Gravitational Schrödinger equation from Ginzburg-Landau equation, and its noncommutative spacetime coordinate representation

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Despite known analogy between condensed matter physics and various cosmological phenomena, a neat linkage between low-energy superfluid and celestial quantization is not yet widely accepted in literature. In the present article we argue that gravitational Schrödinger equation could be derived from time-dependent Ginzburg-Landau (or Gross-Pitaevskii) that is commonly used to describe superfluid dynamics. The solution for celestial quantization takes the same form with Nottale equation. Provided this proposed solution corresponds to the facts, and then it could be used as alternative solution to predict celestial orbits from quantized superfluid vortice dynamics. Furthermore, we also discuss a representation of the wavefunction solution using noncommutative spacetime coordinate. Some implications of this solution were discussed particularly in the context of offering a plausible explanation of the physical origin of quantization of motion of celestial objects.

Keywords: superfluidity, Bose-Einstein condensate, vortices, gravitation, celestial quantization

Introduction

There has been a growing interest in some recent literatures to consider gravity as scalar field from boson condensation [1]. This conjecture corresponds to recent proposals suggesting that there is neat linkage between condensed matter physics and various cosmological phenomena [2,3]. In this regard, it is worth noting here that some authors have described celestial quantization from the viewpoint of gravitational Schrödinger-type wave equation [4]. Considering that known analogy between condensed matter physics and various cosmological phenomena, then it seems also plausible to describe such a celestial quantization from the viewpoint of condensed-matter physics, for instance using Gross-Pitaevskii (GP) or Ginzburg-Landau wave equation.

In the present article, we derived gravitational Schrödinger-type wave equation from various equations known in condensed matter physics, including Gross-Pitaevskii (GP) equation and also time-dependent Ginzburg-Landau (TDGL) wave equation. This method could be regarded as 'inverse' way from method discussed in Berger's article [5], suggesting that it is possible to extend Schrödinger equation to TDGL using De Broglie potential. Provided this neat linkage from TDGL/GPE and Schrödinger equation is verified by observation, then it seems to support a previous conjecture of a *plausible linkage between celestial quantization and quantized vortices* [4]. And then we discuss some issues related to describing cosmological phenomena in terms of diffusion theory of gravitational Schrödinger-type equation, though this issue has been discussed in the preceding articles [3,8,9]. Furthermore, following our argument that it is possible to find noncommutative representation of the wavefunction [4], and then we will discuss a plausible interpretation of the gravitational

Schrödinger equation in terms of *noncommutative spacetime* coordinate. This extension to noncommutative coordinate perhaps will be found useful for further research. And if this proposition corresponds to the astrophysical facts, then it can be used to explain the origin of quantization in astrophysics [7][8].

An alternative method to find solution of gravitational Schrödinger-type equations

The present author acknowledged that the proposed method on relating cosmological phenomena with condensed-matter/low-energy physics has not been widely accepted yet, though some of these approaches have been used to predict phenomena corresponding to neutron stars [12,39]. Furthermore, there is also a deeper question concerning the appropriateness of using and solving gravitational Schrödinger-type equations for depicting cosmological phenomena, beyond what is called as Wheeler-DeWitt (WDW) equation. It should be noted here that our derivation method is somewhat different from Neto *et al.*'s approach [14], because we use Legendre polynomials approach.

Now we are going to find solution of the most basic form of Schrödinger-type equation using Legendre polynomials, from which we will obtain the same expression with known Nottale's quantization equation [11]. We start with noting that Schrödinger equation is derived from a wave of the form:

$$\Psi = a.\sin 2px / l \tag{1}$$

By deriving twice equation (1), then we get the most basic form of Schrödinger equation:

$$d^2\Psi / dx^2 + A.\Psi = 0 \tag{2}$$

where for planetary orbits, it can be shown [13, 5] that we get:

$$A = 4\pi / \lambda^2 = \omega^2 / v^2 = m\omega^2 /(2.KE) \qquad (3)$$

Solution of equation (2) is given by:

$$\chi = C_1 .\exp(\rho / 2) + C \ .\exp(-\rho / 2) \qquad (4)$$

But we shall reject the first term because it will result in infinity for large distance (ρ>>0). This suggests solution of the form [14]:

$$\chi = F(\rho).\exp(-\rho / 2) \qquad (5)$$

Substituting (5) into (2), we get:

$$d^2F / d\rho^2 - dF / d\rho + A.F = 0 \qquad (6)$$

Now we shall find the series solution to (6) and put:

$$F = \sum_{p=1}^{\infty} a_p .\rho^p \qquad (7)$$

The lower limit of this summation is p=1 rather than p=0, otherwise F and therefore χ would not be zero at ρ=0. Thus [14]:

$$dF / d\rho = \sum_{p=1}^{\infty} p.a_p .\rho^{p-1} \qquad (8)$$

$$d^2F / d\rho^2 = \sum_{p=1}^{\infty} (p+1)p.a_{p+1} .\rho^{p-1} \qquad (9)$$

$$F / \rho^2 = a_1 .\rho^{-1} + \sum_{p=1}^{\infty} .a_{p+1} .\rho^{p-1} \qquad (10)$$

By inserting these equations (7), (8), (9), and (10) into equation (6), and observing that *each power of* ρ *mush vanish*, and by inserting our defiition of variable A from equation (3) and inserting the kinetic energy definition $KE = GMm / 2r$, and then we could find the expression for orbital radii which is similar to Nottale's equation [11]:

$$r_o = n^2 .GM / v_o^2 \qquad (11)$$

Therefore we observed that a solution using Legendre polynomials yields the same expression with Nottale's quantization equation [11]. It is also obvious that some assumptions must be invoked in order to find the proper asymptotic solution.

On celestial quantization from GPE and TDGL

In a preceding article we provided simplified derivation of equation of quantization of planetary orbit distance based on Bohr-Sommerfeld hypothesis of quantization of angular momentum [4], which could be considered as 'retro' version of Bohr-Sommerfeld quantization method in microphysics. As shown above, similar quantization result can be derived from generalized Schrödinger-Newton equation suggested by L. Nottale [11].

But this Schrödinger-type wave equation does not exactly correspond to the superfluid theory or condensed matter, therefore in the present article we will derive Schrödinger-type wave equation based on GP/TDGL equation, which is commonly used to describe superfluid medium [3]. It will be shown that the previous solution (11) based on gravitational Schrödinger-type equation is only an approximation of a more general GP/TDGL equation, becauses it neglects nonlinear effects like temperature dependent or screening potential. This conjecture of quantum vortice dynamics also corresponds to hypothesis by Winterberg of superfluid phonon-roton as Planckian quantum vacuum aether [9].

First, we will discuss how to get Schrödinger-type equation from GP equation, and then from TDGL. At subsequent section we will discuss other nonlinear Schrödinger-type equation from Chern-Simons theory.

a. <u>Gross-Pitaevskii equation (GPE)</u>

As we know, superfluid medium is usually described using GP equation, or sometimes known as nonlinear Landau-Ginzburg equation or nonlinear Schrödinger equation (NLSE) [12,2]. In the GP theory the ground state and weakly excited states of a Bose gas are described by the condensate wave function ψ=a.exp(iφ) which is a solution of the nonlinear Schrödinger equation [6]:

$$i\hbar.\partial \psi/\partial t = -\hbar^2/2m.\nabla^2\psi + V|\psi|^2\psi \qquad (12)$$

where V is the amplitude of two-particle interaction.

It has been argued [6], that two-fluid hydrodynamics relations can be derived from the hydrodynamics of an ideal fluid in presence of thermally excited *sound waves*, i.e. phonon scattering by a vortex line. In order to obtain a complete system of equations of the two-fluid theory, one should take into consideration phonon-phonon interaction, which is essential for the phonon distribution function being close to the equilibrium Planck distribution. It was shown in [1], that this sound wave of boson condensate system consists of phonons with *sound velocity* of $c_s^2 = \partial P/\partial(m\rho) = \pi''\rho/m$.

Furthermore, the phonon scattering by a vortex line is analogous to the so-called Aharonov effect for electrons scattered by a magnetic-flux tube, which analogy becomes more evident if one rewrites the sound equation [6] in presence of the vortex as:

$$k^2\phi - \left(-i\vec{\nabla} + k\vec{v}_v/c_s\right)^2\phi = 0 \qquad (13)$$

But the stationery Schrödinger equation for an electron in presence of the magnetic flux confined to a thin tube is given by [6]:

$$E\psi(\vec{r}) = 1/2m.\left(-i\hbar\vec{\nabla} - e\vec{A}/c\right)^2\psi(\vec{r}) \qquad (14)$$

Here ψ is the electron wave function with energy E and the electromagnetic vector potential is connected with the magnetic flux φ by the relation similar to that for the velocity $\vec{v}_v$ around the vortex line [6]:

$$\vec{A} = \Phi.\left[\hat{z}x\vec{r}\right]/2\pi r^2 \qquad (15)$$

In other words, we have outlined a logical mapping [6]: (i) from GP (NLSE) equation to the two-fluid hydrodynamics; (ii) from hydrodynamics to the phonon scattering equation; (iii) from phonon scattering to electron scattered by magnetic-flux tube, and (iv) from electron scattering back to the stationery Schrödinger equation. Now it is worthnoting here, that there is *exact solution* of Aharonov effect for electrons obtained by the partial wave expansion. To find the solution of equation (14), partial-wave amplitudes ψ_l should satisfy equations in the cylindrical system of coordinates (r,φ) [6]:

$$d^2\psi_l/dr^2 + 1/r.d\psi_l/dr - (1-\gamma)^2.\psi_l/r^2 + k^2.\psi_l = 0 \qquad (16)$$

where

$$E = k^2\hbar^2/2m \qquad (17)$$

or

$$k^2 = 2m.KE/\hbar^2 = 1/\lambda^2 \qquad (18)$$

where KE, $\hbar$, λ denotes the kinetic energy of the system, Planck constant and wavelength, respectively. From this equation (16), then we shall find a solution, which at large distances has an *asymptotic* character expressed in exponential form of $\psi=\alpha.\exp(\beta)$, which is typical solution of Schrödinger-type equation; where α and β are functions of some constants.

Because equation (16) is an ordinary differential equation in planar cylindrical system of coordinates, we consider that this equation corresponds to the celestial quantization if we insert proper values of Newtonian equation [4]. Therefore in the subsequent derivation we will not follow the standard partial wave analysis method as described in [6], but instead we will use a method to find solution of ordinary differential equation of Schrödinger equation: $a=n^2.GM/v_o^2$, which is in accordance with Nottale's solution [11]. Here a, n, G, M, v_o, represents semimajor axes, quantum number (n=1,2,3,…), Newton gravitation constant, mass of nucleus of gravitation field, and specific velocity, respectively.

Solution of equation (16) is given by $\psi_1(r,\varphi)= R(r).F(\phi)$. Inserting this relation into (16), and separating the $F(\phi)$ terms, then we get the *ground state* expression of the system (m^2=0 case):

$$d^2R/dr^2 + 1/r.(dR/dr) - [(1-g)^2/r^2 + k^2].R = 0 \tag{19}$$

The solution for R(r) is given by :

$$R(r) = [e^{-a.r} + e^{a.r}] \tag{19a}$$

In order to get the sought-after *asymptotic* solution for equation (16), we only use the negative expression of R(r), otherwise the solution will diverge to infinity at large distance r:

$$R(r) = e^{-a.r} \tag{20}$$

Therefore

$$dR(r)/dr = -a.e^{-a.r} \tag{21}$$

$$d^2R(r)/dr^2 = a^2.e^{-a.r} \tag{22}$$

Inserting (19a)-(22) into equation (19) and eliminating the exponential term $e^{-a.r}$, yield:

$$a^2 = 1/r^2.\{ar + (1-g)^2 - r^2k^2\} \tag{23}$$

Because equation (23) must be right for any value of r, then the right hand side of equation (23) between the {} brackets must equal to zero:

$$ar + (1-g)^2 - r^2k^2 = 0 \tag{24}$$

Maple solution for equation (24) is included in the Appendix section, which yields for g :

$$g = 1 \pm \sqrt{a^2r^2 - ar + k^2r^2} \tag{25}$$

The remaining part is similar to equation (10)-(11), by inserting kinetic energy definition for gravitational potential.

Therefore we conclude that the right term between the {} brackets yields a secondary effect to the equation of celestial quantization, except for some condition where this extra term vanishes. To this author's knowledge, this secondary effect has never been derived before; neither in Nottale [11], nor Neto *et al*. [13]. In our method, the secondary effect comes directly from the partial wave analysis expression of GP equation.

Therefore we obtain a generalised form of the equation of celestial quantization [11], which has taken into consideration the secondary interaction effect of GPE. The expected value for γ can be estimated by equating the right term between the {} brackets to one.[1] However, it is not too clear in what kind of conditions this right term in the bracket will disappear, therefore we are going to discuss another approach for deriving gravitational Schrödinger-type equation, i.e. using TDGL (time-dependent Ginzburg-Landau equation).

b. <u>Time-dependent Ginzburg-Landau equation (TDGL)</u>

It is known that Ginzburg-Landau (TDGL) equation is more *consistent* with known analogy between superfluidity and cosmological phenomena [2][3], and TDGL could also describe vortex nucleation in rotating superfluid [19]. According to Gross, Pitaevskii, Ginzburg, wavefunction of N bosons of a reduced mass m* can be described as [20]:

$$-(\hbar^2/2m^*).\nabla^2 y + k|y|^2 y = i\hbar.\partial y/\partial t \tag{26}$$

It is worthnoting here that this equation is quite similar to Jones' nonlinear Schrödinger equation to describe gravitational systems [21]. For some conditions, it is possible to replace the potential energy term in equation (26) by Hulthen potential. This substitution yields:

$$-(\hbar^2/2m^*).\nabla^2 y + V_{Hulthen}.y = i\hbar.\partial y/\partial t \tag{27}$$

where

$$V_{Hulthen} = -Ze^2 .\boldsymbol{d}.e^{-dr} /(1-e^{-dr}) \qquad (28)$$

This equation (27) has a pair of exact solutions. It could be shown that for small values of $\boldsymbol{d}$, the Hulthen potential (28) approximates the effective Coulomb potential, in particular for large radius:

$$V_{Coulomb}^{eff} = -e^2 / r + \ell(\ell+1).\hbar^2 /(2mr^2) \qquad (29)$$

Inserting (29) into equation (27) yields:

$$-\hbar^2\nabla^2 \boldsymbol{y} /2m^* + \left[-e^2 / r + \ell(\ell+1).\hbar^2 /(2mr^2) \right] \boldsymbol{y} = i\hbar.\partial \boldsymbol{y} / \partial t \qquad (30)$$

While this equation is interesting to describe neutron model, calculation shows that introducing this Hulthen effect (28) into gravitational equation will yield different result only at the order of 10^{-39} m compared to prediction using equation (11), which is of course negligible. Therefore, we conclude that for most celestial quantization problems the result of TDGL with Hulthen potential (28) is *essentially* the same with the result derived from equation (11).

Some implications to cosmology model

The approach described in the previous section using arguments based on condensed matter physics also implies that the linear and point-like topological defects also induce an effective metric, which can be interesting for the theory of gravitation. In this regards, the vortex can be considered as cosmic spinning string.[2]

Another question can be asked here, i.e. to how extent GP equation could be regarded as *exact representation* of cosmological phenomena, because there are arguments suggesting that GP equation is only an approximation [23]. For instance, Castro *et al.* [22] argued that GP equation of NLSE has some weakness, i.e. it does not meet Weinberg homogeneity condition.

Therefore, it becomes obvious that there is also a typical question concerning whether such Schrödinger-type wave function expression corresponds to vortices description in hydrodynamics. In this regard, it seems worth here to consider a more rigorous approach based on Chern-Simons hydrodynamics. Pashaev & Lee [24] reformulated the case of Abelian Chern-Simons gauge field interacting with Nonlinear Schrodinger field as planar Madelung fluid. In this regard, the Chern-Simons Gauss law has simple physical meaning of creation of the local vorticity for the fluid flow; which appears very similar to Kiehn's derivation using Navier-Stokes argument [17,27]. Then Pashaev & Lee [24] obtained the following nonlinear wave equation:

$$iD_0\Psi + D^2\Psi/2m - U\Psi = (1-\hbar^2)/2m.(\Delta|\Psi|.\Psi/|\Psi|) \tag{31}$$

where

$$D_0 = \partial + \in /c.A_0 \tag{32}$$

$$D = \nabla + \in /c.A \tag{33}$$

Then in terms of a new wave function

$$\boldsymbol{c} = \sqrt{\boldsymbol{r}}.\exp(iS/\hbar) \tag{34}$$

they recovered the standard linear Schrödinger equation:

$$i\hbar D_0\boldsymbol{c} + D^2\boldsymbol{c}.\hbar/2m - U\boldsymbol{c} = 0 \tag{35}$$

Thus they concluded that for $\hbar \neq 0$ equation (34) is gauge equivalent to the Schrödinger equation, while for $\hbar = 0$ it reduces to nonlinear wave equation of classical mechanics. The semiclassical limit has been applied to defocusing NLSE [24]:

$$i\hbar\partial_t\boldsymbol{c} + \Delta\boldsymbol{c}.\hbar^2/2m + 2g|\boldsymbol{c}|^2\boldsymbol{c} = 0 \tag{36}$$

which provides an analytical tool to describe shockwave in nonlinear optics and vortices in superfluid. In the formal semiclassical limit $\hbar \to 0$ (before shocks), *one neglects the quantum potential and fluid becomes the Euler system*. Introducing the local velocity field:

$$V = 1/m.[\nabla S + e/c.A] \tag{37}$$

And then they obtained a hydrodynamical model defined by two equations:

$$\partial V/\partial t+(V\nabla)V=-\nabla(-2g\boldsymbol{r}-\hbar^2/2m.\Delta\sqrt{\boldsymbol{r}}/\sqrt{\boldsymbol{r}})/m \qquad (38)$$

$$\nabla xV=e^2\boldsymbol{r}/(m\boldsymbol{k}c^2) \qquad (39)$$

Therefore we concluded that a more rigorous representation of quantum fluid admits vortice configuration. It is perhaps interesting to remark here, that these equations differ appreciably from Nottale's basic Euler-Newton equations [11]:

$$m.(\partial/\partial t+V.\nabla)V=-V(\boldsymbol{f}+Q) \qquad (40)$$

$$\partial\boldsymbol{r}/\partial t+div(\boldsymbol{r}V)=0 \qquad (41)$$

$$\Delta\boldsymbol{f}=-4\boldsymbol{p}G\boldsymbol{r} \qquad (42)$$

which of course neglect vortice configuration.

Upon generalizing the solution derived above, we could expect to see some plausible consequences in cosmology. For instance, that (i) there should be a kind of Magnus-Iordanskii type force observed in astrophysical phenomena, and (ii) that there should be *hollow tubes* inside the center of spinning large celestial bodies, for instance in the Sun and also large planets, including this Earth;[3] (iii) the universe is also very likely to rotate, in accord with recent observation by Nodland & Ralston [25];[4] (iv) the notion of gravitational constant could be related to cosmological temperature [3]; and (v) there exists *ergoregions* in the rotating centers of celestial objects where phonon particles are continuously created [26]. This phenomenon of phonon creation in the ergoregions may offer a rational basis of the observed continuous expansion of the universe. However, it shall be noted here that all of these plausible consequences to cosmology require further research.

Furthermore, some recent observations have concluded that our universe has fractality property. For clarity, the number of galaxies N(r) within a sphere of radius r, centered on any galaxy, is not proportional to r^3 as would be expected of a homogeneous distribution. Instead N(r) is proportional to r^D, where D is approximately equal to 2, which is symptomatic to distribution with fractal dimension D. It is interesting to note, for D=2, the cosmological gravitational redshift gives the linear distance-redshift relation and becomes an observable phenomenon [28]. This property is indicated by its Hausdorff dimension, which can be computed to be within the range of 1.6 ~ 2.0 up to the scale of 200 Mpc. Furthermore, transition to homogeneity distribution has not been found yet. In this regard, P.W.Anderson *et al*. [29] also remarked: *"These findings (of clustering and void formation) have become increasingly difficult to reconcile with standard cosmological theories, in which the approach to homogeneity at large-scales is central element*." It is worth noting here that perhaps this fractality property can be explained using boson condensate model with non-integer dimension. It has been argued that such a boson condensate system exhibits Hausdorff dimension d_H~2 [30]. There is also article arguing in favor of relating the fractal dimension with fluctuation graph [31]:

$$D = 2 - \alpha / 2 \quad \text{for } \alpha \leq 2 \tag{43}$$

where α is the time decay exponent. Furthermore, it was shown recently that an extended version of *GP equation admits self-similar* solutions and also it corresponds to Hausdorff dimension d_H~2 [23], which seems to confirm our hypothesis that there is exact correspondence between cosmological phenomena and condensed matter physics [1,2].

Therefore this Hausdorff dimension argument seems to be a plausible restriction for a good cosmology theoretical model: *Any cosmology theory which cannot exhibit fractality property from its intrinsic parameters perhaps is not adequate to explain inhomogeneity of large scale structures in universe.*

It is also worthnoting here, that an alternative argument in favor of cosmology with d_H~2 has been considered recently by Roscoe [30], which corresponds to Mach principle. While his argument seems very encouraging and perhaps it is also deeply interwoven with arguments presented herein, it shall be noted that his argument suggests the universe *must* have a fractal dimension d_H~2, while in the context of condensed matter physics it can fluctuates around 1.6~2.0 as observed [7]. Furthermore, by making an allusion to Newton's argument, Roscoe also did not consider any physical origin of such fractal distribution of masses in the universe, except that it corresponds to the nature of quantum vacuum aether. Nonetheless, Roscoe's conjecture on the presence of universal clock is very interesting.

Furthermore, if the equation of quantization of celestial motion derived herein from GPE/TDGL equation corresponds to the observed astrophysical facts, then it implies that it seems possible now to conduct a set of laboratory experiments as replica of some cosmological objects [2], provided we take into consideration proper scale modeling (similitude) theories.

Noncommutative spacetime representation

In this section we are going to discuss an alternative representation of the abovementioned Schrödinger equation using noncommutative spacetime coordinate, based on Vancea [33]. According to Vancea, the stationary Schrödinger equation is constructed by analogy with the commutation case an has the following form [33]:

$$H(x,p)*\Psi(x)=E.\Psi(x) \tag{44}$$

Here the wavefunction Ψ belongs to the noncommutative algebra, A_*. If explicit form of Schrödinger equation is given by [33]:

$$\left[-\hbar/2M.\sum_{m=1}^{2N}\partial_m^2+V\right]*\Psi=E\Psi \tag{45}$$

where V(x) is an arbitrary function from A_* and M is the mass of particle. The star product in the kinetic term is equal to the commutative product. Therefore, following the commutative case, the coordinates x^s for k=1,2,…,2N is a variable, and the coordinate x^k for is fixed. Equation (45) could be rewritten in the form [33]:

$$\left[-\hbar^2\partial_k{}^2/2M+V_k*\Psi\right](x)=E\Psi(x) \tag{46}$$

Supposed that there are two solutions of the equation (45) denoted by Ψ_k and $\tilde{\Psi}_k$. Then they are linearly dependent, i.e. there are two nonzero complex numbers c_k and $\tilde{c}_k$, such that the following relations hold simultaneously

$$\Psi_k=-\tilde{c}_k/c_k.\tilde{\Psi}_k \tag{47a}$$

$$\partial_k\Psi_k=-\tilde{c}_k/c_k.\partial_k\tilde{\Psi}_k \tag{47b}$$

Now, by introducing the quantum prepotential defined as in the commutative case by the following relation

$$\tilde{\Psi}_k\equiv\partial F^k\left[\Psi_k\right]/\partial\Psi_k \tag{48}$$

Then the relation between noncommutative coordinate x^k and wavefunction has the following form;

$$x^k=F^k\left[\Psi_k\right]-\tilde{\Psi}_k/2*\Psi_k-f^k\left(x^s\right) \tag{49}$$

This result appears interesting because now our gravitational wavefunction (11) could be given spacetime coordinate representation. This would be interesting subject for further study of the connection between condensed matter wavefunction (GPE/TDGL) and spacetime metric.

Concluding remarks

In the present article, we derived an alternative derivation of celestial quantization equation based on GPE/TDGL equation. It was shown that the obtained solution is also applicable to describe various phenomena in cosmology, including inhomogeneity and clustering formation. In this regard, fractality property emerges naturally from the theoretical model instead of invoked; and it corresponds to the observed value [7] of Hausdorff dimension ranging from 1.6~2.0 in universe up to the scale of 200 Mpc.

It could be expected therefore that in the near future there will be more rigorous approach to describe this fractality phenomena both in boson condensate and also in astrophysics, from which we can obtain a coherent picture of their interaction. Another interesting issue for future research in this regard, is extending the solution derived herein to include superfluid turbulence and also finding its implications in astrophysics.

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Appendix

Thanks to a note by anonymous referee, a Maple solution is included here to find solution of Schrodinger type radial equation from GPE (24). This solution indicates that for an exponential solution to present, this requires that extra term of GPE must vanish.

```
> #Partial Wave analysis
> restart;
>  with (linalg):
```

```
> R:=exp(-(alpha*r));
D1R:=diff(R,r);D2R:=diff(D1R,r);
```

$$R := \mathbf{e}^{(-\alpha\, r)}$$

$$D1R := -\alpha\, \mathbf{e}^{(-\alpha\, r)}$$

$$D2R := \alpha^2\, \mathbf{e}^{(-\alpha\, r)}$$

Formulate the partial wave equation referenced from Sonin[6]

```
> SCHEQ:=D2R+D1R/r-(1-g)^2*R/r^2+(k)^2*R;
```

$$SCHEQ := \alpha^2\, \mathbf{e}^{(-\alpha\, r)} - \frac{\alpha\, \mathbf{e}^{(-\alpha\, r)}}{r} - \frac{(1-g)^2\, \mathbf{e}^{(-\alpha\, r)}}{r^2} + k^2\, \mathbf{e}^{(-\alpha\, r)}$$

```
> XX1:=factor(SCHEQ);
```

$$XX1 := \frac{e^{(-\alpha r)} (\alpha^2 r^2 - \alpha r - 1 + 2 g - g^2 + k^2 r^2)}{r^2}$$

For the assumed exponential solution to be true,
the bracket must vanish.
HENCE: the roots of the quadratic equation are:
EITHER (solving for g)

> **GG:=solve(XX1,g);KK:=solve(XX1,k);AA:=solve(XX1,alpha);**

$$GG := 1 + \sqrt{\alpha^2 r^2 - \alpha r + k^2 r^2},\ 1 - \sqrt{\alpha^2 r^2 - \alpha r + k^2 r^2}$$

or (solving for k)

$$KK := \frac{\sqrt{-\alpha^2 r^2 + \alpha r + 1 - 2 g + g^2}}{r},\ -\frac{\sqrt{-\alpha^2 r^2 + \alpha r + 1 - 2 g + g^2}}{r}$$

or (solving for alpha)

$$AA := \frac{\frac{1}{2} + \frac{1}{2}\sqrt{5 - 8 g + 4 g^2 - 4 k^2 r^2}}{r},\ \frac{\frac{1}{2} - \frac{1}{2}\sqrt{5 - 8 g + 4 g^2 - 4 k^2 r^2}}{r}$$

End note:

[1] Another expression for γ was described in Ref. [37]:

$$g = 16\sqrt{2p}.Ana^{3}.(a_h / a).\sqrt{T_c \ / T}.\sqrt{\hbar w / k_B.T}$$

though it is not yet clear whether this expression could be directly used for cosmological phenomena.

[2] This author acknowledged Prof. C. Castro and Prof. C. Beck for suggesting that there is plausible correspondence between superfluid vortice model and (random) string theory.

[3] X. Song and P. Richards of Columbia University's Lamont-Doherty, http://www.ldeo.columbia.edu/song/pr/html.

[4] Also S. Carneiro, arXiv:gr-qc/0003096; Y.N. Obukhov, arXiv:astro-ph/0008106.