Modified Alcubierre Warp Drive I:

computation II

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Abstract

A solution of general relativity is presented that describes an Alcubierre [1] propulsion system in which it is possible to travel at superluminal speed while reducing the components of the energy-impulse tensor (thus reducing energy density) by an arbitrary value.

1 Introduction:

Alcubierre [1] in 1994 proposed a solution of the equations of general relativity which provides the only viable means to accelerate a spaceship up to superluminal velocities without using wormholes. A problem was soon identified: Pfenning [4] showed that the required energy is comparable to the total energy of the universe and that it is negative. Moreover he used quantum inequalities to show that this energy gets distributed at very short scale (about 100 times the Planck length) up to a multiplicative factor equal to the squared speed.Later Hiscock [10] proved the existence of an event horizon for superluminal travels which would imply the presence of Hawking radiation responsible for the rapid destruction of the spaceship.

Note: In the following we adopt the notation used by Landau and Lifshitz in the second volume ("The Classical Theory of Fields") of their well known Course of Theoretical Physics [12].

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We propose the use of the metric

$$ds^2=\left(1-v^2\frac{f(x,y,z-k(t))^2}{a(x,y,z-k(t))^2}\right)dt^2+2v\frac{f(x,y,z-k(t))}{a(x,y,z-k(t))}dt\,dz-dx^2-dy^2-dz^2 \qquad (1)$$

or in implicit form:

$$ds^2=dt^2-\left[dz-v\frac{f(x,y,z-k(t))}{a(x,y,z-k(t))}dt\right]^2-dx^2-dy^2 \qquad (2)$$

while Miguel Alcubierre solutions [1] is:

$$ds^2=dt^2-[dz-v\,f(x,y,z-k(t))dt]^2-dx^2-dy^2 \qquad (3)$$

- 1)-The Pfenning zone is the zone within the interval: $R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2}$ where $\Delta\ll 1$ R is the radius of the Warp bubble and Δ is the wall thickness of the Warp bubble $R\gg\Delta$.
- 2)- $r=(x^2+y^2+(z-k(t))^2)^{\frac{1}{2}}$ and $\frac{dk(t)}{dt}=v=const$ (4)
- 3)-In the Pfenning zone we let $a(r)=a(x,y,z-k(t))\gg 1$ and $da(r)/dr\leq a(r)$ (there is the source of esotic matter)

2 Energy-impulse tensor in contravariant form in the Pfenning zone:

The components of the energy-impulse tensor are calculated in the Eulerian reference frame, that is moving with the spaceship.We get for each component of the energy-impulse tensor in implicit form the following:

$$(energy\ density)=-k\frac{1}{4}v^2\left[\left(\frac{\partial g}{\partial x}\right)^2+\left(\frac{\partial g}{\partial y}\right)^2\right] \qquad (5)$$

$$(impulse\ density\ x)=-k\frac{1}{2}v\frac{\partial^2 g}{\partial x\,\partial z} \qquad (6)$$

$$(impulse\,density\ y) = -k\frac{1}{2}v\frac{\partial^2 g}{\partial y \partial z} \tag{7}$$

$$(impulse\,density\ z) = -k\frac{1}{2}v\left[\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}\right] \tag{8}$$

$$(stress\ xx) = k\,v^2\left[\frac{1}{4}\left(\frac{\partial g}{\partial x}\right)^2 - \frac{1}{4}\left(\frac{\partial g}{\partial y}\right)^2 - \left(\frac{\partial g}{\partial z}\right)^2 + (1-g)\frac{\partial^2 g}{\partial z^2}\right] \tag{9}$$

$$(stress\ xy) = k\frac{1}{2}v^2\left(\frac{\partial g}{\partial y}\right)\left(\frac{\partial g}{\partial x}\right) \tag{10}$$

$$(stress\ xz) = k\,v^2\left[\left(\frac{\partial g}{\partial z}\right)\left(\frac{\partial g}{\partial x}\right) - \frac{1}{2}(1-g)\frac{\partial^2 g}{\partial x \partial z}\right] \tag{11}$$

$$(stress\ yz) = k\,v^2\left[\left(\frac{\partial g}{\partial z}\right)\left(\frac{\partial g}{\partial y}\right) - \frac{1}{2}(1-g)\frac{\partial^2 g}{\partial y \partial z}\right] \tag{12}$$

$$(stress\ yy) = k\,v^2\left[\frac{1}{4}\left(\frac{\partial g}{\partial y}\right)^2 - \frac{1}{4}\left(\frac{\partial g}{\partial x}\right)^2 - \left(\frac{\partial g}{\partial z}\right)^2 + (1-g)\frac{\partial^2 g}{\partial z^2}\right] \tag{13}$$

$$(stress\ zz) = -k\frac{3}{4}v^2\left[\left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial y}\right)^2\right] \tag{14}$$

where $g = g(x, y, z-k(t)) = \frac{f}{a} = \frac{f(x, y, z-k(t))}{a(x, y, z-k(t))}$ ([13] for explicit form).

and $k = c^4/8\pi G$; c is the speed of light and G is Newton's gravitational constant;

$a(r) = a(x, y, z-k(t)) \gg 1$ and $da(r)/dr \leq a(r)$ in Pfenning zone.

The functions $f(r)=f(x,y,z-k(t))$ and $a(r)=a(x,y,z-k(t))$ assume the following values:

- 1)-inside the warp bubble $(0<r<R-\frac{\Delta}{2})$ $f(r)=1$ and $a(r)=1$
- 2)-outside the warp bubble $(r>R+\frac{\Delta}{2})$ $f(r)=0$ and $a(r)=1$
- 3)-in the Alcubierre warped region $(R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2})$ $0<f(r)<1$ (Pfenning zone [4]) $a(r)=a(x,y,z-k(t))\gg 1$ (possessing extremely large values) and $da(r)/dr\leq a(r)$

Because of the value $a(r)=a(x,y,z-k(t))\gg 1$ (extremely large) and $da(r)/dr\leq a(r)$ the components of T^{ik} can be reduced by an arbitrary value in the Pfenning zone (source of esotic matter) [13].

Einstein Equation:

$$G^{ik}=\frac{8\pi G}{c^4}T^{ik} \qquad T^{ik} \text{ (energy-impulse tensor) [12]} \tag{15}$$

- 1)-Internal metric of the Warp bubble $(0<r<R-\frac{\Delta}{2})$ is:

$$ds^2=dt^2-(dz-vdt)^2-dx^2-dy^2 \tag{16}$$

moving with velocity v (multiple of the speed of light c) along the z-axis.

- 2)-Metric outside of the bubble beyond the Pfenning zone $(r>R+\frac{\Delta}{2})$ is:

$$ds^2=dt^2-dx^2-dy^2-dz^2 \tag{17}$$

3 Explicit form of energy density in Alcubierre rappresentation:

$$(energy\ density)=-\frac{1}{4}k\,v^2\frac{x^2+y^2}{r^2}g(r) \tag{18}$$

where $g(r)$ is given by:

$$g(r) = [\frac{1}{a(r)^2}(\frac{df(r)}{dr})^2 + (\frac{f(r)^2}{a(r)^4})(\frac{da(r)}{dr})^2 - 2\frac{df(r)}{dr}\frac{f(r)}{a(r)^3}\frac{da(r)}{dr}] \quad (19)$$

if $f = f(r) = f(x, y, z - k(t))$ is:

$$f(r) = -\frac{(r - R - \frac{\Delta}{2})}{\Delta} \quad (20)$$

for $(R - \frac{\Delta}{2} < r < R + \frac{\Delta}{2})$, $\Delta \ll 1$,Pfenning zone [4] then for $a = a(r) = a(x, y, z - k(t)) \gg 1$

and $da(r)/dr \leq a(r)$ term dominant (19) is:

$$g(r) \approx \frac{1}{a(r)^2}(\frac{-1}{\Delta})^2 \quad (21)$$

and energy density is:

$$(energy\ density) \approx -\frac{1}{4} k v^2 \frac{x^2 + y^2}{r^2} \frac{1}{a(r)^2}(\frac{-1}{\Delta})^2 \quad (22)$$

Because of the value $a(r) = a(x, y, z - k(t)) \gg 1$ (extremely large) and $da(r)/dr \leq a(r)$, $\Delta \ll 1$,a(r)>1/ Δ ,the $(energy\ density)$ can be reduced by an arbitrary value in the Pfenning zone, $(k = c^4 / 8\pi G)$.

4 Conclusions

These calculations show that the modified Alcubierre propulsion system can achieve superluminal speeds,and the components of energy-impulse tensor,and energy density,can be reduced by an arbitrary value in the Pfenning zone (source of esotic matter)

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