Gravitational Field Equations of the Theory of Self-Variation

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Abstract. In this article we present the gravitational field equations of the Self-Variation Theory. We formulate the differential equation for the gravitational interaction of two bodies and the orbits of the planets. Theory predicts increased stellar velocities on the outskirts of galaxies. It also predicts increased velocities of galaxies on the outskirts of galaxy clusters. A constant of physics appears in the gravitational field Equations. The measurement of this constant can be made from the available observational and experimental data. Knowing the value of the constant we have the exact prediction of Theory for the gravitational field. The first calculations give consistency of the Theory at the distance scales that we have observational data. Further investigation of the Equations of this article will give the complete, accurate prediction of the Theory of Self-Variation for the gravitational interaction.

1. Introduction

The electromagnetic potential of the Self-Variation Theory has two characteristics that other known potentials, such as the Liénard–Wiechert potential, do not. It is the sum of two individual potentials where one does not depend on the distance from the charge-source of the electromagnetic field. However, the field intensity resulting from both potentials depends on the distance. Furthermore, the electromagnetic potential of the Self-Variation Theory gives the correct field intensity whether we consider the charge to be constant or to vary (strictly) according to the principle of self-variation. The Liénard–Wiechert potential gives the same, correct electromagnetic field but considering the electric charge to be constant. If we consider that the electric charge changes, the potential Liénard–Wiechert does not give a correct electromagnetic field (see, [5]).

With the substitutions $-GM \to \dfrac{q}{4\pi\varepsilon_0}$, $-\mathbf{g} \to \boldsymbol{\alpha}$ and $\upsilon \to c$ in the electromagnetic potential of the Theory of Self-Variation we get the corresponding potential V of the gravitational interaction,

$$V = -\frac{GM}{r}\frac{1-\dfrac{u^2}{c^2}}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} + \frac{GM}{\upsilon^3}\frac{\boldsymbol{\upsilon}\cdot\mathbf{g}}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}. \tag{1}$$

In this Equation G is the constant of gravity, M the rest mass-source of the gravitational field, r the distance from the rest mass M, $\mathbf{u}$ the velocity with which M moves, $\boldsymbol{\upsilon}$ the velocity with which the cause of the field moves and $\mathbf{g}$ the intensity of the field. The vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have opposite directions.

2. Potential and intensity of the gravitational field

The vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have opposite directions, $\boldsymbol{\upsilon} = \upsilon \frac{\mathbf{r}}{r}$ and $\mathbf{g} = -g \frac{\mathbf{r}}{r}$, where $\upsilon = \|\boldsymbol{\upsilon}\|$ and $g = \|\mathbf{g}\|$. So we have,

$$\frac{d\boldsymbol{\upsilon}}{dt} = \frac{d\upsilon}{dt}\frac{\mathbf{r}}{r} + \upsilon \frac{d}{dt}\left(\frac{\mathbf{r}}{r}\right) = \mathbf{g} = -g\frac{\mathbf{r}}{r}. \tag{2}$$

From Equation (1) we get,

$$V = -\frac{GM}{r}\frac{1-\frac{u^2}{c^2}}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} + \frac{GM}{\upsilon^2}\frac{g}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}. \tag{3}$$

If $\mathbf{u} = \mathbf{0}$, from Equation (3) we get,

$$V(r) = -\frac{GM}{r} + \frac{GM}{\upsilon^2} g. \tag{4}$$

The gravitational field intensity $\mathbf{g}(r)$ is given by the equation,

$$\mathbf{g}(r) = -\nabla V(r) = -\frac{dV}{dr}\frac{\mathbf{r}}{r}. \tag{5}$$

From Equations (4) and (5) we get,

$$V = -\frac{GM}{r} + \frac{GM}{\upsilon^2}\frac{dV}{dr}. \tag{6}$$

From Equation (2) we get $\frac{d\upsilon}{dt} = -g$ and with Equation (5) and $dr = -\upsilon dt$ (see [5] Equation 2, with the symbolism of this article, the Equation $dr = -cdw$ is written in the form $dr = -\upsilon dt$) we get

$$\upsilon\frac{d\upsilon}{dr} = g = \frac{dV}{dr}, \tag{7}$$

where t we denote the time of the observer. From Equation (7) we get, $\upsilon^2 = 2V + a$ where a is a constant. Taking into consideration that if $V = 0$ then $\upsilon = c$ the speed of light in vacuum we get the following equation,

$$\upsilon^2 = c^2 + 2V. \tag{8}$$

From Equations (6), (7) and (8) we get,

$$\upsilon^2 - c^2 = -\frac{2GM}{r} + \frac{GM}{\upsilon^2}\frac{d\upsilon^2}{dr}. \tag{9}$$

Symbolizing

$$x = \frac{c^2}{GM} r = \alpha r \tag{10}$$

and

$$f\left(x\right)=\frac{\upsilon^{2}\left(x\right)}{c^{2}}, \tag{11}$$

from Equation (9) we get,

$$\frac{df}{dx}-f^{2}+f-\frac{2}{x}f=0. \tag{12}$$

From Equation (12) we get,

$$f\left(x\right)=\frac{x^{2}}{ke^{x}+x^{2}+2x+2}, \tag{13}$$

where k is a constant. From Equations (11) and (13) we obtain the following equation,

$$\upsilon^{2}\left(x\right)=c^{2}\frac{x^{2}}{ke^{x}+x^{2}+2x+2}. \tag{14}$$

From Equation (14) and the Transformation (10) we obtain,

$$\upsilon^{2}\left(x\right)=c^{2}\frac{a^{2}r^{2}}{ke^{ar}+a^{2}r^{2}+2ar+2}, \tag{15}$$

$$a=\frac{c^{2}}{GM}. \tag{16}$$

From Equations (8) and (15) we obtain,

$$V\left(r\right)=-c^{2}\frac{ke^{ar}+2ar+2}{2\left(ke^{ar}+a^{2}r^{2}+2ar+2\right)}. \tag{17}$$

Then, the intensity $\mathbf{g}\left(r\right)$ of the gravitational field is given by Equations (5) and (17).

Alternatively, the gravitational field intensity is calculated as follows. From Equations (5) and (10) we get,

$$\mathbf{g}\left(x\right)=-\frac{adV\left(x\right)}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (8) we get,

$$\mathbf{g}\left(x\right)=-\frac{a}{2}\frac{d\upsilon^{2}\left(x\right)}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (11) we get

$$\mathbf{g}\left(x\right)=-\frac{ac^{2}}{2}\frac{df\left(x\right)}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (12) we get,

$$\mathbf{g}(x) = -\frac{ac^2}{2}\left(f^2 - f + \frac{2}{x} f \right)\frac{\mathbf{r}}{r}. \tag{18}$$

The function f is given by Equation (13).

3. Gravitational interaction of two bodies

We study the case where a body of rest mass m moves in the gravitational field of a stationary body of rest mass $M >> m$. From Equation (17) we get,

$$V(r) = -c^2 \frac{ke^{ar} + 2ar + 2}{2\left(ke^{ar} + a^2r^2 + 2ar + 2\right)} \tag{19}$$

and from Equations (18), (10) we obtain,

$$\mathbf{g}(r) = -\frac{ac^2}{2}\left(f^2(r) - f(r) + \frac{2}{ar} f(r) \right)\frac{\mathbf{r}}{r}. \tag{20}$$

For the application of Equations (19) and (20) the measurement of the constant k is required.

In polar coordinates (r,θ), the orbits $r = r(\theta)$ of the body of rest mass m is given by the solution of the system of equations,

$$L = mr^2 \frac{d\theta}{dt} = constant \tag{21}$$

$$\ddot{r} - r\left(\dot{\theta}\right)^2 = -g(r) \tag{22}$$

and

$$mV(r) + K = E = constant\ . \tag{23}$$

In these Equations t is the time of the observer, L and K the angular momentum and the kinetic energy of the body of rest mass m, $\ddot{r} = \frac{d^2r}{dt^2}$, $\dot{\theta} = \frac{d\theta}{dt}$ and E the mechanical energy of the system of the two bodies.

The solution of the system of equations (19) - (23) gives the orbit $r = r(\theta)$ of the body of rest mass m. The orbits of the planets have been studied in detail. From the comparison of the theoretical prediction of equations (19) - (23) with the observational data for the orbits of the planets the value of the constant k can be measured.

The Gravitational Equations of the Self-Variation Theory predict increased velocities of stars on the outskirts of galaxies, and of galaxies on the outskirts of galaxy clusters. In the case of a circular orbit we avoid the complex calculations required for the solution of the system of Equations (19) - (23). In this case the velocity υ with which the body of mass m moves is given by the equation,

$$\upsilon^2 = gr\ . \tag{24}$$

From Equations (20) and (24) we obtain,

$$\upsilon^2 = \frac{c^2}{2} f(r)\left(arf(r) - ar + 2\right). \tag{25}$$

From Equations (13) and (10) we get,

$$f(r) = \frac{a^2 r^2}{ke^{ar} + a^2 r^2 + 2ar + 2}. \tag{26}$$

Equation (25) gives the increased velocities of stars at the outskirts of galaxies and the increased velocities of galaxies at the outskirts of galaxy clusters. A first estimate for the constant k showed that it has a positive value close to zero, $k \to 0^+$. A more accurate value of the constant can be measured from the already known observational data (see, [1] – [3] and [6] – [13]). For the exact value of the constant $k = 0$, the application of Equations (10) - (23) to the solar system is required.

The equations for the point mass give information about the distance over which the gravitational interaction acts (see, [4]). Indicatively, from equation (19) we get,

$$V(0) = -\frac{c^2}{2} = \lim_{r \to \infty} V(r).$$

We have formulated the Gravitational Field Equations for a point rest mass. By inserting into the Equations we have presented, the mass density or particle density and the current density we obtain the corresponding equations for any distribution of rest mass in space.

With the substitutions $-GM \to \frac{q}{4\pi\varepsilon_0}$, $-\mathbf{g} \to \boldsymbol{\alpha}$ and $\upsilon \to c$ we can define in the gravitational field a pair $(V, \mathbf{A})$ of numerical - vector potential (see [5], Equations (3) and (4)). The potential $(V, \mathbf{A})$ gives a field B with unit 1/second (s^{-1}) corresponding to the magnetic field. It also, predicts gravitational waves, analogous to electromagnetic.

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