Gravitational Field Equations of the Theory of Self-Variation

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Abstract. In this article we present the gravitational field equations of the Self-Variation Theory. We formulate the differential equation for the gravitational interaction of two bodies and the orbits of the planets. Theory predicts increased stellar velocities on the outskirts of galaxies. It also predicts increased velocities of galaxies on the outskirts of galaxy clusters. A constant of physics appears in the gravitational field Equations. The measurement of this constant can be made from the available observational and experimental data. Knowing the value of the constant we have the exact prediction of Theory for the gravitational field. The first calculations give consistency of the Theory at the distance scales that we have observational data. Further investigation of the Equations of this article will give the complete, accurate prediction of the Theory of Self-Variation for the gravitational interaction.

1. Introduction

The Self-Variation principle postulates that the rest masses and charges of the fundamental particles slowly increase (in absolute value) with time, while simultaneously radiating negative energy in the surrounding spacetime in order to balance the energy of their rest mass / charge increase. Through a series of mathematical calculations described in detail in [5], this axiom necessarily involves a modification of the electromagnetic potential. For comparison the classical electromagnetic Liénard–Wiechert potentials are

$$V_{LW}=\frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}$$

$$\mathbf{A}_{LW}=V\frac{\mathbf{u}}{c^2},$$

whereas the corresponding Self-Variation potentials are

$$V=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}+\frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})q}{4\pi\varepsilon_0 c^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2},$$

$$\mathbf{A}=V\frac{\boldsymbol{\upsilon}}{c^2}.$$

The striking difference is that there is a second term in V, which is independent of the distance r, from the source charge and which has significant theoretical implications. Also the Equations for $\mathbf{A}$, are not identical. We shall examine below the implications in detail for the gravitational field. Notice that the field intensity resulting from either of the potentials (Liénard–Wiechert or Self-Variations) depends on the distance r. It is also crucial to realize that the electromagnetic potential of the Self - Variation Theory gives the correct field intensity, whether we consider the charge q, to be constant or varying in time in accordance with the principle of the Self-Variations. In the case of constant charges the field intensity of the Self-Variation potentials is exactly the same as the one implied by the Liénard–Wiechert potentials. However for charges which vary in time while satisfying the Self-Variation principle, the field intensity implied by the Self-Variation potential differs in an essential way.

The Liénard–Wiechert potentials are compatible with the Lorentz–Einstein transformations for constant charges but not for charges that vary in time. In contrast the Self-Variation potentials are compatible with the Lorentz–Einstein transformations both for constant charges and for charges that vary in time. In this sense the Self-Variation potentials present a much more strict formulation of the field potentials.

Gravitational potential

The Self-Variation potential for the gravitational interaction is derived from the above Equations of the electromagnetic Self-Variation potential by substituting the charge q, with the rest mass M, of the source of the gravitational field hence, $\frac{q}{4\pi\varepsilon_0} \to -GM$, where G, is the constant of gravity and by substituting the acceleration of the particle in the electromagnetic field $\boldsymbol{\alpha}$, with the gravitational intensity of the field $\mathbf{g}$, hence, $\boldsymbol{\alpha} \to \mathbf{g}$. Also notice that now $\boldsymbol{\upsilon}$, represents the propagation velocity of the gravitational field, hence we must substitute the speed of light c, with the propagation speed of the gravitational field υ, hence, $c \to \upsilon$. These substitutions lead to the corresponding gravitational potentials of the Self-Variation,

$$V = -\frac{GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} - \frac{GM}{\upsilon^3}\frac{\boldsymbol{\upsilon}\cdot\mathbf{g}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}, \tag{1}$$

$$\mathbf{A} = V\frac{\boldsymbol{\upsilon}}{\upsilon^2}.$$

where $\mathbf{u}$, is the velocity of the rest mass M, and r, is the distance from the rest mass M. Deriving the gravitational potentials in this way, implies that there is a gravitational analog to the magnetic field $\mathbf{B}$, and has units s^{-1} (see [5], Equations (3) – (6)). Notice that in the limit case where the propagation speed of the gravitational field approaches infinity, $\upsilon \to \infty$, we get the limit potential

$$V = -\frac{GM}{r},$$

which is no other than the one of classical mechanics which assumed instant action of gravity at distance r. Below we consider the most simple case of a stationary source mass M ($\mathbf{u} = \mathbf{0}$).

2. Potential, propagation velocity and intensity of the gravitational field caused by a single mass M

Considering that the vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have opposite directions, $\boldsymbol{\upsilon} = \upsilon \frac{\mathbf{r}}{r}$ and $\mathbf{g} = -g\frac{\mathbf{r}}{r}$, $\boldsymbol{\upsilon}\cdot\mathbf{g} = -\upsilon g$, where $\upsilon = \|\boldsymbol{\upsilon}\|$ and $g = \|\mathbf{g}\|$. Then

$$\frac{d\boldsymbol{\upsilon}}{dt} = \frac{d\upsilon}{dt}\frac{\mathbf{r}}{r} + \upsilon\frac{d}{dt}\left(\frac{\mathbf{r}}{r}\right) = \mathbf{g} = -g\frac{\mathbf{r}}{r}. \tag{2}$$

Then from Equation (1) we have

$$V = -\frac{GM}{r}\frac{1-\dfrac{u^2}{\upsilon^2}}{\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} + \frac{GM}{\upsilon^2}\frac{g}{\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}. \tag{3}$$

Assuming the velocity of the mass M, is zero, $\mathbf{u} = \mathbf{0}$, we get the simplified expression

$$V(r) = -\frac{GM}{r} + \frac{GM}{\upsilon^2} g . \tag{4}$$

Here we clearly see the departure from classical mechanics even for the simple case of a single non–moving source mass.

The gravitational field intensity $\mathbf{g}(r)$ is given by

$$\mathbf{g}(r) = -\nabla V(r) = -\frac{dV}{dr}\frac{\mathbf{r}}{r} \tag{5}$$

hence from (4) we have

$$V = -\frac{GM}{r} + \frac{GM}{\upsilon^2}\frac{dV}{dr}. \tag{6}$$

From Equation (2) in [5], we have that, $dr = -cdw$. However using the symbols of the current article this eq is written as, $dr = -\upsilon dt$. From Equation (2) we get $\frac{d\upsilon}{dt} = -g$. Combining these Equations we get

$$\upsilon\frac{d\upsilon}{dr} = g = \frac{dV}{dr}, \tag{7}$$

where with t, we have denoted the time of the observer. From Equation (7) we have, $\upsilon^2 = 2V + a$ where a is a constant. In the limiting case of zero potential, $V \to 0$ the gravitational field propagation speed tends to the speed of light in vacuum, that is, $\upsilon \to c$, hence we conclude that, $a = c^2$. Then

$$\upsilon^2 = c^2 + 2V . \tag{8}$$

From Equations (6), (7) and (8) we have

$$\upsilon^2 - c^2 = -\frac{2GM}{r} + \frac{GM}{\upsilon^2}\frac{d\upsilon^2}{dr}. \tag{9}$$

Let

$$x=\frac{c^2}{GM}r-C=\alpha r-C \tag{10}$$

where C is a constant and let

$$f(x)=\frac{\upsilon^2(x)}{c^2}. \tag{11}$$

Then from (9) we get a differential equation in the function f,

$$\frac{df}{dx}-f^2+f-\frac{2}{x+C}f=0. \tag{12}$$

Solving (12) for f , we have

$$f(x)=\frac{(x+C)^2}{ke^{x+C}+(x+C)^2+2(x+C)+2}, \tag{13}$$

where k, is the integration constant. Then from (11), (13) we have

$$\upsilon^2(x)=c^2\frac{(x+C)^2}{ke^{x+C}+(x+C)^2+2(x+C)+2}. \tag{14}$$

Finally applying the transformation (10) to (14) we get the speed of propagation of the gravitational field, as derived from the Self-Variation gravitational potential, with respect to r,

$$\upsilon^2(r)=c^2\frac{a^2r^2}{ke^{ar}+a^2r^2+2ar+2}, \tag{15}$$

where $a=\frac{c^2}{GM}$. (16)

Then from Equations (8) and (14) we obtain the gravitational potential with respect to x,

$$V(x)=-c^2\frac{ke^{x+C}+2(x+C)+2}{2\left(ke^{x+C}+(x+C)^2+2(x+C)+2\right)}. \tag{17}$$

The gravitational field intensity $\mathbf{g}(r)$ is calculated as follows. From Equations (5) and (10) we get,

$$\mathbf{g}(x)=-\frac{adV(x)}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (8) we get,

$$\mathbf{g}(x)=-\frac{a}{2}\frac{d\upsilon^2(x)}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (11) we get

$$\mathbf{g}(x) = -\frac{ac^2}{2}\frac{df(x)}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (12) we get,

$$\mathbf{g}(x) = -\frac{ac^2}{2}\left(f^2 - f + \frac{2}{x+C}f\right)\frac{\mathbf{r}}{r}. \tag{18}$$

The function f is given by Equation (13).

3. Gravitational interaction of two bodies

We study the case where a body of rest mass m moves in the gravitational field of a stationary body of rest mass $M >> m$. From equations (17) and (10) we obtain the potential of the gravitational field with respect to r,

$$V(r) = -c^2 \frac{ke^{ar} + 2ar + 2}{2\left(ke^{ar} + a^2r^2 + 2ar + 2\right)}. \tag{19}$$

From Equations (18) and (10) we obtain the gravitational field intensity with respect to r,

$$\mathbf{g}(r) = -\frac{ac^2}{2}\left(f^2(r) - f(r) + \frac{2}{ar}f(r)\right)\frac{\mathbf{r}}{r}. \tag{20}$$

For the application of Equations (19) and (20) the measurement of the constant k is required.

In polar coordinates (r,θ), the orbits $r = r(\theta)$ of the body of rest mass m is given by the solution of the system of equations,

$$L = mr^2\frac{d\theta}{dt} = constant \tag{21}$$

$$\ddot{r} - r(\dot{\theta})^2 = -g(r) \tag{22}$$

and

$$mV(r) + K = E = constant. \tag{23}$$

In these Equations t is the time of the observer, L and K the angular momentum and the kinetic energy of the body of rest mass m, $\ddot{r} = \frac{d^2r}{dt^2}$, $\dot{\theta} = \frac{d\theta}{dt}$ and E the mechanical energy of the system of the two bodies.

The solution of the system of equations (19) - (23) gives the orbit $r = r(\theta)$ of the body of rest mass m. The orbits of the planets have been studied in detail. From the comparison of the theoretical prediction of equations (19) - (23) with the observational data for the orbits of the planets the value of the constant k can be measured.

The Gravitational Equations of the Self-Variation Theory predict increased velocities of stars on the outskirts of galaxies, and of galaxies on the outskirts of galaxy clusters. In the case of a circular orbit ($\dot{r} = 0$) we avoid the complex calculations required for the solution of the system of

Equations (19) - (23). In this case the velocity υ with which the body of mass m moves is given by the equation,

$$\upsilon^2 = gr\,. \tag{24}$$

From Equations (20) and (24) we obtain,

$$\upsilon^2 = \frac{c^2}{2} f\left(r\right)\left(arf\left(r\right) - ar + 2\right). \tag{25}$$

From Equations (25) and (10) we get,

$$\upsilon^2 = \frac{c^2}{2} f\left(x\right)\left(\left(x + C\right) f\left(x\right) - x - C + 2\right). \tag{26}$$

From Equations (13) and (10) we get,

$$f\left(r\right) = \frac{a^2 r^2}{ke^{ar} + a^2 r^2 + 2ar + 2}\,. \tag{27}$$

Equation (25) gives the increased velocities of stars at the outskirts of galaxies and the increased velocities of galaxies at the outskirts of galaxy clusters. A first estimate for the constant k showed that it has a small value close to zero, $k \to 0$. A more accurate value of the constant can be measured from the already known observational data (see, [1] – [3] and [6] – [15]).

The graphs of Equations (14), (17) and (26) are displaced in the frame of reference of an observer, which is not in the same point as the rest mass M. Indicatively, the graph of the velocity υ, as given by Equation (26) is shifted up and to the right. The gravitational field distorts the observer's frame of reference.

The equations for the point mass give information about the distance over which the gravitational interaction acts (see, [4]). Indicatively, from equation (19) we get,

$$V\left(0\right) = -\frac{c^2}{2} = \lim_{r\to\infty} V\left(r\right).$$

We have formulated the Gravitational Field Equations for a point rest mass. By inserting into the Equations we have presented, the mass density or particle density and the current density we obtain the corresponding equations for any distribution of rest mass in space.

If the mass M moves, whit velocity $\mathbf{u} \neq \mathbf{0}$, Equation (3) applies. From Equations (3) and (5) we have

$$V = -\frac{GM}{r} \frac{1 - \dfrac{u^2}{\upsilon^2}}{\left(1 - \dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} + \frac{1}{\upsilon^2} \frac{GM}{\left(1 - \dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} \frac{dV}{dr}\,.$$

Then, from this Equations and (8) we obtain,

$$\upsilon^2 - c^2 = -\frac{2GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} + \frac{1}{\upsilon^2}\frac{GM}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}\frac{d\upsilon^2}{dr}. \tag{28}$$

If $\mathbf{u} = \mathbf{0}$, Equation (28) is equivalent to (9).

Compared to Equation (9), in equation (28), the velocity $\mathbf{u}$ has implications for the Newtonian term

$$-\frac{GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}$$

of the potential and the Self-variation term

$$\frac{GM}{\upsilon^2}\frac{g}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}.$$

The Newtonian term can be either greater than the potential

$$-\frac{GM}{r}$$

or less, depending on whether it is

$$1-\frac{u^2}{\upsilon^2} > \left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2$$

or

$$1-\frac{u^2}{\upsilon^2} < \left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2.$$

For the Self-Variation term we have

$$\frac{GM}{\upsilon^2}\frac{g}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} > \frac{GMg}{\upsilon^2}.$$

From the solution of equation (28) we get the velocity of propagation $\boldsymbol{\upsilon} = \boldsymbol{\upsilon}(r)$ of the gravitational field, the potential $V = V(r)$ and the intensity $\mathbf{g} = \mathbf{g}(r)$. Then for the gravitational interaction of the masses M and m, we solve the system of Equations,

$$L = mr^2\frac{d\theta}{dt} = constant,$$

$$\ddot{r} - r\left(\dot{\theta}\right)^2 = -g(r),$$

and

$mV\left(r\right)+K=E=constant$.

The orbits of the planets have been studied in detail. The sun moves in relation to the earth. Hence, for the exact value of the constant k , the application of Equation (28) to the solar system is required.

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