Gravitational Field Equations of the Theory of Self-Variation

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Abstract. In this article we present the gravitational field equations of the Self-Variation Theory. There is a characteristic value in the field action distance. Before and after this distance the field equations are completely different. We formulate the differential equation for the orbits of the planets. Self-Variation Theory predicts increased stellar velocities on the outskirts of galaxies. It also predicts increased velocities of galaxies on the outskirts of galaxy clusters. A constant of physics appears in the gravitational field Equations. The measurement of this constant from the available observational data will give the exact prediction of the Self-Variation Theory for gravitational field of the large structures of matter. Further investigation of the Equations of this article will give the complete, accurate prediction of the Theory for the gravitational interaction.

1. Introduction

With the substitutions $-GM \to \dfrac{q}{4\pi\varepsilon_0}$ and $\mathbf{g} \to \boldsymbol{\alpha}$ in the macroscopic electromagnetic potential of the Theory of Self-Variation (see, [4]) we get the corresponding potential V of the gravitational interaction,

$$V = -\frac{GM}{r}\frac{1-\dfrac{u^2}{c^2}}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} + \frac{GM}{c^3}\frac{\boldsymbol{\upsilon}\cdot\mathbf{g}}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}. \tag{1}$$

In this Equation G is the constant of gravity, M the mass-source of the gravitational field, r the distance from the mass M, $\mathbf{u}$ the velocity with which M moves, $\boldsymbol{\upsilon}$ the velocity with which the cause of the field moves and $\mathbf{g}$ the intensity of the field.

The vector $\boldsymbol{\upsilon}$ is given by the equations $\boldsymbol{\upsilon} = c\dfrac{\mathbf{r}}{r}$, where c is the speed of light in vacuum (see [4], Fig. 1). The vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ may have either the same direction or opposite directions.

2. The vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have opposite directions

In the case that vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have same opposite directions, $\dfrac{\boldsymbol{\upsilon}\cdot\mathbf{g}}{c} = -g = -\|\mathbf{g}\|$ from Equation (1) we get,

$$V = -\frac{GM}{r}\frac{1-\dfrac{u^2}{c^2}}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} - \frac{GM}{c^2}\frac{g}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} \tag{2}$$

and by symbolizing

$$\alpha = \frac{c^2}{GM} \tag{3}$$

we get the following equation,

$$V = -\frac{GM}{r} - \frac{g}{\alpha} . \tag{4}$$

From equations (4) and

$$\mathbf{g}(r) = -\nabla V(r) = -\frac{dV}{dr}\frac{\mathbf{r}}{r} \tag{5}$$

we get

$$V = -\frac{GM}{r} - \frac{dV}{\alpha dr}$$

and equivalently we obtain

$$V + \frac{dV}{\alpha dr} = -\frac{GM}{r} . \tag{6}$$

The potential given by Equation (6) is the same either we consider the mass M to be constant or to vary strictly as predicted by the principle of self-variation. Thus we solve the differential Equation (6) for constant mass M (see, [4]).

By symbolizing

$$x = \alpha r, r = \frac{x}{\alpha}, \tag{7}$$

from Equation (6) we get

$$V + \frac{dV}{dx} = -\frac{GM\alpha}{x}$$

and with Equation (3) we get

$$V + \frac{dV}{dx} = -\frac{c^2}{x}$$

and by symbolizing

$$\frac{V(x)}{c^2} = f(x) \tag{8}$$

we obtain

$$f'(x) + f(x) = -\frac{1}{x} . \tag{9}$$

From Equation (9) we get

$$e^x f'(x) + e^x f(x) = -\frac{e^x}{x}$$

and equivalently we get

$$\left(e^x f(x)\right)' = -\frac{e^x}{x}$$

and equivalently we get

$$e^x f(x) = k - \int \frac{e^x}{x} dx$$

and finally we obtain

$$f(x) = k \cdot e^{-x} - \sum_{n=1}^{\infty} \frac{(n-1)!}{x^n}, \tag{10}$$

where k is a constant.

From Equations (8) and (10) we get

$$V(x) = kc^2 \cdot e^{-x} - c^2 \sum_{n=1}^{\infty} \frac{(n-1)!}{x^n}. \tag{11}$$

From Equation (11) and transformation (7) we obtain the function $V = V(r)$ as given by the following equation,

$$V(r) = kc^2 \cdot e^{-\alpha r} - \sum_{n=1}^{\infty} \frac{c^2 \cdot (n-1)!}{\alpha^n r^n}. \tag{12}$$

From Equations (12) and (5) we obtain the function $\mathbf{g} = \mathbf{g}(r)$ as given by the following equation,

$$\mathbf{g} = \mathbf{g}(r) = -\left(-k\alpha c^2 \cdot e^{-\alpha r} + \sum_{n=1}^{\infty} \frac{c^2 \cdot n!}{\alpha^n r^{n+1}}\right) \frac{\mathbf{r}}{r}. \tag{13}$$

From Equations (13) and (7) we obtain the function $\mathbf{g} = \mathbf{g}(x)$ as given by the following equation,

$$\mathbf{g} = \mathbf{g}(x) = -\left(-k\alpha c^2 \cdot e^{-x} + \sum_{n=1}^{\infty} \frac{\alpha c^2 \cdot n!}{x^{n+1}}\right) \frac{\mathbf{r}}{r}. \tag{14}$$

From Equations (14) and (13) we obtain,

$$g = -k\alpha c^2 \cdot e^{-x} + \sum_{n=1}^{\infty} \frac{\alpha c^2 \cdot n!}{x^{n+1}} = -k\alpha c^2 \cdot e^{-\alpha r} + \sum_{n=1}^{\infty} \frac{c^2 \cdot n!}{\alpha^n r^{n+1}}. \tag{15}$$

From the inequality $g(r) = \|\mathbf{g}(r)\| \geq 0$ and equation (15) we get,

$$k \cdot e^{-\alpha r} \leq \sum_{n=1}^{\infty} \frac{n!}{(\alpha r)^{n+1}} . \tag{16}$$

From Equations (3) and (4) we get,

$$V(r) + \frac{GM}{c^2} g(r) = -\frac{GM}{r} . \tag{17}$$

Equation (17) relates the potential $V(r)$ and the intensity $g(r) = \|\mathbf{g}(r)\|$ of the gravitational field.

In equation (15) the Newtonian term $\frac{c^2}{\alpha r^2} = \frac{GM}{r^2}$ appears in the sum $\sum_{n=1}^{\infty} \frac{c^2 \cdot n!}{\alpha^n r^{n+1}}$ for $n = 1$. This term is the greater of the terms of the sum $\sum_{n=1}^{\infty} \frac{c^2 \cdot n!}{\alpha^n r^{n+1}}$ when $\frac{c^2 1!}{\alpha r^2} > \frac{c^2 2!}{\alpha^2 r^3}$ and equivalently $r > \frac{2GM}{c^2}$. If $r < \frac{2GM}{c^2}$ the Newtonian term is the smallest of the terms of the sum $\sum_{n=1}^{\infty} \frac{c^2 \cdot n!}{\alpha^n r^{n+1}}$. In this case the gravitational field is completely different from the Newtonian. The physics quantity

$$r_0 = \frac{2GM}{c^2} \tag{18}$$

is a fundamental limit for the gravitational interaction.

3. Spherical symmetry

We apply the Gravitational Field Equations of section 2. In order to avoid complex calculations (which can be done in cases where it is essential) we present their simplest application. We present the case where a star moves in a circular orbit on the outskirts of a galaxy or a galaxy moves in a circular orbit on the outskirts of a galaxy cluster with velocity υ. In this case the following equation applies,

$$\upsilon^2 = g \cdot r = \frac{g \cdot x}{\alpha} . \tag{19}$$

We do the application for $r > r_0 = \frac{2GM}{c^2}$.

The mass M of a sphere of radius r and constant density ρ is $M = \frac{4\pi\rho}{3} r^3$. Therefore, from Equations (3) and (7) we get $x = \frac{3c^2}{4G\pi\rho r^2}$. Symbolizing

$$\sigma = \frac{3c^2}{4G\pi\rho} \tag{20}$$

we get

$$\alpha = \frac{\sigma}{r^3} \tag{21}$$

and

$$x = \frac{\sigma}{r^2}. \tag{22}$$

From Equations (15) and (21), (22) we get

$$g(r) = \frac{\sigma c^2}{r^3}\left(-k \cdot e^{-\frac{\sigma}{r^2}} + \sum_{n=1}^{\infty} n!\left(\frac{r^2}{\sigma}\right)^{n+1}\right) \tag{23}$$

and with Equation (19) we obtain,

$$\mathrm{U}^2 = \frac{\sigma c^2}{r^2}\left(-k \cdot e^{-\frac{\sigma}{r^2}} + \sum_{n=1}^{\infty} n!\left(\frac{r^2}{\sigma}\right)^{n+1}\right). \tag{24}$$

For the Newtonian velocity v is,

$$v^2 = \frac{G\pi b\rho}{3} r^2. \tag{25}$$

In Equation (24) is,

$$\frac{r^2}{\sigma} = \frac{4G\pi b\rho r^2}{3c^2} = \frac{v^2}{c^2} << 1.$$

Thus, for $n = 1$ we take the approach,

$$\mathrm{U}^2 = \frac{\sigma c^2}{r^2}\left(-k \cdot e^{-\frac{\sigma}{r^2}} + \left(\frac{r^2}{\sigma}\right)^2\right).$$

In this Equation it is $\mathrm{U} > v$ if $k = -\mu^2$,

$$\mathrm{U}^2 = \frac{\sigma c^2}{r^2}\left(\mu^2 \cdot e^{-\frac{\sigma}{r^2}} + \left(\frac{r^2}{\sigma}\right)^2\right). \tag{26}$$

From Equations (26), (19) and (25) we obtain,

$$\mathrm{U}^2 = \frac{3\mu^2 c^4}{4G\pi b\rho r^2} e^{-\frac{\sigma}{r^2}} + \frac{4G\pi b\rho r^2}{3} = \frac{\mu^2 c^4}{v^2} e^{-\frac{c^2}{v^2}} + v^2. \tag{27}$$

The constant μ can be measured from the already known observational data (see, [1] - [3] and [5] – [12]).

4. Star velocities at the outskirts of galaxies

The gravitational field on the outskirts of a galaxy depends on the distribution of its mass in space. We consider the case where the largest amount of the galaxy's mass M is concentrated near its core. For a star located on the periphery of the galaxy, at a distance r from its center, equation (15) applies,

$$g\left(r\right)=-k\alpha c^2\cdot e^{-\alpha r}+\sum_{n=1}^{\infty}\frac{c^2\cdot n!}{\alpha^n r^{n+1}}\,. \tag{28}$$

In this Equation, the approach we made in section 3 applies. Thus we get

$$g\left(r\right)=-k\alpha c^2\cdot e^{-\alpha r}+\frac{c^2}{\alpha r^2}$$

and finally we get

$$g\left(r\right)=\mu^2 c^2\cdot e^{-\alpha r}+\frac{GM}{r^2}\,. \tag{29}$$

From Equations (19) and (29) we obtain,

$$\mathrm{U}^2=\mu^2 c^2\cdot r\cdot e^{-\alpha r}+\frac{GM}{r}\,. \tag{30}$$

This Equation is valid as an approximation. For the exact theoretical calculation of velocity U in a specific galaxy its mass distribution in space must be taken into account.

5. The differential equation for the orbits of the planets

Applying equation (29) for a planet of mass m we obtain in polar coordinates $\left(r,\theta\right)$ the differential equation,

$$\frac{d^2Y}{d\theta^2}+Y+\frac{\mu^2c^2\alpha m^2}{L^2}\frac{e^{-\frac{\alpha}{Y}}}{Y^2}+\frac{GMm^2}{L^2}=0\,. \tag{31}$$

In this equation we denote M the mass of the sun, $L=mr^2\dfrac{d\theta}{dt}=constant$ the angular momentum of the planet and $Y\left(\theta\right)=\dfrac{1}{r\left(\theta\right)}$. The solution of the differential Equation (31) gives the orbits $r=r\left(\theta\right)$ of the planets. This Equation has an additional term,

$$\frac{\mu^2c^2\alpha m^2}{L^2}\frac{e^{-\frac{\alpha}{Y}}}{Y^2}\,,$$

from the corresponding Newtonian Equation. It is very possible that the solution of the differential equation (31) will give the value of the constant $k=-\mu^2$.

6. The vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have same direction

In the case that vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have same direction, $\dfrac{\boldsymbol{\upsilon}\cdot\mathbf{g}}{c}=g$ from Equation (1) we get,

$$V=-\frac{GM}{r}\frac{1-\frac{u^2}{c^2}}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}+\frac{GM}{c^2}\frac{g}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\,. \tag{32}$$

If $\mathbf{u}=\mathbf{0}$, from Equation (32) we get

$$V(r)=-\frac{GM}{r}+\frac{GM}{c^2}g(r). \tag{33}$$

This equation is the corresponding of (17).

From Equation (32), repeating the proof procedure of section 2 we get the following equations.

$$V(x)=Kc^2\cdot e^x-c^2\sum_{n=1}^{\infty}(-1)^n\frac{(n-1)!}{x^n},$$

$$V(r)=Kc^2\cdot e^{\alpha r}-\sum_{n=1}^{\infty}(-1)^n\frac{c^2\cdot(n-1)!}{\alpha^n r^n},$$

$$\mathbf{g}=\mathbf{g}(r)=-\left(K\alpha c^2\cdot e^{\alpha r}+\sum_{n=1}^{\infty}(-1)^n\frac{c^2\cdot n!}{\alpha^n r^{n+1}}\right)\frac{\mathbf{r}}{r},$$

$$\mathbf{g}=\mathbf{g}(x)=-\left(K\alpha c^2\cdot e^x+\sum_{n=1}^{\infty}(-1)^n\frac{\alpha c^2\cdot n!}{x^{n+1}}\right)\frac{\mathbf{r}}{r},$$

$$g=-K\alpha c^2\cdot e^x-\sum_{n=1}^{\infty}(-1)^n\frac{\alpha c^2\cdot n!}{x^{n+1}}=-K\alpha c^2\cdot e^{\alpha r}-\sum_{n=1}^{\infty}(-1)^n\frac{c^2\cdot n!}{\alpha^n r^{n+1}},$$

$$K\cdot e^{\alpha r}\le\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{(\alpha r)^{n+1}}.$$

where K is a constant. The equations for velocity U are analogous to those in sections 3 and 4.

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