon^{xy0} = 0$. The use of (4.10), (4.11) shows this. This is naturally because the beams of the same handedness, which propagate in the opposite direction, are summed up. So, the Beth's double beam contains spin flux and energy without spin and energy flux.

Another applications of the spin tensor (4.4) are presented in [12, 13] and at web sites www.mai.ru/projects/mai_works/, www.sciprint.org. Absorption and reflection of a circularly polarized beam is calculated there, a radiation of a rotating electrical dipole and other topics are considered in these works.

5. Supplement. Electrodynamics' spin tensor

The standard classical electrodynamics starts from the free field canonical Lagrangian [14]

$$\underset{c}{\mathrm{L}} = -F_{\mu\nu}F^{\mu\nu}/4, \quad F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}, \tag{5.1}$$

Using this Lagrangian, by the Lagrange formalism physicists obtain the canonical energy-momentum tensor

$$\underset{c}{T}^{\lambda\mu} = \partial^\lambda A_\alpha \frac{\partial \underset{c}{\mathrm{L}}}{\partial(\partial_\mu A_\alpha)} - g^{\lambda\mu}\underset{c}{\mathrm{L}} = -\partial^\lambda A_\alpha F^{\mu\alpha} + g^{\lambda\mu}F_{\alpha\beta}F^{\alpha\beta}/4, \tag{5.2}$$

and the canonical total angular momentum tensor

$$\underset{c}{J}^{\lambda\mu\nu} = 2x^{[\lambda}\underset{c}{T}^{\mu]\nu} + \underset{c}{\Upsilon}^{\lambda\mu\nu} \tag{5.3}$$

where

$$\underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda}\delta_\alpha^{\mu]}\frac{\partial \underset{c}{\mathrm{L}}}{\partial(\partial_\nu A_\alpha)} = -2A^{[\lambda}F^{\mu]\nu}, \tag{5.4}$$

is the canonical spin tensor.

Unfortunately, the canonical tensors are not electrodynamics tensors. True electrodynamics tensors must be in accordance with experimental facts. In particular, it should be

$$\partial_\mu T^{\lambda\mu} = -F^{\lambda\mu} j_\mu = F^{\lambda\mu}\partial^\nu F_{\mu\nu} . \tag{5.5}$$

But $\underset{c}{T}^{\lambda\mu}$ has a wrong divergence,

$$\partial_\mu \underset{c}{T}^{\lambda\mu} = -\partial^\lambda A^\mu j_\mu = \partial^\lambda A^\mu \partial^\nu F_{\mu\nu} , \tag{5.6}$$

and is asymmetric. Physicists undertook an attempt to modify these tensors. They “put in by hand” specific addends [15, 16] to the canonical tensors and arrive to the standard energy-momentum tensor $\Theta^{\lambda\mu}$, the standard total angular momentum tensor $\underset{st}{J}^{\lambda\mu\nu}$, and the standard spin tensor $\underset{st}{\Upsilon}^{\lambda\mu\nu}$, which is zero,

$$\Theta^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} - \partial_\nu \underset{c}{\tilde{\Upsilon}}^{\lambda\mu\nu}/2 = -\partial^\lambda A_\nu F^{\mu\nu} + g^{\lambda\mu} F_{\alpha\beta}F^{\alpha\beta}/4 + \partial_\nu (A^\lambda F^{\mu\nu}) ,$$

$$\underset{c}{\tilde{\Upsilon}}^{\lambda\mu\nu} \overset{def}{=} \underset{c}{\Upsilon}^{\lambda\mu\nu} - \underset{c}{\Upsilon}^{\mu\nu\lambda} + \underset{c}{\Upsilon}^{\nu\lambda\mu} = -2A^\lambda F^{\mu\nu} , \tag{5.7}$$

$$\underset{st}{J}^{\lambda\mu\nu} = \underset{c}{J}^{\lambda\mu\nu} - \partial_\kappa (x^{[\lambda} \underset{c}{\tilde{\Upsilon}}^{\mu]\nu\kappa}) , \tag{5.8}$$

$$\underset{st}{\Upsilon}^{\lambda\mu\nu} = \underset{st}{J}^{\lambda\mu\nu} - 2x^{[\lambda}\Theta^{\mu]\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} - \underset{c}{\tilde{\Upsilon}}^{[\lambda\mu]\nu} = 0 . \tag{5.9}$$

But the standard tensors are not true electrodynamics tensors as well:

1. $\Theta^{\lambda\mu}$ obviously contradicts experiments. It is asymmetric and has wrong divergence as well

$$\partial_\mu \Theta^{\lambda\mu} = \partial_\mu \underset{c}{T}^{\lambda\mu} = \partial^\lambda A^\mu \partial^\nu F_{\mu\nu} . \tag{5.10}$$

Tensor Θ is never used. The Maxwell tensor (2.6) is used in the electrodynamics instead of $\Theta^{\lambda\mu}$.

2. The main defect is the absence of spin, $\underset{st}{\Upsilon}^{\lambda\mu\nu} = 0$. In contrast to the canonical pair, $\underset{c}{T}^{\lambda\mu}, \underset{c}{\Upsilon}^{\lambda\mu\nu}$, the standard pair, $\Theta^{\lambda\mu}, \underset{st}{\Upsilon}^{\lambda\mu\nu} = 0$, is defective. Standard energy-momentum tensor is not accompanied by a spin tensor.

Thus the Belinfante-Rosenfeld procedure [15, 16] is not fit for obtaining true electrodynamics tensors. This procedure, (5.7) – (5.9), is

$$\Theta^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} + \underset{st}{t}^{\lambda\mu}, \qquad \underset{st}{t}^{\lambda\mu} = -\partial_\nu \underset{c}{\tilde{\Upsilon}}^{\lambda\mu\nu}/2 = \partial_\nu (A^\lambda F^{\mu\nu}) , \tag{5.11}$$

$$\underset{st}{\Upsilon}^{\lambda\mu\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} + \underset{st}{s}^{\lambda\mu\nu} = 0 , \qquad \underset{st}{s}^{\lambda\mu\nu} = -\underset{c}{\tilde{\Upsilon}}^{[\lambda\mu]\nu} = 2A^{[\lambda}F^{\mu]\nu} . \tag{5.12}$$

Another way of using the canonical pair $\underset{c}{T}^{\lambda\mu}, \underset{c}{\Upsilon}^{\lambda\mu\nu}$ is presented in [11 – 13]. Note that the Maxwell tensor can be gained by adding a term

$$t^{\lambda\mu} = T^{\lambda\mu} - \underset{c}{T}^{\lambda\mu} = \partial_\nu A^\lambda F^{\mu\nu} \tag{5.13}$$

to the canonical energy-momentum tensor $\underset{c}{T}^{\lambda\mu}$. Here a question arises, what term $s^{\lambda\mu\nu}$, instead of $\underset{st}{s}^{\lambda\mu\nu}$, must be added to the canonical spin tensor $\underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda}F^{\mu]\nu}$ for changing it from the canonical spin tensor to an unknown electrodynamics spin tensor $\Upsilon^{\lambda\mu\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} + s^{\lambda\mu\nu}$? Our answer is [11 – 13]: the addends $t^{\lambda\mu}$, $s^{\lambda\mu\nu}$ must satisfy a relationship

$$\partial_\nu s^{\lambda\mu\nu} - 2t^{[\lambda\mu]} = 0 , \text{ i.e. } \partial_\nu s^{\lambda\mu\nu} - 2\partial_\alpha A^{[\lambda}F^{\mu]\alpha} = 0 . \tag{5.14}$$

A simple expression

$$s^{\lambda\mu\nu} = 2A^{[\lambda}\partial^{\mu]}A^\nu \tag{5.15}$$

satisfies Eq. (5.14). So, the suggested electrodynamics spin tensor is

$$2\underset{e}{\Upsilon}^{\lambda\mu\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} + s^{\lambda\mu\nu} = -2A^{[\lambda}F^{\mu]\nu} + 2A^{[\lambda}\partial^{\mu]}A^\nu = 2A^{[\lambda}\partial^{|\nu|}A^{\mu]} . \tag{5.16}$$

The expression (5.16) was obtained heuristically. It is not final one. Spin tensor (5.16) is obvious not symmetric in the sense of electric - magnetic symmetry. It represents only the electric field, $\mathbf{E}$, $\mathbf{A} = -\int \mathbf{E}dt$. A true spin tensor of electromagnetic waves must depend symmetrically on the magnetic vector potential A_α and on an electric vector potential Π_α (4.5). So the spin tensor of electromagnetic waves has the form (4.4).

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