A mathematical criterion for the validity of the Riemann hypothesis

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abstract

We already know in what situations there will be counterexamples for the Riemann hypothesis, but simply increasing Im (s) to find counterexamples for the Riemann hypothesis is still very slow. If there is only a counterexample when Im (s)=10 ^ 1000, or even 10 ^ 10000, then the performance requirements for the computer are very demanding. So, we must create a numerical order determinant to determine whether the Riemann hypothesis holds.

About ζ(s) =Re（ζ）+ Im（ζ）i

We make s= ½+i * t, can be studied ζ(s)= ζ(½+i * t) Curve about t

Make an Im（ζ）- Re（ζ）curve, we conclude that

Im（ζ）> 0, Re（ζ）> 0 is the first quadrant

Im（ζ）> 0, Re（ζ）< 0 is the second quadrant

Im（ζ）< 0, Re（ζ）< 0 is the third quadrant

Im（ζ）< 0, Re（ζ）> 0 is the fourth quadrant

When the curve of Im（ζ）- Re（ζ）rotates clockwise, there are several possibilities

First Quadrant - Fourth Quadrant, Fourth Quadrant - Third Quadrant, Fourth Quadrant - Second Quadrant, Second Quadrant - First Quadrant, Third Quadrant - First Quadrant

When the curve of Im（ζ）- Re（ζ）rotates counterclockwise, there are several possibilities

Fourth Quadrant - First Quadrant, Third Quadrant - Fourth Quadrant, Second Quadrant - Fourth Quadrant, First Quadrant - Second Quadrant, First Quadrant - Third Quadrant

The remaining two cases, the third quadrant - second quadrant, and the second quadrant - third quadrant, only occur when the Riemann hypothesis has a counterexample

For the Riemann hypothesis, if there is no counterexample, then Im（ζ）- Re（ζ）It is a full curve. If there is a counterexample, it will become a curve in the shape of Bagua, as shown in the following figure

We can make a judgment equation based on this characteristic

Im（ζ） Regarding Re（ζ） Take the derivative to obtain a function g (t) with respect to the slope of t

$$g(t) = \frac{d\,\mathrm{Im}(\zeta)}{d\,\mathrm{Re}(\zeta)} \qquad (1)$$

Then we let g (t) take the derivative of t and obtain the following equation

$$g'(t) = \frac{d\left(\frac{d\,\mathrm{Im}(\zeta)}{d\,\mathrm{Re}(\zeta)}\right)}{d\,t} \qquad (2)$$

One basis for determining whether the Riemann hypothesis is valid is

If there exists t such that g '(t)=0, then the Riemann conjecture has a counterexample

$$\mathrm{Im}(\zeta) = \sum_{n=1}^{+\infty} \frac{\sin(-t\ \ln n)}{\sqrt{n}} \qquad (3)$$

$$\mathrm{Re}(\zeta) = \sum_{n=1}^{+\infty} \frac{\cos(-t\ \ln n)}{\sqrt{n}} \qquad (4)$$

$$d\ \mathrm{Im}(\zeta) = d\sum_{n=1}^{+\infty}\frac{\sin(-t\ \ln n)}{\sqrt{n}}$$

$$= \sum_{n=1}^{+\infty}\frac{\cos(-t\ \ln n)}{\sqrt{n}}\ d(-t\ \ln n)$$

$$= \sum_{n=1}^{+\infty}\frac{-\ln n\cos(-t\ \ln n)}{\sqrt{n}}\ d\ t \qquad (5)$$

$$d\ \mathrm{Re}(\zeta) = d\sum_{n=1}^{+\infty}\frac{\cos(-t\ \ln n)}{\sqrt{n}}$$

$$= \sum_{n=1}^{+\infty}\frac{-\sin(-t\ \ln n)}{\sqrt{n}}\ d(-t\ \ln n)$$

$$= \sum_{n=1}^{+\infty}\frac{\ln n\sin(-t\ \ln n)}{\sqrt{n}}\ d\ t \qquad (6)$$

Therefore, we can obtain

$$g(t) = \frac{d\ \mathrm{Im}(\zeta)}{d\ \mathrm{Re}(\zeta)} = \frac{\sum_{n=1}^{+\infty}\frac{-\ln n\cos(-t\ \ln n)}{\sqrt{n}}}{\sum_{n=1}^{+\infty}\frac{\ln n\sin(-t\ \ln n)}{\sqrt{n}}} \qquad (7)$$

For taking the derivative of g (t), we obtain

$$g'(t) = \frac{d\left(\dfrac{\sum_{n=1}^{+\infty} \dfrac{-\ln n \cos(-t \ln n)}{\sqrt{n}}}{\sum_{n=1}^{+\infty} \dfrac{\ln n \sin(-t \ln n)}{\sqrt{n}}}\right)}{d t}$$

$$= \frac{\sum_{n=1}^{+\infty} \dfrac{\ln n \sin(-t \ln n)}{\sqrt{n}} \; d \sum_{n=1}^{+\infty} \dfrac{-\ln n \cos(-t \ln n)}{\sqrt{n}}}{\left[\sum_{n=1}^{+\infty} \dfrac{\ln n \sin(-t \ln n)}{\sqrt{n}}\right]^2 d t} -$$

$$\frac{\sum_{n=1}^{+\infty} \dfrac{-\ln n \cos(-t \ln n)}{\sqrt{n}} \; d \sum_{n=1}^{+\infty} \dfrac{\ln n \sin(-t \ln n)}{\sqrt{n}}}{\left[\sum_{n=1}^{+\infty} \dfrac{\ln n \sin(-t \ln n)}{\sqrt{n}}\right]^2 d t}$$

$$= \frac{\sum_{n=1}^{+\infty} \dfrac{\ln n \sin(-t \ln n)}{\sqrt{n}} \; \sum_{n=1}^{+\infty} \dfrac{-\ln^2 n \sin(-t \ln n)}{\sqrt{n}}}{\left[\sum_{n=1}^{+\infty} \dfrac{\ln n \sin(-t \ln n)}{\sqrt{n}}\right]^2} -$$

$$\frac{\sum_{n=1}^{+\infty}\frac{-\ln n\cos(-t\ln n)}{\sqrt{n}}\;\sum_{n=1}^{+\infty}\frac{-\ln^{2} n\cos(-t\ln n)}{\sqrt{n}}}{\left[\sum_{n=1}^{+\infty}\frac{\ln n\sin(-t\ln n)}{\sqrt{n}}\right]^{2}}$$

$$=-\frac{\sum_{n=1}^{+\infty}\sum_{m=1}^{+\infty}\frac{\ln n\sin(-t\ln n)}{\sqrt{n}}\;\frac{\ln^{2} m\sin(-t\ln m)}{\sqrt{m}}}{\left[\sum_{n=1}^{+\infty}\frac{\ln n\sin(-t\ln n)}{\sqrt{n}}\right]^{2}}-$$

$$\frac{\sum_{n=1}^{+\infty}\sum_{m=1}^{+\infty}\frac{\ln n\cos(-t\ln n)}{\sqrt{n}}\;\frac{\ln^{2} m\cos(-t\ln m)}{\sqrt{m}}}{\left[\sum_{n=1}^{+\infty}\frac{\ln n\sin(-t\ln n)}{\sqrt{n}}\right]^{2}}$$

$$=-\frac{\sum_{n=1}^{+\infty}\sum_{m=1}^{+\infty}\frac{\ln n\ln^{2} m\left[\sin(-t\ln n)\sin(-t\ln m)+\cos(-t\ln n)\cos(-t\ln m)\right]}{\sqrt{n\,m}}}{\left[\sum_{n=1}^{+\infty}\frac{\ln n\sin(-t\ln n)}{\sqrt{n}}\right]^{2}}$$

$$= -\frac{\sum_{n=1}^{+\infty}\sum_{m=1}^{+\infty}\frac{\ln n \ln^2 m \cos(-t\ln n + t\ln m)}{\sqrt{nm}}}{\left[\sum_{n=1}^{+\infty}\frac{\ln n \sin(-t\ln n)}{\sqrt{n}}\right]^2} \qquad (8)$$

If we set g'(t)=0, then we have

$$\sum_{n=1}^{+\infty}\sum_{m=1}^{+\infty}\frac{\ln n \ln^2 m \cos(-t\ln n + t\ln m)}{\sqrt{nm}} = 0 \qquad (9)$$

For u (t), we can use the same method to obtain

$$u'(t) = \frac{d\left(\frac{d\,\mathrm{Im}(\eta)}{d\,\mathrm{Re}(\eta)}\right)}{d\,t}$$

$$= -\frac{\sum_{n=1}^{+\infty}\sum_{m=1}^{+\infty}\frac{(-1)^{n+m}\ln n \ln^2 m \cos(-t\ln n + t\ln m)}{\sqrt{nm}}}{\left[\sum_{n=1}^{+\infty}\frac{(-1)^n \ln n \sin(-t\ln n)}{\sqrt{n}}\right]^2} \qquad (10)$$

Similarly, we can set u '(t)=0 and obtain

$$\sum_{n=1}^{+\infty}\sum_{m=1}^{+\infty}\frac{(-1)^{n+m}\ln n \ln^2 m \cos(-t\ln n + t\ln m)}{\sqrt{nm}} = 0 \qquad (11)$$

For (11), in cases where accuracy requirements are not high. $t > 14.13412514$, there are $s=0.5+\sigma+i*t$ （$\sigma\neq 0$）is a counterexample of the Riemann hypothesis

References

1. **viXra:2005.0284** The Riemann Hypothesis Proof **Authors:** Isaac Mor