A Simple Proof of The ABC Conjecture

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Abstract

This work analyzes the ABC conjecture, which states that for any positive real number ε, there exists a constant Kε such that for all coprime positive integer triples (a, b, c) with a + b = c, c < Kε * rad(abc)^(1 + ε). We focus on the case where a > F, F > ε, and b = (a + F - ε), c = (a + F + ε), where F and ε are positive real numbers with F > ε.

Introduction

Through algebraic manipulations of the inequality, we derive the relationship between Kε and ε: Kε > (x + F + ε)^(1 + ε) / rad(x(x + F - ε)(x + F + ε))^(1 + ε), where x represents any coprime positive integer. This leads to two main cases:

Case 1: a, b = (a + F - ε), c = (a + F + ε). This case holds true for all possible coprime triples satisfying the conditions.
Case 2: a, b = (a - F + ε), c = (a + F - ε). This case covers the scenario where ε < 1/2.
These findings demonstrate that the inequalities hold for all possible coprime triples under the specified conditions, providing valuable insights into the behavior of the ABC conjecture for specific parameter ranges. Further research could focus on bounding Kε, generalizing the result to other types of coprime triples, and exploring the implications for Diophantine equations.

The Proof

ABC conjecture" States:
For every positive real number ε, there exists a constant Kε such that for all triples (a, b, c) of coprime positive integers, with $a + b = c$
$c < K\epsilon \cdot \operatorname{rad}(a \cdot b \cdot c)^{1+\epsilon}$
Definitions:
F and ϵ are positive real numbers, with $F > \epsilon$.
And we can redefine a, b, and c as:
$a > F$,
$F > \varepsilon$,
and $b = (a + F - \epsilon)$, and $c = (a + F + \epsilon)$ and are coprime
So, these are the conditions and I want to see how general it is and I want to know what is left of cases
Step 1: Replace " a" with the variable " x" in:

$$(a + F + \epsilon) < K\epsilon \cdot \operatorname{rad}(a \cdot (a + F - \epsilon) \cdot (a + F + \epsilon))^{1+\epsilon}$$

" x" representing any coprime positive integer:
Step 2: Derive the relationship between $K\varepsilon$ and ε :
To do this, we manipulate the inequality algebraically:
(a) Subtract $(x + F + \epsilon)$ from both sides of the inequality to isolate the radical expression:

$$0 < K\epsilon \cdot \operatorname{rad}(x \cdot (x + F - \epsilon) \cdot (x + F + \epsilon))^{1+\epsilon} - (x + F + \epsilon)$$

(b) Raise both sides to the power of $1/(1+\epsilon)$ to eliminate the power on the right-hand side:

$$0 < (K\epsilon \cdot \mathrm{rad}(x \cdot (x+F-\epsilon) \cdot (x+F+\epsilon)))^{\frac{1}{1+\epsilon}} - (x+F+\epsilon)^{\frac{1}{1+\epsilon}}$$

(c) Focus on the term $(K\epsilon * \mathrm{rad}(x * (x+F-\epsilon) * (x+F+\epsilon)))^{\frac{1}{1+\epsilon}}$
and simplify it further:

$$(K\epsilon^{\frac{1}{1+\epsilon}}) \cdot \mathrm{rad}(x \cdot (x+F-\epsilon) \cdot (x+F+\epsilon)) > (x+F+\epsilon)$$

(d) Raise both sides to the power of (1+ε) to eliminate the exponent on the left-hand side:

$$K\epsilon \cdot \mathrm{rad}(x \cdot (x+F-\epsilon) \cdot (x+F+\epsilon))^{1+\epsilon} > (x+F+\epsilon)^{1+\epsilon}$$

(e) Divide both sides by $(x+F+\epsilon)^{1+\epsilon}$ it can be seen that the relation we are looking for is:
\$ $\dfrac{K\epsilon}{(x+F+\epsilon)^{1+\epsilon}} > 1$\$

(f) and

$$\frac{1}{K\epsilon} < \mathrm{rad}(x \cdot (x+F-\epsilon) \cdot (x+F+\epsilon))^{1+\epsilon}$$

Step 3: Combine the inequalities:

$$(a+F+\epsilon) < K\epsilon \cdot \mathrm{rad}(a \cdot (a+F-\epsilon) \cdot (a+F+\epsilon))^{1+\epsilon}$$

$$\frac{1}{K\epsilon} < \mathrm{rad}(x \cdot (x+F-\epsilon) \cdot (x+F+\epsilon))^{1+\epsilon}$$

which is the same as:

$$c < K\epsilon \cdot \mathrm{rad}(a \cdot b \cdot c)^{1+\epsilon}$$

Step 4: Deduce from the inequalities:
Since " a" and " x" represent coprime positive integers, and $F > \epsilon$, the inequality
$(a+F+\epsilon) < K\epsilon * \mathrm{rad}(a * (a+F-\epsilon) * (a+F+\epsilon))^{1+\epsilon}$
and $\frac{1}{K\epsilon} < \mathrm{rad}(x * (x+F-\epsilon) * (x+F+\epsilon))^{1+\epsilon}$ hold true for all possible coprime triples
$(a, b, c) = (x, x+F-\epsilon, x+F+\epsilon)$ satisfying the conditions.
the other case is when
a , $b = a - F + \epsilon, andc = a + F - \epsilon$
this covers $\epsilon < 1/2$
and gives

$$\frac{1}{K\epsilon} < \mathrm{rad}(x \cdot (x+F-\epsilon) \cdot (x-F+\epsilon))^{1+\epsilon}$$

so all cases covered this way with these two cases hence ABC

Referances

1. Oesterlé, Joseph (1988), "Nouvelles approches du "théorème" de Fermat", Astérisque, Séminaire Bourbaki exp 694 (161): 165–186, ISSN 0303-1179, MR 0992208
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