Dark Energy, MOND and the Mirror Matter Universe.

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Abstract

The purpose of this study is to entrench the Copernican principle into cosmology with regard to dark energy (DE). A dual-universe solution is proposed for both the scale and coincidence problems of DE which is simple and involves no 'fine-tuning'. It is also, in principle, testable and falsifiable. The model enables computation of the total entropy of the universe contained within the horizon expressed holographically projected onto the area of the cosmic horizon in units of Planck area. We subsequently compute the Planck entropy, which takes an irreducibly simple form. A derivation of the relation $[DE] = \sqrt{m_{pl}.H_0}$ is provided and we further show that this relation is valid in all (local i.e. $H'_\tau = H'_0$) observer frames. We prove that the vacuum energy is exactly zero in this dual universe model. Lastly we propose that our analysis implies that the MOND paradigm is due to gravitation interaction of the two universes and we compute the MOND acceleration scale a_0 and scale invariant $.\mathcal{A}_0$ as a consequence of cosmology, completely independent of galaxy dynamics. Significantly, this allows us to bring the MOND paradigm into a cosmological model without modifying General Relativity.

I. INTRODUCTION

The *Copernican Principle*, named after the great Polish astronomer and economist Nicolaus Copernicus, is the idea that human observers are not physically located at the center of the solar system. Copernicus proposed that the Earth rotated around the Sun, rather than the other way around[1]. It is difficult, from our contemporary perspective, to fully appreciate what a shocking notion this must have been to those versed and fully subscribed to the Ptolemaic conceptual framework dominant at the time of Copernicus; for them, it was a self-evident truth that the Earth was stationary. In its modern incarnation, the Copernican Principle is extended to the idea, confirmed by observation, that human observers are not physically located at the center of the galaxy and nor is our Milky Way galaxy in any sense exceptional in terms of its physical location in space, being just one of billions of similar galaxies we can see with telescopes. The discovery of dark energy [1], [2] and the subsequent realization that this energy density constitutes the bulk of the energy budget of the universe at $z = 0$ however introduces a new kind of Copernican issue; the issue of our particular place

[1] The ancient Greek philosopher Aristarchus should be accorded primacy of Heliocentric cosmology however.

in *cosmological time*, as distinct from our particular location in three-dimensional space [3]. One way to express a *temporal* Copernican principle might be the following; "no observer exists at a *time* that confers a privileged view of the universe nor a privileged *time* that permits measurement of the structure of the universe nor any of its intrinsic properties or laws such that they could not be gleaned by observers at different *times*".

Before progressing further we need to clarify what we mean by 'laws of the Universe'. Such 'laws' are most inappropriately named, since they refer not to human legal constructs but rather to intrinsic patterns present in nature itself that, generally, have mathematical representations that reflect simplicity and congruence. Whilst the precise details of the mathematical symbolism used to represent these 'laws' is indeed human generated, the generic feature of 'physical law', indeed its essential feature, is that sentient intelligent beings on some far removed world find precisely the same 'laws of the universe' that we humans do on Earth. The Copernican Principle then becomes a broader and more encompassing expression of the second part of the Principle of Relativity; that all observers find the same 'laws' of the Universe regardless of their relative motions.

The standard ΛCDM cosmology model raises two major fundamental unresolved issues with regard to 'dark energy' (DE). The first is the so-called *scale problem* [4], which is why the measured energy density $[DE]^4 \approx (10^{-3}eV)^4$ at $z = 0$, is so much smaller than the mass scale of the standard model fields (with the exception of the neutrinos) and vastly lower (120 orders of magnitude) than the Planck scale (i.e. $(m_{pl})^4$) [5]. The 'scale' of the DE is also curiously situated as the mean of cosmological parameters since $10^{-3}eV \approx \sqrt{m_{pl}c^2}\sqrt{\hbar H_0}$. A second, related but nevertheless distinct, problem is the *coincidence problem*, which refers to the fact that the dark energy transition from matter dominance (combined putative dark and luminous) to dark energy dominance occurs at $z \approx 0.5$ for $\omega_{DE} \approx -1$, which makes the DE detectable and measurable (i.e can measure the energy density and equation of state) for an observer at $z = 0$ but not for one at $z > 1$ or in the distant future; observers at redshifts $z > 1$ would be able to detect the expansion of the universe but would have great difficulty detecting the *accelerated expansion*, whilst observers in the far future, when extra-galactic structure had been expanded beyond the horizon, would not be able to observe extra-galactic type-1 supernovae. Neither observer type would be able to measure the equation of state of the dark energy, a requisite for characterizing its nature. Thus the Copernican principle is violated since our current perspective appears privileged. Moreover it appears to violate that part of the principle of relativity that requires all observers to find the same laws for the universe, since surely the equation of state of the dark energy is part of the laws of the universe and if only some observers, but not others, exist at an appropriate time in the history of the universe to measure the EOS of the dark energy then different observers have different laws of the universe by omission.

Logically, either the Copernican principle is preserved or it is not. The latter would indicate we do indeed exist at a privileged time in the history of the universe and the Copernican principle is overturned. (The anthropic principle [6], which has been used successfully by Weinberg [7] to account for the scale problem, cannot save us here because there seems no a-priori reason why the ability to measure the DE equation of state should be a requisite for intelligent life). The other alternative is that the Copernican principle is preserved, which is equivalent to saying that *all* observers are in a position to observe and measure the dark energy transition, regardless of the time in the history of the universe in which they exist; i.e. since we appear to live at a privileged time in the history of the Universe able to characterize the DE, the only way to preserve the Copernican principle is to ensure that *all observers* are afforded the same privilege. This requires that the expansion history of the universe is observer dependent. Such an outcome is impossible with preservation of normal causality. The Copernican principle with regard to DE also requires the DE to be the causal agent responsible for ensuring that the expansion history is observer dependent. This, in turn, is only possible if the dark energy is *anti-causal*; that is, it represents a diffuse energy bath literally propagating in the reverse direction of time and changing the past expansion history of the universe in such a way that all observers measure the transition to dark-energy dominance at approximately $z' = 0.5$, where z' represents the red-shift in their local frame of reference. This sequence of logic is unique and exclusive; no other possibilities exist that preserve the Copernican principle with regard to measurement of the dark energy EOS. One could, for example, postulate that the dark-energy is a scalar field 'rolling-down' a potential well [35], [11] such that it matches the measured density for an observed transition to DE dominance at $z \approx 0.5$ in our frame of reference [12], [13], [14]; however, not only does this raise a 'fine-tuning' issue, it exacerbates the conflict with the Copernican principle since it entrenches the privileged observer position that we appear to have at the current time in the history of the universe at the expense of observers in the far past or the far future. Scalar tachyon fields have also been considered [15] [16] [17] as potential dark-energy candidates.

There is also a broader issue regarding causality with the respect to the 'dark sector' (i.e. both putative dark matter and dark energy) as a whole because the foundation of our concept of causality is Special Relativity and this is in turn founded on the basis of the co-ordination of clocks using light beams. It is thus not clear that we can carry over our concept of causality to the dark sector given that it has no apparent interaction with light. Instead, the range of theoretical possibilities considered should include the possibility that causality is violated in the dark sector.

The paper is organized as follows. In section II the basic ideas of Separation Geometry are introduced with a focus on the nature of time. Satisfying the Separation Geometry definition of time underlies the dual universe model we shall invoke to explain the dark energy. Section III introduces a new kind of dual universe model based on the symmetry principles outlined in the earlier section that requires the existence of 'mirror matter'. Section IV exploits the symmetry of the two universes to compute the entropy of the universe. This subsequently allows us to compute the Planck entropy, which takes an extraordinarily simple form. Section V tackles the issue of quantum gravity in the context of dark energy. We derive as a result of this computation the MOND (Modified Newtonian Dynamics [18] [19] [20]) acceleration scale a_0 and the MOND invariant $\mathcal{A}_0$ from fundamental physical principles as a consequence of cosmology. Testing of the theory is discussed in VI. The conclusion is in section VII.

II. SEPARATION GEOMETRY AS A METHOD OF DIMENSIONAL ANALYSIS.

Let us then focus on the possibility that the Copernican principle is preserved. This possibility violates time-reversal symmetry and causality because the only way to abolish the apparent privilege we humans appear to have at the current age of the universe (with respect to the DE) is to ensure all observers enjoy the same perspective which, in turn, requires the past history of the universe to be variable and observer dependent. This requires the DE to literally alter the past on the largest of scales such that all observers find the transition from matter to DE dominance at $z \approx 0.5$ *in their local frame of reference.* Microcausality (i.e. time scales and distances significantly smaller than the horizon) we expect to remain intact.

Separation Geometry (SG) [21] is a novel means of dissecting out the dimensional structure of the universe and is applicable to the time dimension (the material on time presented here is new work not included in the above citation). SG leverages set theory to describe physical structure. A brief exposition is as follows; the *fundamental postulate* of SG is that the reason mathematics can be used to describe physical structure is that the structure of the universe (physical structure) and mathematical structure are isomorphic at the level of the foundation of structure. This foundational element is proven by simple logic in [21] to be a quantized duality consisting only of two separate points. Geometrically it is a one-dimensional 'irreducible interval' (I.I.). No points exist on the interval except at the two terminations. The I.I. represents a set of finite cardinality (the cardinality of the set is the number of elements in the set). The I.I., when expressed as a quantum field, is a massless spin-2 object [21] which mandates General Relativity as the correlative classical description [22] of gravity within the SG model. Thus Separation Geometry differs fundamentally from string theory in that the 'length' scale of the 'string' is quantized in a manner that makes it unmeasurable; the 'length' of the 'string' is unity regardless of the location of the two terminal points that define the interval, which can be the size of the horizon or the space spanning the Planck length. Iterative feedback of the concept of separation based on tiers of cardinality of number fields is then used to generate additional dimensions. A two-dimensional area bounded by intervals represents the cardinality $\aleph_0$, the transfinite cardinality of the field of rational numbers and, assuming the continuum hypothesis (i.e. that no number $\aleph_1$ exists, such that $\aleph_0 < \aleph_1 < c$, where c is the cardinality of the real number field), a third order of separation defines three-dimensional volumes bounded by irreducible intervals and irreducible areas. The volume of space so defined would then constitute a continuum space based on the real number field provided the continuum hypothesis is applied. In SG, the continuum hypothesis is the source of the three space dimensions of the universe. However, in keeping with the *fundamental postulate*, Gödel's incompleteness theorem [24] requires that we also admit the negation of the continuum hypothesis into physical structure, since the continuum hypothesis is a formally undecidable proposition [25] [26]. This then requires the dual but distinct definition of the real continuum to be four-dimensional (i.e. finite, $\aleph_0, \aleph_1, \&$ c orders of cardinality). It is the requirement to physically manifest both the continuum hypothesis and its negation which is the source of the time dimension in SG. The direction of the time dimension is also intrinsically embedded in this structure, and is a function of whether $\aleph_1 < c$ (forward time) or $\aleph_1 > c$ (reverse time, where c is the transfinite cardinality of the continuum but in this context can conceptually be correlated with the speed of light). This structure thus explicitly breaks time reversal symmetry.

Let us unpack this structure further. In SG a three-dimensional bounded volume (3-form) represents a massive quantum field of some kind. As a forward-time object (negation of the continuum hypothesis) the cardinality of its internal space is thus $\aleph_1$ and to define a continuum space (assumed a requirement for all observable states) requires that it has continuous motion in the fourth dimension of time; that is, a co-moving clock must never stop. By providing a 3-form with continuous motion in the time dimension the cardinality of the contained bounded space is elevated from $\aleph_1$ to c, the cardinality of the continuum. A photon in SG is a two-dimensional object (2-form) that defines cardinality $\aleph_0$ but is never stationary in the frame of any 3-form, thus sweeping-out a finite volume equivalent space of cardinality c in a finite time (i.e. the cardinality level of $\aleph_1$ is bypassed by the photon, which is equivalent to saying co-moving clocks would be stopped in the frame of the photon; the continuum hypothesis is preserved in the photon frame). Since the photon can never be stationary in the intertial frame of any 3-form but must always have finite relative velocity regardless of how any 3-form changes its velocity with respect to any other 3-form, one can infer that the relative velocity of the photon must always be a finite constant with respect to any observer 3-form frame of reference, consistent with the first part of the principle of special relativity (in this case mandated by geometry rather than an input assumption). Three-forms with reverse time propagation, i.e. $\aleph_1 > c$, define a space of intrinsic cardinality $\aleph_1 (\equiv \aleph_2$ in this context) that is 'more numerically dense' (represents a number field of higher cardinality) than the continuum from the perspective of the forward-time observer. This indicates that it is a tachyon, and 'sweeping-out' a higher-density space than the photon or any subluminal forward-time massive field quanta due to propagation faster than light. There is thus an issue with the dimensionality of a 3-form with cardinality $\aleph_1 > c$ but this issue is resolved with the dual symmetry which we will subsequently develop.

There are thus five possible tiers of cardinality in this system; $\{finite, \aleph_0, \aleph_1, c, \aleph_2\}$ corresponding to at most a 5-dimensional Universe. $\aleph_2$ here replaces supra-luminal $\aleph_1$ in five dimensions and is treated as the *supreme cardinal*, whilst c is the cardinality of the continuum (or equivalently the cardinality of the real number field). Geometry is built-up step-wise based on self-dual forms that are invariant under affine transformations (for a formal definition of affine transformations see [21]); there is one such self-dual form for each tier of dimension. These can be represented by Platonic geometries (irreducible Platonic forms are invariant under affine transformations); they are, in order of cardinality, the I.I., the irreducible triangle, the tetrahedron, the 4-dimensional analogue of the tetrahedron (which we shall call a 'Penton') and the 24-cell geometry in 4-dimensions which constitutes a 5-dimensional space with a four dimensional boundary in SG. Only three space dimensions are possible in SG as this is the most reduced (irreducible) expression of the continuum (i.e. continuum hypothesis defines space). All other dimensions are time-related. This means that the two additional dimensions in the 24-cell 5-D geometry represent two orthogonal time dimensions [2]. The evolution from a 5-dimensional space to a 4 dimensional space-time involves the sandwiching of the two time directions into a single time dimension with two possible directions in time. At the level of 5 dimensions these two dimensions of time are completely symmetrical, but this symmetry is broken by the decomposition to a 2×4 space-time with the direction of time in our universe emerging as a result of the breaking of this symmetry. The bulk of the material in this study is directed towards entrenching this symmetry in our description of the Universe and exploiting the implied symmetry to calculate quantities such as entropy and gravitational energy.

To place this definition of time on a more metric-based footing, and to understand better the transition from 5 to 4 dimensional space-time (which we shall show in due course corresponds to the transition from supra-Planckian scales to sub-Planckian scales) we begin with a very unusual 5-dimensional metric based on the quaternions describing the two time dimensions as a substitution for the Lorentz metric [3];

$$\mathscr{G}_{\Theta\Lambda} = \begin{pmatrix} K & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -K \end{pmatrix} \tag{1}$$

where the central 3x3 square-root of identity sub-matrix represents the space dimensions and the quaternion components satisfy $I^2 = J^2 = K^2 = -1$. Thus $det(\mathscr{G}_{\Theta\Lambda}) = +1$. The two distinct time dimensions, represented by the K index, ultimately will code for different directions in time. An explicit representation is furnished by the Pauli spin matrices; $I =$

[2] Sakharov was the first to consider the possibility of two time dimensions [23].

[3] This metric is restricted to *inside* black holes, to scales beyond the Planck mass and may possibly also be relevant for inflation theory.

$i\tau_1,\ J = i\tau_2,$ and $K = i\tau_3$ with the commutation relation; $[I, J]_- = 2\varepsilon_{IJK}K;\ \varepsilon_{IJK} = +1.$

A corresponding 5-component coupling unit vector is given by;

$$V^{\Theta} \equiv \begin{pmatrix} I.t_1 \\ x \\ y \\ z \\ J.t_2 \end{pmatrix} \quad \text{or, for row vectors;} \quad (I.t_1, x, y, z, J.t_2) \tag{2}$$

where $t_{1,2}$ parametrizes the two time dimensions and {x, y, z} are the three space dimensions. We use units where $c \equiv 1$ for the speed of light.

We will require that, in the formation of a scalar product, the I quaternion containing component of expression (2) contracts with the J containing component. To achieve this new kind of scalar product we now define a new mathematical operation called *mirror conjugation*, a mathematical innovation developed specifically to deal with quaternion based metrical systems [27], which acts on a matrix M with elements $a_{i,j}$ which has r rows and c columns symbolized with the superscript $\tilde{m}$ as follows;

$$\left(M^{\tilde{m}}\right)_{k,l} = \left[(a_{i,j})^{\tilde{m}}\right]_{k,l} \equiv (a^{\tilde{m}})_{k=(c-j+1),l=(r-i+1)}$$

If the elements $a_{i,j}$ of matrix M are c-numbers, c say, then $c^{\tilde{m}} = c^*$ where $*$ is the complex conjugate. Thus for a 3 x 3 square matrix whose elements are c-numbers we then have, for example, for the first row of components,

$$(a_{1,1})^{\tilde{m}} = \left(a^*_{1,1}\right)_{3,3},\ (a_{1,2})^{\tilde{m}} = \left(a^*_{1,2}\right)_{2,3}, (a_{1,3})^{\tilde{m}} = \left(a^*_{1,3}\right)_{1,3}$$

where $*$ is the complex conjugate; and so-on for the other rows. More generally;

$$(\alpha, \beta, ..., \delta, \gamma)^{\tilde{m}} = \begin{pmatrix} \gamma^{\tilde{m}} \\ \delta^{\tilde{m}} \\ . \\ . \\ . \\ \beta^{\tilde{m}} \\ \alpha^{\tilde{m}} \end{pmatrix}, \text{ and } \begin{pmatrix} \gamma \\ \delta \\ . \\ . \\ . \\ \beta \\ \alpha \end{pmatrix}^{\tilde{m}} = (\alpha^{\tilde{m}}, \beta^{\tilde{m}}, ..., \delta^{\tilde{m}}, \gamma^{\tilde{m}})$$

that is, $(M^{\tilde{m}})^{\tilde{m}} = M$.

The easiest way to visualize $\tilde{m}$ for a square matrix is a reflection over the diagonal orthogonal to that upon which the trace of the matrix is based (e.g., in a 2x2 matrix the (1,1) (i.e. 1^{st}row, 1^{st}column), entry exchanges with the (2,2) entry and the (1,2) and (2,1) entries remain in position) followed by mirror conjugate of each individual element of the matrix; if these are c-numbers then the mirror conjugate is just complex-conjugation of the c-number.

The m-transpose is consistent for any order of square matrix and any order column or row vector (it can also be applied consistently to other non-square matrix products). The following relations are valid to all orders;

$$(A+B)^{\tilde{m}} = A^{\tilde{m}} + B^{\tilde{m}},\ (AB)^{\tilde{m}} = B^{\tilde{m}}A^{\tilde{m}},\ (A^{\tilde{m}})^{\tilde{m}} = A.$$

Mirror conjugation is one possible solution to the problem of defining a transpose for quaternion systems.

Note that with the modified Pauli matrix representation of the quaternions we have $I^{\tilde{m}} = -I,\ J^{\tilde{m}} = J$, and $K^{\tilde{m}} = K$. The sign on the K-entries in metric eq.(1) however changes under an m-transpose, which is an effective interchange of the roles of t_1 and t_2. The I and J quaternions are multiplied by a time co-ordinate to represent their time base; the m-transpose interchanges t_1 with t_2. Thus $(I.t_1)^{\tilde{m}} = -I.t_2$ etc. We have;

$$(\mathscr{G}_{\Theta\Lambda})^{\tilde{m}} = \begin{pmatrix} -K & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K \end{pmatrix} \quad and \quad (I.t_1, x, y, z, J.t_2)^{\tilde{m}} = \begin{pmatrix} J.t_1 \\ z \\ y \\ x \\ -I.t_2 \end{pmatrix} \tag{3}$$

(note the sign change of the I quaternion under the m-transpose and the time index switch) and we have $\mathscr{G}\mathscr{G}^{\tilde{m}} = \mathscr{G}^{\tilde{m}}\mathscr{G} = I_5$ (a metric with this property we will call *m-symmetric*; note that the 3x3 space-dimension sub-matrix in eq.(1) is *m-invariant*, i.e. $S^m = S$). Using the vectors defined in eq.(3) and the metric eq.(1) we have;

$$s^2 = V^{\Theta}\mathscr{G}_{\Theta\Omega}V^{\tilde{\Omega}} = t_1^2 - x^2 - y^2 - z^2 - t_2^2 = (t_1^2 - t_2^2) - x^2 - y^2 - z^2 \tag{4}$$

where $V^{\tilde{\Omega}} \equiv (V^{\Omega})^{\tilde{m}}$. We see that in the 5-D space with two time dimensions there is no speed limit, since we can set $t_2 > t_1$, which permits faster-than light propagation; in that circumstance however, s^2 is negative and the interval s is pure imaginary. However, there is complete symmetry between the two time dimensions since forming s^2 with $\mathscr{G}^{\tilde{m}}$ interchanges the roles of t_1 and t_2.

Dimensional reduction can be represented by setting one K-entry to zero in the metric $\mathscr{G}$ and forming an m-product. Call matrix $\mathscr{G}_{\Theta\Omega}$ with the top left K (and any other first row or first column entries) set to zero $({}^{0}\mathscr{G})_{\Theta\Omega}$ and if we set the bottom right-hand corner metric entry (as well as any remaining 5th row and 5th column entries) similarly to zero as $({}_{0}\mathscr{G})_{\Theta\Omega}$, then (setting ${}_{0}\tilde{\mathscr{G}} \equiv {}_{0}(\mathscr{G}^{\tilde{m}})$);

$$({}^{0}\mathscr{G})_{\Theta\Omega})\{({}_{0}\tilde{\mathscr{G}})\,{}^{\Omega}{}_{\Delta}\}^{\tilde{m}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \tag{5}$$

which is a 3+1 dimensional space-time but with the direction of time indicated by the zeroed first dimension. Alternatively we can similarly contract $\mathscr{G}$ as $({}_{0}\mathscr{G})_{\Theta\Omega})\{({}^{0}\tilde{\mathscr{G}})\,{}^{\Omega}{}_{\Delta}\}^{\tilde{m}}$ which creates the other possible time orientation with zeros in the fifth row and fifth column entries and -1 in the first row, first column entry.

The interpretation of this is that the lack of time asymmetry in the laws of physics reflects a deficiency in the construction of the mathematical foundation rather than the nature of reality. Two universes emerge from the decomposition of the supra-Planckian space-and-dual-time, each with its own copy of the Lorentz metric but with different time dimensions. There are thus two universes in this theory of time and a representation of their respective Lorentz metrics in the embedding 5-D geometry are;

$$Universe\ R = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1_{x'} & 0 & 0 & 0 \\ 0 & 0 & 1_{y'} & 0 & 0 \\ 0 & 0 & 0 & 1_{z'} & 0 \\ 0 & 0 & 0 & 0 & -1_{t2} \end{pmatrix} \quad \text{and} \quad Universe\ L = \begin{pmatrix} -1_{t1} & 0 & 0 & 0 & 0 \\ 0 & 1_{x} & 0 & 0 & 0 \\ 0 & 0 & 1_{y} & 0 & 0 \\ 0 & 0 & 0 & 1_{z} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{6}$$

Each universe is subluminal *in its own frame of reference* with precisely the same physical structure of standard model fields and the same laws of physics. The 'fifth' dimension imposes a direction on the time dimension by virtue of the fact that t_1 and t_2 code for tachyons and subluminal states with these roles inter-changable by an m-transformation on the metric (corresponding to the two possible choices for sign pairing of K's in the 5-metric eq.(1) which interchanges the signs on t_1 & t_2 in eq.(4); we recover the conventional Lorentz metric by eliminating the zeroed 1st or 5th row and column entries consistent with the 5th dimension not being within the observer's frame). An observer in one universe however can only observe one dimension of time and thus can only perceive the other universe as an m-transposed state with respect to his or her time dimension. We cannot however naively impose an m-transformation on Universe R's metric in eq.(6) because we have zeroed all the first row and first column entries, eliminating the t_1 time. We must instead use a reduction that forces t_2 into the metric frame of t_1, which here is $({}_0\mathscr{G})\{({}^0\mathscr{G})^{\tilde{m}}\} = (U_R)^{\tilde{m}}$;

$$U_L \otimes (U_R)^{\tilde{m}} = \begin{pmatrix} -1_{t1} & 0 & 0 & 0 & 0 \\ 0 & 1_x & 0 & 0 & 0 \\ 0 & 0 & 1_y & 0 & 0 \\ 0 & 0 & 0 & 1_z & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1_{t1} & 0 & 0 & 0 & 0 \\ 0 & 1_{z'} & 0 & 0 & 0 \\ 0 & 0 & 1_{y'} & 0 & 0 \\ 0 & 0 & 0 & 1_{x'} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{7}$$

Eq.(7) describes the viewpoint of an observer in Universe L; they inhabit a four-dimensional space-time with Lorentz metric with their direction of time fixed relative to the un-measurable 5th dimension as t_1. The transformed Universe R metric into the time frame of t_1 now represents tachyons in the frame of Universe L but, because it is the identity in the 4D frame of reference, the tensor product of metrics leaves Universe L metric invariant. Thus we see that the appropriate treatment for an m-transpose on Universe R's metric in eq.(6) is to set $(1_{t2})^{\tilde{m}} = -1_{t1}$ (even though this is a scalar identity it carries a specific time coupling which must change its identity under an m-transpose; here it is behaving as a pure imaginary number when shifting between zeroed rows and columns; note however, that the energy follows the form of the $0,0$th (res. $5,5$th) entry in the metric and is real rather than imaginary in both universes [4]. We have accordingly also transposed the primed space co-ordinates in the second matrix in eq.(7) as they would under a direct m-transpose of metric U_R indicating that U_R and U_L carry opposite parity. Thus if the weak interaction is left-handed in U_L then it will be right-handed in U_R. To conform the space co-ordinates

[4] Imaginary masses and energy for 'tachyons' are the result of analytic continuation of the equations of special relativity of sub-luminal states into the supra-luminal domain; here we see that the energy of representations native to the supra-luminal domain are real if those native to the sub-luminal domain are also real.

in the two metrics in eq.(7), we simply set $x = z', y = y'$ and $z = x'$ and the identity subscripts can then be dropped throughout. The m-transpose is thus a generalized parity transform that involves both time and space, with the direction of time being the 'parity' of time. Metric eq.(1) is *m-symmetrical.* The Lorentz metric is not m-symmetrical and thus the decomposition from 5 to 4 dimensions represents the breaking of m-symmetry. As has frequently been the case in physics, the finding of previously unrecognized mathematical symmetries reveals new physical insights. The existence of m-symmetry explains why it is that parity is fundamental in nature and relevant to the foundation of space-time structure, as it must be given that it dictates the structure of the weak interaction.

The second universe (Universe R) is now purely tachyonic in the time frame of Universe L but, as we shall find in subsequent sections, the potential energy of this second universe is measurable in Universe L as a diffuse *positive energy* fluid propagating in reverse time. The negative energy tachyon fields generating this potential energy are not directly measurable but their stability is protected by symmetry.

Lastly in this section, note that as a consequence of the invariance of the tensor product of the metrics of the two universes referred to a single time base eq.(7), we shall find that the invariant scale between the two universes is the square of the dark-energy $[DE]^2$ rather than the linear scale $[DE]$. This fact will become important when computing the MOND parameters.

III. THE DUAL MIRROR UNIVERSE MODEL.

If the Copernican principle is valid and the dark-energy is indeed secondary to the positive potential energy complement of a negative energy diffuse bath of tachyonic fields with genuine reverse-time evolution [5], then the SG definition of time requires that our universe has a schizophrenic identity both as a subluminal conventional universe and also as a diffuse bath of tachyonic negative energy density (the means by which both the continuum

[5] One needs to distinguish between reversal of direction of motion but still in the frame of a forward-time moving observer, which constitutes the usual mechanical definition of 'time reversal' in physics, and genuine reversal of the direction of time; the latter involves an actual inversion of causality so that effect precedes cause for all phenomenology, both macro and micro. More generally, the lack of time asymmetry in the laws of physics reflects the underlying treatment of dimensional structure; the conventional Lorentz metric, for example, does not incorporate the time direction so it should be no surprise that laws derived from such a metric as an underlying assumption show no time asymmetry. Assuming this demands that nature is also time symmetrical is thus circular logic.

hypothesis and its negation are made physically manifest). The time symmetry requirement can be satisfied, but only if our universe has a dual twin propagating in the opposite direction in time, in the second component of the two blended time dimensions folded-together from the 5-dimensional precursor space as described in II. The basic features of tachyon fields were described long ago [32], [33], [34] and include anomalous spin-statistics, reverse time evolution, faster-than-light propagation with the velocity of light a lower limit velocity for tachyons, just as it is an upper limit for subluminal states and imaginary masses (see footnote 4).

Let us consider the issue of quantum uncertainty for tachyon states. We are interested in a relation analogous to $\Delta q \Delta p \geq \frac{1}{2}\hbar$. The two scales involved are the cosmic horizon (or Hubble parameter, which has the dimension of inverse time) and $[DE]_0$, the observed dark energy scale parameter at $z = 0$, which has the dimension of energy. Therefore, to have the correct dimension of energy x time, let us propose that for tachyon fields the uncertainty relation transforms to;

$$\Delta q' \Delta p' \leq \frac{1}{2}\left(\frac{[DE]_0}{H_0}\right) = \frac{1}{2}\sqrt{\frac{m_{pl}c^2\hbar}{H_0}} = \frac{1}{2}\frac{\hbar}{\sqrt{t_{pl}.H_0}} \tag{8}$$

where H_0 is the Hubble parameter, m_{pl} is the Planck mass, t_{pl} is the Planck time and we have used the relation $[DE]_0 = \sqrt{m_{pl}c^2.H_0\hbar}$ (this relation is exact under a 'crossing symmetry' and is derived explicitly in IV). The L.H.S are the uncertainty of the tachyon field position and momentum. The $\leq$ sign is chosen because this relation should flip the sub-luminal expression in terms of scale (another way of understanding this inequality sign choice is to consider a completely uniform cosmological constant; the energy density is constant with no variation, so the uncertainty is zero - similarly, the $\leq$ choice in eq.(8) limits the amount of observable substructure that a tachyonic non-cosmological constant form of DE can have). With the conventional uncertainty relation, if the energy or momentum is known precisely, then the time or position uncertainty can be as large as the Hubble time or the cosmic horizon size, at least in principle, which leads to a field-theory description. The matter fields nevertheless in practice manifest as highly localized phenomena, such as electrons or protons. Essentially the opposite is seen with tachyons; eq.(8) shows that the quantum uncertainty spread of any reverse-time evolving tachyonic field is limited, in some sense to the average energy scale of the dark energy. We expect tachyon fields to be highly non-local and evolve towards a state of uniform energy density, wherein individual tachyons cannot be detected as localized entities but manifest to a subluminal observer as mixed entities. We will subsequently find that eq.(8) reduces to $\Delta q' \Delta p' = 1/2\hbar = \Delta q \Delta p$ exactly at the Planck scale (where the unprimed p & q are the subluminal fields). Note that mixed non-local tachyon fields cease to represent a means of transmitting information in reverse time, which has long been one concern regarding faster-than-light states. Tachyon fields are best conceived of as a diffuse energy fluid causally connected over space-like separations rather than 'particles' and represent a logical extension of the QFT concept of sub-luminal fields but as collective rather than discrete entities.

'Dual Universes' are a perennial topic of science fiction stories but here we are driven to consider a dual universe to restore total time symmetry, in keeping with the definition of time given by the metric structure in II, and by our demand that the Copernican principle be preserved; that is, all observers observe the same laws and structure of nature regardless of their time or location (space-time) perspective. We now impose a symmetry which interchanges the tachyonic fluid and the sub-luminal fluid (both luminous and dark, and radiation fluid) and their associated potential energies. This symmetry requires the presence of two universes, each being the tachyonic transform of the other, and propagating in opposite directions in time but within the same 3D space (this last point, which is entrenched in the metric structure eq.(1), is critical because it constrains the future evolution of each tachyonic-transformed partner universe to lie within the causal horizon of the other). Imposition of mass degeneracy between the two universes is required for symmetry and is a natural supposition (and also demanded by conservation of total energy given that both evolve from the Planck geometry; discussed in IV). This symmetry is called a *crossing symmetry* (or 'mirror symmetry') of two universes. Observers define the 'crossing' time; the two universes are always 'just passing' at the time an observer experiences physical existence and is able to make observations. The crossing time evolves into the future in both universes. In this picture, the matter and radiation fields in *our* universe manifest as a diffuse energy bath of *negative energy* tachyonic fluid in the other universe but propagating in the reverse direction of time in the matter frame of the other universe. Similarly, the associated negative potential energy in *our* universe, dominated by gravitational potential energy, transforms into a *positive energy* density propagating with reverse evolution in the other universe from the crossing symmetry. Precisely the same process causes the negative potential energy in the tachyverse to manifest in our universe as a positive energy evolving in the reverse direction of time as the 'dark energy' we can measure. Since sub-luminal states manifest as discrete entities (like the electron or proton fields) whilst the tachyonic states manifest as a collective, or mixed, structure of the order of the cosmic horizon, such a symmetry must integrate over the entire sub-luminal universe within the cosmic horizon and convert it to a diffuse energy bath also of the order of the cosmic horizon. Since the visible sub-luminal universe lies along the light cone of any observer, the transformation is into that space-time which lies on and outside of that observer's light cone. Thus the tachyon energy density, both its negative component corresponding to the matter fields of the tachyverse and its positive transformed potential energy, at $z = 0$ is back-projected along the past light-cone of our universe and the energy density we measure at $z = 0$ dictates the energy density which we measure projected anti-causally onto our past light cone of observation (with the caveat that the actual value measured will be a function of ω_{DE} at the time of the crossing symmetry and its reverse-time evolution, if any).

An equivalent way of understanding this is that at all times the net potential energy of the universe and its net matter and radiant energy are equivalent but with opposite sign; the overall energy sums to zero universe wide. Thus, under a crossing symmetry, we can use the evolution of the total matter component in the tachyverse to determine the evolution of the total potential energy component, because these two should have the same equation of state. Matter in the tachyverse evolves (in its own sub-luminal frame) in an identical fashion to matter in our universe $\bar{\rho}_{\bar{M}} \propto \bar{a}^{-3}$, (where the presence of a 'bar' indicates the tachyverse frame/tachyon state and where we have used the equation of state $\rho(a)_i = a(t)^{-3(1+\omega_i)}$, with $\omega_M = 0$ for any matter component). Translated into our own (non-tachyverse) frame, this would imply an equation of state $\rho_{\overline{TM}} \propto a^{+3}$ (where $TM =$ 'tachyverse matter'). However the tachyverse frame is back-projected on our sub-luminal past light-cone observer frame where it is subjected to a forward time dilution a^{-3}, which cancels-out the $\rho_{\overline{TM}}$ scaling. Thus, the equation of state of that component of the tachyverse which corresponds to a crossing symmetry of 'dust' in our universe, expressed as the equivalent positive potential energy, is $\omega = -1$, which is the same as a cosmological constant. This seems an odd result but is in fact entirely expected from metric eq.(1), which forces both universes to share the same 3-space. This constrains the available space for the future evolution of each universe to lie within the horizon of the partner universe. Thus, to the extent that the 'future' of our universe may be considered to exist, it does so only within the context of being compressed into the available space of the partner universe's horizon, which can be treated as a fixed volume. The total matter and energy content of our universe, expressed as an energy density in this volume, is then what the partner universe sees projected anti-causally into its past as a constant value. In the absence of radiation in our universe, this would dictate that the equation of state of the dark energy is $\omega = -1$ exactly. The radiation component of our universe however scales as a^{-4}, where a is the scale factor and in apparent contradiction with global energy conservation. The corresponding component in the tachyverse identically scales as $\rho_{\bar{\gamma}} = \bar{a}^{-4}$ but under back-projection onto our universe, and picking-up of an expansion factor of a^{-3}, scales like $\rho_{\bar{\gamma}} \propto a(t)$ which gives the tachyverse radiation component an equation of state in *our* universe of $\omega_{\bar{\gamma}} = -4/3$, strongly in the phantom range. The radiation component of our universe at $z \approx 0$ is however only $\Omega_\gamma \approx 5 \times 10^{-5}$, and if we add this component under an exact crossing symmetry we obtain a net equation of state of the dark-energy as $\omega_{DE} \approx -1.00006$, which is in the 'phantom' range, but only weakly so. Nevertheless, it is a generic prediction of the 'tachyverse crossing symmetry' model that the equation of state of the dark-energy must be in the phantom range, making the theory in principle falsifiable. Should there exist additional long-range gauge forces between any dark-matter in the universe which scale like radiation, this would push the DE equation of state further into the phantom range, making it experimentally more tractable, since measuring ω_{DE} to 6 digit accuracy is observationally a daunting task.

Note also that total energy conservation is restored, since although in our frame radiation evolves like a^{-4}, in apparent contradiction with energy conservation, the equivalent radiation component in the tachyverse induces the small phantom modification to the dark-energy equation of state that restores total energy conservation. The constraint of the dominant energy condition for phantom dark-energy is thus bypassed in this model as a result of total energy conservation.

We must explain why we observe $\Omega_{DE} \approx 2\Omega_M$ at the current epoch. We have already noted that this fact is crucial for the dark energy to be not only observable but comprehensible in the analytical sense and is an integral part of the coincidence problem. If the crossing symmetry is the defining event at our instant of existence, then the same wave of dark energy propagating into the past also produces its (anti)causal influence visible on our past light cone. Thus the energy density we can see in the past is determined by that observable at the present instant. Because we observe the effects of the energy density in

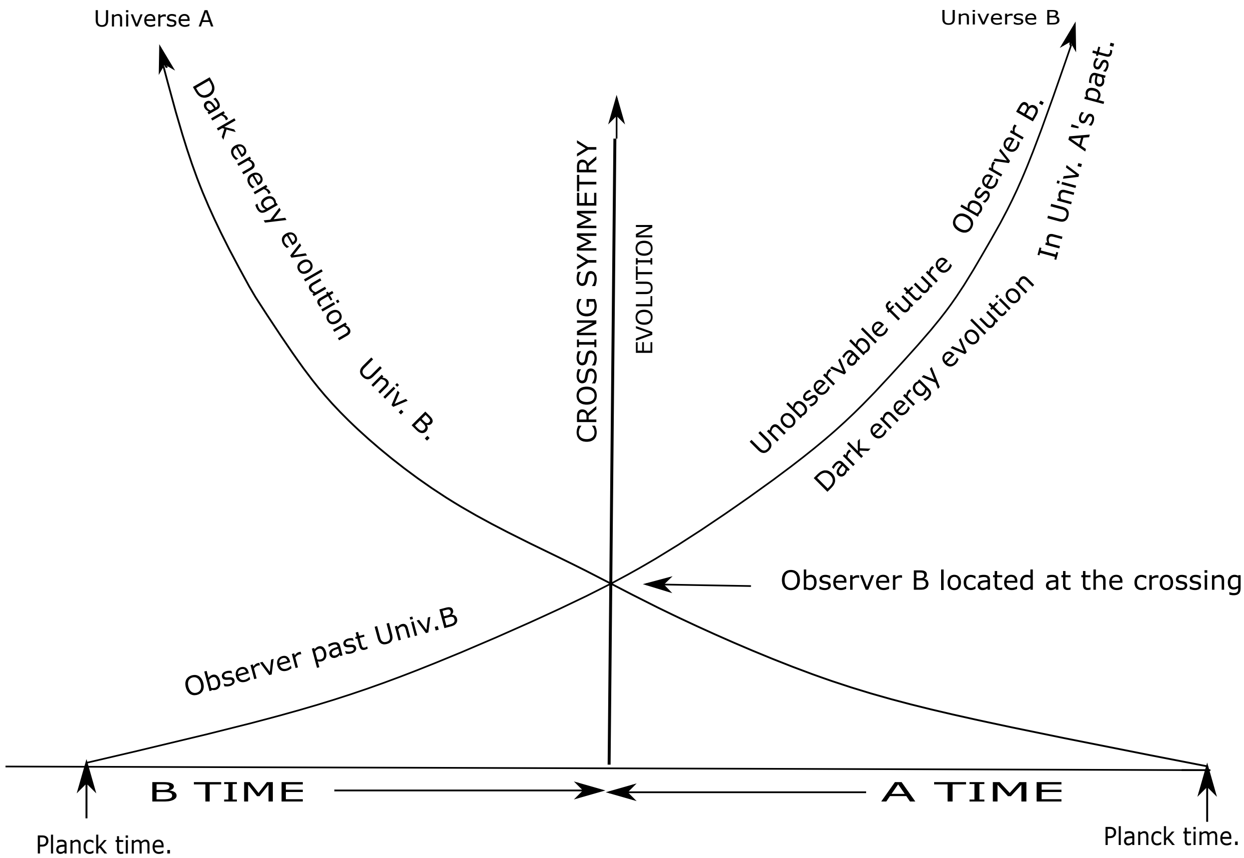

FIG. 1. Schematic representation of a crossing symmetry evolution of two universes with orthogonal time dimensions projected on the same one-dimensional line. The crossing symmetry (and hence the scale factor) evolves in the vertical direction whilst time evolution is represented on the horizontal axis, both curves beginning at the central location and diverging as time progresses. As the baseline diverges the crossing time (the 'now') moves higher. The future evolution of universe A's potential energy thus lies in the past of universe B with respect to a universe B observer, who always is located along the central axis at the point of crossing of the two universes. No 'big rip' ever eventuates; instead, the observed dark energy $[DE]_a$ falls as the inverse square-root of the scale factor a. All observers will see a past history of the universe consistent with his or her particular crossing time with the observed equation of state of the dark energy dictated by the future evolution of the partner universe potential energy.

the past the wave appears to have passed us.

Another way to understand this is that the universe visible to an observer at $z = 0$ includes components as they were up to ≈ 13.7 billion years ago, not as they are with respect to a space-like sphere instantaneously defined at $z = 0$, which would be very much larger than 13.7 billion light-years in radius (approximately two times larger). Structure corresponding to the last scattering surface that forms the CMB, for example, will have expanded to form galaxies and clusters of galaxies that are now far outside our horizon. The dark-energy that is anti-causal with respect to our visible component of the universe however we assume is constrained by our current horizon size to constitute a space-like 3-sphere of radius $\approx 13.7 \times 10^9$ light years at $z = 0$ and project back in time with the same fixed radius (assuming $\omega_{DE} \approx -1$). We can think of this in terms of t_1 and t_2 time being orthogonal dimensions; if our time is t_1, as we look-back everything we see is at a fixed t_2 time so we see a fixed snap-shot of the other universe projected into our past as a constant energy density at fixed horizon size dictated by the crossing. As such, Ω_{DE} is perceived as larger than Ω_M at very low redshift and smaller than Ω_M at very high red-shift. Some intermediate red-shift will be identified as the time of $\Omega_{DE} = \Omega_M$ by the observer at $z = 0$. This is computed as follows; we define a particular observer location p_0 at $z = 0$, a_0, and trace the world-line of p_τ backwards in time to approximately $\tau = 0$ where τ is the cosmic time. Consider each slice of time as a crossing-symmetry in the past defined at that particular instant. The light from those events in the past propagates forward in time to eventually reach the observer. Thus the panorama of the past seen by the observer at p_0 is effectively a sum over past crossing symmetries. According to our theory of retro-causal dark energy however, what the observer actually sees is modified by the dark-energy density at z_o propagating backwards in time and altering the photons in transit such that the observer sees an expansion history dictated by the energy density at his or her locally defined crossing-symmetry at z_0. The dark energy is causally connected over space-like separations and at scale-factor a_τ, the dark-energy occupies a spherical region of space at a common time instant (i.e. entirely space-like separation from the center of the sphere of local horizon size a_τ) surrounding p_τ. The total energy at each instant integrated over the scale factor (ignoring the inflationary epoch) is thus a representation of the sum over crossing symmetries in the past and is approximately;

$$\begin{aligned}\int_0^{a_0} \Lambda_\tau \frac{4}{3}\pi (a_\tau)^3 da_\tau &= \frac{4}{3}\pi \int_0^{a_0} m_{pl}^2 c^4 H_\tau^2 \hbar^2 (a_\tau)^3 da_\tau \\ &= \frac{4}{3}\pi \int_0^{a_0} m_{pl}^2 c^6 \hbar^2 (a_\tau) da_\tau \\ &= \frac{2}{3}\pi m_{pl}^2 c^6 \hbar^2 (a_0)^2 \\ &= \frac{2\pi}{3}\Lambda_0 (a_0)^4 \qquad (9)\end{aligned}$$

where Λ_0 is the dark energy density at the present epoch, $\Lambda_\tau = m_{pl}^2 c^4 H_\tau^2 \hbar^2$ (this relation is derived in IV), a_τ is the scale factor in a ΛFLRW universe (i.e., we have implicitly assumed that $\omega_{DE} = -1$) and used the relation $a_\tau = c/H_\tau$. The energy sum-over-history eq.(9) must be a conserved quantity but, according to theory, is plastic (deformable). Thus there should be some crossing time $\tau_{eq.}$ with local scale factor $a_{eq.}$, when viewed from the perspective of the observer at p_0, that defines an equivalent energy to eq.(9) but which is defined over a volume of space of constant horizon size and the corresponding red-shift will then define where $\Omega_m = \Omega_{DE}$ is *observed* to occur (i.e., $\Omega_m = \Omega_{DE}$ is by definition the crossing symmetry of two mass equivalent universes but the observer sees their own crossing-symmetry at $z = 0$ being projected into the past). The energy density in this cylindrical-shaped 4-volume will be the same density as at scale factor a_0 (i.e. Λ_0) because this is the crossing-symmetry being imaged in the past;

$$\frac{4}{3}\pi\Lambda_0(a_{eq.})^3 \int_0^{a_0} da = \frac{4\pi}{3}\Lambda_0(a_{eq.})^3.a_0 \tag{10}$$

Eq.s (9) and (10) are equivalent descriptions of a mass-degenerate crossing symmetry viewed from the perspective of an observer at z_0. Therefore, the observer at z_0 sees equivalence of the total matter and total DE content (i.e. $\Omega_m = \Omega_{DE}$) at $2a_{eq.}^3 = a_0^3$ which in turn gives $z \approx 0.26$ for $\Omega_m = \Omega_{DE}$. This is the red-shift that the mass-degenerate crossing symmetry at $z = 0$, viewed from p_0, is back-projected to. All observers, regardless of their temporal location in the universe, see the same thing; they always find $\Omega_m = \Omega_{DE}$ at $z \approx 0.26$ in their local observer frame of reference.

This is, however, not the red-shift at which the DE begins to dominate the expansion of the universe (i.e. $\ddot{a}_\tau > 0$, with the derivative with respect to cosmic time τ). The latter is defined as the red-shift when $\ddot{a}_\tau = 0$ as the cosmos transits from matter domination, $\ddot{a}_\tau < 0$, to DE domination, $\ddot{a}_\tau > 0$. Using $\dot{H}_\tau = \ddot{a}/a - H_\tau^2 = -H_\tau^2(1-q)$, where $q = -a\ddot{a}/\dot{a}^2$ is the standard deceleration parameter [8];

$$q = \frac{1}{2E_z^2}\frac{dE_z^2}{dz}(1+z) - 1 \tag{11}$$

where in a ΛFLRW universe, $E_z^2 = H_z^2/H_0 = \Omega_m(1+z)^3 + \Omega_{DE}(1+z)^{3(1+\omega)}$, ($\Omega_k = 0$ by input assumption), which gives, setting $q = 0$ and $\Omega_m = 2\Omega_{DE}$, $z_{acc.} \approx 0.62$ as the transition red-shift to DE dominance. Alternatively eq.(11) can be solved [8] to obtain;

$$\Omega_{DE} = -\frac{\Omega_m}{(1+3\omega_{DE})(1+z_{acc.})^{3\omega_{DE}}} \tag{12}$$

where $z_{acc.}$ is the transition red-shift to $\ddot{a}_\tau > 0$. Setting the transition red-shift to 0 and setting $\omega_{DE} = -1$ we obtain $2\Omega_{DE} = \Omega_m$ at the transition to DE dominance concordant with the previous input assumption.

Now the 'center' of the crossing symmetry from $z = 0$, which we can think of as a spherical space-like surface surrounding p_0, is back-projected to $z \approx 0.26$ but the transition to DE dominance, $z_{acc.}$, is also the result of back-projection from the universe at $z = 0$ and there is symmetry between the two components on either side of the 'center' at $z \approx 0.26$ (i.e. there is a symmetry relationship between $z_{acc.}$ and $z = 0$ about the center of the back-projection $\Omega_m = \Omega_{DE}$). Ignoring any contribution to $\Omega_{Tot.}$ that evolves like radiation, the imposition of this symmetry on eq.(12) gives the relative matter to DE density at $z = 0$ simply by interchange of Ω_{DE} with Ω_m;

$$\Omega_m = -\frac{\Omega_{DE}}{(1 + 3\omega_{DE})(1 + z_0)^{3\omega_{DE}}} \tag{13}$$

which gives $2\Omega_m = \Omega_{DE}$ at $z_0 = 0$. Further, using the symmetry between $z = 0$ and $z = a_{acc.}$ about z_{eq}, we can compute the transition red-shift to DE dominance $z_{acc.}$ by putting $\Omega_m = 2\Omega_{DE}$ in eq.(13) which gives $z_{acc.} \approx 0.6$, consistent with the previous computation.

Eq.(9) is the sum over a forward-time evolution history of DE and is equated to the corresponding reverse-time evolution of DE (eq.(10)) of fixed horizon size to find the back-projected red-shift of the crossing symmetry equivalence relation. If however we substitute a fixed dark-energy density Λ_0 in eq.(9), instead of summing over a variable evolving DE history, we maximize the red-shift at which the DE can be seen to have an effect, i.e., we define $z_{acc.}$, then equating to eq.(10) with $a_{acc.}$ instead of $a_{eq.}$ gives;

$$\frac{4}{3}\pi\Lambda_0(a_{acc.})^3 \int_0^{a_0} da_\tau = \frac{4}{3}\pi\Lambda_0 \int_0^{a_0} (a_\tau)^3 da_\tau \tag{14}$$

which gives $1.6\, a_{acc.} \approx a_0$ or $z_{acc.} \approx 0.6$. For a universe that is 13 billion years old $z = 0.6$ is about 6.5 billion years ago; broadly in agreement with observation. A similar relationship will occur for all observers regardless of their time in the history of the universe; that is, they always see the transition to dark energy dominance occur at a scale factor $a_\tau \approx a'_0/1.6$ where a'_0 is the observer's local scale factor.

To better understand the symmetry of $z_{acc.}$ and z_0 about $z_{eq.}$ we can apply an integral technique similar to eq's. (9) & (10) by replacing $a_{acc.}$ with a_0 in eq.(14) and an unknown limit of integration;.

$$\frac{4}{3}\pi\Lambda_0(a_0)^3\int_0^{a_x} da_\tau = \frac{4}{3}\pi\Lambda_0\int_0^{a_x}(a_\tau)^3 da_\tau \tag{15}$$

which gives $1.6a_0 \approx a_x$ or $z_x = -0.6$ (using $a_0/a_x = (1-z_x)$ for times where $\tau = x$ lies in the future with respect to the observer at a_0, i.e. negative red-shift). By symmetry, $z = -0.6$ corresponds to $\bar{z}_{acc.}$, the red-shift of transition from matter to DE dominance in the mirror universe. Thus under a crossing symmetry, z_0 maps to $\bar{z}_{acc.}$, $z_{eq.}$ maps to $\bar{z}_{eq.}$ and $z_{acc.}$ maps to $\bar{z}_0$. This confirms the symmetry of $z_{acc.}$ and z_0 about the 'mean' of the back-projected crossing symmetry, $z_{eq.}$.

Finally, to obtain the equivalent description of the transition between $z_{eq.}$ and $\bar{z}_{eq.}$, we replace $a_x \equiv \bar{a}_{acc.}$ in eq.(15) with $(\sqrt[3]{2}a_0)$ $(\equiv \bar{a}_{eq.})$ and a_0 by $\sqrt[3]{2}a_{eq.}$ $(\equiv a_0)$.

An alternative computation can also be used to confirm that $\Omega_m = 2\,\Omega_{DE}$ at $z \approx 0.6$. The dark-energy density in our space-like balls scales as $[DE]^3_\tau \propto a^{-3/2}$ and at the transition red-shift to DE dominance gives $[DE]^3_{acc.} = 2[DE]^3_{z=0}$. The volume of space, from the perspective of an observer at $z = 0$, is reduced by a factor $(a_0/1.6)^3 \approx (a_0)^3/4$. Thus, assuming mass degeneracy between the two universes, at the transition to DE dominance $\Omega_m \approx 2.\Omega_{DE}$ (since the matter density scales inversely with the volume). Using $a^3_{eq.} = 2a^3_{acc.}$ in a FLRW ΛCDM Universe, where $a_{eq.}$ is the scale factor at which there is DE and matter equivalence we have [9];

$$\frac{(1+z_{acc.})}{(1+z_{eq.})} = 2^{1/3} \tag{16}$$

we find $\Omega_m = \Omega_{DE}$ at $z = 0.27$, consistent with the previous calculation and we have already computed that at the current epoch ($z = 0$), $\Omega_{DE} = 2 \times \Omega_M$. The factor of 2 multiples that occur at both $z_{acc.}$ and z_0 in a reciprocal fashion are a unique marker of retro-causal dark-energy and are a consequence of the 'crossing-symmetry' projected into 'the past'. It allows to write the equivalent symmetry relation for z_0;

$$\frac{(1+z_{eq.})}{(1+z_0)} = 2^{1/3} \tag{17}$$

which makes clear the pattern of the crossing symmetry in relation to $z_{acc.}$, $z_{eq.}$ and z_0, and gives the prediction $\Omega^0_{DE} = 2/3$ and $\Omega^0_m = 1/3$ (this is exact if we include all species that evolve like radiation in the term Ω_m and assume $\Omega_{Tot} = 1$ and $\omega_{DE} = -1$ exactly), which indicates a need for dark-matter. All observers, regardless of their time in the history of the Universe, see the transition to DE dominance at $z \approx 0.6$ with the implied red-shift of matter and DE equivalence $z \approx 0.3$ *in their local frame.* Our analysis here is however simplified by the approximation that $\omega_{DE} = -1$. Phantom DE, evolving into the past from the fixed value crossing symmetry at $z = 0$, cannot alter the red-shift for matter / DE equivalence, since this is just a geometrical projection of the equivalence at $z = 0$ and is fixed at $z \approx 0.26$ regardless of the equation of state of the DE; i.e., the center of the crossing is invariant. However, phantom DE will *lower* the red-shift of the observed transtion to accelerated expansion, since this will be the result of reverse evolution from equivalence, which will be altered for the phantom case. The 'law of 2's' eq.'s (16), since it is due to the crossing symmetry, should nevertheless be valid for all observers. For an exact cosmological constant the theory predicts $\Omega_\Lambda^0 = 2/3$ (i.e. at $z = 0$) by the law-of-two's. Equivalently, by the law-of-two's, for phantom DE, we infer that $\Omega_\Lambda^0 > 2/3$ by symmetry and the deviation away from $\Omega_\Lambda^0 = 2/3$ measures the deviation of ω_{DE} away from a cosmological constant. If empirically an observer infers $\Omega_{DE}^0 = 0.7000$, for example, this requies an equation of state of the DE of $\omega_{DE} \approx -1.030$ using the law-of-two's, replacing z_0 in eq.(17) with the redshift at which $\Omega_{DE} = 2/3$, which is $z \approx 0.0145$ for an observed $\Omega_{DE}^0 = 0.70$. Alternatively, an observed DE density of $\Omega_{DE}^0 = 0.720$ requires an equation of state $\omega_{DE} \approx -1.051$. More generally, the crossing symmetry thus imposes a testable consistency constraint on the DE E.O.S. and thus on the amount of energy that evolvess like radiation.

A hand-waving interpretation of this unusual result, which is quite counter-intuitive given that the wave of dark-energy propagating in reverse time is mass degenerate with the subluminal universe at $z = 0$, is that we cannot treat the DE as *instantaneously* causally connected over space-like separations of the order of the horizon; which is to say the DE is causally connected over space-like separations but nevertheless always invovling a finite non-zero interval of time. The speed of light is the same in both frames so that the (anti) causal frame of the DE is back-projected onto the past light-cone of the observer.

We are thus now in a position to account for the basic features of the dark energy, including the transition time and the dark-energy scale, which is dictated by the mass degeneracy of the crossing symmetry of two universes. There is still however the need to account for the discrepancy with the expected vacuum energy based on the standard model fields. A crossing symmetry thus far described has tacitly assumed that the vacuum energy is zero absolutely and that the residual diffuse energy bath that is observed is due to a second universe expressed as a tachyonic reverse-time evolving featureless bath of (transformed potential) energy density. Eq.(8) gives us the vacuum structure of the negative-energy tachyonic fluid. Under a crossing symmetry, as we will subsequently prove by evolving Eq.(8) to the Planck scale, we recover the conventional quantum uncertainty principle for subluminal states. Equivalently, for every quantum vacuum fluctuation that is possible in the subverse (sub-luminal universe) there must be a corresponding equivalent vacuum fluctuation in the tachyverse because it contains exactly the same standard model (and dark matter, if it exists) fields with the same masses. The tachyverse fields, when viewed in the subverse frame, undergo a sign-change of the potential energy so that the negative potential energy in the native tachyverse frame appears as positive energy in our (subverse) frame. Since the negative energy must exactly balance the positive energy, the native tachyverse fields (i.e. the standard model + DM fields native to the tachyverse sub-luminal frame) must have negative energy when expressed as tachyons. Thus the associated vacuum fluctuations must therefore transform to negative energy also. Thus the vacuum energy is rendered zero exactly by cancellation; zero vacuum energy is built into the model by symmetry. (A similar mechanism to render the vacuum energy zero was proposed by Linde in 1988 [35] using mirror matter with negative energy, but not involving a tachyonic transform of states). To reinforce that this is still true under a sub-luminal-tachyon interchange, consider the following schematic representation of the vacuum energy density of the subverse (or equivalently of the tachyverse in its own local matter frame);

$$E_{subvac} = \int_{\hbar H_0 . c^{-1}}^{m_{pl} c} dp \, \Sigma \, (\text{virtual fields}) \tag{18}$$

where the sum over 'virtual fields' includes all standard model fields and any dark matter fields (i.e., sum over all possible vacuum Feynman diagrams), and the integration measure is over momenta from the smallest possible momentum, defined by the horizon, to the largest possible, defined by the Planck mass. Now, in both the subverse and the tachyverse local matter frames, the relativity expression $E^2 = p^2c^2 + m^2c^4$ applies. Thus, the energy of the fields has the same sign as the momenta. When the equivalent tachyverse fields are transformed into the subverse time frame both the tachyverse fields and the measure of integration change sign and thus the integrand in eq.(18) does not change sign when projected from the frame of one universe to the other. Write the transformed tachyverse vacuum energy in the subverse frame time as:

$$E'_{tachyvac} = \int_{\hbar H_0 . c^{-1}}^{m_{pl} c} dp' \, \Sigma \, (\text{virtual fields})' \tag{19}$$

where the prime $'$ indicates a tachyonic-transformed component. Now, in section IV a relation is derived which links the sub-luminal vacuum structure and the equivalent tachyonic transformed vacuum (eqs.(22), (25) & (26)),which defines the transformation between the two vacuua as equivalent to interchange of the Planck scale with the horizon scale. The crossing symmetry thus interchanges the limits of integration and the field definition. Hence $E'_{tachyvac}$ in eq.(19) may be redefined as;

$$E'_{tachyvac} = \int_{m_{pl}c}^{\hbar H_0.c^{-1}} dp\, \Sigma\, (\text{virtual fields}) = -E_{subvac} \tag{20}$$

and the two vacuum energies thus cancel out since they contain exactly the same 'virtual fields' [6].

There is yet another problem which a 'crossing symmetry' solves; that of the Universe-wide parity choice of the weak interaction. Unlike antimatter asymmetry, which is not exact since we see, for example, CP violation in the decay of neutral K^o, the parity sign of the weak interaction appears fixed; we always find only left-handed weak interactions in the matter frame and only right-handed weak interactions in the anti-matter frame. We never detect a right-handed weak-interaction in the matter frame. This asymmetry entrenched in the structure of the universe requires explanation [36]. Since the matter in the tachyverse in its own frame must have a weak interaction, if this were left-handed then both the sub-luminal and supra-luminal frames would be left-handed, which is only possible if this chirality is embedded somehow in the initial state of the Universe. However, we shall prove in the next section that the initial state of the Universe is irreducibly simple and the 5-D analysis proved in II that the two universes have opposite parity assignments which, in turn, proves that the initial state had no intrinsic chirality. Therefore the weak interaction in the matter frame of the tachyverse is right-handed and the tachyverse is made of 'mirror matter'. We just happen to inhabit the left-handed side of the pair. Note here that assuming the tachyverse is made of anti-matter does not solve this problem, since anti-matter appears in our universe with a right-handed weak interaction and thus a right-handed anti-matter weak interaction in the tachyverse frame would simply entrench the existing asymmetry. Thus to resolve this asymmetry, the tachyverse must be made of the equivalent of matter (even though it is negative energy when referenced to our subverse frame, this is the result of a

[6] This result is unaltered if we treat tachyon fields as having imaginary masses and imaginary energy; since there are two re-definitions of the fields in the derivation, the imaginary factors cancel-out.

tachyonic transform - it is positive matter density in its own frame) but with a right-handed weak interaction. It is a 'Mirror-Matter' Universe. (The concept of 'Mirror matter' can be traced back to the work of Lee and Yang [37] in the 1950's and Kobzarev *et. al.* in the 1960's [38]. It enjoyed some popularity as a potential explanation for dark matter [39] [40] and neutrino anomalies over the last three decades [41]. Interest as an explanation for the solar and atmospheric neutrino anomalies however lapsed with improved data which showed consistency with three species oscillations, although there is still some interest as a DM candidate [42]).

Lastly in this section we need to consider how to incorporate the negative energy tachyon fields (i.e. the transposed tachyverse standard model fields + any tachyverse dark-matter fields) in the framework of General Relativity (GR). In GR the negative potential energy of the gravitational field is not incorporated in the stress-energy tensor but is instead embedded in the metric, where it can be transformed-away in a free-fall (inertial) frame. It is proposed that, similarly, the diffuse negative energy tachyverse fields must be incorporated in the metric structure rather than in the stress-energy tensor. Since this negative energy is diffuse and featureless, it can be treated as a gauge transformation of the background potential (i.e., there should still exist an inertial frame at each space-time location in which no acceleration is experienced). The transformed tachyverse potential energy, manifesting as positive 'dark energy' in the sub-luminal universe frame of observation, must be incorporated into the stress-energy tensor (since it is not a constant) as diagonal entries with negative pressure but positive energy (off diagonal entries are vanishingly small in the circumstance that the tachyonic fields have relatively uniform distribution within the horizon and little temporal variation for a given observer due to the equation of state very close to $\omega_{DE} = -1$).

IV. COMPUTING THE ENTROPY OF THE UNIVERSE.

The crossing-symmetry has a further element of great utility; it enables us to determine the 'temperature' of the universe. The tachyverse (supra-luminal universe) total entropy must be the same as the subverse (subluminal universe) total entropy at the crossing time (observer time). The two universes have the same horizon size at crossing and have the same total energy content within the same volume of space, with equal division between positive and negative energy; therefore they have the same average temperature. The dark-energy is the (gravitational) potential energy of the tachyverse expressed as a diffuse positive energy reverse-time evolving tachyonic fluid, but this energy density is identical to the energy density (modulo a sign change) of the negative energy tachyon fields and thus can be used to define the temperature of the tachyverse standard model fields. The temperature of the (subverse) universe is hard to assess; but it is very simply to determine the temperature of the tachyverse and these two values must be equivalent due to the energy-degenerate crossing symmetry. Using $\hbar^2 H_0^2 \times m_{pl}^2 c^4 = [DE]_0^4$ (this relation is derived at the end of IV) we can write the temperature of the positive energy tachyonic fluid $T_{DE} = [DE]_0.(k_B)^{-1} = \hbar^2 H_0^2 m_{pl}^2 c^4 [DE]_0^{-3} k_B^{-1}$ where $[DE]_0^{-3} = (\hbar c)^{-3} l_{DE}^3$ is a volume of space characteristic of the dark energy density at $z = 0$ and k_B is the Boltzmann constant. Under a crossing symmetry, ignoring the radiation component (which is very small at the current epoch, but should be included in a complete calculation), the dark energy within the horizon is equivalent to the total matter content of the subverse universe expressed as energy $M_u \times c^2$ (which we will assume is constant), and the total entropy of either universe (within the horizon) is given by (integrating $dE = TdS$ over the volume of the universe within the horizon);

$$S_{Univ.} = \frac{a_0^2 M_u (\hbar c)^3 k_B}{\hbar^2 m_{pl}^2 c^4 . l_{DE}^3 .} = \frac{a_0^2 [M_u] l_{pl}^2 (\hbar c) k_B}{\hbar^2} \tag{21}$$

where we have defined the mass of the universe as mass within a co-moving cosmic boundary per unit volume of dark-energy equivalence $[M_u] = M_u . l_{DE}^{-3}$ and set the scale factor a_0 as $a_0^2 = c^2/H_0^2$. Eq.(21) relates the total entropy of the universe to the area of the horizon and is equivalent for the two universes by symmetry. The Planck area in the numerator can be considered the 'units' of entropy; thus the maximum possible entropy is proportional to the number of Planck area units that can be fitted into the area of the horizon and rises in proportion to the area of the horizon (an expression of the holographic principle [43] [44]).

What is the measurement of the universe's entropy by observers at $z = 0$? These observers measure the equation of state of the dark-energy very close to $\omega_{DE} = -1$ and thus, given that M_u is due to the matter content of the universe predominantly (both luminous and dark with equation of state $\omega_m = 0$), they assess $[M_u]$ to be a constant over their observable history of the universe and therefore conclude that the entropy of the universe is proportional to the horizon area and has risen smoothly from the past in proportion to the square of the scale parameter consistent with the second law of thermodynamics.

Now consider observers at scale parameter $a_0' = 2a_0$. They measure the dark-energy scale parameter l_{DE} increased by a factor of $\sqrt{2}$ (using $[DE]_\tau \propto \sqrt{H_\tau} \propto a_\tau^{-1/2}$ and $l_{DE} \propto [DE]^{-1}$), and thus the factor $[M_u]'$ measures $2\sqrt{2}$ times smaller. The horizon area is 4 times larger and thus they find the total entropy of the universe to have increased by a factor of $\sqrt{2}$ in comparison with measurements that observers obtained at the scale factor $a_0'/2$ in their far past (should they have access to such ancient measurements). However, they too measure $\omega_{DE} \approx -1$ and conclude that $[M_u]'$ is constant in their observable past and that the entropy of the universe has increased smoothly in proportion to the square of the scale factor *in their frame of reference*. They would retrodict that the entropy of the universe at one-half their scale factor to be lower by a factor of 4 rather than lower by a factor of $\sqrt{2}$ actually obtained by ancient observers should such ancient records be available for comparison. The total entropy of the universe thus becomes observer dependent.

It is a generic feature of a crossing-symmetry of two universes propagating in the opposite direction in time within the same 3-space that the second law of thermodynamics is preserved in local frames but different observers making observations at different times in the history of the universe do not measure the same dark-energy density nor agree upon the total entropy of the universe.

Putting aside the issue of the existence of observers in extreme early times, can we ask what is the behavior of the entropy as the scale factor approaches the Planck length? We can answer this question with eqs.(8) & (21). In the circumstance that the Hubble time equals the Planck time, eq.(8) becomes (converting to an energy-time uncertainty); $\Delta E' \Delta t' \leq \frac{1}{2}\hbar$ for the tachyverse referenced from the subverse frame. By contrast, within its own subverse frame, the uncertainty principle is $\Delta E \Delta t \geq \frac{1}{2}\hbar$. At the Planck time both universes occupy the same uncertainty space at the same time and thus these two relations must both be simultaneously true. This is only possible if;

$$\Delta E' \Delta t' = \Delta E \Delta t = \frac{1}{2}\hbar \tag{22}$$

exactly, at the Planck scale, for both universes in this model (note that if we define both the time and the energy as negative for the tachyon field, the equality still holds). At the Planck time the scale factor $a_{pl} = l_{pl}$ and the $[M_u]$ term contains a l_{pl}^{-3} (since at the Planck scale this is the maximum available space to scale as a DE volume) times a mass. The fact that the gravitational potential energy of the universe is equal but opposite to the net matter and radiant energy content [45] might suggest we set this mass to zero; however, the uncertainty relation eq.(22) indicates we should set this mass instead to the Planck mass rather than zero. Thus we finally obtain, using eq.(21) with $[M_u]_{pl} = m_{pl}c^2c^{-2}/l_{pl}^3 = \hbar/l_{pl}^4\ c$, the expression for the total entropy of the universe at the Planck time becomes;

$$S_{Planck} = k_B \tag{23}$$

i.e. unity or one quantum unit of 'Planck entropy', which is irreducibly simple. An important point is that it is finite and non-zero, implying that there is no singularity at the beginning of the universe. This formula is very similar to the Hawking formula [46], [47] for black-hole entropy. If we insert the identity, using $l_{pl}^2 = \hbar G/c^3$ we obtain;

$$S_{Planck} = k_B \frac{l_{pl}^2\ c^3}{\hbar\ G} \tag{24}$$

with l_{pl}^2 playing the role of the area. The factor of 1/4 in the Hawking formula that arises from the black hole temperature calculation is missing; we can reasonably assume the energy and temperature is at a minimum in the absence of pair-production on the event horizon (i.e. one cannot define a fraction of one Planck unit of entropy, which is the minimal definable entropy) thus eq.(23) is consistent. This result thus also provides an internal check on the consistency of eq.(21).

Thus the initial state of the Universe 1: has non-zero entropy as a one quantum of Planck entropy and 2: has non-zero total energy. The Planck geometry effectively splits in half and produces two universes; both have equal but opposite amounts of total potential and total matter/radiant energy but the split is cross-symmetrical; thus for observers in our universe, our matter content is positive energy and the other universe's potential energy is positive energy. Our potential energy we find is negative and their matter and radiant energy, referenced to ours, is negative energy. Observers in the other universe see exactly the same pattern.

Eq.(22) defines the fundamental invariance between the two universes at the Planck time. To be consistent with the definition of $\hbar$ we must add the two uncertainty expressions [7] to obtain;

$$m_{pl}.c^2 \times t_{pl} = \hbar \tag{25}$$

where $m_{pl}.c^2$ is the Planck mass and t_{pl} is the Planck time. We can track the evolution of this invariance for later times by multiplying both sides of eq.(25) by a Hubble time at scale

[7] We could alternatively use the sum over the two universes' uncertainties at the Planck time to define $\hbar$.

factor a_τ, $\hbar H_\tau$ (i.e. converted to an energy) and dividing by the Planck time;

$$m_{pl}c^2\hbar H_\tau = \frac{\hbar^2 H_\tau}{t_{pl}} = \frac{\hbar^2 c^2}{l_{pl}.a_\tau} = \left(\frac{\hbar c}{l_{pl}}\right)\left(\frac{\hbar c}{a_\tau}\right) \tag{26}$$

where we have used $H_\tau = c/a_\tau$. The R.H.S. of eq.(26) relates Compton waves of the shortest and longest possible wavelengths in the both the Subverse and Tachyverse local frames (here assumed defined by the horizon a_τ; defining the maximal possible Compton wavelength as $2a_\tau$ would exclude the local observer frame). This implies that these Compton wavelengths interchange under a crossing symmetry between subluminal and tachyonic states. The product defines a mean effective energy-squared which in that circumstance is invariant under a crossing symmetry, which is therefore the dark-energy scale. Thus we have;

$$m_{pl}c^2\hbar H_\tau = ([DE]_\tau)^2 \tag{27}$$

Thus observers at scale factor τ always find that the dark-energy scale is given by $[DE] = \sqrt{m_{pl}c^2\hbar H_0'}$ in their local frame of reference where $H_\tau = H_0'$. Note however that the invariant scale is $([DE])^2$, as this fact is an input into the MOND computations in the next section.

V. MOND AS QUANTUM CORRECTIONS TO GENERAL RELATIVITY.

The *negative* gravitational potential energy in the tachyverse transforms into our subverse frame as *positive* dark-energy but structurally distinct to the negative-energy diffuse tachyonic fluid which represents the energy associated with the transformed standard model fields (+ dark matter fields if they exist) arising from the tachyverse. We will treat the gravitational potential like a radiation field, since the gauge boson (the graviton) propagates at light speed and the potential energy content may be considered to be a function of the gravitational field itself.

Since the dark energy is a diffuse energy field causally connected over space-like separations, the wavelength and hence energy of the exchanged gravitons is set at the scale of the $[DE]$, with the dark-energy intrinsic graviton wavelength $\lambda_{Tachy.grav}$ a function of the order of the dark-energy scale in order to couple to the field;

$$\lambda_{Tachy.grav} \approx \frac{\hbar c}{[DE]_0} = \sqrt{\frac{\hbar c}{m_{pl}c^2}\frac{\hbar c}{\hbar H_0}} = \sqrt{\frac{\hbar}{m_{pl}H_0}} \tag{28}$$

i.e., interpolates between the Planck scale and Horizon, which is typical for a crossing symmetry. The average energy of the exchanged gravitons, expressed as a quantum bulk of the order of the dark-energy scale, using $E = h.\nu = h.c/\lambda$ is;

$$E_{Tachy.grav} = 2\pi\sqrt{m_{pl}c^2\hbar H_0} = 2\pi[DE]_0 \tag{29}$$

The energy scale that is invariant under a crossing symmetry at the current epoch from eq.(27) is $([DE]_0)^2$ so under a crossing symmetry energy eq.(29) becomes $E = (2\pi)^{-1}[DE]_0$ in the subverse frame (so that the product of energies is fixed at the crossing scale). This energy is associated with a characteristic acceleration; using the characteristic length scale as $\lambda \equiv \lambda_{Tachy.grav}$ (since the energy and wavelength are a dependent function of each other we do not modify the wavelength input) and $U = M_s G_N(E)/\lambda$ for the generated gravitational equivalent potential U in the subverse frame ;

$$U = M_s G_N \frac{[DE]_0}{2\pi\lambda} = M_s(2\pi)^{-1} m_{pl} G_N c H_0 = M_s m_{pl} G_N a_0 = M_s m_{pl} \mathcal{A}_0 \tag{30}$$

where $a_0 \equiv cH_0/2\pi$ is the MOND acceleration scale, $\mathcal{A}_0 = G_N a_0$ the scale-invariant MOND parameter [48], [49] and M_s is some arbitrary source. It is significant that these parameters derive from very general considerations and are a function of the crossing symmetry and the treatment of gravitational potential energy in the two universes. Eq.(30) also indicates that the expression $a_0 = cH_0/(2\pi)$ is exact rather than approximate. MOND was originally derived heuristically from the Tully-Fisher relation [50] and examination of galaxy rotation curves and these parameters were empirically determined [18], [19], [20]. The fact that they now can be derived from a fundamental cosmological model indicates that the MOND parameters represent truly fundamental structure.

Eq.(30) contains a coupling between a source M_s and the Planck mass under a (presumed) scale invariant parameter $\mathcal{A}_0$. The Planck mass is the vacuum scale for gravity and the appearance of the Planck mass in this expression suggests a vacuum modification for gravity in the infra-red i.e. the onset of quantum gravity vacuum corrections in the low acceleration regime (as has been suggested by others [51] [52] [53] [54] [55] or as emergent gravity [56] [57]). The source M_s is arbitrary. [8].

[8] The scale $[DE]_0$ is actually a 'mean' or average energy level in the Universe and thus the onset of the MOND dynamical region is 'below the mean global Universe acceleration' rather than overtly in the 'infra-red' regime

Armed with this idea, can offer an explanation for the MOND behavior, widely observed in galaxy rotation curves [63]? A vacuum modification of the Planck geometry will cause the Planck mass to 'run'. Since the gravitation coupling $G_N \propto m_{pl}^{-2}$, if the Planck mass 'runs' then so does the gravitational coupling; but runs only as $m_{pl}^{'-1}$ since the Planck mass also appears as a separate entry in the numerator in eq.(30). Thus the 'effective' acceleration $a_l = \sqrt{a_N a_0}$ at low acceleration scale $a_N < a_0$, has gravitational coupling G_l which varies with the 'running' Planck mass m'_{pl} as a function of $m_{pl}^{'-1}$. If we assume that as the local acceleration *decreases* the vacuum coupling to the Planck geometry *increases* in strength, then the Planck mass falls and G_l becomes larger as local acceleration drops. Since G_l dictates the strength of coupling of gravitons to matter, in the infra-red the strength of gravity increases, violating the strong equivalence principle. In order for this 'running' of the Planck mass to fit the MOND phenomenology we require;

$$\frac{G_l}{G_N} \propto \sqrt{\frac{a_0}{a_N}} \tag{31}$$

where G_N is the standard gravitational coupling at acceleration scales larger than a_0 and G_l is the 'running' coupling at low acceleration $a_N < a_0$. This enables us to rewrite $\sqrt{a_N a_0} = a_l \propto (G_l/G_N).a_N$, thus absorbing the MOND acceleration scale into the running of the coupling. The scale invariant becomes;

$$\mathscr{A}_0 = G_l \sqrt{a_N a_0} \tag{32}$$

Either way, the utility of the analysis in this section is that due to the gravitational interaction of the two universes, we have, at least in principle, a means of explaining MOND phenomenology without having to recourse to modifying General Relativity, which has always been one of the impediments to acceptance of MOND as a viable fundamental theory.

A proper treatment will probably need a formal quantum theory of gravity. The quantum field evolution of the running of G_l may also be non-linear and could, for example, slowly increase as the local acceleration drops, which might account for the discrepancy between MOND in cluster observations (this would however require a running of $\mathscr{A}_0$), but this possibility is speculative and needs further work. It is interesting to think that we may already be seeing quantum gravity effects in MOND phenomenology and this would mean that MOND is a good laboratory to try to create a viable theory of quantum gravity. It is also a further demonstration of the utility of the 'crossing symmetry' of two universes.

Whether this obviates the need for dark matter is a contentious issue. Suffice to say that MOND is very successful at predicting galaxy rotation curves without any DM [63], [64] with the tight-fit and low-scatter of the 'Baryonic Tully-Fisher' relation [65] [66] very hard to explain with CDM, but there are issues with galaxy clusters (including the Bullet cluster [67], [68], [69]) and with large-scale cosmology in the early universe, where the flatness of the observed Universe ($\Omega_{TOT} = 1$ which requires $\Omega_M \approx 0.3$ whilst the baryons are only about 20% of this matter content [70]) indicates a need for DM. One should remain open to the possibility that DM and MOND may co-exist but that DM may be only relevant at scales larger than those typical of galaxies. Certainly the finding that the MOND parameters can be derived from a cosmology model consistent with General Relativity represents a challenge to the applicability of the dark-matter paradigm (at the level of galaxies at least), as anticipated by Milgrom [49].

VI. TESTING THE THEORY.

The first and most direct test of the theory is that the equation of state of dark-energy lies in the phantom range (in this theory the dominant energy condition is bypassed due to the fact that total energy is always conserved in the two-universe system; see Sec. III). Thus if it is proven that the EOS of the dark-energy lies in the range $-1 \leq \omega_{DE} < 0$, then the theory is falsified. ('Phantom' DE will also result in the late time measured Hubble parameter being larger than a value based on early universe evolution but a detailed discussion of this issue is outside the scope of this study).

The EOS of the dark-energy is computable precisely if one has a complete ledger of all species that evolve like radiation, combined with their relative abundances at $z = 0$. We have already computed the contribution from electro-magnetic radiation but in the SG theory, neutrinos in a dual-universe system also evolve like radiation (i.e. their density contribution evolves as a^{-4}) in spite of the presence of non-zero mass-squared oscillations [27]. This leads to a further test of the theory since it predicts that the neutrino propagation velocity (for all three species) equals that of light, which can be tested by detection of extra-galactic neutrinos and comparing their arrival time with the arrival time of electro-magnetic radiation from the same source; for sources of similar magnitude and physical nature, the difference in arrival time should be independent of the distance to the source [27]. Finding alternatively that neutrinos have conventional masses would provide strong evidence against the dual-universe model, although would not be sufficient to falsify it. Similarly, finding that the neutrinos (all three species) propagate exactly at the speed of light over astrophysical distances would provide strong support for the dual-universe model but not sufficient to verify it.

Since the theory predicts precisely the value of the dark-energy density at $z = 0$ as $[DE]^4 = (m_{pl}c^2\hbar H_0)^2$ this provides yet a further test of the theory. The transition red-shift from matter dominance to DE dominance is thus precisely computable provided one has a complete ledger of all species that evolve like radiation such that the E.O.S. of the DE is known accurately (there may exist, for example, additional species that evolve like radiation associated with dark-matter which would push the E.O.S. further into the phantom range - these would lower the transition red-shift to DE dominance). Alternatively, accurate measurement of the transition red-shift to DE dominance can be used to compute the DE E.O.S. within the framework of the theory and thus predict whether any such additional 'dark-radiation' exists. Additionally, high precision measurements of the discrepancy of the Hubble parameter between the early and late universe can be used to constrain the phantom component of the DE E.O.S.

A further potential avenue for testing the theory lies in its early-universe inflation predictions. It is outside the scope of this study, primarily devoted to the issue of the Copernican principle in cosmology, to detail the inflation predictions of the theory (which it is hoped will be the subject of another study) but a few comments are warranted. It is believed that early universe inflation occurs within the domain of metric eq.(1) either totally or predominantly. Rather than the four-component dual universe picture (balanced positive mass-energy and negative potential energy in each of two universes within their own frame of reference gives a total of four components) pertaining after early universe inflation has ended, in the inflation epoch there should be two balanced energy components only, one positive and one negative (frame dependent with exact symmetry), corresponding to the two time dimensions in metric eq.(1). Each is the tachyonic transform of the other. In the tachyonic representation, quantum perturbation evolution is governed by eq.(8), which contains a scaling factor $(\sqrt{H_\tau})^{-1}$. This exactly cancels the energy scaling factor $\sqrt{H_\tau}$ in the time-dependent dark-energy density, thus rendering the evolution of perturbations scale-invariant in the circumstance that the tachyonic representation evolves to the corresponding sub-luminal positive energy representation at the end of inflation. The process of re-heating thus corresponds to the splitting of metric eq.(1) from five down to two $\times$ four dimensions and requires contributions from both precursor energy densities to each of the two evolving universes to maintain total energy conservation.

There is a further utility of eq.(8) since it contains an inverted $\leq$ sign with respect to the conventional quantum uncertainty principle and will thus suppress long wavelength perturbations and therefore should suppress r, the tensor to scalar ratio (since gravitational waves have large wavelengths).

Finally the computation of the MOND parameters from the dual-universe model means that continued generic validation of MOND provides yet another test of the theory.

VII. CONCLUSION

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In conclusion, to preserve the Copernican principle with regard to dark energy we postulate that dark energy is evolving in genuine reverse time evolution and with reverse causality with respect to observers. This naturally leads to a dual-universe model where each universe is a tachyonic transformation of the other. The dark energy represents the transformed total potential energy of the second universe, which has a sign change under a sub-luminal-to-tachyon interchange to become *positive* energy in our universe, but retains reverse-time evolution characteristic of the other universe. It appears diffuse and featureless because the negative-energy tachyon fields which generate it have this characteristic. This is a radical solution but the problem is otherwise intractable. We have addressed the issue of the stability of the tachyonic fluid. The past expansion history of the universe then becomes variable and observer dependent. An observer always finds the transition to dark energy dominance at $z \approx 0.6$ in his or her frame. This solves the coincidence (Copernican) problem. The scale problem is solved by the crossing symmetry of mass degenerate universes. There is no 'fine-tuning'. We have further analyzed the issue of gravitational potential energy under a 'crossing symmetry' and find that it leads to a simple method for calculating the MOND parameters and it is proposed that MOND arises as a result of the interaction of the two universes.

Thus, at the price of imposing a single additional symmetry, that of a tachyonic twin universe, the following problems are solved;

1: The nature and origin of dark energy.

2: The coincidence and scale problems of dark energy.

3: The nature of time and the source of the direction of time.

4: The vacuum energy problem is solved.

5: Total parity symmetry of the dual universe is restored.

6: The initial state of the universe is proven not to be a singularity.

7: The total entropy of the universe and the entropy of the initial state of the universe are rendered computable.

8: The total energy of both universes is conserved, restoring energy conservation in cosmology.

9: The MOND parameters are derived directly from cosmology completely independent of galaxy dynamics.

The imposed symmetry is also very simple and arguably natural. The 'tachyverse solution' does not completely solve the dark matter problem and there is still a requirement for a mechanism to drive inflation in the early universe (although there is the suggestion of a mechanism already present in the computation, since in the early universe the scaling volume for the dark-energy content evolves from the Planck volume according to eq.(21) and eq.(23) and a further brief surmise of inflation has been canvassed in Sec.VI). Unlike other dual universe models [73] [74], it does not provide an explanation for CP violation, since an explanation is needed for the relative lack of abundance of anti-matter in our universe, quite independent of what may constitute the tachyverse. Thus an additional mechanism is required to generate the antimatter asymmetry in the early universe in the tachyverse model. Scale invariant CMB fluctuations are expected with this model, due to the nature of the new uncertainty principle eq.(8), but a detailed discussion is postponed for future work.

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