A Modified Born-Infeld Model of Electrons with Intrinsic Angular Momentum

Martin Kraus (kraus.martin@gmail.com)

March 1, 2024

Abstract

This work analyzes a recently reported numerical solution to a modified Born-Infeld model of electrons by computing its energy, momentum, invariant mass, and intrinsic angular momentum. While the invariant mass is used to improve the model's Born-Infeld parameter, the computed energy and intrinsic angular momentum provide new insights into the model. Specifically, the computed energy is negative, which might be a consequence of the model describing a bound state in the form of a massive particle. The computed intrinsic angular momentum agrees with the spin of electrons within the accuracy of the numerical approximations; however, the actual predictive power of the model remains unclear because one of the parameters of the model is the reduced Compton wavelength of electrons.

1 Introduction

Born and Infeld proposed a classical field theory, which considers particles of matter "as singularities of the field" and where "mass is a derived notion to be expressed by field energy (electromagnetic mass)" [BIF34]. In the same publication, they proposed a model of electron-like particles without magnetic moment [BIF34]. This model was later modified [Kra23] to include a realistic magnetic moment and an internal clock, which was hypothesized by de Broglie [dB25]. Section 2 provides a brief review of this modified model.

In this work, the reported numerical field solution of the modified model [Kra23] is analyzed by computing its canonical stress-energy tensor (see Section 3) and quantities that can be derived from it; namely the solution's total energy, total momentum, invariant mass (see Section 4), and total intrinsic angular momentum (see Section 5).

The invariant mass is used to improve the estimated Born-Infeld parameter of the model by iteratively adjusting it until the computed invariant mass of the model matches the observed invariant mass of electrons. Section 6 discusses results based on the improved Born-Infeld parameter. In particular, the computed total energy is negative, which might be surprising for a field energy that—for low energies—approximates the standard (positive-definite) electromagnetic field energy. One interpretation of this negative energy—or, more precisely, negative energy level—is that it describes a bound state that is energetically more stable than states of positive energy level. Lastly, the computed value of the total intrinsic angular momentum of the rotating field solution does match the spin of electrons within the accuracy of the numerical model.

As discussed in Section 6, these numerical results provide further evidence of the consistency of the model with real electrons. However, the predictive power of the model remains unclear; specifically, because the Planck constant enters the model as a parameter in the form of the reduced Compton wavelength of electrons.

2 Modified Born-Infeld Field Theory

Here, the Lagrangian density $\mathscr{L}$ of the modified Born-Infeld field theory is defined in SI units as

$$\mathscr{L} \stackrel{\text{def}}{=} \frac{b^2}{\mu_0}\left(1 - \sqrt{1 - \frac{1}{b^2}(\partial^\mu A^\nu)(\partial_\mu A_\nu)}\right), \tag{1}$$

which differs from previous work [Kra23] by an overall factor of $-b^2/\mu_0$ with the Born-Infeld parameter b specifying the maximum magnetic field strength and the vacuum permeability μ_0. The motivation for this choice is that this Lagrangian density approximates the standard electromagnetic Lagrangian density for low-energy, electrostatic fields. As in previous work [Kra23], basic Ricci calculus is used as well as the Minkowski metric tensor η in the form $\mathrm{diag}(+1, -1, -1, -1)$, and the electromagnetic four-potential $(A^0, A^1, A^2, A^3) = (\phi/c, A_x, A_y, A_z)$, which is also used to define electric field strength $\mathbf{E}$ as:

$$\mathbf{E} \stackrel{\text{def}}{=} -\nabla\phi - \frac{\partial}{\partial t}\mathbf{A}. \tag{2}$$

Magnetic field strength $\mathbf{B}$ is defined as

$$\mathbf{B} \stackrel{\text{def}}{=} \nabla \times \mathbf{A}. \tag{3}$$

More details about the notation are provided in previous work [Kra23] and references therein.

For the modified Born-Infeld model of electrons, the corresponding Euler-Lagrange equations were solved numerically in previous work [Kra23] resulting in a rotating field solution with a peak moving at the speed of light on a circular orbit with a radius equal to an electron's reduced Compton wavelength. While most features of electrons (electric charge, magnetic moment, Compton frequency) were imposed on the solution, the total field energy of the solution was (somewhat naively) approximated by the volume integral over $(|\mathbf{E}|^2/c^2 + |\mathbf{B}|^2)/(2\mu_0)$ and matched to the rest mass energy of an electron by adjusting the Born-Infeld parameter. Part of the motivation for the present work was to apply a more appropriate definition of field energy.

3 Stress-Energy Tensor

Preferably, the field energy that specifies the "electromagnetic mass" of a particle model should be a Lorentz invariant that is conserved over time. To this end, the stress-energy tensor is computed in this section, and an invariant mass is derived from its components in the next section.

Specifically, the canonical stress-energy tensor $T^{\mu\nu}$ is defined in this work as

$$T^{\mu\nu} \stackrel{\text{def}}{=} \frac{\partial \mathscr{L}}{\partial(\partial_\mu A_\gamma)}(\partial^\nu A_\gamma) - \delta^{\mu\nu}\mathscr{L} \tag{4}$$

such that it satisfies the four continuity equations

$$\partial_\mu T^{\mu\nu} = 0. \tag{5}$$

Inserting the definition of the Lagrangian density $\mathscr{L}$ into the definition of $T^{\mu\nu}$ leads to:

$$T^{\mu\nu} = \frac{b^2}{\mu_0}\left(\frac{\frac{1}{b^2}(\partial^\mu A^\gamma)(\partial^\nu A_\gamma) + \delta^{\mu\nu}\left(1 - \frac{1}{b^2}(\partial^\alpha A^\beta)(\partial_\alpha A_\beta)\right)}{\sqrt{1 - \frac{1}{b^2}(\partial^\alpha A^\beta)(\partial_\alpha A_\beta)}} - \delta^{\mu\nu}\right) \tag{6}$$

In this form, it becomes clear that the canonical stress-energy tensor $T^{\mu\nu}$ for $\mathscr{L}$ is symmetric. Note that this is not the case for the Lagrangian density of standard electromagnetism, which is a complication that may be cured by employing the Belinfante-Rosenfeld stress-energy tensor or the (in that case identical) Hilbert stress-energy tensor instead of the canonical stress-energy tensor [Bel40, Ros40].

Numerically evaluating $T^{\mu\nu}$ at a specific point in space-time is straightforward in a framework such as described previously [Kra23] since the square root in the denominator has to be evaluated already for the field equations and all other terms are either constant (for specific values of μ and ν) or partial derivatives of the form $\partial^\alpha A^\beta$, which are part of the mentioned square root. Numerically integrating $\int T^{\mu\nu}\mathrm{d}^3\mathbf{x}$ works analogously to integrating the energy density $(|\mathbf{E}|^2/c^2 + |\mathbf{B}|^2)/(2\mu_0)$ as described previously [Kra23].

4 Energy, Momentum, and Invariant Mass

The component T^{00} of the stress-energy tensor may be considered a field energy density, such that the volume integral $\int T^{00}\mathrm{d}^3\mathbf{x}$ represents the total energy of the system at a specific point in time. More precisely spoken, it is an energy level *relative* to a state with energy level 0, for example, an unbound electromagnetic wave of infinitesimal intensity. Therefore (and to distinguish it from an electric field strength), this energy level is denoted by ΔE with the definition

$$\Delta E \stackrel{\text{def}}{=} \int T^{00} \mathrm{d}^3\mathbf{x}. \tag{7}$$

For the numerical field solution representing an electron [Kra23], values of T^{00} far from the peak of the solution are positive (as expected based on the analogy to standard electromagnetism); however, values of T^{00} within femtometers of the peak are negative and dominate the volume integral such that ΔE is also negative. As discussed in Section 6, this might suggest that the model's field solution represents a bound state.

The vector formed by T^{01}, T^{02}, and T^{03} (or T^{10}, T^{20}, and T^{30}) of the (symmetric) stress-energy tensor may be considered a momentum density multiplied by c. Therefore, the volume integral over this vector divided by c may be considered the total momentum of the system at a specific point in time, which is denoted by $\mathbf{p} = (p_x, p_y, p_z)$ with the definition

$$\mathbf{p} \stackrel{\text{def}}{=} \frac{1}{c} \left(\int T^{01} \mathrm{d}^3\mathbf{x}, \int T^{02} \mathrm{d}^3\mathbf{x}, \int T^{03} \mathrm{d}^3\mathbf{x} \right). \tag{8}$$

In the case of the rotating field solution representing an electron [Kra23], $|\mathbf{p}|$ is approximately $1.4 \times 10^{-22}\,\mathrm{kg\,m/s}$. However, $\mathbf{p}$ is rotating (with the Compton frequency) such that its time average approximates 0 (provided that the rotation center of the field solution is at rest).

Furthermore, the energy level ΔE and the momentum $\mathbf{p}$ form a four-momentum ($\Delta E/c$, p_x, p_y, p_z) with the Lorentz-invariant Minkowski norm $m_0 c$, where the invariant mass m_0 is defined this way:

$$m_0 \stackrel{\text{def}}{=} \frac{1}{c} \sqrt{\Delta E^2/c^2 - p_x^2 - p_y^2 - p_z^2}. \tag{9}$$

Thus, it is straightforward to compute m_0 from ΔE and $\mathbf{p}$ for a given field solution. This allows us to adjust the Born-Infeld parameter b such that m_0 matches the experimentally observed invariant mass of electrons. In comparison to previous work [Kra23], this increased the estimated value for $b \times c$ from $5.0 \times 10^{22}\,\mathrm{V/m}$ to $5.3 \times 10^{22}\,\mathrm{V/m}$. This value of b was also used to compute the result of the next section.

5 Intrinsic Angular Momentum

The total intrinsic angular momentum $\mathbf{s} = (s_x, s_y, s_z)$ of field solutions rotating about the origin of an xyz-coordinate system is defined here as

$$\mathbf{s} \stackrel{\text{def}}{=} \frac{1}{c} \left(\int \left(y\,T^{03} - z\,T^{02} \right) \mathrm{d}^3\mathbf{x}, \int \left(z\,T^{01} - x\,T^{03} \right) \mathrm{d}^3\mathbf{x}, \int \left(x\,T^{02} - y\,T^{01} \right) \mathrm{d}^3\mathbf{x} \right). \tag{10}$$

Thus, given the values of T^{01}, T^{02}, and T^{03} at each point of the field, it is straightforward to compute the total intrinsic angular momentum of the field. For the rotating field solution representing an electron [Kra23], the result is $5.3 \times 10^{-35}\,\mathrm{J\,s} \approx \hbar/2$ as expected for an electron. The predictive power of this result is discussed in the next section.

6 Discussion

Two results of this work stand out: a negative value of ΔE and a good approximation for the spin of electrons in a classical field theory. This section discusses these two results in more detail.

A negative value of ΔE would not be possible in standard electromagnetism because T^{00} of the standard electromagnetic stress-energy tensor is positive definite. On the other hand, negative energy levels of bound states, e.g., an electron orbiting a proton, are very common in physical models. At the time of writing, it is unclear whether or in how far a negative value of ΔE implies the existence of a bound state, but the possibility is certainly tantalizing. If true, the negative value of ΔE might even imply that the modified Born-Infeld model analyzed in this work describes an energetically more stable electron than the original Born-Infeld model ever could.

The agreement between the computed value of the model's intrinsic angular momentum and the (explicitly quantum-mechanical) value of $\hbar/2$ requires some context: the field solution is constructed such that most of its mass and momentum is concentrated in a peak that is moving with the speed of light on an orbit with a radius equal to the reduced Compton wavelength $\hbar/(m_0 c)$. Approximating its intrinsic angular momentum $|\mathbf{s}|$ by $r\,|\mathbf{p}|$ with radius $r = \hbar/(m_0 c)$ shows that the value of $\hbar$ enters the model as a parameter via the reduced Compton wavelength. Nonetheless, it is still remarkable that the computed total momentum $|\mathbf{p}|$ is in fact very close to $m_0 c/2$ such that the model's intrinsic angular momentum is consistent with the spin of real electrons. However, it does not mean that this classical field theory can predict the value of $\hbar$.

7 Conclusion

In this work, the rotating field solution of a modified Born-Infeld model of electrons was analyzed by computing its canonical stress-energy tensor and derived quantities; specifically, energy, momentum, invariant mass and intrinsic angular momentum. This led to a new estimate for the value of the Born-Infeld parameter, and a computed intrinsic angular momentum that is in good agreement with an electron's spin. This raises (but does not answer) the question of the predictive power of the model. Another open question is whether the negative total field energy is related to the existence of a bound state of the field.

References

[Bel40] Frederik Jozef Belinfante. On the current and the density of the electric charge, the energy, the linear momentum and the angular momentum of arbitrary fields. *Physica*, 7(5):449–474, 1940.

[BIF34] Max Born, Leopold Infeld, and Ralph Howard Fowler. Foundations of the new field theory. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 144(852):425–451, 1934.

[dB25] Louis de Broglie. Recherches sur la théorie des Quanta. *Annales de Physique*, 10(3):22–128, January 1925. English translation by A. F. Kracklauer.

[Kra23] Martin Kraus. A modified Born-Infeld model of electrons and a numerical solution procedure. viXra:2312.0149, 2023.

[Ros40] Léon Rosenfeld. *Sur le tenseur d'impulsion-énergie*. Académie royale de Belgique. Classe des sciences. Mémoires. Collection in-8°. Tome 18. Fasc. 6. G. Thone, 1940.

A Revisions

- Original version submitted to vixra.org on March 1, 2024.