Co-Moving Coordinates Cannot Maintain Their Co-Moving Status in the Spatially Non-Flat Cosmological Models

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Abstract

It is thought that consideration of the General Relativity force law demonstrates that particles will retain their stationary status in the standard cosmological models. However this argument neglects the effects of pressure-dependent gravitational forces. When these forces are correctly included, what actually happens is that in spatially non-flat universes particles do not really remain co-moving, and indeed develop motion that is not consistent with the very symmetry condition these models were designed to manifest.

1. Introduction

It is standardly argued [Weinberg, Adler, Ohanian, Hobson], that the vanishing of Γ^i_{00} (where $i = 1$, 2, or 3) in the cosmological models leads to particles staying in place because the $\frac{d^2x^i}{ds^2}$ is $-\Gamma^i_{00}\frac{dx^0}{ds}\frac{dx^0}{ds}$ and this vanishes because of the vanishing of the Γ^i_{00}. However, $\frac{d^2x^i}{ds^2} = -\Gamma^i_{00}\frac{dx^0}{ds}\frac{dx^0}{ds}$ is not the full force equation. The full equation is $\frac{d^2x^i}{ds^2} = -\Gamma^i_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}$–it includes terms of the form $-\Gamma^1_{ii}\frac{dx^i}{ds}\frac{dx^i}{ds}$ (where $i = 1$, 2, or 3). These additional, neglected, terms cause particles to not remain co-moving, contrary to what has previously been concluded.

2. The Structure of Cosmological Models

Let us consider concrete examples. One form of cosmological models is where g_{11} varies with time, and the other metric components are stationary

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in time. We will call this the Type 1 model. Another form is where g_{11}, g_{22} and g_{33} vary in time and g_{00} is stationary in time. We will call this the Type 2A model. And another form is where the g_{11}, g_{22}, g_{33} and also the g_{00} components vary in time. We will call this the Type 2B model.[1]

We will choose the case of a positive spatial curvature. The negative curvature cases are completely analogous to the positive spatial curvature cases for the behavior we are examining, and thus need not be explicitly examined separately.

For positive spatial curvature the metric for the Type 1 version is

$$g_{00} = 1 \tag{1a}$$
$$g_{11} = -1/(1-(r/a(t))^2) \tag{1b}$$
$$g_{22} = -r^2 \tag{1c}$$
$$g_{33} = -(rsin\theta)^2 \tag{1d}$$

For positive spatial curvature the Type 2A metric is

$$g_{00} = 1 \tag{2a}$$
$$g_{11} = -a(t)^2 \tag{2b}$$
$$g_{22} = -(a(t)sin\chi)^2 \tag{2c}$$
$$g_{33} = -(a(t)sin\theta sin\chi)^2 \tag{2d}$$

For positive spatial curvature the Type 2B metric is

$$g_{00} = a(t) \tag{3a}$$
$$g_{11} = -a(t)^2 \tag{3b}$$
$$g_{22} = -(a(t)sin\chi)^2 \tag{3c}$$
$$g_{33} = -(a(t)sin\theta sin\chi)^2 \tag{3d}$$

[1] It is thought that all these models are equivalent, derivable from each other by changes of variables. Thus it would be sufficient to only examine the Type 1 model – or only examine any one model alone – but we will nevertheless examine the behavior of all three.

3. The Dynamics of Particle Motion in Spatially Non-Flat Cosmological Models

Suppose a particle is initially on average at rest with respect to the "co-moving" coordinates. The criterion for it to stay at rest on average with the coordinate system is usually given as $\Gamma^i_{00} = 0$ [Weinberg, Adler, Ohanian, Hobson], under the belief that this insures the particle has no acceleration and thus can remain at rest. However the true full force law is $\frac{d^2x^i}{ds^2} = -\Gamma^i_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}$. Being that that matter in the Universe has pressure, the $\left(\frac{dx^i}{ds}\right)^2$ quantities are in reality non-zero.

In the Type 1 model the non-vanishing of T^{11} interacting with the non-vanishing of Γ^1_{11} causes the $-\Gamma^1_{11}\frac{dx^1}{ds}\frac{dx^1}{ds}$ acceleration term to be non-zero. In the Type 2 models, the non-vanishing of T^{22} and T^{33} interacting with the non-vanishing of Γ^1_{22} and Γ^1_{33} causes the $-\Gamma^1_{22}\frac{dx^2}{ds}\frac{dx^2}{ds}$ and $-\Gamma^1_{33}\frac{dx^3}{ds}\frac{dx^3}{ds}$ force terms to be non-zero.[2]

Thus we see that in all the spatially non-flat models, the matter in the Universe cannot remain on average co-moving. Indeed, because in the early Universe the pressure was large–indeed in the Radiation Dominant Era the $-\Gamma^1_{ii}\frac{dx^i}{ds}\frac{dx^i}{ds}$ terms were non-trivial being that $\left(\frac{v}{c}\right)^2$ was very non-trivial, and because the relevant Γ^1_{ii} quantities were large, the destruction of a co-moving status would be substantial, rather than neglectable.

Worse yet, these forces would cause particles to pick up significant velocities in a particular direction– the "r" direction – leading to the Universe not having the isotropic symmetry that these models were developed to manifest.

References

[1] S. Weinberg *Cosmology.* (Oxford University Press) 2008; page 4

[2]If we were to calculate G^{01}, which is proportional to T^{01}, by inserting the various Type 1 and Type 2 metrics into the Einstein equations differential equation for G^{01} we would find that all the Type 1 metrics give a non-vanishing G^{01} and that all the Type 2 metrics give a vanishing G^{01}. This is peculiar for two reasons. Firstly, since the Type 2 formulations are supposedly equivalent to the Type 1 formulations it is puzzling why they should differ in the implied T^{01} functions. Secondly, since the work in the text proves that T^{01} cannot remain constant for the spatially curved Type 2 metrics, it should not be possible for T^{01} to remain zero for Type 2 situations. These two peculiarities are resolved in future papers.

[2] Adler, Bazin, and Schiffer *Introduction to General Relativity*, (McGraw-Hill, Inc.) 1965, 1975; Page 408

[3] H. Ohanian, R. Ruffini *Gravitation and Spacetime*. (W. W. Norton & Company, Inc.) 1994; Page 412, eq 9.42

[4] M. P. Hobson, G. P. Efstathiou and A. N. Lasenby *General Relativity: An Introduction for Physicists* (Cambridge University Press) 2006; page 358