On the interpretation of the metric signature of temporal dimensions

By **Thomas Emilio Villa (**thomas.villa@gmail.com)

Abstract:
We suggest in this work a modest yet quite speculative idea: what if the negative sign in front of time-like dimensions is not just a mathematical tool after all? What if it follows the natural behaviour of time flowing? In this work we are suggesting an unorthodox philosophical interpretation and an explaination of the way that causality is conserved in equations, using the negative sign in front of temporal dimensions. To do so, we use non-conventional approaches like inductive dimension, Lebesgue covering dimension and Minkowski-Bouligand dimension to better understand the nature of time.

<u>NOTE TO THE KIND READER:</u> ***this work is <u>NOT</u> intended to be a scientific research paper. This analysis is intended to address the progresses of the high energy physics in terms of phylosophical speculations. This work is intended by the author as a philosophy of science analysis and, despite the mathematical formalism appearing in some parts, this is a work about the mere interpretation of the already existing theories of Conformal Cyclic Cosmology and the Holographic approach. This present work has to be read -as the title explictly says – as a philosophical speculation.***

Try to visualize a different number of dimensions than the one we appreciate with our senses daily it's a challenging experience. One way to visualize a higher number of dimension is an operation known in mathematics as *suspension*, while for negative dimensions, known as “spectra”, the operation to do is a *desuspension*. In this speculative work, we try to consider the option that the usual notation of the metric signature with space-like dimensions indicated with positive sign and time-like dimension(s) indicated with the opposite sign is indeed a profound clue about what makes time so special as a dimension. What is time is literally a *spectrum*, a *negative* dimension? What if time flowing is literally the operation of a desuspension of a 3-D sphere whose Minkowskian-like foliations are being peeled away?

It is well known that a D-dimensional sphere should be defined as embedded into a D+1 isometric hypersurface. So, a 2-sphere is embedded into a 3-dimensional hypersurface, a 1-sphere into a 2-dimensional hypersurface and a 0-sphere (a dot) is embedded into a 1-dimensional hypersurface. Mathematically, a D-sphere can be defined as the set of points in (D+1) - dimensional Euclidean space that are a fixed distance (the radius) from a central point. We can see it as a *continuous transformation*, an operation which implies a time-like dimension where we can transform a point into a set of points at a fixed distance from the central radius, like we would do drawing a circunference with a compass. This implies that in the embedding the additional dimension can be seen as a temporal one. So,we could see that a 1-sphere as embedded into a 1-dimensional isometric hypersurface with an extra, time-like dimension where we can rotate the point around the central point and draw the 1-sphere (a circumference) within a 2-dimensional hypersurface (a plane).

So, what happen when we consider a 0-dimensional hypersurface? What kind of D-sphere can be drawn there? The maximally applied logic would tell you that a (-1)-sphere should be embedded into a 0-dimensional hypersurface. The (-1) terms here -we suggest- can be seen as a time-like negative dimension, and its rotation through D+1 dimension draw a 0-dimensional hypersurface.

Negative dimensionality can be euristically interpreted as a diminishing in the number of the foliations $\boldsymbol{S_t}$ or the bidimensional sheets. Every step ($\|\mathbf{v}\| = 1$) forward taken in the dimension (-1) would lead me at x'=x-1. For example, we know that by definition a point has no parts, it's a scalar. By definition the perfect, geometrical point has dimension 0.
So, for a 0 dimensional point, its dimensionality can be described as the boundary of the open subset of dimensionality D=-1.

$$\operatorname{ind}(\varnothing) = \operatorname{Ind}(\varnothing) = -1$$

This is true in Lebesgue covering dimension (or Large Inductive Dimension, expressed with the operation ***Ind(x)***) and in the Brouwer dimension, or Small Inductive Dimension, whose operation is expressed by ***ind(x)***.

While the approximation of the open subset D to D+1 for D>0 happen with adding layers, or D-balls, from the boundary of the dimension D adding gradually infinitesimal degrees of freedom until a new boundary is reached at the limit of D+1, for D<0 the situation is the other way round: here euristically we can see that the motion from D to D+1 happens with *removing* layers, or D-balls. This removal of layers happens from the boundary of D upwards, removing layers from the bottom and getting closer and closer to the upper boundary of D+1, that is, when no layer or D-ball of dimension D remained. In a sense, we can say that in negative dimension, rungs are *removed* as you go up the ladder, until the ground floor.

Another way to address this problem is to ask what can be the dimensionality of time when it comes to using to it a quite unorthodox approach like the one given by the Minkowski-Bouligand "Box counting" dimension (see ***Appendix 1***). In this approach, quite empirical indeed, in order to determine the fractal dimensionality of an object or a drawing you have to cover it with a grid or progressively shrinking boxes or squares. The equation goes like this:

$$N(\varepsilon) \sim \left(\tfrac{1}{\varepsilon}\right)^{D}$$

Where N(ε) is the number of boxes with side lenght ε, D is the dimensionality and $\lim_{\varepsilon\to 0} f(\varepsilon)$. that can be generalized in this way:

$$N(\varepsilon) = C \cdot \varepsilon^{-D}$$

Here, we have to keep in mind a peculiar feature of the dimensionality of time: as far as we know, it has a continuous structure, it is not coarse, at least not at the observational level of today. This is peculiar, because 3D object always shows patterns or structures, irregularities and so on. That is, they are not scale-invariant. The smaller the box, the largest is the difference between the number of boxes with a side lengh of ε and the 1/ ε ratio. Many objects even shows fractal behaviour, that is: they can be described by a fractional dimensionality of ½, ¼, 1/137 and so on. But time is actually scale-invariant: the size of ε does not matter, the choosen lenght of fragment of the norm of time will always have one box with a side lenght of ε in it (see ***Appendix 2***). Expressed mathematically, one could write this equivalence like this:

$$N(\varepsilon) = \varepsilon^{-D}$$

That can be transformed, since we are considering the dimensionality of time *t*, like this:

$$N(\varepsilon) = \varepsilon^{-\boldsymbol{t}}$$

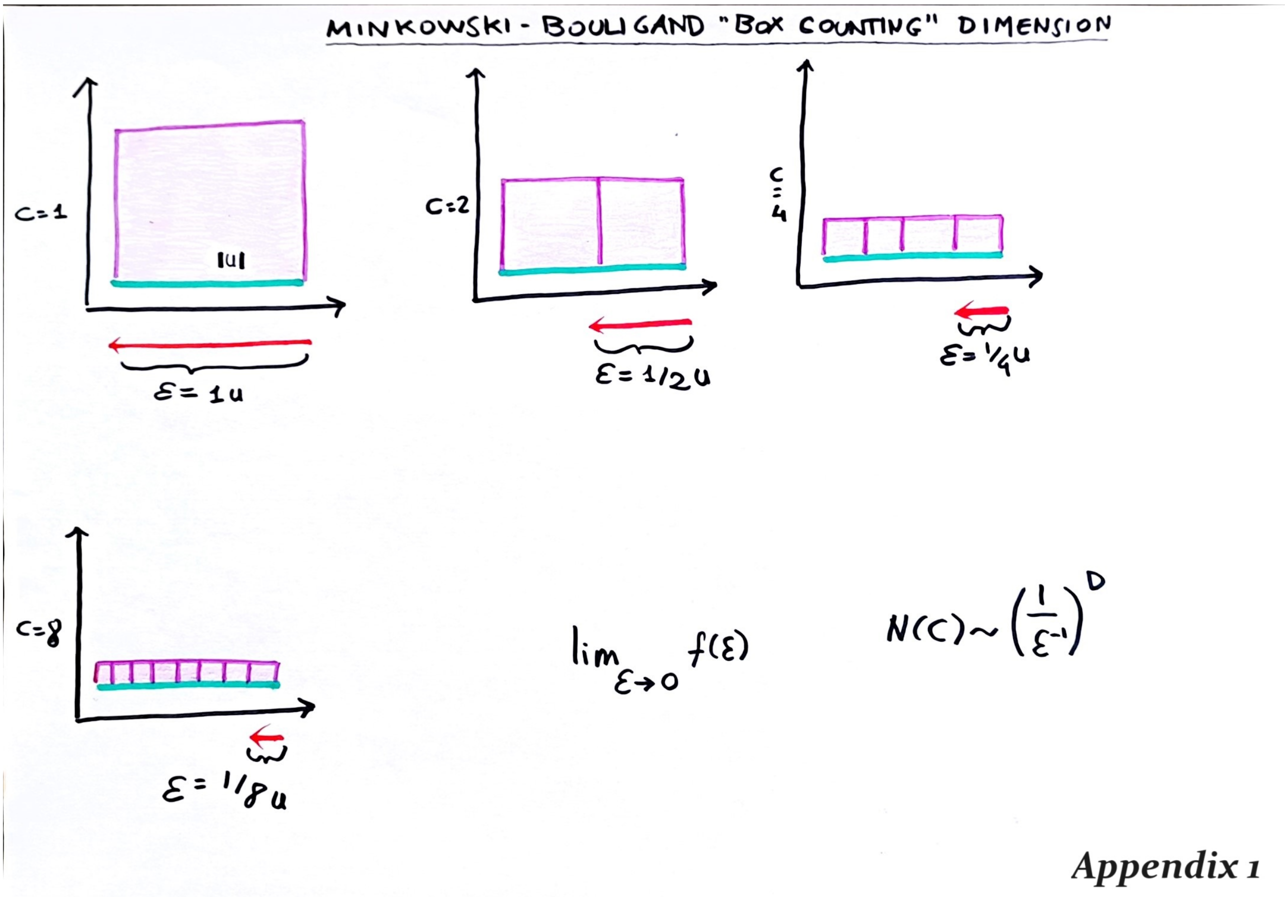
Where one can see that the dimensionality of time can be seen actually as *t*<0.

As a matter of fact, it's worth noting that even in the simplest special relativity equation of the spacetime interval (2), the time is always interpreted with a negative sign (or, to be more precise, with the opposite sign of spatial dimension in order to preserve causality) and therefore the metric is (-,+,+,+).

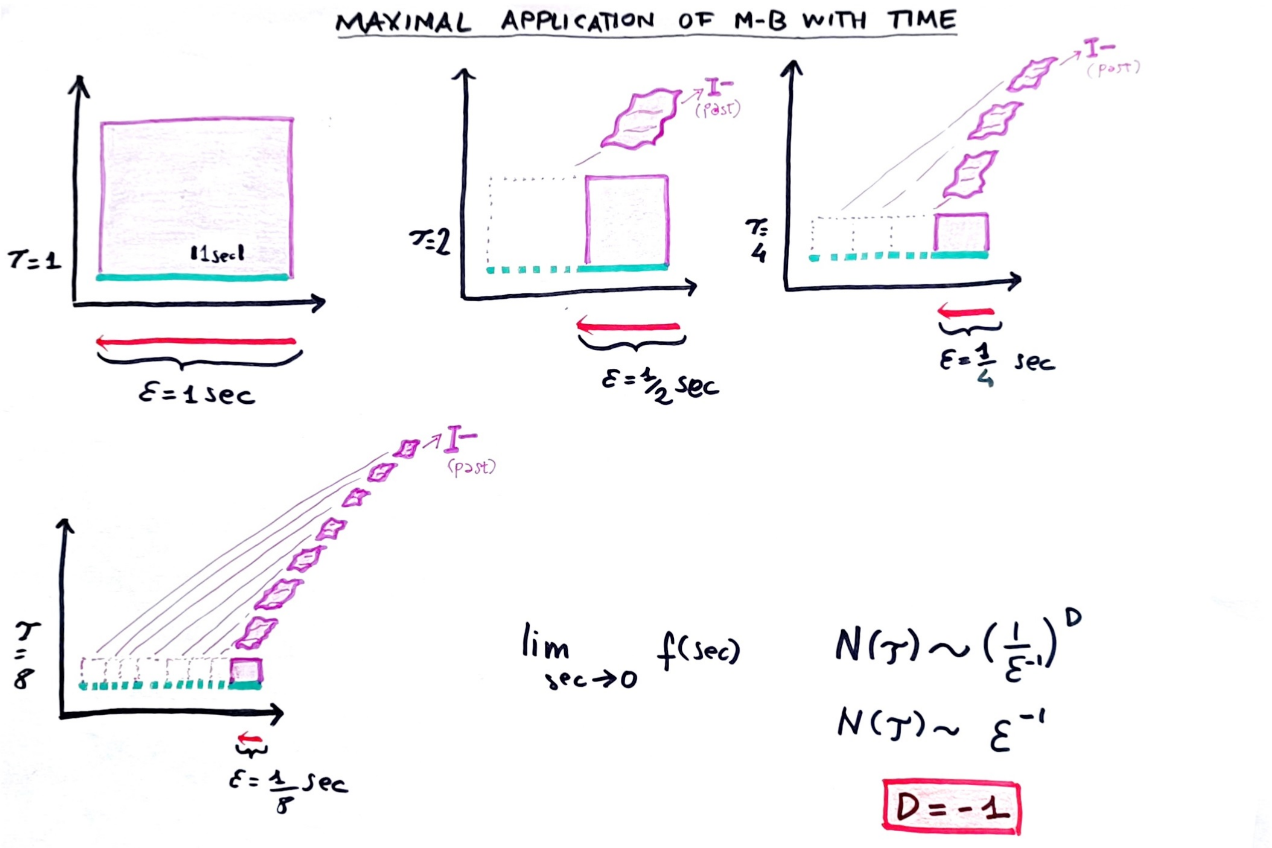
$$ds^2 = -c^2dt^2+dx^2+dy^2+dz^2 \qquad (2)$$

The Shannon information theory (3), which deals with entropy in terms of the growing information needed to describe the evolution of the system in the future, also shows a minus sign in front of the equation:

$$\mathrm{H}(X) := -\sum_{x\in\mathcal{X}} p(x)\log p(x) \quad (3)$$



Appendix 1



Appendix 2: time as a Minkowski-Bouligand "Box counting" dimension