Collatz Conjecture integer series has no looping except one.

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Abstract

If the series of Collatz Conjecture integer has looping in it, it is sure the members of the looping cannot reach to value 1. Here it is proven that the possibility of looping is zero except one case.

1. Introduction

Procedure of Collatz Conjecture is recognized as following operations.

It starts with positive odd integer n_0.

It continues following calculation up to $n_i = 1$.

- Compute $n_w = 3 \times n_{i-1} + 1$. (1)
- n_w is divided by 2, m_i times until it becomes positive odd integer.

$$n_i = \frac{n_w}{2^{m_i}} = \frac{3 \times n_{i-1} + 1}{2^{m_i}} \quad (2)$$

n_i becomes n_{i-1} for (1).

2. Looping

Collatz conjecture procedure ith iteration calculation is (3) based on (2).

$$n_i = \frac{3 \times n_{i-1} + 1}{2^{m_i}} = \frac{n_{i-1}\left(3 + \frac{1}{n_{i-1}}\right)}{2^{m_i}} \quad (3)$$

$(i-1)$th iteration calculation is (4) same as (3).

$$n_{i-1} = \frac{n_{i-2}\left(3 + \frac{1}{n_{i-2}}\right)}{2^{m_{i-1}}} \quad (4)$$

In turn calculating n value down to n_0 and inserting these, we can get (5).

$$n_i = n_0 \left(\frac{3 + \frac{1}{n_0}}{2^{m_1}}\right)\left(\frac{3 + \frac{1}{n_1}}{2^{m_2}}\right)\left(\frac{3 + \frac{1}{n_2}}{2^{m_3}}\right) \cdots \left(\frac{3 + \frac{1}{n_{i-1}}}{2^{m_i}}\right) \quad (5)$$

If $n_0 = n_i$ in Collatz Conjecture integer series, it makes looping from n_0 to n_{i-1}. Therefore, the condition for looping is (6).

$$\left(\frac{3 + \frac{1}{n_0}}{2^{m_1}}\right)\left(\frac{3 + \frac{1}{n_1}}{2^{m_2}}\right)\left(\frac{3 + \frac{1}{n_2}}{2^{m_3}}\right) \cdots \left(\frac{3 + \frac{1}{n_{i-1}}}{2^{m_i}}\right) = 1 \quad (6)$$

About the parts of (6),

$$3 < 3 + \frac{1}{n_{j-1}} \leq 4 \quad \text{(j; for all i).} \quad (7)$$

Equal condition is satisfied when $n_{j-1} = 1$.

Regarding to the structure of (6), left side is

$$\left(\frac{3+\text{'below the decimal point or 1'}}{\text{power of 2}}\right)\left(-\right)\left(-\right)\cdots\left(-\right). \qquad (8)$$

In the case of $n_0 \neq 1$, value of 'below the decimal point or 1' of (8) is not 1. Therefore, (6) is not satisfied because same kind of fractions multiplication cannot make result of value 1. Then $n_i = n_0$ cannot be realized in this case although $n_i < n_0$ or $n_i > n_0$ can be.

In the case of $n_0 = 1$, value of 'below the decimal point or 1' of (8) is 1, then

$$3 + \frac{1}{n_0} = 4. \qquad (9)$$

Also, actually result is (10) in this case, then (6) is satisfied.

$$n_j = 1,\ m_j = 2 \qquad \text{(j; for all i)} \qquad (10)$$

Considering all above situations, only possible looping has result (10).
This means that this looping is only one member looping or self-looping when n=1.
Therefore, n=1 can be terminal point of Collatz Conjecture operation.

3. Consideration

No looping proof in this report could be used with *1 and *2 which investigate Collatz Conjecture Space. These show that the space expectation value of 2^{m_i} in (2) is $2^2 = 4$. Also, this no looping report could be used with *3 which investigates the series of Collatz Conjecture integer.
These combinations show Collatz Conjecture is correct.

*1) viXra:2204.0151
*2) viXra:2304.0182
*3) viXra:2302.0015