Gravitational Quantization States in Solar Systems

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ABSTRAC' We apply the new gravitational wave equation (GWE) derived from the General Theory of Relativity to determine quantization states in solar systems. The GWE has one ad hoc assumption: gravitational quantization states depend solely on the gravitationally-bound system's total angular momentum and its total mass. From the predicted planet and satellite equilibrium orbital distances we make comparisons to the empirical values. To our surprise, we find that the angular momentum in the Oort Cloud determines the planetary spacings! We derive also a mass relationship for orbiting bodies in any planetary system, including exoplanetary systems. We suggest a laboratory experiment with a torsion bar near a rotating mass.

I. Introduction

From the General Theory of Relativity (GTR), H. G. Preston and F. Potter[1] have derived a new gravitational wave equation (GWE) for gravitational interactions that predicts energy and angular momentum quantization states *per mass* for gravitationally-bound systems. The GWE is derived from GTR via a simple transfornation that defines a mass wave function Ψ analogous to the wave function of quantum mechanics. We report here some applications of the GWE in the Solar System, thereby providing direct comparisons to the standard Newtonian and GTR results and to the scale relativity approach introduced by L. Nottale[2-4]. Gravitational quantization states in galaxies and the universe are reported elsewhere[5]. The GWE should not be considered as the equation for quantum gravity at the Planck scale.

The single ad hoc assumption is that large-scale physical properties of gravitationally-bound systems, such as energy states and angular momentum states, depend only on the bound system's total mass M and total angular momentum H_Σ expressed as the ratio

$$H = \frac{H_\Sigma}{\mathrm{Mc}} \tag{1}$$

where c is the speed of light. Our subsequent time-independent Schrödinger-like equation for stationary states is an approximation derived in the Schwarzschild metric from the GWE which is generated from the general relativistic Hamilton-Jacobi equation by the transformation $\Psi = e^{iS/\mu cH}$, where μ is the test particle mass and S is the action.

For comparisons to quantum mechanics, E. Schrödinger derived his equation from the non-relativistic Hamilton-Jacobi equation via the transformation $\Psi = e^{iS/\hbar}$. Note that a local constant μcH for gravitational systems replaces Planck's universal constant $\hbar$. In the non-relativistic limit of the GWE in the Minkowski metric, one recovers the Schrödinger equation of quantum mechanics, i.e., μcH becomes $\hbar$, a hint that our GWE includes standard quantum mechanics.

Some major consequences for solar systems in the Schwarzschild metric approximation are

- Gravitational quantization states are defined by quantization of angular momentum *per mass* and energy *per mass* for orbiting bodies such as planets in solar systems and satellites of planets.
- The effective potential has angular momentum quantization terms instead of the classical angular momentum.
- Orbiting bodies in planetary systems have specific equilibrium orbital distances, with radial equilibrium orbital spacings proportional to the square of small integers, whereas all orbits are equilibrium orbits in Newtonian mechanics.
- The existence of a lower limit on the minimum energy state for gravitationally bound systems is in sharp contrast to the Newtonian case where the minimum energy diverges at $r = 0$.

We concentrate in this report on determining the quantization states around a massive central body and comparing the predicted results to the actual orbital spacings for the planets in the Solar System, for satellites around the Jovian planets, and for planets in exoplanetary systems. Any slow accelerations toward equilibrium orbits, the existence of equilibrium states within massive bodies, and the possibility of hierarchical gravitationally-bound systems will be discussed in a future article.

II. Gravitational Wave Equation (GWE)

The new scalar gravitational wave equation (GWE) derived from the general relativistic Hamilton-Jacobi equation is

$$g^{\alpha\beta} \frac{\partial^2 \Psi}{\partial x^\alpha \, \partial x^\beta} + \frac{\Psi}{H^2} = 0 , \tag{2}$$

where the wave function Ψ is the mass probability amplitude, the x^α are coordinates, and $g^{\alpha\beta}$ is the GTR metric tensor. Its derivation from the relativistic Hamilton-Jacobi equation ensures that the GWE dictates the same acceleration for all orbiting bodies of different small masses. From its solution in the appropriate metric, one can show that the corresponding wave number **k** and frequency ω for the body in orbit are independent of its mass. In the limit of a free particle in the Minkowski metric, when the total mass reduces to μ and the total angular momentum is the spin of the particle, $\Psi = C \exp[i\,(\mathbf{kx}\text{-}\omega t)]$ is a solution of the GWE.

In order to apply the GWE, we need to choose the appropriate GTR metric for the test particle of mass μ in the particular gravitationally-bound system of interest. Solar systems of planets can be described very well using the Schwarzschild metric:

$$c^2\, d\tau^2 = \left(1 - \frac{r_g}{r}\right) c^2\, dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2\, d\theta^2 - r^2 \sin^2\theta\, d\phi^2 . \tag{3}$$

Expanding the GWE in the Schwarzschild metric using the contravariant metric tensor $g^{\alpha\beta}$ produces

$$\left(1-\frac{r_g}{r}\right)^{-1}\frac{\partial^2\Psi}{c^2\,\partial t^2}-\frac{\partial}{r^2\,\partial r}\left(r^2\left(1-\frac{r_g}{r}\right)\frac{\partial\Psi}{\partial r}\right)-\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial\Psi}{\partial\theta}\right)-\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Psi}{\partial\phi^2}+\frac{\Psi}{H^2}=0 \tag{4}$$

where the Schwarzschild radius $r_g = 2GM/c^2$. This equation is separable and the appropriate substitution is the product wave function $\Psi = \Psi_t\Psi_r\Psi_\theta\Psi_\phi$. Separation of variables leads to the coordinate equations [see Appendix A].

For the test particle, its relativistic energy E_0 and angular momentum L are conserved quantities or cyclic variables of the gravitationally-bound system. Requiring Ψ_ϕ to be a single-valued function dictates the angular momentum quantization condition

$$L = m\,\mu c H \quad \text{or} \quad L = m\,\mu\, H_\Sigma / M \tag{5}$$

with integer m = 0, ±1, ±2, etc. Therefore, the angular momentum *per mass* is quantized, i.e., $L/\mu = m\, H_\Sigma/M$. The equation for Ψ_θ determines the azimuthal quantum number ℓ = 0, 1, 2, etc. with $|m| \le \ell$. When $|m| = \ell$ the maximum probability occurs at $\theta = \pi/2$, i.e., around the equatorial plane.

With $E_0 = \mu c^2 + E$ and $E \ll \mu c^2$, the radial equation becomes the Schrödinger-like equation

$$\frac{d^2\Psi_r}{dr^2}+\frac{2}{r}\frac{d\Psi_r}{dr}+\frac{2}{H^2c^2}\left(\frac{E}{\mu}+\frac{GM}{r}-\frac{\ell(\ell+1)H^2c^2}{2r^2}\right)\Psi_r \approx 0 \tag{6}$$

which has solutions that represent mass probability amplitudes. In this approximation, terms for the advance of the perihelion of an orbit, for corrections to the period of orbit leading to modification of Kepler's 3rd Law[6], and several other GTR effects have been dropped. In the scale relativity approach, Nottale[7] uses $\hbar$ in his Schrödinger-like equation where we have $\mu c H$.

The middle term of the large bracket includes the classical Newtonian gravitational potential -GM/r but the total *effective* potential

$$V_{\text{eff}} = -\frac{GM}{r}+\frac{\ell(\ell+1)H^2c^2}{2r^2} \tag{7}$$

is non-classical because the quantization of the total angular momentum given by $\sqrt{\ell(\ell+1)}\,Hc$ replaces the classical angular momentum. This fundamental change results in observable effects. For example, for a particle beginning at rest above the Earth's atmosphere with ℓ = 0 (i.e., not in orbit), its free fall acceleration will be the Newtonian value. But if the particle is in a circular orbit with $\ell > 0$, there will be a non-Newtonian radial acceleration (and a ϕ-aceleration) toward an equilibrium orbit defined by a particular quantization state.

The stationary state radial solutions Ψ_r are similar to the Laguerre functions that are obtained for the hydrogen atom with the traditional time-independent Schrödinger equation of quantum mechanics, except that they represent mass density amplitudes instead of charge density amplitudes. Solving for the energy states from the radial solution gives

$$E_n = -\mu c^2\frac{r_g^2}{8n^2H^2} \qquad \text{or} \qquad \frac{E_n}{\mu} = -\frac{G^2M^4}{2n^2H_\Sigma^2} \tag{8}$$

where E_n is on the order of $10^{-6}\,\mu c^2$ for most cases of interest. The principal quantum number, $n = n_r + \ell + 1$, is always a positive integer. We see that the energy *per mass* is quantized. Because the minimum value for n is 1, there is a limit on the minimum energy state for these gravitationally bound systems, in sharp contrast to the Newtonian case where the minimum energy diverges at $r = 0$.

For simplicity, we concentrate on circular or near-circular orbits only. When $\ell = n - 1$, the orbit is circular with a single peak in the radial probability distribution, and the Laguerre function reduces to a constant. We define a ‘gravitational Bohr radius’

$$r_0 = \frac{2H^2}{r_g} \qquad \text{or} \qquad r_0 = \frac{H_\Sigma^2}{GM^3} \tag{9}$$

which establishes the quantization radial distance scale for the gravitationally-bound system. The peak in the radial wave function occurs at $n^2 r_0$ for each n. The GWE therefore predicts a Schwarzschild metric *equilibrium* orbital radius r_{eq} calculated from the negative gradient of V_{eff} for a particle in circular orbit to be located at

$$r_{eq} = n(n-1)\, r_0 \tag{10}$$

for each $n > 1$, in contrast to Newtonian mechanics in which all orbits are equilibrium orbits. Only at these equilibrium radii do the GWE physical values agree with their Newtonian values, e.g., the angular momentum and tangential velocity values. All massive particles such as satellites, planets, and orbiting binary stars not at an equilibrium radial distance will have a small acceleration toward an equilibrium radius in a Schwarzschild metric approximation. The n = 1 state has no equilibrium radius, so any mass in this state can collect radially into a central body such as a star or a planet.

III. Solar Systems

We apply the GWE in the Schwarzschild metric to the planets of the Solar System, to the satellites of the Jovian planets, and to exoplanetary systems. We analyze in detail only the Solar System planetary orbits and simply comment on the orbital equilibrium radius results for the Jovian satellites. Then we derive a general mass relationship that applies to all gravitationally-bound systems, including the exoplanetary systems.

If the majority of the angular momentum of the Solar System is determined by the Sun and the nine planets, i.e., L ~ 4 x 10^{43} kg-m^2 s^{-1}, as is traditionally stated, then using our algebraic expression for r_{eq} reveals that all the predicted planetary orbital radii would be inside the Sun! This result would be disasterous for our GWE approach. To our surprise, we learned that the large angular momentum contribution of the Oort Cloud[8], at least forty times the total angular momentum of the planets, actually determines the planetary orbital spacings for the Solar System! Here is the analysis.

For the planets of the Solar System we assume that they have reached their equilibrium orbital radii over billions of years, in which case the planet’s Newtonian angular momentum $L = \mu\sqrt{GMr}$ equals the quantization state angular momentum given by $L = m\,\mu H_\Sigma/M$, with the total mass M being very nearly the mass of the Sun. A stringent linear regression fit requiring $R^2 > 0.999$ in the plot of $\sqrt{r}$ vs. H_Σ/M for the nine planets determines an acceptable set of m values, i.e., a set having small integers. The corresponding set of n values follows from $n = m + 1$, and they are: 4, 5, 6, 7, 12, 16, 22, 27, 31.

The plot of $\sqrt{r}$ vs n is shown in Figure 1. The slope of this excellent linear fit determines the total angular momentum $H_\Sigma \simeq 1.86$ x 10^{45} kg-m^2 s^{-1} of the Solar System, making H ≃ 3.1 x 10^6 m and $r_0 \simeq$ 6.4 x 10^9 m, i.e., a distance outside the Sun, and verifying the the need for the enormous angular momentum contribution of the Oort Cloud. This set of small integers is not unique, however, because another set of small integers with $R^2 > 0.999$ exists. Our several fits of $\sqrt{r}$ vs n agree with statistical fits done by Nottale's group who derived a one-parameter Schrödinger-like equation from chaos theory, i.e., by not using $\hbar$ in his Schrödinger-like equation.

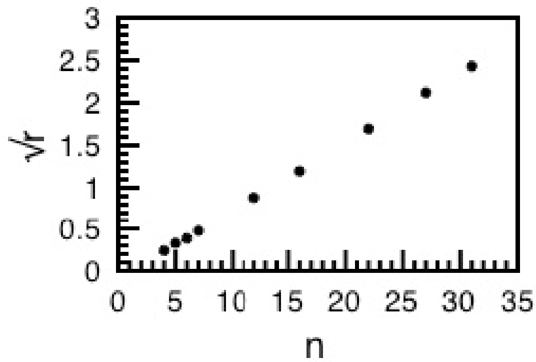

FIG. 1: Square root of planet orbital radii (x10^6 meters) versus principal quantum number *n* for the nine planets of the Solar System determined by the angular momentum fit with $R^2 > 0.999$. Earth has n = 6.

The GWE applies also to the orbital distances of the satellites of the Jovian planets Jupiter, Saturn, Uranus, and Neptune. The total angular momentum of each planetary system is essentially in the rotation of the planet, as is its total mass. We find that our GWE fits agree with Nottale's fits for the satellite orbital spacings for the Jovian systems. However, even though the moments of inertia for the Jovian planets are known to within about 1%, the total angular momentum is not known to within 10% because of differential mass rotations within each planet. Therefore, we find that several sets of small integers favorably fit the empirical data for each of these Jovian satellite systems. The planets Earth, Mars, and Pluto also have satellites with radial distances that can fit the theory.

Exoplanetary solar systems have no conflict with the GWE nor with Nottale's statistical approach. However, these systems with so few known planets permit excellent fits of many different sets of small integers for the *m* and *n* values, and our analyses suggest that many of them will have more orbiting mass at further distances from their stars. Some exoplanetary systems may have the equivalent of the Oort Cloud.

We can use the angular momentum quantization condition to derive a surprising relationship among the masses of all the orbiting bodies in any system, including the exoplanetary systems. For the general case, sum over all the angular momenta, the spin of the central body L_0 plus the orbital angular momentum of the *i*-th body, but ignoring the spins of the orbiting bodies, to produce the angular momentum relationship

$$H_\Sigma = L_0 + \frac{H_\Sigma}{M} \sum_i m_i \, \mu_i \, . \tag{11}$$

Expressing the central body's spin as $L_0 = \beta \, H_\Sigma$, the above relationship then relates the total mass M to its mass components by

$$M = \frac{1}{1-\beta} \sum_i m_i \, \mu_i \, . \tag{12}$$

For the Solar System, $\beta \sim 0$, and one finds agreement with this mass relationship only when the estimated mass and the approximate quantum number of the Oort cloud are included. At the other limit, for Jovian satellite systems, $\beta \sim 1$, so the relationship isn't very useful. But for the exoplanetary solar systems when $\beta \sim 0$, one can use the relationship to estimate how much more mass might be in orbiting bodies.

We find therefore that the applications of the GWE to the gravitationally bound systems in the Solar System produce some remarkable results. The large angular momentum of the Oort Cloud is now understood to be an essential ingredient in determining the behavior of our planetary system, and all the planets orbit the Sun at radial equilibrium distances. Even Pluto fits the scheme. Satellites of the Jovian planets agree also. Not all orbital equilibrium radii are populated by a massive body in orbit, probably a consequence of the history of the Solar System.

The large uncertainty in the total angular momentum of the Solar System and in each of the other systems investigated leaves us without a definitive test of the GWE in spite of its successful applications. Therefore, we have investigated even larger systems[5] such as galaxies and clusters of galaxies, but again the angular momentum uncertainties are too large to provide a definitive test. In the next section we suggest a possible laboratory experiment which may prove to be a definitive test of the GWE.

IV. Possible laboratory experiment

The GWE predicts very small non-linear accelerations of a test particle in circular orbit toward an equilibrium radius. These accelerations would bring the particle into an equilibrium orbit with possible small oscillations about the radial equilibrium distance. Therefore, we predict changes in the acceleration of a torsion bar nearby a rotating spherical central mass when the rotation speed of the central mass changes. Essentially, if this torsion bar system is truly a gravitationally-bound system in the plane defined by the bar and the rotating mass, there will be equilibrium radii. Changing the angular momentum of this bound system would change the radii of the equilibrium orbits and the torsion bar will respond as these equilibrium radii pass through.

A laboratory experiment to test the GWE predictions would require measurable gravitational forces greater than about 10^{-12} N and a 'gravitational Bohr radius' r_0 of about one meter or less so that equilibrium radii for $n > 1$ exist within room dimensions. Suppose we have a 10^{-2} kg mass 'in orbit' about a spinning 10^3 kg central mass at a 0.5 m orbit radius. Since $r_g = 1.5 \times 10^{-24}$ m, in order to have an r_0 of 0.10 m, say, we need an H = 2.7×10^{-13} m and $H_\Sigma = 8.2 \times 10^{-2}$ kg-m^2 s^{-1}. A 10^3 kg spinning sphere would need to spin at about 5.1×10^{-3} radians per second, or about 3 revs/hr. The classical turning points for the total kinetic energy dictate that only the $n = 2$ and $n = 3$ eigenstates can be bound states before the spin changes. For $n = 2$, the GWE radial acceleration at 0.5 m is about -7.2×10^{-9} m s^{-2} compared to the Newtonian radial acceleration of -2.7×10^{-7} m s^{-2}, a measurable effect. One would slowly increase the spin rate of the central sphere to detect the sign change of the GWE accelerations on different sides of the equilibrium radii.

V. Final comments

We apply the new large-scale gravitational wave equation (GWE) derived from the General Theory of Relativity to the Solar System in order to understand any predicted differences from classical celestial mechanics. We find that the GWE in the Schwarzschild metric agrees extremely well with the empirical data. The fact that quantization energy states depend upon the ratio of the system's total angular momentum to its total mass dictates that the Oort Cloud in the Solar System determines the planetary radial spacings. We suggest a laboratory torsion bar experiment with a nearby rotating mass as a possible definitive test.

VI. Acknowledgments

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APPENDIX A:

For the GWE in the Schwarschild metric, separation of variables produces the coordinate equations, with the primes representing division by μc. In the t-coordinate:

$$\frac{d^2\,\Psi_t}{c^2\,d\,t^2} = -\frac{E_0^{'2}}{H^2\,c^2}\,\Psi_t\,. \tag{A1}$$

In the ϕ-coordinate:

$$\frac{d^2\,\Psi_\phi}{d\,\phi^2} = -\frac{L^{'2}}{H^2}\,\Psi_\phi\,. \tag{A2}$$

In the θ-coordinate:

$$\frac{1}{\sin\theta}\,\frac{d}{d\,\theta}\left(\frac{\sin\theta\;d\,\Psi_\theta}{d\,\theta}\right) - \frac{m^2\,\Psi_\theta}{\sin^2\theta} = -\ell(\ell+1)\,\Psi_\theta\,. \tag{A3}$$

In the radial coordinate:

$$\left(1-\frac{r_g}{r}\right)\frac{\mathrm{d}^2\,\Psi_r}{\mathrm{d}\,r^2} + \frac{\left(2-\frac{r_g}{r}\right)}{r}\,\frac{\mathrm{d}\,\Psi_r}{\mathrm{d}\,r} + \left(\left(1-\frac{r_g}{r}\right)^{-1}\frac{E_0^{'2}}{H^2\,c^2} - \frac{1}{H^2} - \frac{\ell\,(\ell+1)}{r^2}\right)\Psi_r \;=\; 0\,. \tag{A4}$$

The ϕ-equation dictates angular momentum quantization. The radial equation has a singularity at $r = \mathrm{r_g}$ that is transformed to the origin by the standard substitution $r(r - \mathrm{r_g}) = r'^2$, where r' is a new radial coordinate. In all cases of interest, $\mathrm{r_g} \ll r'$, typically $\mathrm{r_g}/r' < 10^{-8}$, so we choose to ignore terms proportional to r_g/r'^2, r_g^2/r'^2, and smaller. For notational simplicity, we drop the prime on the new r. We make the traditional substitution for the relativistic energy $\mathrm{E_0} = \mu\mathrm{c}^2 + \mathrm{E}$. Since $\mathrm{E} << \mu\mathrm{c}^2$, the radial equation becomes the Schrödinger-like equation

$$\frac{\mathrm{d}^2\,\Psi_r}{\mathrm{d}\,r^2} + \frac{2}{r}\,\frac{\mathrm{d}\,\Psi_r}{\mathrm{d}\,r} + \frac{2}{H^2\,c^2}\left(\frac{E}{\mu} + \frac{\mathrm{GM}}{r} - \frac{\ell\,(\ell+1)\,H^2\,c^2}{2\,r^2}\right)\Psi_r \;\approx\; 0 \tag{A5}$$

which has solutions that represent mass probability amplitudes. Among the many very small terms dropped are the advance of the perihelion of an orbit and other GTR effects.

The solution for the radial wave function Ψ_r is

$$\Psi_r = A\, r^{\ell} \exp\left(\frac{-\sqrt{-2E/\mu}}{H\,c}\, r\right) {}_1F_1\left(\frac{-r_g\, c}{2H\sqrt{-2E/\mu}} + \ell + 1,\, 2\ell + 2;\; 2\,\frac{\sqrt{-2E/\mu}}{H\,c}\, r\right) \tag{A6}$$

with A the normalization constant. These stationary-state radial solutions lead to the same Laguerre functions that are obtained for the hydrogen atom with the traditional time-independent Schrödinger equation of quantum mechanics except that they represent mass density probabilities instead of charge density probabilities.

If the first parameter in the confluent hypergeometric function ${}_1F_1$ (g, h; r) in the wave function is set equal to -n_r, a negative integer or zero, then ${}_1F_1$ (g, h; r) reduces to a polynomial with a finite number of terms which is multiplied by the negative exponential and a power of r. The net result is a wave function that does not diverge at infinity for all negative integers or zero. Additionally, the second parameter must be a positive integer for the function to be single-valued. Equivalently, ${}_1F_1$ (g, h; r) can be related to a Laguerre function with the same result. The second solution with U(g, h; r) does not yield a satisfactory wave function[9]. The development continues in the main text.

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