A Modified Born-Infeld Model of Charged Leptons, Part 1: Foundations

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Abstract

This work proposes and outlines a generalization of a modified Born-Infeld model of electrons that includes all charged leptons, i.e., electron, muon, tau, and their antiparticles. In the proposed model, all charged leptons are based on linearly scaled versions of a single field solution. Due to the nonlinear nature of the modified Born-Infeld field equations, these linearly scaled versions themselves do not satisfy the field equations. A quantized excitation is hypothesized to compensate for the nonlinear effects and satisfy the nonlinear field equations such that electron and positron correspond to the ground state of the excitation, and heavier leptons correspond to excited states. While the proposed model is assumed to be testable, an actual test is beyond the scope of this work.

1 Introduction

Born and Infeld proposed a classical field theory, in which "particles of matter are considered as singularities of the field and mass is a derived notion to be expressed by field energy (electromagnetic mass)" [BIF34]. This led to the Born-Infeld model of electron-like particles without magnetic moment [BIF34], which was modified [Kra23] to include a realistic magnetic moment and an internal clock, which was hypothesized by de Broglie [dB25]. (See Section 2 for a brief review of this model.)

Born and Infeld's model of electrons also inspired Dirac's model of charged leptons [Dir62]. Similarly, this work proposes a generalization of the modified Born-Infeld model of electrons [Kra23] to other charged leptons. To this end, it is noteworthy that if the field solution of the modified Born-Infeld model of electrons is linearly scaled in a specific way (see Section 3), it can approximately fit the features of all charged leptons (including field energy, Compton frequency, charge, and magnetic moment) but cannot satisfy the nonlinear effects in the field equations. It could fit these features almost perfectly if the Born-Infeld parameter is adapted for each particle.

If, however, the Born-Infeld parameter is assumed constant, linearly scaled versions of the field solution for electrons do not satisfy the nonlinear field equations of the model. Specifically, for a heavier lepton than the electron, the equations require reduced field strengths compared to a linearly scaled field solution representing a lepton with the same mass (as expressed by its Compton frequency). This corresponds to a reduced field energy, which means that the field energy of an actual solution would be less than the rest mass energy of the corresponding particle, which would contradict the assumption that the two energies are equal.

In the proposed model, this energy gap is filled by a quantized excitation (see Section 4) such that the electron corresponds to the ground state of the excitation, the muon to the first excited state, and the tau to the second excited state. Thus, one could think of the scaling as an integral part of the excitation. In this work, however, the scaling is considered different from the excitation (but inseparable for any actual solution of the field equations) in order to emphasize nonlinear effects of the field equations.

While the proposed model is testable (as discussed in Section 5), an actual test is part of future work as mentioned in Section 6, which concludes this work.

2 Modified Born-Infeld Field Theory

As in previous work [Kra23], the dimensionless Lagrangian density of the modified Born-Infeld field theory is

$$\mathscr{L}(A_\nu, \partial_\mu A_\nu) \stackrel{\text{def}}{=} \sqrt{1 - \frac{1}{b^2}(\partial^\mu A^\nu)(\partial_\mu A_\nu)} - 1 \tag{1}$$

using basic Ricci calculus, the Minkowski metric tensor η in the form diag$(+1, -1, -1, -1)$, the electromagnetic four-potential $(A^0, A^1, A^2, A^3) = (\phi/c, A_x, A_y, A_z)$ in SI units, and the Born-Infeld parameter b specifying the maximum magnetic field strength.

The electromagnetic four-potential is also used to define electric field strength $\mathbf{E}$, magnetic field strength $\mathbf{B}$, and four-current density $(J^0, J^1, J^2, J^3) = (c\rho, J_x, J_y, J_z)$:

$$\mathbf{E} \stackrel{\text{def}}{=} -\nabla\phi - \frac{\partial}{\partial t}\mathbf{A} \tag{2}$$

$$\mathbf{B} \stackrel{\text{def}}{=} \nabla \times \mathbf{A} \tag{3}$$

$$J^\nu \stackrel{\text{def}}{=} \frac{1}{\mu_0}\left(\partial_\mu \partial^\mu A^\nu - \partial_\mu \partial^\nu A^\mu\right) \tag{4}$$

More details about the notation are provided in previous work [Kra23] and references therein.

The corresponding Euler-Lagrange equations are:

$$0 = \partial_\mu \left(\frac{\partial \mathscr{L}(A_\nu, \partial_\mu A_\nu)}{\partial\left(\partial_\mu A_\nu\right)}\right) - \frac{\partial \mathscr{L}(A_\nu, \partial_\mu A_\nu)}{\partial A_\nu} = \partial_\mu \frac{-1}{b^2} \frac{\partial^\mu A^\nu}{\sqrt{1 - \frac{1}{b^2}(\partial^\alpha A^\beta)(\partial_\alpha A_\beta)}}. \tag{5}$$

Thus:

$$0 = \partial_\mu \frac{\partial^\mu A^\nu}{\sqrt{1 - \frac{1}{b^2}(\partial^\alpha A^\beta)(\partial_\alpha A_\beta)}} \qquad \text{with } \nu = 0, \ldots, 3. \tag{6}$$

For the modified Born-Infeld model of electrons, the field equations were solved numerically in previous work [Kra23] resulting in a rotating field solution with a peak moving at the speed of light on a circular orbit with a radius equal to the electron's reduced Compton wavelength. While most features of electrons (electric charge, magnetic moment, Compton frequency) were imposed on the solution, the field energy of the solution was matched to the rest mass energy of an electron by adjusting the Born-Infeld parameter.

3 Linear Scaling of Rotating Field Solution

The numerical solution of the field equations for an electron [Kra23] could be denoted by $\phi(m_\text{e}, \mathbf{x}, t)$ for the electric potential and $\mathbf{A}(m_\text{e}, \mathbf{x}, t)$ for the magnetic vector potential at position $\mathbf{x}$ and time t with the rotation center of the rotating field solution at the origin of the coordinate system. The m_e indicates that this is the field solution for mass m_e of an electron (where the mass is identified with the electron's Compton frequency apart from a constant factor). With this field solution, consider the scaled fields $\phi(m, \mathbf{x}, t)$ and $\mathbf{A}(m, \mathbf{x}, t)$ for a charged lepton of arbitrary mass m (more precisely, the corresponding Compton frequency) defined this way:

$$\phi(m, \mathbf{x}, t) \stackrel{\text{def}}{=} \frac{m}{m_\text{e}} \phi\left(m_\text{e}, \frac{m}{m_\text{e}}\mathbf{x}, \frac{m}{m_\text{e}}t\right) \tag{7}$$

$$\mathbf{A}(m, \mathbf{x}, t) \stackrel{\text{def}}{=} \frac{m}{m_\text{e}} \mathbf{A}\left(m_\text{e}, \frac{m}{m_\text{e}}\mathbf{x}, \frac{m}{m_\text{e}}t\right) \tag{8}$$

For a heavier charged lepton than the electron with $m > m_\text{e}$, $\phi(m, \mathbf{x}, t)$ and $\mathbf{A}(m, \mathbf{x}, t)$ describe smaller (by factor m_e/m), faster rotating (by factor m/m_e) fields than $\phi(m_\text{e}, \mathbf{x}, t)$ and $\mathbf{A}(m_\text{e}, \mathbf{x}, t)$. This is consistent with a smaller Compton wavelength (by factor m_e/m) and greater Compton frequency (by factor m/m_e) for a charged lepton of mass m.

Assuming that the field energy is proportional to a volume integral over a sum of squared derivatives of ϕ and $\mathbf{A}$, the field energy is greater by factor m/m_e (as expected) since the volume is smaller by factor $(m_\mathrm{e}/m)^3$, and derivatives of ϕ and $\mathbf{A}$ (at corresponding locations) are greater by factor $(m/m_\mathrm{e})^2$ such that squared derivatives are greater by factor $(m/m_\mathrm{e})^4$.

For large distances $|\mathbf{x}|$, $\phi(m_\mathrm{e}, \mathbf{x}, t)$ is proportional to $1/|\mathbf{x}|$ and independent of t. Therefore, $\phi(m, \mathbf{x}, t)$ approximates $\phi(m_\mathrm{e}, \mathbf{x}, t)$ for large distances $|\mathbf{x}|$. In other words, both describe the far field of the same electric charge, as expected based on the known charged leptons. $|\mathbf{A}(m_\mathrm{e}, \mathbf{x}, t)|$ is proportional to $1/|\mathbf{x}|^2$ and also independent of t for large distances $|\mathbf{x}|$. Thus, $|\mathbf{A}(m, \mathbf{x}, t)|$ is smaller than $|\mathbf{A}(m_\mathrm{e}, \mathbf{x}, t)|$ by factor m_e/m as expected based on the known magnetic moments of heavy leptons (apart from the quantum mechanical anomaly of magnetic moments, which is outside the scope of the proposed model).

While linearly scaled fields $\phi(m, \mathbf{x}, t)$ and $\mathbf{A}(m, \mathbf{x}, t)$ are satisfying linear approximations of the field equations (for $(\partial^\alpha A^\beta)(\partial_\alpha A_\beta)/b^2 \approx 0$), they do not satisfy the actual nonlinear field equations, which limit the maximum field strength based on the Born-Infeld parameter b, which is assumed to be a constant of nature. The effect of the nonlinearity of the field equations is to require smaller field strengths compared to the linearly scaled fields for the same mass as defined above, which implies a smaller field energy, which would contradict the assumption that the rest mass energy of a particle equals the field energy of the corresponding field solution. In the proposed model, this energy gap is addressed by a quantized excitation, which is discussed next.

4 Quantized Excitation

As mentioned in the previous section, the energy levels of the proposed excitation are supposed to explain an energy gap between linearly scaled fields and actual field solutions for specific masses (i.e., Compton frequencies). To this end, any model of this excitation has to satisfy some requirements; for example:

1. The excitation has to be consistent with the field equations; i.e., it has to be a modification of the scaled field such that the modified field satisfies the field equations for the same mass (i.e., Compton frequency) but with an increased field energy compared to the scaled field. Since the field equations are wave equations, this requirement appears to imply that the peak of the modified field still moves (at least approximately) at the speed of light.

2. The excitation has to be "internal" to the field, i.e., on a scale at or below the Compton wavelength, because the known charged leptons show no differences on larger scales (apart from their mass, stability, and anomalous magnetic moment).

3. The energy levels of the excitation have to be quantized to explain why there are only a few charged leptons (electron, muon, and tau).

These requirements are insufficient to determine the nature of the excitation. As a preliminary hypothesis, the overall structure of the excitation is assumed to be a vertical oscillation of the peak of the rotating field above and below its orbital plane. This is similar to how our solar system is oscillating above and below the galactic plane while orbiting around the center of the Milky Way. It appears reasonable to assume that such a vertical oscillation is possible in the proposed model, and that it would contribute field energy based on its amplitude and frequency.

Furthermore, a quantization of the frequency of this vertical oscillation is hypothesized with the condition that the period of the orbit of the peak is an integer multiple of the period of the vertical oscillation. This is similar to the quantization of an electron's orbit in the Bohr model of atoms with the condition that its circumference is an integer multiple of the electron's de Broglie wavelength. (The hypothetical reason for this condition in the proposed model is that other periods are (more) unstable because they result in (stronger) emission of field energy.) While the frequency of the vertical oscillation is restricted by this quantization, its amplitude is assumed to be restricted by the requirement that the energy contributed by the vertical oscillation matches the energy gap mentioned above.

The ground state of this vertical oscillation is then associated with the electron and positron, the first excited state with the muon and its antiparticle, and the second excited state with the tau and its antiparticle. This implies that the rest mass energy of the field solution with a planar orbit, which is used in Section 3, should be reduced such that the inclusion of the ground state of the vertical oscillation brings the field energy to the rest mass energy of the electron.

This completes the description of the foundations of the proposed model. More (quantitative) details are beyond the scope of this work and have to be left to future work.

5 Discussion

While this work proposes a new model of charged leptons and describes its foundations, many details of the model are still unknown at the time of writing. The foundational ideas are presented here to make them publicly available and inspire work by other researchers, including numerical simulations of the model to test it. The main challenge for this line of work is that the hypothetical excitation (see Section 4) breaks the pure rotational movement, which is a required feature of the field solution in the previously employed numerical simulations [Kra23]. There are, however, various potential approaches to overcome this challenge, for example, a full four-dimensional numerical simulation, which would, however, require considerably increased computational resources. Nonetheless, the presumed existence of a brute-force approach is reason to be optimistic about the possibility of testing the proposed model.

6 Conclusion

This work outlines a new model of charged leptons based on a recently presented modified Born-Infeld model of electrons [Kra23]. While many details of the model are still unknown, numerical tests of the model appear to be possible in principle and are part of ongoing current work. If successful, the proposed model would allow to quantitatively describe features of all known charged leptons based on a limited number of parameters, including the Born-Infeld parameter and possibly further parameters of the hypothesized internal excitation.

References

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A Revisions

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