Improved Newton's Formula of Universal Gravitation

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Abstract By using the movement equation of planet derived by general relativity, this paper presents the improved formula of universal gravitation as follows

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4}$$

By using this formula, the accurate solution for the problem of gravitational deflection of photon orbit around the sun and the problem of advance of Mercury's perihelion can be given.

Key words: general relativity, moving equation of planet, improved formula of universal gravitation

The formula of universal gravitation presented by Newton has been widely used in many fields, while it cannot be used for solving the problems of advance of Mercury's perihelion and gravitational deflection of photon orbit around the sun. By using the movement equation of planet derived by general relativity, this paper presents the improved formula of universal gravitation, it can be used for solving the above mentioned problems.

1 Improved Formula of Universal Gravitation

As discussing the problem of planet's movement around the sun according to the general relativity, the following equation can be given (Hu, 2000 and Fu, 1989)

$$u''+u = \frac{1}{p} + \frac{3GMu^2}{c^2} \qquad (1)$$

where

$u = \frac{1}{r}$; G – gravitational constant; M – mass of sun; c – velocity of light;

p - half normal focal chord.

p = a(1-e^2) (for ellipse)

p = a(e^2-1) (for hyperbola)

p = y^2/2x (for parabola)

According to the central force, the orbit differential equation(Binet's formula) reads

$$h^2u^2(u''+u) = -\frac{F}{m} \qquad (2)$$

where h^2 – a constant.

Substituting Eq.(1) into Eq.(2), we have

$$F = -mh^2u^2\left(\frac{1}{p} + \frac{3GMu^2}{c^2}\right) \quad (3)$$

The existing formula of universal gravitation reads

$$F = -\frac{GMm}{r^2} = -GMmu^2 \quad (4)$$

For Eqs.(3) and (4), comparing the terms including u^2, we have

$$h^2 = GMp$$

Substituting h^2 into Eq.(3), we have

$$F = -GMmu^2 - \frac{3G^2M^2mpu^4}{c^2} \quad (5)$$

Substituting $u = \frac{1}{r}$ into Eq.(5), the improved formula of universal gravitation reads

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4} \quad (6)$$

It should be noted that if the movement of the two objects in the manner of center to center(including relative motionless), it can be treated as the case of $p = 0$, then the improved formula of universal gravitation has the same form as the existing one.

In addition, as handling the problem of advance of Mercury's perihelion, the second term in Eq.(6) is a very small quantity to compare with the first term, while for the problem of gravitational deflection of photon orbit around the sun, for the reason that the half normal focal chord p is a very large quantity, therefore the second term in Eq.(6) is not a small quantity to compare with the first term. In fact it gives the same value of deflection as given by the first term. For this reason, in other case concerned with gravitation, the effect of the second term in Eq.(6) may not be neglected.

2 Application of the Improved Formula of Universal Gravitation

Because the improved formula of universal gravitation is derived from the planet's movement equation given by the general relativity, of course it can be used for solving the problem of advance of Mercury's perihelion. In fact, substituting Eq.(6) into Eq.(2)(Binet's formula), the Eq.(1), i.e., the equation for solving the problem of advance of Mercury's perihelion given by the general relativity, can be reached.

By using the improved formula of universal gravitation instead of the general relativity, the problem of gravitational deflection of photon orbit around the sun can be solved by using the method of Newton's mechanics presented in reference (Kittel, 1973).

Supposing that m represents the mass of photon; r_0 represents the nearest distance to the center of the sun as shown in Fig.1, on point （r_0，y）, the force acted on photon reads

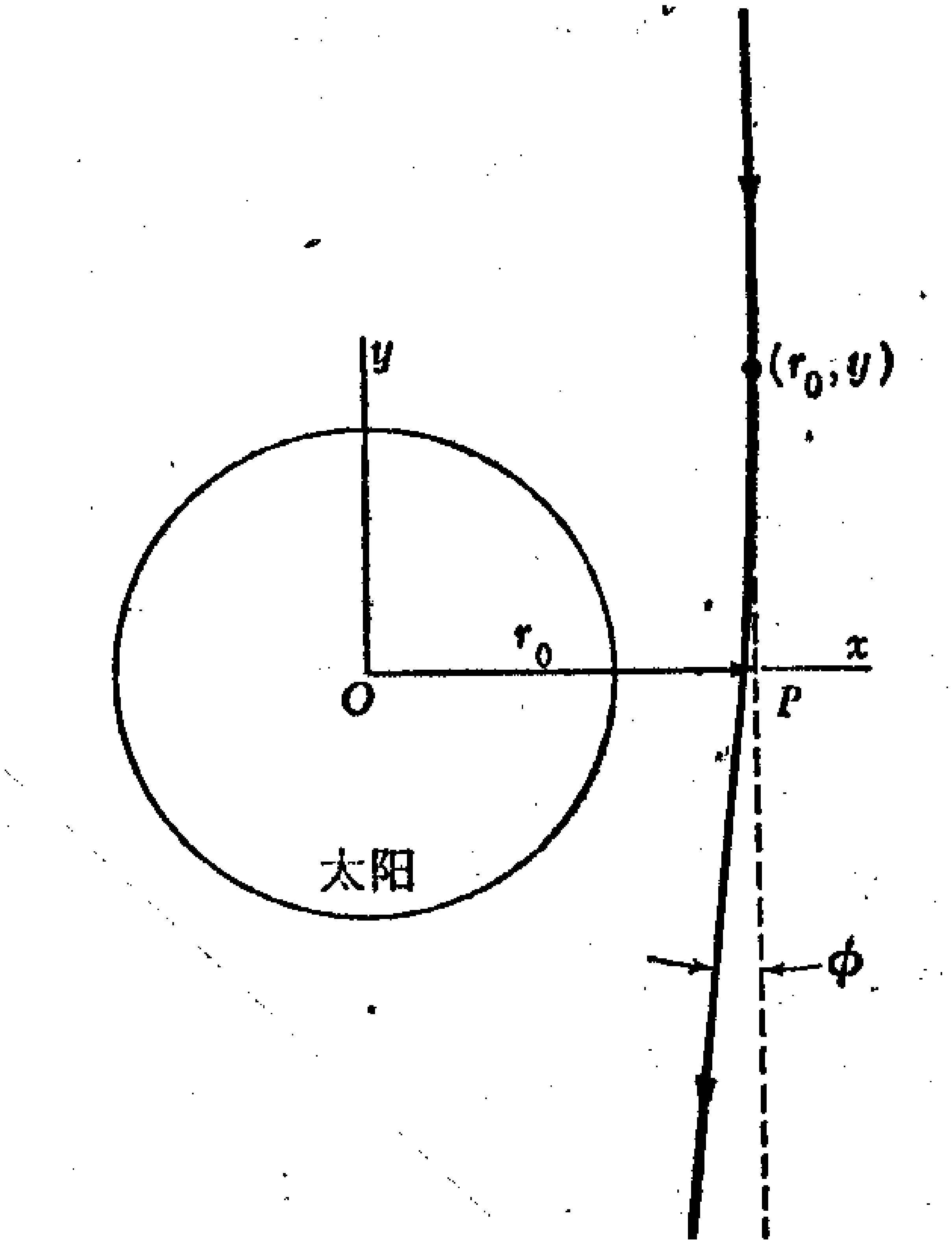

Fig.1 gravitational deflection of photon orbit around the sun

$$F_x = \frac{Fr_0}{(r_0^2 + y^2)^{1/2}} \qquad (7)$$

where $F = -\frac{GMm}{r_0^2 + y^2} - \frac{3G^2M^2mp}{c^2(r_0^2 + y^2)^2}$

Because

$$mv_x = \int F_x dt = \int F_x \frac{dy}{v_y} \approx \frac{1}{c}\int F_x dy$$

Hence

$$v_x \approx -\frac{GMr_0}{c}\int_{-\infty}^{\infty}\frac{dy}{(r_0^2 + y^2)^{3/2}} - \frac{3G^2M^2pr_0}{c^3}\int_{-\infty}^{\infty}\frac{dy}{(r_0^2 + y^2)^{5/2}}$$

Then we have

$$v_x \approx -\frac{2GM}{cr_0} - \frac{4G^2M^2p}{c^3r_0^3} \qquad (8)$$

The deflection angle reads

$$\phi \approx tg\phi \approx \frac{|v_x|}{c} = \frac{2GM}{c^2r_0} + \frac{4G^2M^2p}{c^4r_0^3} \qquad (9)$$

The value of ϕ has to be solved by using the method of iteration, while firstly we will validate that the value of the second term in Eq.(9) is equal to the value of the first term, that means that the deflection given by the second term in Eq.(6) is equal to that given by the first term.

By using the general relativity to solve the problem of gravitational deflection of photon orbit around the sun, the track of the light can be decided as a hyperbola, the equation reads

$$u = u_0\cos\varphi + \frac{GMu_0^2(1+\sin^2\varphi)}{c^2} \qquad (10)$$

where $u_0 = \frac{1}{r_0}$

Hence

$$\frac{1}{p} = u\big|_{\varphi=\pi/2} = \frac{2GM}{c^2r_0^2} \qquad (11)$$

Substituting the value of p into Eq.(9), we have

$$\phi = \frac{4GM}{c^2r_0} = \frac{4GM}{c^2R_s} \qquad (12)$$

where R_s - radius of the sun

That means the value of the second term in Eq.(9) is really equal to that of the first term.

Now we will solve the value of ϕ by using the method of iteration.

Supposing that

$$\phi = \frac{K\,G\,M}{c^2 R_s}. \tag{13}$$

Firstly, we apply the result given by the existing formula of universal gravitation(Kittel, 1973), then the value of K reads

$$K_0 = 2$$

From this value of ϕ, the corresponding value of the half normal focal chord p can be decided, substituting this value of p into Eq.(9), then we have

$$K_1 = 6$$

By using the same method continuously, the values of K_2, K_3 and the like can be decided as follows: 3.3333, 4.4, 3.8182, 4.0952, 3.9535, 4.0234, 3.9883, 4.0059, 3.9971, 4.0015, 3.9993, 4.0004, 3.9998, 4.0001, 4.0000, 4.0000

Hence

$$K = 4$$

That means that the accurate result for the problem of gravitational deflection of photon orbit around the sun can be given by the improved formula of universal gravitation.

3 Conclusions

This paper presents the improved formula of universal gravitation. The example given in this paper shows that by using this formula for solving some problems, the better results may be reached.

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