Quantum States as Physical Possibilities

Armin Nikkhah Shirazi *

Abstract

There is a common back-of-the-mind idea, prompted by the counterintuitiveness of quantum phenomena, that quantum states may represent "mere possibilities" in some vague sense. Yet, this idea reflects itself neither in the quantum formalism nor in the way we use it. In this paper, I explore what it entails to take this idea seriously.

I begin by showing that already axiomatic probability fails to formally distinguish between possibilities and outcomes, even though it is conceptually supposed to be a unit measure over possibilities. I propose an axiomatic enrichment which introduces the distinction into Kolmogorov's axiomatization and then demonstrate how this provides a more faithful model of reality. Next, I explore an analogous mathematical modification of the standard quantum formalism in which Hilbert Space elements are no longer states of physical systems but mere physical possibilities, each linked to one of a set of actual states of systems we can observe if the possibility is actualized.

The modified formalism leaves the rules of quantum mechanics the same as before, but makes it possible to distinguish between what Heisenberg called "potentialities or possibilities" and "things and facts" at the level of the mathematics. I call this the *Heisenberg Interpretation*, and while it is not testable within the domain of quantum mechanics itself, it does erect a mathematically well-defined separation between quantum and classical physics. If the classical domain is identified with that of general relativity, then the formalism can be tested by checking for the existence of the boundary via proposed experiments like the BMV effect or by measuring the gravity field of an ultra high energy laser. Finally, I argue that physical possibilities are an overlooked conceptual ingredient critical to all of physics.

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*armin@umich.edu

1 Introduction

How to precisely interpret quantum states has been an open question since the inception of quantum mechanics[1]. Phenomena such as quantum superposition, quantum contextuality and quantum entanglement at spacelike distance seem to foreclose the possibility of taking them to be ordinary system states, meaning states of systems as we would encounter in our direct experience.
A long-standing notion, most forcefully articulated by Werner Heisenberg, considers them to be a kind of possibility. In his 1958 book *Physics and Philosophy*, he wrote [2]:

> *"In the experiments about atomic events we have to do with things and facts, with phenomena that are just as real as any phenomena in daily life. But the atoms or the elementary particles themselves are not as real; they form a world of potentialities or possibilities rather than of things or facts."* (p. 160)

This view has been considered by others over the ensuing decades, but a curious feature of these discussions is that they have largely remained at a verbal, as opposed to a mathematical one (see, e.g. [3]). In fact, my work preceding this paper showed that if Heisenberg's categories are considered as ontological equivalence classes, then his distinction cannot be formally incorporated into the standard quantum formalism without rendering it incoherent, and I called this *Heisenberg's equality of inequivalents problem*[1][4].
In this paper I will build on prior investigations [5] to develop a formalization of the notion of a quantum state as "potentiality or possibility" which avoids this problem, guided by the following principle: I will modify the formalism, *not* to change the rules, but only to add sufficient mathematical structure that possibilities can be teased apart from facts.
I will apply this approach first to axiomatic probability theory, and in what I call the *axiomatic enrichment* to probability, I show how this extra structure permits the formal separation of possibilities from outcomes. This, in turn, permits a formal definition of the concept of a classical possibility, something which to my knowledge had not been previously done.
I will then apply the same kind of approach to the quantum formalism, resulting in a modified formalism I call the *Heisenberg Interpretation* (HI). Like the axiomatic enrichment, it does not change the rules of the game, but permits Heisenberg's distinction to be formally incorporated into the mathematical formalism.
In the HI, Hilbert space vectors are demoted from system states to quantum possibilities, in contrast to states under measurement, which are collected in a set disjoint from Hilbert space. Thus, the quantum-classical divide becomes a function of the divide between possibilities and facts, leading this formalism to have a well-defined Heisenberg cut, unlike the standard quantum formalism. Despite a reinterpretation of a number of aspects of quantum mechanics, the Heisenberg interpretation is to my best knowledge predictively identical to standard quantum mechanics within its domain. However, the erection of a definite quantum-classical separation can, with a suitable mild assumption, lead to highly unexpected predictions for certain experiments which may be feasible in the near future. Thus, formalizing quantum states as mere physical possibilities is not just a philosophical matter but has real scientific consequences.

[1] The problem is surprisingly simple: for the distinction between "things and facts" and "potentialities or possibilities" to be mathematically represented in a meaningful way, they must be considered to be mutually exclusive, and better yet, members of distinct equivalence classes. However, the structure of Hilbert space always allows a state under measurement (=a "thing or fact") to be expressed in terms of a combination of states in a different measurement basis, a basis which at that point is merely hypothetical and therefore reflects "potentialities or possibilities". An equality between members of distinct equivalence classes leads, of course, to contradiction.

2 The Axiomatic Enrichment to Probability

In the standard axiomatization of probability [6], outcomes *are* possibilities; the theory has no separate category of outcomes as facts. I will now introduce a modified formalism pursuant to the guiding principle that the rules of probability are to be left unchanged, but that possibilities are to be teased apart from facts by adding extra mathematical structure. This modified formalism will be called the *axiomatic enrichment to probability*:

> Let $\Omega = \bigcup_{i=1}^{N} E_i$ be a set where N is either finite or countably infinite, $\mathcal{A} \subseteq \mathcal{P}(\Omega)$ a set of its mutually exclusive subsets E_i, and call the pair $(\Omega, \mathcal{A})$ a measurable space. Let $\Gamma = \{\gamma | \gamma = f(\omega)\}$ be a set where $f : \Omega \to \Gamma$ is a bijection and let G be a family of functions $g : \Omega \to \Gamma$ each of which maps every element of Ω to one distinct element of Γ such that each $\gamma \in \Gamma$ lies in the range of exactly one map g. A real-valued function $P : \mathcal{A} \to \mathbb{R}$ satisfying
>
> - Axiom 0: $P(g^{-1}(\gamma)) = P(\Omega)$
> - Axiom 1: $P(\Omega) = 1$
> - Axiom 2: $0 \leq P(E_i) \leq 1$
> - Axiom 3: $P(\bigcup_{i=1}^{N} E_i) = \sum_{i=1}^{N} P(E_i)$
>
> is called a *probability*.

A more economical formulation could have combined axioms 0 and 1 into one:

$$P(g^{-1}(\gamma)) = 1 \tag{1}$$

but for didactic purposes, I have kept them separate.
Compared to the standard axiomatization, the enriched axiomatization has additional structure which consists of:

1. a set Γ, called the *outcome set*, which represents the collection of outcomes, to be distinguished from possibilities which are represented by the sample space Ω. The collection of possibilities and outcomes into disjoint sets is the key feature here

2. a bijective map f called the *possibility-actuality correspondence*, which brings each element of the sample space into a one-to-one correspondence with an element of the outcome set, in order to ensure that possibilities match outcomes.

3. an axiom which says that probability is a unit measure over a *fiber* g^{-1} on each element of the outcome set such that this fiber is just what we call the sample space. The map $g \in G$ is called the *Unactualized certainty-actuality (UC-A) correspondence*, because it conceptually reflects a correspondence between an outcome and a certainty which has not actualized yet (since certainty means $P(\omega) = P(\Omega) = 1$).

Figure 1 shows diagrammatically the extra mathematical structure of the axiomatic enrichment:

$$\begin{tikzcd} \Omega \arrow[r, "f"] \arrow[r, bend left=60, no head, "G"] & \Gamma \end{tikzcd}$$

Figure 1: A diagram which shows how the sample space and outcome set are related to each other. The map f conceptually represents the correspondence between an unactualized possibility and an outcome, whereas the maps $g \in G$ conceptually each represent the correspondence between an unactualized certainty and an outcome.

Standard probability has none of this structure beyond Ω, yet the rules of probability are the same for both.

3 The Classical Concept of a Possibility

Separating outcomes from possibilities in a formal manner permits a formulation of the *concept of a possibility*, something for which, surprisingly, I was unable to find any attempts in the literature which did not conflate *interpretation* with *concept*. For example, the concept of a possibility may be conflated with its ontological status, which belongs to its interpretation [7].

To set this point straight, let me first briefly review the distinction between interpretation and concept, and between those of *probability* and *possibility*:

- The *interpretation* of something tells us its nature, what it is.
- The *concept* of something is the idea that tells us how it fits within our overall understanding of how the world works.

Applying this distinction to probability and possibility:

- the *interpretation* of a probability gives an account of its nature, such as whether it is a frequency, a degree of belief, a propensity, a quasi-logical framework for dealing with uncertainty, and so on.
- the *concept* of a probability reflects what it does: as a unit measure over a set of possibilities, it assigns a size to each possibility pursuant to the usual rules.
- the *interpretation* of a possibility gives an account of how the world or some aspect of it could be. It has long been understood, as, for instance, in ancient Greece, the possible was considered "that which is or will be true". It has also been formalized, as, for instance, in the possible world semantics of modal logic. Modal notions which relate possibility to necessity also reflect an account of the nature of a possibility, and therefore its interpretation. Alternative interpretations, including those which do not make any inherent distinction between possibilities and actualities, such as Lewis' modal realism, are possible.
- the *concept* of a possibility has to my knowledge not yet been elucidated. In fact, I am under the impression that there is either a widespread failure to even appreciate that there is an unresolved question here, or that its concept has been repeatedly conflated with its interpretation, or possibly both.

To formulate the concept of a possibility, consider the approach of Democritus to defining the concept of an *atom*. Recall, he said that if we were to keep cutting down a chunk of matter, we would eventually be left with something that we could cut no further. He called this an "atom" and took it to be the most elementary building block of material objects.

I will now take an analogous approach, but note that this approach applies only to *classical* possibilities. Quantum possibilities are conceptually very similar, but there is a critical distinction which will be discussed in section 4.

So, consider a set of possibilities over which a probability measure is defined, and imagine that we were to "cut it down", by which I mean that we change the circumstances so that progressively fewer alternatives are available. Eventually, we will arrive at a single possibility with unit probability.

Now notice, while there was still a larger set of possibilities, they all functioned identically as elements of the sample space, even though the *interpretation* of each possibility was distinct. As far as how possibilities function within probability theory, any possibility is as good as any other.

I will therefore use Occam's razor to strip possibilities of any intrinsic properties or characteristics: these belong properly to the corresponding *outcome*, which now belongs to a collection disjoint from the sample space!

Thus, a possibility has to be conceptually represented as a featureless thing, and I choose to call it a *monad*: a featureless unit which cannot be broken down into further constituents and is thus the possibility counterpart of a Democritean atom.

The monad may be labeled, however, by the outcome which obtains *if* the classical possibility actualizes, and hence we arrive at the concept of a classical possibility as a *labeled monad*, characterized intrinsically solely by its capacity to actualize into an outcome:

$$\text{Monad} \circ \text{label} = \text{classical possibility} \tag{2}$$

This reflects itself in the notation of possibilities in the axiomatic enrichment:

$$\begin{aligned} \Omega &= \{1_{\gamma_1}, 1_{\gamma_2}, ...\} \\ \Gamma &= \{\gamma_1, \gamma_2, ...\} \end{aligned} \tag{3}$$

where 1 indicates the monad, and its subscript is a label which indicates the outcome that obtains if this possibility actualizes. The label has no intrinsic significance, it is merely a useful way for us to know which outcome corresponds to that possibility, analogous to how labels on Democritean atoms or a coordinate system on a space have no intrinsic significance, but can be a useful in describing them.

A long-standing problem in the foundations of probability is that the mathematical formulation fails to satisfy its concept: the standard axioms of probability can be equally well applied to many things which are not possibilities, such as regions on a unit stick or of a unit area. The discussion above shows that this problem arises out of the identification of possibilities with outcomes. Whenever there is such an identification, the axioms can be applied to things which are not possibilities. Hence, the standard mathematical formulation of probability does *not* conceptually describe a probability at all, but merely a unit measure!

Another way of seeing this is that the axiomatic enrichment, but *not* standard probability, is a unit measure over a collection of objects which actually satisfy the concept of a possibility: under the axiomatic enrichment, probability assigns sizes to labeled monads pursuant to the usual rules, and because labeled monads stand in conceptual distinction to everything else which is not a classical possibility, this formalism is inapplicable to things which are not possibilities, unlike standard probability.

It is instructive to compare standard probability with the axiomatic enrichment in a couple of simple situations. Consider a situation in which a coin is to be flipped which is rigged to yield heads with probability one. Since in standard probability outcomes are possibilities, the model treats the situation prior to the coin flip as identical to the situation afterwards:

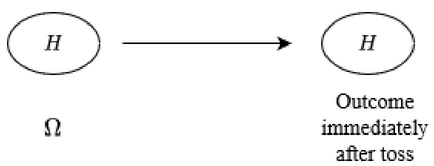

Figure 2: Since $\Omega = \{H\}$ here, there is in the standard probability model no difference between the situation before and after the coin flip, contrary to reality.

But we know that in real-life these situations are not identical: before flipping the coin, we did not yet have an outcome, but merely a possibility for obtaining that outcome *if* we flipped it. After

the coin flip, we have an actual outcome.
The axiomatic enrichment has the expressive power to model this distinction:

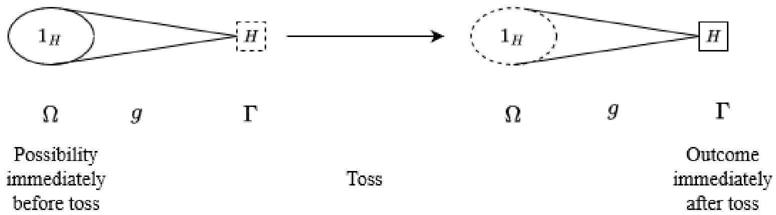

Figure 3: Since $\Omega = \{1_H\}$ and $\Gamma = \{H\}$ here, the axiomatic enrichment can model the coin flip as the replacement of the possibility 1_H by the outcome H. Solid lines represent aspects of the model which represent reality in the given situation, and dotted lines represent aspects which are ignored in the given situation.

Similarly, if we consider an ordinary coin flip followed by getting ready to flip it again, standard probability models the coin flip as a "collapse" of two alternatives into one, followed by a "spreading" out into two alternatives as we get ready to flip the coin again:

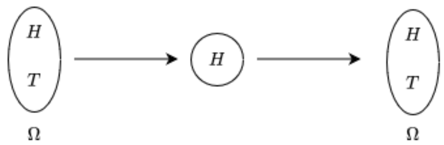

Figure 4: The "collapse" and subsequent "spreading out" of outcomes in standard probability of a coin flip which is followed by getting ready to flip it again.

On the other hand, the axiomatic enrichment models this as the replacement of two possibilities with an outcome, followed by the replacement of the outcome with two possibilities:

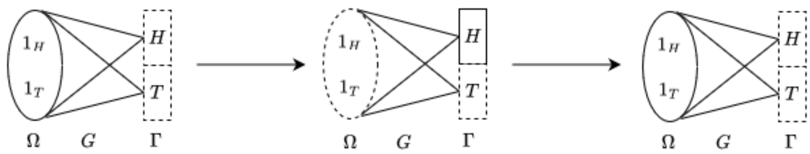

Figure 5: The replacement of two possibilities by an outcome, followed by the replacement of that outcome by the same two possibilities, under the axiomatic enrichment. Solid vs. dotted lines function as in figure 3.

A good model of reality captures all and only aspects of reality relevant to the purpose to which the model is deployed. Standard probability fails to properly model possibilities because under the contemporary paradigm, we do not deem them to be sufficiently important as an aspect of a fundamental description of the world. But if we wish to seriously consider quantum states as a kind of possibility, we need to take possibilities as a fundamental ingredient in our models of reality seriously.

4 The Quantum Concept of a Possibility

The first postulate of the interpretation to be given in the next section demotes the vectors in Hilbert space from representations of states of actual systems to mere labels for what I call an *actualizability.* So, before I discuss that interpretation, I need to clarify what an actualizability is. In some interpretations of quantum mechanics, quantum states are assigned a concept (or an interpretation) in an ad hoc manner, but it is much more convincing if it can be shown *within the formalism* that the quantum state actually satisfies the concept we wish to assign to it. This is what I will set out to do now.

We found that a classical possibility can be conceptualized as a labeled monad. A quantum state can be shown to be something very similar:

$$\begin{aligned} \text{Quantum State} &= |\Psi\rangle \\ &= |\Psi\rangle \langle\Psi|\Psi\rangle \\ &= \text{label} \circ \text{Actualizability} \\ &= \text{Quantum Possibility} \end{aligned} \tag{4}$$

Thus, the physical aspect, i.e. the *ontology* of the quantum state, is the actualizability $\langle\Psi|\Psi\rangle$, and $|\Psi\rangle$ is merely a label which tells us what we can observe if we perform a measurement[2]. Just as is the case for labels of classical possibilities, it has no intrinsic physical significance!

Like a monad, an actualizability is a featureless unit devoid of any intrinsic features except for the capacity to actualize into a classical state. However, unlike a monad, which cannot be decomposed any further, an actualizability can be decomposed into a linear combination of *other* actualizabilities. It is a mereologically atomless unit. The statement that an actualizability can be decomposed into a linear combination of other actualizabilities is nothing other than a familiar equality

$$\langle\Psi|\Psi\rangle = \sum_k |c_k|^2 \langle\psi_k|\psi_k\rangle \tag{5}$$

This view of Hilbert space elements drastically changes how we are to understand the Born rule. A long-standing difficulty in the foundations of quantum mechanics has been that quantum probabilities seem to be very different from classical probabilities, but under the reconceptualization of a quantum state as a labeled actualizability, this merely reflects a misinterpretation: the Born rule is not about probability at all, but a certain kind of possibility which *mimics* probability because it can be directly expressed in terms of a linear combination of other possibilities of the same kind and because, like a probability, it is a featureless unit! Thus, portions of an actualizability which represent other actualizabilities can be interpreted as if they were probabilities.

Similarly, the "time evolution of a quantum state" of a system has to be reinterpreted in a completely different way than usual, because *before any measurement there simply is no system!* Rather, the time evolution has to be interpreted as a continuous process by which an actualizability changes its associated label. That the actualizability is *everywhere* in Hilbert space but "hidden" can be

[2]This changes the relationship between the mathematical representation and "what is in the physical world" in yet another, more subtle way: in standard quantum mechanics, the physical aspect, i.e. the physical state, is really represented by a *ray* in Hilbert space, since multiplying an element of $\mathcal{H}$ by any scalar returns the same state. In the HI it is the unit magnitude (=actualizability) which is fundamental, but since it is not characterized by any intrinsic absolute scale, multiplying it by a scalar simply returns back the same unit magnitude, but now at a different scale. The lack of intrinsic absolute scales for possibility sizes is actually what permits normalization in probability, so from the perspective of the HI, even after accounting for the Born Rule being about possibility rather than probability, the normalization argument in favor of rays as representations of physical states in standard quantum mechanics has it backward.

made explicit by suggestively writing

$$|\Psi\rangle\langle\Psi|\Psi\rangle = \sum_k c_k |\psi_k\rangle\langle\psi_k|\psi_k\rangle \tag{6}$$

which is now interpreted as a relation between differently-oriented labeled actualizabilities in Hilbert space. So, now (as will be formalized in the first postulate) Hilbert space is a space of labeled actualizabilities such that the change in the orientation of a unit vector over time in it does not reflect any material process or change in a system state at all, but just what we will observe *if* we perform a measurement at that moment.

5 The Heisenberg Interpretation

What I call the *Heisenberg Interpretation of Quantum Mechanics (HI)* is a modification of the standard formalism which makes it possible to formally distinguish between physical possibilities and facts, by which I mean here unmeasured quantum states and states under measurement, respectively.

The guiding principle is the same as before: the rules are to be left unchanged, and the modification consists only of adding sufficient mathematical structure which makes it possible to tease possibilities and facts apart.

The usual statement of the postulates of standard quantum mechanics presupposes some basic knowledge of the relevant mathematical structures, which I will not elaborate on here. However, the HI postulates also presuppose knowledge of some very basic but new mathematical structures which I will briefly discuss now, before presenting the postulates themselves.

- A key structure not found in the standard quantum formalism is a family $\mathfrak{E}$ of partial maps I call the *eigenvector maps* $\mathfrak{e} : \mathcal{H} \longrightarrow \mathcal{H}$, each with a domain of definition that includes a subset of a complete set of eigenvectors in a given measurement basis, and its range containing a single element, namely a vector to which each of these eigenvectors contributes in linear combination. If the complete set of eigenvectors contributes to the element in its range, then its domain of definition includes all of them.

  Obviously, a given Hilbert space element can be expressed in a great many different bases, and, conversely, a set of eigenvectors in a given basis can be used to construct many different elements via linear combination, so even for a 2-dimensional Hilbert space, $\mathfrak{E}$ has an infinite number of members, one for each case.

- States under measurement are collected in a set which is disjoint from Hilbert Space, and will be called the *classical States Set* $\mathcal{C}$. This set has no further structure, but is related to the Hilbert Space via two sets of partial maps. These maps reflect the same concepts as the maps f and g in the axiomatic enrichment, albeit expressed in very different mathematical form, and will be therefore symbolized by $\mathfrak{f}$ and $\mathfrak{g}$, respectively.

- Each member of the family $\mathfrak{F}$ of partial maps $\mathfrak{f} : \mathcal{H} \longrightarrow \mathcal{C}$, is called (like f) the *possibility-actuality correspondence*, and has as its domain of definition a subset of a complete set of eigenvectors in a given measurement basis α which is identical to the domain of definition of at least one eigenvector map $\mathfrak{e}$. Its range $\mathcal{B}_\alpha \subset \mathcal{C}$ includes the corresponding states under measurement, which it relates via a one-to-one correspondence. There are as many partial maps $\mathfrak{f}$ as eigenvector maps $\mathfrak{e}$, and conceptually, $\mathfrak{f}$ ensures that each possibility *in a measurement basis*[3] of $\mathcal{H}$ matches exactly one actual system in $\mathcal{C}$.

[3] I emphasize that the correspondence only applies to elements in a measurement basis because it is possible to

- Each member of the family $\mathfrak{G}$ of partial maps $\mathfrak{g}$ is called (like g) the *unactualized certainty-actuality correspondence* and has as its domain of definition a subset $\mathcal{B}_\alpha \subset \mathcal{C}$ which is identical to the range of at least one partial map f, and includes in its range a single element of $\mathcal{H}$, namely that which is related to a set of eigenvectors in basis α which stand in a one-to-one correspondence to the elements of $\mathcal{B}_\alpha$.
  Conceptually, $\mathfrak{g}$ establishes a correspondence between an unactualized certainty (since $\langle\Psi|\Psi\rangle = 1$) and actuality, in this case one of a set of states under measurement.

The point of this extra mathematical structure is to be able to express a given eigenvector map inside Hilbert space in terms of a composition of two partial maps which relate elements of Hilbert space to those of the Classical states set. Thus, for any given vector in a particular measurement basis, we can find partial maps $\mathfrak{e} \in \mathfrak{E}$, $\mathfrak{f} \in \mathfrak{F}$ and $\mathfrak{g} \in \mathfrak{G}$ such that

$$\mathfrak{e} = \mathfrak{g} \circ \mathfrak{f} \tag{7}$$

holds. The HI postulates themselves will actually use the inverses of these relations.
After these preliminaries, I can now state the postulates. The aspects which stay the same as in standard quantum mechanics [8] are stated in informal language because my focus here will be on what is new:

1. The elements of quantum mechanical Hilbert space $\mathcal{H}$ represent *labeled actualizabilities*
2. The time evolution of the elements of $\mathcal{H}$ obeys the Schrödinger equation.
3. Measurements are represented by linear Hermitian operators that are functions of the position and/or momentum operator acting on elements of $\mathcal{H}$
4. Quantum collapse is expressed in terms of the *collapse relation* $\mathfrak{e}^{-1} : \mathcal{H} \longrightarrow \mathcal{H}$, which can be expressed as a composition of two relations
   (a) The *actualization relation* $\mathfrak{g}^{-1} : \mathcal{H} \longrightarrow \mathcal{C}$
   (b) The *deactualization map* $\mathfrak{f}^{-1} : \mathcal{C} \longrightarrow \mathcal{H}$

   where the co-domain $\mathcal{C}$ is the classical states set. The range for each relation is the complete set $\mathcal{B}_\alpha \subset \mathcal{C}$ of those elements which are classical counterparts to the elements of $\mathcal{H}$ in a given measurement basis α.
5. The actualization relation $\mathfrak{g}^{-1}$ is subject to the Born Rule[4].

The formal distinction between unmeasured states and states under measurement also necessitates on logical grounds a splitting in the concept of mass:

Auxilliary Postulate:

- *actualizable mass* $\mathfrak{m}$ characterizes elements of $\mathcal{H}$
- *actual mass* m characterizes elements of $\mathcal{C}$.

This can be intuitively seen by considering that if a system which existed as a mere possibility was characterized by the same concept of mass as one which existed as an actuality, then this would render the modal distinction incoherent.
The first postulate explicitly characterizes elements of the Hilbert space as a kind of possibility I

express Hilbert space elements in other bases. For example, it is possible, via partial Fourier transform, to express position or momentum states in a mixed position-momentum basis, but since there are no corresponding measured states, this does not constitute a measurement basis.

[4]this postulate might be rigorously derivable from the actualizability concept, but to be safe it is included.

call an *actualizability*, as was explained in the previous section. Other than that, the first three postulates, which establish the rules of the quantum formalism, are the same as those of standard quantum mechanics.
The other major difference to standard quantum mechanics beside the reconceptualization of a quantum state as a labeled actualizability is that the collapse postulate has been replaced by a postulate which frames quantum collapse as a relation which can be decomposed into two relations representing the process of actualization (while the measurement takes place) followed by deactualization (once the measurement ceases). Thus, we have

$$\mathfrak{e}^{-1} = \mathfrak{f}^{-1} \circ \mathfrak{g}^{-1} \tag{8}$$

as illustrated in figure 6.

$$\begin{tikzcd}
\mathcal{H} \arrow[rr, "\mathfrak{g}^{-1}"] \arrow[d, "\mathfrak{e}^{-1}"'] & & \mathcal{C} \arrow[dll, "\mathfrak{f}^{-1}"] \\
\mathcal{H} & &
\end{tikzcd}$$

Figure 6: The collapse relation $\mathfrak{e}^{-1}$ as a composition of the deactualization map $\mathfrak{f}^{-1}$ after the actualization relation $\mathfrak{g}^{-1}$.

As a concrete example, consider a measurement on a spin 1/2 system in the $x-$ basis:

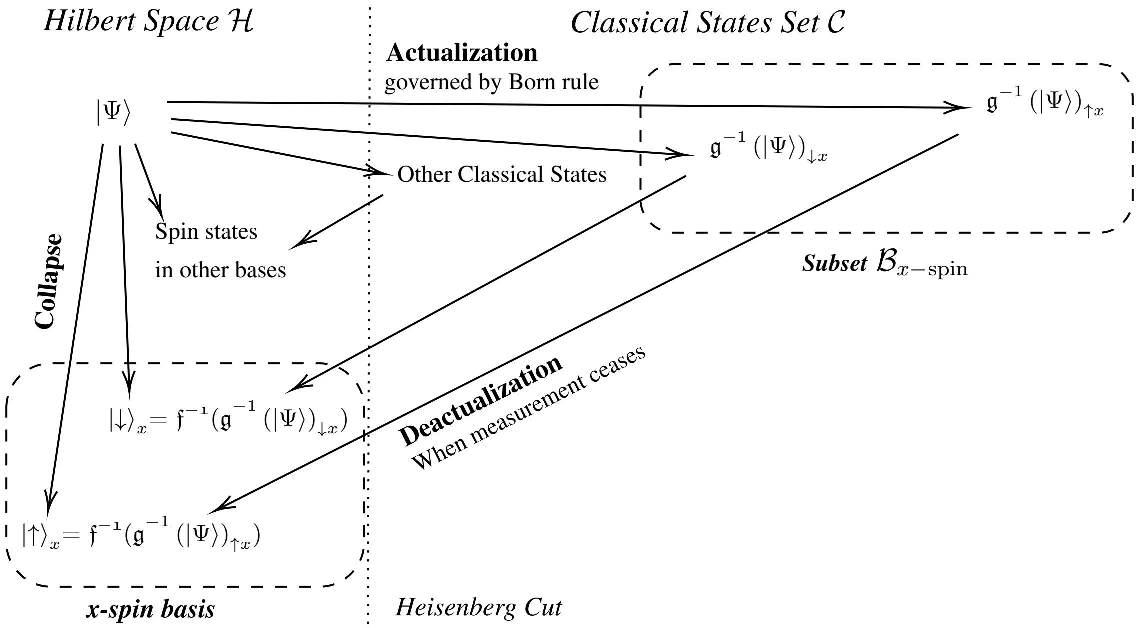

Figure 7: The composition $\mathfrak{e}^{-1} = \mathfrak{f}^{-1} \circ \mathfrak{g}^{-1}$ illustrated by a spin 1/2 state for the x-spin basis.

Upon a measurement, the original spin state (upper left) actualizes into one of two classical states which stand in a one-to-one correspondence with its eigenvectors in the $x-$spin measurement basis. Once measurement ceases, the classical state deactualizes to its corresponding eigenstate and the net result is collapse of the original state to one of its eigenvectors in that basis.
Note that even though the composition involves two one-to-many relations, the one-to-one correspondence $\mathfrak{f}^{-1}$ ensures that it still holds. Also note that, unlike standard quantum mechanics,

the HI has a well-defined *Heisenberg cut*: a definite separation between the quantum and classical realms.

6 Implications of the Heisenberg Interpretation

A detailed discussion of the manifold consequences and implications of the Heisenberg interpretation awaits future treatment. But here, I wish to at least summarize some of these.
The Heisenberg interpretation radically changes our understanding of certain aspects of quantum mechanics while potentially permitting a deeper understanding of many of its counterintuitive aspects:

1. **Quantum state:** In standard quantum mechanics, even when there is no quantum measurement, there is assumed to be a system, however fuzzy, undefined or ephemeral it may be. The Heisenberg interpretation does away with this assumption altogether, and in that respect can be considered to be maximally anti-realist (where actuality is taken to be a necessary aspect of reality) while interpreting Hilbert space elements still ontologically: unless there is a quantum measurement, there is only a quantum possibility and nothing else (in spacetime[5]).

2. **Quantum Superposition:** Although Heisenberg evidently did not explictly say it, the original basis for him considering "possibilities or potentialities" as an altogether different kind of family than "things and facts" may have been that for the former it is in principle possible to have a combination of mutually incompatible members of the family at the same time whereas for the latter it is not. At any rate, taking this to be the case at an ontological level is the basis for my formalization which collects them in distinct sets. Once we accept that quantum states are merely physical possibilities, quantum superposition is no more strange than the fact that before the flip of coin, there exist two mutually incompatible possibilities at the same time[6].

3. **Quantum Collapse:** In standard quantum mechanics, quantum collapse is an utterly mysterious process, and under some interpretations, most notably the Everettian or many Worlds interpretation, it is even denied altogether[9]. While the HI does not completely clarify it, I would argue that it still advances our understanding because

   (a) It turns an informal statement of the postulate into a mathematical representation, namely the collapse relation. This in turn, can be examined and manipulated in ways that the standard formulation could not, such as indeed its decomposition into the actualization and deactualization relations.

   (b) It does clarify that the process involves the actualization and deactualization of possibilities, analogous to what already happens in the axiomatic enrichment to classical probability.

[5] It turns out the distinction between "nothing else" and "nothing else in spacetime" is critical to investigating further questions outside the scope of this paper, such as whether a system really "materializes from nothing" when a quantum measurement takes place.

[6] Actually, this is overstating it slightly: if we consider classical systems to be truly subject only to deterministic laws, then there can be no mutually incompatible possibilities at an ontological level. However, the constituents of classical systems obey quantum mechanics, so one cannot exclude that some kind of influence due to indeterminacy at the quantum level "propagating upward", however slight, could open the door for mutually incompatible ontic possibilities even within the classical realm. Chaotic systems with their sensitive dependence on initial conditions most clearly exemplify the difference between a system which is classical "all the way down" and a system which hits quantum indeterminacy along the way.

The central aspect left unclear is the set of details about what actually happens during a quantum measurement. The HI, like standard quantum mechanics, treats a "measurement" as a black box. My view is that its proper elucidation will require a change of paradigm.

4. **Born Rule:** As mentioned, under the HI, the Born rule is not fundamentally a postulate about probability but about a certain kind of possibility which mimics probability. One may still ask, why can quantum possibilities but not classical possibilities mimic probability? Recall, the monad of a classical possibility cannot be broken into smaller parts. The answer is that there is always a system which grounds a classical possibility, whereas there is none for a quantum possibility. So, for example, the possibilities "heads" and "tail" are grounded in a coin, those of "on" and "off" in a switch, and so on. But there is no system (in spacetime[7]) which grounds a "quantum state". Hence, quantum possibilities have a kind of "flexibility in representation" altogether alien to classical possibilities, a flexibility which makes them conceptually very similar to probabilities.

5. **Quantum Entanglement:** The strangeness of quantum entanglement if we take Hilbert space elements to represent actual physical system states is perpetually understated. Already phrases like "system A becomes entangled with system B" minimize that what we should really say is that both systems disappear and are replaced by something which is or contains neither system but is a kind of holistic conditional combination of both. The Schrödinger cat paradox is often used as an analogy to illustrate the apparent strangeness of quantum measurements, but it is arguably even better suited to illustrate the apparent strangeness of quantum entanglement, since as soon as we close the box, the cat and the poison mechanism have to be considered to disappear and be replaced by an inseparable conditional combination. It is very difficult to wrap one's mind around what that really means if we take Hilbert space elements to be system states, but arguably, if we think of entanglement as something that occurs only between possibilities, then due to their immateriality (which is distinct from their physicality!) it becomes easier to intuit it.

6. **Quantum Contextuality:** Again, if we consider Hilbert space elements to be actual physical system states, then quantum contextuality, the idea that quantum systems have no pre-existing intrinsic properties independent of a measurement context, is very difficult to make sense of. But under the HI, this is an almost trivial consequence of the fact that quantum possibilities have no intrinsic system properties whatsoever.

7. **Quantum Non-locality:** The HI can help clarify how to understand this phenomenon by identifying two possible ways of thinking about it as conceptual dead ends:

    (a) **Transmission of some kind of influence:** The successful transmission of anything requires that a receiver be subjected to it. But if the purported receiver does not exist until it is measured, then that precludes this as a potential explanation of quantum non-locality.

    (b) **A non-local correlation between systems:** Under the HI, the observed quantum correlations are emphatically not between systems but between *events*, namely the actualization events for the systems; they reflect *correlated actualizations*. The difference is as follows: if we think of the correlations in terms of actualization events, then *before the events, there are no systems, and after the events, there is no correlation.* On the

[7] The answer to how to make sense out of this without outright taking it to mean that each time we observe a system we create it requires ideas which are outside of the current paradigm, and are outside the scope of this paper.

other hand, thinking about the correlations in terms of a relationship between systems naturally invites hypotheses which are bound to come into conflict with relativity.

The HI by itself does not explain how or why there can be correlated actualization events at spacelike distance, but being able to recognize some ways of thinking about this as potentially false is arguably still of value.

8. **Conservation laws in a quantum context:** If the Hilbert space elements really are mere physical possibilities, then conservation laws in a quantum context must be radically re-interpreted. Consider that prior to a measurement, there is nothing (in spacetime). Then, upon a measurement, the system appears, and once the measurement concludes, it disappears. Under these circumstances, conservation laws must be interpreted as *Emergence/Submergence Specifications*, basically, constraints on what kind of system can emerge upon a measurement, and what kind of a possibility remains after the measurement concludes and the system "submerges". A more detailed consideration beyond the idea that the HI points to quantum collapse as a kind of emergence process is under investigation.

Beside the prospect of a deeper understanding of quantum phenomena, the HI can also under a mild assumption be turned into a rival theory to standard quantum mechanics. If we assume, in addition to the postulates above, that

$$\text{classical} = \text{in the domain of general relativity} \tag{9}$$

then the Heisenberg cut turns into a definite boundary between the quantum realm and that of general relativity. For ease of reference, I will refer to (9) as the "boundary assumption". So, under the boundary assumption, the elements of $\mathcal{H}$ lie in the quantum domain, while the elements of $\mathcal{C}$ lie in the domain of general relativity, and these domains are mutually exclusive.
The existence of such a boundary can be tested in principle in at least two ways:

1. **by testing for gravitationally mediated quantum phenomena:** If the boundary exists, then nothing about gravity is quantum, and there should be no such phenomena.

2. **by testing the gravity field of a quantum system:** If the boundary exists, then the gravity field of a quantum system should be zero, since the objects in general relativity which produce gravity fields are all actual objects in spacetime.

The BMV (Bose-Marletto-Vedral) experiment is a proposed test which may be performed in the near future [10][11]. It involves the gravitationally mediated entanglement of a pair of quantum systems, and as such is a test of the first type. Under the contemporary paradigm, it is widely expected to be observable, though a failure to observe it may require subsequent tests at higher precision or accuracy. The HI together with the boundary assumption predicts a null result.
Testing the gravity field of quantum systems is overwhelmingly difficult due to the weakness of gravity. In addition, measures would have to be taken to ensure that the test itself does not actualize a possibility, which would then result in a non-zero gravity field. For this reason, the conceptually cleanest test of the second type would be the measurement of the gravity field of an ultra high energy laser beam. While the expected gravity field of a laser was already calculated back in 1931[12], the HI together with the boundary assumption predicts a null result. Unfortunately, it appears that such a test lies further in the future.

7 A Proposed Paradigm Which Takes Possibilities Seriously

Above, I indicated that certain important questions raised by the HI can only be answered within a future paradigm. In this section, I wish to ascertain that I am not just a waving off the need for the clarification of questions raised by the HI, but that I do indeed have something concrete in mind when I refer to a possible "future paradigm". It is outside the scope of this paper to describe this proposed paradigm in detail, and many parts of it have indeed not been worked out yet. However, it may be worthwhile to provide some tantalizing examples which suggest what it may look like.

This paper presents a fragment of a broader research program in which physical possibilities are taken to be a critical conceptual ingredient in all of physics. The central idea is that physical possibilities are virtually everywhere, that we are surrounded by them like fish in the ocean, but that we do not "see" them because they do not appear to us as such in the equations. Equation (4) gives the paradigmatic example of this state of affairs.

But it is one thing to make a general claim and support it with only one example, and another to draw on multiple examples from disparate areas. So, here I will briefly summarize other areas of physics where physical possibilities are proposed to appear without having been recognized as such:

1. **Quantum Field Theory:** Going from a non-relativistic to a relativistic regime should not change the ontological nature of Hilbert space elements. It follows then that if they are mere physical possibilities in the former, then they have to be the same in the latter. This implies that under the Heisenberg interpretation a quantum field is a *possibility distribution on all of space*. Local excitations in the field would have to be re-interpreted as fluctuations in the availability of certain physical possibilities within the neighborhood of a point in space.

2. **Maxwell's Theory:** For quantum electrodynamics, Maxwell's theory is taken as it classical approximation, and it is indeed not difficult to show that classical electromagnetic fields can be considered a *a possibility distribution on all of space with unit probability*: *if* an electric charge is put in an electromagnetic field, then it is *certain* (=unit probability) that it will experience a given acceleration. The current universal conflation of possibilities with actualities, and especially possibilities characterized by unit probability, as already reflected in its standard axiomatization, has likely done much to obfuscate this simple observation and led to the idea that electromagnetic fields are "actual" objects.

3. **Information theory:** In recent years and inspired by the famous slogan that "information is physical", the theory of information has gained tremendous importance in physics. From the perspective of the proposed paradigm, in contexts in which information is the only "physical entity" under consideration, there has been a conflation of the label with the ontology. In other words, since in those contexts, information is under the proposed paradigm taken to be the label associated with a physical possibility, contemporary ideas about information are taken to unjustifiably reify the label when the physically significant part is actually the monad/actualizability labeled by it.

4. **Arrow of Time:** the directionality of time is quite evident in our direct experience, but reflects itself nowhere in the equations of physics. From the perspective of the proposed paradigm, this is due to the failure of our models of reality to represent the distinction between a future filled with possibilities and a past filled with facts. The directionality of time would naturally appear in a model of reality which faithfully represents this distinction. Since in such a model, the directionality is due to an ontological transformation of possibilities into facts, I call it the *ontic arrow of time*. In the present pradigm, the ontic arrow is invisible.

5. **Energy**: a central argument under development is that behind the Noetherian concept of energy as a conserved charge under time translation symmetry there lies a much more intuitive one: that energy is a capacity for transforming possibilities into facts. As a very loose example, consider the difference between explosives which differ in the amount of energy

stored. If the attempt to define this concept mathematically is successful, then possibilities will appear everywhere energy appears which, since energy is everywhere in physics, implies that possibilities are ubiquitous in physics.

The examples provided above do not exhaust all the proposed situations in which possibilities appear in unrecognized form under the present physics paradigm. But they suffice to show that the HI is not just one of dozens interpretions of quantum mechanics which do not necessarily connect to other parts of physics, but that it is in fact the one that most naturally fits within a broader proposed paradigm for all of physics.

8 Conclusion

In this paper, I have explored how possibilities can be taken seriously by adding mathematical structure to existing theories. I found that a classical possibility is conceptually a labeled monad, while a quantum possibility is conceptually a labeled actualizability, and that the latter has a natural place within a modification of the standard quantum formalism which I call the Heisenberg interpretation.

I argued that the HI has the potential to help us understand a number of counterintuitive quantum phenomena both more clearly and more deeply, and that with the supplementation of what I have called the boundary assumption, it can be transformed into a rival theory to standard quantum mechanics. The differences in prediction all pertain to the question of whether there is a definite boundary between the quantum realm and the domain of general relativity or not. I there is, then effects which presuppose the absence of such a boundary should not occur.

Finally, I argued that the HI is the most natural interpretation of the quantum formalism within a proposed paradigm that recognizes physical possibilities virtually everywhere in physics. I gave several examples of this, and further work on this proposed paradigm is in progress.

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