# Uchida's Identities and Simple Results of $1/0 = 0/0 = \tan(\pi/2) = \cot(\pi/2) = 0$

Saburou Saitoh
**saburou.saitoh@gmail.com**

December 26, 2023

**Abstract:** In this note, we would like to show the simple results $1/0 = 0/0 = \tan(\pi/2) = \cot(\pi/2) = 0$ based on the simple identities that are discovered by Keitaroh Uchida. The logic and results are all reasonable and exceptionally pleasant lookings for high school students.

**Key Words:** Division by zero, division by zero calculus, circular functions, trigonometric function, Pythagorean theorem.

**2010 Mathematics Subject Classification:** 30A10, 30H10, 30H20, 30C40.

## 1 Results

We assume the elementary properties of the division by zero and division by zero calculus. See the basic references. However, here, we would like to point out the simple results

$$\frac{1}{0} = \frac{0}{0} = \tan\frac{\pi}{2} = \cot\frac{\pi}{2} = 0,$$

as follows:

from the identities

$$\cos^2 x + \sin^2 x = 1$$

and

$$\sin 2x = 2\cos x \sin x,$$

we obtain the identity

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = 2\frac{1}{\sin 2x}.$$

By putting $x = \pi/2$, we have

$$\frac{1}{0} + \frac{0}{1} = 2\frac{1}{0}.$$

This means the desired results $1/0 = \tan(\pi/2) = \cot(\pi/2) = 0$.

In addition, by setting $x = 0$, we obtain

$$\cot 0 = 0.$$

(No. 1298: Keitaroh Uchida 2023.12.5)

Meanwhile, from

$$\sin^2 x = (1 + \cos x)(1 - \cos x),$$

we have the identity

$$\frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}.$$

By setting $x = 0$, we have

$$\frac{0}{0} = \frac{0}{2} = 0.$$

By setting $x = \pi$, we have

$$\frac{2}{0} = \frac{0}{0} = 0.$$

(No. 1299: Keitaroh Uchida 2023.12.8)

Furthermore, from the identity

$$\frac{\cos x}{\sin x}\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}\frac{\sin x}{\sin x} = \frac{1}{\sin x}\frac{1}{\sin x},$$

by setting $x = 0$, we obtain

$$\frac{1}{0} + \frac{0}{0} = \frac{1}{0}$$

and so

$$\frac{0}{0} = 0.$$

Note that, in general, for the product $f(z)g(z)$ of two analytic functions $f(z)$ and $g(z)$, for the value of $f(z)g(z)$ at a singular point $z = a$, we can consider its value in the both senses; that is,

$$f(z)g(z)|_{z=a}$$

and

$$f(z)|_{z=a} \cdot g(z)|_{z=a}.$$

Those values are, in general, different, in the division by zero calculus. We should consider separately in the above formula. And for the function

$$\frac{\sin x}{\sin x}$$

at $x = 0$, not by the division by zero calculus

$$\frac{\sin x}{\sin x}(0) = 1,$$

but we here should consider it as follows:

$$\frac{\sin x}{\sin x}(x = 0) = \frac{\sin 0}{\sin 0} = \frac{0}{0} = 0.$$

0f course, for the identity

$$\cot^2 x + 1 = \frac{1}{\sin^2 x},$$

this identity is valid for $x = 0$, by the division by zero calculus, as

$$\frac{-2}{3} + 1 = \frac{1}{3}.$$

**Acknowledgements**

The author would like to express his sincere thanks Mr. Keitaroh Uchida for some pleasant communication with this topics. Indeed, the author is gathering examples on the division by zero and division zero calculus, and these examples were listed with No. 1298 on 5th, December 2023 and No. 1299 on 8th, December 2023, respectively.

# References


[1] H. Okumura, *Geometry and division by zero calculus,* International Journal of Division by Zero Calculus, **1**(2021), 1-36.

[2] S. Saitoh, *Introduction to the Division by Zero Calculus*, Scientific Research Publishing, Inc. (2021), 202 pages.

[3] S. Saitoh, *History of Division by Zero and Division by Zero Calculus*, International Journal of Division by Zero Calculus, **1** (2021), 1-38.

[4] S. Saitoh, *Division by Zero Calculus - History and Development*, Scientific Research Publishing, Inc. (2021.11), 332 pages.