Electrodynamics' spin

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We solve the angular momentum problems of the electrodynamics. We show that a desire of using a spin density proved to be correct for evaluating of the electrodynamics spin because the moment of momentum is an orbital angular momentum density and does not encompass the spin. However, the expression $\mathbf{E}\times\mathbf{A}$ is not a true expression for the spin density. This expression yields correct results in simplest cases randomly. Its improvement is hopeless. Instead we present a true spin density of electromagnetic waves and demonstrate a way for its deducing. Our result can be expressed by a sum of the orbital and spin angular momentums.

PACS: 75.10.Hk; 12.20.Fv; 03.05.De
Keywords: Field theory; spin tensor

I. INTRODUCTION

There is a scandalous situation in the modern electrodynamics.

On the one hand,

$$\mathbf{s}=\mathbf{E}\times\mathbf{A} \tag{1.1}$$

is considered as a spin volume density of an electromagnetic field and

$$\mathbf{S}=\int_V \mathbf{E}\times\mathbf{A}dV \tag{1.2}$$

is considered as the spin of an electromagnetic field in the volume *V*. (Here **E** and **A** are the electric field strength and the magnetic vector potential, respectively). I present here a series of quotations for a confirmation of my statement.

1) Jackson [1] divides the angular momentum of a distribution of electromagnetic fields

$$\mathbf{J}=\int\mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV \tag{1.3}$$

into a spin and an orbital parts,

$$\mathbf{J}=\int[\mathbf{E}\times\mathbf{A}+E^i(\mathbf{r}\times\nabla)A_i]dV \tag{1.4}$$

(for short I set $\mu_0=1$, $c=1$). He wrote, "The first term is sometimes identified with the 'spin' of the photon".

2) Also Ohanian [2] expresses the angular momentum by a sum of two terms:

$$\mathbf{J}=\int\mathbf{r}\times(E^i\nabla A_i)dV+\int\mathbf{E}\times\mathbf{A}\,dV\ . \tag{1.5}$$

He wrote, "The first term in Eq. (1.5) represents the orbital angular momentum, and the second term the spin".

3) The expression $\mathbf{E}\times\mathbf{A}$ is used by Friese et al. [3] for a plane electromagnetic wave.
If $\mathbf{E}=\Re\breve{\mathbf{E}}$, $\breve{\mathbf{E}}=\breve{\mathbf{E}}_0\exp[i(kz-\omega t)]$, we have $\breve{\mathbf{A}}=-i\breve{\mathbf{E}}/\omega$ because

$$\breve{\mathbf{A}}=-\int\breve{\mathbf{E}}\,dt \tag{1.6}$$

(symbol 'breve' marks complex vectors and numbers except *i*). The authors [3] wrote, "The angular momentum can be found from the electric field $\breve{\mathbf{E}}$ and its complex conjugate $\overline{\mathbf{E}}$ by integrating over all spatial elements dV giving

$$\breve{\mathbf{J}}=\int\overline{\mathbf{E}}\times\breve{\mathbf{E}}\,dV/2i\omega\text{"} \tag{1.7}$$

(for short I set the permittivity $\varepsilon=1$).

4) Nieminen et al. [4] wrote that, "The Cartesian components of the time-averaged spin angular momentum flux density **s** are

$$s_x = \Im(\breve{E}_y \overline{E}_z)/\omega, \quad s_y = \Im(\breve{E}_z \overline{E}_x)/\omega, \quad s_z = \Im(\breve{E}_x \overline{E}_y)/\omega\text{''}. \tag{1.8}$$

5) Crichton & Marston [5] claimed, "The spin angular momentum density,

$$s_i = \frac{1}{8\pi\omega} \overline{E}_j (-i\varepsilon_{ijk}) \breve{E}_k, \tag{1.9}$$

is appropriately named in that there is no moment arm."

Please note that a corollary of this definition of the spin density is a circularly polarized plane wave,

$$\breve{\mathbf{E}} = E_0 \exp[i(kz - \omega t)](\mathbf{x} + i\mathbf{y}), \quad \breve{\mathbf{A}} = -i\breve{\mathbf{E}}/\omega, \tag{1.10}$$

has the spin density

$$\mathbf{s} = \mathbf{E}\times\mathbf{A} = \mathbf{z}(\breve{E}_x \overline{A}_y - \breve{E}_y \overline{A}_x)/2 = \mathbf{z}\Re(\breve{E}_x i \overline{E}_y)/\omega = \mathbf{z}E_0^2/\omega, \tag{1.11}$$

which ratio to the energy density $U = E_0^2$, $s_z / U = 1/\omega$, is that the quantum theory prescribes.

At the same time, however, on the other hand, all physicists insist that a circularly polarized plane wave (1.10) does not carry spin. It is a matter of common opinion that

$$\mathbf{j} = \mathbf{r}\times <\mathbf{p}> = \mathbf{r}\times <\mathbf{E}\times\mathbf{B}> \tag{1.12}$$

encompasses both the spin and orbital angular momentum density of electromagnetic field. (Here $\mathbf{p} = \mathbf{E}\times\mathbf{B}$ is the linear momentum volume density or flux density of electromagnetic mass-energy, i.e. the Poynting vector). Therefore a circularly polarized plane wave carry neither orbital angular momentum nor spin in direct contradiction to the quantum theory. I present here a series of quotations for a confirmation of my statement.

1) Heitler [6] wrote, "In Maxwell's theory the Poynting vector is interpreted as the density of momentum of the field. We can then also define an angular momentum relative to a given point or to a given axis

$$\mathbf{J} = \int \mathbf{r}\times <\mathbf{E}\times\mathbf{B}> dV\ . \tag{1.13}$$

A plane wave traveling in the *z*-direction and with infinite extension in the *xy*-directions can have no angular momentum about the *z*-axis, because $\mathbf{E}\times\mathbf{B}$ is in the *z*-direction and $\mathbf{r}\times <\mathbf{E}\times\mathbf{B}> = 0$.

2) Simmonds and Guttmann [7]: "For the plane wave states, **E** and **B** were mutually perpendicular and their cross product was parallel to the direction of propagation. It follows that the total angular momentum of a plane wave can have no component of angular momentum parallel to the direction of propagation".

3) Stewart [8]: "The angular momentum of a classical electromagnetic plane wave of arbitrary extent is predicted to be, on theoretical grounds, exactly zero".

Naturally, this contradiction shows strong evidence for a defect of the classical field theory, and this defect causes many conflicts, vagueness, and paradoxes concerning electrodynamics angular momentum. This was recognized long ago, and I present here a series of quotations for a confirmation of my statement.

1) Zambrini, and Barnett [9]: "Experimental observations appear to be in conflict with theoretical considerations".

2) Nieminen et al. [4]: "If the above expression (1.12) was in fact the correct angular momentum flux density, then the angular momentum of a circularly polarized plane would be zero. Since the correct classical angular momentum density must agree with the classical limit of the quantum angular momentum density, this must be incorrect."

3) Allen and Padgett [10]: "A circularly polarised plane wave has a linear momentum density only in the z-direction. When this is crossed with *r* to give the angular momentum density, there is no contribution in the z-direction. Thus, such a beam has no angular momentum to transfer to a waveplate. Yet, Beth was able to make such a transfer – a paradox."

4) Khrapko [11]: "The classical experiment of Beth [12, 13] was carried out almost 70 years ago. However, this experiment raises questions. The Poynting vector was everywhere equal to zero in the experiment. How is it therefore that the plate experienced a torque and rotated?"

5) Khrapko [14]: "Suppose that a quasiplane wave is absorbed by a round flat target which is divided concentrically into outer and inner parts. According to previous reasoning, the inner part of the target will not perceive a torque. Nevertheless R. Feynman [15] clearly showed how a circularly polarized plane wave transfers

a torque to an absorbing medium. What is true? And if R. Feynman is right, how can one express the torque in terms of ponderomotive forces?"

We intend to solve the angular momentum problems of the electrodynamics in this paper. We show that the desire of using a spin density proved to be correct for evaluating of the electrodynamics spin because the moment of momentum (1.12) is an orbital angular momentum density and does not encompass the spin. However, the expression $\mathbf{E}\times\mathbf{A}$ is not a true expression for the spin density. This expression yields correct results in simplest cases randomly. Instead we present a true spin density of electromagnetic waves, $\Upsilon^{\lambda\mu\nu}$, and demonstrate a way for its deducing. Our result can be expressed by a formula for a sum of the orbital and spin angular momentums of an electromagnetic wave in a volume dV,

$$d\mathbf{J} = [\mathbf{r}\times(\mathbf{E}\times\mathbf{B}) + \Upsilon^{ij0}]dV\,, \tag{1.14}$$

or, in the general case,

$$dJ^{\lambda\mu} = (2x^{[\lambda}T^{\mu]\nu} + \Upsilon^{\lambda\mu\nu})dV_\nu \tag{1.15}$$

(here $T^{\mu\nu}$ is the Maxwell energy-momentum tensor).

II. THE CANONICAL TENSORS

Unfortunately, authors [1 – 5] do not explain that the expression $\mathbf{E}\times\mathbf{A}$ is a component of the canonical spin tensor. The point is that the classical field theory starts from a free field Lagrangian. In the case of the standard classical electrodynamics this Lagrangian is the canonical Lagrangian

$$\underset{c}{\mathsf{L}} = -F_{\mu\nu}F^{\mu\nu}/4\,,\quad F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}\,,\quad \mu,\nu,\ldots = 0,1,2,3\,. \tag{2.1}$$

Using this Lagrangian, by the Lagrange formalism physicists obtain the canonical energy-momentum tensor

$$\underset{c}{T}{}^{\lambda\mu} = \partial^\lambda A_\alpha \frac{\partial \underset{c}{\mathsf{L}}}{\partial(\partial_\mu A_\alpha)} - g^{\lambda\mu}\underset{c}{\mathsf{L}} = -\partial^\lambda A_\alpha F^{\mu\alpha} + g^{\lambda\mu}F_{\alpha\beta}F^{\alpha\beta}/4\,, \tag{2.2}$$

and the canonical total angular momentum tensor

$$\underset{c}{J}{}^{\lambda\mu\nu} = 2x^{[\lambda}\underset{c}{T}{}^{\mu]\nu} + \underset{c}{\Upsilon}{}^{\lambda\mu\nu} \tag{2.3}$$

where

$$\underset{c}{\Upsilon}{}^{\lambda\mu\nu} = -2A^{[\lambda}\delta^{\mu]}_{\alpha}\frac{\partial \underset{c}{\mathsf{L}}}{\partial(\partial_\nu A_\alpha)} = -2A^{[\lambda}F^{\mu]\nu}\,, \tag{2.4}$$

is the canonical spin tensor.

Here $F^{\mu\nu} = -F^{\nu\mu}$, $F_{\mu\nu} = F^{\alpha\beta}g_{\mu\alpha}g_{\nu\beta}$ is the field strength tensor. The sense of its components is

$$F^{0i} = -E^i\,,\quad F_{0i} = E_i\,,\quad F^{ij} = -B^{ij}\,,\quad F_{ij} = -B_{ij}\,,\quad B_k = B^{ij}e_{ijk}\,,\quad B^k = B_{ij}e^{ijk}\,,\quad i,\ j,\ldots = 1,2,3\,. \tag{2.5}$$

For example,

$$F^{x0} = F_{0x} = E^x = E_x\,,\quad F^{xy} = F_{xy} = -B^z = -B_z\,. \tag{2.6}$$

The component

$$\underset{c}{\Upsilon}{}^{ij0} = -2A^{[i}F^{j]0} = -2A^{[i}E^{j]} = E^iA^j - E^jA^i = \mathbf{E}\times\mathbf{A} \tag{2.7}$$

is a volume density of spin. This means that

$$dS^{ij} = \underset{c}{\Upsilon}{}^{ij0}\,dV \tag{2.8}$$

is spin of electromagnetic field inside the spatial element dV. The component

$$\underset{c}{\Upsilon}{}^{ijk} = -2A^{[i}F^{j]k} = 2A^{[i}B^{j]k} \tag{2.9}$$

is a flux density of spin in the direction of the x^k axis. For example,

$$\underset{c}{\Upsilon}^{xyz} = 2A^{[x}B^{y]z} = A^x B^{yz} - A^y B^{xz} = A^x B_x + A^y B_y\,, \tag{2.10}$$

and

$$dS_z = dS^{xy} = \underset{c}{\Upsilon}^{xyz}\, da_z = (A^x B_x + A^y B_y)da_z \tag{2.11}$$

is *z*-component of spin passing through the surface element da_z per unit time.

However, the canonical tensors (2.2), (2.4) are not electrodynamics tensors. They obviously contradict experiments. In particular, $\underset{c}{T}^{\lambda\mu}$ is nonsymmetrical and has a wrong divergence:

$$\partial_\mu \underset{c}{T}^{\lambda\mu} = \partial^\lambda A_\sigma \partial_\kappa F^{\sigma\kappa}\,. \tag{2.12}$$

We show in the next Sections that $\underset{c}{\Upsilon}^{\lambda\mu\nu}$ contradicts experiments as well. For this purpose we apply the expression to a plane wave and to a standing wave.

III. PLANE WAVE

Let a right-circularly polarized electromagnetic plane wave, which propagates in *z*-direction, takes the form

$$E^x = \cos(z-t),\quad E^y = -\sin(z-t),\quad B^x = \sin(z-t),\quad B^y = \cos(z-t) \tag{3.1}$$

(for short we set $k = \omega = 1$). Because $\mathbf{A} = -\int \mathbf{E}dt$, we have

$$A^x = \sin(z-t),\quad A^y = \cos(z-t),\quad \underset{c}{\Upsilon}^{xy0} = 1,\quad \underset{c}{\Upsilon}^{xyz} = 1\,. \tag{3.2}$$

This result is adequate because the Poynting vector is $E^x B^y - E^y B^x = 1$, and the ratio of spin to energy, $S/U = 1/\omega$, holds. But a calculation of other components of the spin tensor yields

$$\underset{c}{\Upsilon}^{zxy} = A^x B_x = \sin^2(z-t)\,,\qquad \underset{c}{\Upsilon}^{yzx} = A^y B_y = \cos^2(z-t)\,. \tag{3.3}$$

This result is absurd, because it means that there are spin flux in the direction, which is transverse to the direction of the wave propagation.

Let us take now the sum of the wave (3.1)

$$E_1^x = \cos(z-t),\quad E_1^y = -\sin(z-t),\quad B_1^x = \sin(z-t),\quad B_1^y = \cos(z-t)\,. \tag{3.4}$$

and a wave reflecting off a perfect conductive plane $z = 0$

$$E_2^x = -\cos(z+t),\quad E_2^y = -\sin(z+t),\quad B_2^x = -\sin(z+t),\quad B_2^y = \cos(z+t)\,. \tag{3.5}$$

The total field is

$$E^x = E_1^x + E_2^x = 2\sin z \sin t\,,\qquad E^y = E_1^y + E_2^y = -2\sin z\cos t\,, \tag{3.6}$$

$$B^x = B_1^x + B_2^x = -2\cos z\sin t\,,\qquad B^y = B_1^y + B_2^y = 2\cos z\cos t\,. \tag{3.7}$$

Magnetic vector potential, according to $\mathbf{A} = -\int \mathbf{E}dt$, is

$$A^x = 2\sin z\cos t,\quad A^y = 2\sin z\sin t\,. \tag{3.8}$$

So, we can calculate components of the spin tensor:

$$\underset{c}{\Upsilon}^{xy0} = 4\sin^2 z,\quad \underset{c}{\Upsilon}^{xyz} = 0\,. \tag{3.9}$$

The result $\underset{c}{\Upsilon}^{xyz} = 0$ is adequate because there is no spin flux to the conductive plane, but $\underset{c}{\Upsilon}^{xy0} = 4\sin^2 z$ raises a doubt because there is no cause of dividing electromagnetic spin into layers. As is known, energy density is constant: $(E^2 + B^2)/2 = 2$.

Unfortunately, the calculation of other components of the spin tensor yields the absurd result as well

$$\underset{c}{\Upsilon}^{zxy} = A^x B_x = -\sin 2z\sin 2t\,,\qquad \underset{c}{\Upsilon}^{yzx} = A^y B_y = \sin 2z\sin 2t\,. \tag{3.10}$$

IV. A MAGNETIC ADDITION TO THE CANONICAL SPIN TENSOR

The canonical spin tensor (2.4), (2.7) is obvious not symmetric in the sense of electric - magnetic symmetry. It represents only the electric field, $\mathbf{E}$, $\mathbf{A} = -\int \mathbf{E}dt$. It calls forth the unsatisfactory result (3.9) for Υ^{xy0}. So, it makes sense to symmetrize the spin tensor by adding a term

$$\underset{c.m}{\Upsilon}^{\lambda\mu\nu} = -\Pi_*^{[\lambda} F_*^{\mu]\nu} . \tag{4.1}$$

The point is that the electrodynamics is asymmetric. Magnetic induction is closed, but magnetic field strength has electric current as a source:

$$\partial_{[\lambda} F_{\mu\nu]} = 0, \quad \partial_\nu F^{\mu\nu} = -j^\mu . \tag{4.2}$$

So, a magnetic vector potential A_ν exists, but, generally speaking, an electric vector potential does not exist. However, when currents are absent the symmetry is restored, and a possibility to introduce an electric multivector potential $\Pi^{\lambda\mu\nu}$ appears. The electric multivector potential satisfies the equation

$$\partial_\nu \Pi^{\lambda\mu\nu} = F^{\lambda\mu} . \tag{4.3}$$

A covariant pseudovector, dual relative to the multivector potential,

$$\Pi_\kappa^* = e_{\kappa\lambda\mu\nu} \Pi^{\lambda\mu\nu} , \tag{4.4}$$

is an analog of the magnetic vector potential A_κ. We name it the electric vector potential. It is inserted into (4.1). Pseudotensor $F_*^{\mu\nu}$ is dual to the field strength tensor $F^{\mu\nu}$,

$$F_*^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} e_{\alpha\beta\gamma\delta} F^{\gamma\delta} / 2 . \tag{4.5}$$

The sense of its components is

$$F_*^{0i} = B^i , \quad F_*^{ij} = -e^{ijk} E_k . \tag{4.6}$$

For example,

$$F_*^{0x} = B^x , \quad F_*^{yz} = -E_x . \tag{4.7}$$

So, accordingly with (4.1), omitting $*$ of Π, we have,

$$\underset{c.m}{\Upsilon}^{ij0} = -\Pi^{[i} F_*^{j]0} = (\Pi^i B^j - \Pi^j B^i)/2 = (\Pi \times \mathbf{B})/2 . \tag{4.8}$$

$$\underset{c.m}{\Upsilon}^{ijk} = -\Pi^{[i} F_*^{j]k} . \tag{4.9}$$

For example,

$$\underset{c.m}{\Upsilon}^{xyz} = -\Pi^{[x} F_*^{y]z} = (\Pi^x E_x + \Pi^y E_y)/2 , \qquad \underset{c.m}{\Upsilon}^{zxy} = -\Pi^{[z} F_*^{x]y} = (\Pi^z E_z + \Pi^x E_x)/2 . \tag{4.10}$$

A relation between Π and F can be readily obtained in the vector form as follows. If $\mathrm{div}\mathbf{D} = 0$, then $\mathbf{D} = \mathrm{curl}\Pi$. If also $\partial\mathbf{D}/\partial t = \mathrm{curl}\mathbf{H}$, then $\mathbf{H} = \partial\Pi/\partial t$, but we set $\mathbf{H} = \mathbf{B}$, so

$$\partial\Pi/\partial t = \mathbf{B} . \tag{4.11}$$

We consider now a total spin tensor corresponding to the canonical spin tensor (2.4):

$$\underset{tot}{\Upsilon}^{\lambda\mu\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu}/2 + \underset{c.m}{\Upsilon}^{\lambda\mu\nu} = -A^{[\lambda} F^{\mu]\nu} - \Pi_*^{[\lambda} F_*^{\mu]\nu} . \tag{4.12}$$

For the plane wave (3.1) we have the same adequate result (3.2),

$$\Pi^x = \cos(z-t), \quad \Pi^y = -\sin(z-t), \quad \underset{tot}{\Upsilon}^{xy0} = 1, \quad \underset{tot}{\Upsilon}^{xyz} = 1 \tag{4.13}$$

But the calculation of other components of the spin tensor yields the absurd result as well

$$\underset{tot}{\Upsilon}^{zxy} = (A^x B_x + \Pi^x E_x)/2 = 1/2 , \qquad \underset{tot}{\Upsilon}^{yzx} = (A^y B_y + \Pi^y E_y)/2 = 1/2 . \tag{4.14}$$

However, the magnetic part of spin tensor flattens the layers (3.3) of spin flux in the direction, which is transverse to the direction of the wave propagation.

For the standing wave (3.6) - (3.8) we have, instead of (3.8) - (3.10),

$$\Pi^x = 2\cos z\cos t, \quad \Pi^y = 2\cos z\sin t. \quad \underset{tot}{\Upsilon}^{xy0} = 2, \quad \underset{tot}{\Upsilon}^{xyz} = 0. \quad \underset{tot}{\Upsilon}^{zxy} = 0 \quad \underset{tot}{\Upsilon}^{yzx} = 0 \tag{4.15}$$

The results are adequate because the energy density is $(E^2 + B^2)/2 = 2$, and the ratio of spin to energy, $S/U = 1/\omega$, holds.

Thus the magnetic part of spin tensor flattens the spin layers and eliminates the transverse spin flux in the case of standing waves. This proves usefulness of adding the magnetic term (4.1) to the canonical spin tensor (2.4). Nevertheless the transverse spin flux in the case of plane waves proves that the canonical spin tensor is inadequate even with the magnetic addition.

V. THE STANDARD TENSORS

We must recognize that the canonical energy-momentum tensor (2.2), $\underset{c}{T}^{\lambda\mu}$, is inadequate as well as the canonical spin tensor (2.4). It obviously contradicts experiments. It is not symmetric and has a wrong divergence

$$\partial_\mu \underset{c}{T}^{\lambda\mu} = \partial^\lambda A_\sigma \partial_\kappa F^{\sigma\kappa}. \tag{5.1}$$

And so, physicists are forced to modify the canonical tensors. Following [16, 17], physicists accomplish a Belinfante-Rosenfeld procedure. They add specific terms to the canonical tensors and arrive to the standard energy-momentum tensor $\Theta^{\lambda\mu}$, the standard total angular momentum tensor $\underset{st}{J}^{\lambda\mu\nu}$, and the standard spin tensor $\underset{st}{\Upsilon}^{\lambda\mu\nu}$, which is zero,

$$\Theta^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} - \partial_\nu \underset{c}{\tilde{\Upsilon}}^{\lambda\mu\nu}/2 = -\partial^\lambda A_\nu F^{\mu\nu} + g^{\lambda\mu} F_{\alpha\beta} F^{\alpha\beta}/4 + \partial_\nu (A^\lambda F^{\mu\nu}),$$

$$\underset{c}{\tilde{\Upsilon}}^{\lambda\mu\nu} \overset{def}{=} \underset{c}{\Upsilon}^{\lambda\mu\nu} - \underset{c}{\Upsilon}^{\mu\nu\lambda} + \underset{c}{\Upsilon}^{\nu\lambda\mu} = -2A^\lambda F^{\mu\nu}, \tag{5.2}$$

$$\underset{st}{J}^{\lambda\mu\nu} = \underset{c}{J}^{\lambda\mu\nu} - \partial_\kappa (x^{[\lambda} \underset{c}{\tilde{\Upsilon}}^{\mu]\nu\kappa}), \tag{5.3}$$

$$\underset{st}{\Upsilon}^{\lambda\mu\nu} = \underset{st}{J}^{\lambda\mu\nu} - 2x^{[\lambda}\Theta^{\mu]\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} + 2A^{[\lambda}F^{\mu]\nu} = 0. \tag{5.4}$$

But the standard energy-momentum tensor $\Theta^{\lambda\mu}$ obviously contradicts experiments as well. It is not symmetric and has a wrong divergence

$$\partial_\mu \Theta^{\lambda\mu} = \partial_\mu \underset{c}{T}^{\lambda\mu} = \partial^\lambda A_\sigma \partial_\kappa F^{\sigma\kappa}. \tag{5.5}$$

Tensor Θ is never used. The Maxwell tensor,

$$T^{\lambda\mu} = -F^{\lambda\sigma} F^{\mu\kappa} g_{\sigma\kappa} + g^{\lambda\mu} F_{\sigma\kappa} F^{\sigma\kappa}/4, \tag{5.6}$$

is used in the electrodynamics instead of $\Theta^{\lambda\mu}$. For example, it is the Maxwell tensor that is used in the standard expression for the angular momentum of electromagnetic field (1.3),

$$\underset{st}{J}^{\mu\nu} = 2\int x^{[\mu} T^{\nu]\alpha} dV_\alpha, \quad \text{i.e.} \quad \underset{st}{\mathbf{J}} = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV, \tag{5.7}$$

$$\text{rather than} \quad \underset{\Theta}{J}^{\mu\nu} = 2\int x^{[\mu} \Theta^{\nu]\alpha} dV_\alpha, \quad \text{i.e.} \quad \underset{\Theta}{\mathbf{J}} = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B} - \mathbf{A}\,\mathbf{j}) dV. \tag{5.8}$$

The main defect of the standard tensors is the absence of spin, $\underset{st}{\Upsilon}^{\lambda\mu\nu} = 0$. Neither Eq. (5.7), nor Eq. (5.8) contains a spin term. In contrast to the canonical pair, $\underset{c}{T}^{\lambda\mu}, \underset{c}{\Upsilon}^{\lambda\mu\nu}$, the standard pair, $\Theta^{\lambda\mu}, \underset{st}{\Upsilon}^{\lambda\mu\nu} = 0$, is defective. Standard energy-momentum tensor is not accompanied by a spin tensor.

The absence of spin in the standard electrodynamics implies an absurd corollary: a circularly polarized plane wave has no angular momentum at all [1, 2, 6 – 8, 18, 19] because $\mathbf{E} \times \mathbf{B}$ is parallel to the direction of propagation and Eq. (5.7) gives zero. But this corollary is in direct contradiction to quantum theory [20]. In accordance with this absurdity, the author [8] uses mystical concepts of “angular momentum in an *actual* form” and “angular momentum in an *potential* form”.

Integral (1.3), (5.7) is zero in another important case. $\underset{st}{\mathbf{J}} = 0$ for the Beth experiment [12, 13] because $\mathbf{E} \times \mathbf{B} = 0$. In the Beth experiment a beam of circularly polarized light exerted a torque on a doubly refracting plate, which changes the state of polarization of the light beam. But, it is evident that the Poynting vector equals to zero in the experiment because the passed beam is added with the reflected one [11]. Therefore the result of the Beth experiment cannot be understood in the frame of the standard electrodynamics without spin. So, we must add a concept of spin to the standard electrodynamics.

VI. THE FICTITIOUS DIVISION OF THE ORBITAL ANGULAR MOMENTUM

The absence of the standard spin tensor provokes physicists into a search of spin inside the orbital angular momentum (5.7). They try to decompose (5.7) into an “orbital” and “spin” parts (1.4), (1.5),

$$\underset{st}{\mathbf{J}} = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV = \mathbf{L} + \mathbf{S}. \tag{6.1}$$

For this purpose

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{6.2}$$

or

$$\mathbf{E} = \nabla \times \mathbf{F}, \qquad \mathbf{F} = -\int \frac{\partial_t \mathbf{B} dV}{4\pi r}, \tag{6.3}$$

is substituted into (5.7) for an electromagnetic beam [2, 21, 22, 8]. As a result, Eq. (6.2) gives (1.5)

$$\underset{st}{\mathbf{J}} = \int \mathbf{r} \times (E^i \nabla A_i) dV + \int (\mathbf{E} \times \mathbf{A}) dV, \tag{6.4}$$

and Eq. (6.3) gives

$$\underset{st}{\mathbf{J}} = \int \mathbf{r} \times (B^i \nabla F_i) dV + \int (\mathbf{F} \times \mathbf{B}) dV. \tag{6.5}$$

But I think these decompositions do not give grounds to interpret the summands as orbital and spin components of the angular momentum of the beam.

Firstly, neither $\mathbf{E} \times \mathbf{A}$ nor $\mathbf{F} \times \mathbf{B}$ are spin tensors.

Secondly, for a circularly polarized beam without an azimuth phase structure the contribution to the integral arises from the skin of the beam where **E** and **B** fields have a component parallel to the wave vector (the field lines are closed loops) and the mass-energy whirls around the bulk of the beam [2, 7]. It confirms the orbital character of the angular momentum. And I think that the transformation of the integral (6.1) over skin of the beam into an integral over bulk of the beam proves nothing. For example, consider an analogous integral $\int \mathbf{r} \times \mathbf{j} dV$ where **j** is an electric current density of a long solenoid. We have

$$\int \mathbf{r} \times \mathbf{j} dV = \int \mathbf{r} \times (\nabla \times \mathbf{H}) dV = \int (r^i \partial_k H_i - r^i \partial_i H_k) dV = \int [\partial_k (r^i H_i) - \partial_k r^i H_i - \partial_i (r^i H_k) + \partial_i r^i H_k] dV = \int 2\mathbf{H} dV$$

The equality between the moment of electric current and an integral of **H** proves nothing.

VII. TRUE ELECTRODYNAMICS SPIN TENSOR

It was explained [11, 23, 24] that the Belinfante-Rosenfeld’s modification [16, 17] of the canonical pair (2.2), (2.4),

$$\underset{c}{T}^{\lambda\mu} = -\partial^\lambda A_\nu F^{\mu\nu} + g^{\lambda\mu} F_{\alpha\beta} F^{\alpha\beta} / 4, \qquad \underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda} F^{\mu]\nu},$$

does not lead to true energy- momentum and spin tensors. We must change this standard procedure. We must use another addends. Our addends are

$$t^{\lambda\mu} = \partial_\nu A^\lambda F^{\mu\nu}, \qquad s^{\lambda\mu\nu} = 2A^{[\lambda} \partial^{\mu]} A^\nu. \tag{7.1}$$

The standard addends,

$$\underset{st}{t}^{\lambda\mu} = \partial_\nu (A^\lambda F^{\mu\nu}), \quad \underset{st}{s}^{\lambda\mu\nu} = 2A^{[\lambda} F^{\mu]\nu},$$

lead to the defective standard pair (5.2), (5.4)

$$\Theta^{\lambda\mu} = -\partial^\lambda A_\nu F^{\mu\nu} + g^{\lambda\mu} F_{\alpha\beta} F^{\alpha\beta} / 4 + \partial_\nu (A^\lambda F^{\mu\nu}), \qquad \underset{st}{\Upsilon}^{\lambda\mu\nu} = 0.$$

Our addends (7.1) satisfy the equation

$$2t^{[\lambda\mu]} = \partial_\nu s^{\lambda\mu\nu} \tag{7.2}$$

as well as the standard addends satisfy. But using our addends we get, instead of (5.2), (5.4), the Maxwell energy-momentum tensor

$$T^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} + t^{\lambda\mu} = -F^\lambda{}_\nu F^{\mu\nu} + g^{\lambda\mu} F_{\alpha\beta} F^{\alpha\beta} / 4 \tag{7.3}$$

and a tensor, which was proved to be a doubled electric part of the spin tensor

$$2\underset{e}{\Upsilon}^{\lambda\mu\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} + s^{\lambda\mu\nu} = 2A^{[\lambda} \partial^{|\nu|} A^{\mu]}. \tag{7.4}$$

This result was submitted to “JETP Letters” on May 12, 1998. But, this result was not final one. The true spin tensor must depend symmetrically on the magnetic vector potential A_α and on the electric vector potential Π_α (4.4). So the spin tensor of electromagnetic waves has the form

$$\Upsilon^{\lambda\mu\nu} = \underset{e}{\Upsilon}^{\lambda\mu\nu} + \underset{m}{\Upsilon}^{\lambda\mu\nu} = A^{[\lambda} \partial^{|\nu|} A^{\mu]} + \Pi^{[\lambda} \partial^{|\nu|} \Pi^{\mu]}, \tag{7.5}$$

and the total angular momentum has the form

$$J^{\lambda\mu} = \int (2x^{[\lambda} T^{\mu]\nu} + \Upsilon^{\lambda\mu\nu}) dV_\nu, \ \text{ or } \ \mathbf{J} = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV + \int \Upsilon^{ij0} dV \tag{7.6}$$

instead of (1.3).

A new spin tensor (7.5) was presented and applied in a series of works [25] and also at web sites http://www.sciprint.org, http://www.mai.ru/projects/mai_works/.

When calculating this tensor, we must take account of $\partial^z = g^{zz} \partial_z = -\partial_z$. For the plane wave using (3.2) and (4.13) yields

$$\Upsilon^{xy0} = 1, \quad \Upsilon^{xyz} = 1, \quad \Upsilon^{zxy} = \Upsilon^{yzx} = 0. \tag{7.7}$$

For the standing wave using (3.8), (4.15) yields

$$\Upsilon^{xy0} = 2, \quad \Upsilon^{xyz} = 0, \quad \Upsilon^{zxy} = \Upsilon^{yzx} = 0, \tag{7.8}$$

which was to be demonstrated.

CONCLUSION, NOTES and ACKNOWLEDGEMENTS

This paper conveys new physics. We briefly review existing works concerning electrodynamics spin and indicate that existing theory is insufficient to solve spin problems because spin tensor of modern electrodynamics is zero. Then we show how a change of the Belinfante-Rosenfeld procedure resolves the difficulty by introducing a true electrodynamics spin tensor. Our spin tensor, in particular, doubles a predicted angular momentum of a circularly polarized light beam without an azimuth phase structure and explains the Beth experiment.

Unfortunately, materials of this paper were rejected more than 350 times by scientific journals. For example (I show an approximate number of the rejections in parentheses): JETP Lett. (8), JETP (13), TMP (10), UFN (9), RPJ (70), AJP (14), EJP (4), EPL (5), PRA (3), PRD (4), PRE (2), APP (5), FP (6), PLA (9), OC (2), JPA (4), JPB (1), JMP (4), JOPA (3), JMO (2), CJP (1), OL (1), NJP (2), MREJ (3), arXiv (70). In particular, PRA rejected a paper “Beth's experiment modification” submitted on Sun, 16 Nov 2003 06:36:00.

I am deeply grateful to Professor Robert H. Romer for publishing my question [14] (was submitted on Oct. 7, 1999) and to Professor Timo Nieminen for valuable discussions (Newsgroups: sci.physics.electromag). Unfortunately, Jan Tobochnik, the present-day Editor of AJP, rejected my papers more than 20 times.

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[1] J. D. Jackson, *Classical Electrodynamics* (Wiley, 1999)
[2] H. C. Ohanian, Amer. J. Phys. **54**, 500-5 (1986)
[3] M. E. J. Friese, T. A. Nieminen, N. R. Heckenberg & H. Rubinsztein-Dunlop, Nature **394,** 348–350 (1998)
[4] A. Nieminen et al. arXiv:physics/0408080.
[5] J. H. Crichton and P. L. Marston, Electronic Journal of Differential Equations **Conf. 04**, 37 (2000).
[6] W. Heitler, *The Quantum Theory of Radiation* (Oxford: Clarendon, 1954)
[7] J. W. Simmonds, M. J. Guttmann, *States, Waves and Photons* (Addison-Wesley, Reading, MA, 1970)
[8] A. M. Stewart, Eur. J. Phys., **26**, 635 (2005)
[9] R. Zambrini, S. M. Barnett, J. Mod. Optics **52** 1045 (2005)
[10] L. Allen, M. J. Padgett, Opt. Commun. **184**, 67 (2000).
[11] R. I. Khrapko, Measurement Techniques **46** No. 4, 317 (2003)
[12] R. A. Beth, Phys. Rev., **48**, 471 (1935)
[13] R. A. Beth, Phys. Rev., **50**, 115 (1936)
[14] R. I. Khrapko, Amer. J. Phys., **69**, 405 (2001)
[15] R. P. Feynman et al., *The Feynman Lectures on Physics* (Addison–Wesley, London, 1965) V. 3, p. 17–10.
[16] F. J. Belinfante, Physica **6,** 887 (1939).
[17] L. Rosenfeld, Memoires de l'Academie Royale des Sciences de Belgiques **8** No 6 (1940).
[18] D. E. Soper, *Classical Field Theory* (N.Y.: John Wiley, 1976).
[19] F. Rohrlich, *Classical Charged Particles* (Addison-Wesley, Mass. 1965)
[20] L. H. Ryder, *Quantum Field Theory* (Cambridge, 1985)
[21] J Humblet, Physica, **10**, 585 (1943)
[22] J. Crichton et al., Gen. Relat. Grav., **22**, 61 (1990)
[23] R. I. Khrapko, Gravitation & Cosmology, **10**, 91 (2004)
[24] R. I. Khrapko, physics/0102084, physics/0105031
[25] R. I. Khrapko mp_arc@mail.ma.utexas.edu 03-307, 03-311, 03-315