A new paradox involving the Lorentz transformation

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1 Abstract

A previously unrecognized / unpublished paradox about the Lorentz transformation as interpreted and used by Einstein is presented. This new paradox shows the ultimate futility of explaining reality with Einstein's interpretation of the Lorentz transformation and his use of the Lorentz transformation to derive the equations of time dilation. (Similar agruments would apply to length contraction.) It does not appear that this paradox has been discussed in the literature in the past; however, given the vast amount of research and speculation about the Lorentz transformation that has been published in various languages in the last hundred years, it is very difficult to verify this claim.

2 The Lorentz transformation

The Lorentz transformation is so well known that it can be discussed without reference to any historical literature; an internet search of this topic will reveal literally thousands of references to this topic. Although proceeding from the work of Voigt, FitzGerald, Lorentz and others, it was "co-opted" (i.e., appropriated) by Einstein (1905) and interpreted in a manner foreign to the interpretation of Lorentz, the scientist whose name is associated with the transformation. Einstein used the transformation to develop his famous special (restricted) theory of relativity in which the new idea of time dilation appeared.

The criticisms of the Lorentz transformation herein refer only to the tranformation only when used in the manner advocated by Einstein. The criticisms do not apply to the use of the Lorentz transform as envisaged by Lorentz. In parrticular, these criticisms do not apply to "Lorentz ether theory".

The Lorentz transformation between events ($\vec{r}$, t) in a "stationary" or laboratory frame and events ($\vec{r'}, t'$) in a moving frame and the "reverse" Lorentz transformation between the events in the moving frame and the events in the laboratory frame is given by the equations

$$
\begin{aligned}
x' &= \gamma\,(x - v \cdot t) & \qquad x &= \gamma\,(x' + v \cdot t') \\
y' &= y & \qquad y &= y' \\
z' &= z & \qquad z &= z' \\
t' &= \gamma\,(t - \beta x/c) & \qquad t &= \gamma\,(t' + \beta x'/c)
\end{aligned}
$$

where

$$\beta \equiv v/c$$

$$\gamma \equiv (1 - \beta^2)^{-1/2}$$

and the moving frame is moving along the laboratory frame's x axis with speed v in the positive x direction.

From these equations, Einstein's equations of time dilation, length contraction and proper time can be derived; nearly all contemporary physics textbooks display these equations along with their derivations. For a typical example of these equations and their derivations, see Dalarsson and Dalarsson (2015, pages 105 - 108).

The usual discussion of Einstein's application of the Lorentz transformation involves two – and only two – reference frames. These reference frames are in all – or nearly all – cases assumed to be true inertial reference frames in which Newton's three laws are true. However, in the vast majority of cases no rule is given for defining a prototype (or "primordial" or "boilerplate") inertial frame from which all other inertial frames can be derived by a Galilean boost. But this is another issue, unrelated to the paradox to be presented immediately below.

The "new" paradox relating to Einstein's use of the Lorentz transformation is this. While two – and only two – reference frames are discussed in the context of the Lorentz transformation to derive time dilation (and length contraction), it is possible to envision three or more reference frames, all moving with uniform translational velocity with respect to each other along the common axis of motion (usually specified as the x axis).

In this case, it is possible to label (or specify) the reference frames in a variety of ways. For example, if there are n reference frames all in uniform translational motion, it is possible to designate $n-1$ frames as moving frames and the other remaining frame as a laboratory frame. It is also possible to simply number the frames and to consider them members of the set $\{1, 2, \ldots, n\}$.

Let us adopt the second protocol for identifying the frames. Then the frames will be identified by a number ranging from 1 to n. The paradox is that the clock in frame n can be displaying time at $n - 1$ rates depending on which of the other $n - 1$ frames is being considered. However, it is impossible for a clock to "run" at several different rates simultaneously. This illustrates the absurdity of the use by Einstein of the Lorentz transformation to derive equations for time dilation.

Also, we may ask the questions two related questions:

1) What aspect of reality controls the rate of flow of time in the reference frame identified with the letter n ?

and

2) What controls the rate of flow of time in *any* reference frame?

As was just observed, the rate of flow of time in any reference frame can assume any value (according to Einstein) depending on the motion of a reference frame related to it by a Lorentz transformation.

Similar arguments could be given criticizing the validity of length contraction as a fundamental fact of reality. What determines the "real" length of a meter in any reference frame if various lengths can be vindicated based on the presence of multiple other reference frames ?

It should be noted that this paradox is totally unrelated to the infamous *twin paradox*. The twin paradox arises because events in any reference frame can be interpreted and analyzed in two conflicting ways:

1) The events in any two frames can be related by the two sets of transformations displayed immediately above, i.e., by the Lorentz transformation and the inverse Lorentz transformation.

2) The events in any two frames can be related by simply viewing the moving frame as the stationary frame and vice versa, that is, by considering the "laboratory" frame to be in motion while the "moving frame" is stationary. These two ways of viewing the events are equally valid since all motion is relative.

3 Conclusion

The Lorentz transformation as interpreted and used by Einstein to derive time derivation is totally inadequate to describe reality. It cannot give a reason for the rate of flow of time in any reference frame. It also leads to an unresolvable paradox that renders its use problematic, that is, that a clock in any reference frame must be running simultaneously at multiple different rates.

References

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