Worldlines and Bordisms II

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December 9, 2023

1 Preamble

Let us begin by defining the worldvolume of a kinetically active particle by the formula

$$\mathcal{F}_{\mathfrak{W}_x} = Vol(\mathfrak{W}_{\{*\}} \times_{\mathbb{C}^n} \sum_{i=0}^{\infty} p_i)$$

in which it is equated with a foliation about a fixed worldline in complex space. Recall that $\mathcal{F}_{\mathfrak{W}_x}$ is a stratification of a bordism W, such that there is an exact sequence

$$Col^{n,m}_{\mathbb{R}} \longrightarrow ... \longrightarrow Col^{n,m}_{\mathbb{R}}$$

indexing the set of fibers $\int_{i=\emptyset}^{|x|} fib_i(d\varphi(n,m))$. This gives us the partioning of a d-brane $\mathcal{B}^d \cong W$ into $Strat_W^{fib(W)}$. We then identify every infinitesimal open cover $\{U_i\}_{i\in\mathcal{I}}$ with a section of a fiber $x_0 \longrightarrow x_{max(n,m)}$, and compute the directional derivative $\vec{\partial}(x)$ fiberwise.

1.1 Holonomy

Let $\Gamma^d : x \longrightarrow y$ denote a C^d-continuous path on objects, and let ∇_x^y denote the connection $\nabla : x \xrightarrow{+k} y$. We then define a *generalized transport cell* to be a sextuple: $\{\Gamma, \nabla, k, x, y, d\}$, and we write $\overset{\Gamma}{\nabla}(k,d)$ for short.

Let $\mathscr{G}_x$ be the isotropy group of a point-like object x, and let $\mathcal{G}_0 = id_{\tilde{x}}$ for some $\tilde{x} \in \mathfrak{X}$ with $\mathfrak{X}$ a topological stack. First, identify the topological realization $|\tilde{x}| = x$ with a differential k-form acting on the kth section of a fiber p of an anonymous stack. Denote by $Hol^{\mathscr{G}}_{x\cong p_k}$ the holonomy groupoid of a particle p_k, or simply $Hol^{\mathscr{G}}_x$.

1.1.1 Holonomy ∞-Groupoid

Let $Hol^{\mathscr{G}}_x$ be the holonomy groupoid, as described above. Recall that a groupoid is a group in which every member is invertible. By extension, $(Aut(Hol^{\mathscr{G}}_x))^{-1}$ exists and is a subset of the class of endomorphisms of the holonomy groupoid.

Let $\mathscr{G}_x^\infty$ denote an infinite isotropy group. We now extend $Hol_x^{\mathscr{G}}$ to an ∞-groupoid $Hol_x^{\mathscr{G}^\infty}$ by replacing every section of the fiber x with an i-cell. This gives us the map $x \xrightarrow{d,i} fib(\varphi_i(x)$, where d is the dimension of the ambient space, and i is the codimension of the section.

It is here that we obtain the isometry

$$fib(\varphi_i(x)) \cong \overset{\Gamma}{\nabla}(k,d) \times_{\mathfrak{St}^{\mathfrak{et}}} \tilde{x}$$

giving us a lucid, totally lossless projective resolution $\mathfrak{X} \longrightarrow \mathfrak{X}^{op} \sim \mathfrak{Y}$.

1.2 Free Loop Superspace

It was shown by Rivera [1] that to any space Y, one can associate a new space LY by equipping the set of continuous maps $S^1 \longrightarrow Y$ with a compact-open topology. In the present discussion, we extend this to a superspace, $L^\sharp Y$, by equipping the set of maps $Map(S^{n,m}, LY)$ with the subspace topology, thereby obtaining a split functor

$$S^d \underset{n}{\overset{m}{\rightrightarrows}} LY$$

where S^d is a d-brane whose underlying Lie group is the d-dimensional (C^d-continuous) circle group.

In the case where we are working with an infinite-dimensional Hilbert space, we obtain the canonical map

$$C_\bullet^\infty \longrightarrow \pi_\infty(LY)$$

where

$$\pi_\infty(LY) = \int_{i=0}^{\infty} d\pi_i(Y)$$

Let $\mathbb{K}$ be a noetherian ring with a $\mathbb{Z}_p$ grading, for p a prime number. Let $\tau(\langle x_{ij}|x_{ji}\rangle)$ be the truth value of a quantum state $q_\diamond$. Fixing the free loop superspace $L^\sharp Y$, one obtains, via a surjection onto the $\mathbb{Z}_p$-modules of $\mathbb{K}$, an ℓ-adic cohomology taking its values in the Novikov ring $Nov_{\mathbb{K}}$.

Thus, $dom(\ell_{coh}) = Nov_{\mathbb{K}}$, and there is an adjunction

$$\forall k \in Nov_{\mathbb{K}} \;\; k \vdash k^\flat \in Tor(\omega_d)$$

In other words, a differential d-form acting as a torsor for a Lie superalgebra $\mathscr{L}_Y$ is the right adjoint to the Novikov localization functor $Nov(\Gamma)$.

2 Bordisms: Revisited

Let $Sing(\mathscr{O})$ be the singularity of an orbifold $\mathscr{O}$, and let $Diff_{\mathscr{P}}$ denote the class of diffeomorphisms of a universal covering $\mathscr{P}$ of some portion of $\mathscr{O}$ under the relationship $\overset{dif}{\sim}$. A bordism, W, is an atlas A_ρ consisting of charts $\phi_i \in \rho$ whose closure $\bar{\phi}_i$ is a proper cover of a set of collars Col_A^ρ.

Recall that, for the "pants diagram", there is a categorification

$$\begin{tikzcd}
 & S^1 \arrow[d] & \\
 & Sing(p) \arrow[dl] \arrow[dr] & \\
S^1 \arrow[uur] & & S^1 \arrow[uul]
\end{tikzcd}$$

which extends to

$$\begin{tikzcd}
 & & S^1_{full} \arrow[dll, bend right, "pr_1"'] \arrow[d, two heads] \arrow[drr, bend left, "pr_2"] & & \\
T_xS^1 \arrow[rr, mapsto, two heads, "sing"] & & \hat{p} \arrow[dl, hook] \arrow[dr, hook] & & T_x\mathcal{S}^1 \arrow[ll, mapsto, two heads, "sing"'] \\
 & S^1 \arrow[ul, hook, bend left, "ind"] & & S^1 \arrow[ur, hook, bend right, "ind"'] &
\end{tikzcd}$$

such that $im(S^1_{full}) = (S^1)^{-1}$, and such that the pushout of every object is either the tangent space of S^1 or the terminal object (one-point compactification). For our purposes, we want to envision $\hat{p}$ as a quantum of some field $\mathcal{P}$, and S^1_{full} as the absolute frame $\mathscr{A}$. Then, each of S^1 are mixed states, and T_xS^1 are permutations of eigenstates; in other words, rings of quasi-quanta. The map $T_xS^1 \twoheadrightarrow \hat{p}$ collapses the wave function via holography by lowering the genus and dimension of the kernel to zero.

One may envision the complex $\hat{\mathcal{P}}$, consisting of the above diagram, as a relatively closed system in which multiple simultaneously evolving thermodynamic states may persist. Under this perspective, the "pants diagram" resembles a multiverse on the order of $\hbar_\mu \times 10^{\frac{1}{2}}$, where $\hbar_\mu$ is the Planck scale.

One very interesting program for studying the topology of bordisms is by constructing a map

$$\mathbb{L}^4 \longrightarrow \hat{\mathcal{P}}$$

from the 4-dimensional Minkowski lightcone to the pants diagram. Shown below is a modal lightcone model of causality:

$$
\begin{tikzcd}
\diamond^2 & & \diamond^2 & & \diamond^2 \\
& \diamond \arrow[ul, dotted] \arrow[ur, dotted] & & \diamond \arrow[ul, dotted] \arrow[ur, dotted] & \\
& & \Box \ni \hat{p} \arrow[ul, dotted] \arrow[ur, dotted] & & \\
& \diamond^{-1} \arrow[ur, squiggly] & & \Box^{-1} \arrow[ul] & \\
\diamond^{-2} \arrow[ur, "|" marking] & & \Box^{-2} \arrow[ul, "|" marking] \arrow[ur] & & \diamond^{-2} \arrow[ul, squiggly]
\end{tikzcd}
$$

where solid lines represent the paths that are actually taken by the bulk of a spacetime, zig-zag lines represent paths that could have possibly been taken, dotted lines represent possible future perturbations of the mechanical system, arrows with lines through them represent the cancellation of potential from a universe, i.e., time-lines that "were never really possible." Squares represent necessity, which is equivalent with actualization, the strictest form of topological realization, which is purely physical or kinetic, while diamonds represent possibility, potential, and complexity.

The set of all solid lines represents the idealized evolution of the entire universe, or in other words,

$$\sum_{i=0}^{\infty} \mathcal{F}_{\mathfrak{W}_i}$$

the set of all possible wordvolumes of all particles in the observable universe.

For every $\hat{p} \in \Box$, there is a linearly-ordered, time-ordered chain

$$... \longrightarrow p'' \longrightarrow p' \longrightarrow \longrightarrow \hat{p}$$

of particles undergoing creation and annihilation operators, which successively result in the evolution of a quantum state towards the physical realization of $\hat{p}$ at the time $t = 0$. Physically speaking, this is represented by the evolution:

$$\underset{t \to 0}{lim}\ t(\rho_{mn})$$

Recall from [2] the definition of ρ_{mn}, which is due to the second named author.

Geometrically, it is enticing to visualize the creation of a closed system $\hat{\mathcal{P}}$ out of a universal[1] lightcone $\mathbb{L}^4$ by first *twisting* the lightcone, then *folding* it. This gives us a closed envelope of wave packets.

Write $Env(\hat{\mathcal{P}})$ for the enveloping algebra of a closed system $\hat{\mathcal{P}}$, where $\hat{p} \in Env(\hat{\mathcal{P}})$ is the stabilizer, and $\tilde{p} \in S^1_{full}$ is the uniformizer. By applying the

[1]Perhaps, more aptly, multiversal

creation map to $Env(\hat{\mathcal{P}})$, one obtains a bundle of self-dual chains of holonomy groupoids, $Bun_{Hol_p^{\mathscr{G}}}$.

Proposition 1. *The map*

$$Bun_{Hol_p^{\mathscr{G}}} \longrightarrow Hol_x^{\mathscr{G}}$$

is an epimorphism when x belongs to the algebra of observables.

A proof of the above proposition is outside the scope of the current paper, but it is essential to our understanding of the *creation operator*, Cr_x.[2]

2.1 Twisted D-branes

Let Ψ_θ denote the set of θ-diffeomorphic Schrodinger invariants. One may pick an element $\theta_k \in \Psi_\theta$, and assign to it a small, smooth category $\mathscr{C}$ of functors whose essentially image is open in some topological stack $\mathfrak{X}$. Then, for every convex subset of a topological space X with the subspace topology, one can induce, via isogeny, the Dirac operator of a Tate module over the vect-enrichment of $\mathscr{C}$ by writing

$$\mathscr{C}_{Vect} \ltimes \delta_i^{Tate}(X)$$

Essentially, we take some unramified space $\mathscr{A}$, with enough surjections onto the super-circle $S^{n,m}$, and we "mod out" by an anonymous homotopy invariant, which is the tensor product of 2-torsion terms over the underlying algebra of $\mathscr{A}$. In effect, we end up with some "locally small"[3] (δ_i-small) measurable space, call it $\#\mathcal{B}^d$.

We may want to make some further finiteness assumptions about $\#\mathcal{B}^d$. Assume that it is a slice of $\mathbb{L}^\infty$. Then, there must be some exact sequence

$$Pur \longrightarrow \mathbb{L}^\infty \longrightarrow \#\mathcal{B}^d \rightsquigarrow sSets$$

such that we can regard the twisted brane as a simplicial set. This sets us up[4] quite nicely to discuss an interior algebra[5] acting on $\#\mathcal{B}^d$.

Write $sk_{\#\mathcal{B}^d}$ for the skeleton of a twisted d-brane, and write $\mathcal{N}^n_{\#\mathcal{B}^d}$ for its nth nerve. Recall that the nth nerve of a skeleton is a fiber of rank n which is bijective for both the source and target of the map. The first nerve of the skeleton is called the *Koszul nerve*, and it is an absolute bijection.

[2]Ibid

[3]With respect to some ring $\mathbb{K}$, where $i \in \mathcal{I}$ is an element of the Dirac index of a particle's wave function, and δ is the Kronecker delta

[4]Pun quasi-intentional

[5]Perhaps an algebra of observables

Example 1. *The map* $sk_{\#\mathcal{B}^d} \longrightarrow q$ *to a quantum* q *consisting of* n *entangled Bloch spheres is the simplicial realization*

$$\mathcal{N}^n_{\#\mathcal{B}^d} = \hat{q}^n \times S^{n,m}$$

Example 2. *For a wordline* $\mathfrak{W}_x$ *located in* $\mathcal{B}^d \cong S^{n,m}_{conf}$, $\#\mathcal{B}^d$ *is analogous to an extension of* $\hat{\mathcal{P}}$ *into its ambient space. The correspondence*

$$\#\mathcal{B}^d \cong X \subset CFT$$

is the stringy AdS*-*CFT *correspondence.*

3 References

[1] M. Rivera, *An algebraic model for the free loop space*, (2023)
[2] R.J. Buchanan, P. Emmerson, *Scattering of Wordlines Along a Bordism* (2023)