MOVEMENT OF SPACE VERSUS MOVEMENT IN SPACE

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Abstract. Tensor calculus is particularly suitable for plotting the path of an object or wave through a stationary or slowly evolving space time. It is less easily used where the space time itself is violently evolving. I introduce an unusual metric which has some surprising properties and which can cover a vast volume of space time in a single chart. Such a chart suggests that the internals of a collapsed star are often misunderstood, and suggests a practical way for analysing those internals. One surprising property of the metric may have implications for wave theories of matter.

1. Contents

2. Assumptions, conventions, and terms

I assume that space time can be described as curved 3 space varying over time. Coordinate time is everywhere normal to the 3 space. A value of coordinate time identifies a 3 space: I call that 3 space a 'time slice' . If a line element for the time slice at time t is:

$$ds^2 = \sum_{i=1}^{3}\sum_{j=1}^{3} g_t(x_i, x_j)dx_i dx_j$$

then a line element for the space time is:

$$ds^2 = -e^{\nu} dt.dt + \sum_{i=1}^{3}\sum_{j=1}^{3} g_t(x_i, x_j)dx_i dx_j. \tag{1}$$

The same notation can be used to express the magnitude of any one-form, thus if dn is a unit one-form, then $g(n, n) = 1$.

I shall call that space time metric a point orientated metric, or simply a point metric.

3. Properties of the metric

The rate of physical time, e^{ν} in equation (1), is a scalar field on the time slice, so if we change coordinates to (say) $\{X, Y, Z\}$ then each point in the time slice has the same 'rate of time' in the new coordinate system as it had in the old. Less obviously, the value of Ke^{ν}, where K is an arbitrary constant for the space time, can be calculated from the line element of the time slice: this and allied statements are proved in an appendix.

3.1. **The 3 space predicts the future.** The fact that the curvature of a time slice predicts e^ν means that any prediction of future evolution is fully contained in the curvature of the 3 space. The value of e^ν is just an intermediate value, albeit an immensely useful one if the space time is effectively stationary. I will return to this in section 4 item 2.

3.2. **Surfaces with equal e^ν are closed.** They can be thought of as 'contour surfaces', a three dimensional analog of contour lines. The value of e^ν falls as you go 'down hill', into a gravitational well. $e^\nu = 0$ is the surface at which physical time 'stops'. This surface is usually called the event horizon[1].

3.3. **Moving space through the coordinate system.** We have a curved space that evolves over time, and we want a coordinate system that addresses the resulting space time. I break down the task into the following steps.

(1) Choose a mapping from an initial time slice to a 3 dimensional coordinate system. Such a mapping may seek to reflect symmetries inherent in the curvature by symmetries expressed in the coordinate system: consider for example a time slice of Schwarzschild space.

 This step may be much the same for a point metric as for any other metric.

(2) Choose a definition of movement that expresses the first order change of the curvature of a time slice as a definition of the movement of the time slice through coordinate space: consider for example the evolution of a time slice of Kerr space.

 The point metric treats the movement as that of a point in curved space moving through a coordinate system. A more usual metric rolls the motion into the 4 dimensional space time metric, which has the disadvantage of expressing the movement in physical rather than coordinate terms,

(3) Calculate the change in that movement that expresses the second order change of the curvature of the time slice as a change in the movement: consider for example a Kerr space being modified by the approach of a second massive star

 This step is viable for a point metric, and I expect it to help identify where the future of a time slice is dictated by the curvature of the time slice, or equivalently, what possible futures can be explored. I find it hard to see this being practical other than by the use of a point metric.

4. Implications

The characteristics of the point metric suggest two very interesting subjects for research.

(1) The curvature of the surface where $g(t,t) = 0$ remains constant for all time in a point metric, so an event horizon cannot form in a point metric. If any exist, they are ab initio. Any that do form, form after infinite coordinate time. It follows that when collapsing stars collide and merge, their composition can still be described all the way down to their centre. The properties of a point metric show it is possible to explore that composition, and suggest a tool for doing so.

[1]I use that name, but some authors use a different definition of an event horizon.

(2) If the time coordinate adds no further constraint on the future evolution of a time slice, then perhaps quantum mechanics and particle theory would be better expressed in terms of the curvature of space, rather than in terms of the components of the Einstein tensor.

5. Appendix: deriving the rate of physical time from the curvature of a timeslice

This appendix considers a 3 space that is evolving over time, where the space is a timeslice at time $t = T$ of a space time in which special relativity holds at every point on the time slice. I show that the rate of physical time relative to coordinate time at every point on the time slice can be calculated from the curvature of the time slice.

5.1. **Consider a 3 dimensional curved space.** L is any smooth line through this space, and I want a metric that can describe the curvature at every point on the line. I define 3 mutually orthogonal 1-forms, dx, dy and dz, where dx lies along L. The line element at points along L consists of

$$ds^2 = e^\lambda dx^2 + dy^2 + dz^2$$

A fourth 1-form du is parallel to dx, but is a unit 1-form, so $(\frac{\partial x}{\partial u})^2 = e^\lambda$.

If it helps, you can think of x as a coordinate, then $\int_{L_1}^{L_2} dx$ is the coordinate distance between L_1 and L_2, whereas $\int_{L_1}^{L_2} du$ is the physical length of the line from L_1 to L_2.

The relative change of e^λ with unit length along L is $e^{-\lambda}\frac{\partial e^\lambda}{\partial u} = \frac{\partial \lambda}{\partial u}$ which I will write λ'.

5.2. **Allow the space to change over time.** We add in a timelike 1-form dt which is orthogonal to the 3 space, and add $-e^\nu dt^2$ into the line element:

$$ds^2 = -e^\nu dt^2 + e^\lambda dx^2 + dy^2 + dz^2$$

The relative change of e^ν with unit length along L is $-e^{-\nu}\frac{\partial -e^\nu}{\partial u} = \frac{\partial \nu}{\partial u}$ which I will write ν' At any point on the line the square of the speed of light in local physical units is $e^{(\lambda-\nu)}$, and that is constant and well defined along the line L if and only if λ' and ν' are equal and finite. Integrating along the line determines the value of Ke^ν where K is the constant of integration. Thus the curvature of a time slice of space time dictates the rate of physical time relative to normal coordinate time at every point on the three dimensional space.

Although I suggested the reader might find it helpful to think of x as a coordinate, in general x would make a poor coordinate. x is not necessarily monotonic. To put that another way, although $(\frac{\partial x}{\partial u})^2$ is always non-negative, $(\frac{\partial x}{\partial u})$ can change sign depending on whether the line L goes (metaphorically) 'up' or 'down' a gravitational well. Extending the metaphor, the value of ν at a point can be thought of as the gravitational depth of the point relative to some arbitrary zero for the space time.

6. An appendix further describing the advantage of the point metric.

There are no 'correct' coordinate representations of a time slice, but some are more convenient than others. The advantage of the point metric is that it separates the changes of curvature over time from the changes to the coordinate system made for convenience. It is convenient to separate the rate of change with coordinate from the rate of change with time. This can be done if we identify an entity whose curvature remains constant over changes to the coordinate system and whose identity remains constant over time. In a point metric the entity chosen is the point at which a time coordinate intersects the time slice, since that entity has a time line that is independent of the coordinate system. This gives meaning to such ideas as the movement of a point through the coordinate system, and the various derivatives of those properties over space and time.

The properties of a point are curvature, velocity and orientation. There are no 'correct' choices for the velocity and orientation of a point, but again, some are more convenient than others. The curvature at a point on the time slice largely dictates the change of that curvature over time[2], and that change suggests how to modify the velocity and orientation of the point.

A traditional metric represents velocity and orientation in the metric, along with the curvature. This makes it **much** more complicated to adapt the metric reactively.

[2]Only 'largely' because there remain unresolved ambiguities.