The Vacuum Catastrophe Solved by Taking into Account Hawking-Bekenstein Black Hole Entropy

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Abstract

We will demonstrate that the vacuum catastrophe can be solved by utilizing Bekenstein-Hawking entropy and applying it to black hole type cosmology models, as well as to a large class of $R_h = ct$ models. Additionally, we will examine a recent exact solution to Einstein's field equation and explore how it may potentially resolve the vacuum catastrophe rooted in both steady-state universe and possibly growing black hole universe scenarios.

Keywords: Planck energy, Planck mass, Vacuum energy, Cosmology constant problem.

1 Background on the vacuum catastrophe

The vacuum catastrophe, also known as the cosmological constant problem, is related to the fact that the vacuum energy from observations is estimated to be approximately $\rho_{\text{vac}} \approx 5.96 \times 10^{-27}\ kg/m^3$, as reported by the Planck Collaboration [1]. However, according to quantum field theory, a predicted vacuum energy (on mass equivalent form) is given by

$$\rho_{\text{vac}} = \frac{m_p}{\frac{4}{3}\pi l_p^3} \approx 1.23 \times 10^{96}\ kg/m^3, \tag{1}$$

where m_p is the Planck mass and l_p is the Planck length, initially described by Max Planck [2, 3]. This implies that the vacuum energy is overestimated by an order of 120, as highlighted by, for example, [4, 5]. This substantial disparity between predictions and observations is the reason it is termed the vacuum catastrophe, and not merely the vacuum problem. The difference in predictions and observations is remarkably vast. Although this remains an unsolved problem, the following section will explore potential solutions.

2 The black hole entropy solution

Even though the Λ-CMB model stands as the prevailing cosmological model today, the notion that the Hubble sphere can be perceived as a type of black hole presents an alternative theory. This concept was initially proposed in 1972 by Pathria [6] and later revisited by Stuckey in

1994 [7]. The idea of the universe as a black hole remains a topic of active discussion to this day, as evidenced by discussions in literature such as [8–10].

Various interpretations connect the Hubble sphere to a black hole. One possibility is that it functions as a growing black hole, suggesting that the observable universe originated as a small black hole, evolved into today's universe, and continues to expand. An alternative to the Λ-CDM model discussed actively to this date is so called $R_h = ct$ cosmological models (see [11–16]), where the Hubble radius grows at a rate proportional to cosmic time, denoted by t since the universe's inception. A special underclass of $R_h = ct$ models is growing black hole models, see [17, 18]. Another conception of a black hole universe proposes that the black-hole horizon serves as a form of information horizon everywhere, something we soon will get back to.

If the Hubble sphere indeed represents any type of black hole, whether a growing black hole or a steady-state information horizon black hole then the Bekenstein-Hawking entropy [19] or similar entropies may aid in addressing the vacuum catastrophe. The Bekenstein-Hawking black hole entropy is expressed as

$$S_{BH} = \frac{4\pi r^2}{4l_p^2} = \frac{\pi r^2}{l_p^2}, \tag{2}$$

and when applied to the Hubble sphere, the Bekenstein-Hawking entropy becomes

$$S_{BH,H} = \frac{\pi R_h^2}{l_p^2}, \tag{3}$$

where R_h represents the Hubble radius defined as $R_h = \frac{c}{H_0}$.

Somewhat speculatively Haug [20] (in a brief section 6, November 9), suggested that the predicted Planck energy quantum field vacuum energy likely must be adjusted by the entropy within the black hole Hubble sphere. After all, entropy is inherently connected to how energy disperses over time. Haug proposed the following adjustment:

$$\rho_{vac} = \frac{\frac{m_p}{\frac{4}{3}\pi l_p^3}}{S_{BH,H}} K_b = \frac{\frac{m_p}{\frac{4}{3}\pi l_p^3}}{\frac{k_b \pi R_h^2}{l_p^2}} k_b = \frac{m_p}{\frac{4}{3}\pi^2 l_p R_h^2} \approx 5.31 \times 10^{-27}\ kg/m^3 \tag{4}$$

Where k_b is the Boltzmann constant. This formula provides predictions quite close to the measured vacuum density of $\rho_{\text{vac}} \approx 5.96 \times 10^{-27}$. Equation (4) can be explained from a physical standpoint under different models. The exact value could then change, as the black hole entropy of Hawking used here is rooted in the Schwarzschild metric. The entropy could simply represent how energy spreads out over time in a growing black hole $R_h = ct$ model. The formula that also covers earlier times of the cosmic epoch would then be:

$$\rho_{\text{vac}} = \frac{\frac{m_p}{\frac{4}{3}\pi l_p^3}}{S_{BH,H}} = \frac{\frac{m_p}{\frac{4}{3}\pi l_p^3}}{\frac{\pi (ct)^2}{l_p^2}} = \frac{m_p}{\frac{4}{3}\pi^2 l_p (ct)^2} = \frac{m_p}{\frac{4}{3}\pi^2 l_p (nct_p)^2} \tag{5}$$

where n is the number of Planck times and t_p is the Planck time. Today, $R_h = ct = \frac{c}{H_0}$, but if we look back in time, it will be $R_h = ct$, where t is the time since the beginning of the black hole observable universe, one such model likely fitting this view is the Tatum et al growing black hole model rooted in Schwarzschild type black holes. We will soon also look at another new metric from Einstein's field equation.

3 Steady state black hole universe with center everywhere and information horizon equal to the Hubble radius everywhere used to solve the vacuum catastrophe

Another solution to the vacuum catastrophe can be derived from the steady-state black hole view that we will introduce here. For thousands of years, it was assumed that the universe extended infinitely in both time and space. This perspective was held by Einstein, Lorentz, Poincaré, and other great physicists until around 1930 when the cosmological red-shift observations by Lemaître [21] and Hubble [22] were interpreted as indicating the universe's expansion. However, there could be alternative explanations for the cosmological red-shift, which we will touch upon shortly, even though it is not the focus here.

Let's consider a universe extending infinitely in space and time, assuming there was no Big Bang and no expansion of space. However, just as in the standard model, we assume there is a vacuum energy density. As long as there is a nonzero energy density, every point in such a universe will have an event horizon. The specific nature of the event horizon depends on the metric solution used. Let's begin with the Schwarzschild metric. Here, we can rewrite the Schwarzschild radius as a function simply of the equivalent mass density. We use the term "equivalent" because energy can also be treated as rest mass, given that we have $M = \frac{E}{c^2}$. Thus, we must have:

$$\begin{aligned}
R_s &= \frac{2GM}{c^2} \\
R_s &= \frac{8\pi G \frac{M}{\frac{4}{3}\pi R_s^3} R_s^3}{3c^2} \\
R_s &= \frac{8\pi G\rho R_s^3}{3c^2} \\
R_s &= \frac{8\pi\rho R_s^3}{3c^2} \\
3c^2 &= 8\pi G\rho R_s^2 \\
\frac{3c^2}{8\pi G\rho} &= R_s^2 \\
R_s &= \sqrt{\frac{3c^2}{8\pi G\rho}} \qquad (6)
\end{aligned}$$

For example the density in the critical Friedmann universe is given by

$$\begin{aligned}
\rho_{cr} &= \frac{M_c}{\frac{4}{3}\pi R_s^3} \\
\rho_{cr} &= \frac{\frac{R_H c^2}{2G}}{\frac{4}{3}\pi R_s^3} \\
\rho_{cr} &= \frac{3H_0^2}{8\pi G} \approx 8.38 \times 10^{-27}\ kg/m^3 \qquad (7)
\end{aligned}$$

Inserted in equation 6 we get

$$
\begin{aligned}
R_s &= \sqrt{\frac{3c^2}{8\pi G\rho_c}} \\
R_s &= \sqrt{\frac{3c^2}{8\pi G\frac{3H_0^2}{8\pi G}}} \\
R_s &= R_H \approx 1.38\times 10^{26}\ m
\end{aligned} \tag{8}
$$

In the recent new exact solution to Einstein's [23] field equation given by Haug and Spavieri [24], the energy density of the observable universe is exactly twice that of the critical Friedman universe:

$$\rho_{HS} = \frac{3H_0^2}{4\pi G} \approx 1.68\times 10^{-27}\ kg/m^3 \tag{9}$$

However, the event horizon for twice the mass density is equal to the event horizon of the Schwarzschild metric because the event horizon, as a function of energy density in this model, is given by:

$$R_h = \sqrt{\frac{3c^2}{8\pi G\rho_{HS}}} \approx 1.38\times 10^{26}\ m \tag{10}$$

The Haug-Spavieri metric, when applied to a steady-state universe, predicts that the black hole mass increases exactly by the Planck mass for every Planck length moved from the center (the observer). In a steady-state black hole university, there is a center everywhere with an information horizon equal to R_h everywhere. This is due to a density limitation arising from the metric when one seeks to avoid imaginary event horizons, as discussed in [25]. This corresponds to the black hole increasing by a Planck mass for every Planck time t_p moved with the speed of light away from the central singularity. This leads to a current prediction of vacuum density:

$$\rho_{vac} = \frac{m_p}{\frac{4}{3}\pi R_h^3 - \frac{4}{3}\pi(R_h - l_p)^3} \approx \frac{m_p}{4\pi R_h^2 l_p} \approx 5.57\times 10^{-27} kg/m^3 \tag{11}$$

With a one-standard deviation of $5.34\times 10^{-27}\ kg/m^3$ to $6.32\times 10^{-27}\ kg/m^3$ when using the Hubble parameter value found by the recent study by Kelly et al. [26] of $66.6^{+4.1}_{-3.3}\ (km/s)/Mpc$. Equation 11 can either be seen as the current (now) vacuum density in a growing black hole model or as the vacuum density close to the observer in a steady-state black hole universe. Equation (11) can also be approximated as:

$$\rho_{\text{vac}} = \frac{m_p}{\frac{4}{3}\pi R_h^3 - \frac{4}{3}\pi(R_h - l_p)^3} \approx \frac{m_p}{4\pi R_h^2 l_p} \times 5.56\times 10^{-27}\ kg/m^3 \tag{12}$$

We can then see the only difference between this and our other equation (4) to predict the vacuum energy based on Bekenstein-Hawking entropy is that in the denominator, we have 4π instead of $\frac{4}{3}\pi^2$. When we talk about "close to the observer", we are naturally in this context talking about cosmic distances relative to the Hubble radius. This model can also be extended to observations further away in time as we then must have:

$$\rho = \frac{m_p}{\frac{4}{3}\pi(ct)^3 - \frac{4}{3}\pi(ct - l_p)^3} \tag{13}$$

where ct is the distance the photons used for observations have traveled to reach us, and t is the time from when the photon was sent to reach us. Additional gravitational time dilation effects may occur over long distances. The following equation could also be used

$$\rho = \frac{m_p}{\frac{4}{3}\pi R_h^3 - \frac{4}{3}\pi (R_h - nl_p)^3} \tag{14}$$

where nl_p is the distance the observations are coming from that we use to find the vacuum energy.

Instead of a steady state black hole universe, one can also try to formulate t the Haug-Spavieri metric consistent with a growing black hole universe. The mass would then start with the Planck mass and grow at the Planck mass for every Planck time, similar to the Tatum et al. [17] model. However, the latter model is rooted in the Schwarzschild metric and does not automatically give constraints on the mass density from the metric solution, as all the mass in a Schwarzschild metric can end up in the central singularity. However Tatum et. al has added an extra assumption that the Black-Hole grow with a Half a Planck mass per Planck time. That all the mass can end in the center sigularity seems to be an impossibility in the Haug-Spavieri metric. Actually the Haug-Spavieri metric indicates that at the Planck length distance from the center of the "black-hole" there can only be a Planck mass inside, due to the density constrain given by the metric to get real values of the horizon radius. And the singularity itself should simply be interpreted as no mass can be inside a zero volume, so then there are no gravitational effect as mass causes space-time to curve in general relativity theory, so without mass there is no space-time curvature and no gravity.

A Haug-Spavieri growing black hole would mean today's mass (energy-equivalent mass) is exactly identical to twice the mass in the critical Friedman universe. However, at the current state, I personally lean more towards a steady-state universe, despite the consensus theory still being the Λ-CDM model. Actually it seems possible impossible to distinguish from observations a growing black-hole or steady state black hole when one interpret through the Haug-Spavieri metric.

An important question in a steady state cosmological model is how we then can explain the cosmological red-shift. We notice that:

$$z \approx \frac{dH_0}{c} = \frac{1}{\frac{GM_u}{c^2 d}} \tag{15}$$

so one possible explanation is that cosmological red-shift has nothing to do with expanding space, but possibly is just a special kind of gravitational red-shift due to how close the photons are sent out relative to the information horizon, which is the Hubble radius. This red-shift will over longer distances be:

$$z \approx \frac{dH_t}{c} = \frac{1}{\frac{G(M_u - m_p n)}{c^2 d}} \tag{16}$$

where $n = \frac{d}{l_p}$.

4 Conclusion

We have examined how the utilization of Hawking-Bekenstein black hole entropy can potentially explain the vacuum catastrophe. Entropy describes the dispersion of energy over time and distance. Inside a black hole, it seems that when considering Hawking-Bekenstein black hole entropy, the quantum field-predicted Planck energy leads to an energy level close to the observed vacuum energy. We have also shown that both steady-state and growing black-hole universes, when analyzed through the Haug-Spavieri metric, also seem to solve the vacuum catastrophe.

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