NAVIER–STOKES EQUATIONS ARE NEITHER FIRST NOR SECOND ORDER APPROXIMATION

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Abstract. One of the Millennium Problems is fluid movement. If the movement is lightspeed-like, then we take Einstein's Relativity into account. If not, then we don't need complicated Navier-Stokes formulas. They are not a first-order approximation at low speeds. However, they has to be first-order approximation since they are meant to be classical.

MSC Class: 35Q30, 00A27, 00A05, 00A30, 03F40, 03F99, 00A35.

1. Introduction

To cite an Encyclopedia of 2023 AD: "*Since understanding the Navier–Stokes equations* [1] *is considered the first step to understanding the elusive phenomenon of turbulence, the Clay Mathematics Institute in May 2000 made this problem one of its seven Millennium Prize problems in mathematics. It offered a prize to the first person providing a solution for a specific statement of the problem: Prove or give a counter-example of the following statement: In three space dimensions and time, given an initial velocity field, there exists a vector velocity and a scalar pressure field, which are both smooth and globally defined, that solve the Navier–Stokes equations.*"

The above formulation of Navier–Stokes problem has terms from Physics: velocity (in the following, $\vec{u}$), space (in the following, coordinate vector $\vec{r}$), time t, and pressure (in the following, the influence of pressure is hidden within $\vec{f}$). The density field is ρ. Therefore, having contradictions with the Physical picture, I have found countless counter-examples against these equations

The Navier-Stokes equations are [1]

$$\rho(\vec{r},t)\left(\frac{\partial \vec{u}(\vec{r},t)}{\partial t} + \vec{u}\,\nabla\vec{u}\right) = \vec{f}(\vec{r},t)\,, \tag{1}$$

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where

$$\nabla\vec{u} \equiv \frac{\partial\vec{u}(\vec{r},t)}{\partial\,\vec{r}}\,, \tag{2}$$

The second-order term is $\vec{u}\,\nabla\vec{u}$, since the u is written twice. But since we need a first-order equation, this term has to be deleted. Moreover, this term causes the violation of the Galilean Relativity Principle: if you replace velocity $\vec{u}$ with $\vec{U} + \vec{C}$, where $\vec{C}$ is the transformation constant velocity, then you have term $\vec{C}\,\nabla\vec{U} \neq 0$ left. It means that transformed

$$\rho(\vec{r},t)\left(\frac{\partial\vec{U}(\vec{r},t)}{\partial t} + \vec{U}\,\nabla\vec{U} + \vec{C}\,\nabla\vec{U}\right) = \vec{f}(\vec{r},t) \tag{3}$$

does not match Eq. (1) due to $\vec{C}\,\nabla\vec{U} \neq 0$.

This means that all solutions of Navier-Stokes equations with $\vec{u}\,\nabla\vec{u} \neq 0$ are counter-examples against the theory of the Navier-Stokes equations.

Galilean invariance or Galilean relativity states that the laws of motion are the same in all inertial frames of reference. Galileo Galilei first described this principle in 1632 in his Dialogue Concerning the Two Chief World Systems using the example of a ship travelling at constant velocity, without rocking, on a smooth sea; any observer below the deck would not be able to tell whether the ship was moving or stationary.

Because Galilean relativity is a first approximation of Einstein's relativity, then classical matter (i.e., non-relativistic) must observe the Galilean principle. This term in the Navier-Stokes theory, which is not linear relative to speed, belongs to the second approach (relative to low speed). Since the remaining second-order terms are absent in the N-S theory, N-S theory is not a second-order approach. The N-S theory is also not classical (since one second-order term is inside).

References

[1] Navier. Mémoire sur les lois du mouvement des fluides. Mémoires de l'Académie des sciences de l'Institut de France. 1822. Vol. 6; Stokes. On the theories of internal friction of fluids in motion, and of the equilibrium and motion of elastic solids. Transactions of the Cambridge Philosophical Society. 1845. Vol. 8.