A New Attempt to Check Whether Ramanujan's Formula $\pi^4 \approx 97.5 - 1/11$ is a Part of Some Completely Accurate Formula

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Abstract

Intuitively, it seems that Ramanujan's formula $\pi^4 \approx 97.5 - 1/11$ is an approximation for some perfectly accurate formula for π. Here is one attempt to prove this. The principle of proof, however, is based on closeness of the every rest term to the inverse of integers. Although it is indeed somewhat closer to integers than it is on average, this proof is not complete. So we cannot say for sure whether this proves or disproves that this Ramanujan's formula has higher approximations; however, it gives hints and opens up space for further research. Moreover, this attempted proof is quite original. Also, such a method could also help in physics.

1 Introduction

I came across Ramanujan's formula [1–3]

$$\pi^4 \approx 97.5 - 1/11. \tag{1}$$

The formula is very simple and very accurate, maybe even the most simple and accurate among the formulas that are approximate for π and which are not supposed to be part of the completely exact formula for π. With such a precise and simple formula, intuition hints us that it is only a part of some completely exact formula for π.

Similarly, our intuition hints at some physical formulas, such as Eq. (8) in Ref. [4]. It would be well to check such intuitions, i.e. in general it would be well to develop some system where we would estimate for both such types of formulas what these probabilities are, [3]. Physical formulas have a disadvantage because the measurements of physical constants are limited in accuracy, but this disadvantage does not exist with mathematical constants such as π. So, for the sake of the development of such a system, it would be good to find out for the above formula if either it is just random, or it is a part of some completely accurate formula for π.

I asked an expert on Ramanujan, and he said that to the best of his knowledge of this Ramanujan mathematics, Eq. (1) is not a part of a perfectly precise formula. But such experts should have said more here.

However, I undertook some such analysis, which is described below. The approach is statistical. It is not exact, but it indicates the way how this could be proved.

2 The first calculation

I am trying to find a continuation with the following formula

$$22\pi^4 = Z_0 + 1/363893\bullet918594827538226091114823084696230459280615070 0... \tag{2}$$

where $Z_0 = 2143 = 22 \times (97.5 - 1/11)$, so Equation "$22\pi^4 \approx Z_0$" is only Eq. (1). (Sign $\bullet$ means decimal point, so it is easier to notice it. This will be used where necessary.)

I noticed that the denominator is close to an integer. The deviation is only:

$o_1 = -0.081405172461773908885176915303769540719384929939696468590477263...$

So I make the next step of this formula

$22\pi^4 = Z_0{+}1/Z_1{+}1/1626663387702\bullet140856794990826442569779214847497211534792897518343030620...$

Where $Z_1 = 363894$, so here we round to an integer and so on.

Note: The improvements follow this pattern throughout the document.

Now, the deviation from integer is:

$o_2 = 0.140856794990826442569779214847497211534792897518343030620...$

And the next step is

$22\pi^4 = Z_0 + 1/Z_1 + 1/Z_2 - 1/187852760462333684369632 09\bullet86446262128910816644904993895658...$

Where $Z_2 = 1626663387702$.

Now, the deviation from integer is:

$o_3 = 0.135537378710891833550950061043417161292349747104890748783280236 1...$

And the next step is

$22\pi^4 = Z_0 + 1/Z_1 + 1/Z_2 + 1/Z_3 -$
$1/260361089678378941981412618163377292566171432249389 5\bullet053407...$

where $Z_3 = -18785276046233368436963210$.

Now, the deviation from integer is:

$o_4 = -0.0534078448001961428979052813476865592761075319257260691219214748 14$
$905734963456215869206398823293641457491071680355917077695678778466613401$
$377559077525823684969399090887435202593095786415152643 5...$

And the next step is

$22\pi^4 = Z_0 + 1/Z_1 + 1/Z_2 + 1/Z_3 + 1/Z_4$
$+ 1/126924981287138415481675593868682369933942128756399303728126677115953896 75$
$4998173958397444503914207333464 9\bullet787697293921722272732341176200284509874558059 4$
$361373929164017018523152400443644986994137823574428001837713568414466380088014$
$483678474079202720140258511101642923743604874980621796322212395086306990789431$
$511793982278206486245046207802396925853004281318326783455832771560754753614604$
$112873677491981111808388279558158789438557781483049691646732221783083736376878$
$220907737918950880113219644971774668972970734312 8...$

where $Z_4 = -260361089678378941981412618163377292566171432249389 5$.

Now, the deviation from integer is:

$o_5 = -0.21230270607827772726765882379971549012544194056386260708359 82981...$

And the next step is

$22\pi^4 = Z_0 + 1/Z_1 + 1/Z_2 + 1/Z_3 + 1/Z_4 + 1/Z_5$
$+ 1/7588198554944734228608449194765350755037135170454562852075574028370 8062463$
0805556423188116825068547128100807174352595050980602118788520245372302489 79738
09510634854315455312116270359360859806755297674477226048 0•829528086133692 56371
4383672429098496940327258854005071430393493603588846315508992752161216019 30517
2043402804255751409175653295126176612625884511507009671014518842000171947 22222
7607305251214755238999294902447965121738536649102277673781859531296898898 61689
0397697940681131975554521332476575870259212764463842257972461404058894085 75073
5427831512290562...
where $Z_5 = 1269249812871384154816755938686823699339421287563993037281266771 1595$
38967549981739583974445039142073 34650.

The deviation from integer is

$o_6 = -0.170471913866307436285616327570901503059672741145994928569606 50639$
641115368449100724783878398069482795659719574424859082434 6705...

And the next step is

$22\pi^4 = Z_0 + 1/Z_1 + 1/Z_2 + 1/Z_3 + 1/Z_4 + 1/Z_5 + 1/Z_6$
$+ 1/3377726923065053984138453836446897293028629633132074891719442894910 9122551$
6144763231121328382962277166594718697774314193803966847942384975670062033 87446
2838233397538367853112867558302658415046857860683850558652945400325728033 5631
4352754736983532588358777107051663471136064566156756006566762887722282270 6021
5845409099794670494336386727653485623462124264855519018070858372448943859 0530
852243133058122237128932305362892908•2150722135423029860882021904868899709 609
4027244954367213571548963813247676014377738623818978442871114303887225938 9117
2399043321470483025914814970882230787313236248466041453450641979411638299 9938
2809024014809112646392854618166462271062414127606016417420152899339283849 4345
696164870800578780816538089809135448843228698964 4048...
Where $Z_6 = 7588198554944734228608449194765350755037135170454562852075574 02837$
0806246308055564231881168250685471281008071743525950509806021187885202453 7230
2489797380951063485431545531211627035936085980675529767447722604 81.

The deviation from integer is:

$o_7 = 0.2150722135423029860882021904868899709609402724495436721357154 89638132$
4767601437773862381897844287111430388722593891172399043321470483025914 8149
7088223078731323624846604145345064197941163829999382809024014809112646 3928
5461816646227106241412760601641742015289933928384943...

The next step is:

$22\pi^4 = Z_0 + 1/Z_1 + 1/Z_2 + 1/Z_3 + 1/Z_4 + 1/Z_5 + 1/Z_6 + 1/Z_7$
$- 5304748102457433565472294347977546279471809171040973527125537 96001475$
0320382692150792679451353255129484946855059194076660529202020436853080 8943
0122732417030718105167917913930168734962632929205967841245316068150900 0019
2796761666797026151313360895774972665977659086946888103899934323675113 5668
4407565365385053277206061769539035803886743043532435700449631275150252 0095
0202020312287395291839233914283925501119047727905277922193507033910461 2294
9259450280056456176505165777466042052836195406783963444788956278088210 0828

4311112141613491759718647838363498423266221733653762475827179585339355987 4
3366078638232181411981722617946289584316830517509437576705727543153540802 5
3089460072408564995526463562319481429074132911205251440573345160041688110 4
2131828279467325164798606272366107648913938448812032119828125092204430703 1
1298508884901331541390364198 3●388427806753183053943518736668419045021663 31
8273077149020935510459253515151256704310254358595157683958754305993450771 2
6885558907927560981864671255829768313116745642151288841392255743151411300 3
7591313970047064916843831977180762451014923545571231629924263037810853867 5
0013387095316384808793650585214758054787565536503986169740717708073368327 5
8080362198016991501433395984429538586120392643742296322662946028659047237 8
9510009070258227401884995024411681616238770757992939855460509681771584390 4
7167042571018217114724530296291022604458827173225260949794098531616163911 3
6515524497308351383553282243461654201885447588335826306176592446870404325 9
2942032900432319492244426894068390634464450391250244663232625443850693768 7
7192123888343360939191785165073489328544910504496879885015248158367216407 1
8920993524323070627872472656371676019576040094050376146922573493946704373 0
2889975801174214791502014380006087491277249499428863084702382963325061987 8
8524128861915658099508955302083820520223563039309876365235921491012675541 8
686362420882626235491215399741...
Where $Z_7 =$ 3377726923065053984138453836446897293028629633132074891719442894 91
0912255161447632311213283829622771665947186977743141938039668479423849756 7006
2033874462838233397538367853112867558302658415046857860683850558652945400 3257
2803356314352754736983532588358777107051663471136064566156756006566762887 7222
8227060215845409099794670494336386727653485623462124264855519018070858372 4489
43859053085224313305812223712893230536289 2908.

The deviation from integer is:

$o_8 =$
$-$ 0.38842780675318305394351873666841904502166331827307714902093551045 9253
5151512567043102543585951576839587543059934507712688555890792756098 1864
6712558297683131167456421512888413922557431514113003759131397004706 4916
8438319771807624510149235455712316299242630378108538675001338709531 6384
8087936505852147580547875655365039861697407177080733683275808036219 8016
99150143339598442953858612039264374229 63226...

Where $Z_8 =$
$-$ 53047481024574335654722943479775462794718091710409735271255379600 1475
0320382692150792679451353255129484946855059194076660529202020436853080 8943
0122732417030718105167917913930168734962632929205967841245316068150900 0019
2796761666797026151313360895774972665977659086946888103899934323675113 5668
4407565365385053277206061769539035803886743043532435700449631275150252 0095
0202020312287395291839233914283925501119047727905277922193507033910461 2294
9259450280056456176505165777466042052836195406783963444788956278088210 0828
4311112141613491759718647838363498423266221733653762475827179585339355 9874
3366078638232181411981722617946289584316830517509437576705727543153540 8025
3089460072408564995526463562319481429074132911205251440573345160041688 1104
2131828279467325164798606272366107648913938448812032119828125092204430 7031
12985088849013315413903641983, etc.

3 Analysis of results of o_i

We are mainly interested here in analysis of the values of o_i, although maybe in future the correlations among the values of Z_i could also be interesting for analysis. If the values of o_i were random, then their average of $|o_i|$ should be close to 0.25 for a large number of o_is. Here comes the question of how to evaluate deviation from random distribution of o_is. Namely, it is interesting that o_1 to o_7 are all smaller than 0,25. I simply ended here when $o_8 > 0.25$, because I assume that once $o_8 > 0.25$ happens, then the values of o_i at $i > 8$ will no longer be very small, so the probability of randomness, p, will no longer decrease.[1] However, there is a question as to what exactly would be the best statistical calculation of p.

Namely, there are several options, and the essential ones are:

1. This series may actually consist only of terms $1/Z_i$. (Where any Z_i is an integer, except for 0. i means a counter.)
2. $1/Z_i$ can be a good approximation only at the beginning, then not anymore.
3. It can be some other formula independent of $1/Z_i$.
4. This formula may not exist at all, and o_is are just random.

Now it is necessary to make such analysis, that it will find probability for every one of such options. In these calculations, o_1 should not be taken into account in principle, because we obtained this hypothesis with its help. (But this is not 100% sure, this can also be an indicator that the exact background formula exists.) As one estimate, I chose to calculate the average of $|o_2|$ to $|o_8|$, which is 0.188011..., and simulated the probability that such a low value is just a coincidence in these 7 locations, which is, in other words, the calculation of the p value, and it gives $p \approx 0.15$. However, this is really just the beginning of a model that maybe would be calculated correctly once in future. But we do not know how to choose these intervals.

For every option of 1 to 3 it is difficult to choose the right statistical estimate, i.e. the right model for calculation of p. Even if we only assume option 1, these $|o_i|$s need not always be very small.

So the models for three ps are not clear.

4 Repeated calculation with factor 6

Value $|o_1|$ decreases if we take into account the factors 2, 3, and 6. Namely, 363894 is divisible by 6. The cofactor 60649 is a prime. This way we get a smaller deviation from integer.

$6{\times}22\pi^4 = 6 \times 2143 + 1/60648\bullet98643247125637101518580384744937174321343584501005058856825 3...$

Now the denominator is 6 times closer to integer than in Eq. (2).

The deviation from integer is only

$o_1 = -0.0135675287436289848141961525506282567865641549899494114317 46210...$

I do the next step of this formula

$132\pi^4 = 12858 + 1/60649 + 1/271110564617\bullet02347613249847107376162986914124953525579 8816253...$

Deviation from integer is only:

$o_2 = 0.02347613249847107376162986914124953525579 8816253...$

Again, I do the next step of this formula

[1] Of course, it would be interesting to have calculated terms of o_i at $i > 8$. Maybe someone will calculate it.

$132\pi^4 = 12858 + 1/60649 + 1/271110564617 - 1/31308793410388947394938 68\bullet3107437702148513$ $61074841656492763...$

The deviation is:

$o_3 = 0.31074377021485136107484165649 2763...$

The next step of this formula is

$132\pi^4 = 12858 + 1/60649 + 1/271110564617 - 1/3154497817081469488072331720172069374158290 83$ $71773\bullet72887672212172496724354800183240007121412321671398$

The deviation is:

$o_4 = 0.2711232778782750327564519981675999287858767832 8602$

5 Conclusions about the second calculation

So here a smaller number of consequtive terms with $|o_i| < 0.25$ is, but it could prove significant that $|o_1|$ and $|o_2|$ are much smaller than before, they are very close to zero.

Otherwise, it is possible to try factors 2 and 3 here.

6 Conclusions

So we wonder how much to believe in this derivation. I have not yet provided tangible statistical proof. In my intuitive opinion, the upgrade for Eq. (1) is not so credible as the physical background of Eq. (8) in Ref. [4]. But I posted this derivation here, so that it is published, and so that better analyzes will follow, i.e. that someone will find an answer to this problem.

Otherwise, it would be good if $|o_i| < 0,25$ were true at all is. This would mean that $p \approx 0$. But they break up at $i = 8$.[2] Now the question is how to assess this statistically, how much to take into account in the sequence, and either the initial ones or all of them are important. Additionally, we have to ignore the $|o_1|$, but not always.

Indeed, here the real formula in the background (if it exists) can be based on either of points 1, 2, or 3 in Section 3. Even at point 1 it is possible that these integers can only be known at the beginning, but then they hide to something else. Therefore, it is difficult to evaluate this statistically.

I admit that I have never seen such fast convergence. So, maybe this will be also one control for 4 points in section 3.

Otherwise, the main goal for this formula is to check whether or not it is a part of a completely exact formula for π. A broader goal is to develop a system where we would more accurately estimate the probability of the existence of exact and simple formulas for π that are not part of a completely exact formula. At the same time, in a more general way, the aim is to develop a similar system, where for the guessed physical formulas we will find the probability that they have a physical basis, for instance Eq. (8) in Ref. [4].

However, whatever is already with Eq. (1), this paper is already proof that many digits of π can be practically used in physics. Namely, this paper is a part of an attempt to help count how many exact and simple formulas for π there are that are just random, i.e., are not a part of some exact formula for π. So this paper tries to find out for Eq. (1), if either it is random, or is a part of some exact formula

[2] At the beginning it even looked like this, with a large number of i's was valid $|o_i| < 0,25$, I used software [5]. Then I realized that the software was using a rounded value for π, so the calculations were wrong. The calculations published here are probably correct, because I started to combine the result with [6].

for π. (Intuitively, it seems to be a part of an exact formula, but one would have to prove either this or the opposite.) This would once give the probability p that some formula for π is random. Analogously, we are interested in guessed simple and very accurate formulas in physics, for example for Eq. (8) in Ref. [4], what is the probability p that this formula has no physical basis. In any case, some system for determining such probabilities should be built, e.g. Ref. [3].

Maybe one day a similar method will be found that will also prove or disprove my claim. Perhaps there will also be a supplement to this method of mine and a final assessment of what we get from it. There is also AI software that may do this one day, [7, 8].

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