An improved belief entropy measuring total uncertainty in the evidence theory

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Abstract

Dempster-Shafer evidence theory is widely used in many fields due to its advantages in dealing with uncertain information. However, measuring the uncertainty of information in evidence theory remains an open question. In recent years, Deng entropy has gained attention as a representative measure of uncertainty. However, Deng entropy cannot express the uncertainty of conflict well. Inspired by Jousselme's mathematical expressions for non-conflict, this paper proposes an improved belief entropy that generalizes Deng entropy and can better express the uncertainty of conflict. Additionally, this paper highlights the difference between generalized condition and traditional condition in which DS evidence theory is used. Based on this difference, the paper also discusses the basic principles proposed by Klir and Wierman. These research findings are significant for further advancing the application and development of DS evidence theory.

Keywords: evidence theory, belief entropy, uncertainty measure, evidence distance

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1. Introduction

DS evidence theory[1][2] is a generalization of Bayesian theory[3]. Different from Bayesian theory, DS evidence theory does not need to know prior probability and can express uncertainty well. Because of its powerful ability to express uncertain information, DS evidence theory is widely used in: information fusion, expert system, intelligence analysis, legal case analysis, multi-attribute decision analysis, etc. However, how to accurately describe the uncertainty in a basic probability distribution (*BPA*) is still an urgent problem to be solved, which has important significance for the quantitative description of the information included in a *BPA*.

Shannon entropy is a recognized method for quantitatively describing uncertainty in probability distributions, and has been widely used. Shannon's effectiveness has been repeatedly verified in practical applications Therefore, when the concept is expanded from probability distribution to DS evidence structure, a number of uncertainty measures based on Shannon entropy are formed to describe the uncertainty of *BPA* quantitatively. They are collectively called entropy-like uncertainty measures. Such as Dubois & Prade's weighted Hartley entropy[4] Hohle's confusion measure[5], Yager's dissonance measure[6], Klir & Ramer's discord[7], Klir & Parviz's strife[8], Deng entropy[9] and its variants[10][11][12].

Klir and Wierman pointed out that the uncertainty measure of DS evidence theory must be able to accurately describe conflict and non-specificity[13]. At the same time, it should have the following five basic properties: *Probabilistic consistency*, *Set consistency*, *Range*, *Subadditivity*, *Additivity*. In the traditional application of evidence theory, the answer to the problem can only be a single element of the frame of discernment. However a different scenario was given by Deng[9]:assume that in a test there are 32 participants. And we want to know who is(are) the top 1 participant(s) who get(s) the highest score(s). There is a possibility that there are participants tie for first. Higher non-specificity uncertainty will be obtained by introducing the condition of a tie for first.

Therefore, the non-specificity uncertainty of the new system is inevitable higher than that of previous condition. When Klir and Wierman proposed the five basic properties, Deng's academic views are not yet formed, so this paper believes that the two properties of *Set consistency* and *Range* may need to be amended accordingly. The condition described by Deng is more consistent with the situations encountered in real-life engineering, which represents a significant expansion in application of DS evidence theory.

Deng introduced a new scenario[9] that several participants can tie for first, and proposed corresponding non-specific expression. However, Deng's expression of conflict measure is flawed. Therefore, many scholars proposed some variants based on Deng entropy, partially making up for the deficiency in Deng's conflict measure of *BPA*. However, those belief entropies have their own problems. For example, Cui *et al*'s belief entropy introduced a modification based on Deng entropy, but the modification is too tiny, therefore, its performance is only marginally different from that of Deng entropy. Wang *et al* 's belief entropy may result in negative entropy in some cases, which does not conform to the actual meaning of entropy. Zhou *et al*'s belief entropy performs well? in most cases and conform basic properties of uncertainty measures. However, its mathematical formula is not concise, and may lead to counter-intuitive result under some cases.

In order to solve the above problems and provide a more consistent and comprehensive uncertainty measure for *BPA*, this paper proposes an improved belief entropy to quantitatively express *BPA* uncertainty in DS evidence theory. The main contributions of this paper can be summarized as follows:

(1) This paper presents an uncertainty measure which takes into account the effect of the intersection between focal elements and uses $inverse - conflict$ $(D(A,B))$ to describe this relationship quantitatively. The proposed entropy satisfies several properties after amendment, and has better performance than other methods under several cases.

(2) The difference between the conditions that only one single element can be the answer to the problem and that one or several elements can be the answer to the

problem is analyzed. Moreover, the basic properties of uncertainty measures are discussed and amended accordingly, which extend the properties and be more applicable to the new conditions. The condition transforms from 'the final answer can only identify a single element of the framework' to 'the final answer can identify one or more elements of the framework at the same time' and in response to that question, The $Set - Consistency$ and $Range$ are also amended accordingly, which is also in line with subjective feelings and the actual situation.

The rest of this article is organized as follows: In Section 2, some basic principles of DS evidence theory and the entropy formula proposed by scholars are introduced. In Section 3, a new belief entropy is proposed to solve the problems of the existing belief entropy. The five basic principles proposed by Klir and wierman are discussed and validated in Section 4. In Section 5, some representative examples are given to prove the rationality of the proposed entropy compared with the entropy proposed by other scholars.

2. Preliminaries

2.1. Basic Concepts in Evidence Theory[1][2]

Let $X = \{H_1, H_2, ..., H_N\}$ be an exhaustive set of all possible values of a variable, and the elements in X are mutually exclusive, then X is the frame of discernment. Let X have N elements, then the power set of X is $P(X)$, the power set has 2^N elements, and each element corresponds to the proposition of a case of a variable.

$$P(X) = \{\emptyset, \{H_1\}, \{H_2\}, ..., \{H_N\}, \{H_1 \cup H_2\}, \{H_1 \cup H_3\} ...X\} \qquad (1)$$

For any subset A belonging to X, let it correspond to a number in the interval [0,1], and satisfy:

$$\sum_{A \in P(X)} m(A) = 1 \qquad (2)$$

$$m(\emptyset) = 0 \tag{3}$$

The function m is called the basic probability distribution function (BPA) on $P(X)$, and $m(A)$ is the basic probability of A. When $A \in X$ and $m(A) \neq 0$, A is called a focal element of m.

The belief function belief is defined as:

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{4}$$

The plausibility function Pl is defined as:

$$Pl(A) = \sum_{B \cap A \neq \emptyset} m(B) \tag{5}$$

2.2. Some Typical Entropies of BPA

In this paper, entropy before Deng and Entropy after Deng will be studied separately because in Deng's condition that one or several elements can be the answer to the problem, the influence of non-specificity is undoubtedly greater than it in previous research conditions, so this paper believes that it is necessary to distinguish it from the previously proposed research conditions. In this paper, the research premise of this kind of scenario is summarized as generalized condition, and the previous research condition is called traditional condition accordingly.

2.2.1. Entropises in Traditional Condition

Dubois & Prade's weighted Hartley entropy[4]:

$$I_{DP}(m) = \sum_{A \subseteq X} m(A) \log_2 |A| \tag{6}$$

Hohle's confusion measure[5]:

$$C_H(m) = -\sum_{A \subseteq X} m(A) \log_2 Bel(A) \tag{7}$$

Yager's dissonance measure[6]:

$$E_Y(m) = -\sum_{A \subseteq X} m(A) \log_2 Pl(A) \tag{8}$$

Klir & Ramer's discord[7]:

$$D_{KR}(m) = -\sum_{A \subseteq X} m(A) \log_2 \sum_{B \subseteq X} m(B) \frac{|A \cap B|}{|B|} \tag{9}$$

Klir & Parviz's strile[8]:

$$S_{KP}(m) = -\sum_{A \subseteq X} m(A) \log_2 \sum_{B \subseteq X} m(B) \frac{|A \cap B|}{|A|} \tag{10}$$

2.2.2. Entropies in Generalized Condition

Deng entropy[9]:

$$E_d(m) = -\sum_{A \subseteq X} m(A) \log_2 \frac{m(A)}{2^{|A|} - 1} \tag{11}$$

Cui *et al*'s belief entropy[10]:

$$E\,(m) = -\sum_{A \subseteq X} m\,(A) \log_2 \left(\frac{m\,(A)}{2^{|A|} - 1} e^{\sum\limits_{B \subseteq X, B \neq A} \frac{|A \cap B|}{2^{|X|} - 1}} \right) \tag{12}$$

Wang *et al*'s belief entropy[11]:

$$E_{Id}\,(m) = -\sum_{A \subseteq X} m\,(A) \log_2 \left(\frac{m\,(A)}{2^{|A|} - 1} e^{\sum\limits_{B \subseteq X, B \neq A} \frac{|A \cap B|}{|X|}} \right) \tag{13}$$

Zhou *et al*'s belief entropy[12]:

$$\tilde{E}(m) = \begin{cases} \frac{1}{2(|\mathfrak{B}|-1)} \sum\limits_{A_i \subseteq X} \sum\limits_{A_j \subseteq X} SC_{A_i,A_j} \cdot \left[-m(A_i) \log_2 \frac{m(A_i)}{2^{|A_i|}-1} - m(A_j) \log_2 \frac{m(A_j)}{2^{|A_j|}-1} \right], |\mathfrak{B}| \geq 2 \\ -m(A) \log_2 \frac{m(A)}{2^{|A|}-1}, |\mathfrak{B}| = 1 \end{cases} \tag{14}$$

3. Proposed New Belief Entropy

This paper studies the generalized condition. For a problem whose frame of discernment is X, the power set of all subsets in X may be the final answer to the problem, and there is no possibility beyond it. The information boundary for this problem is $\log_2\left(2^{|X|}-1\right)$. The maximum value of the Deng entropy, however, will exceed this information boundary.

This paper suggests that the possible reason is that the relationships between sets in BOE is not well-considered in Deng entropy.Assume that the frame of discernment is $X = \{1,2,3,4,5,6\}$ and we have tow BPAs, $m1$:$m(\{1,2,3\}) = 0.6$, $m(\{4,5,6\}) = 0.4$ and $m2$: $m(\{1,2,3\}) = 0.6$, $m(\{3,4,5\}) = 0.4$. $m1$ and $m2$ have the same non-specificity, but $m2$ excludes the distribution of the final result at $'6'$, so the entropy of $m2$ must be less than that of $m1$. In other words, there is a correlation between $\{1,2,3\}$ and $\{3,4,5\}$ at the element $'3'$, so they cannot be regarded as two completely unrelated cases, and thus there would be less uncertainty in $m2$ than that in $m1$. But contrary to expectations, if the Deng entropy is applied in $m2$, the same value with $m1$ will be obtained. It will inevitably lead to the wrong increase of the calculated $m2$ entropy if the correlation between $\{1,2,3\}$ and $\{3,4,5\}$ is not considered.

We can then see that Deng's variant entropy is designed for this problem.For example, Cui *et al*'s belief entropy and Wang *et al* 's belief entropy are both multiplied by a correction term related to the intersection in the logarithm term, thereby reducing the value of the Deng entropy; after simplifying Zhou *et al*'s belief entropy, it can be found that Zhou *et al*'s belief entropy subtracted a correction term related to intersection from Deng entropy, thereby reducing the value of Deng entropy. The entropy proposed in this paper also achieves the purpose of revising the entropy value by introducing the concept of inverse-conflict.

The proposed entropy is as follows:

$$E_J(m) = -\sum_{A \subseteq X} m(A) \log_2 \frac{\sum\limits_{B \subseteq X} m(B) \cdot D(A,B)}{2^{|A|} - 1} \tag{15}$$

Among them, $D(A,B)$ first comes from the concept of $Inverse-conflict$ proposed by Jousselme[14], which is used to quantitatively express the degree of correlation between set A and B. Its mathematical expression is as follows

$$D(A,B) = \frac{|A \cap B|}{|A \cup B|} \tag{16}$$

This theorem has three properties:

(a)$D(A,B) \leq 1$ and $D(A,B) = 1$ if and only if $A = B$.

(b)The "closer" to each other A and B are, the nearer $D(A,B)$ must be to unity.

(c)The "farther" from each other A and B are, the nearer $D(A,B)$ must be to zero.

4. Properties Of The Proposed Entropy

Klir and Wierman pointed out[13]: the function donated by aggregate uncertainty is supposed to capture,in an aggregate fashion, both $non-specificity$ and $conflict$-the two types of uncertainty that coexist in evidence theory.

In addition to qualify as a meaningful measure of aggregate uncertainty in evidence theory, function AU(aggregate uncertainty) must satisfy certain requirements that are generally considered essential on intuitive grounds.

4.1. *Probabilistic Consistency*

Definition: When all focus elements in a BOE are singletons, AU must be equal to Shannon entropy.

$$AU(m) = -\sum_{A \subseteq X} m(A) \log_2 m(A) \tag{17}$$

Proof of the proposed entropy: When all focus elements in a BOE are singletons, $\sum\limits_{B \subseteq X} m(B) \cdot D(A,B) = m(A)$ and $2^{|A|} - 1 = 1$. Thus, the proposed belief entropy can be reduced to

$$E_J(m) = -\sum_{A \subseteq X} m(A) \log_2 m(A) \tag{18}$$

4.2. *Set Consistency*

Definition: Whenever belief focuses on a single set $m(A) = 1$ for some $A \subseteq X$, AU assumes the form of the Hartley measure[4].

$$AU(m) = \log 2\,|A| \tag{19}$$

In the original text of Klir and Wierman, the proof of this property is as follows:

$m(A) = 1$ means that every probability distribution that sums to one for elements x in A and is zero for all x not in A is consistent with belief. And *Klir* argues that in this case AU should be equal to the maximization of the uncertainty in the set A, i.e. the uniform distribution of all possible outcomes in A. The traditional condition holds that the possible result must be the identification of a particular individual element in the frame, so Klir comes to this conclusion.

$$AU(m) = -\sum_{x \in A} \frac{1}{|A|} \log_2 \frac{1}{|A|} = \log_2 |A| \tag{20}$$

In the generalized condition envisioned by Deng, the maximization of the uncertainty of $m(A) = 1$ should be distributed in $2^{|A|} - 1$ subsets of A. In the previous example of 32 students who can take the first place, for an answer $m(A) = 1$, the possible result is that one person in A takes the first place, and two people in A tie for the first place... All $|A|$ individuals in A are tied for first place, so there are $2^{|A|} - 1$ cases. If it is evenly distributed over A to maximize

AU, there is

$$AU(m) = -\sum_{B\subseteq A} \frac{1}{2^{|A|}-1} \log_2 \frac{1}{2^{|A|}-1} = \log_2 \left(2^{|A|}-1\right) \tag{21}$$

Since the proof procedure of Klir and Wierman is not violated, the set consistency of a generalized condition is defined here as follows: Whenever belief focuses on a single set $m(A) = 1$ for some $A \subseteq X$, AU assumes the form as below:

$$AU(m) = \log_2 \left(2^{|A|}-1\right) \tag{22}$$

Proof of the proposed entropy:
When belief focuses on a single set $m(A) = 1$for some $A \subseteq X$, there is $\sum_{B\subseteq X} m(B) \cdot D(A,B) = 1$,

$$E_J(m) = \log_2 \left(2^{|A|}-1\right) \tag{23}$$

4.3. *Range*

Definition: The range of AU is $[0, \log_2 |X|]$ when belief is defined on $P(X)$ and AU is measured in bits.

This article's definition to Range in generalized condition is as follows: The range of AU is $[0, \log_2 |X|]$ when belief is defined on $P(X)$ and AU is measured in bits.

The range represents the number of bits of information boundary. In traditional conditions, the information boundary is $|X|$, which means there are $|X|$ possible results in total. Therefore, the range is $[0, \log_2 |X|]$. In generalized conditions, there are $2^{|A|}-1$ possible results. Thus, the information boundary is $\log_2 \left(2^{|A|}-1\right)$, and the range should be modified to $[0, \log_2 |X|]$. Proof of the proposed entropy:
Whenever belief focuses on sets As for any $A \subseteq X$, Because $0 \leq D(A,B) \leq 1$
Hence

$$\sum_{B\subseteq X} m(B) \cdot D(A,B) \leq \sum_{B\subseteq X} m(B) \tag{24}$$

$$\sum_{B\subseteq X} m(B)\cdot D(A,B) \leq 1 \tag{25}$$

Therefore

$$E_J \geq 0 \tag{26}$$

4.4. *Subadditivity* and *Additivity*

Some scholars proposed in their own papers the additive and sub-additive studies on existing belief entropy. For example, in analyzing properties of Deng entropy in the theory of evidence, Joaquin Abellan proposed that Deng entropy does not satisfy *subadditivity* and *additivity*.

This may be because most of the mathematical functions proposed by the existing belief entropy are for one-dimensional frame of discernment. *Additivity* and *subadditivity* are the theories proposed for the spatial structure of multidimensional frame of discernment, which require additional defined mathematical functions to express such problems. Therefore, this paper does not carry out research here.

4.5. Discussion

In this chapter, the basic properties of each belief entropy are summarized. As shown in Table 1, where TC represents a traditional condition and GC represents a generalized condition.

We can see from Table 1 that Dubois & Prade's entropy does not conform to *Probabilistic Consistency* and $Non-negativity$, Hohle's and Yager's entropies do not conform *Set Consistency* and $Non-negative$. Klir & Ramer's and Klir & Parviz's entropies do not conform *Set Consistency*, Deng entropy and Cui *et al*'s belief entropy do not conform *Maximum*, Wang *et al* 's belief entropy do not conform $Non-negativity$ and *Maximum*, Zhou *et al*'s entropy and the entropy proposed in this paper conform all the principles discussed above.

Table 1: Belief entropy analysis of five basic principles

	Probabilistic consistency	Set consistency	Range (Non-negativity)	Range (Maximum)
Dubois & Prade's	no	yes	no	yes(TC)
Hohle's	yes	no	no	yes(TC)
Yager's	yes	no	no	yes(TC)
Klir & Ramer's	yes	no	yes	yes(TC)
Klir & Parviz's	yes	no	yes	yes(TC)
Deng entropy	yes	yes(GC)	yes	no
Cui *et al*'s	yes	yes(GC)	yes	no
Wang *et al*'s	yes	yes(GC)	no	no
Zhou *et al*'s	yes	yes(GC)	yes	yes(GC)
Proposed	yes	yes(GC)	yes	yes(GC)

• *TC* represents a traditional condition and *GC* represents a generalized condition.

5. Comparative Analysis

In this subsection, the advantages of our improved entropy in measuring uncertainty with intersections in focal elements can be revealed by comparing it with other existing entropy functions based on several numerical examples.

5.1. Example 1

Let the frame of discernment be $X = \{1, 2, \cdots, 15\}$, and a BPA is given as $m(\{3, 4, 5\}) = 0.05$, $m(\{6\}) = 0.05$, $m(A) = 0.8$, $m(X) = 0.1$, where proposition A is a variable that varies from $A = \{1\}$ to $A = \{1, 2, \cdots, 14\}$.

This example is used to test the non-specificity of belief entropies common sense tells us that as the cardinality of A increases, the uncertainty of m increases; *i.e.*, the entropy increases. Now, we examine whether the uncertainty measures of various entropy functions for m conform to this expectation. Since the study has been divided into traditional condition and generalized condition previously, the ranges of the two conditions are marked with dotted lines respectively in Figure 1.

It is shown in Figure 1 that all images of entropies are within a range of their own conditions. However, some function images have a downward trend with the increase of size of A like Hohle's, Yager's, Klir & Ramer's, Klir & Parviz's,

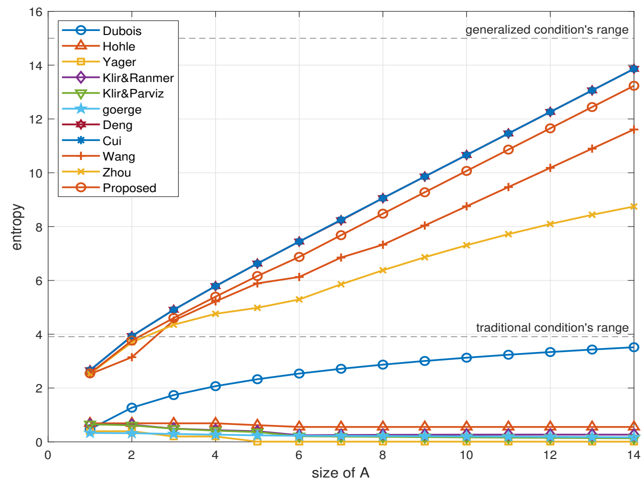

Figure 1: Comparison between proposed entropy and other uncertainty measures

Dubois, which is obviously unreasonable. The function image of Dubois & Prade's entropy have a increase trend, eventually approaching the traditional condition's range. Here we argue that Dubois& Prade's weighted Hartley entropy is an effective way to express non-specificy in traditional conditions. In addition, Deng entropy and all his variants showed a good increase trend, because they all use the form of $2^{|A|}-1$ to express non-specificity.

5.2. Example 2

Let the frame of discernment be $X=\{1,2,3,4,5,6\}$, and a BPA is given as $m1$:$m(\{1,2,3\})=0.4$, $m(\{4,5,6\})=0.6$; $m2$:$m(\{1,2,3\})=0.4$, $m(\{3,4,5\})=0.6$; $m3$:$m(\{1,2,3\})=0.4$, $m(\{2,3,4\})=0.6$. Common sense tells us that $m2$ has more information on 3 and 6 compared to $m1$, therefore, the uncertainty of $m2$ should be lower than $m1$. Similarly, the uncertainty of $m3$ should be lower than that of $m2$. Now, we examine whether the uncertainty measures of various entropy functions for m conform to this expectation.

As can be seen from Table 2, The reduction of uncertainty from $m1$ to $m2$ to $m3$ can not be represented by some entropies like Dubois & Prade's, Hohle's and Deng's entropies. Yager's entropy value even reduce to 0 when BPA is

Table 2: Belief entropy values in the case study

	m1	m2	m3
Dubois & Prade's	1.5850	1.5850	1.5850
Hohle's	0.9710	0.9710	0.9710
Yager's	0.9710	0	0
Klir & Ramer's	0.9710	0.5633	0.2526
Klir & Parviz's	0.9710	0.5633	0.2526
Deng entropy	3.7783	3.7783	3.7783
Cui *et al*'s	3.7783	3.7554	3.7325
Wang *et al*'s	3.7783	3.5379	3.2974
Zhou *et al*'s	3.7783	3.0226	1.8892
Proposed	3.7783	3.5186	3.2063

given as $m2$ and $m3$. The entropy value is equal to 0 only when there is no uncertainty, that is, the result is certain but $m2$ and $m3$ do not fit this case.

5.3. Example 3

We assume in the frame of discernment for $X = \{1, 2, 3, 4, 5\}$, the entropy of $m(\{1, 2, 3, 4\}) = 1$ is obviously less than the entropy of $m(X) = 1$. If belief is assigned to the two sets like $m(\{1, 2, 3, 4\}) = 0.5$ and $m(X) = 0.5$, we can assume that its entropy would be less than $m(X) = 1$ which means blank information, and greater than $m(\{1, 2, 3, 4\}) = 1$, because the latter has less uncertainty in non-specificity and conflict. To put it another way, when the belief assigned to the X reduces from 1 to 0, we will become increasingly convinced that $'5'$ is not the answer, which means we have more information to eliminate uncertainty. Let $m(X) = a$ and $m(\{1, 2, 3, 4\}) = 1 - a$, and calculated when a increasing from 0 to 1, the change trend of entropies, as shown in Figure 2

The four dotted lines in Figure 2 correspond to the values $log_2(4)$, $log_2(5)$, $log_2(15)$, and $log_2(31)$, respectively. The values of the four dotted lines are $Set-Consistency$ for the values specified in the traditional condition and generalized condition when $m(1)$ or $m(2)$ equals 1, respectively. The function images of Cui *et al*'s belief entropy and Deng entropy completely overlap. Looking at Dubois, it can be found that the values at both ends of the function image are exactly equal to $log_2(4)$ and $log_2(5)$, respectively, and when $0 < a < 1$, the function

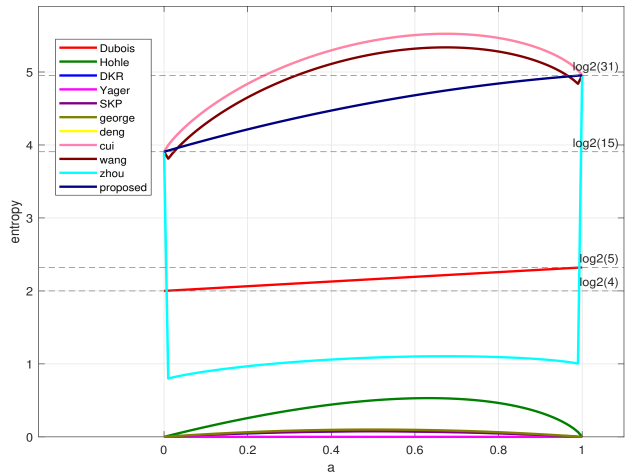

Figure 2: Entropies as functions of $m(X) = a$

image monotonically increases, which also supports the rationality of Dubois's non-specific expression of entropy. Hohle's, Yager's, Klir & Ramer's and Klir & Parviz's are completely outside $[log_2(4), log_2(5)]$, which is not consistent with the above analysis. The values of Deng entropy and its variants when $a = 0$ and $a = 1$ are all equal to $log_2(15)$ and $log_2(31)$respectively, which is consistent with the prediction in this paper. Zhou *et al*'s entropy drops suddenly when $0 < a < 1$. When a is in a certain interval, Deng entropy Cui *et al*'s belief entropy and Wang *et al* 's belief entropy can exceed $log_2(31)$, which is also a manifestation that these belief entropy formulas do not conform to the Maximum of *Range*. In this example, the value of the proposed belief entropy increases monotonically as a increases and never goes beyond the range of $[log_2(15), log_2(31)]$.

6. Conclusion

This paper presents an improved Deng entropy for measuring the uncertainty of mass function in evidence theory. Based on the Deng's non-specificity measure of entropy and Jousselme's method of expressing set similarity. Then, the rationality of the proposed method is demonstrated by discuss the five basic principles proposed by Klir and Wieman and analyzing three examples. The entropy proposed in this paper, retains the excellent characteristics of Deng entropy and can better describe the influence of intersections in focal element on uncertainty.

A major innovation of this paper is that it emphasizes the difference between the scenario hypothesis proposed by Deng and the condition studied by previous scholars. In this paper, they are called generalized condition and traditional condition respectively. This paper explains the reasons for the differences between the two conditions, and argues that the two conditions should be studied separately and not mixed together.

The entropy proposed in this paper needs further verification and research in theory, and also needs further exploration in practice.

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